

What Can a Bayesian Say About y/x ?

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Purpose of presentation

- ▶ I'll show how to do Bayesian inference in a context that's simple but non-trivial.
- ▶ Talk will be pedagogical and mostly mathematical.
- ▶ Prior knowledge of Bayesian ideas is useful but not necessary.
- ▶ Thanks to Niall Jeffrey for help.

The Original Cosmological Motivation (1)

- ▶ The *overdensity* of dark matter (DM), denoted δ_{DM} , is the percentage difference between the local density (in some volume) and the universal average density.
- ▶ Overdensities have existed since early times. Gravity draws DM into overdense regions, so the overdensities get bigger with time.
- ▶ There is a similar definition for baryons. Baryons fall into DM overdensities, so we get baryon overdensities in the same place.

The Original Cosmological Motivation (2)

- ▶ Physical effects mean that the baryon overdensity need not equal the DM overdensity; a simple model assumes a linear relationship:

$$b = \frac{\delta_{\text{baryon}}}{\delta_{\text{DM}}}$$

- ▶ In a recent paper (<https://arxiv.org/abs/2509.18967>), we (Ellen, Niall, LW, Ofer, Josh, et al.) measured the baryon overdensity (from galaxy counts) and the DM overdensity (from weak-lensing mass maps). But how then to infer something about b ?

Restate as a Statistics Problem

- ▶ We make noisy measurements of x and y and we want to infer $b = y/x$.
- ▶ The answer should be probabilistic (due to the uncertainty arising from the measurement noise).

Noise model

- ▶ Let's assume a simple model for the measurement noise:

Measurement noise in x and measurement noise in y are uncorrelated Gaussian with standard deviations σ_x and σ_y (which we will assume to both be unity).

- ▶ We make one observation of x (call the measured value \tilde{x}) and one of y (call the measured value \tilde{y}).
- ▶ Assume that we are in the low signal-to-noise regime, so \tilde{x} and \tilde{y} are 'a few' (e.g. not 'a few thousand'). For example, $\tilde{x} = 3$ and $\tilde{y} = 4$.

Naive calculation

- ▶ So here's a calculation that might appear reasonable to do.
- ▶ By subtracting the (unknown) noise from (\tilde{x}, \tilde{y}) we get a Gaussian distribution of the true values (x, y) that is centred on (\tilde{x}, \tilde{y}) . For each possible true (x, y) there is a true $b = y/x$, hence we have a distribution of the true b .