Scientific Software Report: Assignment 1

Daniel Loscos Barroso
<daniel.loscosbarroso@student.kuleuven.be>

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`Time spent: 20 hours`

Discussion of the results

Matrix A1

There is not much to say about the first system. Both algorithms work fine in single and double precision. The double precision version provides more accurate results.

This good behavior can be explained by the regularity of the A1 matrix. All the eigenvalues are in a similar order of magnitude (low condition number) and the diagonal elements that the not-pivoting algorithm uses are all different from 0.

Matrix A2

For the second system our four algorithms present the same curious result: a very small backwards error and a much bigger forward error (a million times bigger for Gauss-pivot in double precision). It is almost as if $x \neq x^*$ but $Ax = Ax^*$.

That is the case indeed. Our matrix is not a max rank matrix (you can see that one of the eigenvalues is almost 0, although not exactly 0 because of machine precision). Thus, we can interpret geometrically matrix A2 as a projection instead of and affine transformation of the space we are working on. The result is two different vectors x mapping to the same point v. The system $Ax = v$ does not have one solution but infinite, our algorithms just find one of them.

Matrix A3

For this matrix the pivoting methods work but the non-pivoting algorithms do not. This is because there is a 0 in the diagonal of A3 that the pivoting methods avoid dividing by.

Since the condition number of the matrix is reasonable, the pivoting methods provide good solutions to the system.

Matrix A4

Matrix A4 has a big condition number. There is a large difference between the order of magnitude of two subsets of eigenvalues (one is around E+10 and the other around E+5 for single precision and 1 for double precision). The big numbers numbers explain why the absolute error is much bigger than the relative error.

However the most curious result is that working with different precisions yields different eigenvalues. Not only is the order of magnitude of the small eigenvalues different when we change the working precision but also all the eigenvalues are real in double precision and two of the eigenvalues are complex in single precision.

since the calculation of the eigenvalues is done on a separate module whose code I cannot read I cannot pinpoint the exact point at which this error is introduced into our system. My assumption is that we are dealing with big numbers and because of machine representation a large error is introduced when operating with them. This error does not matter when analyzing the big eigenvalues (it is relatively small) but is much more noticeable on the small ones.

Double precision provide much more accurate results since the error introduces while working at the E+10 scope are much more smaller. We can encounter these issues when the condition number of the matrix is big.

Raw Results

Single Precision

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 Gauss 1:
Absolute 2-norm of the backward error = 2.37522818E-06
Relative 2-norm of the backward error = 2.36343865E-07
2-norm of the forward error = 1.43894610E-07
Gauss Pivot 1:
Absolute 2-norm of the backward error = 5.59141142E-07
Relative 2-norm of the backward error = 5.56365833E-08
2-norm of the forward error = 6.14390601E-08
Eigenvalues:
( 9.99999523 , 0.00000000 )
( 20.0000153 , 0.00000000 )
( 30.0000248 , 0.00000000 )
( 39.9999886 , 0.00000000 )
( 50.0000000 , 0.00000000 )
Condition number = 12.9457054
Sum of the eigenvalues = 150.000031
Gauss 2:
Absolute 2-norm of the backward error = 6.56471588E-04
Relative 2-norm of the backward error = 1.77148399E-06
2-norm of the forward error = 1.20563018
Gauss Pivot 2:
Absolute 2-norm of the backward error = 5.22000591E-05
Relative 2-norm of the backward error = 1.40861488E-07
2-norm of the forward error = 0.957422495
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Eigenvalues:
( -2.28881836E-05 , 0.00000000 )
( 10.0000267 , 0.00000000 )
( 20.0000114 , 0.00000000 )
( 99.9999313 , 0.00000000 )
( 89.9999542 , 0.00000000 )
( 30.0000381 , 0.00000000 )
( 79.9999847 , 0.00000000 )
( 39.9999847 , 0.00000000 )
( 50.0000267 , 0.00000000 )
( 60.0000000 , 0.00000000 )
( 69.9999847 , 0.00000000 )
Condition number = 16280.1758
Sum of the eigenvalues = 549.999939
Gauss 3:
Absolute 2-norm of the backward error = NaN
Relative 2-norm of the backward error = NaN
2-norm of the forward error = NaN
Gauss Pivot 3:
Absolute 2-norm of the backward error = 3.51697709E-05
Relative 2-norm of the backward error = 3.81305370E-08
2-norm of the forward error = 1.27815274E-05
Eigenvalues:
( -39.4374237 , 0.00000000 )
( 3.70014167 , 0.00000000 )
( 52.2164192 , 35.0767365 )
( 52.2164192 , -35.0767365 )
( 100.375175 , 0.00000000 )
( 22.8969707 , 0.00000000 )
( 32.5654640 , 0.00000000 )
( 45.0288620 , 0.00000000 )
( 80.6666565 , 0.00000000 )
( 65.0037460 , 0.00000000 )
( 71.2391891 , 0.00000000 )
Condition number = 761.441589
Sum of the eigenvalues = 486.471649
Gauss 4:
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Absolute 2-norm of the backward error = 143415.750
Relative 2-norm of the backward error = 4.74749498E-07
2-norm of the forward error = 5.99658298
Gauss Pivot 4:
Absolute 2-norm of the backward error = 785090.312
Relative 2-norm of the backward error = 2.59888634E-06
2-norm of the forward error = 74.4654770
Eigenvalues:
( 4.99999539E+10 , 0.00000000 )
( -4.99999580E+10 , 0.00000000 )
( 4.00000041E+10 , 0.00000000 )
( -4.00000328E+10 , 0.00000000 )
( 3.00000174E+10 , 0.00000000 )
( -3.00000195E+10 , 0.00000000 )
( -1.99999693E+10 , 0.00000000 )
( 2.00000041E+10 , 0.00000000 )
( 1.00000072E+10 , 0.00000000 )
( -9.99998771E+09 , 0.00000000 )
( -10091.5586 , 11313.8447 )
( -10091.5586 , -11313.8447 )
( 4535.71680 , 0.00000000 )
( 20997.8848 , 0.00000000 )
Condition number = 22554.8555
Sum of the eigenvalues = 24806.4844
```

Double Precision

```
 Gauss 1:
Absolute 2-norm of the backward error = 6.7386056314926527E-015
Relative 2-norm of the backward error = 6.7051587721426181E-016
2-norm of the forward error = 4.3466507513761424E-016
Gauss Pivot 1:
Absolute 2-norm of the backward error = 1.7342238036525468E-015
Relative 2-norm of the backward error = 1.7256160377712542E-016
2-norm of the forward error = 3.6880726988814704E-016
Eigenvalues:
( 9.9999999999999858 , 0.0000000000000000 )
( 19.999999999999986 , 0.0000000000000000 )
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( 29.999999999999996 , 0.0000000000000000 )
( 40.000000000000014 , 0.0000000000000000 )
( 49.999999999999993 , 0.0000000000000000 )
Condition number = 12.945703004095062
Sum of the eigenvalues = 149.99999999999997
Gauss 2:
Absolute 2-norm of the backward error = 9.4261230274631840E-011
Relative 2-norm of the backward error = 2.5436327713806479E-013
2-norm of the forward error = 57.895051836086047
Gauss Pivot 2:
Absolute 2-norm of the backward error = 2.2595370717450927E-013
Relative 2-norm of the backward error = 6.0973451408336525E-016
2-norm of the forward error = 1.1991270841302213
Eigenvalues:
( -6.2172489379008766E-014 , 0.0000000000000000 )
( 10.000000000000052 , 0.0000000000000000 )
( 99.999999999999844 , 0.0000000000000000 )
( 20.000000000000032 , 0.0000000000000000 )
( 90.000000000000057 , 0.0000000000000000 )
( 29.999999999999897 , 0.0000000000000000 )
( 39.999999999999886 , 0.0000000000000000 )
( 80.000000000000014 , 0.0000000000000000 )
( 49.999999999999844 , 0.0000000000000000 )
( 60.000000000000107 , 0.0000000000000000 )
( 70.000000000000000 , 0.0000000000000000 )
Condition number = 326510207.73801190
Sum of the eigenvalues = 549.99999999999955
Gauss 3:
Absolute 2-norm of the backward error = NaN
Relative 2-norm of the backward error = NaN
2-norm of the forward error = NaN
Gauss Pivot 3:
Absolute 2-norm of the backward error = 1.0658141036401503E-013
Relative 2-norm of the backward error = 1.1555396400147836E-016
2-norm of the forward error = 1.0284632206957708E-013
Eigenvalues:
( -39.437442106626229 , 0.0000000000000000 )
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( 3.7001335510433089 , 0.0000000000000000 )
( 52.216501054899247 , 35.076705364192165 )
( 52.216501054899247 , -35.076705364192165 )
( 100.37501343836480 , 0.0000000000000000 )
( 22.896985632690402 , 0.0000000000000000 )
( 32.565466393222465 , 0.0000000000000000 )
( 45.028853601744387 , 0.0000000000000000 )
( 80.666615395142941 , 0.0000000000000000 )
( 65.003779827503379 , 0.0000000000000000 )
( 71.239183563707215 , 0.0000000000000000 )
Condition number = 760.69942694991676
Sum of the eigenvalues = 486.47159140659113
Gauss 4:
Absolute 2-norm of the backward error = 1.8079612712035355E-004
Relative 2-norm of the backward error = 5.9848985712601486E-016
2-norm of the forward error = 2.2009378438558701E-004
Gauss Pivot 4:
Absolute 2-norm of the backward error = 3.1355442472026053E-005
Relative 2-norm of the backward error = 1.0379599709408397E-016
2-norm of the forward error = 4.5169313818688477E-005
Eigenvalues:
( 50000000000.000076 , 0.0000000000000000 )
( -50000000000.000076 , 0.0000000000000000 )
( -40000000000.000046 , 0.0000000000000000 )
( 40000000000.000061 , 0.0000000000000000 )
( 29999999999.999928 , 0.0000000000000000 )
( -29999999999.999966 , 0.0000000000000000 )
( 20000000000.000008 , 0.0000000000000000 )
( 10000000000.000032 , 0.0000000000000000 )
( -20000000000.000004 , 0.0000000000000000 )
( -9999999999.9999809 , 0.0000000000000000 )
( 3.9999558863180700 , 0.0000000000000000 )
( 3.0000168154271858 , 0.0000000000000000 )
( 1.0000127986548584 , 0.0000000000000000 )
( 1.9998981031480512 , 0.0000000000000000 )
Condition number = 155201516.35587636
Sum of the eigenvalues = 9.9999141211262899
```