Bayesian physical reconstruction of initial conditions from large-scale structure surveys

paper by Jens Jasche & Benjamin D. Wandelt

Lorenzo Cavezza - 2130648

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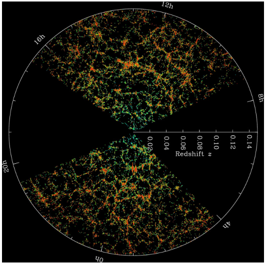
- Tracers' Surveys (Galaxy Surveys)



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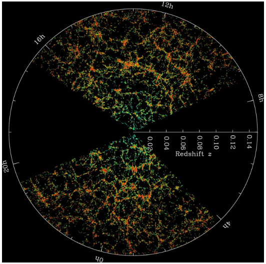

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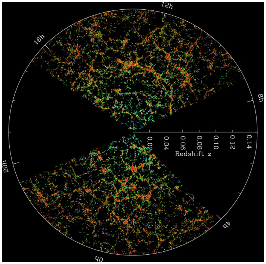

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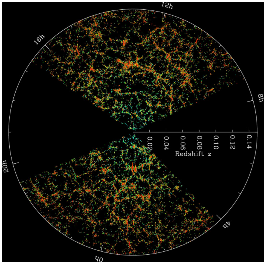

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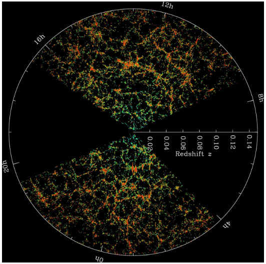

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- 2LPT dynamically evolved final density field prior
- Hamiltonian Monte Carlo exploration of initial field conditioned on mock observations

Bayesian Framework

Given a model of **LSS** formation $G\left(a, \delta^{i}\right)$ (a *Second Order Lagrangian Perturbation Thoery* model (**2LPT**) in our case), we obtain the final density contrast δ^{f} prior as:

$$\mathcal{P}\left(\{\delta_l^f\}\right) = \int d\{\delta_l^i\}\mathcal{P}\left(\{\delta_l^i\}, \{\delta_l^f\} \,\middle|\, \mathbf{S}\right) =$$

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$$= \int d\{\delta_l^i\}\prod_l \delta^D\left(\delta_l^f - G\left(a, \delta^i\right)_l\right)\left(\frac{e^{-\frac{1}{2}\sum_{lm}\delta_l^i \mathbf{S}_{lm}^{-1}\delta_m^i}}{det(2\pi\mathbf{S})}\right)$$

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with $\mathbf{S}$ representing the initial contrast covariance matrix.

This can be done through sampling of $\mathcal{P}\left(\{\delta^f_l\}, \{\delta^i_l\} \,\middle|\, \mathbf{S}\right)$ and discarding the δ^i_l realization

The LSS Likelihood

We can model the observed tracers, namely galaxies, through an *inhomogeneous* **Poisson** *Likelihood*:

$$\mathcal{P}\left(\{N_k^g\} \,\middle|\, \{\lambda_k\}\right) = \prod_k \frac{(\lambda_k)^{N_k^g} e^{-\lambda_k}}{N_k^g!}$$

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  where:

  - R_k: survey geometries and selection functions linear operator
  - $B(x)_k$: non linear and non local bias operator

The 2LPT - Poisson Posterior

The joint posterior of the final and initial contrast densities is:

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Benefits:

- Forward Model Evaluation
- All uncertaintes accounted (survey geometry, selection biases, galaxy distribution noise and cosmic variance)

Hamiltonian Monte Carlo

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- *HMC* moves smoothly inside *typical volumes* by exploiting well defined equations of motion through the *Hamiltonian*:

$$H = \sum_i \sum_j \frac{1}{2} p_i \mathbf{M}_{ij}^{-1} p_j + \psi(x) = \sum_i \sum_j \frac{1}{2} p_i \mathbf{M}_{ij}^{-1} p_j - \ln \mathcal{P}(x)$$

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- Momenta randomly drawn and easily marginalized as nuisance parameters. After dynamical evolution the *acceptance probability* is

$$\mathcal{P}_A = \min\left[1, \exp\left(-\left(H\left(\{x_i'\}, \{p_i'\}\right) - H\left(\{x_i\}, \{p_i\}\right)\right)\right)\right]$$

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$$= -\sum_j \mathbf{S}_{pj}^{-1} \delta_j^i + D^1 J_n - D^2 \sum_{a>b}\left(\tau_n^{aabb} - \tau_n^{bbaa} - 2\tau_n^{abab}\right)$$

where the τ_m^{abcd} accounts for the gradient of the second order lagrangian potential.

The Leapfrog Method

As an integration scheme the Leapfrog Method is used:
$\Longrightarrow$ simplest integration method that conserves the energy and phase space area and is time reversible

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$$p_m\left(t+\frac{\epsilon}{2}\right) = p_m(t) - \frac{\epsilon}{2}\frac{\partial\psi\left(\{\delta^i_k\}\right)}{\delta^i_l}\Bigg|_{\delta^i_m(t)}$$

$$\delta^i_m\left(t+\epsilon\right) = \delta^i_m(t) - \frac{\epsilon}{m_i}p_m\left(t+\frac{\epsilon}{2}\right)$$

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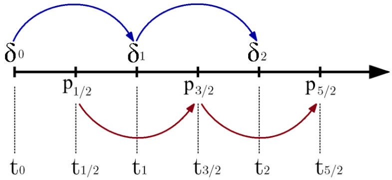

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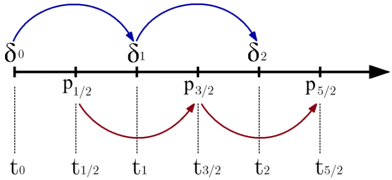

which translates to:

$$p_m(t+\epsilon) = p_m(t) - \tfrac{\epsilon}{2}\left(\left.\frac{\partial\Psi(\delta^i)}{\partial\delta^i_i}\right|_{\delta^i(t)} + \left.\frac{\partial\Psi(\delta^i)}{\partial\delta^i_m}\right|_{\delta^i(t+\epsilon)}\right)$$

$$\delta_m(t+\epsilon) = \delta_m(t) + \epsilon\sum_j M^{-1}_{mj}\, p_j(t) - \frac{\epsilon^2}{2}\sum_j M^{-1}_{mj}\left.\frac{\partial\Psi(\delta^i)}{\partial\delta^i_j}\right|_{\delta^i(t)}$$

Testing the Model

Mock Observations

δ_i Generation

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$\downarrow$

Expansion by D^+

Mock Observations

δ_i Generation

↓

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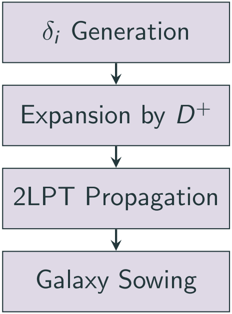
2LPT Propagation

Mock Observations



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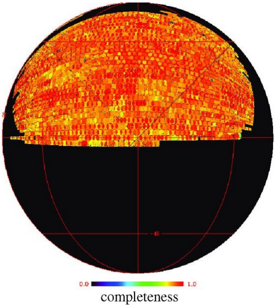

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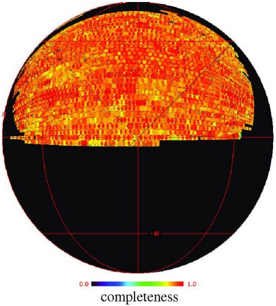
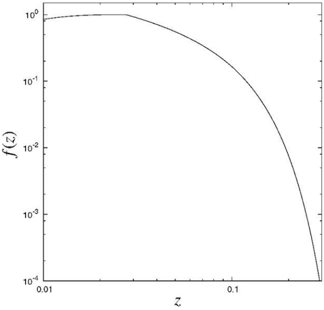
Schechter Selection





with: $f(z) = \frac{\int_{M_{min}(z)}^{M_{max}(z)} \Phi(M)dM}{\int_{M_{min}}^{M_{max}} \Phi(M)dM}$

where $\Phi(M)$ is the *Schechter* luminosity function.

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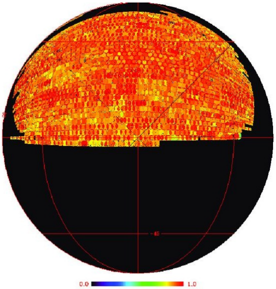
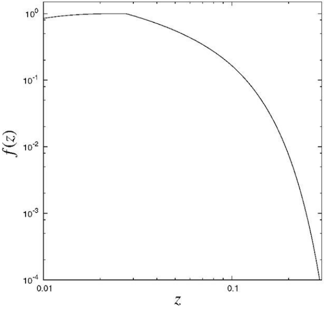
Schechter Selection





with: $f(z) = \frac{\int_{M_{min}(z)}^{M_{max}(z)} \Phi(M)dM}{\int_{M_{min}}^{M_{max}} \Phi(M)dM}$

where $\Phi(M)$ is the *Schechter* luminosity function. The linear operator used in Poisson Sampling is:

$$R_i = M_i F_i = M(\alpha_i, \delta_i) f^I(z_i)$$

Burn - In

- Arbitrarily lower strength δ_l^i by one tenfold
- Quantify power spectra drift with:

$$\xi\left(P_i(K)\right) = 1 - \frac{P_i(k)}{P^0(k)}$$

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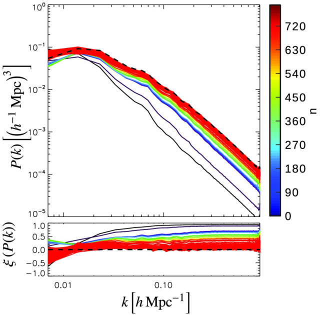

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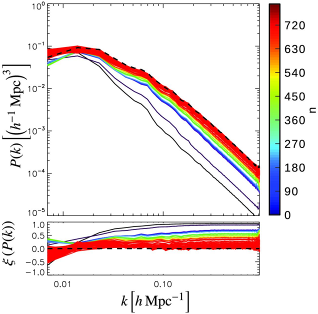

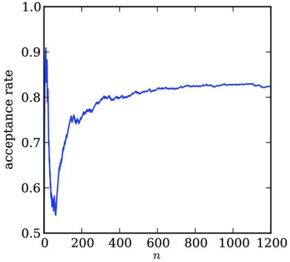

- Acceptance takes a dip after manual drift
- Back to normal ($\sim$ 84%) after $\sim$ 600 epochs

Autocorrelation

Autocorrelation is computed to determine the number of independent samples and to assess good mixing:

$$C(\delta)_n = \left\langle \frac{\delta^i - \langle\delta\rangle}{\sqrt{Var(\delta)}} \frac{\delta^{i+n} - \langle\delta\rangle}{\sqrt{Var(\delta)}} \right\rangle$$

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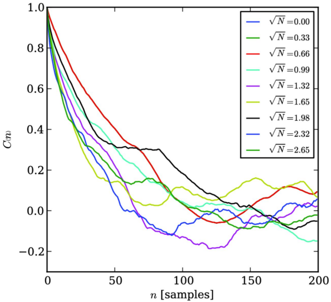
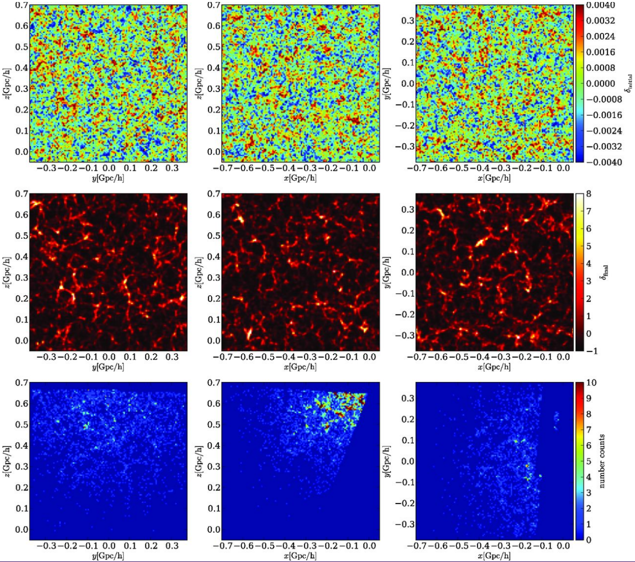
We see such correlations plotted from different signal-to-noise ratios $\sqrt{N}$



$$C(\delta)_n < 0.1 \text{ when } n \sim 200 \implies \text{correlation length is about 200}$$

Inferred Density Fields

Samples from the sampled density fields and mock data:



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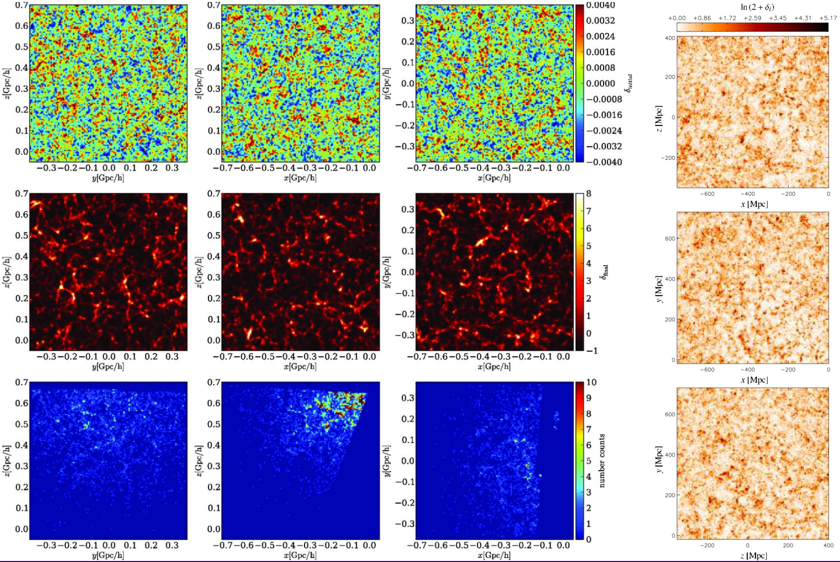
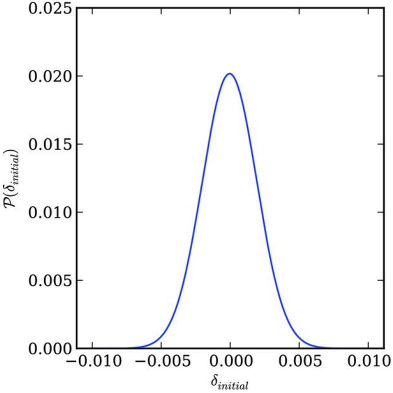
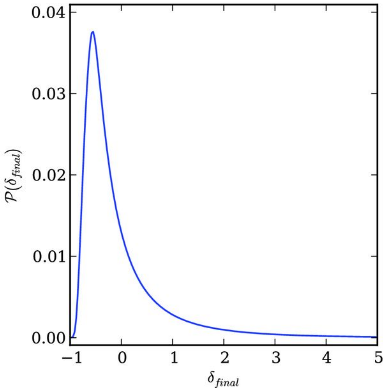


One-point distribution of density contrasts:

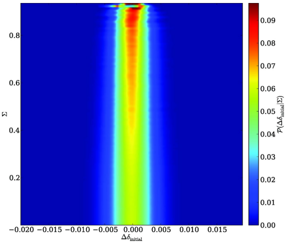




Accuracy

Inferred initial density accuracy is assessed with the posterior of $\Delta\delta_{initial} = \delta_{initial}^{true} - \delta_{initial}$ conditioned to a signal-to-noise ratio proxy:

$$\Sigma = \frac{|\langle\delta_{initial}\rangle|}{\sqrt{\langle(\delta_{initial} - \langle\delta_{initial}\rangle)^2\rangle}}$$



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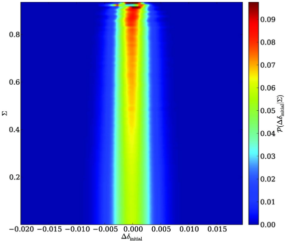
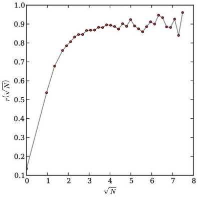
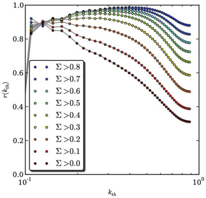




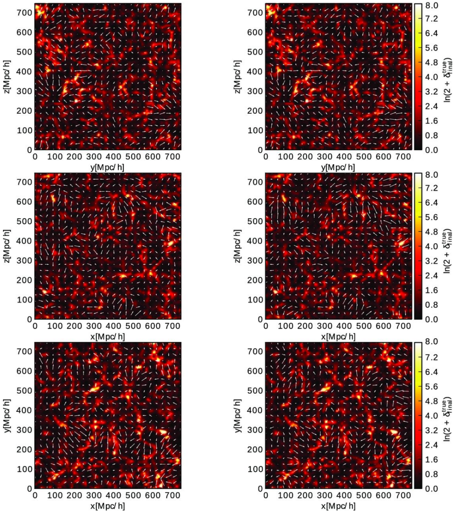


The correlation coefficient between inferred and real densities is computed as a function of $\sqrt{N}$ for the final and of k for different Σ for the initial:

$$r(k_x) = \frac{\langle\delta_0^x\langle\delta\rangle^x\rangle}{\sqrt{\langle(\delta_0^x)^2\rangle}\sqrt{\langle(\delta^x)^2\rangle}}$$

Inferred Dynamics

True vs inferred dynamics:



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- Possible further improvements include better model errors and accuracy of 2LPT, real galaxy surveys tests, and incorporation of halo-mode-based galaxy bias models

Thank you for your attention