AST6245: Radiative Processes and Stellar atmospheres
Fall 2023
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Final Project Assignment - Part I
(due November 15, 2023)

The aim of this final project is to explore the physical properties of an optically thin gas in a plane-parallel stellar atmosphere with a gradient in T, ρ, and P as a function of atmospheric height, and then code up various fundamental formulae that are at the base of any radiative transfer problem and level populations exercise you will encounter in your classwork or research.

For stars, the photons are generated in the optically thick interior (the nuclear burning core for most stars, excepting pre-main sequence objects, white dwarfs, and neutron stars). These photons propogate through the $\tau >> 1$ stellar interior, and after the order of millions of years, have their first chance to escape at the bottom of the stellar atmosphere, defined by $\tau = 1$. However, the gas remaining at the $\tau < 1$ region is the place where all of our diagnostics about the star are imprinted on the Planck function that forms the input spectrum for the radiative transfer, specified by $T = T(\tau = 1)$. Note that this is not the same T as T_{eff}, which is defined as $T = T(\tau = 2/3)$.

The project is divided into three parts, exploring a realistic model atmosphere of the Sun and using it to diagnose the Solar spectrum. In this assignment, you will tackle Part I.

Please turn in a report that outlines the logic of your foray into understanding the stellar atmosphere problem, along with the requested figures including appropriately numbered (according to the corresponding part), labeled, clear and detailed figures and figure captions. Making good descriptive figures with captions and writing solid abstracts is one of the most important skills you can develop while in graduate school. Please hand in your project report typed along with tables, figures, and any other relevant materials. Utilizing a peer-review paper style journal template for your report such as ApJ or MNRAS will count for extra credit. Neatness and organization count so please pay attention to details. DO NOT turn in your Jupyter Notebooks.

Part I- Let's start with a model for the radial profiles of various physical quantities in the $\tau < 1$ region. First find the file `falc.dat.txt`, on Canvas. This is a solar atmosphere model by Fontenla et al. (1993), derived using the plane-parallel and hydrostatic equilibrium assumptions. **Parts [a.] through [k.] below ask you to explore this model.** You're going to plot[1] a few quantities of interest in the atmosphere to diagnose some of the physical properties of the Sun (and stars in general).

a. Plot the temperature T vs. the height h, and describe the various regimes in the atmosphere. Where do absorption lines typically form? Where do emission lines typically form?

b. Plot the distribution of the temperature as a function of the optical depth τ at 5000 Å, $T(\tau_{5000\,\text{Å}})$ for this realistic Solar atmosphere model. Compare

[1] Make sure to use logarithms when appropriate so as to see the phenomena on both small and large scales.

it to the distribution $T(\tau) = T_{\text{eff}}(\frac{3}{4}\tau + 1/2)^{1/4}$ which is known as the "grey atmosphere" solution that we derived analytically in class using the Eddington-Milne approximation. Describe any differences.

c. Make a plot showing the number densities of protons, electrons and hydrogen (n_p, n_e, and n_H, respectively) versus h. Explain the difference in behavior of these curves. You may wish to consider the complementary plots of T vs h and/or y (ionization fraction) vs h in your assessment.

d. Plot the stratification of the density ρ vs h. Estimate the density scale height H_ρ, (the distance over which the pressure drops by 1/e) defined by $\rho = \rho_0 e^{-h/H_\rho}$ where $\rho_0 = \rho(h = 0)$. Compare this scale height to the radius of the Sun and commment on the validity of the plane parallel atmosphere assumption. How many "scale heights" high is the Solar atmosphere?

e. Plot the atmospheric pressure in the Sun P vs h. Compare to the gas pressure in the Sun, P_{gas}, assumed to be an ideal gas. <u>Hint:</u> This will require you computing P_{gas} (starting from the ideal gas law) as a function of the stratification properties. Describe any differences between $P = P_{total}$ that comes from the model and the $P = P_{gas}$ that you computed and explain why these differences might arise.

f. Plot β vs h from the Sun's atmospheric model, where $\beta = P_{gas}/P_{total}$. How does this compare to your answer and plot in part e? What could explain these differences?

g. Now investigate the P$_{total}$ vs m relationship, where m is the mass column density in units of g/cm^2. You should find that it is linear with P$_{total}$ = $m \times g$, where g is the surface gravity. What value of the Sun's surface gravity was assumed in the code that produced the atmospheric model?

h. One assumption in the model is of complete mixing, i.e., the same proportion of each element is mixed in at all heights. Check this by plotting the ratio of the hydrogen mass density to the total mass density against height. Then add helium via an appropriate helium-to-hydrogen ratio using their relative abundance and mass ratios – N(He)/N(H) = 0.1, m(He)/m(H) = 3.97 – and estimate the fraction of the total mass density made up by the remaining elements in the model mix (i.e. the "metals").

i. Plot v_{turb} vs h. What does this plot reflect about conditions in the atmosphere? The pressure from this turbulence can be found as $P_{turb} = 1/2\rho v_{turb}^2$; how does it compare to other pressure terms discussed above?

j. <u>OPTIONAL:</u> Now you may want to compare the particle density to the photon density, n_{phot}. For the particle density, we can take <u>n_H from above</u>. For the photon density, if we can assume thermal equilibrium, then the radiation field is isotropic and $I_\lambda = B_\lambda$, so the energy density

$$u_\lambda = \frac{1}{c} \int \int I_\lambda \; d\nu d\Omega = \frac{4\sigma}{c} T^4.$$

Thus

$$n_{phot} = \int_0^\infty \frac{u_\nu}{h\nu} d\nu = \frac{1}{hc} \int_0^\infty \int \frac{B_\nu}{\nu} d\Omega d\nu \approx \; 20T^3,$$

which given $T(h)$, will be a function of height in the atmosphere. Where in the atmosphere is the above expression more valid (deep or shallow part)?