

# What Can a Bayesian Say About $y/x$ ?

Lorne Whiteway  
lorne.whiteway.13@ucl.ac.uk

Astrophysics Group  
Department of Physics and Astronomy  
University College London

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# Purpose of presentation

- ▶ I'll show how to do Bayesian inference in a context that's simple but non-trivial.
- ▶ Talk will be mostly mathematical
- ▶ Existing knowledge of Bayesian ideas is useful but not necessary.
- ▶ Thanks to Niall Jeffrey for help.

# The Original Cosmological Motivation (1)

- ▶ The *overdensity* of dark matter (DM), denoted  $\delta_{\text{DM}}$ , is the percentage difference between the local density (in some volume) and the universal average density.
- ▶ Overdensities have existed since early times. Gravity draws DM into overdense regions, so the overdensities get bigger with time.
- ▶ There is a similar definition for baryons. Baryons fall into DM overdensities, so we get baryon overdensities in the same place.

# The Original Cosmological Motivation (2)

- ▶ Physical effects mean that the baryon overdensity need not equal the DM overdensity; a simple model assumes a linear relationship:

$$b = \frac{\delta_{\text{baryon}}}{\delta_{\text{DM}}}$$

- ▶ In a recent paper (<https://arxiv.org/abs/2509.18967>), we (Ellen, Niall, LW, Ofer, Josh, et al.) measured the baryon overdensity (from galaxy counts) and the DM overdensity (from weak-lensing mass maps). But how then to infer something about  $b$ ?

# Restate as a Statistics Problem

- ▶ We make noisy measurements of  $x$  and  $y$  and we want to infer  $b = y/x$ .
- ▶ The answer should be probabilistic (due to the uncertainty arising from the measurement noise).

# Noise model

- ▶ Let's assume a simple model for the measurement noise:

Measurement noise in  $x$  and measurement noise in  $y$  are uncorrelated Gaussian with standard deviations  $\sigma_x$  and  $\sigma_y$  (which we will assume to both be unity).

- ▶ We make one observation of  $x$  (call the measured value  $\tilde{x}$ ) and one of  $y$  (call the measured value  $\tilde{y}$ ).
- ▶ Assume that we are in the low signal-to-noise regime, so  $\tilde{x}$  and  $\tilde{y}$  are 'a few' (e.g. not 'a few thousand'). For example,  $\tilde{x} = 3$  and  $\tilde{y} = 4$ .

# Naive calculation

So here's a calculation that might appear reasonable to do.