What Can a Bayesian Say About y/x?

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Purpose of presentation

- I'll show how to do Bayesian inference in a context that is straightforward but non-trivial.
- Talk will be pedagogical and mostly mathematical.
- Prior knowledge of Bayesian ideas is useful but not necessary.
- Thanks to Niall Jeffrey for help.

The Original Cosmological Motivation (1)

- The *overdensity* of dark matter (DM), denoted δ_{DM}, is the percentage difference between the local density (in some volume) and the universal average density.
- Overdensities have existed since early times. Gravity draws DM into overdense regions, so the overdensities get bigger with time.
- There is a similar definition for baryons. Baryons fall into DM overdensities, so we get baryon overdensities in the same place.

The Original Cosmological Motivation (2)

- Physical effects mean that the baryon overdensity need not equal the DM overdensity; a simple model assumes a linear relationship:

$$b = \frac{\delta_{\text{baryon}}}{\delta_{\text{DM}}}$$

- In a recent paper (`https://arxiv.org/abs/2509.18967`), we (Ellen, Niall, LW, Ofer, Josh, et al.) measured the baryon overdensity (from galaxy counts) and the DM overdensity (from weak-lensing mass maps). But how then to infer something about b?

Restate as a Statistics Problem

- We make noisy measurements of x and y and we want to infer $b = y/x$.
- The answer should be probabilistic (due to the uncertainty arising from the measurement noise).

Noise model

- Let's assume a simple model for the measurement noise:

  The measurement noise in x and the measurement noise in y have uncorrelated Gaussian distributions with zero means and with standard deviations σ_x and σ_y (which we will assume to both be unity).

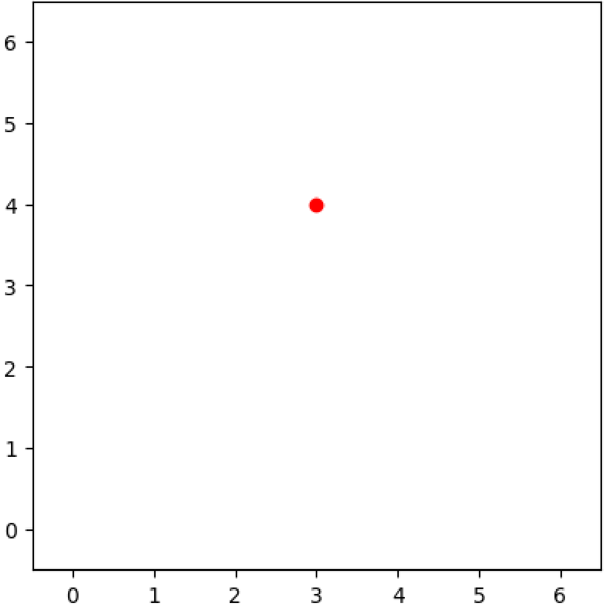
Data

- We make one observation of x (call the measured value $\tilde{x}$) and one of y (call the measured value $\tilde{y}$).
- Assume that we are in the low signal-to-noise regime, so $\tilde{x}$ and $\tilde{y}$ are 'a few' (e.g. not 'a few thousand'). For example, $\tilde{x} = 3$ and $\tilde{y} = 4$.

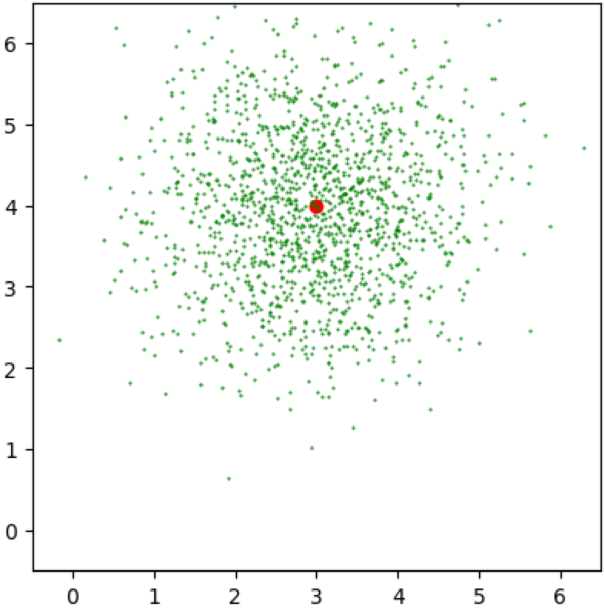
Naive calculation

- So here's a naive calculation that appears reasonable to do:
- By subtracting the (unknown) noise from $(\tilde{x}, \tilde{y})$ we get a Gaussian distribution of the true values (x, y) that is centred on $(\tilde{x}, \tilde{y})$. For each possible true (x, y) there is a true $b = y/x$, hence we have a distribution p of the true b.

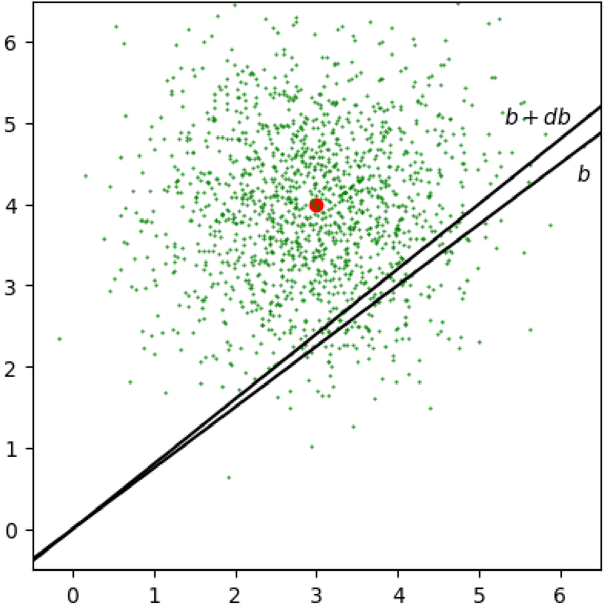
Naive calculation



Naive calculation



Naive calculation



Naive calculation

- So to calculate $p(b)\mathrm{d}b$ we need to calculate the probability mass within the thin pie-shaped region.
- Note that this region doesn't have constant angular width!
- Intuition: should we expect the distribution of b to be Gaussian? symmetric?
- Do you expect its mode to be at $\tilde{b} = \tilde{y}/\tilde{x}$?

Naive calculation

The calculation we need is

$$C \int_{\mathrm{pie}} \mathsf{exp}(-\frac{1}{2}(x - \tilde{x})^2 + (y - \tilde{y})^2)) \, \mathrm{d}x \, \mathrm{d}y \tag{1}$$

Naive calculation

Tricks for doing this calculation:

- Use polar coordinates $\mathrm{d}x\ \mathrm{d}y = r\ \mathrm{d}r\ \mathrm{d}\theta$.
- Don't forget the opposite quadrant! So $-\infty < r < \infty$.
- Complete the square.