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Physics Notes

EDUQAS A-level

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1 Newtonian Physics

1.1 Basic Physics

- **Base Quantities**
    - *MASS – [M] – Kilogram (kg)*
    - *LENGTH – [L] – Metre (m)*
    - *TIME – [t] – Second (s)*
    - *CURRENT – [I] – Ampere (A)*
    - *TEMPERATURE – [T] – Kelvin (K)*
    - *MOLAR QUANITITY – [n] – Mole (mol)*
    - *LUMINOUS INTENSITY – [L] – Candela (cd)*
- **HOMOGENOUS –** of the same kind – so an equation is homogenous if both sides have the same units on both sides >> Dimensional Analysis
- **Vectors vs Scalars**
    - Vectors: magnitude and direction
    - Scalars: just magnitude
    - Must be able to perform some basic vector operations
- **Density**
    - $\rho = \frac{m}{V}$
    - Density = Mass / Volume
    - **Measuring Density**
        - Two necessities – mass and volume
        - *Mass is normally determined using an electronic balance – though in the case of liquids, one needs to be aware that the masses of the containers must be discounted.*
            - *Otherwise the mass can be easily be found using a lever and a known mass – if such a balance does not exist – by varying the distance at which the system is at equilibrium*
        - **Volume**
            - For a regular shape, by measuring the exact sizes of the shape, the volume of the shape can normally be calculated.
            - For an irregular shape: measure displacement of the water is measured
                - Object **suspended by a thread**
                - Placed in water until entirely submerged,
                - Change in height of water in the measuring cylinder and then the volume of the object can be found – since the water has a known density.
    - USEFUL Densities

Material	**ρ / kgm^{-3}**
Air	1.29
Water	1000
Brick	2300
Petroleum	880
Steel	7900
Aluminium	2800
Mercury	13600

- **MOMENTS**
  - The moment of a force about a point is the product of the force and the ***PERPENDICULAR*** distance from the distance to the line of action of the force
  - $m = F * d$
  - **An object is at equilibrium exists when there is no resultant force acting on the object as well as there being no moment acting on the object about any point.**
  - *CENTRE OF GRAVITY*
    - The centre of gravity is the point at which we can consider the entire weight of an object to act. In a uniform gravitational field, the Centre of Gravity will lie on an any plane of symmetry.
    - In the case of a tipping object, if the centre of gravity is above the point of contact with the ground (hence the weight is acting through this point), then the object will not tip. Otherwise, the object will tip.

1.2 Kinematics

- $Speed = \frac{Distance}{Time}$
- $Mean\ Speed = \frac{Distamce\ Travelled}{Time\ Taken}$
- $Mean\ Velocity = \frac{Displacement}{Time\ Taken}$
- $Acceleration = \frac{\Delta V}{\Delta T}$
- $\boldsymbol{SUVAT}$
  - $v = u + at$
  - $\frac{v+u}{2} = \frac{s}{t}$
  - $v^2 = u^2 + 2as$
  - $s = ut + \frac{1}{2}at^2$
  - $s = vt - \frac{1}{2}at^2$
- Know how to interpret s,v,t graphs
- How to derive suvat eqns
- Describe motion of bodies falling in a uniform g-field
  - NO AIR RESISTANCE: steady acceleration
  - AIR RESISTANCE: acceleration steadily decreases until terminal velocity is reached
- Independence of horizontal and vertical motions

1.3 Dynamics

- **NEWTON'S LAWS**
  1. A body will remain at constant velocity unless an external force acts.
  2. The rate of change of momentum is equal to the net force
     - $F = \frac{dp}{dt}$
     - $F = m\frac{dv}{dt}$
     - $\boldsymbol{F = ma}$
  3. All forces come in pairs that are
     - Of the same type
     - Equal in magnitude
     - Opposite in direction
     - Act on different bodies
- **Momentum**
  1. $p = mv$
  2. Mass is a scalar quantity, which is a measure of the body's intertia
  3. CONSERVED
     - The vector sum of the momenta in a system is constant
     - Provided no external forces act
  4. COLLISIONS
     - INELASTIC
       - Just momentum is conserved.
     - ELASTIC
       - Momentum and Kinetic Energy is conserved.
  5. IMPULSE
     - $\Delta p = F\Delta t$
- **Forces between materials in Contact**
  - NORMAL FORCE
    - If an object rests against a surface, the surface exerts a force on the object
    - Because the molecules in the two bodies are placed in close contact – the electrons in the outer shells repel one another.
  - FRICTION
    - Static Friction
      - $F \leq \mu N$
      - Force acts to oppose motion
    - Limiting Friction
      - $F = \mu N$
      - Friction is just enough to prevent motion
    - Dynamic Friction
      - Friction is less than the force acting in the direction of motion.
      - *Arises from temporary bonds which form as molecules in the surface move past each other*
        - Bonds stretch and break – the stored energy in the bonds is converted to vibrational energy of the molecules.
  - AIR RESISTANCE
    - Example of **viscous drag**

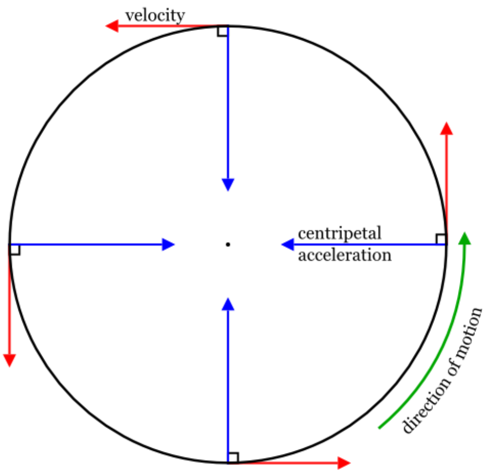
    - It opposes relative motion between the object and the fluid.
    - Molecules of the fluid bounce off a moving object moves slightly faster than they hit it – momentum increase in the fluid. Therefore force on the fluid in the direction of motion. BY NIII the fluid exerts an equal and opposite force on the body.
    - $\boldsymbol{D = C_d \frac{1}{2}\rho v^2 A}$
- **Gravitational Force**
  - $W = mg$

1.4 Energy Concepts

- **Work Done**
  - Work Done = Force x Distance Moved <u>(in the direction of the Force)</u>
  - $W.D. = F * d$
    - If F and d not parallel >> W = Fd cosθ
  - Work is done when a force moves its point of application
  - Energy is the ability to do work.
- **Energy is conserved**
- **Internal Energy**
  - Total Energy Possessed by an Object = Kinetic Energy + Potential Energy
- **Kinetic Energy**
  - $K.E. = ½mv^2$
- **Gravitational Potential Energy**
  - $G.P.E. = mg\Delta h$
- **Elastic Potential Energy**
  - $E.P.E. = ½\,kx^2$
- **POWER**
  - $Power = \frac{Energy\ Transfer}{Time}$
  - $P = \frac{W.D.}{t}$
  - Therefore as, $P = F\frac{d}{t}$
  - $P = Fv$
    - Must be component of velocity in direction of force (or vice versa)
- **Efficiency**
  - $\eta = \frac{Useful\ Energy\ out}{Total\ Energy\ in}$

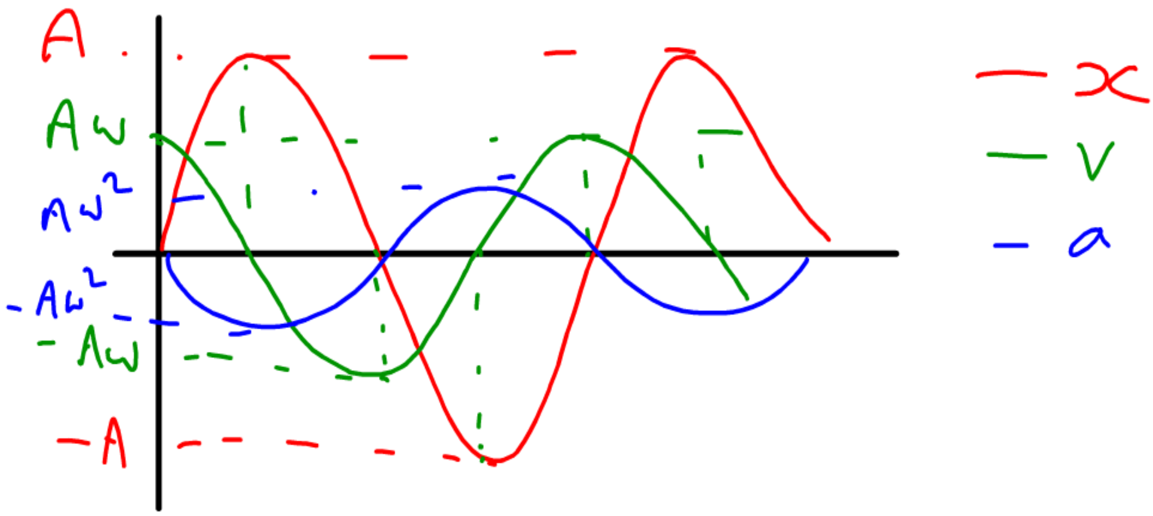
1.5 Circular Motion

- **Definitions**
  - Circular Motion
    - Motion at a constant speed but a varying velocity – with the motion occurring around a circle of radius $\boldsymbol{r}$
  - Period $\boldsymbol{T}$
    - The time to complete one cycle
  - Frequency $\boldsymbol{f}$
    - The number of cycles per second
  - Angular Displacement **θ**
    - The angle (mostly in radians) turned through
  - Angular Velocity $\boldsymbol{\omega}$
    - The rate of change of angle – rad/s => s^{-1}
  - Angular Acceleration
    - The rate of change of angular velocity
  - Tangential Speed $\boldsymbol{v}$
    - The instantaneous speed of the object
  - Centripetal
    - Going towards the centre
    - Centripetal Acceleration $\boldsymbol{a}$
      - The acceleration of the object (pointed inwards)
    - Centripetal Force $\boldsymbol{F}$
      - The resultant force on the object (pointed inwards)
- **Equations**
  - $T = \frac{1}{f}$
  - $\omega = \frac{d\theta}{dt}$
  - $\omega = 2\pi f$
  - $v = \omega r$
  - $a = \frac{v^2}{r}$
  - $a = \omega v$
  - $a = \omega^2 r$
  - $F = ma$

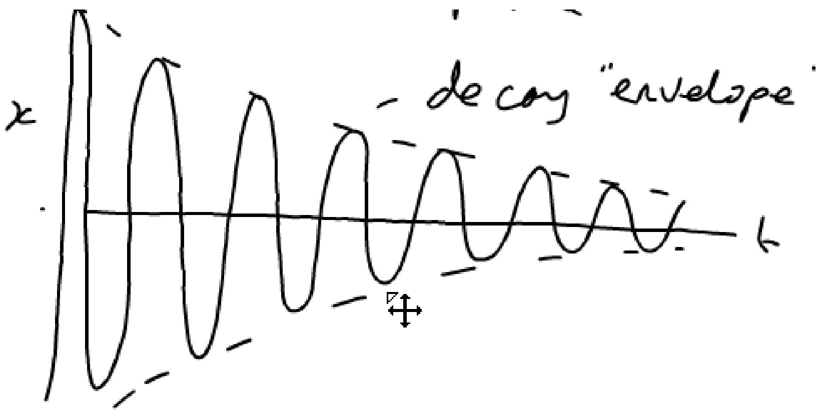


1.6 Simple Harmonic Motion

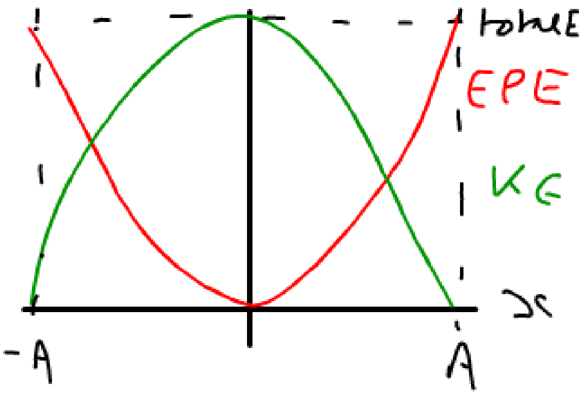
- **SHM Definition**
  - Periodic motion at some amplitude $\boldsymbol{A}$
  - Acceleration $\boldsymbol{a}$ is proportional to displacement $\boldsymbol{x}$ (from a fixed point)
  - But in the opposite direction. Hence:
    - $\boldsymbol{a} \propto -\boldsymbol{x}$
    - $\boldsymbol{F} \propto (-\boldsymbol{x})$
- **Basic Maths**
  - For a SPRING
    - $\omega = \sqrt{\frac{k}{m}}$
    - $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$
  - For a PENDULUM
    - $\omega = \sqrt{\frac{g}{L}}$
    - $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$
- **Maths with trig**
  - Can differentiate to get between x, v and a
  - Therefore,
    - $x = A\sin(\omega t + \varepsilon)$
    - $v = A\omega\cos(\omega t + \varepsilon)$
    - $a = -A\omega^2\sin(\omega t + \varepsilon)$
  - Where ε is a phase factor to account for when arbitrary choice of t = 0
  - Therefore, for maximum values
    - $x_{max} = A$
    - $v_{max} = A\omega$
    - $a_{max} = A\omega^2$
- **Conditions for SHM**
  - Period is independent of amplitude (ISOCHRONOUS)
  - Energy in the system will remain constant



- **Damping**
  - Damping is a force which removes energy from an oscillator, resulting in the amplitude decaying exponentially.
  - $x = Ae^{-\gamma t} sin(\omega t)$
    - $\omega = \sqrt{w_0 - \gamma^2}$



  - **Underdamped**
    - Still oscillates but amplitude reduces to zero over time. $\{\gamma < \omega\}$
  - **Critical Damping**
    - Quickly returns to equilibrium without further oscillation $\{\gamma = \omega\}$
  - **Overdamped**
    - Time taken to return to equilibrium is greater than that of critical damping as damping is too large. $\{\gamma > \omega\}$
- **ENERGY IN SHM**
  - All oscillators store multiple forms of energy and interchange between them.
  - **SPRING**
    - EPE and KE
    - $\frac{1}{2} kx^2 + \frac{1}{2} mv^2$

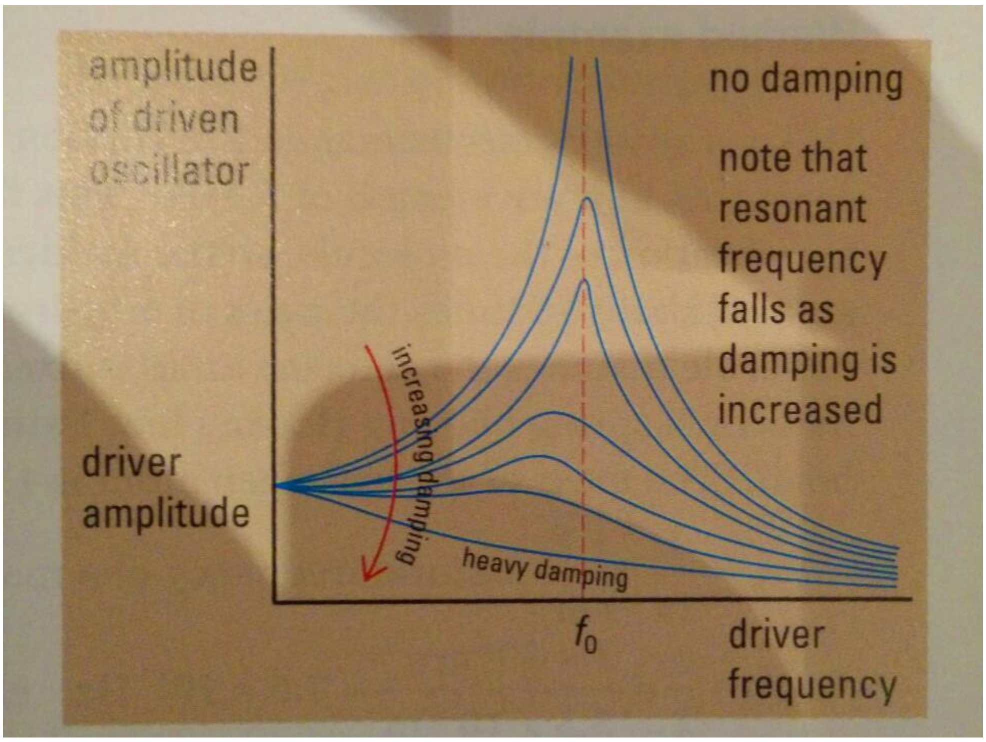


- **FORCED OSCILLATIONS**
    - Systems are driven by a sinusoidal external force
    - Driven oscillator responds at the driving frequency (not it's natural frequency) with a phase and amplitude which vary with the driving frequency.

Transient solution | Steady-state solution

$$x(t) = A_h e^{-\gamma t}\sin(\omega' t + \varphi_h) + A\cos(\omega t - \varphi)$$

Determined by initial position and velocity | Determined by driving force



- The effects of damping are shown in the graph above, with the graph also showing how the nearer to the resonant frequency that the driver frequency is, the greater the amplitude is:
- Resonance
    - Useful
        - Radio tuning circuits

            - Microwaves – tuned to resonate with water molecules
        - Not useful
            - ‘buzzing’ of speaker cases
            - Bridges
                - Tacoma Narrow Bridge which resonated with the wind.
                - Soldiers walking on a bridge at frequency resonated with bridge which could cause it to collapse.
    - If driving frequency << resonant frequency
        - the driven oscillator is in phase with the driver
    - If driving frequency >> resonant frequency
        - the driven oscillator is out of phase with the driver
    - The resonant frequency occurs
        - when the two are 90 degrees out of phase

1.7 Kinetic theory

- **Ideal gas:** $pV = nRT$, where
  - p = pressure
  - V = volume
  - n = mols
  - R = molar gas constant = $8.31\, J\, mol^{-1}\, K^{-1}$
- OR $pV = Nk_BT$, where
  - N = number of molecules of gas = $n\, N_A$
  - k_B = Boltzmann constant = $1.38 \times 10^{-23}\, JK^{-1} = \frac{R}{N_A}$
    - N_A = Avogadro's constant = $6.02 \times 10^{23}\, mol^{-1}$
    - N_A number of molecules is 1 mol
- **Assumptions of Kinetic theory**
  - Random distribution of energy (speeds)
  - Random distribution of direction
  - Molecules are far apart compared to their size
  - Molecules undergo perfectly elastic collisions with each other and container but do not interact otherwise
  - Transfer of kinetic energy is heat
- **Molecular movement causes collisions >> pressure given by**
  - $p = \frac{1}{3}\rho\langle c^2\rangle$
    - p = pressure
    - ρ = density
    - $\langle c^2\rangle$ = r.m.s speed of molecules
  - OR $pV = \frac{1}{3}Nm\langle c^2\rangle$
    - N = number of molecules
    - M = mass of each molecule
    - V = volume
- **Avogadro's constant:** $N_A = 6.03 \times 10^{23}\, mol^{-1}$
  - Is the number of Carbon-12 molecules that give a mass of 12g
  - The mole is N_A lots of stuff
  - => 1 mol of C-12 is 12g
  - $n = \frac{M}{M_r}$, where
    - M = total mass (g)
    - M_r = molar mass (g mol^{-1})
- **Rearranging equations**
  - $pV = \frac{1}{3}Nm\langle c^2\rangle$ & $pV = nRT = Nk_BT$
  - >> $\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}k_BT$
  - >> for one mole we multiply by N_A>>$k_BN_A = R$
    - For a single molecule
      - $K.E._{molecule} = \frac{3}{2}k_BT$
    - For a mol
      - $K.E._{mol} = \frac{3}{2}RT$
  - From this we can see >> $K.E. \propto T$

1.8 Thermal physics

- **Internal Energy U**
  - Internal energy of a gas is the sum of the K.E. from its motion and the P.E. from its bonding
  - U = K.E. + P.E.
- **Absolute Zero**
  - At 0 K there is no molecular motion => U is at a minimum (as P.E. is generally fixed)
- **Calculating U**
  - If P.E is 0 (monatomic gas) then it is easy to calculate its internal energy, from previous equations
    - $U = \frac{3}{2}nRT$
  - If P.E & n are constant
    - $\Delta U = \frac{3}{2}nR\Delta T$
- **Heat transfer**
  - Heat enters/leaves a system through its boundary
  - Heat is energy flow
- **Work transfer**
  - Energy can also enter/ leave a system by work
  - >> work is also a form of energy transfer
- **Thermal Equilibrium**
  - If no heat flows between two bodies in contact >> in thermal equilibrium >> same temperature (follows on from T0)
- **Laws of Thermodynamics**
  - **T0:** If A is in thermal equilibrium B, and B is so with C >> A is also in thermal equilibrium with C >> defines temperature as a state variable
  - **T1:** Energy is conserved >> $\Delta U = Q - W$ where
    - ΔU = **increase** in internal energy
    - Q = heat supplied **to** system
    - W = work done **by** system
  - **T2:** Heat flows from hot to cold. Or entropy of the universe always increases
  - **T3:** The entropy of a perfect crystal at absolute zero is exactly equal to zero (not A-level)
- **Calculating Work**
  - $W = p\Delta V$ at constant pressure (isobaric)
  - >> if pressure non-constant W given by area under p-V graph
    - $W = \int pdV$
- **Using T1**
  - For a solid, W is usually negligible >> $\Delta U = Q$
- **Specific Heat capacity**
  - It pays to define a shorthand variable for the specific heat capacity (c) = $J\ kg^{-1}\ K^{-1}$
  - $Q = mc\Delta\theta$
    - Q = heat supplied
    - m = mass of substance
    - c the specific heat capacity
    - $\Delta\theta$ = temperature rise

2 Electricity and the Universe

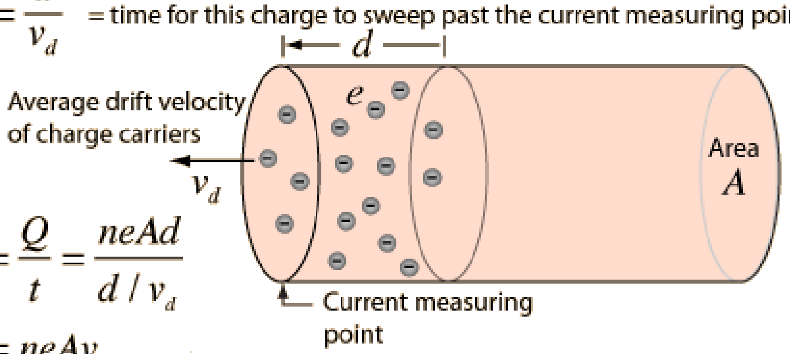
2.1 Conduction of Electricity

- **Coulomb C**
  - S.I. unit of charge Q
  - $e = 1.6 \times 10^{-19} C$ is the charge of one electron
- **Ampere A**
  - S.I. unit of current I = 1 Cs^{-1}
  - $I = \frac{\Delta Q}{\Delta t}$ = rate of flow of charge
  - Current can flow through some materials – called conductors
  - In metals current is carried by the drifting of electrons
  - Know how to derive $I = nAve$ where
    - I = current
    - n = electron number density (electrons / m^3)
    - v = drift velocity
    - e = electronic charge

n = number of charges e per unit volume

$Q = neAd$ = total mobile charge in length d of the conductor

$t = \frac{d}{v_d}$ = time for this charge to sweep past the current measuring point.

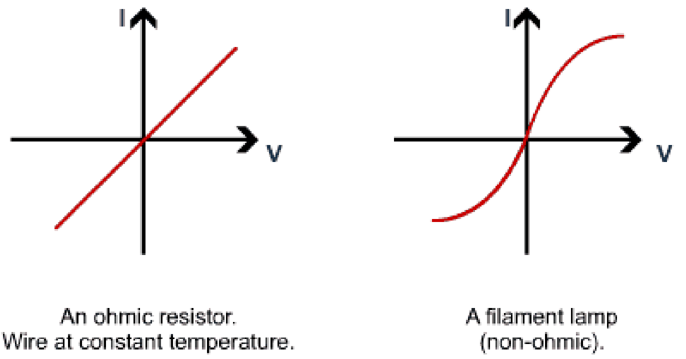


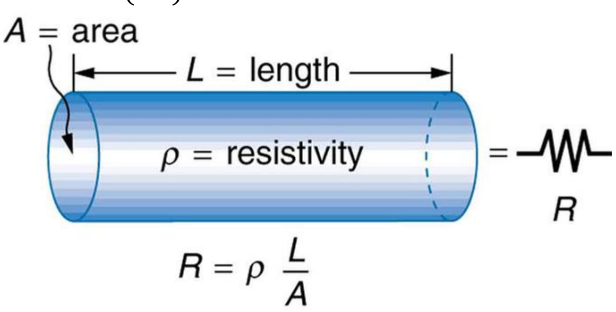
$$I = \frac{Q}{t} = \frac{neAd}{d / v_d}$$

$$I = neAv_d$$

-

2.2 Resistance

- **Potential Difference ΔV**
    - The voltage difference between two points
    - The energy imparted to one coulomb of charge moving from A to B
    - Measured in volts
    - 1 Volt = 1 Joule / Coulomb
- **Characteristic I-V graphs**
    - 

        - For the wire, constant resistance >> $I \propto V$ >> ohmic
        - For the bulb, as I ↑ R ↑ >> $I\ not \propto V$ >> non-ohmic
- **Ohm's Law**
    - $V = IR$
    - Definition of Resistance (units of Ohms Ω)
    - 1 Ohm = 1 Volt / Amp
- **Power**
    - $P = VI = I^2R = \frac{V^2}{R}$
- **Resistivity**
    - Resistance comes from collisions between free electrons and ions >> dissipate energy as heat
    - Resistance increases with temperature as more vibrations of ions
    - $R = \frac{\rho l}{A}$
        - $R = resistance\ (\Omega)$
        - $\rho = resistivity\ (\Omega m)$ – material property
        - $l = length\ (m)$
        - $A = area\ (m^2)$
        - 

- **Temperature effects**
    - For most materials $R \propto T$ where T is temperature (K) over a large range of T
    - But for some materials, when T goes below a critical temperature, R = 0

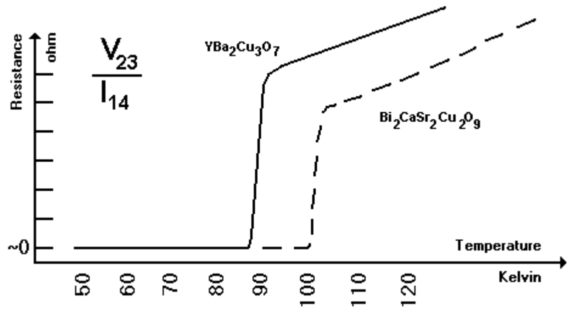
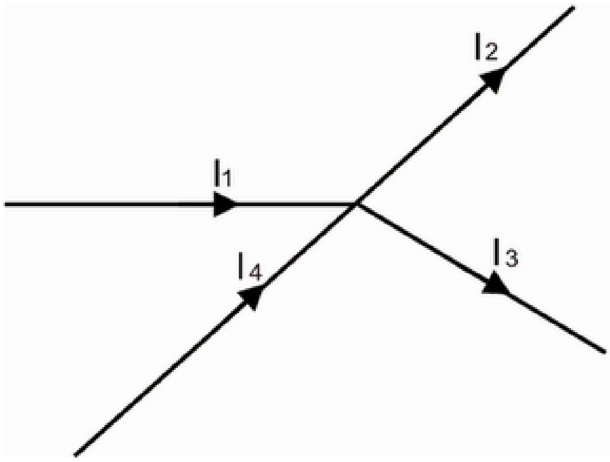
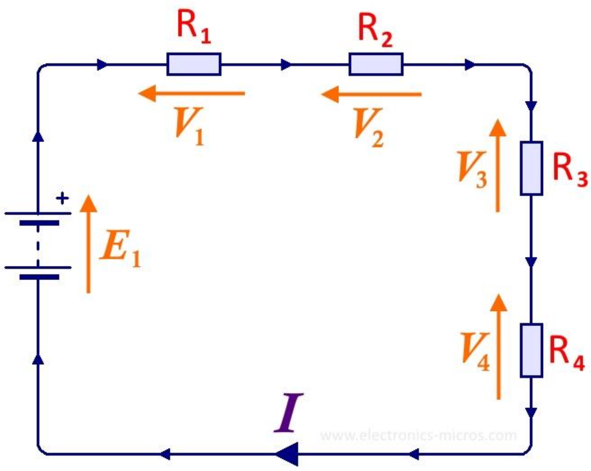


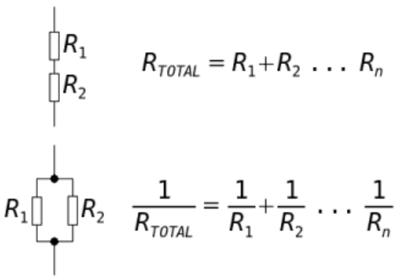
Figure 4: **Resistance versus Temperature**

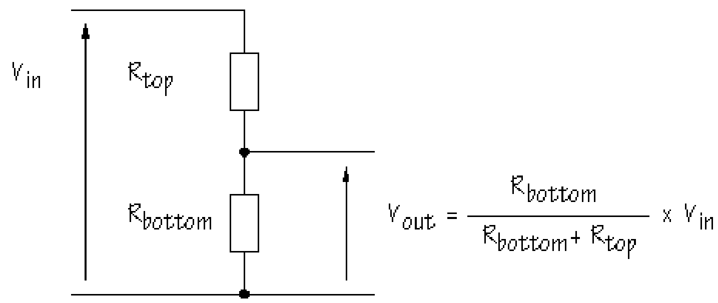
- These are called superconductors
- Most metals have transition temperatures at a few degrees above 0K
- But some exotic ceramics have quite high ones (80 K), which is above the boiling point of nitrogen
- Uses of superconductors:
  - Powerful magnets
    - MRI
    - Maglev
    - Particle accelerators
  - Reducing electrical losses

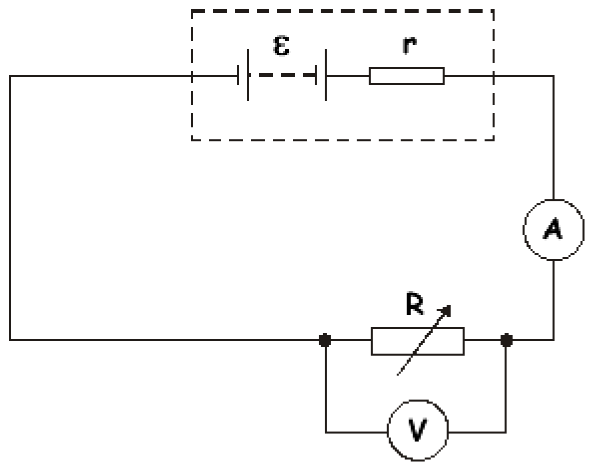
2.3 D.C circuits

- **Kirchhoff's Laws**
  - **Current**: Sum of currents into a node = sum of currents going out
    - 

    - >> $I_1 + I_4 = I_2 + I_3$
  - **Voltage:** The sum of the potential differences around a closed loop equals the sum of the EMF's (battery supply)
    - 

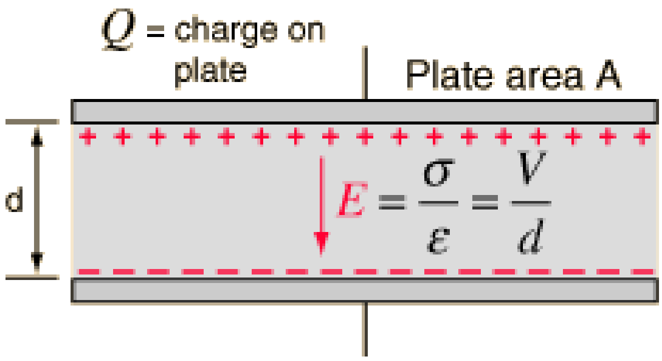
    - >> $E_1 = V_1 + V_2 + V_3 + V_4$
  - This is a consequence of conservation of energy as an electron that moves around the circuit must not gain energy in a closed loop
- Potential differences equal for parallel components
- **Combining resistors**
  - **Series**
    - $R_{tot} = R_1 + R_2 + \cdots R_n$
  - **Parallel**
    - $\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_n}$
  - 

- **Potential divider**
    - 

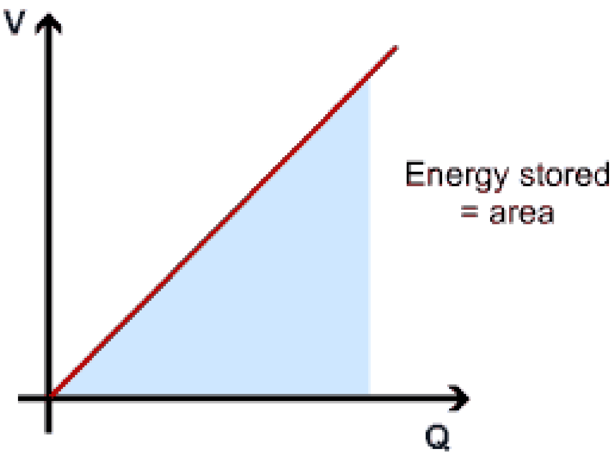
    - Often used in sensing circuits >> LDR or thermistor >> voltage out related to temperature /light intensity
- **Electromotive Force (EMF)**
    - The emf of a source is the potential difference measured across its terminals
    - It is the change in electric potential imparted to a charge carrier moving through the source
    - emf has units of Volts
- **Internal resistance**
    - Cells have an internal resistance which opposes the flow of charge through them
    - >> When a current is drawn, some potential is dropped across this internal resistor >> terminal voltage falls
    - 

    - When a current I flows, a potential Ir is dropped across the internal resistor
    - >> $V = \varepsilon - Ir$ where
        - V = terminal voltage
        - ε = emf of source
        - I = current
        - r = internal resistance
- Know how to calculate currents and potential differences for simple circuits

2.4 Capacitance

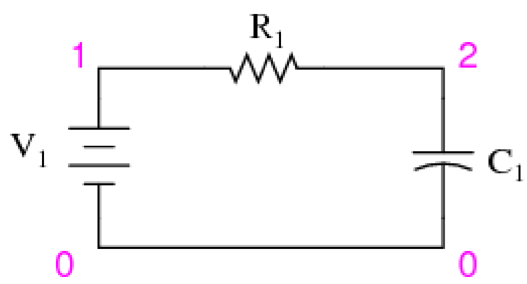
- **What is a capacitor**
  - A capacitor is a break in a circuit
  - Simplest form is a parallel plate capacitor, which consists of a pair of parallel metal plates separated by air
  - A capacitor stores energy by transferring charge from one plate to another
    - >> plates carry equal but opposite charges
- **Capacitance (symbol C, units F)**
  - The Farad F is defined as the amount of charge stored in the capacitor per volt across it
  - >> $Q = CV$
- **Calculating capacitance**
  - For simple parallel plate capacitors, for no dielectric (filling of sandwich)
  - $C = \frac{\varepsilon_0 A}{d}$
    - C = capacitance
    - $\varepsilon_0 = 8.85 \times 10^{-12}\ Fm^{-1}$ = permittivity of free space
    - A = plate areas
    - d = plate separation
    - 

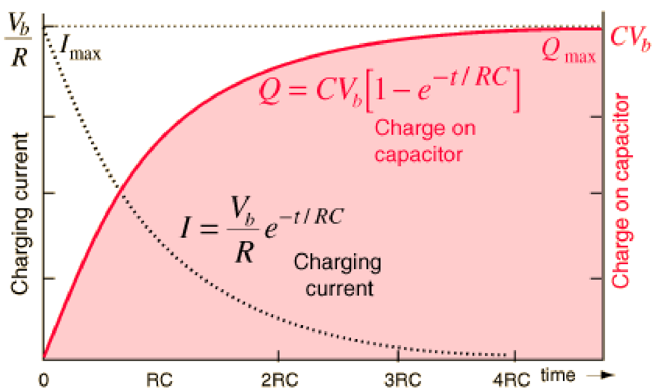
- **Dielectric**
  - Often insert a dielectric (material sandwiched between plates) which can
    - Increase the breakdown voltage
    - Increase capacitance by decreasing the strength of the electric field >> more coulombs per volt
- **Electric fields**
  - The E-field within capacitor is uniform (as field lines must be perpendicular to surface)
  - >> $E = \frac{V}{d}$
    - E = electric field strength
    - V = potential difference
    - d = separation
- **Energy stored**
  - Energy stored is area under Q-V graph

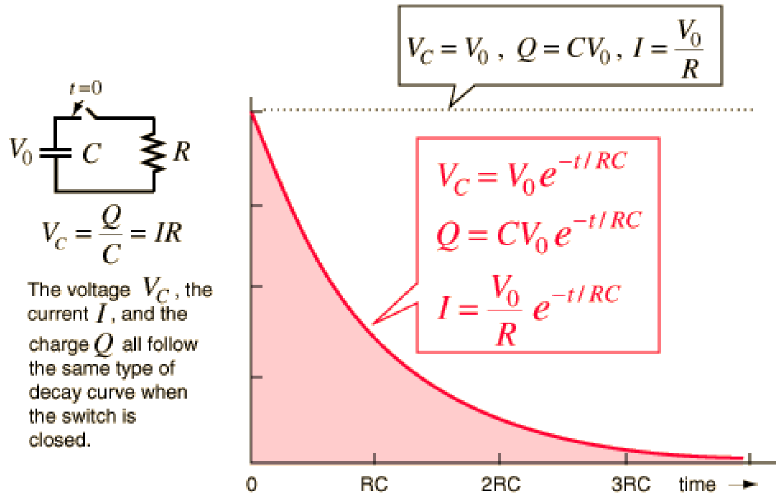


- >> $E = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$

- **Combining capacitors**
  - Opposite to resistors
  - **Series**
    - $\frac{1}{C_{tot}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots \frac{1}{C_n}$
  - **Parallel**
    - $C_{tot} = C_1 + C_2 + \cdots C_n$
  - Putting two identical caps in series >> doubling d >> halves C
  - Putting two identical caps in parallel >> doubling A >> double C
- **Charge/Discharge**
  - **Charge**
    - Constant P.D. applied >> current flows
    - But then capacitor builds up potential difference of its own in opposite direction >> effective P.D. decreases
    - >> Current decreases exponentially >> Rate of charge deposition decreases and capacitor is never 100% charged, but it gets asymptotically close



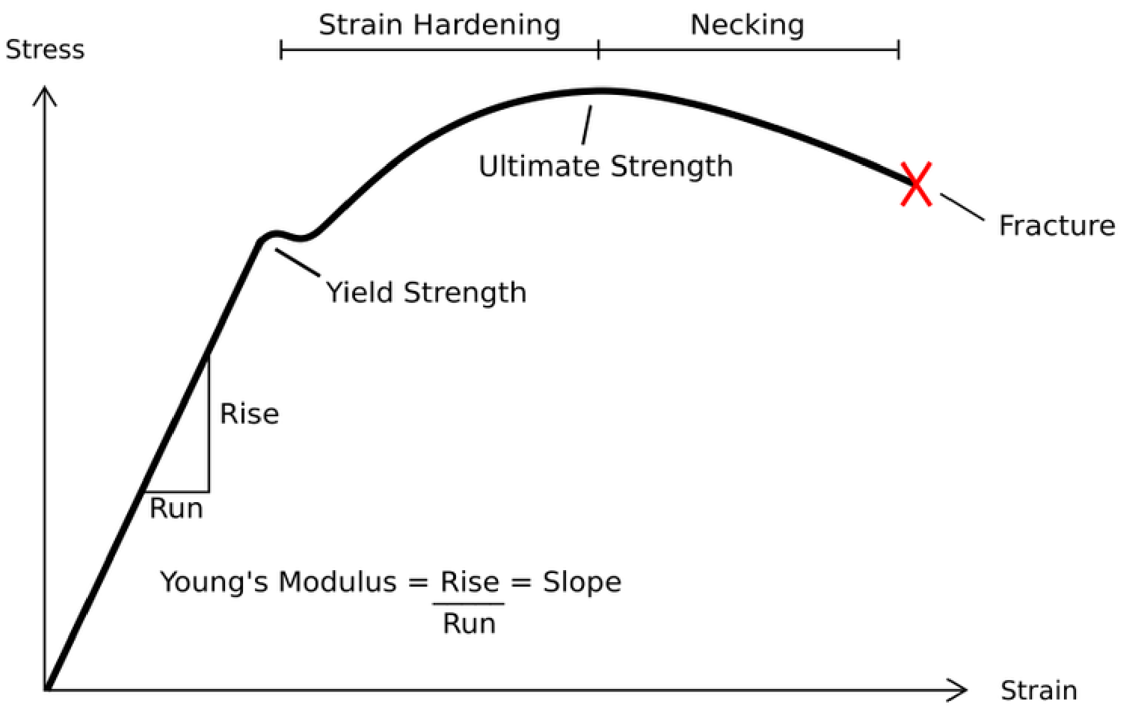


- **Discharge**
  - Same idea, as current flows, Q decreases >> V decreases >> I decays exponentially
    - 

- **Equations**
  - **Charge up**
    - $Q = Q_0\left(1 - e^{-\frac{t}{RC}}\right)$
  - **Discharge**
    - $Q = Q_0 e^{-\frac{t}{RC}}$
  - Where
    - Q = charge at time t
    - Q_0 = max capacitance
    - t = time
    - R = resistor
    - C = capacitance
  - RC is the time constant >> time taken to decrease by a factor of e or get to within 1/e of final value

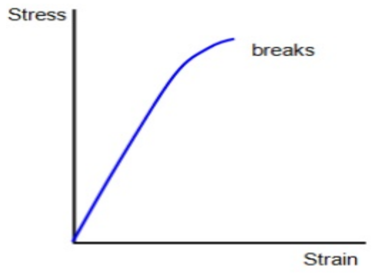
2.5 Solids under stress

- If equal and opposite forces are applied to the opposite ends of an object, its particles will be forced into new equilibrium positions with respect to one another
- 3 types of stress: *COMPRESSIVE, TENSILE & SHEAR*
- GASES cannot be put under tension because the molecules are not bound together.
- LIQUIDS cannot withstand a shear because they have no rigidity to their shape.
- **HOOKE'S LAW**
  - When an object is subject to a tensile force, it stretches., for most objects, the degree of the stretch is directly proportional to the tension.
  - **F = kx**
    - k is the spring constant; F is force; x is extension
  - If an object is subject to too much stress, it will fracture, but before that happens, the tension-extension graph ceases to be linear – entering a plastic region {elastic limit}
  - **Work done is the area under the graph**
  - $W = \frac{1}{2}Fx = \frac{1}{2}kx^2$ if Hookean
- **Stress, Strain and E**
  - Stress (σ) - Force / Cross-Sectional Area
    - $\sigma = \frac{F}{A}$
  - Strain (ε) - Change in Length / Total Length
    - $\varepsilon = \frac{\Delta l}{l_0}$
  - Young's Modulus (E) – Stress / Strain
    - $E = \frac{\sigma}{\varepsilon}$
  - Therefore,
    - $\boldsymbol{\sigma = E \times \varepsilon}$
  - Work Done = Area under stress strain graph
    - $\boldsymbol{W.D. = \int \sigma \, d\varepsilon}$
- Classification of solids
  - **Crystalline** – regular 3D repeating structure e.g. metals, diamond
  - **Amorphous** – no regular structure e.g. glass, ceramics
  - **Polymeric** – made up of monomer building blocks >> 1D repetition e.g. rubber
- **Ductile Crystalline (copper)**
  - **Structure**
    - Many metals can be drawn out into wires – they are ductile
    - Metals are **crystalline** – made of a lattice structure – containing positive ions and a sea of delocalised electrons.
    - When moving from molten to solid, crystallisation occurs in multiple points
    - => irregularities form which weaken the structure
    - In ductile metals, there are a number of irregularities within the lattice:
      - **Edge dislocation** – additional ½ plane of ions is present
      - **Point defect –** lattice ion is missing or a 'foreign atom' or just an additional ion is present
      - **Grain boundaries –** where two crystals meet
    - All these resist movement of ions >> stiffer but more brittle



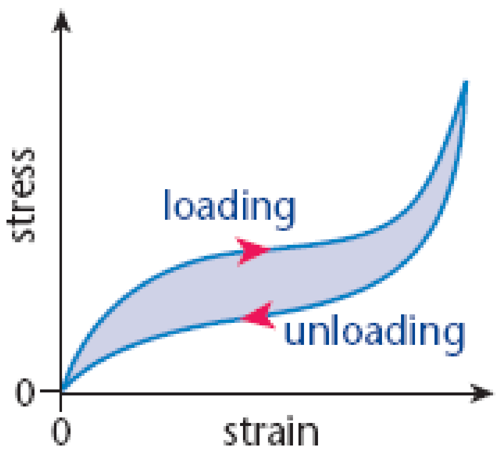
- **Properties**
  - When under low strain
    - Elastic deformation >> returns to original configuration upon removal of stress
  - Past elastic limit
    - Irreversible rearrangement of particles – **due to edge dislocations**
    - Under the influence of the forces, the placement of the force can break bonds between the bonds and reform them – with the individual ions themselves moving very minimal amounts.
    - What happens depends on other factors:
      - EDGE LOCATIONS CAN GET ENTANGLED
      - SIZE OF GRAINS
      - PRESENCE OF POINT DISLOCATIONS
  - Ductile Fracture
    - As material gets drawn out it starts to neck (reduce in CSA) >> stress increases >> ductile fracture

- **Brittle Amorphous (glass)**
  - **Structure**
    - Amorphous solids are a random arrangement of atoms >> no ability for layers to slide past one another >> brittle



- **Properties**
  - Straight until breakage
  - High Young's Modulus
  - Low Breaking strain >> brittle
  - **Brittle Fracture**
    - A small crack is present >> very high local stress >> crack widens and propagates at the speed of sound >> brittle fracture
  - **Increase Breaking stress**
    - Can remove surface imperfections by remelting very thin fibres and protecting by embedding in a resin >> fibreglass
    - Cracks only propagate under tension >> can put material under compression >> pre-stressed concrete, prince rupert's drop

- **POLYMERS**
  - **Rubber**
    - Molecule can rotate around each of the single C-C bonds – so the shape of rubber molecules is randomly tangled.
    - Under tension, molecule is straightened out >> Low Young's Modulus – since the bonds are not being stretched, simply rotated
    - The molecules want to kink back up as that has higher entropy (being hit in the side by random collisions) >> called thermal opposition
    - Some energy is lost as thermal energy through random intramolecular collisions
  - **Hysteresis**
    - In rubber, loading and unloading curves are different >> known as hysteresis
    - Therefore, the area between the curves represents the mechanical energy loss in the cycle => converted into thermal energy

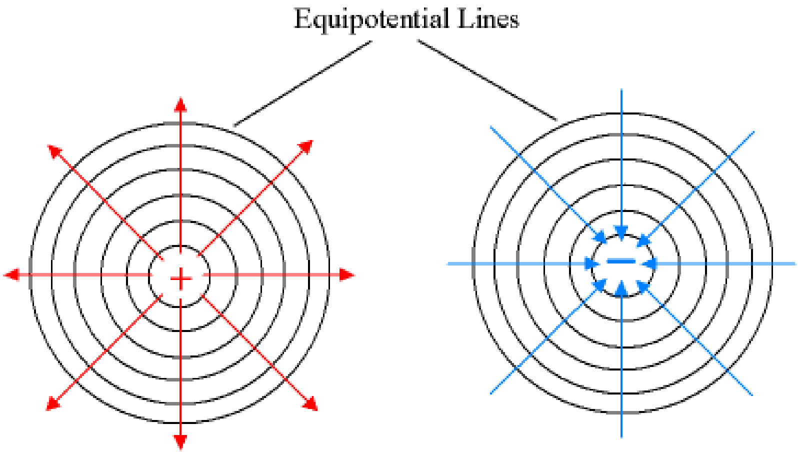


2.6 Electrostatic and Gravitational Fields

- Memorise

ELECTRIC FIELDS	GRAVITATIONAL FIELDS
Electric field strength, E, is the force per unit charge on a small positive test charge placed at the point	Gravitational field strength, g, is the force per unit mass on a small test mass placed at the point
Inverse square law for the force between two electric charges in the form $F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2}$ (Coulomb's law)	Inverse square law for the force between two masses in the form $F = G\frac{M_1M_2}{r^2}$ (Newton's law of gravitation)
F can be attractive or repulsive	F is attractive only
$E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}$ for the field strength due to a point charge in free space or air	$g = \frac{GM}{r^2}$ for the field strength due to a point mass
Potential at a point due to a point charge in terms of the work done in bringing a unit positive charge from infinity to that point	Potential at a point due to a point mass in terms of the work done in bringing a unit mass from infinity to that point
$V_E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$ and $PE = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r}$	$V_g = -\frac{GM}{r}$ and $PE = -\frac{GM_1M_2}{r}$
Change in potential energy of a point charge moving in any electric field $= q\Delta V_E$	Change in potential energy of a point mass moving in any gravitational field $= m\Delta V_g$
Field strength at a point is given by E = - slope of the $V_E - r$ graph at that point	Field strength at a point is given by g = - slope of the $V_g - r$ graph at that point
Note that $\frac{1}{4\pi\varepsilon_0} \approx 9 \times 10^9\,\mathrm{F^{-1}m}$ is an acceptable approximation	

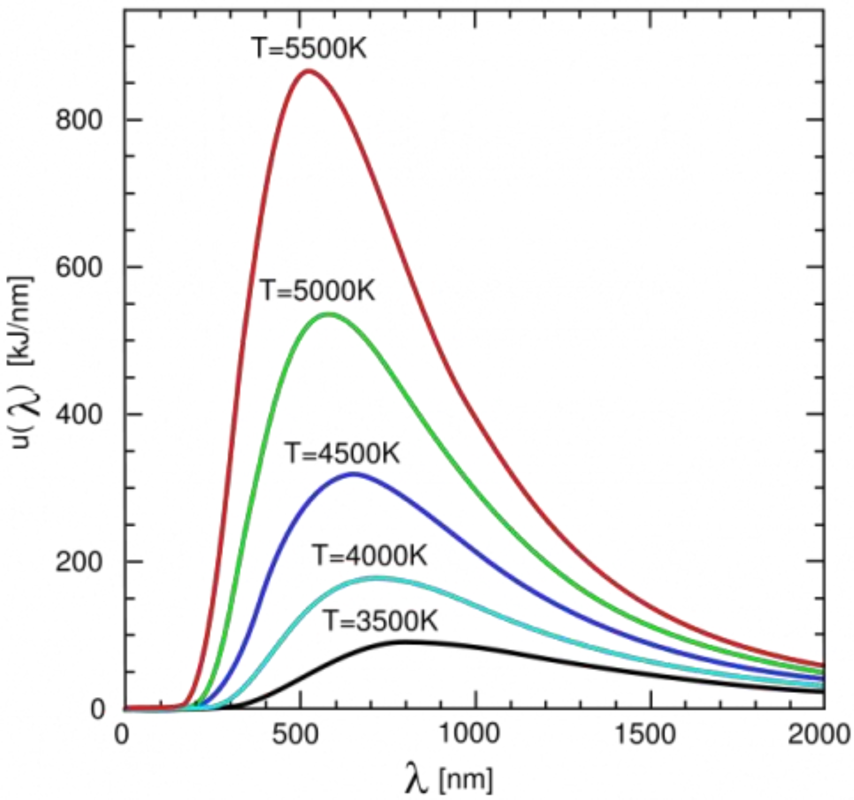
  -
  - $\varepsilon_0 = 8.85 \times 10^{-12}\ Fm^{-1}$ = permittivity of free space
- **Shortcuts for G-fields**
  - The G-field outside spherical bodies (e.g. planets) is the same as if all mass were concentrated at the centre
  - If variation in g is negligible
    - >> $\Delta U = mg\Delta h$
- **E-Field lines**
  - Give the direction of the field at a point >> give direction of force that would act on a +ve test charge. Field lines aka lines of force.
    - for an isolated +ve point charge field lines point radially outwards
    - Opposite is true for -ve charges (filed lines inwards)
- **Equipotentials**
  - Lines/surfaces joining points of equal potential
  - Must cross field lines at right angles, as field line act along changes of potential
  - >> for a point charge, equipotential surfaces are spherical



- **How to calculate net potential and resultant field strength**
  - Potential = scalar
    - >> total potential at a point is the simple sum of all potentials due to charges around it
    - >> $V_{tot} = \sum V_i$
  - Field strength = vector
    - >> resultant field strength is the vector sum of all field strengths
    - >> $\boldsymbol{E_{res}} = \boldsymbol{E_1} + \boldsymbol{E_2} + \cdots \boldsymbol{E_n}$ Vectors

2.7 Using radiation to investigate stars

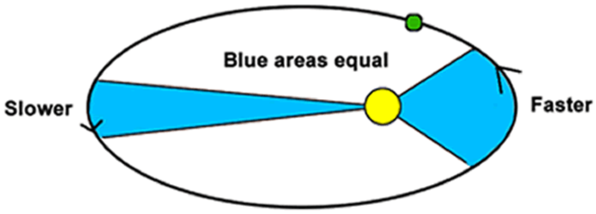
- **Stellar spectrum**
  - Consists of
    - a continuous emission spectrum (blackbody) from the dense gas at the surface of the star
    - & a line absorption spectrum from when the light passes through the star's atmosphere and gets absorbed in places
- **Blackbodies**
  - Something that absorbs all incident radiation >> any radiation coming from it is emitted by itself
  - Stars are very good approximations to blackbodies as they are very bright and isolated
  - **The Blackbody spectrum**
    - Shape:



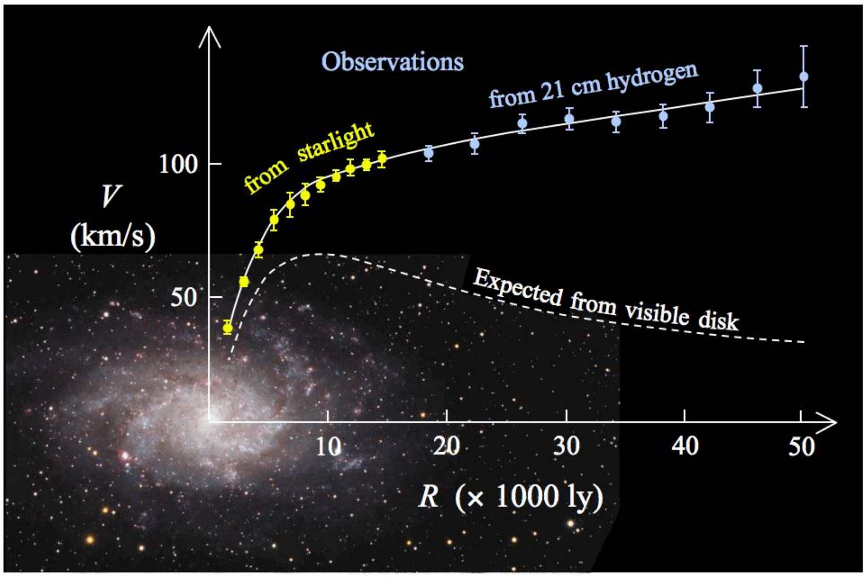
  -
- **Investigating properties of stars**
  - **Wien's displacement law**
    - Peak value of wavelength in BBS inversely proportional to absolute temperature
    - >> $\lambda_{max} \propto \frac{1}{T}$
    - Precise statement:
    - $W = \lambda_{max} T$
      - W is Wien's constant = $2.90 \times 10^{-3}\ mK$
  - **Stefan-Boltzmann Law**
    - Aka Stefan's law
    - Emission power from a blackbody proportional to 4th power of absolute temperature
    - $P = A\sigma T^4$

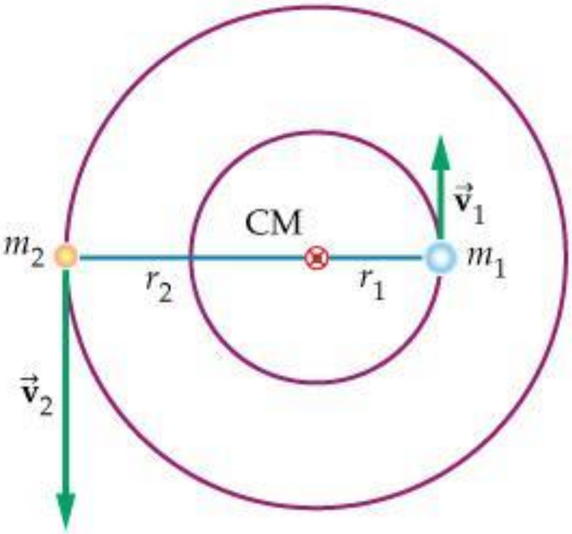
        - P = emissive power
        - A = area of object
        - $\sigma = 5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$= Stefan-Boltzmann constant
- **Inverse square law**
    - Intensity (power per unit area) drops off with the inverse square of distance from emitter
    - $I \propto \frac{1}{r^2}$
        - I = intensity
        - r = separation
    - Useful relation, when distance r from star of power P, intensity given by
    - $I = \frac{P}{4\pi r^2}$
- Must be able to apply the above three to investigate properties of stars
    - Luminosity – Total power emitted P = **L**
    - Size – often Surface Area of star **A**
    - Temperature – surface temperature of star **T**
    - Distance – distance from star to observer **r**
- **Multiwavelength astronomy**
    - Different processes emit photons of different energies (wavelengths) >> investigating different wavelengths gives us a wider window

2.8 Orbits and the wider Universe

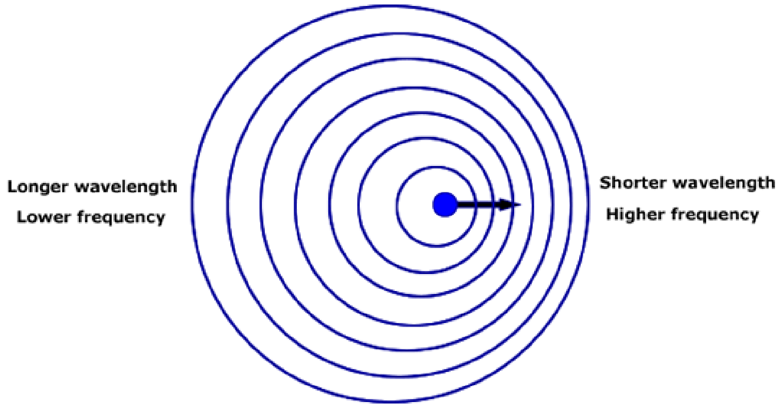
- **Kepler's laws**
  - **K1:** all orbits are elliptical
  - **K2:** a radial line joining planet to sun sweeps out equal area in equal time >> vr constant
    - 

  - **K3:** $T^2 \propto r^3$
    - T = orbital period
    - r = orbital radius
- **Newton's Law of gravitation**
  - $F = \frac{GMm}{r^2}$
    - F = force
    - $G = 6.67 \times 10^{-11}\ N\ m^2\ kg^{-2}$ = universal gravitational constant
    - M = mass of body A
    - m = mass of body B
    - r = separation
- **Derivation of K3**
  - Assume circular orbit
  - >> can derive From Newtons' law of gravitation and centripetal acceleration
    - Equate two forces
    - $\frac{GMm}{r^2} = m\omega^2 r$
    - $\left(\frac{2\pi}{T}\right)^2 = \frac{GM}{r^3}$
    - $T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$
    - $T^2 \propto r^3$
- How to use data on orbital motion (such as period or orbital speed) to calculate mass of central object
- **Dark matter**
  - Galaxy rotation curves do not behave like they should, according to the visible matter that is there
  - >> there must be some dark matter to contribute extra mass and gravity



- **Higgs Boson**
  - It is the field that gives objects mass
  - Recently discovered
  - Could be related to dark matter as it is this invisible thing that is related to mass
- **Mutual Orbits**
  - Be able to calculate CoM and orbital period given masses and separation (d) of both bodies >> consider frame in which CoM is not moving
  - 

  - **Centre of Mass:** consider moments about each planet
    - $r_1 = \frac{m_2}{m_1+m_2} d$
    - $r_2 = \frac{m_1}{m_1+m_2} d$
  - **Orbital period:** can derive by resolving on m1
    - $\frac{Gm_1m_2}{d^2} = m_1\omega^2 r_1$
    - $\frac{Gm_1m_2}{d^2} = \frac{m_1m_2d}{m_1+m_2}\left(\frac{2\pi}{T}\right)^2$
    - $\left(\frac{T}{2\pi}\right)^2 = \frac{d^3}{G(m_1+m_2)}$
    - $T = 2\pi\sqrt{\frac{d^3}{G(m_1+m_2)}}$

- **Doppler Shift**
  - Caused by relative motion between source and observer
  - 

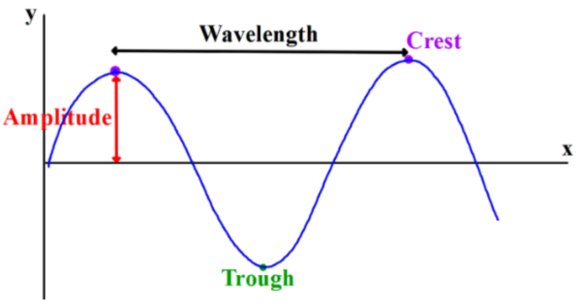
  - $\frac{\Delta\lambda}{\lambda_0} = \frac{v}{c}$
    - v = relative speed
    - c = speed of light
    - λ = wavelength
  - Know how to use Doppler shift to find radial velocity of stars (component of velocity along line from star to observer)
  - Use data on the variation of radial velocities in a binary system (e.g. star and planet) and period to determine masses of bodies, in the case of and edge on circular orbit
- **Hubble Constant**
  - The universe is expanding (dark matter?) due to an expansion of space itself
  - The Hubble constant H_0 is the current rate of expansion
  - >> an object 1m away is moving at H_0 away from you
  - $v = H_0 D$
    - v = speed away from observer
    - D = separation
  - >> if an object at 1m away is moving at H_0, then assuming the speed has not changed much >> age of the universe must be approximately $\frac{1}{H_0}$ >> then everything must be concentrated at the same point >> big bang follows
- **Critical densities**
  - Consider energy of an object of mass m at the edge of a circular universe with uniform mass distribution
  - The object is moving away from us within a gravitational potential
  - To get potential energy (-ve)
    - $P.E. = -\frac{GMm}{r}$
    - & $M = \frac{4}{3}\pi r^3 \rho$
    - >> $P.E. = -\frac{4}{3}\pi G m r^2 \rho$
  - To get kinetic energy
    - $K.E. = \frac{1}{2}mv^2$
    - & $v = H_0 r$
    - >> $K.E. = \frac{1}{2}mr^2 H_0^2$
  - Therefore, total energy:
    - $E = \frac{4}{3}mr^2\left(\frac{3}{8}H_0^2 - G\pi\rho\right)$

        - At critical density, E = 0
        - >> $\boldsymbol{\rho_c = \frac{3H_0^2}{8\pi G}}$
    - If density is {blah} then critical density then
        - **Greater:** P.E. > K.E. => universe collapses => Big Crunch
        - **Equal:** P.E. = K.E. => universe reaches a fixed size and stops (SHM) => steady state
        - **Less:** P.E < K.E. => universe carries on expanding forever => Big Rip

3 Light, Nuclei and Options

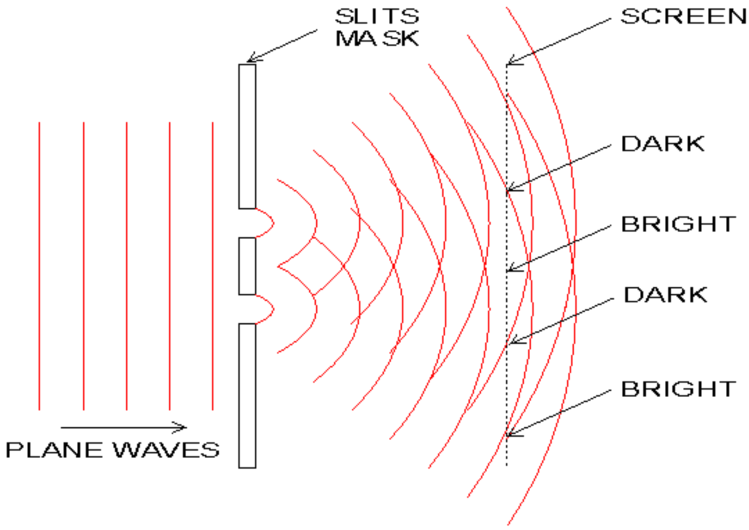
3.1 Nature of waves

- **Progressive Wave**: an oscillation that transfers energy but not matter
    - **Transverse Wave** – direction of oscillation perpendicular to propagation
    - **Longitudinal Wave** – direction of oscillation parallel to propagation
- <u>Polarisation</u>
    - **Polarised** – oscillations only in one plane,
    - **Unpolarised** – oscillations in multiple planes
- <u>Definitions</u>
    - **Cycle:** Smallest portion of an oscillation, starting at any point, which repeats exactly
    - **Amplitude:** Maximum value of the displacement
    - **Period:** Time for one cycle of oscillation
    - **Frequency:** The number of cycles per unit time
    - **Phase:** The position in the cycle (expressed as an angle)
        - **In phase:** at the same stage of the cycle always
        - **Antiphase:** half a cycle out pf phase
    - **Wavelength:** Distance between consecutive particles that are oscillating in phase
    - **Wavefront:** Line connecting all points of a wave cycle that are in phase
    - **Coherent:** All oscillations are in phase
    - **Monochromatic:** Single frequency
- **Wave Equation**
    - $\boldsymbol{v = f\lambda}$
- Direction of propagation is perpendicular to wave fronts



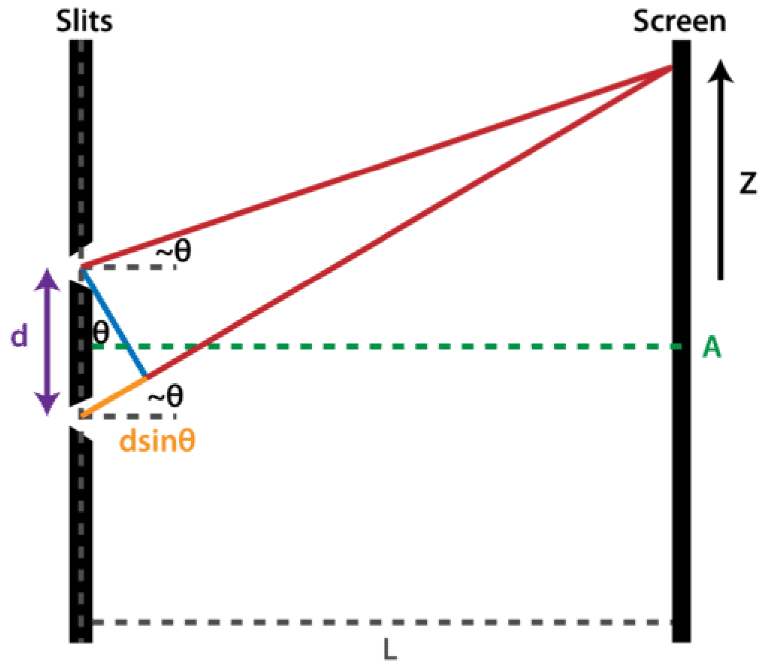
3.2 Wave properties

- **Diffraction**
  - Spreading of waves around obstacles in their way.
  - Maximum diffraction when obstacles are of same size as wavelength
  - >> semi-circular wavefronts
  - Very little diffraction if wavelength << size of object
- **Superposition**
  - **Principle of Superposition:** The resultant displacement at a point is the vector sum of all the displacements of the waves at that point.
  - Therefore
    - In phase waves constructively interfere => increase in intensity
    - Out of phase destructively interfere => decrease in intensity
  - **Constructive interference**
    - For this, waves must be in phase
    - => assuming equal frequency
    - => path difference = whole number of wavelengths
    - => $P.D. = n\lambda$
- **Young's Slits Experiment**



  - $\lambda = \frac{ay}{D}$
  - Where
    - a is the distance between the slit and the screen
    - y is the vertical distance between the midpoint and the maximum
    - D is the distance between the slits
  - This relies on the small angle approximation
  - => assumes a >> D
  - Historical importance: light can behave as a wave

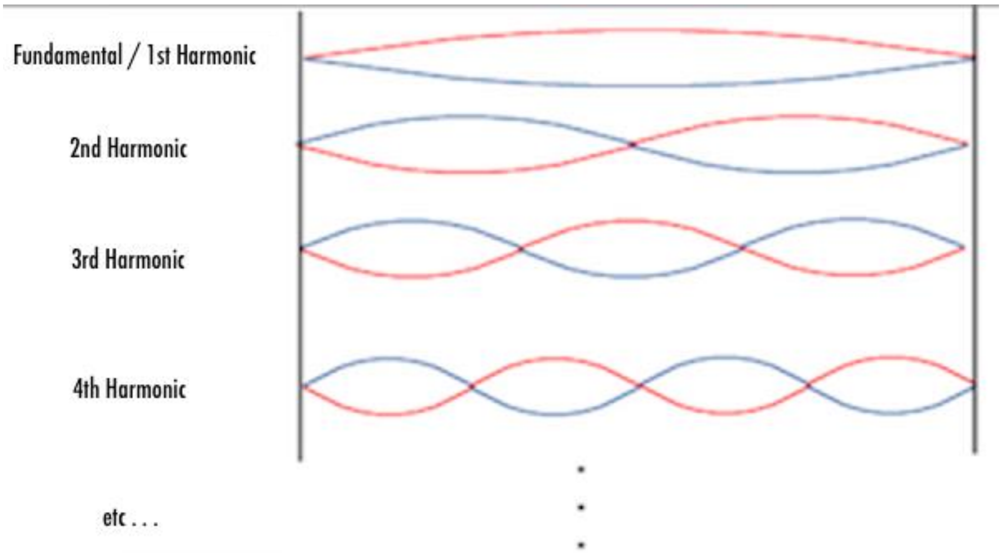
- **Diffraction Grating**



  - $n\lambda = d\sin(\theta)$
  - Where
    - n is the order of the maxima
    - λ is the wavelength
    - d is the distance between each line
    - θ is the angle, measured perpendicular to the grating
  - Small d >> maxima further apart than in Young's
  - Large number of slits, make maxima sharper
- **Coherence**
  - Coherent source:
    - All of beam is in phase
    - Monochromatic (same wavelength)
    - Wavefronts continuous across width of beam
  - Coherent source: laser, led
  - Incoherent source: light bulb
- For two source-interference to occur, the sources must have a constant phase difference and have oscillations in same direction
- **Stationary Waves**
  - Opposite to progressive waves => no energy transfer
  - In a stationary wave, all points between a pair of neighbouring nodes oscillate in phase
    - In a progressive wave phase changes with position
  - In a stationary wave, the amplitude of vibration varies smoothly from zero, at the nodes, to a maximum, at the antinodes
    - In a progressive wave, all points oscillate with the same amplitude
  - Nodes are ½ wavelength from each other.
  - **Formation of stationary waves:**
    - Energy is put into forming progressive waves
    - A progressive wave is produced, which travels along the string

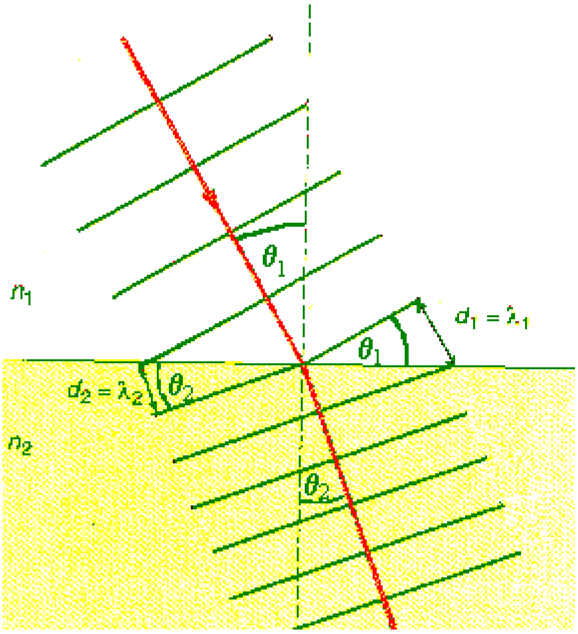
- The waves reflect at the end
- Reflected wave interferes with the original to produce a standing wave

- **Harmonics:**
  - 1st Harmonic: $\frac{1}{2}\lambda$ (i.e. half a wavelength is seen)
  - 2nd Harmonic: λ
  - 3rd Harmonic: $\frac{3}{2}\lambda$
  - Etc

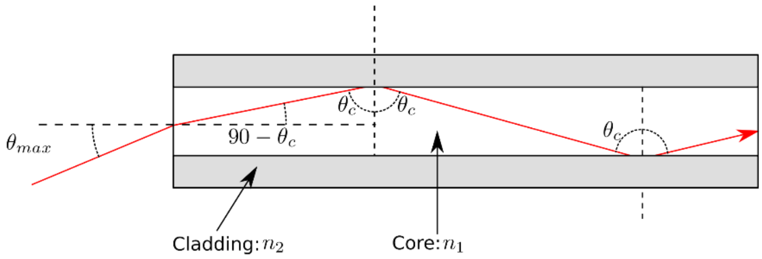


- **Spectrometer**
  - Instead of simply using a diffraction grating allows the person to get a very high angular resolution.
  - Non-coherent light can also be used.
  - The spectral lines are very narrow as the collimator is effective at producing narrow slits.
  - It isolates light from the outside.

3.3 Refraction

- Waves change speed when transitioning between media (of different optical density)
- Therefore they change direction also
- Snell's law:
  - $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$
- Calculating refractive index
  - n is ratio of speed of light in vacuum, compared to medium
  - n > 1
  - $n = \frac{c}{v}$
  - $\gg n_1 v_1 = n_2 v_2 = c$
- Physical analogy
  - 

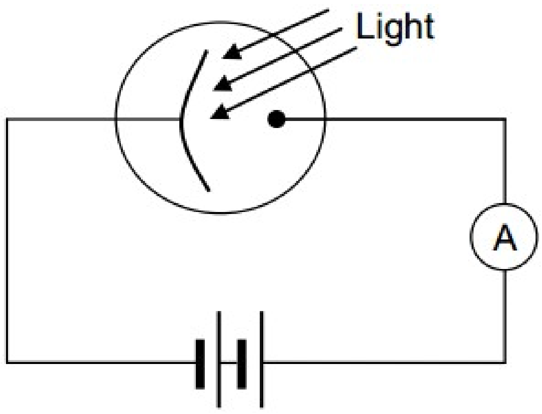
- **Total Internal Reflection**
  - To find critical angle, sub in 90 degrees for θ_2
  - Therefore,
    - $n_1 \sin(\theta_c) = n_2$
    - $\sin(\theta_c) = \frac{n_2}{n_1}$
  - **Uses of TIR**
    - Totally Reflecting Prisms
      - Binoculars, Microscopes and Prisms
    - Optical Fibres
      - Used for data transmission (LAN and longer distance)
      - Used also in remote imaging systems – endoscopy
      - **Structure**
        - SINGLE GLASS THREAD – contains a core containing the light signal and the cladding, which keeps the light signal in the core.
        - Around this, there is a coating – of about 0.25mm
        - Cable consists of lots of these fibres.

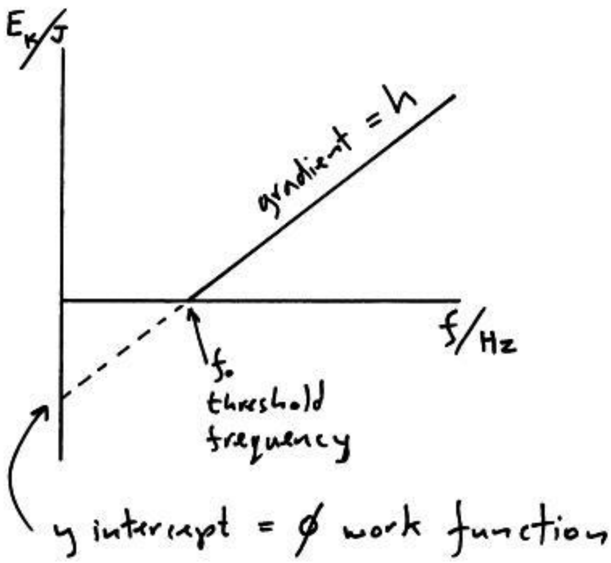
○ 

- Relies upon the core having a much higher refractive index than the cladding.
- **Multimode fibres**
  - In multimode fibres, signals can be sent at different angles, including a number of light rays at the same time.
  - However, this suffers the problem of having a time difference, since something at a shallower angle would move further in the same time than a steeper angle.
  - This is a large issue when the distances become much larger **(MULTIMODE DISPERSION)** – therefore use of monomode fibres.
  - **Multimode dispersion** – broadening of signal as beams take different paths and so travel further >> limits baud rate
- **Monomode fibres**
  - The core diameter is made very small - <10um – as this ensures that the light wave cannot take multiple paths and is forced to travel parallel to the axis of the fibre.
  - Therefore, there are no problems with multimode dispersion and baud rate can be increased

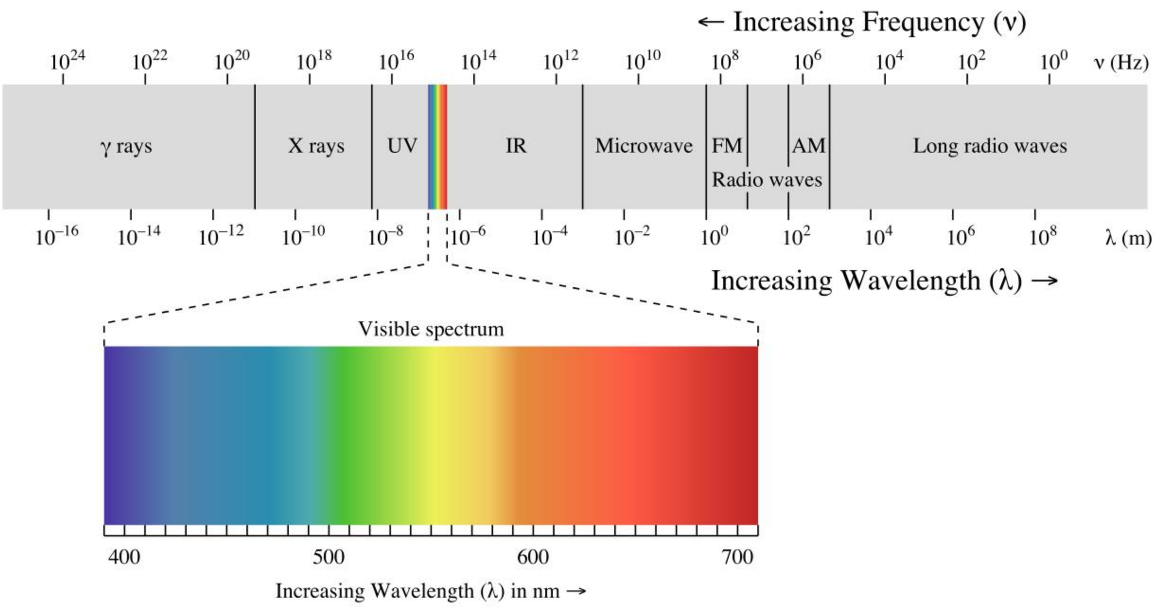
3.4 Photons

- Light consists of discrete packets of energy (photons)
- Einstein showed the energy of photons was proportional to the frequency of the light, with the constant of proportionality being the Planck Constant (6.63 x 10^{-34})
  - $\boldsymbol{E = hf}$
- The photoelectric effect provides evidence for the particle nature of light (in discrete energy packets – photons)
- **PHOTOELECTRIC EFFECT**
  - Experimental setup:

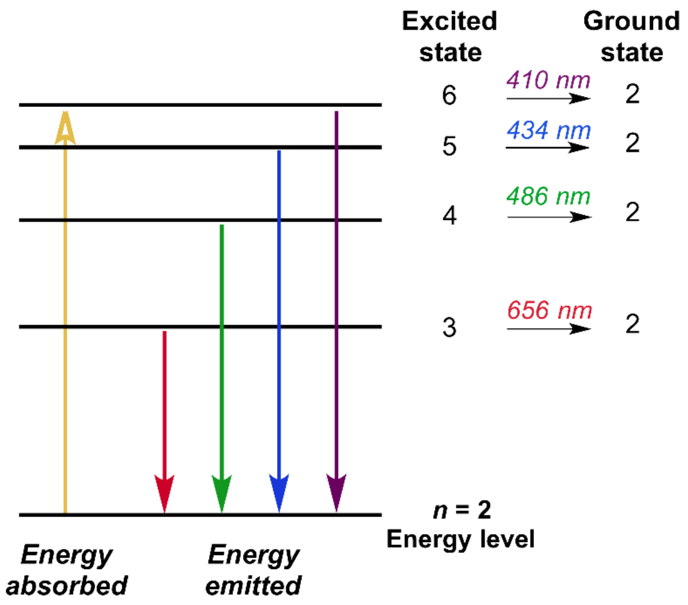


-
    - Alter voltage across photocell until current is just 0 >> get stopping voltage >> know maximum K.E. of electrons
  - Shows that light consists of discrete packets of energy (photons)
  - These photons have energy: $E = hf$
  - $h = 6.63 \times 10^{-34} Js$ = Planck's constant
  - When a photon impacts the surface it interacts with a single electron
  - This electron absorbs the photon => gaining its energy
  - There is a minimum energy which is needed to remove an electron from a specific surface (called the work function – ϕ). The remainder is kinetic energy of the electron.
  - **Einstein's Photoelectric equation**
    - $K.E._{max} = hf - \phi$
    - Where K.E. is the kinetic energy of the released electron
    - 

- **WAVE-PARTICLE DUALITY**
  - **LIGHT**
    - Wave
      - Diffracts
      - Refracts
      - Interference
    - Particle
      - Emission and absorption of light appears to show light as a particle
      - Photoelectric effect shows light consists of discrete packets
      - Can travel without a medium – travel through a vacuum.
      - Instead of particles oscillating as an EM wave travels, it is the Electric Field Strength and the Magnetic Field Strength that oscillate at each point
- **EM SPECTRUM** - LEARN



- **ATOMIC SPECTRA**
  - **Atomic Energy Levels**
    - Atoms can only take a discrete set of energy states – corresponding to the energy levels of the electron(s)
    - Conventionally, electrons outside of an atom have 0 energy
      - In the atom, they have a negative energy, since energy must be given to the electron to allow it to escape the atom
      - GROUND STATE is the minimum (most negative) energy state of the electron
      - The negative value of the ground state is known as the **ionisation energy** of the atom (energy to rip off electron)

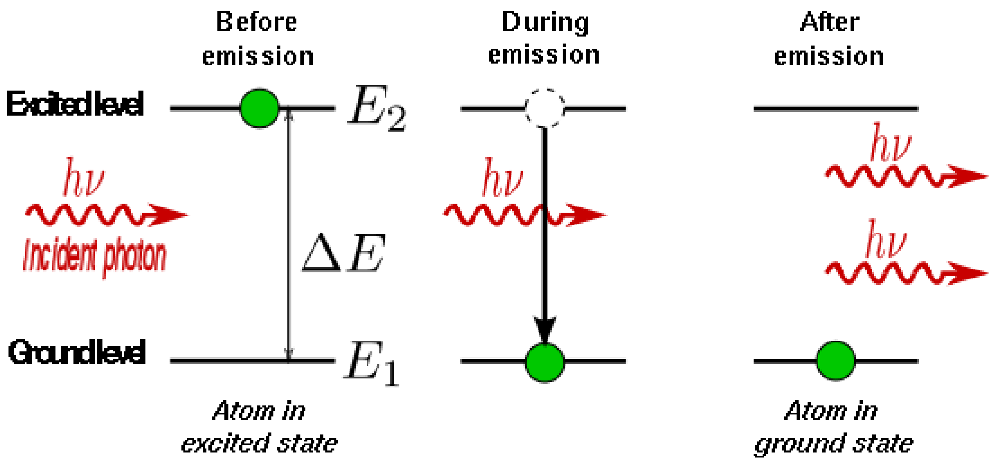


- Atomic Absorption Spectra
  - The light which can be absorbed by atoms correspond to the various differences in energy levels between the different energy levels of the atom.
- Atomic Emission Spectra
  - The photons emitted have energy equal to the energy difference between energy levels >> absorption and emission spectra are symmetric
- The atomic spectra can be viewed by using a gas discharge tube – with the ability for a potential difference to be applied to the tube – this partially ionises the gas and allows it to be raised to a higher energy level – and then drop down (when voltage is removed), hence emitting the colours of the Atomic Emission Spectra
- Hence the formation of the Aurorae (Borealis and Australis)
- To query the spectra can put through a diffraction grating or a prism >> separates wavelengths
- Electrons can also diffract. In fact all matter has a wave-like aspect

- **MOMENTUM**
  - **De Broglie Wavelength**
    - Applies to any object
    - $\lambda = \frac{h}{p}$
      - p is momentum
      - h is Planck's Constant
  - **Momentum of Photons**
    - $p = \frac{h}{\lambda} = \frac{hf}{c} = \frac{E}{c}$
  - **Radiation pressure**
    - Know how to calculate given intensity $W\ m^{-2}$
    - >> get number of photons per second
    - Multiply by photon momentum
    - Multiply by two if reflected rather than absorbed

3.5 Lasers

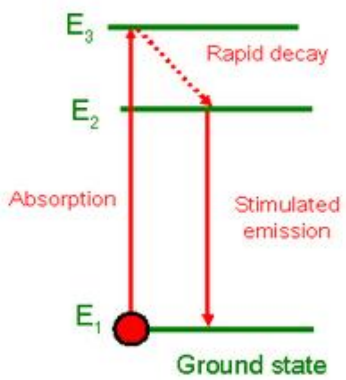
- **L**ight **A**mplification by **S**timulated **E**mission of **R**adiation
- **STIMULATED EMISSION**



$$E_2 - E_1 = \Delta E = h\nu$$

Note v = f

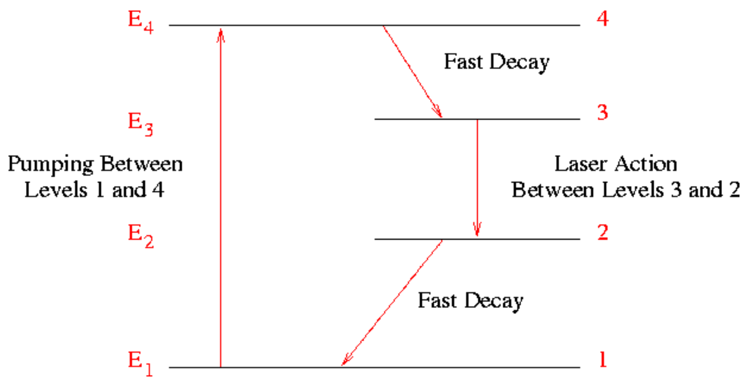
  - Atom in an excited energy state
  - Incident photon stimulates it to drop an energy level
  - Energy difference is used to create a photon
  - of equal energy (frequency) that is in phase
  - Assuming population inversion, this can go on to start a chain reaction
  - => produces intense and coherent light
- **ACHIEVING A POPULATION INVERSION**
    - **Population Inversion –** when the number of atoms on upper energy states is greater than the number of atoms in the ground state.
    - The PI is required for a chain of stimulated emissions to occur
    - Achieving the population inversion is known as **PUMPING**
  - **Three-State Laser Systems**



      - E3 = Pumped State (P)
      - E2 = Upper State (U)
      - E1 = Ground State (G)

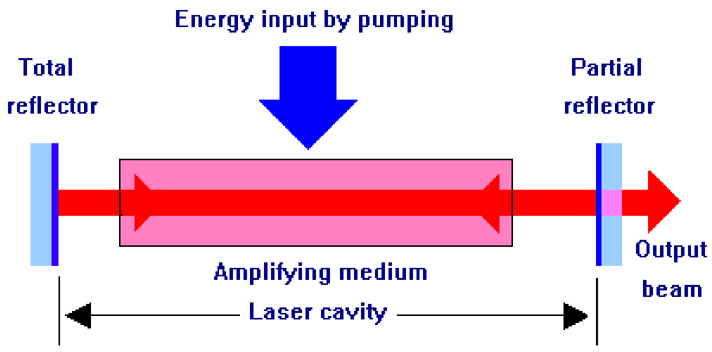
- Optical Pumping involves raising the atoms from the ground state to the Pumped state
- Once in this pumped state, they decay rapidly to the Upper state, ready for Stimulated Emission
- Pumping is done in many ways:
  - using photons of Energy $(E_P - E_G)$
  - electrical
  - chemical
  - nuclear
- If the pumping is fast enough, the number of atoms in the U state will be greater than the number in the G state, therefore achieving PI.
- However, this is highly inefficient, and so, mostly, four state laser systems are used.

- **FOUR STATE LASER SYSTEMS**



- In this, E2 will always be empty, since there is a fast decay to E1
- Therefore, population of E3 > E2
- Therefore, there is a higher probability of stimulated emission, rather than the incident photon being used for further pumping

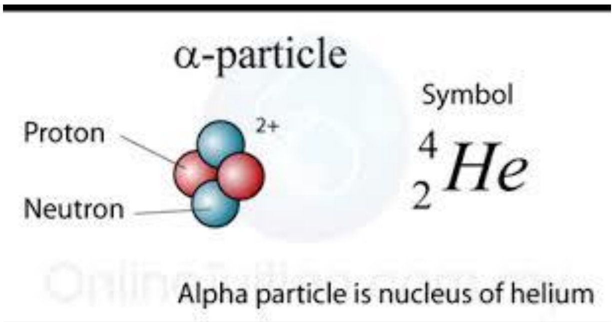
- **LASER CONSTRUCTION**

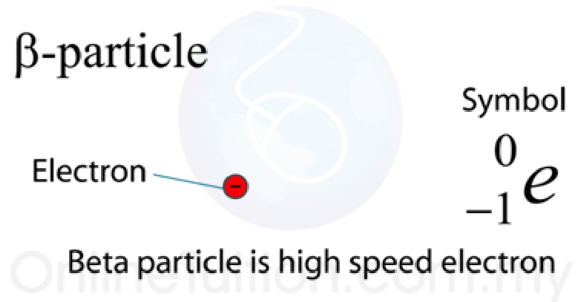


- Amplifying medium for stimulated emissions between two mirrors
- One mirror is totally reflective, other is 95% reflective
- Therefore, light passed back and forth through amplifying medium several times until building enough intensity to emit at a steady intensity through the partial reflector.

- There exist semiconductor lasers, which are small, efficient and rapidly switching >> used in CD/DVD readers and optical fibres

3.6 Nuclear Decay

- Spontaneous
    - Nuclear decay is spontaneous
    - 3 main types:
        - **Alpha α**
            - Helium nucleus
            - 2 protons, 2 neutrons
            - Mass of 4, Charge of +2
            - 

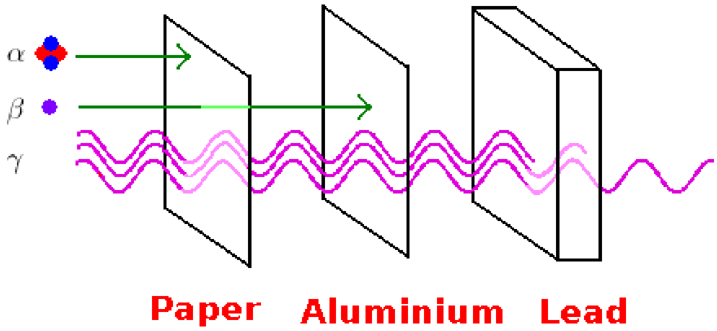
        - **Beta β**
            - β^- is an electron | β^+ is a positron (for A-level only electron is needed
            - Negligible mass, Charge of -1
            - 

        - **Gamma γ**
            - Very high frequency (energy) photons (E.M. radiation)
- **Nuclear transformation notation**
    - ^A_ZX
        - A = mass (in u)
        - Z = charge (in e)
        - X = type of particle
    - In nuclear equations, must conserve mass-energy & charge
    - Example decays:

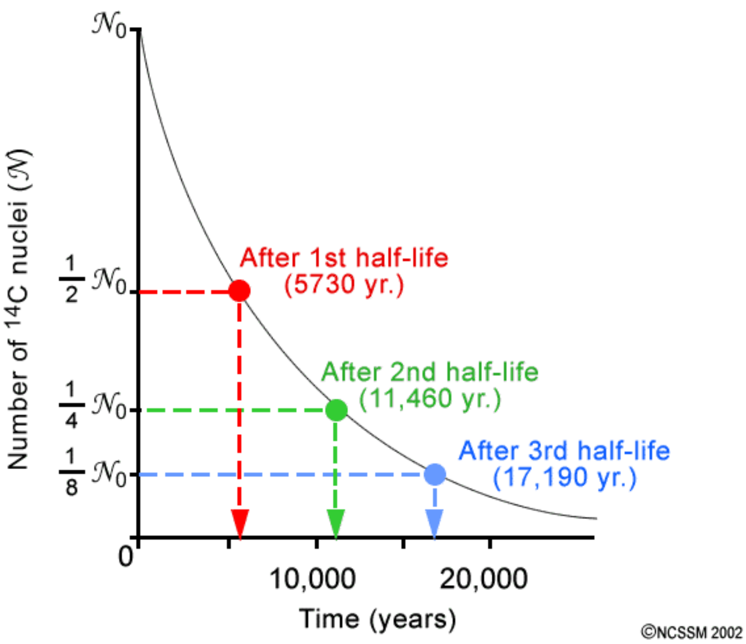
$$^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\alpha$$

$$^{234}_{90}\text{Th} \rightarrow {}^{234}_{91}\text{Pa} + {}^{0}_{-1}\beta + {}^{0}_{0}\gamma$$

$$^{234}_{91}\text{Pa} \rightarrow {}^{234}_{92}\text{U} + {}^{0}_{-1}\beta + {}^{0}_{0}\gamma$$

- **Penetration**
    - Alpha particles have the highest charge and move the slowest >> ionize (rip electrons out of shells) surroundings very quickly >> lose energy and are absorbed
    - Beta particles are more penetrating
    - Gamma particles are the most penetrating as have no charge >> seldom interact
    - Absorption
        - Alpha – few cm of air
        - Beta – few mm of aluminium
        - Gamma – few m of lead
    - 

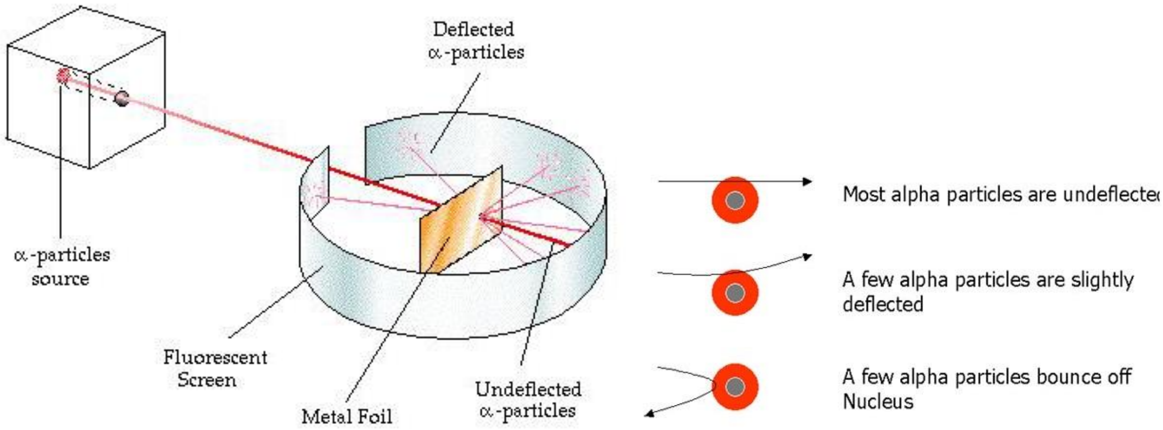
- **Background Radiation**
    - There exists background radiation from
        - radioactive gases such as Radon
        - Radioactive minerals such as those found in granite
        - Cosmic radiation
    - In any nuclear decay experiment must subtract background count from that measured when the source is in place
- **Half life**
    - Nuclear decay is exponential >> any atom has a fixed probability to decay within a given period of time >> for a large number of atoms this averages out to a very nice exponential decay curve
        - Half-life $\left(T_{\frac{1}{2}}\right)$ is the time taken for half the atoms in the sample to decay



- **Activity (A)**
  - This is the number of decays per second
  - Can be measured easily with a Geiger counter
  - Units of Becquerel (Bq) = 1 decay / s
  - $A = \lambda N$
  - Where λ is the decay constant (probability of an atom decaying in a unit of time)
- **Maths – exponentials**
  - **Population**
    - $N = N_0 e^{-\lambda t}$
      - N = population at time t
      - N_0 = original population at t = 0
      - λ = decay constant
  - **Activity**
    - $A = -\frac{dN}{dt} = \lambda N_0 e^{-\lambda t} = \lambda N$
    - And at t = 0, $A_0 = \lambda N_0$
    - >> $A = A_0 e^{-\lambda t}$
  - **Decay Constant from half life**
    - Let half-life = T
    - At t = T >> $N = \frac{1}{2} N_0$
    - >> $e^{-\lambda T} = 1/2$
    - >>$\lambda T = \ln 2$
    - $\lambda = \frac{\ln 2}{T}$

3.7 Particles and Nuclear Structure

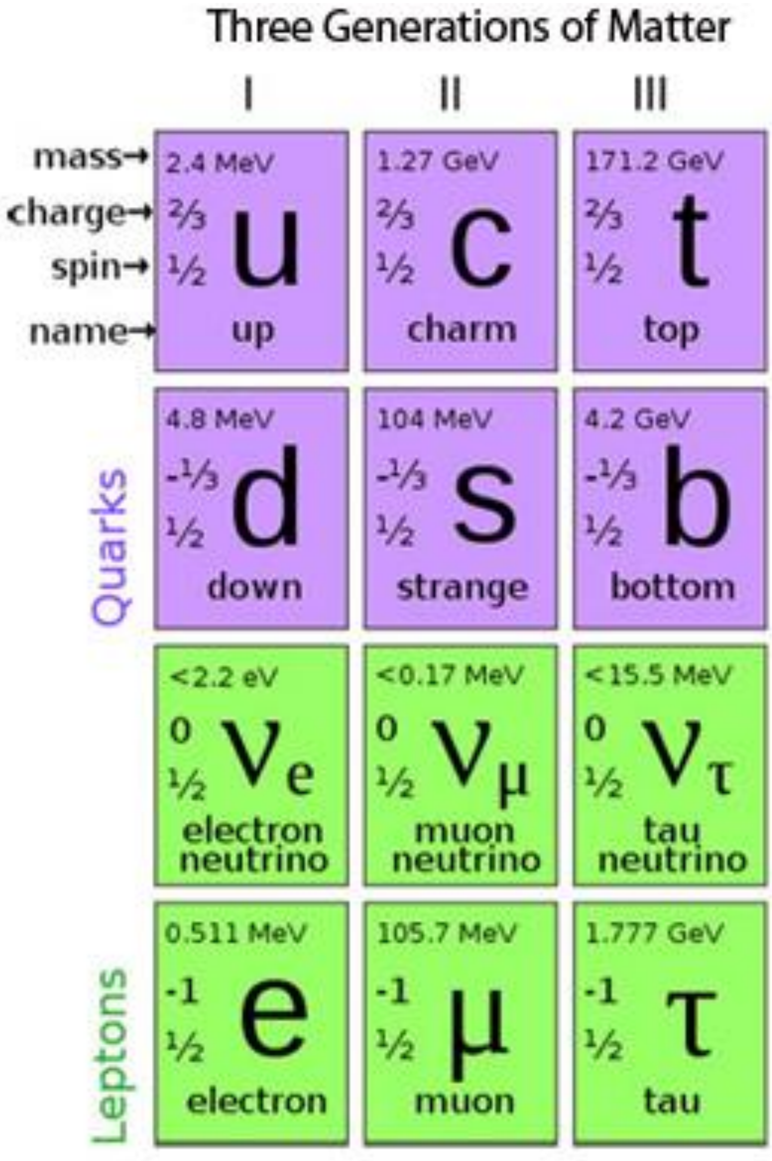
- Rutherford alpha scattering



  - Results highlight that the atom consists of a dense central positive charge (nucleus) surrounded by a sparse negative electron cloud
  - To estimate maximum coulomb repulsion force:
    - Assume head on collision >> CoE
    - $\frac{1}{2}mv^2 = \frac{kQq}{r}$
    - >> find r (this r is much less than the radius of the atom >> plum pudding not valid)
    - $F_{max} = \frac{kQq}{r^2} = \frac{mv^2}{2r}$
- Elementary particles
  - **LEPTON**
    - Low mass, elementary particles – e.g. electron, neutrino
  - **QUARK**
    - Elementary particle, not found in isolation, which combines to form hadrons
- Antiparticles
  - For every particle there exists a particle of equal mass, opposite charge >> a particle and its anti-particle annihilate upon contact into energy (photons)
  - Denoted with a bar on top
- HADRON
  - Particles consisting of multiple quarks or anti-quarks
  - Quarks are never observed in isolation
  - **BARYON**
    - Baryon has 3 quarks
    - Anti-Baryon has 3 anti-quarks
    - EXAMPLES
      - Proton p = uud
      - Neutron n = udd
  - **MESON**
    - quark anti-quark pair
    - EXAMPLES
      - Pion $\pi^+ = u\bar{d}, \pi^- = \bar{u}d$
- **Units of Mass and Energy**
  - Small energies normally expressed in electron Volts – eV

- An electron accelerated through a P.D. of 1V gains a kinetic energy of 1eV, since it has a charge of e.
    - **E = VQ**
    - **1eV = 1.602 x 10^{-19} J**
- **ALSO**
    - $E = mc^2$
    - Mass can be written in eV/c^2
    - Electron has an electronic mass of $0.511MeV/c^2$

- **Only need to know first generation:**

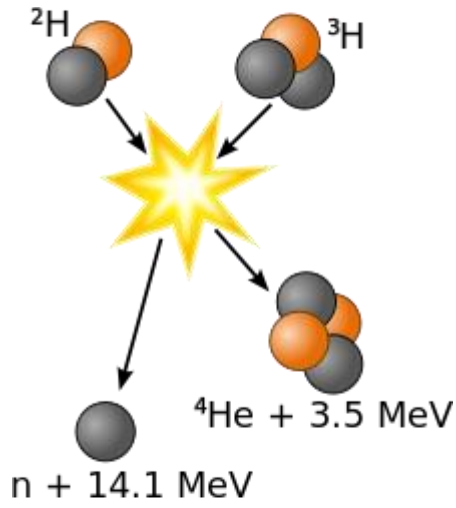


- **Neutrinos**
    - Very low mass – only interact via the weak force
    - Interactions very rarely happen
- **PAIR PRODUCTION**
    - The energy from a high-energy photon can be used to create a pair of antiparticles, thus destroying the photon
    - Most commonly: photon -> electron + positron

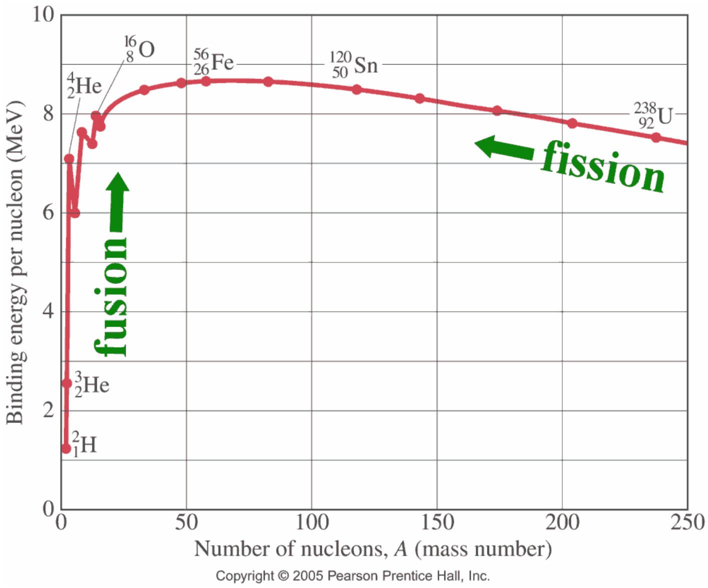
INTERACTION	AFFECTS	RANGE	COMMENTS	Particle
Gravitational	All matter	Infinite	Negligible except for massive objects	Graviton
Weak	All particles	~10^-18m	Negligible in comparison to EM and strong – significant otherwise Neutrino involvement or quark flavour changes >> weak for sure	W^+, W^-, Z^0 bosons
EM	All charged particles	Infinite	Also affects neutral hadrons because quarks have charges	Photons γ
Strong	All quarks	~10^-15m	Also affects interactions between hadrons. Becomes stronger as the distance increases Holds the nucleus together – opposes EM repulsion between two protons.	Gluons

- **CONSERVATION LAWS**
    - Lepton Number
    - Baryon Number
    - Quark Number
    - Charge
    - Mass-Energy
- N.B. an anti-lepton as lepton number -1

3.8 Nuclear Energy

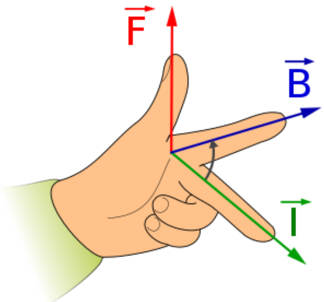
- **Mass is Energy**
  - $E = mc^2$
- **Unified Atomic Mass Unit (u)**
  - The mass of a proton and neutron is assigned 1u >> easier arithmetic
- **Binding Energy**
  - In compound nuclei, the mass of the nucleus is smaller than the sum of the constituent nucleons (protons & neutrons)
  - This mass defect comes from some mass-energy being stored as potential energy used to bind the nuclei together
  - Energy needs to be expended to split the nuclei apart >> regain the mass defect
  - This energy which needs to be input is called the binding energy
- **Calculating binding energy**
  - Know mass of a proton and neutron
  - Know mass of nucleus
  - Know number of protons and neutrons in said nucleus
  - Add together constituents and subtract nuclear mass
  - >> can divide through by the number of nucleons (protons + neutrons) to get binding energy per nucleon
- **Energy is conserved**
  - Nuclear fission
    - some mass is lost when a neutron decays into a lighter proton >> this mass is released as energy
    - $$\boxed{{}^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{90}_{36}\mathrm{Kr} + {}^{143}_{56}\mathrm{Ba} + {}^{1}_{0}\mathrm{n} + {}^{1}_{0}\mathrm{n} + {}^{1}_{0}\mathrm{n}}$$
  - Nuclear fusion
    - Some mass is lost to the mass defect as a helium nucleus has a higher mass defect than two hydrogen atoms >> this mass is released as energy
    - 

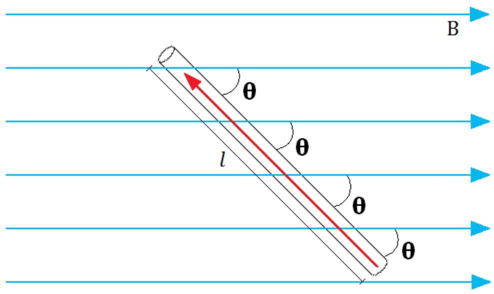
- **Nuclear Binding energy**
  - Amount of energy needed to break apart the nucleus
  - Nuclear bindning energy per nucleon different for different sizes of nuclei

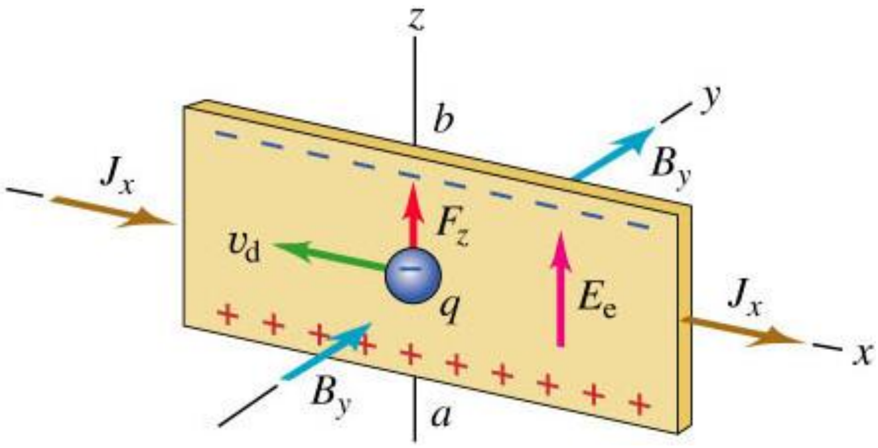
- 

- As diagram shows, fusion or fission towards Iron-56 releases energy
- This is because it has minimum potential energy (in the deepest well)

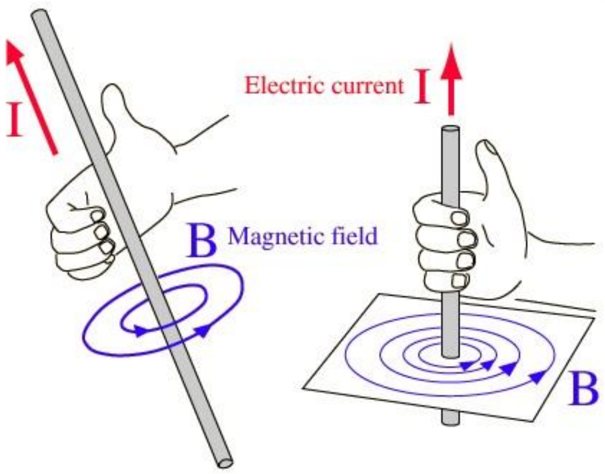
3.9 Magnetic Fields

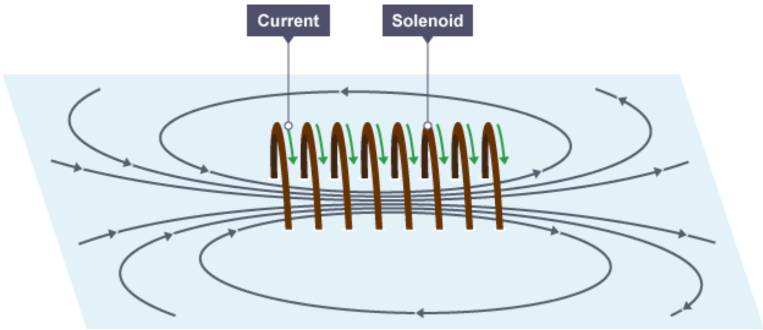
- **Fleming's Left Hand Rule**
  - 

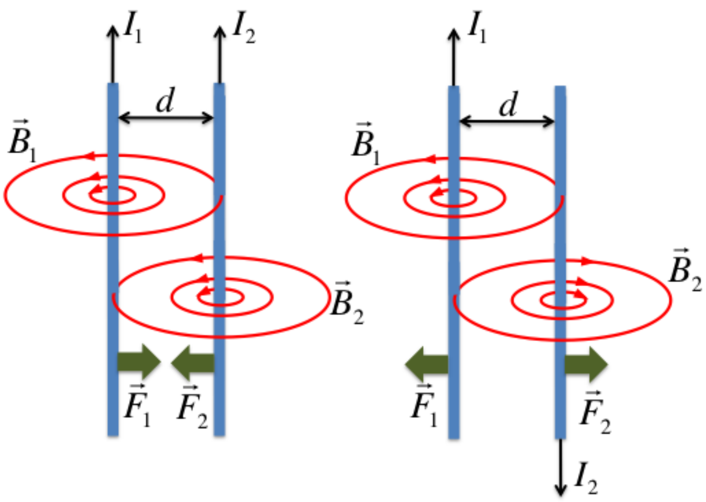
  - Used to determine direction of force on a current carrying conductor within a B-field
- **Force on a current carrying wire**
  - $F = BIl \sin\theta$
    - F = Force
    - B = magnetic field strength (Tesla – T)
    - I = current
    - L = length of conductor in field
    - Θ = angle between B and I vectors
    - 

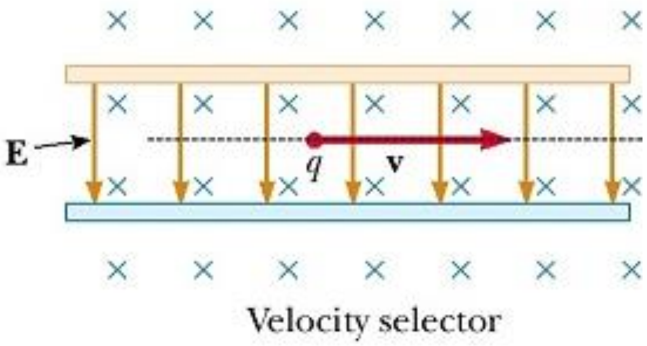
  - Can also be used to calculate B from known F, I , L and theta
- **Force on a moving charge**
  - $F = Bqv \sin\theta$
    - q = charge
    - v = speed
- **Hall Effect**
  - 

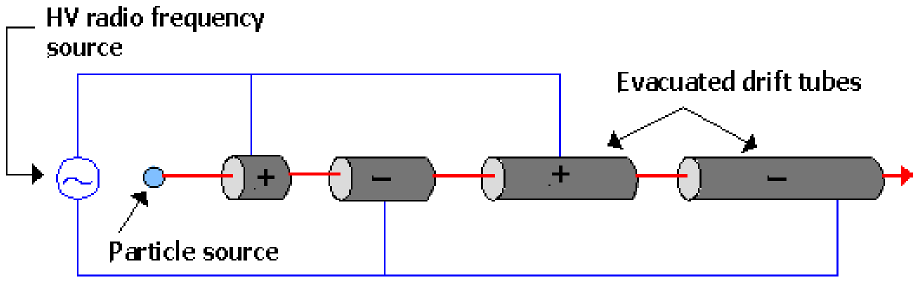
  - As electrons move in a uniform B-Field they experience a force perpendicular to current and field >> sets up a hall voltage. Equilibrium reached when magnetic force equals electric force
  - Hall voltage proportional to B-field (fixed current) $V_H \propto B$

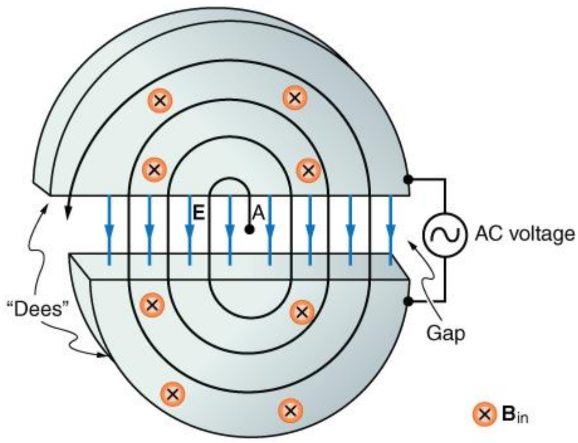
- **Shapes of magnetic fields**
  - Can use right hand grip rule to determine directions (thumb is current, fingers are field)
  - Dot **.** is out of page; Cross **x** is into page (arrow)
  - **Long Straight wire**
    - 

    - $\boldsymbol{B = \frac{\mu_0 I}{2\pi a}}$
      - a is distance from wire to point
  - **Long solenoid**
    - 

    - $\boldsymbol{B = \mu_0 n I}$
      - n is turns per unit length
    - Adding an iron core to a solenoid increases the field strength
- **Two wires**
  - Like currents attract. Opposite currents repel
    - 

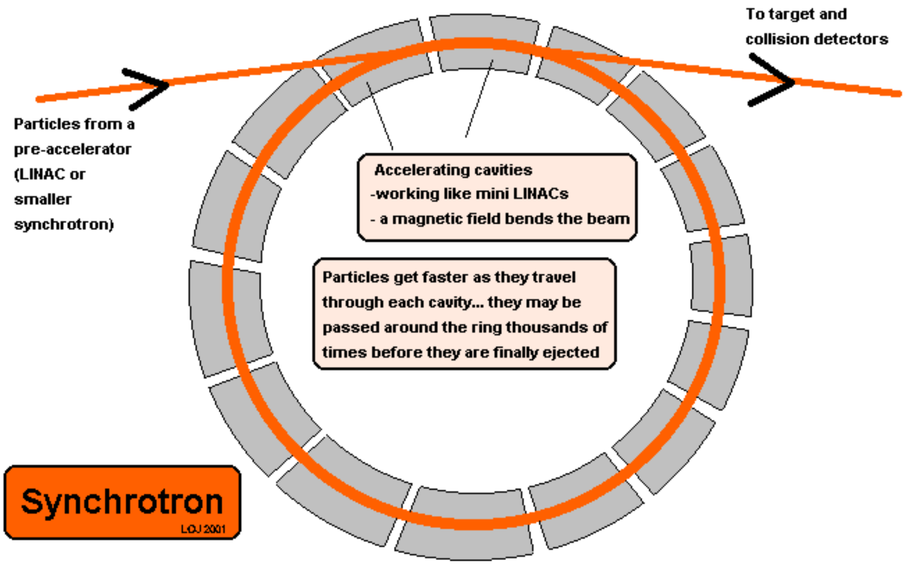
- **Ion beams**
    - Beams of charged particles inside uniform E and B-fields >> experience a force due to each field >> can be used to select a very specific velocity for which magnetic and electric fields balance
        - 

- **Particle accelerators**
    - **Linear Accelerator (LINAC)**
        - 

Linear Accelerator
        - E-field within a uniformly charged cylinder is 0 >> drifts within tubes (no acceleration)
        - Potential difference between tubes >> accelerated when transitioning between the tubes
        - Drift tubes get longer such that drift time stays constant >> can use constant switching frequency
    - **Cyclotron**
        - 

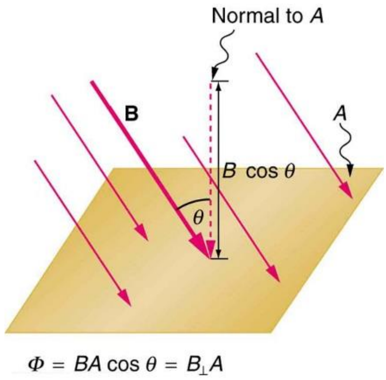
        - Uniform B-Field into page >> particles travel in a spiral
        - Accelerated between Dees by P.D.
        - Arc time stays constant >> can use constant switching frequency

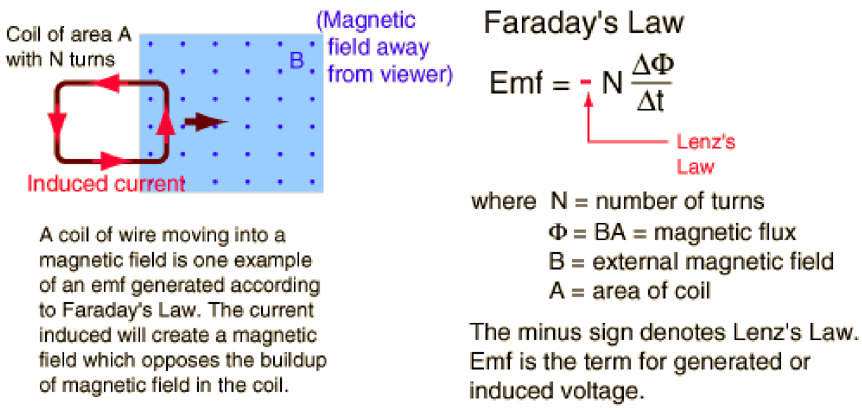
- Synchrotron



- A circular LINAC
- Uniform B-field into page >> makes charged particles travel in circle
- As particles accelerate Switching frequency must increase and so too must B-Field to keep radius constant

3.10 Electromagnetic induction

- **Magnetic Flux**
  - $\varphi = AB\cos\theta$
    - A is area of interest
    - B is magnetic field strength (uniform)
    - θ is the angle between the normal and the field
  - Amount of magnetic field lines that go through an area
  - 

- **Flux Linkage**
  - $\Phi = N\varphi$
  - Used when a coil has N turns >> find total effective flux through coil
- **Faraday's Law**
  - Induced e.m.f is proportional to rate of change of magnetic flux linkage
  - $\varepsilon = -N\frac{d\varphi}{dt}$
  - 

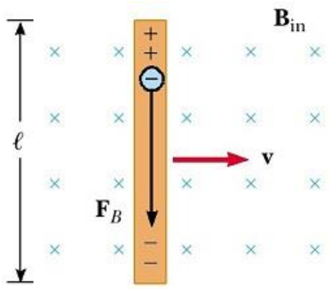
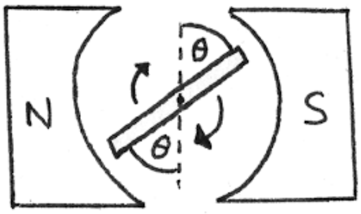
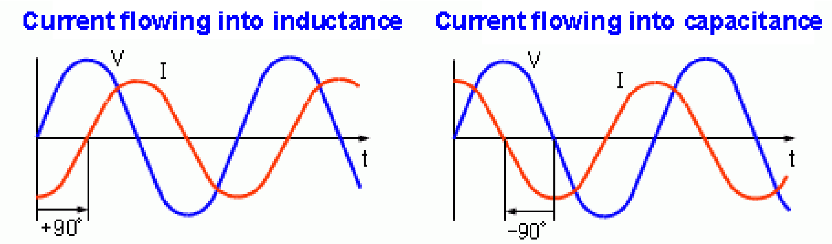
- **Lenz's Law**
  - The induced e.m.f is such as to oppose the change causing it
  - Denoted by the –ve sign in Faraday's Law
- **Simple example:**
  - 

    Figure 31.8 A straight electrical conductor of length ℓ moving with a velocity $\mathbf{v}$ through a uniform magnetic field $\mathbf{B}$ directed perpendicular to $\mathbf{v}$. A potential difference $\Delta V = B\ell v$ is maintained between the ends of the conductor.
- Coil rotating in uniform B-field >> covered in AC theory

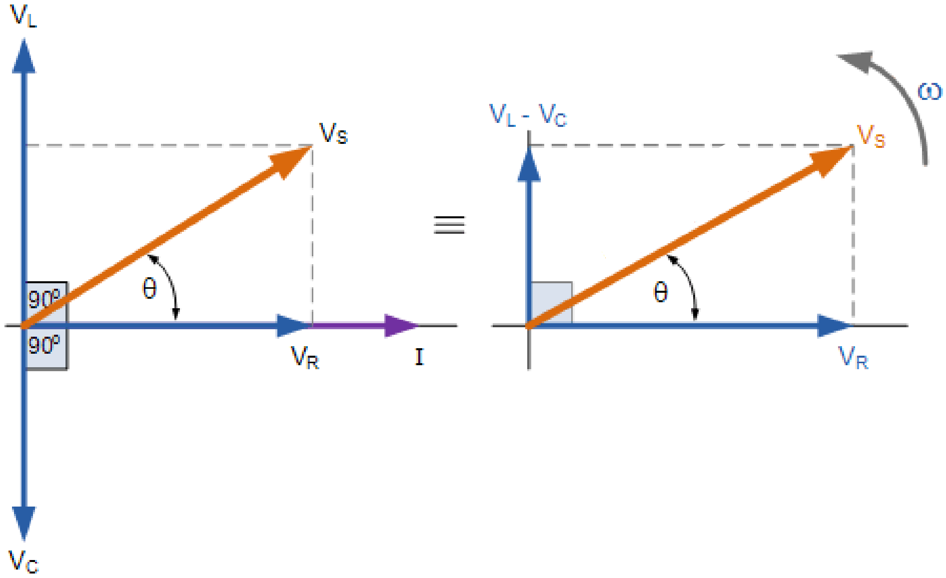
3.11 OPTION: Alternating Currents

- **Rotating coil in magnetic field**
  - 

  - $\varphi = ABcos\theta = AB\cos\omega t$
    - Where θ is the angle between normal and field
  - $\varepsilon = N\frac{d\varphi}{dt} = -\omega BAN\sin\omega t$
    - -ve sign is arbitrary due to Lenz's law
    - Get an a.c. voltage
- **Definitions**
  - Frequency f – cycles per second / Hz
  - Period T – time between cycle / s
  - Peak value V_0 – max value of whatever
  - RMS value V_{rms} – root means square value
    - when applied to ac V and I, gives equivalent power dissipated by dc circuit
    - $V_0 = V_{rms}\sqrt{2}$ & $I_0 = I_{rms}\sqrt{2}$
    - $P_{av} = V_{rms}I_{rms}$ power dissipated by a resistor
- **How to measure ac signals**
  - Use an oscilloscope (CRO) >> can discern peak values and frequencies
- **Time lags**
  - $\boldsymbol{\varepsilon LI\ the\ IC\varepsilon\ guy}$
  - In an inductor, e.m.f leads current by 90°
  - In a capacitor, current leads e.m.f by 90°



- **Reactances**
  - A resistor has a resistance /Ω which dissipates energy
  - Capacitors and Inductors also oppose current flow but do not dissipate energy (they return what they take out) >> called a reactance /Ω
  - **Inductor** – opposes high frequencies
    - $X_L = \omega L$
      - Where L is the inductance / Henry (H)
      - And $\omega = 2\pi f$
  - **Capacitor** – opposes low frequencies
    - $X_C = \frac{1}{\omega C}$
      - Where C is the capacitance / Farads (F)

- **Phasor diagrams**
  - Know how to use phasor (vector) diagrams to combine potential differences in RLC circuits
  - 

  - Basically, draw a vector for current that rotates counter-clockwise and draw potential differences accordingly
    - $V_R = IR$ & $V_L = IX_L$ & $V_C = IX_C$
    - >> $V_S = I\sqrt{R^2 + (X_L - X_C)^2}$; Pythagoras
  - **Phase angle** – angle between current and resultant voltage
    - $\theta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$
  - **Impedance** – ratio of applied voltage to resultant current (either peak or rms) >> V = IZ
    - $Z = \sqrt{R^2 + (X_L - X_C)^2}$ /Ω
- **Resonance**
  - Resonance occurs when Voltage and Current are in phase OR when impedance at a minimum => $X_C = X_L$ => $\frac{1}{\omega C} = \omega L$ => $\omega = \frac{1}{\sqrt{LC}}$
  - $f_0 = \frac{1}{2\pi\sqrt{LC}}$ = resonant frequency
- **Q-Factor** – determines sharpness of resonance curve
  - $Q = \frac{V_L}{V_R} = \frac{V_C}{V_R}$ at resonance
  - Higher Q means resistor has less of a role >> sharper peak
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