1 Ocean Waves

This document presents a summary of the theory of ocean waves and is mainly based on [1].

The outstanding visible characteristic of an open sea surface is its irregularity. The waves on the surface do not repeat periodically in time or space. Yet, over a wide area and for a period of time, the sea surface maintains a characteristic appearance. Study on wave data have shown that even though the sea surface is irregular, the wave elevation readings is Gaussian in nature and is statistically a constant for a given area for a certain period of time. It is therefore possible to define a sea condition, for a region for a (short) period of time, using statistical parameters such as mean elevation and variance. The mean elevation will however be zero, since wave does not change the water level in the sea. Which means, considering Gaussian distribution for wave elevations, the sea condition can be defined using variance alone.

For the purpose of this research we consider waves generated due to the interaction of wind and water surface. The two main physical process involved in the generation of storm waves are the friction between air and water and the local variation of pressure field due to wind. Even though there are many processes that will affect the growth and propagation of waves, for waves of small amplitude, it is primarily governed by the principle of superposition. So if $\zeta_1(x, y, t)$ and $\zeta_2(x, y, t)$ are functions that represent two wave systems then $\zeta_1(x, y, t) + \zeta_2(x, y, t)$ is also a wave system. However, it should be noted that the assumption regarding the linear superimposability of waves fails when the wave system is too steep and wave breaking occurs.

1.1 Regular sea waves

A regular sea wave is a harmonic wave with crests that are infinitely long, parallel and equally spaced and having constant wave heights. The general equation of a regular long crested wave travelling at a angle μ to the x-axis is:

$$\zeta(x, y, t) = \zeta_a \cos[k(x \cos \mu + y \sin \mu) - \omega t + \epsilon] \tag{1}$$

where:
ζ_a is the wave amplitude at the water surface
$k = \frac{2\pi}{L_w}$ is called the wave number.
L_w is the wave length
x is the distance along the x-coordinate
y is the distance along the y-coordinate
μ is the direction of propagation of the wave as an angle with respect to positive direction of x-axis.
ω is the circular frequency of the simple harmonic wave
t is time
ϵ is a phase angle.

The wave system travels perpendicular to the line of crests with a velocity V_c. It is assumed that water is incompressible and has zero viscosity. Based on these assumptions the motion of water particle in the wave can be described using a quantity called *velocity potential* which is defined as a function whose negative derivative in any direction yields the velocity component of the fluid in that direction. Given below is a simplified equation for velocity potential:

$$\phi = -\zeta_a V_c \frac{\cosh k(z + h)}{\sinh kh} \sin k(x - V_c t) \tag{2}$$

where:
h is the water depth (distance from sea surface to seabed)
V_c is the wave velocity (or celerity)
z is the vertical distance of the water particle from the surface and is measured positive in the upward direction
For deep water, ie. where $h \gg \frac{L_w}{2}$,

$$\frac{\cosh k(z + h)}{\sinh kh} \approx e^{kz} \tag{3}$$

Substituting equation 3 in equation 2 we get:

$$\phi = -\zeta_a V_c e^{kz} \sin k(x - V_c t) \tag{4}$$

The wave causes variation in the distribution of pressure below the water surface and the equation for variation of pressure head due to waves is:

$$\zeta = \frac{k\zeta_a {V_c}^2}{g} \frac{\cosh k(z+h)}{\sinh kh} \cos k(x - V_c t) \tag{5}$$

For deep water the above equation can be approximated as:

$$\zeta = \zeta_a e^{kz} \cos k(x - V_c t) \tag{6}$$

For simple harmonic motion:

$$\omega = \frac{2\pi}{T_w} = kV_c \tag{7}$$

where T_w is the time period of the simple harmonic wave.
Substituting equation 7 in equations 5 and 6:
Pressure head variation due to wave for any water depth:

$$\zeta = \frac{k\zeta_a {V_c}^2}{g} \frac{\cosh k(z+h)}{\sinh kh} \cos(kx - \omega t) \tag{8}$$

Pressure head variation due to wave for deep water:

$$\zeta = \zeta_a e^{kz} \cos(kx - \omega t) \tag{9}$$

The pressure at any point is given by:

$$p = \rho g(-z + \zeta) \tag{10}$$

$$p = -\rho g z + \zeta_a \rho g e^{kz} \cos(kx - \omega t) \tag{11}$$

The total wave energy per unit area is:

$$E = \frac{1}{2} \rho g {\zeta_a}^2 \tag{12}$$

The variance, or mean-square value of surface elevation as a function of time is:

$$S = \langle \zeta(t)^2 \rangle = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \zeta^2(t) dt \tag{13}$$

For simple harmonic motion of frequency ω, the above equation for variance can be written as:

$$S(\omega) = \langle \zeta(t)^2 \rangle = \frac{1}{2} {\zeta_a}^2 \tag{14}$$

Based on equation 14, equation 12 can be written as:

$$E = \rho g S(\omega) \tag{15}$$

1.2 Irregular sea and wave spectrum

Oceanographers have found that an irregular sea wave can be resolved into a sum of regular waves of various length and direction using Fourier Integral techniques. Or alternatively it can be said that the visible system of waves on the sea surface is a result of super positioning of many regular component waves with each travelling is different direction with a different amplitude, frequency, wave length and phase. Since the component waves have different direction and celerities, the wave patterns keeps changing with time. It is convenient to begin with a simple case of wave pattern observed at a single point (ie. $x = y = 0$) and assuming that all component waves are travelling in the same direction ($\mu = 0$). Based on equation 1, the simplified equation of a compound wave (ie. a wave that consist of many component waves) is:

$$\zeta(t) = \sum_i (\zeta_a)_i \cos(-\omega_i t + \epsilon_i) \tag{16}$$

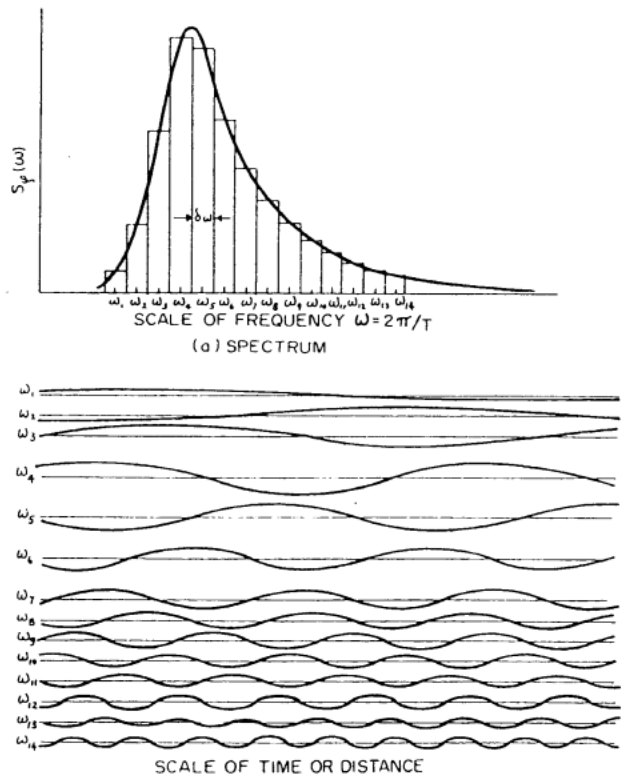

Figure 1: Example of variance spectrum from [1]

It is convenient to define wave components in terms of a function called *variance spectrum,* $S(\omega)$. A typical example of a plot of variance spectrum is shown in figure 1. For any particular wave frequency, ω_i, the variance of the wave components for a narrow band of frequency, $\delta\omega$, centred on ω_i is given by:

$$\langle \zeta_i(t)^2 \rangle = S(\omega_i)\delta\omega \tag{17}$$

The total variance of the system is given by:

$$\langle \zeta(t)^2 \rangle = \sum_i \langle \zeta_i(t)^2 \rangle = \int_0^\infty S(\omega)d\omega \tag{18}$$

That is, the total variance of the system is obtained by finding the area under the variance spectrum. Also, as $\delta\omega \to 0$, it means the wave system is composed of only one frequency which means it becomes a simple harmonic wave. The mean value for a wave elevations for a simple harmonic wave is 0 (wave does not increase the mean water level) and the variance is given by:

$$\langle \zeta_i(t)^2 \rangle = \frac{1}{2}{(\zeta_a)_i}^2 \tag{19}$$

Applying this is equation 17, we get:

$$\frac{1}{2}{(\zeta_a)_i}^2 = S(\omega_i)\delta\omega \tag{20}$$

$$(\zeta_a)_i = \sqrt{2S(\omega_i)\delta\omega} \tag{21}$$

The above equation is useful to get the amplitude of a component wave within a frequency band.

A fair finite-sum model of a unidirectional sea can be obtained by taking about 20 different frequency bands for a single direction. This is because any particular rectangle in figure 1 represents the variance in that band of frequencies. A regular wave of the indicated finite amplitude would have the same variance as the infinite number of component within that band. Hence, the addition of these components (shown at the bottom of the figure 1) will give a pattern that has the same total variance and closely resemble the record from which the spectrum was obtained.

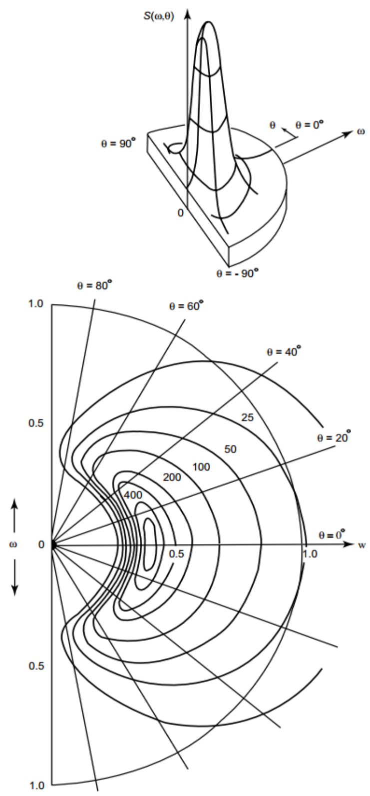

Figure 2: Example of directional spectrum from [2]

A point spectrum does not take into account the direction of propagation of component waves. A *directional spectrum* gives a more complete representation of the sea. Figure 2 show a typical example of a directional spectrum. The general equation for the total wave system of components moving in different direction, μ is:

$$\zeta(x,y,t) = \sum_i \sum_j (\zeta_a)_{ij} \cos[k_i(x\cos\mu_j + y\sin\mu_j) - \omega_i t + \epsilon_{ij}] \tag{22}$$

And the total variance of the system is:

$$S = \int_0^\infty \int_0^{2\pi} S(\omega,\mu) d\mu d\omega \tag{23}$$

where $S(\omega,\mu)$ is the function defining the directional spectrum such as the one shown in figure 2. The following section looks into various spectral families.

1.3 Idealized spectral families

This section describes three idealised point spectrum followed by a section which describes how to convert point spectrum into a directional spectrum.

1.3.1 Pierson-Moskowitz spectrum

This spectral form requires only one parameter, wind speed, as input and was developed primarily for oceanographic use. It is intended to represent point spectrum of a fully developed sea, that is fetch and duration are great, and there is no contaminating swell from other areas.

$$S(\omega) = \frac{\alpha g^2}{\omega^5} e^{-\beta\left(\frac{g}{V_\omega}\right)^4} \tag{24}$$

where:
ω is the frequency in radians/sec

$\alpha = 8.10 \cdot 10^{-3}$
$\beta = 0.74$
$g =$ acceleration due to gravity in cm/sec^2
$V_\omega =$ wind speed in cm/sec measured 19.5m above the surface.

While its oceanographic importance is great, this spectral model is good only for extreme storm condition and is inappropriate for general use because it is based on the assumption of full developed sea state reached after an extended time period of steady wind with no contaminating swell.

1.3.2 Bretschneider spectrum

This spectrum takes two input parameters - wave period and wave height. It has the form:

$$S(\omega) = \frac{A}{\omega^5} e^{\frac{-B}{\omega^4}} \tag{25}$$

where the two parameters A and B depend on the modal frequency, ω_m, and variance, S. The modal frequency is

$$\omega_m = \left[\frac{4}{5}B\right]^{1/4} \tag{26}$$

and total variance is

$$S = A/4B \tag{27}$$

The International Towing Tank Conference (ITTC, 1978) recommends the use of a form of Bretschneider spectrum when more specifically appropriate spectral forms are not known.

$$A = 173 \frac{{H_{1/3}}^2}{{T_1}^4} \tag{28}$$

$$B = \frac{691}{{T_1}^4} \tag{29}$$

$H_{1/3}$ is the significant wave height and T_1 is the time period; both are inputs.

1.3.3 JONSWAP spectrum

This spectral function takes two parameters as input - fetch and wind speed. The preceding two spectrum models represent open-ocean wave conditions. However, this might not always be the case where geographic boundaries limit the fetch in generating areas. North Sea is such an area and extensive oceanographic measurements were taken under the Joint North Sea Wave Project (JONSWAP). The spectral function derived from the recorded data is of the form:

$$S(\omega) = \frac{\alpha g}{\omega^5} e^{\left[-\frac{5}{4}\frac{\omega}{\omega_m}\right]^{-4}} \gamma^{e^{-\frac{(\omega-\omega_m)^2}{2\sigma^2 {\omega_m}^2}}} \tag{30}$$

where:
γ is 3.3
σ is 0.07 for $\omega < \omega_m$ and is 0.09 for $\omega > \omega_m$
α is $0.076\tilde{x}^{-0.22}$
ω_m is ${}^{2\pi \tilde{f_m} g}/_{V_{W10}}$ (modal frequency)
$\tilde{x}$ is ${}^{gx}/_{{V_{W10}}^2}$
$\tilde{f_m}$ is $3.5\tilde{x}^{-0.33}$
x is fetch
V_{W10} is wind speed at 10m (32ft) above sea level

Note: JONSWAP is simply a form of Bretschneider spectrum, multiplied by a frequency dependent factor (the γ term).

Appendix A of [3] provides a more through listing of different wave spectrum.

1.3.4 Converting point spectrum to directional spectrum

According to [3] even though there are several unidirectional spectral models, there are only a few multidirectional spectral models. But, even for the multidirectional wave models that are available, the data documentation has been lacking or is uncertain. In addition [3] also mentions that for modelling purpose, the directional characteristics of waves are assumed to be uncoupled from their spectral properties, and the spectrum of waves travelling within a given range of headings is taken to be some proportion of that measured at a point. On this basis, the directional spectrum is of the form:

$$S(\omega, \mu) = S(\omega)G(\mu) \tag{31}$$

$G(\mu)$ is called the *spreading function* and is of the form:

$$G(\mu) = F(s)\cos^{2s}\frac{1}{2}(\mu - \mu_1) \tag{32}$$

where:
μ_1 is the predominant wave direction
s is an index that determines the width of the directional spread.

$$F(s) = \frac{2^{2s-1}}{\pi}\frac{\Gamma^2(s+1)}{\Gamma(2s+1)} \tag{33}$$

Γ is the Gamma function. The function $F(s)$ ensures that the total variance of the directional spectrum $S(\omega, \mu)$ is same as that of the point spectrum $S(\omega)$.
Note: Could not find appropriate value for the index s in ITTC recommendations.

DNV-GL-H103 [4] (section 2.2.7) also suggests a similar approach for converting a point spectrum to a directional spectrum.

$$S(\omega, \mu) = S(\omega)G(\mu) \tag{34}$$

$$G(\mu) = \frac{\Gamma(1 + n/2)}{\sqrt{\pi}\Gamma(1/2 + n/2)}\cos^n(\mu - \mu_p) \tag{35}$$

where μ_p is the prevailing wind direction.

An alternative to the above two formulation is the 15^{th} ITTC (1978) recommendation mentioned in [1].

$$G(\mu) = k\cos^n\mu \tag{36}$$

where:
$-\pi/2 < \mu < \pi/2$
$n = 2$
$k = 2/\pi$

2 Simulation of ocean waves

This section presents a rough structure of the classes and how the data are grouped under each class. It also captures the main functions that will be called when simulating the ocean waves. The aim of this documentation is only to provide a rough overview and does not aim to mirror the actual implementation. The source code documentation will contain the most accurate and in-depth description of the classes and its members.

The ocean is represented by a surface. The contour of the surface is controlled by an $n \times n$ matrix of points called the *control points*· The sea surface is represented by the class *SeaSurface* (section 2.2) and the control points are represented by the class *SeaSurfaceControlPoint* (section 2.5). Random waves are generated on the surface by the interaction of several regular waves which are represented by the class *RegularWave* (section 2.4). The statistically property of the random sea generated through the regular waves is controlled by a directional spectrum, which is represented by the class *DirectionalSpectrum* (section 2.3). The directional spectrum is a collection of point spectrum for each direction ranging from $-\pi/2$ to $\pi/2$. The directional spectrum contains a table where each row represents a point spectrum for a direction and the columns of the table relate to each frequency.

2.1 Steps for simulating the random sea

Following are the main steps to initialise the sea surface to a wave spectrum and start simulation.

- Create an object of the class *SeaSurface*.
- Initialise the seas surface to a wave spectrum by calling *SeaSurface.Init()*. A call to the method *SeaSurface.Init()* will also generate and initialise all the regular waves in the sea surface.
- Start simulation by calling SeaSurface.Simulate().

2.2 Class SeaSurface

The class *SeaSurface* is used to define a square patch of the sea surface.

Member variables in the class:

- seaPatchSize - size of the square sea patch represented by *SeaSurface*.
- seaSurfaceControPoints - an array of points to control the contour of the sea surface.
- directionalSpectrum - contains a table of variance values for each direction and frequency.
- regularWaves - array of all regular waves on the sea surface.

Methods in the class:

- SetCtrlPointMatSize() - sets the number of control points on the surface.
- Init()
  > Initialise the directional spectrum.
  > Create the regular waves in the sea as described below:
  >> for each row in directional spectrum (ie: direction):
  >>> for each column in directional spectrum (ie: frequency):
  >>>> Get the variance value from the directional spectrum.
  >>>> Calculate the amplitude from variance.
  >>>> Generate a random value for wave phase.
  >>>> Create a regular wave with the amplitude, phase, frequency and direction.
  >>>> Insert the regular wave created into the array regularWaves.
- Simulate()
  > Initialise time = 0.
  > for each point in the array seaSurfaceControlPoints:

$>>$ for each wave in array regularWaves:
$>>>$ Calculate elevation due to each wave.
$>>$ Sum the elevations for all waves.
$>>$ Set SeaSurfaceControlPoint.z = sum of elevations.
$>$ Increment time and continue above loop.

2.3 Class DirectionalSpectrum

Class to generate a directional spectrum.
Member variable in the class:

- countDirections - the number of directions for which point spectrum data will be generated.
- countFrequencies - the number of frequencies bands that will be used for each point spectrum.
- directions - array containing direction angles.
- frequencies - array containing the frequency intervals.
- spectrum - table containing variance values. Each row corresponds to a direction and each column corresponds to a frequency band.

Methods in the class:

- SetCountDirections() - sets value for member variable countDirections.
- SetCountFrequencies() - sets value for member variable countFrequencies.
- Init()
  $>$ Set values for the array directions.
  $>$ Set values for the array frequencies.
  $>$ For each angle:
  $>>$ For each frequency:
  $>>>$ Calculate variance $S(\omega)G(\mu)$
  $>>>$ Enter the variance into the table spectrum.

2.4 Class RegularWave

Class to represent a regular wave.
Member variables in the class:

- direction - direction of travel of the wave.
- phase - the random generated phase value of the wave.
- frequency - the circular frequency of the regular wave.
- amplitude - the amplitude of the regular wave.

2.5 Class SeaSurfaceControlPoint

Class to contain the geographic coordinates of a control point on the sea surface.
Member variables in the class:

- x - the x coordinate
- y - the y coordinate
- z - the z coordinate

3 Hydrodynamics

This section presents a summary of the theory of ASV response motion in waves and and is mainly based on [1], [5] and [6].

ASV in waves experiences oscillatory motions and these motions have six degrees of freedom that is three transitional components - surge (in longitudinal direction, x), sway (in transverse direction, y) and heave (in vertical direction, z), and three angular components - roll (about the longitudinal axis, x), pitch (about the transverse axis, y) and yaw (about the vertical axis, z). Of the six motions, only three are purely oscillatory in nature- heave, pitch and roll. This is because these motions causes a change in the equilibrium between gravitational force and buoyancy force acting on the vessel resulting in a restoring force which brings the vessel back to the equilibrium position. Surge, sway and yaw does not produce a restoring force and hence these motions are not oscillatory in nature unless the exciting force itself changes direction and brings it back to the initial state.

At first, this section explores motion in each degrees of freedom independently of others. That is, for example, it is assumed that the heave motion is not affect by and does not affect any other motion. In reality this is not true. Since the ASV is longitudinally asymmetric, the heave motion will also induce pitching motion. The relationship between each motion in each degrees of freedom is explored in the section

To do: Link to section for coupled motion

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3.1 Equation of motion

The equation of motion is based on Newton's second law of motion. For each transitional motion component, this means that the force acting on the vessel should be equal the product of mass and acceleration in that direction and for each angular motion components it means that the moment acting on the vessel equals the product of mass moment of inertia and angular acceleration. The general equation of motion for forced oscillation is:

$$M\ddot{x} + C\dot{x} + Kx = F_0 \cos \omega t \tag{37}$$

where, x is the displacement (Note: x here referrers to displacement in any particular direction and not just along the X-coordinate.). Kx is the restoring force and K is the hydrostatic stiffness. A body oscillating in a viscous medium will experience damping force due to dissipation of energy to the surrounding medium. The term $C\dot{x}$ referrers to the damping force experienced by the body. C is the damping coefficient and $\dot{x}$ is the velocity. $M\ddot{x}$ is the inertia force where M is the mass and $\ddot{x}$ is the acceleration. F_0 is the magnitude of the periodic force that act on the body with a frequency .

It is more convenient to represent the cyclic force term using complex numbers as shown below:

$$F_0(\omega, t) = F_0 \cos \omega t + iF_0 \sin \omega t = F_0 e^{i\omega t} \tag{38}$$

The observed value of force at any instant of time is the real part of F_0. It is assumed that the wave excitation forces and the resultant oscillatory motions are linear and hormonic. Also the frequency of the motion is assumed to be same as the frequency of wave encounter but with phase lag of ϕ. Therefore at any instant of time t, the displacement for an encounter wave frequency of ω is:

$$x(\omega, \phi, t) = x_0 \cos(\omega t - \phi) + ix_0 \sin(\omega t - \phi) = x_0 e^{i(\omega t - \phi)} \tag{39}$$

$$\dot{x} = i\omega x_0 e^{i(\omega t - \phi)} = i\omega x \tag{40}$$

$$\ddot{x} = i^2 \omega^2 x_0 e^{i(\omega t - \phi)} = -\omega^2 x \tag{41}$$

Applying equations 38, 39, 40 and 41 in 37, we get:

$$-M\omega^2 x + iC\omega x + Kx = F_0 e^{i\omega t} \tag{42}$$

Therefore:

$$(-M\omega^2 + iC\omega + K)x = F_0 e^{i\omega t} \tag{43}$$

$$x = \frac{F_0 e^{i\omega t}}{-M\omega^2 + iC\omega + K} \tag{44}$$

Or:

$$x = HF_0 e^(i\omega t) \tag{45}$$

where:

$$H = \frac{1}{-M\omega^2 + iC\omega + K} \tag{46}$$

H is called the *complex transfer function*, because it transfers input force to output deflection and maintains phase information. The phase lag:

$$\phi = \tan^{-1}\left(\frac{C\omega}{K - M\omega^2}\right) \tag{47}$$

and amplitude:

$$x_0 = \frac{F_0}{\sqrt{(K - M\omega^2)^2 + (C\omega)^2}} \tag{48}$$

A key point to note is that when the ASV is moving at velocity of U_0 at an angle μ with respect to wave then the frequency of oscillation will shift from the wave frequency to the encountered wave frequency. In this case the ω term in the above equations should be replaced by encountered wave frequency,ω_e:

$$\omega_e = \omega_0 - \frac{\omega_0^2}{g} U_0 \cos\mu \tag{49}$$

3.2 Exciting force and moment

Since the focus of this section is vessel response motion due to waves, the forces and moments considered are only the fluid forces and moments due to waves acting on the vessel. Fluid forces and moments can be subdivided into two types - Froude-Krylov and diffraction excitation. Froude-Krylov excitation force and moment is obtained by integrating the pressure due to wave on the wetted surface area of the hull. Froude-Krylov does not consider the effects vessel on the incident wave. On the other hand, diffraction excitation are forces and moments due to modification of incident wave due to the presence of the vessel. In cases where the length of the incident wave is longer than the vessel length, the diffraction excitation forces and moments will be of small magnitude and hence is often ignored.

Froude-Krylov force it obtained by integrating the pressure due to wave along the wetted surface of the hull. For a regular sea wave, equation 9 in section 1.1 provides the formula for pressure head variation due to the wave. The Froude-Krylov force on the hull can then be obtained as given below:

$$\int_{\frac{-L}{2}}^{\frac{L}{2}} \int_{\frac{-B}{2}}^{\frac{B}{2}} \rho g \zeta_a e^{kz} \cos(kx - \omega t)\, dx\, dy \tag{50}$$

Since we consider each component motion separately, instead of considering the total pressure force, we consider the component of the pressure force acting in component direction. So for hear the above formula will be modified only to take the vertical component of the pressure force. So, if the surface $dx\, dy$ has an angle ϕ with the vertical plane, them the formula for Froude-Krylov force for heave is:

$$\int_{\frac{-L}{2}}^{\frac{L}{2}} \int_{\frac{-B}{2}}^{\frac{B}{2}} \rho g \zeta_a e^{kz} \cos(kx - \omega t) \cos\phi\, dx\, dy \tag{51}$$

Similarly, for surge and sway motion (which are not harmonic motion) the Froude-Krylov force can be obtained by taking the component in each direction.

The exciting moments can similarly be calculated by taking the moments of the forces about the axis of rotations. For pitch, the exciting moment about the transverse axis of the ASV is:

$$\int_{\frac{-L}{2}}^{\frac{L}{2}} \int_{\frac{-B}{2}}^{\frac{B}{2}} \rho g \zeta_a e^{kz} \cos(kx - \omega t) \cos\phi x\, dx\, dy \tag{52}$$

The moments for roll and yaw can also be calculated similarly.

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