UNIT 3 PROBLEM SET (PS3)

Due: 11 pm CT on Monday, March 15, 2021.
Submission instructions are on Canvas.

All **textbook problems** refer to problems from *Introduction to Probability*, Second Edition, by Bertsekas and Tsitsiklis. Problems labeled with "(Textbook)" have solutions available at this link and will be graded for effort and completion; the remaining problems will be graded for effort and correctness.

Lesson 3.1: PDF, Mean & Variance, CDF

1. Consider r.v. X with PDF $f(x) = \begin{cases} \frac{3}{8}\left(4x - 2x^2\right) & \text{for } 0 < x < 2, \\ 0 & \text{otherwise.} \end{cases}$
   (a) Find $P(X > 1.3)$.
   (b) Find $P(X = 1.3)$.
   (c) Find $P(0.3 \leq X \leq 2.3)$.
   (d) Find $E[X]$.
   (e) Find $Var(X)$.
   (f) State the CDF of X. Be sure to define it for the whole real line, not just over the support of X.
2. (Textbook) Ch. 3, Problem 19, parts (a) and (b), BUT for part (b) you only need to find $P(A)$, not the conditional PDF.
3. For the same X defined in Ch. 3, Problem 19, find:
   (a) $E[X]$
   (b) $Var(X)$

Lesson 3.2: Named Distributions

4. Let $U \sim \text{Unif}(2, 9)$.
   (a) State the PDF of U. Don't forget the support.
   (b) Find $P(U > 4.5)$.
   (c) Find $Var(U)$.
   (d) Find $E[ln(U^4)]$, where ln is natural log.
5. (Textbook) Ch. 3, Problem 7, part (a) only. *Hint: Start with the CDF of X. What is the area of a circle with radius x?*
6. Let X be the length of time, in hours, that a laptop will work without breaking down. Suppose X follows the exponential distribution with parameter $\lambda = 1/100$.
   (a) How long do we *expect* the laptop to work without breaking down? Include units.
   (b) Find the standard deviation of X.

(c) Find the probability the laptop will work more than 50 hours.

(d) Given the laptop has already worked 100 hours, what's the probability it will work at least an additional 50 (that is, at least 150 total hours)? In other words, find $P(X > 150 \mid X > 100)$. *Hint: Use the definition of conditional probability. The answer should end up being the same as the answer to (c). This is called the "memoryless" property of the exponential distribution.*

(e) Find the *median* time the laptop will work, i.e., the amount of time t such that $P(X \leq t) = 0.5$. *Hint: Use the CDF of X.*

7. Assume that IQ scores are normally distributed with a mean of 100 and a standard deviation of 15. I suggest drawing a rough sketch of the PDF. Use the 68-95-99.7 Rule to find the approximate probability that a randomly chosen person has an IQ:

   (a) less than 85?

   (b) between 115 and 145?

   (c) that is more than 30 away from 100?

8. Consider $X \sim \text{Normal}(\mu = 10, \sigma^2 = 9)$.

   (a) Roughly sketch the PDF of X and shade the area that corresponds to $P(X \leq 8)$.

   (b) Why can't we use the 68-95-99.7 Rule to find $P(X \leq 8)$?

   (c) Find $P(X \leq 8)$. Your answer should be in Φ notation.

   (d) Again find $P(X \leq 8)$, this time using an online applet, calculator, or normal table. You should provide a final numerical answer.

9. (Textbook) Ch. 3, Problem 11. *Hint for part (b): What happens to a normal r.v. when we subtract its mean and divide by its standard deviation?*

10. (Textbook) Ch. 3, Problem 13. *Hint:* If C is temperature in Celsius and F is temperature in Fahrenheit, they are related by $\frac{5}{9}(F - 32) = C$.

Lesson 3.3: Functions of RVs, MGFs

11. (Textbook) Ch. 3, Problem 18 part (b) only.

12. (Textbook) Ch. 4, Problem 1. *There is an optional video explaining this one; it's posted on the Lesson 3.3 page under Additional Resources.*

13. (a) Suppose r.v. X has MGF $M_X(s) = (0.3e^s + 0.7)^{10}$. What named distribution does X follow? State the name and parameter value(s).

    (b) Suppose r.v. Y has MGF $M_Y(s) = \frac{e^{5s}}{4s} - \frac{e^s}{4s}$. What named distribution does Y follow? State the name and parameter value(s).

14. (Textbook) Ch. 4, Problem 29. *Remember that "transform" is another name for "moment generating function".*

15. (Textbook) Ch. 4, Problem 36 parts (b) and (c) only.

Lesson 3.4 will be on the Unit 4 Problem Set.