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Radiative Transfer: Homework 2

1) $d = 10\ pc$

$T_\star = 5500\ K$, $R_\star = R_\odot = 6.96 \times 10^8\ m$

$T_p = 273\ K$, $R_p = R_\oplus = 6.38 \times 10^6\ m$

$\frac{F_p}{F_\star} = ?$ at $\lambda_{max,p}$

Find $\lambda_{max,p}$ using wein's law,

$$\lambda_{max} = \frac{2.898 \times 10^{-3}}{T} \left[\frac{m}{K}\right] = \frac{2.898 \times 10^{-3}}{273}$$

$$\Rightarrow \lambda_{max,p} = 10615.385\ nm$$

Find intensity of both objects at $\lambda_{max,p}$ using the Planck fxn,

$$B_\lambda(T) = \frac{\frac{2hc^2}{\lambda^5}}{e^{hc/\lambda kT} - 1}$$

$$B_\star = \frac{\frac{2hc^2}{(1.06 \times 10^{-5})^5}}{e^{hc/(1.06 \times 10^{-5})(5500)k} - 1} = 1.19 \times 10^{-6}\ erg \cdot cm^{-2} \cdot s^{-1} \cdot Hz^{-1} \cdot str^{-1}$$

$$B_p = \frac{\frac{2hc^2}{(1.06 \times 10^{-5})^5}}{e^{hc/(1.06 \times 10^{-5})(273)k} - 1} = 2.33 \times 10^{-9}\ erg \cdot cm^{-2} \cdot s^{-1} \cdot Hz^{-1} \cdot str^{-1}$$

Convert intensity to flux,

$$F = B_\lambda \cdot \pi \left(\frac{R}{D}\right)^2$$

$$F_* = (1.19 \times 10^{-6})\pi \left(\frac{6.96 \times 10^8}{10\,pc}\right)^2 = 1.190\ Jy$$

$$F_P = (2.33 \times 10^{-9})\pi \left(\frac{6.38 \times 10^6}{10\,pc}\right)^2 = 3.13 \times 10^{-7}\ Jy$$

$$\Rightarrow \boxed{\frac{F_P(\lambda_{max,p})}{F_*(\lambda_{max,p})} = 1.65 \times 10^{-7}}$$

measurement would need to be accurate up to 10^7 orders of magnitude!

2. $l = ?$, distance through the atmosphere

$\lambda = 500$ nm

$K_{500} = 0.264\ g^{-1} cm^2$

Let's use the equation for mean free path,

$$l_\nu = \frac{\langle \tau_\nu \rangle}{\alpha_\nu} = \frac{1}{\alpha_\nu} \quad (\tau = 1)$$

and $\alpha_\nu = K_\nu \rho$

$\rho = 1.225\ kg/m^3$, density of Earth's atmosphere (wikipedia)

$\rho = 1.225 \times 10^{-3}\ g/cm^{-3}$

$\Rightarrow l_\nu = \dfrac{1}{\kappa_\nu \rho}$

$l_\nu = \dfrac{1}{(0.264)(1.225\times10^{-3})}$

$\Rightarrow \boxed{l_\nu = 3092 \text{ cm}}$

3) Supernova remnant

$\theta = 4.3''$, angular diameter $= 0.072^\circ$
$= 0.0013$ rad

$\nu = 100$ MHz

$F_{100} = 1.6\times10^{-19}$ erg·cm^{-2}·s^{-1}·Hz^{-1}

assume LTE

a. $I_\nu = ?$, total intensity

In general, $F_\nu = \int I_\nu \cos\theta \, d\Omega$

$d\Omega = \sin\theta \, d\theta \, d\phi$

$\Rightarrow F_\nu = \int_0^{2\pi}\int_0^{\theta/2} I_\nu \cos\theta \sin\theta \, d\theta \, d\phi$

$F_\nu = \int_0^{2\pi} I_\nu \left(\frac{1}{2} - \frac{\cos^2\theta/2}{2}\right) d\phi$

$F_\nu = 2\pi I_\nu \left(\frac{1}{2} - \frac{\cos^2\theta/2}{2}\right) = 2\pi I_\nu \frac{\sin^2(\theta/2)}{2}$

small angle approximation,

$F_\nu = \pi I_\nu \left(\frac{\theta}{2}\right)^2$

$$F_\nu = \pi I_\nu \left(\frac{\theta}{2}\right)^2$$

$$\Rightarrow I_\nu = \frac{F_\nu}{\pi}\left(\frac{2}{\theta}\right)^2$$

$$I_\nu = \frac{(1.6 \times 10^{-19})}{\pi}\left(\frac{2}{0.0013}\right)^2$$

$$\boxed{I_\nu = 1.21 \times 10^{-13} \ \mathrm{erg \cdot s^{-1} \cdot cm^{-2} \cdot Hz^{-1} \cdot str^{-1}}}$$

b. $T = ?$

LTE so $I_\nu = B_\nu$ = Planck fxn

$h\nu << kT$ is the Rayleigh-Jeans limit,

$$I_\nu^{RJ}(T) = \frac{2\nu^2}{c^2} kT$$

part a

$$T = \frac{I_\nu^{RJ}(T)\, c^2}{2\nu^2 k}$$

$c = 3 \times 10^{10} \ \mathrm{cm \cdot s^{-1}}$

$$T = \frac{(1.21 \times 10^{-13})\, c^2}{2(1 \times 10^8)^2 \, k}$$

$k = 1.38 \times 10^{-16} \ \mathrm{cm^2 \cdot g \cdot s^{-1} \cdot K^{-1}}$

$$\boxed{T = 3.95 \times 10^7 \ \mathrm{K}}$$

c. from my equation in part a,

$$I \propto \left(\frac{1}{\theta}\right)^2$$

so a more compact region would result in a larger I_ν.

and in the Rayleigh-Jean limit;

$$I \propto T.$$

So, brightness temperature would increase.

d. The maximum brightness occurs at the peak wavelength which we can determine with Wein's law,

$$\lambda_{max} = \frac{0.290}{T} \left[\frac{cm}{K}\right]$$

↑ part b

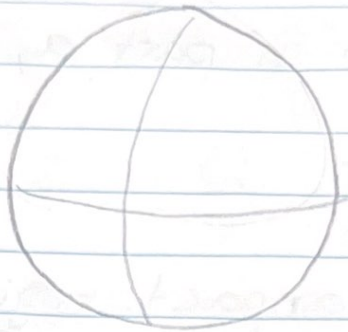
$$\lambda_{max} = \frac{0.290}{3.95\times10^7}$$

$$\Rightarrow \lambda_{max} = 7.34\times10^{-9}\ cm$$

$$= 0.0735\ nm$$

$$\boxed{\text{You would probably need an X-Ray telescope.}}$$

4)



Blackbody radiation,
- $T = 4\times10^{3}$ K, temperature of each particle
- $d = 0.1$ pc $= 3.086\times10^{17}$ cm
- $\nu_0 = 1.3\times10^{15}$ Hz,

$\alpha_\nu = 5.51\times10^{-20}$ cm^{-1}

$\sigma_\nu = 9.51\times10^{-18}$ cm^{-1}

$I_{\nu_0} = ?$, isotropic

Brainstorm some eqn's,

$$\frac{dI}{ds} = -(\alpha_\nu + \sigma_\nu)(I_\nu - S_\nu')$$, RTE including scattering

$$S' = \frac{\alpha_\nu S_\nu + \sigma_\nu J_\nu}{\alpha_\nu + \sigma_\nu}$$

$$d\tau_\nu = (\alpha_\nu + \sigma_\nu)\,ds$$

$$\Rightarrow \tau_\nu = (\alpha_\nu + \sigma_\nu)\left(\frac{D}{2}\right)$$ for isotropic

This is a "pure" extinction case ($j_\nu = 0$) which we know the solution for,

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu$$

$$\Rightarrow I_\nu(S) = I_\nu(0)e^{-\tau(S)}$$

$$I_\nu(S) = I_\nu(0)\,e^{-(\alpha_\nu+\sigma_\nu)D/2}$$

and the particles emit like a blackbody so,

$$I_\nu(0) = B_\nu(\nu_0, T)$$

Let's calculate the initial intensity,

$$I_\nu(0) = B_\nu = \frac{2(1.3\times10^{15})^3/c^2}{e^{h(1.3\times10^{15})/k_B(4\times10^3)}-1}$$

$$I_\nu(0) = 5.45\times10^{-9}\ \mathrm{erg\cdot s^{-1}\cdot cm^{-2}\cdot Hz^{-1}\cdot str^{-1}}$$

Back to the RTE solution,

$$I_\nu = I_\nu(0)\, e^{-(\alpha_\nu+\sigma_\nu)D/2}$$

$$I_\nu = (5.45\times10^{-9})\, e^{-(5.51\times10^{-20}+9.51\times10^{-18})3.086\times10^{17}/2}$$

$$\Rightarrow \boxed{I_\nu = 1.25\times10^{-9}\ \mathrm{erg\cdot s^{-1}\cdot cm^{-2}\cdot Hz^{-1}\cdot str^{-1}}}$$