AST6245: Radiative Processes and Stellar atmospheres
Fall 2023
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Final Project Assignment - Part II
(due November 29, 2023)

The aim of this final project is to explore the physical properties of an optically thin gas in a plane-parallel stellar atmosphere with a gradient in T, ρ, and P as a function of atmospheric height, and then code up various fundamental formulae that are at the base of any radiative transfer problem and level populations exercise you will encounter in your classwork or research.

For stars, the photons are generated in the optically thick interior (the nuclear burning core for most stars, excepting pre-main sequence objects, white dwarfs, and neutron stars). These photons propogate through the $\tau >> 1$ stellar interior, and after the order of millions of years, have their first chance to escape at the bottom of the stellar atmosphere, defined by $\tau = 1$. However, the gas remaining at the $\tau < 1$ region is the place where all of our diagnostics about the star are imprinted on the Planck function that forms the input spectrum for the radiative transfer, specified by $T = T(\tau = 1)$. Note that this is not the same T as T_{eff}, which is defined as $T = T(\tau = 2/3)$.

The project is divided into three parts, exploring a realistic model atmosphere of the Sun and using it to diagnose the Solar spectrum. In this assignment, you will now tackle Part II.

Please turn in a report that outlines the logic of your foray into understanding the stellar atmosphere problem, along with the requested figures including appropriately numbered (according to the corresponding part), labeled, clear and detailed figures and figure captions. ***This is a continuation of the same project. Therefore, do not start another report. Simply, resubmit your full project up until this point including your previous answers to Part I, as well as your new answers to Part II, clearly distinguishing between the two.*** *Making good descriptive figures with captions and writing solid abstracts is one of the most important skills you can develop while in graduate school. Please hand in your project report typed along with tables, figures, and any other relevant materials. Utilizing a peer-review paper style journal template for your report such as ApJ or MNRAS will count for extra credit. Neatness and organization count so please pay attention to details. DO NOT turn in your Jupyter Notebooks.*

Part II- Now that we have the physical parameters and their variation in the atmosphere understood, let's start to consider what we can actually observe. Let's first examine the broad solar spectrum (or spectral energy distribution) between 0.2-5μm. Find the file `solspect.dat.txt`, taken from Allen (1976) in your Canvas. There are four different quantities, both the astrophysical flux and the radially emergent intensity, each given with and without the inclusion of spectral lines (units are in cgs).

k. Plot the four columns against wavelength, all on the same Figure. Check that the continuum intensity peak is reached at $\lambda = 0.41\mu$m.

To proceed with our modelling we are going to assume a plane-parallel atmosphere that consists of two parts: (i) a semi-infinite lower part at temperature T_l that lies below (ii) an upper part at temperature T_u. We will also assume a source function $S_\lambda = B_\lambda(T)$. The surface of the atmosphere at optical depth $\tau = 0$ is at the top of the upper part, and the optical depth of the upper part is τ_u.

The idea of this so-called Schuster-Schwarzschild model, is that the continuous radiation, without spectral lines, is emitted by the semi-infinite lower layer having T_l, and irradiates a separate upper layer with intensity $I_\lambda(0) = B_\lambda(T_l)$.

The upper layer with T_u sits as a shell around the star, and causes attenuation as well as local emission only at the wavelengths of spectral lines. The shell is thus made up exclusively of line-causing atoms or ions. The shell may be optically thin or thick at the line wavelength, depending on the atom concentration which is expressed as a density or an abundance.

l. Solve (analytically) the radiative transfer equation of this atmosphere to show that the emergent intensity $I(\tau = 0, \theta)$ as a function of viewing angle θ can be expressed as:

$$I_\lambda = B_\lambda(T_l)e^{-\tau_\lambda} + B_\lambda(T_u)(1 - e^{-\tau_\lambda}) \tag{1}$$

m. Code a function that outputs B_λ for any input temperature T using the Planck formula. Test it for some temperatures relevant to stellar atmospheres.

n. Fit your Planck function to the continuum solar spectrum from above. What value of T do you infer for the Sun?

Now regarding the *line opacity* and spectral lines, the two primary considerations are: how much of any given element there is, and then, for a given temperature of the gas, how the electrons are distributed in the different energy levels (and thus available to move to other energy levels). For this we need both the Saha equation, telling us about the ionization state, and the Boltzmann equation, telling us about the excitation states for the bound electrons.

Next we are going to consider the optical depth term that goes into **Equation 1**, i.e. $\tau_\lambda(h) = -\int_\infty^h \alpha_\lambda dl$. Where the total broadened spectral line profile comes from broadened optical depth $\tau_\lambda = \tau_{\lambda_o}\ \phi(\lambda - \lambda_o)$.

τ_{λ_o} is the opacity at line center, which depends on α_{λ_o}, the linear extinction coefficient, which for a spectral line looks like:

$$\alpha_{\lambda_o} = \frac{\sqrt{\pi}e^2}{m_e c}\frac{\lambda_o^2}{c}\frac{n_l}{n_E}\ n_H\ A_E\ f_{lu}(1 - e^{-hc/\lambda_o kT}) \tag{2}$$

In this equation, n_l is the electron number density of the lower level and n_E is the total number density of element "E", while A_E is the abundance of element "E". The n_l/n_E quantity is what comes out of the consideration of the Saha + Boltzmann equations. f_{lu} is the oscillator strength for the specific transition.

Now let's apply these equations to a specific line example, to model the line absorption strength of the famous Na I D lines. This is a doublet, meaning there are actually two spectral lines separated by a small difference in energy (and thus wavelength) due to level splitting. The relevant atomic parameters for the D1, D2 pair are shown in Table 1. Additionally, take the partition functions $U_{\mathrm{NaI}} = 1$ and $U_{\mathrm{NaII}} = 6$.

λ_0 (Å)	f_{lu}	g_l	g_u	E_l (eV)	E_u (eV)
5889.95	0.641	2	4	0.0000	2.1044
5895.92	0.320	2	2	0.0000	2.1022

Table 1: Atomic level and transition properties of the Na I doublet D1 (5889.95 Å) and D2 (5895.92 Å) lines.

o. Code each of the Boltzman and Saha functions, that use the relevant parameters of the Na I D1 line.

p. Now code up a function `alpha_NaD` using the formula for α_{λ_o} in Equation 2 above, taking the Solar Na abundance to be $A_{\mathrm{Na}} = 1.8 \times 10^{-6}$.

q. Plot your linear extinction coefficient as a function of temperature, just to see how it behaves. Over what range of temperatures might we expect to see Na I D absorption lines?

You could imagine generalizing some of the steps above, by having a function for α that takes as input λ_o and f_{lu}, as well as outputs from the Saha-Boltzmann function, and an element abundance. But sometimes it is best to not be too clever, too fast, when developing new code.

With α_{λ_o} now in hand, we have an easy path to τ_{λ_o}. But you will recall from above that the total prescription for optical depth of an absorption line is $\tau_\lambda = \tau_{\lambda_o}\ \phi(\lambda - \lambda_o)$. So we have just one more function to specify!

The $\phi(\lambda - \lambda_o)$ component is the "broadening profile" of the absorption line. "Lines" are not delta functions, but have some broadening to them. The opacity

τ_λ varies not just over broad wavelength ranges, but also on small scales, indeed across individual spectral lines. For the bound-bound component of the opacity, when atoms absorb or emit a photon due to a valence electron jumping between two bound energy levels with $\Delta E = hc/\lambda_o$, the effect is spread out in wavelength around the line center λ_o. This line broadening is due to various effects that we learnt about in class, but for practical purposes here, we just need to know how to describe the distribution in wavelength. You can use a Gaussian, but usually a Voigt profile provides a better match and greater understanding. We thus define

$$\phi(\lambda - \lambda_o) = \frac{1}{\sqrt{\pi(\Delta\lambda_D)}} V(a, u) \tag{3}$$

where $\Delta\lambda_D$ is the Doppler width, $\Delta\lambda_D = (\lambda_o/c)\sqrt{2kT/m + v_{turb}^2}$. The Voigt function $V(a, u)$ is defined as a convolution[1], but can be approximated with the following python code:

```
def Voigt(a, u):
    I = scipy.integrate.quad(lambda y: np.exp(-y**2)/(a**2 + (u - y)**2),-np.inf, np.inf)[0]
    return (a/np.pi)*I

a = 0.1
u_range = np.linspace(-10,10,30)

plt.plot(u_range, [Voigt(a, u) for u in u_range])
```

Where a is called the impact parameter and depends on the broadening coefficients contributing to the spectral lines.

r. Use the code above to plot the Voigt function $V(a, u)$ for $a = 0.1$ and $u = -10$ to $u = +10$. Now vary the value of a between $a = 1$ and $a = 0.001$ to see the effect of this parameter. Use a log scale to inspect the far wings of the profile. Explain what you see.

[1] https://scipython.com/book/chapter-8-scipy/examples/the-voigt-profile/