EXAM 1 STUDY GUIDE

This is literally a guide of *what* to study. This document **does not contain all the details** about these topics. It is more like a checklist for you to go through to make sure you didn't forget to study something. If you are not comfortable with a topic, it is your responsibility to **review the corresponding course materials**.

LESSON 1.1: SETS

1. Set notation
2. Terminology (e.g., set, element, subset)
3. Venn diagrams: how to draw, label, shade
4. Set operations and properties: complement, union, intersection, disjoint, partition, De Morgan's laws

LESSON 1.2: PROBABILITY MODELS & CONDITIONAL PROBABILITY

1. Terminology: experiment, outcome, sample space, event
2. Visualization techniques: write out sample space, tree diagram, grid diagram, Venn diagram
3. Probability axioms: non-negativity, additivity, normalization
4. Discrete uniform probability law
5. Interpreting probability: long-run relative frequency or degree of belief
6. Probability properties: complement, subset, probability of union (inclusion-exclusion formula)
7. Conditional probability definition
8. Multiplication rule
9. Using visualizations (Venn diagram, tree diagram, etc.) to calculate probabilities

LESSON 1.3: TOTAL PROBABILITY, BAYES' RULE, & INDEPENDENCE

1. Total probability theorem
2. Bayes' rule
3. Independence of two events: definition, how to check it, how to use it
4. Conditional independence of two events
5. Independence of three events
6. Pairwise independence of three events
7. Independence of more than three events: probability of intersection is the product of separate probabilities

LESSON 1.4: COUNTING

1. Visualizations: tree diagrams, blanks "diagram"
2. The counting principle
3. Permutations (selecting k objects from n objects when order matters)

4. Combinations (selecting k objects from n objects when order doesn't matter)
5. Choose notation (aka binomial coefficient)
6. Partitions (splitting n objects into r distinct groups)
7. Using counting techniques to calculate probabilities

LESSON 2.1: DISCRETE RVS, PMFS, SPECIAL DISTRIBUTIONS

1. What is a random variable (r.v.) (a function that maps the sample space to $\mathbb{R}$)
2. Terminology: discrete r.v., support (or range) of an r.v.
3. Notation: $P(X = x)$ and $P(X \in T)$
4. Probability mass functions (PMFs): stating it, plotting it, sums to 1, use to make calculations
5. Special (named) distributions. For each: know/identify setting and parameter values, state PMF including support, use PMF to make calculations
   (a) Bernoulli(p)
   (b) Binomial(n, p)
   (c) Geometric(p)
   (d) Poisson(λ)
6. When n is large and p is small, Binom(n, p) can be approximated by Pois($\lambda = np$).

LESSON 2.2: FUNCTION OF AN RV; MEAN AND VARIANCE

1. Find PMF of function of single discrete RV (function could be one-to-one or not)
2. Expected value (aka mean) interpretation: center of distribution or long-run average
3. Expected value definition formula
4. Expected value rule for function of RV
5. Variance interpretation: spread or dispersion of distribution
6. Variance definition formula
7. Alternative formula for variance
8. Standard deviation
9. Mean and variance of linear function of RV
10. Mean and variance formulas for Bernoulli, binomial, geometric, and Poisson distributions

LESSON 2.3: JOINT PMFS, CONDITIONING, INDEPENDENCE

1. Joint PMF
2. Marginal PMF
3. Conditional PMF
4. Calculate probabilities using joint, marginal, conditional PMFs
5. Conditional expectation of X or $g(X)$
6. Independence of RVs: definition, use it, check it
7. Functions of two RVs: find PMF, mean, variance of new RV
8. Linearity of expectation
9. Variance of sum of independent RVs