Physics
of Data

An information-theoretic quantification of the content of communication between brain regions

Work based on *Celotto et al*, 2023

Information Theory & Inference

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Outline

01

FIT, a new method

framework and FIT definition

02

Validation

creation of a mock dataset to verify FIT analysis

03

Real data analysis

real data analysis of two EEG datasets and results

01

General framework

understand brain function ⟷ quantifying the **amount**, **content** and **direction** of **communication between brain regions**

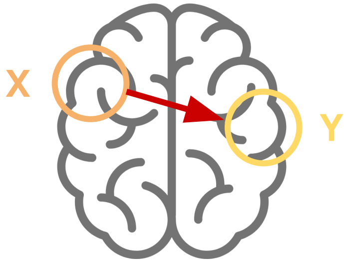
Wiener - Granger causality principle

sending region **X**	receiving region **Y**
past activity	present activity

information flow = causality prediction

01

Introduction



Wiener - Granger causality principle

Transfer Entropy (TE)

$$\boxed{TE(X \rightarrow Y) = I(X_{past}; Y_{pres} | Y_{past})}$$

overall information propagated across regions
mutual information conditioned on the receiver **past** activity
does **not** depend on a specific stimulus feature

01

Introduction

Partial Information Decomposition (PID)

$I(S;\underline{X})$

information atoms

decomposes the joint mutual information that a set of *N* source variables carries about a **target** variable **S** into non-negative components

01

Introduction

Partial Information Decomposition (PID)

$$I(S; \underline{X})$$

information atoms

decomposes the joint mutual information that a set of N source variables carries about a **target** variable **S** into non-negative components

example: for $N = 2$ the 4 information atoms are

1. **SI(S: X_1, X_2)** → **shared**/redundant information that both X1 **and** X2 encode about S
2. **UI(S: $X_1 \backslash X_2$)**
3. **UI(S: $X_2 \backslash X_1$)**

   → **unique** information that X1 **or** X2 encodes about S
4. **CI(S: X_1, X_2)** → complementary/**synergistic** information encoded by the combinations of the variables

01

Introduction

Partial Information Decomposition (PID)

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information atoms

decomposes the joint mutual information that a set of N source variables carries about a **target** variable **S** into non-negative components

How to **evaluate** the information atoms?

$$I_{min}$$

expected value of the minimum information that any source provides about each outcome of S

redundancy = information common to all sources

+

sources may provide information about different outcomes of S

01

Introduction

$$I(S;\underline{X})$$

information atoms

Partial Information Decomposition (PID)

decomposes the joint mutual information that a set of N source variables carries about a **target** variable **S** into non-negative components

with I_{min} the **shared** information becomes

$$SI(S:X_1,X_2)=\sum_{s\in S}p(s)\min_{X_i\in\{X_1,X_2\}}I(S=s;X_i)$$

where $$I(S=s;X_i)=\sum_{x_i\in X_i}p(x_i|s)\left[\log\frac{p(s|x_i)}{p(s)}\right]$$

01

Introduction

Partial Information Decomposition (PID) → **Feature - specific Information Transfer (FIT)** NEW

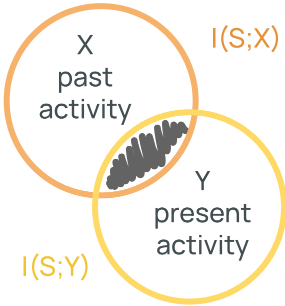
$I(S;\underline{X})$

↓

information atoms

decomposes the joint mutual information that a set of *N* source variables carries about a **target** variable **S** into non-negative components

non-negative measure that quantifies the directed flow of information about a **specific feature S** of interest between neural populations



Introduction

Feature - specific Information Transfer (FIT)

non-negative measure that quantifies the directed flow of information about a **specific feature S** of interest between neural populations

FIT definition

a. natural candidate: PID atom $SUI(S : X_{past}, Y_{pres} \setminus Y_{past})$ but has to be smaller than TE and has to depend on

$$P(X_{past}, Y_{pres})$$

b. considering PID atom $SUI(Y_{pres} : X_{past}, S \setminus Y_{past})$

target

$$FIT = \min[SUI(S : X_{past}, Y_{pres} \setminus Y_{past}), SUI(Y_{pres} : X_{past}, S \setminus Y_{past})]$$

01

Introduction

Feature - specific Information Transfer (FIT)

non-negative measure that quantifies the directed flow of information about a **specific feature S** of interest between neural populations

FIT properties

$$FIT = \min[SUI(S : X_{past}, Y_{pres} \setminus Y_{past}), SUI(Y_{pres} : X_{past}, S \setminus Y_{past})]$$

a. upper bounded by $\begin{cases} I(S; X_{past}) \\ I(S; Y_{pres}) \\ TE(X \rightarrow Y) \end{cases}$

feature information encoded in the past of X and in the present of Y

b. depends on $P(S, X_{past}, Y_{pres})$

through $\begin{cases} P(S, X_{past}) \\ P(S, Y_{pres}) \\ P(X_{past}, Y_{pres}) \end{cases}$

01

Introduction

Feature - specific Information Transfer (FIT)

non-negative measure that quantifies the directed flow of information about a **specific feature S** of interest between neural populations

FIT calculation

$$Y_{pres} = Y_t$$

$$X_{past} = X_{t-\delta}$$

$$Y_{past} = Y_{t-\delta}$$

probabilities

empirical occurrences after discretizing both features and neural activities

02

Validation: mock data

subjects: 50	**S**: [1,2,3,4]	**trials** per subject: 2000

Validation: mock data

subjects: 50	**S**: [1,2,3,4]	**trials** per subject: 2000

sender activity

$$\begin{cases} X(t)_{noise} = N(0, \sigma) \\ X(t)_{stim} = S(t)(1 + N(0, \sigma_{stim})) \end{cases}$$

Validation: mock data

subjects: 50	**S**: [1,2,3,4]	**trials** per subject: 2000

sender activity

$$\begin{cases} X(t)_{noise} = N(0, \sigma) \\ X(t)_{stim} = S(t)(1 + N(0, \sigma_{stim})) \end{cases}$$

between 200 and 250 ms

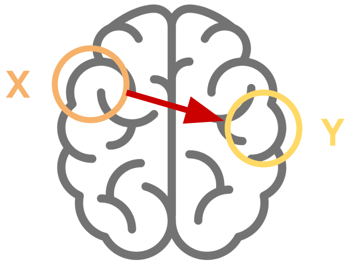
02

Validation: mock data

subjects: 50	**S**: [1,2,3,4]	**trials** per subject: 2000

sender activity

$$\begin{cases} X(t)_{noise} = N(0,\sigma) \\ X(t)_{stim} = S(t)(1 + N(0,\sigma_{stim})) \end{cases}$$



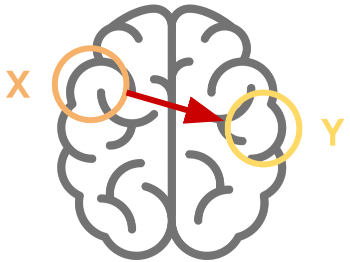
receiver activity

$$Y(t) = w_{stim} X_{stim}(t-\delta) + w_{noise} X_{noise}(t-\delta) + N(0,\sigma)$$

02

Validation: mock data

subjects: 50	**S**: [1,2,3,4]	**trials** per subject: 2000



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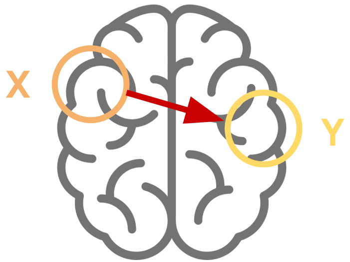


receiver activity

$$Y(t) = w_{stim} X_{stim}(t - \delta) + w_{noise} X_{noise}(t - \delta) + N(0, \sigma)$$

where

$\sigma = 2$	$\sigma_{stim} = \sigma/5$	δ~U[40,60] *ms*

02

Validation: mock data

example

02

Validation: FIT/TE from mock data

FIT / TE ← I_{min} ← $p(X_{past}, Y_{pres}, Y_{past}, S)$ ← **empirical occurrences**

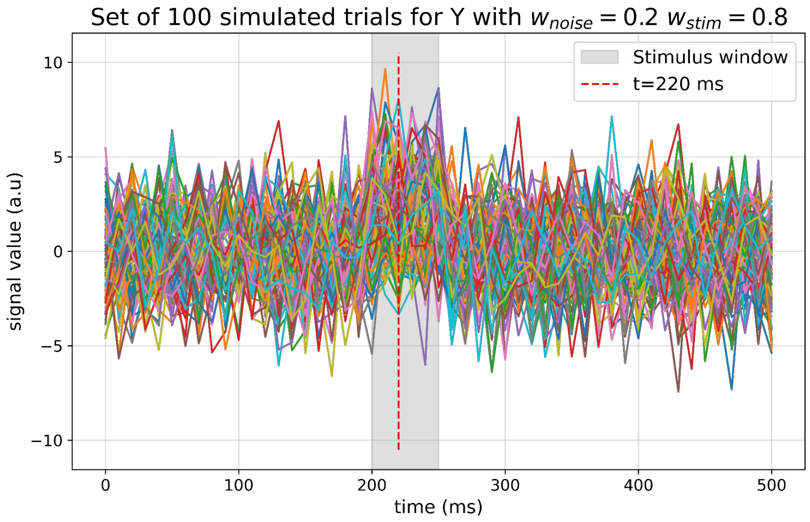
joint probability distribution

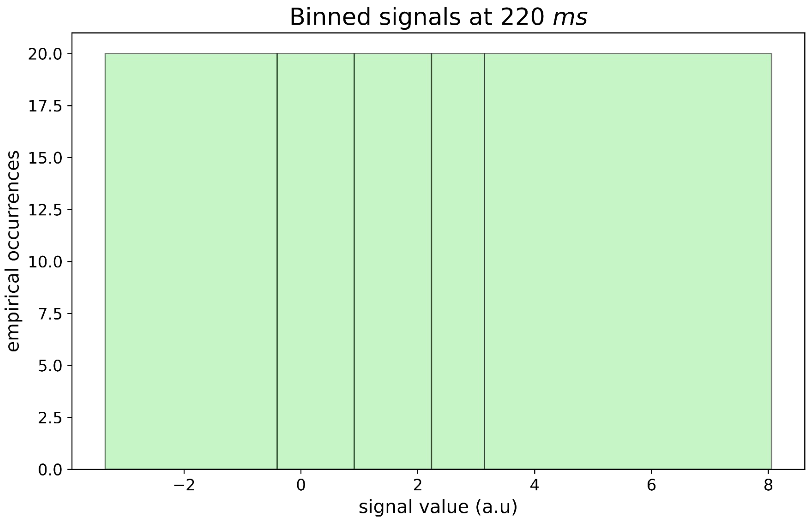
02

Validation: FIT/TE from mock data

FIT / TE ← I_{min} ← $p(X_{past}, Y_{pres}, Y_{past}, S)$ ← **empirical occurrences**

joint probability distribution



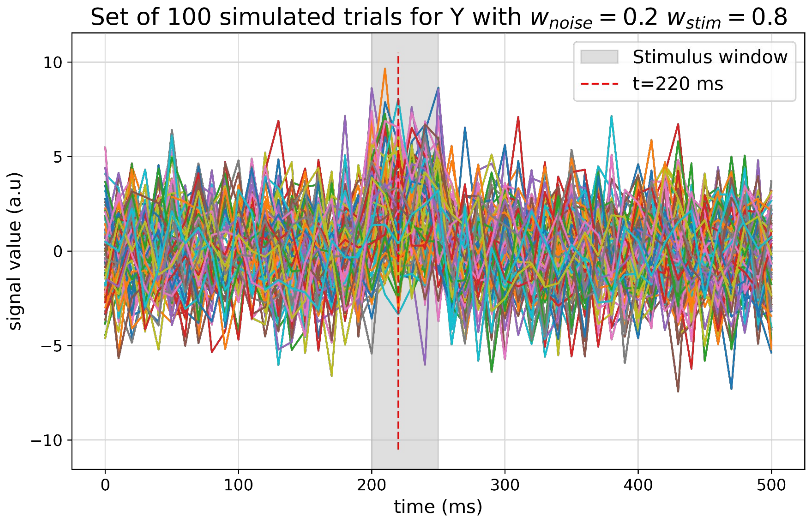


02

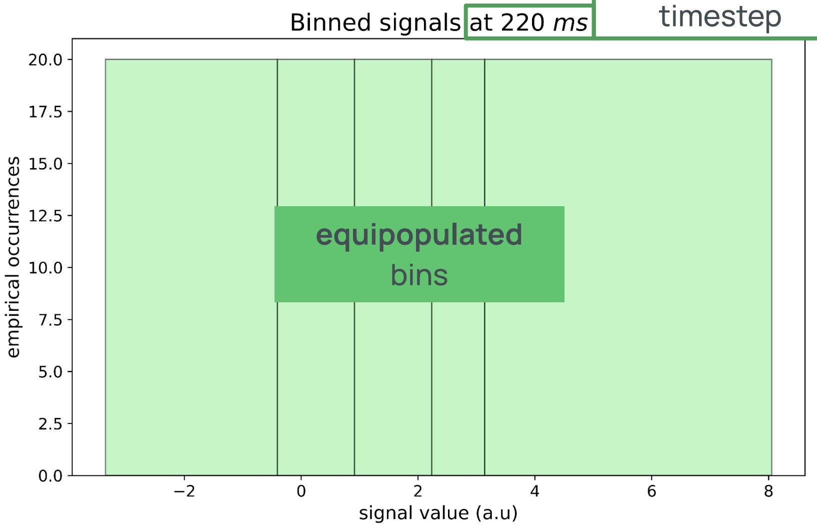
Validation: FIT/TE from mock data

FIT / TE ← I_{min} ← $p(X_{past}, Y_{pres}, Y_{past}, S)$ ← **empirical occurrences**

joint probability distribution



binning at each timestep



equipopulated bins

02

Validation: binning methods

However, we adopted two different **binning methods** for the sender activity X:

paper method

[unjustified]

$$X^{bin} = (X_s^{bin}(t) - 1) \cdot N_{bins} + X_n^{bin}(t)$$

→ #bins(X) > #bins(Y)

alternative method

[natural choice]

$$\mathrm{X}^{bin} = (X_s(t) + X_n(t))^{bin}$$

→ #bins(X) = #bins(Y)

02

Validation: FIT/TE from mock data

consequently from $p(X_{pres}, Y_{pres}, Y_{past}, S)$ we can compute

- $FIT = \min[SUI(S : X_{past}, Y_{pres} \setminus Y_{past}), SUI(Y_{pres} : X_{past}, S \setminus Y_{past})]$

02

Validation: FIT/TE from mock data

consequently from $p(X_{pres}, Y_{pres}, Y_{past}, S)$ we can compute

- $FIT = \min[\boxed{SUI(S : X_{past}, Y_{pres} \setminus Y_{past})}, \boxed{SUI(Y_{pres} : X_{past}, S \setminus Y_{past})}]$

$$\boxed{I_{min}(S; X_{pres}, Y_{pres}) - I_{min}(S; X_{pres}, Y_{pres}, Y_{past})}$$

$$\boxed{I_{min}(Y_{pres}; X_{pres}, S) - I_{min}(Y_{pres}; X_{pres}, S, Y_{past})}$$

02

Validation: FIT/TE from mock data

consequently from $p(X_{pres}, Y_{pres}, Y_{past}, S)$ we can compute

- $FIT = \min[SUI(S : X_{past}, Y_{pres} \setminus Y_{past}), SUI(Y_{pres} : X_{past}, S \setminus Y_{past})]$

- $TE(X \to Y) = \boxed{I(X_{past}; Y_{pres}|Y_{past})}$

$$\boxed{H[Y_{pres}, Y_{past}] - H[Y_{past}] - H[X_{past}, Y_{past}, Y_{pres}] + H[X_{past}, Y_{past}]}$$

Validation: significance test

non parametric permutation test

information transmission from X to Y about S = stimulus presence + (temporal) correlation between X and Y activities

02

Validation: significance test

non parametric permutation test

information transmission from X to Y about S = ~~stimulus presence~~ + (temporal) correlation between X and Y activities

First data shuffle:

randomly **shuffling S** within trials

02

Validation: significance test

non parametric permutation test

information transmission from X to Y about S = stimulus presence + ~~(temporal) correlation between X and Y activities~~

Second data shuffle:

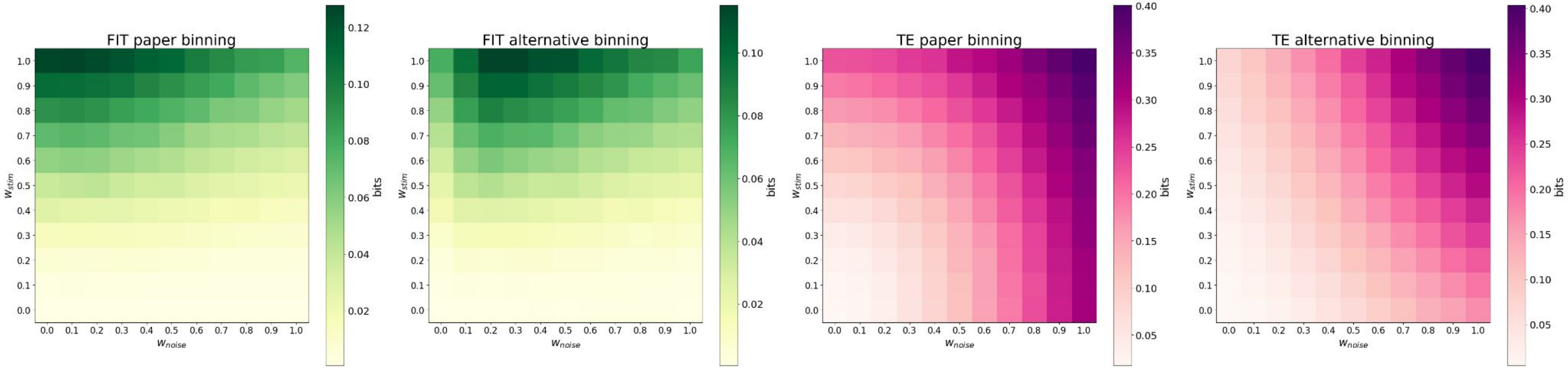
randomly **shuffling X activity with the same S value** within trials

02

Validation: results

how FIT and TE from X to Y depend on the amount of stimulus-feature-related transmission and of unrelated transmission

w_{stim} w_{noise}



02

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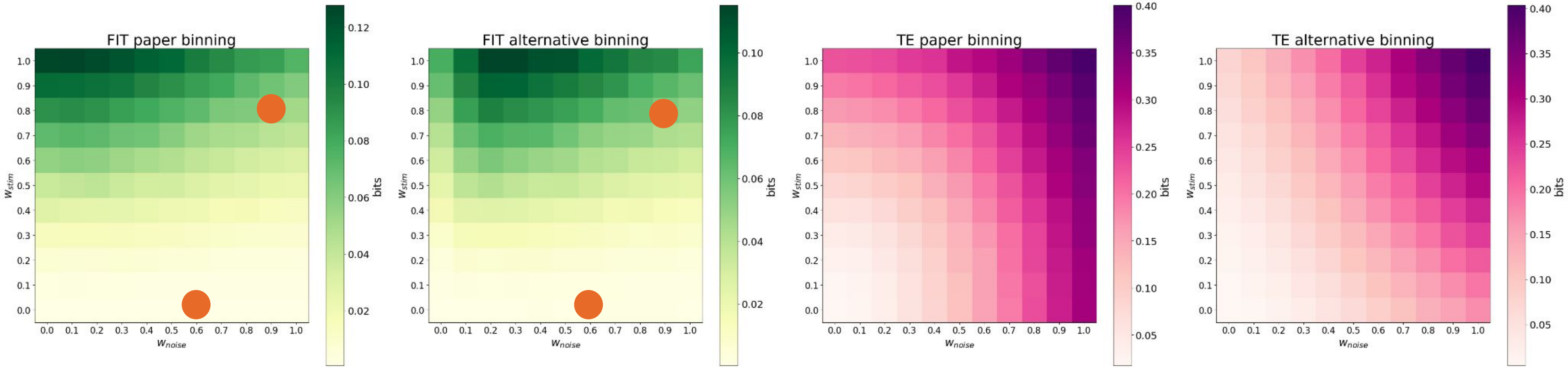
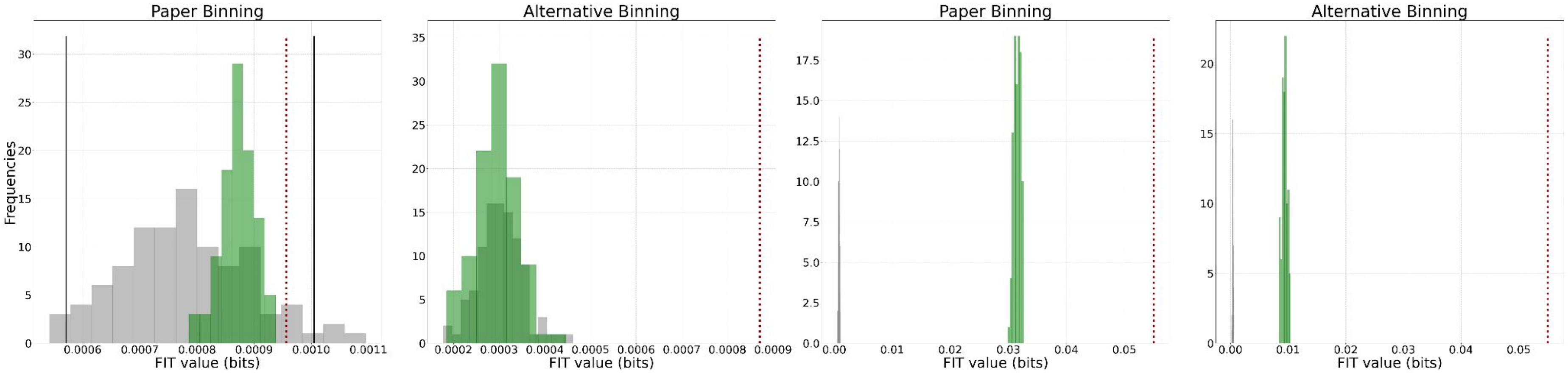


02

Validation: results

FIT significance on the non-parametric permutation test



w_{stim}	0.0
w_{noise}	0.6

S-shuffled null distribution

S-fixed null distribution

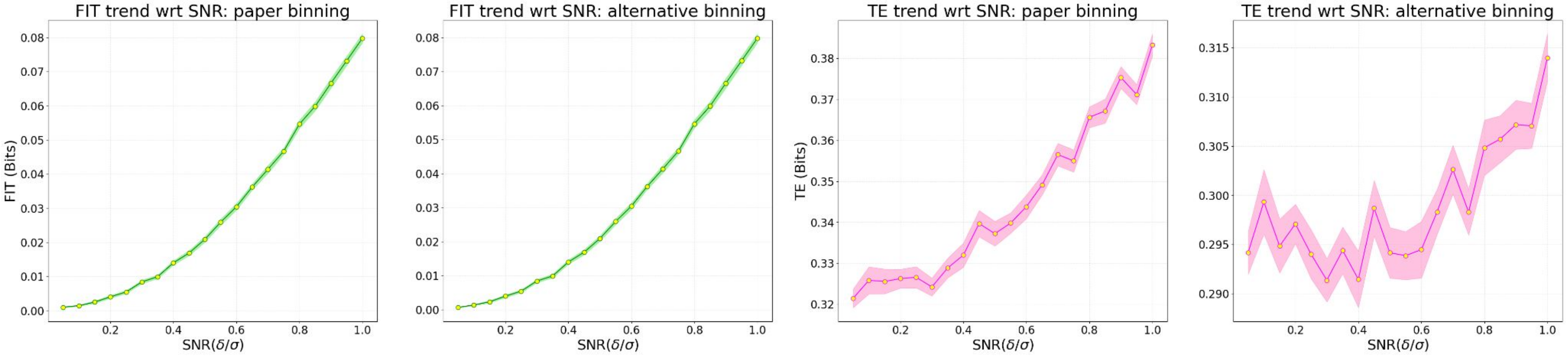
Measure

w_{stim}	0.8
w_{noise}	0.9

02

Validation: results

FIT/TE as a function of SNR = δ/σ



mean FIT

SEM FIT

w_{stim}	0.5
w_{noise}	1.0

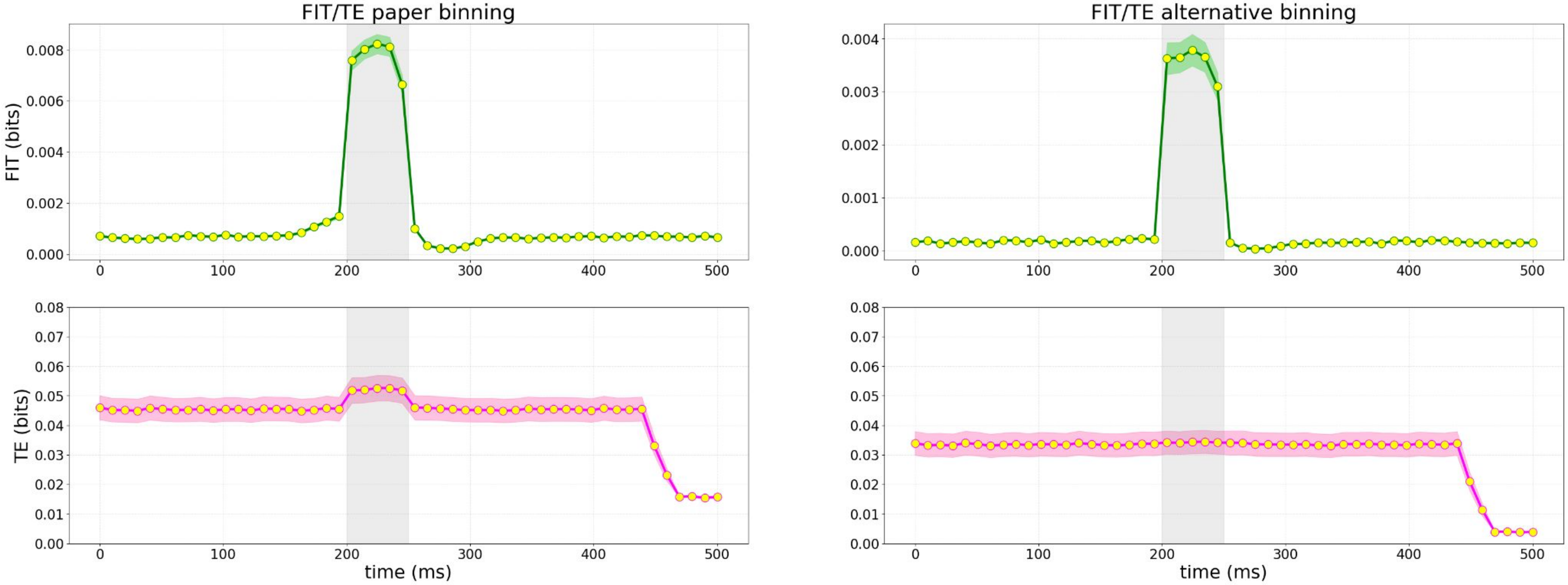
mean TE

SEM TE

02

Validation: results

how well TE and FIT **temporally localize** the stimulus-feature-related information transmitted from X to Y



03

EEG Data Analysis

two EEG datasets from experiments with human participants

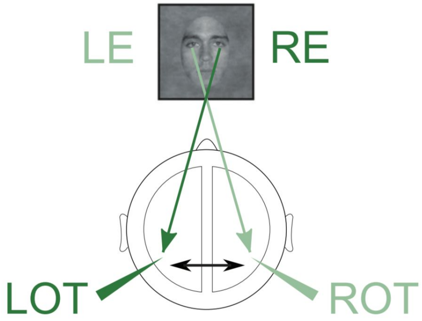
aggregate activity of neural populations

dataset	**1**	**2**
task	face detection	image classification
subjects	15	1
trials	~1000	800

03

EEG data analysis method

test the ability of FIT to **detect feature-specific information flow between brain hemispheres**



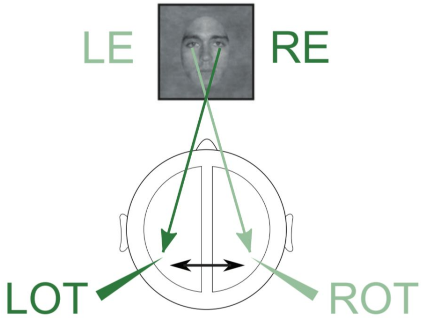
03

EEG data analysis method

test the ability of FIT to **detect feature-specific information flow between brain hemispheres**

because we only know that eye-specific information appears

- first at **~120 ms** post image presentation in the **contralateral** OT
- then after **~20-40 ms later** in the **ipsilateral** OT



03

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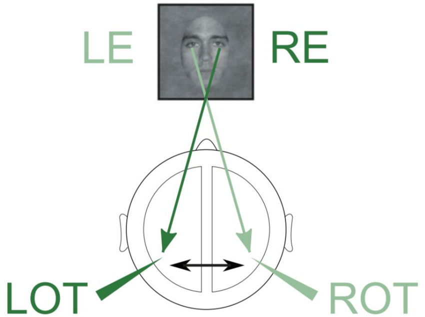
- then after **~20-40 ms later** in the **ipsilateral** OT

is the eye information in the ipsilateral hemisphere received from the contralateral hemisphere?



03

EEG data analysis method

FIT and TE transmission of eye-specific information calculated between **L**eft **O**ccipito - **T**emporal and **R**ight **O**ccipito - **T**emporal regions

03

EEG data analysis method

FIT and TE transmission of eye-specific information calculated between **L**eft **O**ccipito - **T**emporal and **R**ight **O**ccipito - **T**emporal regions

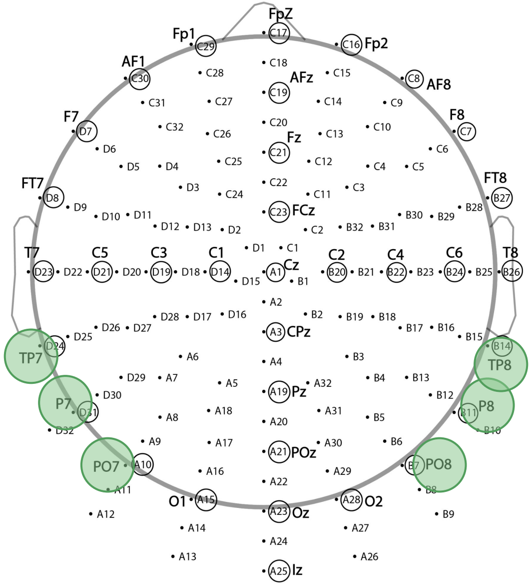
- **stimulus:** eye visibility (in the image)

  proportion of image pixels in the eye region visible through the mask

- **selected electrodes**:

  highest mutual information about the visibility of the contralateral eye from the highlighted ones

BIOSEMI 128 electrodes locations



03

EEG data analysis method

FIT and TE transmission of eye-specific information calculated between **L**eft **O**ccipito - **T**emporal and **R**ight **O**ccipito - **T**emporal regions

both as a function of **peri-stimulus time** and of **delay** between sender and receiver

both for **Left** and **Right eyes** of the subjects

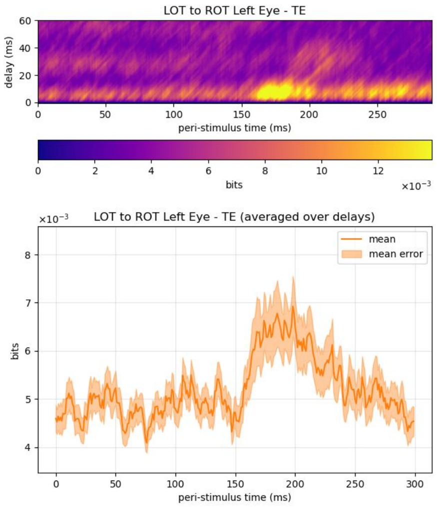
- **limited sampling bias**

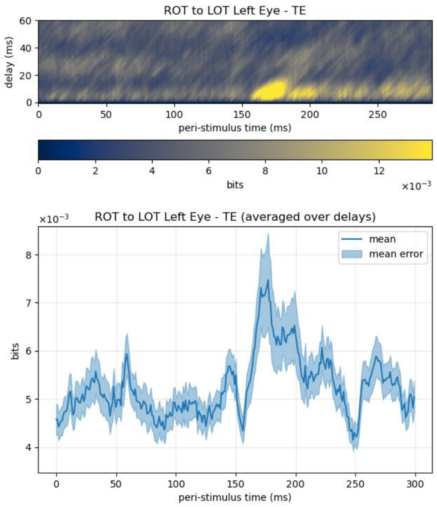
  probabilities are estimated from a limited number of experimental trials → **bias correction** (**linear** or **quadratic**)

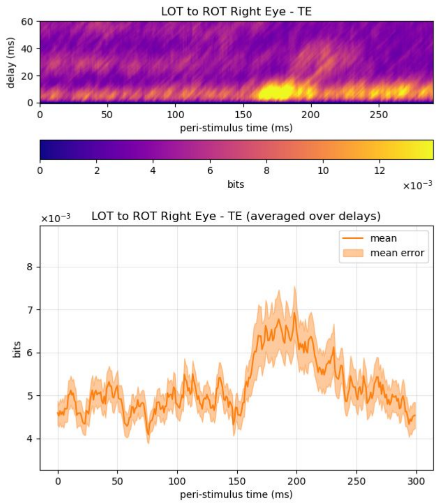
- proper binning and noise treatment

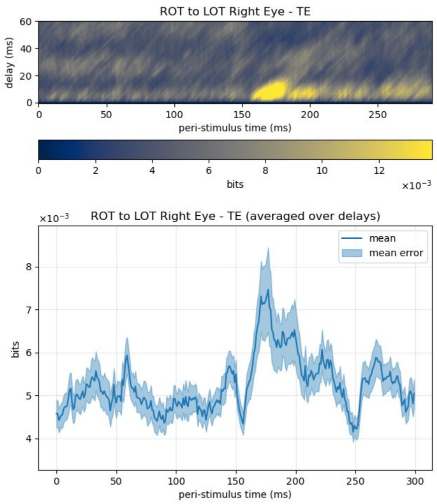
03

TE for eye visibility







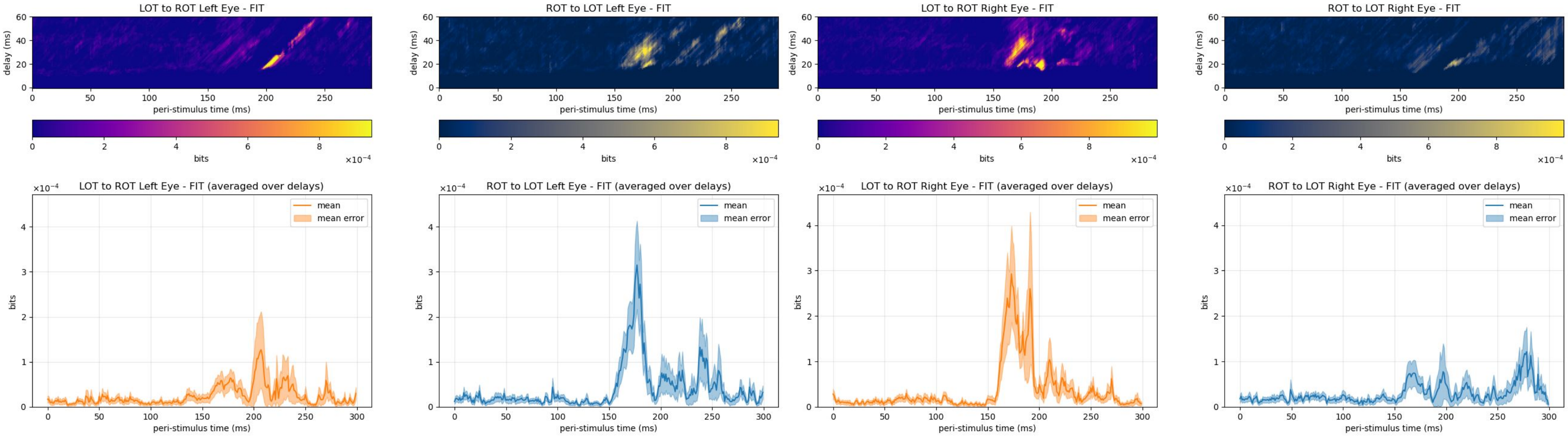


left eye

right eye

03

FIT for eye visibility



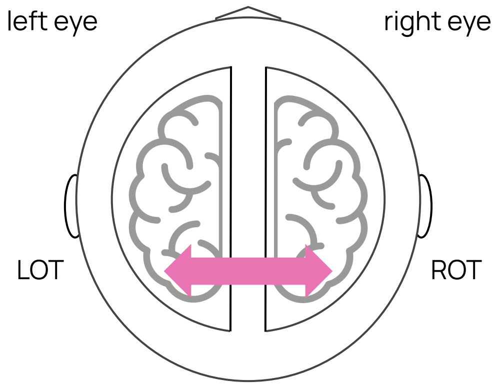
left eye

right eye

03

Results

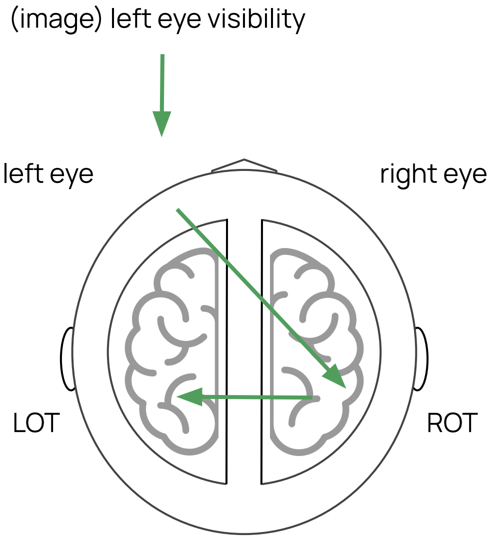
- **TE** identifies the overall information flow independently from the received stimulus



03

Results

- **TE** identifies the overall information flow independently from the received stimulus
- **FIT** identifies
  - the information flow from ROT (contralateral) to LOT (ipsilateral) for the left eye

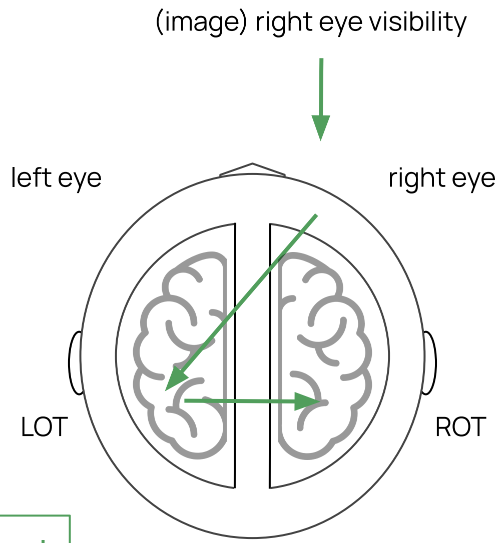


03

Results

- **TE** identifies the overall information flow independently from the received stimulus
- **FIT** identifies
  - the information flow from ROT (contralateral) to LOT (ipsilateral) for the left eye
  - the information flow from LOT (contralateral) to ROT (ipsilateral) for right eye

the eye information in the ipsilateral hemisphere is indeed received from the contralateral hemisphere



03

Bias Corrections

FIT and TE suffer from limited sampling bias

we can determine their true value with the following approximation: **quadratic extrapolation**

$$FIT_{true} = \lim_{n\to\infty} FIT(n) =$$

$$\lim_{n\to\infty}\left(a_l \frac{1}{n} + b_l\right) = b_l$$

linear approximation

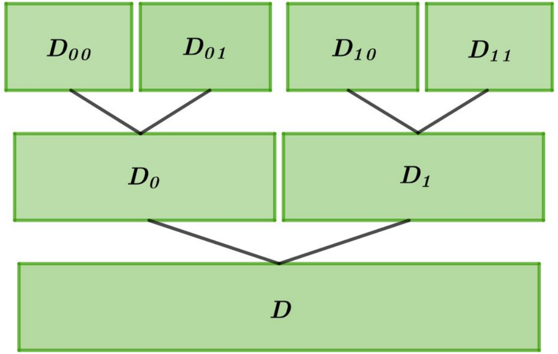
$$\lim_{n\to\infty}\left(a_q \frac{1}{n^2} + b_q \frac{1}{n} + c_q\right) = c_q$$

quadratic approximation

03

Bias Corrections

dataset → N subsets → FIT/TE mean values → linear/quadratic fit



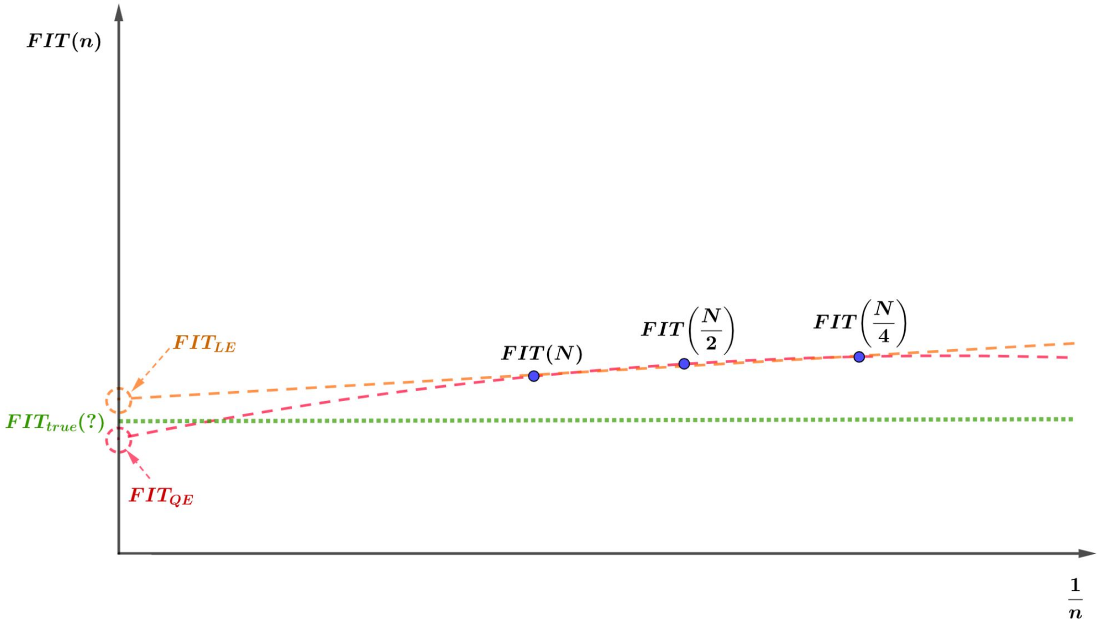
$$FIT\left(n = \frac{N}{4}\right) = \frac{\sum_{ij} FIT(F_{ij})}{4}$$

$$FIT\left(n = \frac{N}{2}\right) = \frac{\sum_{i} FIT(D_{i})}{2}$$

$$FIT(n = N) = FIT(D)$$

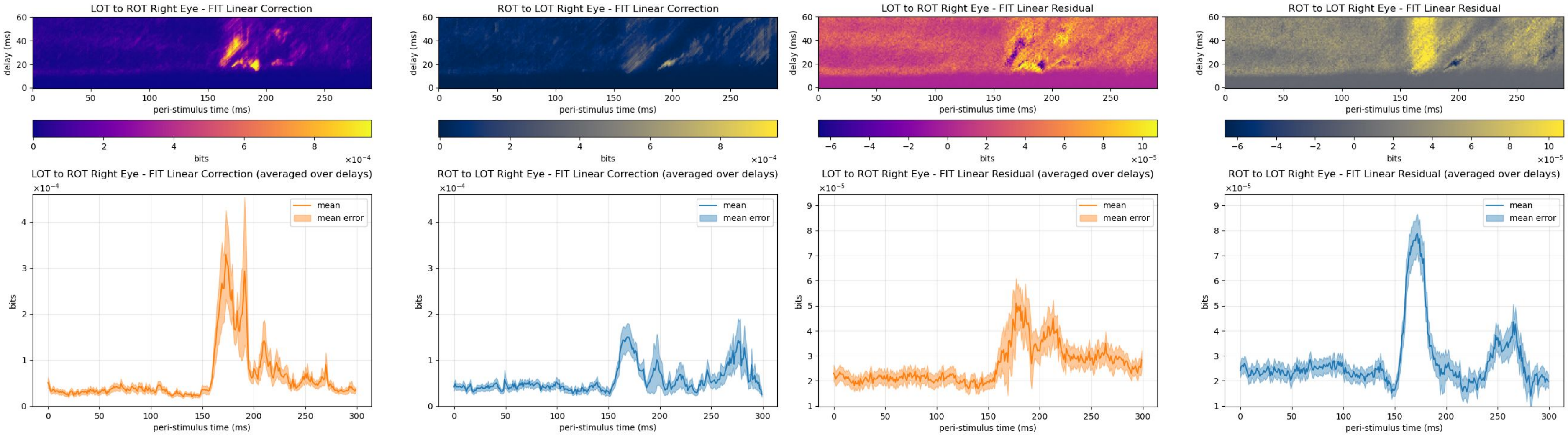
03

Bias Corrections



03

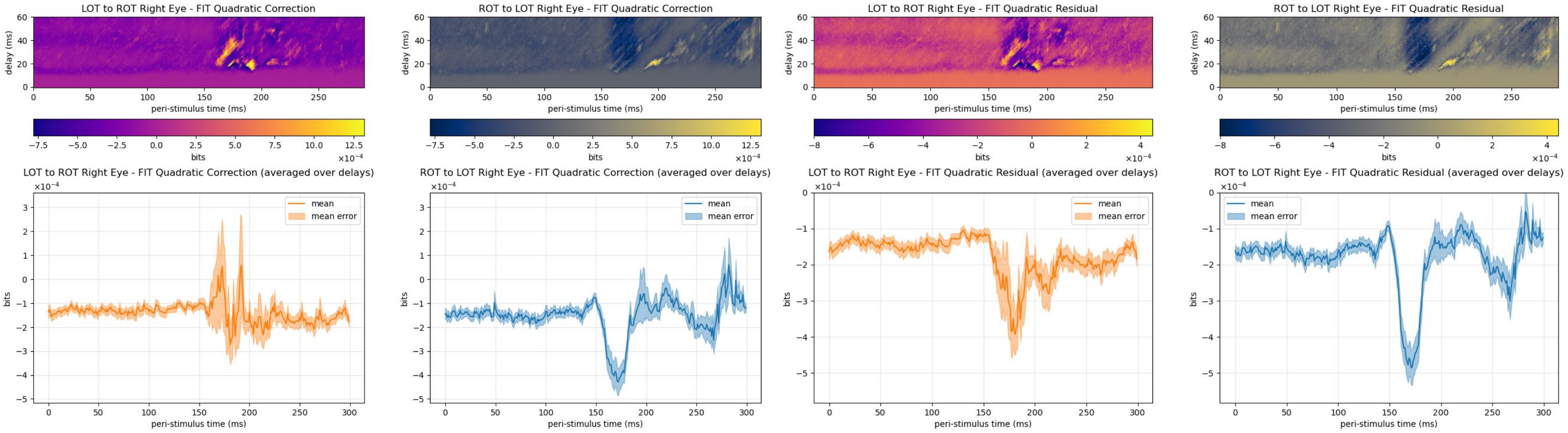
FIT Linear Correction



correction

residuals

FIT Quadratic Correction

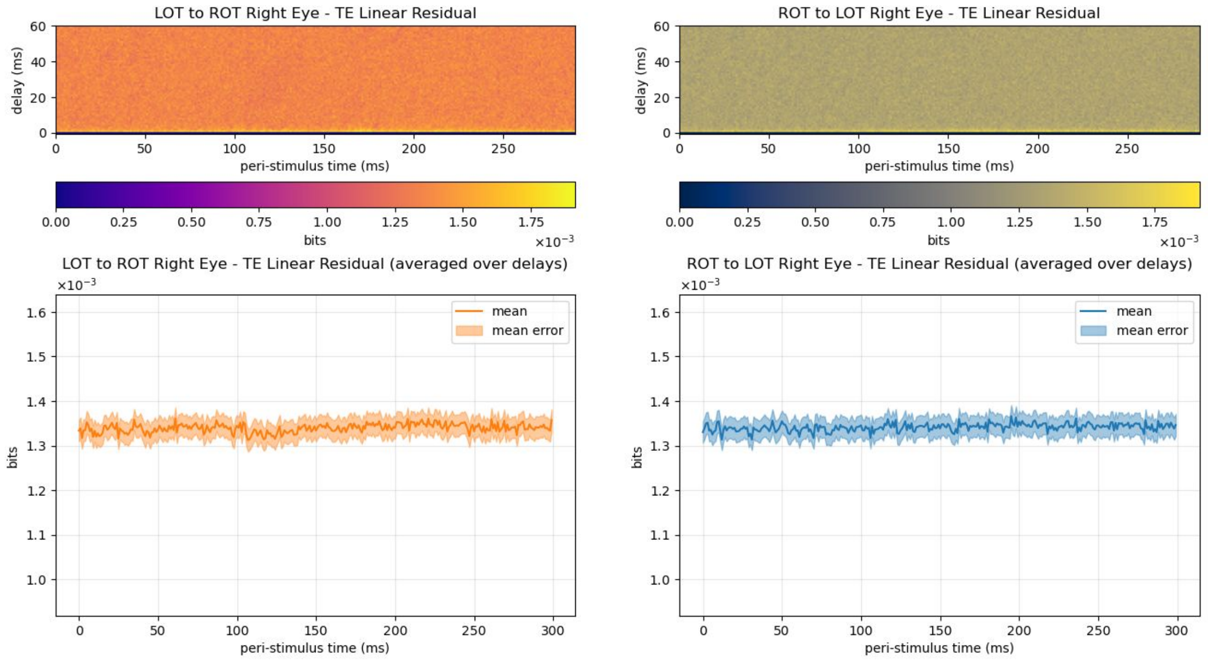


correction

residuals

03

TE Bias Corrections



03

EEG dataset 2 analysis

same analysis
as for dataset 1

- **selected electrodes**:

  relevant electrodes for visual and motor areas in OT

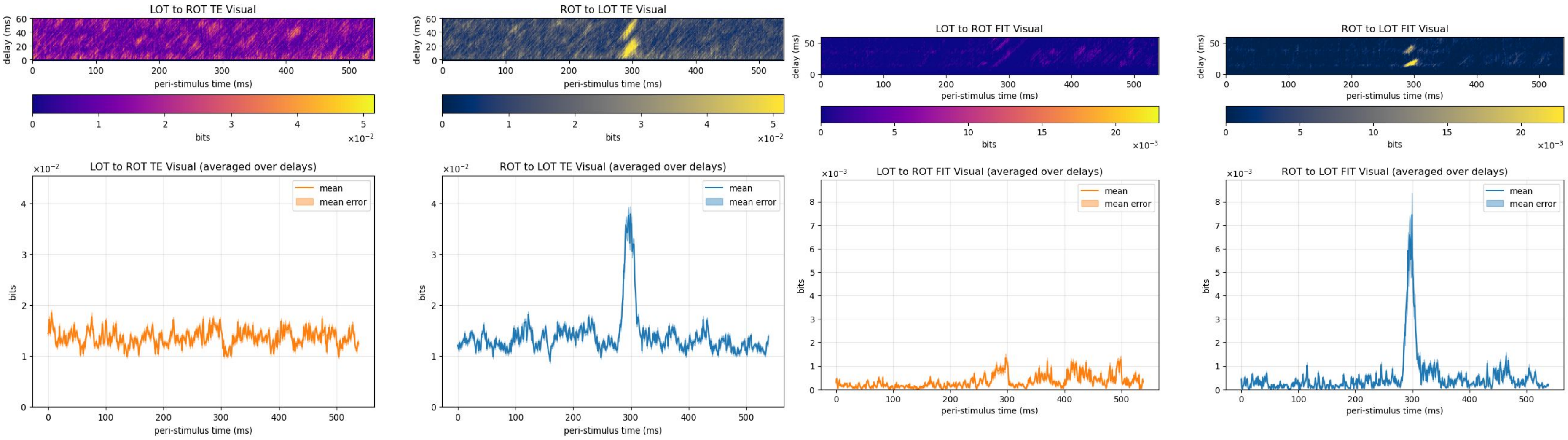
  → FIT and TE between visual areas and between visual and motor areas

- **stimulus:**

  image categories (faces, scenes, bodies, tools, scrambled)

03

Visual to Visual

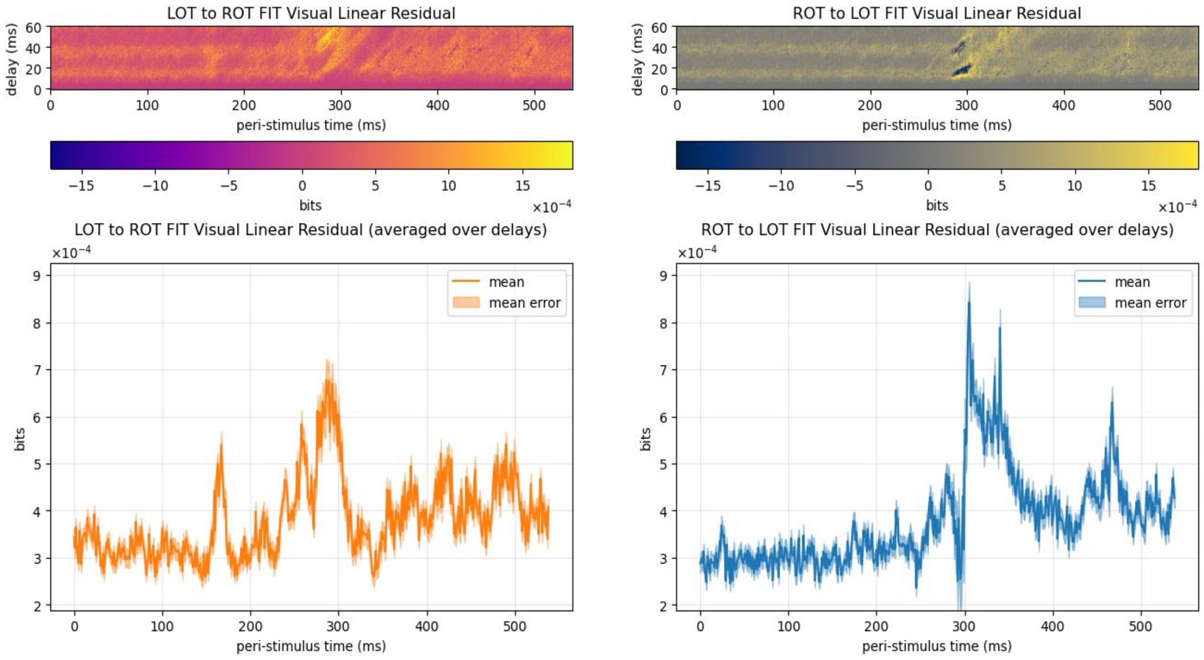


TE

FIT

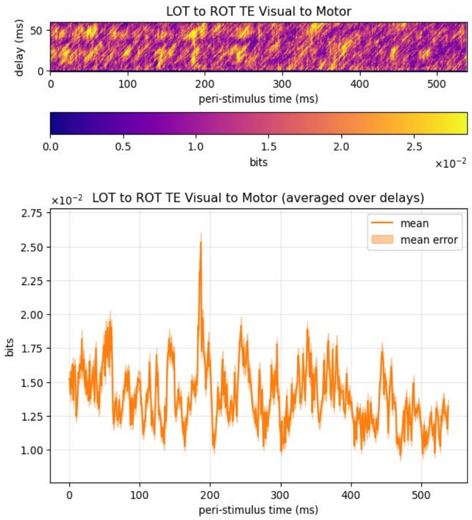
03

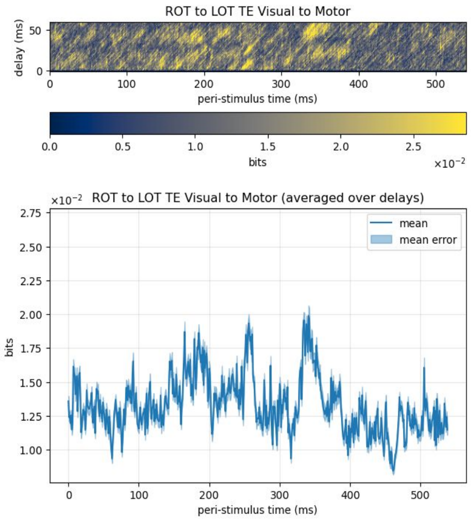
Visual to Visual: FIT corrections

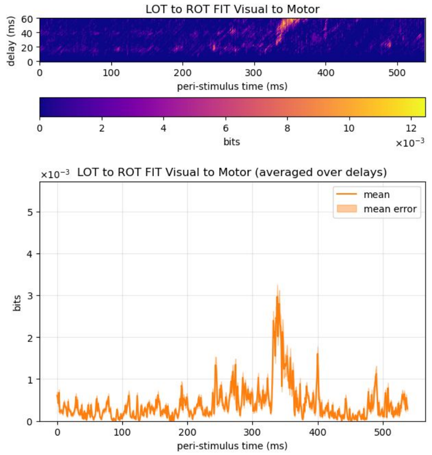


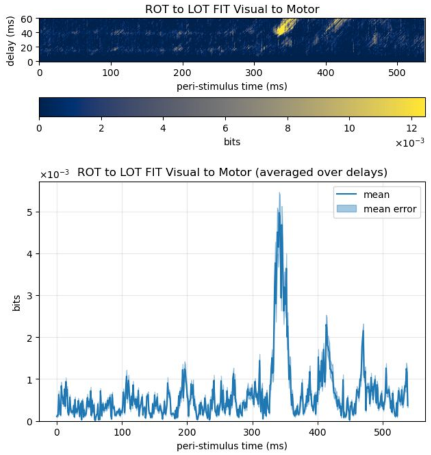
03

Visual to Motor







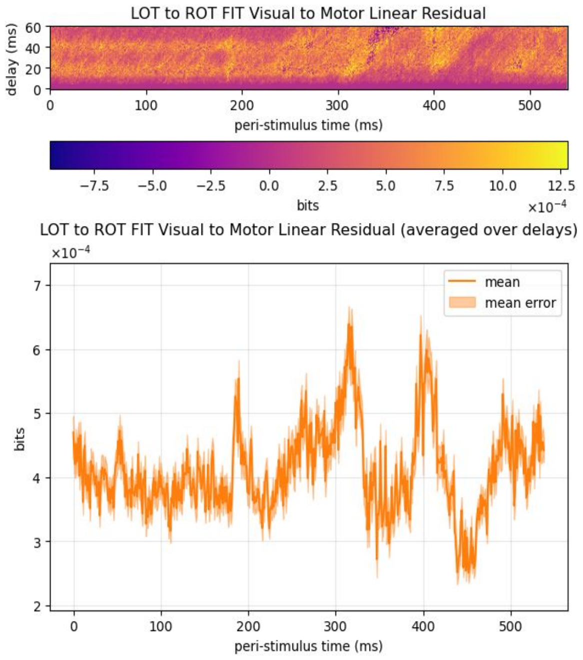


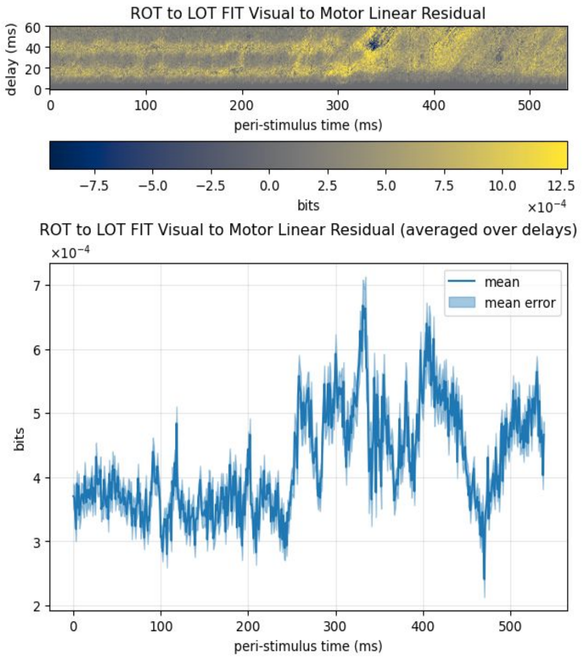
TE

FIT

03

Visual to Motor: FIT corrections





Conclusions

- successfully reproduced paper results
- investigated some unjustified procedures both for simulated and real data
- successfully applied the new method proposed in the paper to another dataset

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Physics of Data

Thank you for your attention!

Information Theory & Inference

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