**AST6245: Radiative Processes and Stellar atmospheres**
**Fall 2023 - Prof. Rana Ezzeddine**

**Assignement 1 (due: September 22, 11:59pm)**

**1.** Derive the Stephan-Boltzmann law, $F = \pi \int_0^\infty B_\nu d\nu = \sigma T^4$, from Planck's law. Show all steps in your solution. Hint: You can use the integral relations:

$$\int_0^\infty \frac{x^{n-1}}{e^x - 1} = \Gamma(n) \left( \frac{1}{1^n} + \frac{1}{2^n} + \frac{1}{3^n} + \dots \right) \tag{1}$$

where $\Gamma(n) = (n-1)!$ is the Gamma Function. Also, it's useful to note that:

$$\left( \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots \right) = \frac{\pi^4}{90} \tag{2}$$

**2.** Plot (using python) a Planck function at $T = 100\,\mathrm{K}$ (a typical temperature for warm interstellar dust), and $T = 10,000\,\mathrm{K}$ (representative of a hot star). Show that a hotter object is brighter than a cold object at all wavelengths (say, in this example, from $912\,\text{Å} - 1\,\mu\mathrm{m}$).

**3.** Find the shortest wavelength photon emitted by an emission electron transition on the Lyman ($n = 1$), Balmer ($n = 2$) and Paschen ($n = 3$) series. These wavelengths are know as the *series limits*. In which regions of the electromagnetic spectrum do each of these emit?

**4.** Consider a stellar atmosphere composed of pure helium. The aim is to find the temperature at the middle of the He I partial ionization zone, where half of the He I atoms have been ionized (such as the atmosphere of a =white dwarf). Assume for simplicity that the electron pressure is a constant $P_e = 200\,\mathrm{dyne\,cm^{-2}}$. The ionization energies of neutral helium and singly ionized helium are respectively: $\chi_I = 24.6\,\mathrm{eV}$ and $\chi_{II} = 54.4\,\mathrm{eV}$. The partition functions are $U_I = 1$, $U_{II} = 2$ and $U_{III} = 1$ for the neutral, first ionized and second ionized atoms, respectively.

(a) Find $N_{II}/N_I$ and $N_{III}/N_{II}$ for temperatures of 5000 K, 15,000 K and 25,000 K. How do they compare?

(b) Show that $N_{II}/N_{\mathrm{total}} = N_{II}/(N_I + N_{II} + N_{III})$ can be expressed in terms of the ratios $N_{II}/N_I$ and $N_{III}/N_{II}$.

(c) Plot (in python) $N_{II}/N_{\mathrm{total}}$ for the range of temperatures 5000 K to 25,000 K. What is the temperature at the middle of the He I partial ionization zone? What does that tell you about DB white dwarf atmospheres?