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Radiative Processes: Homework 1

1) Derive stefan-Boltzmann law from Planck fxn,

$$F = \pi \int_0^\infty B_\nu \, d\nu = \sigma T^4$$, stefan-Boltzmann law

generally Planck's law,

$$B_\nu(T) = \frac{8\pi h \nu^3 / c^3}{e^{h\nu/kT} - 1}$$

$$F = \pi \int_0^\infty B_\nu \, d\nu = \pi \int_0^\infty \frac{8\pi h \nu^3 / c^3}{e^{h\nu/kT} - 1} \, d\nu$$

$$= \frac{8\pi^2 h}{c^3} \int_0^\infty \frac{\nu^3}{e^{h\nu/kT} - 1} \, d\nu$$

Let $x = h\nu/kT \rightarrow dx = h/kT \, d\nu$

$\rightarrow d\nu = kT/h \, dx$

applying change of variables,

$$F = \frac{8\pi^2 h}{c^3} \int_0^\infty \left(\frac{(xkT/h)^3}{e^x - 1} \, dx \cdot \frac{kT}{h} \right)$$

$$= \frac{8\pi^2 h}{c^3} \left(\frac{kT}{h} \right)^4 \underbrace{\int_0^\infty \frac{x^3}{e^x - 1} \, dx}_{= \pi^4/15 \text{ (from hint)}}$$

$$\Rightarrow F = \frac{8\pi h}{c^3} \left(\frac{kT}{h} \right)^4 \cdot \frac{\pi^4}{15} = \frac{8\pi^5 \cancel{h} (kT)^4}{15 c^3 h^{\cancel{4}}}$$

$$= \frac{8\pi^5 (kT)^4}{15 c^3}$$

$\longrightarrow$

and the Stephan-Boltzmann constant is,

$$\sigma_{SB} \equiv \frac{8\pi^5 k^4}{15\, c^3}$$

$$\Rightarrow \boxed{F = \sigma_{SB} T^4}$$

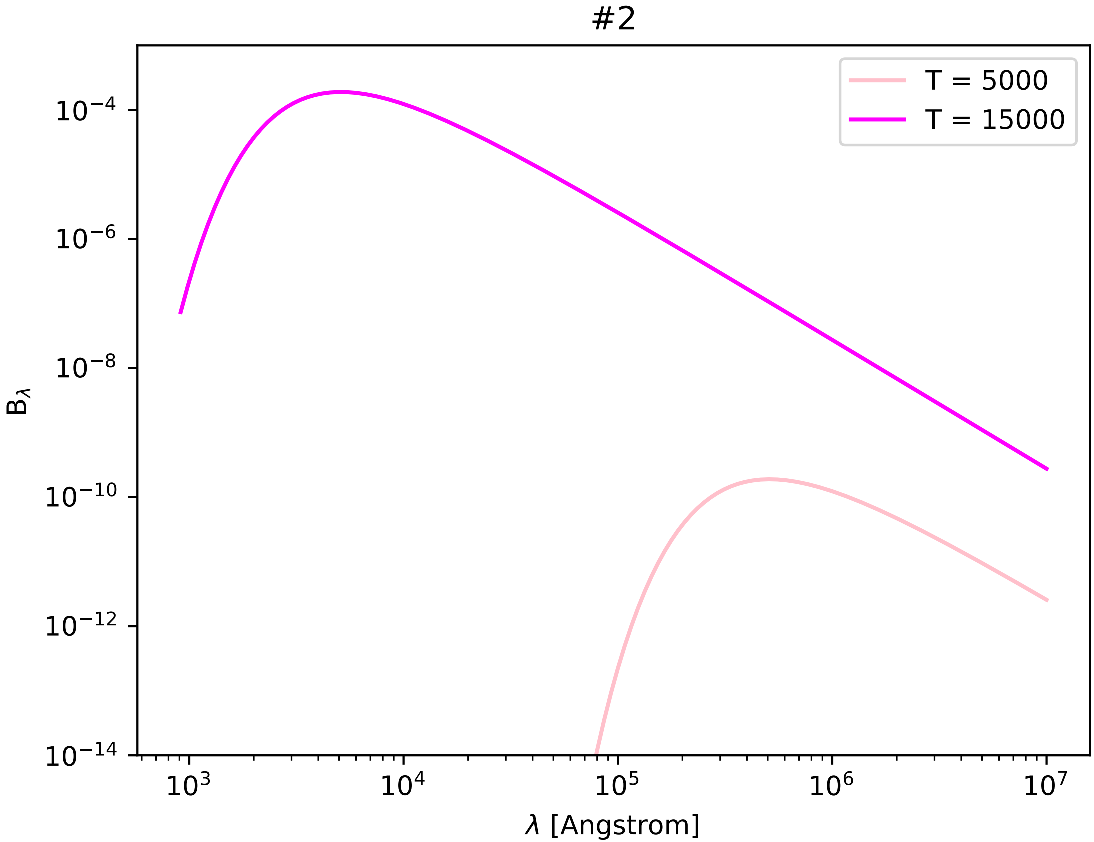
2) Plot the Planck fxn

a. T = 100 K, warm interstellar dust

b. T = 10 000 K, hot star

For the hot star, B_ν peaks around 10^{-4}. But for the warm interstellar dust B_ν peaks much lower $\sim 10^{-10}$.

(see plots on next page)


#2
T = 5000
T = 15000
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
B_λ
10^3
10^4
10^5
10^6
10^7
λ [Angstrom]

3) $\lambda_{min} = ?$ (series limits)

a. Lyman $(n=1)$

b. Balmer $(n=2)$

c. Paschen $(n=3)$

$$\Delta E = -\frac{13.6}{Z^2}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

$c = \lambda \nu$ and $E = h\nu$ and $E = \frac{hc}{\lambda}$

It can be helpful to write this equation in terms of Rydberg constant,

$$R_H = \frac{13.6}{hcZ^2} \sim 109678 \text{ cm}^{-1} = 1.09 \times 10^5 \text{ cm}^{-1}$$

$$\Rightarrow E = \frac{1}{\lambda} = R\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right), \text{ Rydberg formula}$$

a. Lyman series

$n_1 = 2,\ n_2 = 1$

$$\frac{1}{\lambda} = R\left(\frac{1}{1} - \frac{1}{4}\right) = \frac{3}{4}R$$

$$\Rightarrow \lambda = \frac{4}{3R} = 1.2 \times 10^{-5} \text{ cm}$$

$\boxed{\lambda = 120 \text{ nm}}$ → $\boxed{\text{Vaccum UV (EUV)}}$

→

b. Balmer series

$n_1 = 3$, $n_2 = 2$

$$\frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5}{36}R$$

$$\lambda = \frac{36}{5R} = 6.61 \times 10^{-5} \text{ cm}$$

$\boxed{\lambda = 661 \text{ nm}}$ $\boxed{\text{Optical}}$

c. Paschen series

$n_1 = 4$, $n_2 = 3$

$$\frac{1}{\lambda} = R\left(\frac{1}{9} - \frac{1}{16}\right) = \frac{7}{144}R$$

$$\lambda = \frac{144}{7R} = 1.89 \times 10^{-4} \text{ cm}$$

$\boxed{\lambda = 1890 \text{ nm}}$ $\boxed{\text{near infrared}}$

$= 0.00189 \text{ mm}$

4) pure He stellar atmosphere

$T = ?$, where half of the HeI atoms have been ionized

$P_e = 200 \text{ dyne} \cdot cm^{-2}$, constant electron pressure

$\chi_I = 24.6 \text{ eV} = 3.9 \times 10^{-11}$ erg

$\chi_{II} = 54.4 \text{ eV} = 8.7 \times 10^{-11}$ erg

$$\left.\begin{array}{l} U_I = 1 \\ U_{II} = 2 \\ U_{III} = 1 \end{array}\right\} \text{partition fxns}$$

a. $\frac{N_{II}}{N_I} = ?$ and $\frac{N_{III}}{N_{II}} = ?$

a. $T = 5000$ K

b. $T = 15000$ K

c. $T = 25000$ K

The Saha eqn,

$$\frac{N_+}{N_i} = \frac{2kT}{P_e} \frac{g_+}{g_i} \left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_i/kT}$$

So generally,

$$\frac{N_{II}}{N_I} = \frac{2kT}{P_e} \frac{U_{II}}{U_I} \left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_I/kT}$$

$$\frac{N_{III}}{N_{II}} = \frac{2kT}{P_e} \frac{U_{III}}{U_{II}} \left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_{II}/kT}$$

$\longrightarrow$

now just plug in numbers,

for T = 5000 K,

$$\frac{N_{II}}{N_I} = \frac{2k(5000)}{P_e} \frac{2}{1} \left(\frac{2\pi m_e(5000)}{h^2}\right)^{3/2} e^{-\chi_I/k(5000)}$$

$$\Rightarrow \boxed{\frac{N_{II}}{N_I} = 3.62 \times 10^{-18}, \ T = 5000 \text{ K}}$$

$$\frac{N_{III}}{N_{II}} = \frac{2k(5000)}{P_e} \frac{1}{2} \left(\frac{2\pi m_e(5000)}{h^2}\right)^{3/2} e^{-\chi_{II}/k(5000)}$$

$$\Rightarrow \boxed{\frac{N_{III}}{N_{II}} = 5.74 \times 10^{-49}, \ T = 5000 \text{ K}}$$

for T = 15000 K,

I followed a similar process as above to get,

$$\boxed{\frac{N_{II}}{N_I} = 1.22, \quad \frac{N_{III}}{N_{II}} = 2.62 \times 10^{-11}, \quad \text{for } T = 15000 \text{ K}}$$

for T = 25000 K,

$$\boxed{\frac{N_{II}}{N_I} = 8160.38, \quad \frac{N_{III}}{N_{II}} = 1.86 \times 10^{-3}, \quad \text{for } T = 25000 \text{ K}}$$

b. $$\frac{N_{II}}{N_{TOT}} = \frac{N_{II}}{(N_I + N_{II} + N_{III})}$$

express in terms of $\frac{N_{II}}{N_I}$ and $\frac{N_{III}}{N_{II}}$

Total number density,

$$N_{TOT} = N_I + N_{II} + N_{III}$$

let's say, $$N_{II} = \left(\frac{N_{II}}{N_I}\right) \cdot N_I$$

$$N_{III} = \left(\frac{N_{III}}{N_{II}}\right) \cdot N_{II}$$

Substitute into N_{TOT};

$$N_{TOT} = N_I + \left(\frac{N_{II}}{N_I}\right) \cdot N_I + \left(\frac{N_{III}}{N_{II}}\right) N_{II}$$

$$N_{TOT} = N_I + \left(\frac{N_{II}}{N_I}\right) N_I + \left(\frac{N_{III}}{N_{II}}\right)\left(\frac{N_{II}}{N_I}\right) N_I$$

$$N_{TOT} = N_I \left[1 + \frac{N_{II}}{N_I} + \left(\frac{N_{III}}{N_{II}}\right)\left(\frac{N_{II}}{N_I}\right)\right]$$

$$\Rightarrow \boxed{\frac{N_{II}}{N_{TOT}} = \frac{\frac{N_{II}}{N_I}}{\left[1 + \frac{N_{II}}{N_I} + \left(\frac{N_{III}}{N_{II}}\right)\left(\frac{N_{II}}{N_I}\right)\right]}}$$

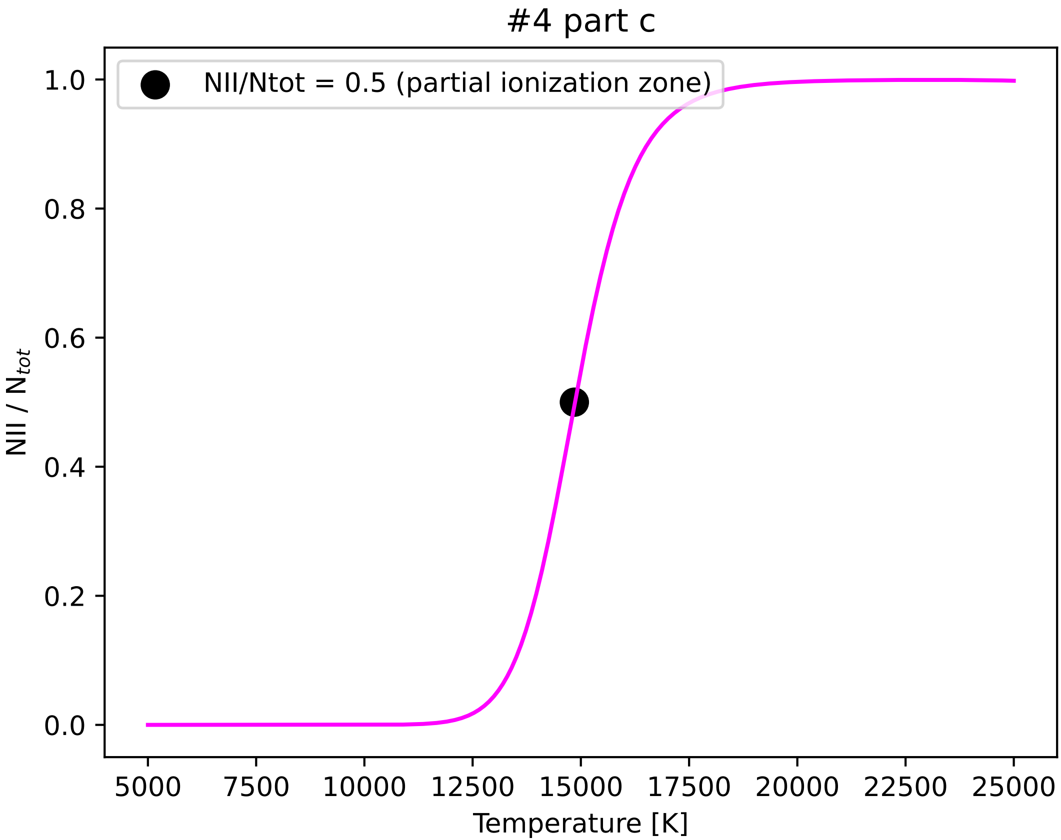
c. Plot $\frac{N_{II}}{N_{TOT}}(T)$

→ See plot on next page.

He I partial ionization zone when $\frac{N_{II}}{N_{TOT}} = 0.5$

$\boxed{\sim 14850\ K}$

⇒ a white dwarfs atmosphere is at least partially ionized.


#4 part c
NII/Ntot = 0.5 (partial ionization zone)
1.0
0.8
0.6
0.4
0.2
0.0
NII / N$_{tot}$
5000
7500
10000
12500
15000
17500
20000
22500
25000
Temperature [K]