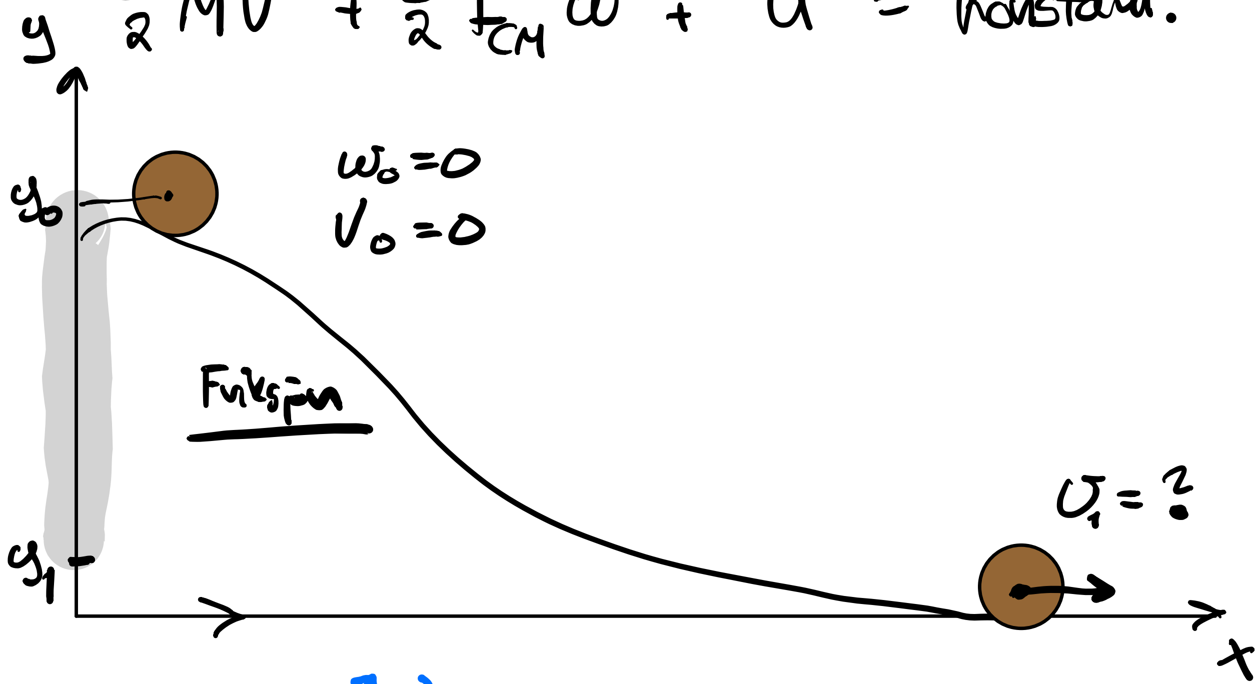
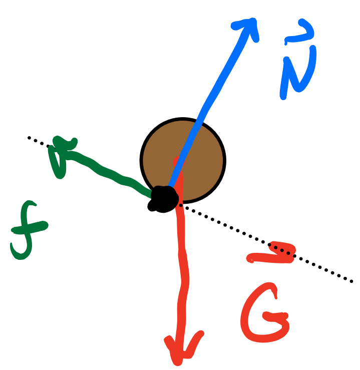
Rotasjon Energi

$$\frac{1}{2}MV^2 + \frac{1}{2}I_{CM}\omega^2 + U = \text{Konstant.}$$





$$W = \int \vec{F} \cdot \vec{v} \, dt$$

I kontaktpunkt er $\vec{v} = 0$

Friksjonskraft gjør derfor ikke arbeid på legemet.

(rulling uten å gli)

Energibevaring

$$\underbrace{\frac{1}{2}MV_0^2}_{=0} + \underbrace{\frac{1}{2}I_{cm}\omega_0^2}_{=0} + Mgy_0 = \frac{1}{2}MV_1^2 + \frac{1}{2}I_{cm}\omega_1^2 + Mgy_1$$

$$Mg(y_0 - y_1) = \frac{1}{2}MV_1^2 + \frac{1}{2}I_{cm}\omega_1^2$$

Rullebetingelse: $\boxed{\omega \cdot R = V}$

$$Mg(y_0 - y_1) = \frac{1}{2}MV_1^2 + \frac{1}{2}I_{cm}\left(\frac{V_1}{R}\right)^2$$

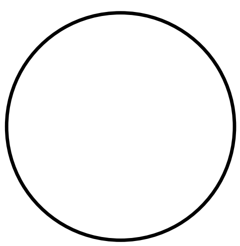
$$Mg(y_0 - y_1) = V_1^2\,\frac{1}{2}\left(M + \frac{I_{cm}}{R^2}\right)$$

$$V_1 = \sqrt{\frac{2\cancel{M}g(y_0 - y_1)\cdot\frac{1}{M}}{\cancel{\frac{1}{2}}\left(\cancel{M} + \frac{I_{cm}}{MR^2}\right)\cdot\frac{1}{M}}}$$

$$V_1 = \sqrt{\frac{2g(y_0 - y_1)}{1 + \frac{I_{cm}}{MR^2}}}$$

$$\frac{I_{cm}}{MR^2}$$

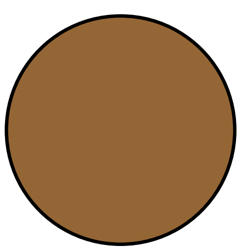
Sylinderskall



$$I_{cm} = MR^2$$

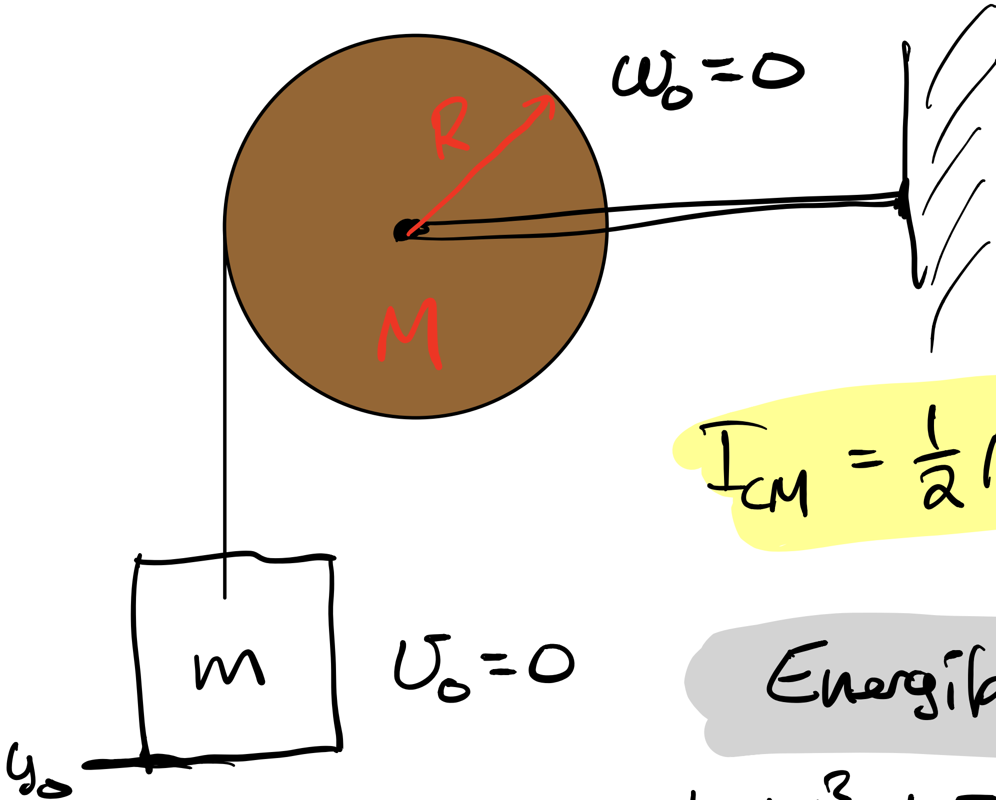
$$\frac{I_{cm}}{MR^2} = \frac{MR^2}{MR^2} = 1$$

Sylinder



$$I_{cm} = \frac{1}{2}MR^2$$

$$\frac{I_{cm}}{MR^2} = \frac{\frac{1}{2}MR^2}{MR^3} = \frac{1}{2}$$

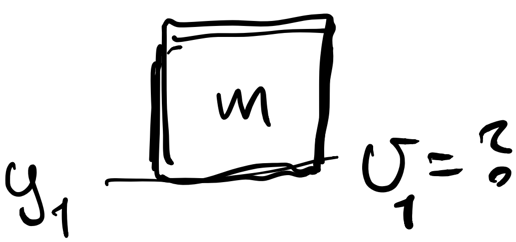


$$I_{CM} = \frac{1}{2} M R^2$$

Energikonservering

$$\frac{1}{2} M v^2 + \frac{1}{2} I_{cm} \omega^2 + U =$$

$$\underline{\text{konstant.}}$$



$$\underbrace{\frac{1}{2} m v_0^2}_{=0} + \underbrace{\frac{1}{2} I_{CM} \omega_0^2}_{=0} + mgy_0 = \frac{1}{2} m v_1^2 + \frac{1}{2} I_{cm} \omega_1^2 + mgy_1$$

$$mg(y_0 - y_1) = \frac{1}{2} m v_1^2 + \frac{1}{2} I_{cm} \omega_1^2$$

Rullebetingelse $v = R \cdot \omega$

$$\omega_1 = \frac{v_1}{R}$$

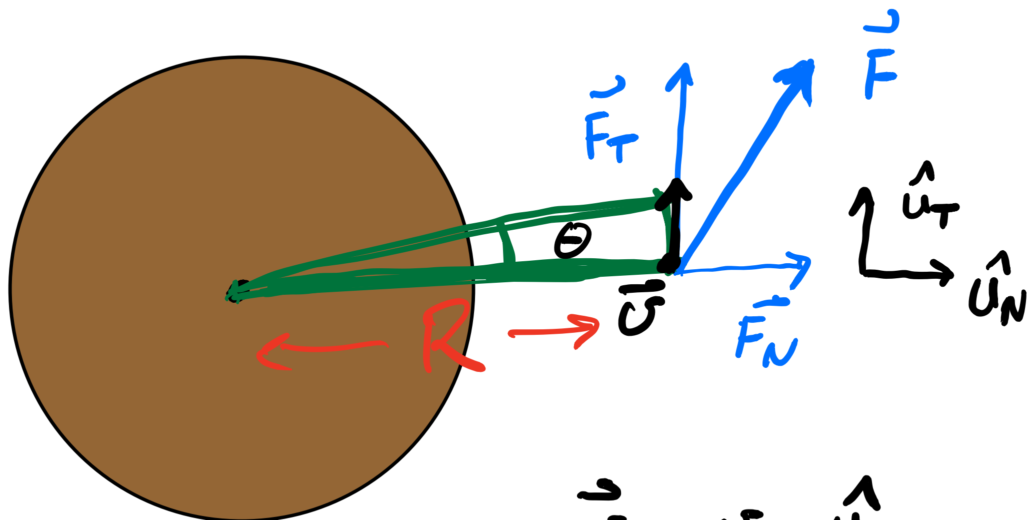
$$mg(y_0 - y_1) = \frac{1}{2} m v_1^2 + \frac{1}{2} \cdot \frac{1}{2} M \cancel{R^2} \cdot \left(\frac{v_1}{\cancel{R}}\right)^2$$

$$mg(y_0 - y_1) = \left(\frac{1}{2} m + \frac{1}{4} M\right) v_1^2$$

$$v_1 = \sqrt{\frac{mg(y_0 - y_1)}{\frac{1}{2} m + \frac{1}{4} M}}$$

Stive legemers Dynamikk kap 16

Hvorfor roterer ting?



$$W = \int \underline{\vec{F} \cdot \vec{v}}\, dt$$

$$\vec{v} = \underline{v \cdot \hat{u}_T}$$

$$\vec{F} = \underline{F_T \hat{u}_T} + F_N \hat{u}_N$$

$$W = \int F_T \cdot v\, dt$$

$$S = \theta \cdot R$$

$$W = \int F_T \cdot ds = F_T \cdot S = \underline{F_T \cdot \theta \cdot R}$$

$$\underline{W} = \Delta K = \underline{\tfrac{1}{2} I_{CM} \cdot \omega^2} - \underbrace{\tfrac{1}{2} I_{CM} \omega_0^2}_{=0}$$

$$W = \theta \cdot F_T \cdot R = \frac{1}{2} I_{CM} \cdot \omega^2$$

$$\boxed{\omega^2 = \frac{2 \cdot \theta \cdot F_T \cdot R}{I_{CM}}}$$

$\underline{\tau} = F_T \cdot R$ - dreiemoment Nm (Torque)

Arbeid $W = \Delta\theta \, R \cdot F_T$

Endring Rotasjonskinetisk energi $\Delta K = \frac{1}{2} I_{cm} \omega_1^2 - \frac{1}{2} I_{cm} \omega_0^2$

$$\frac{R \cdot F_T \cdot \Delta\theta}{\Delta t} = \frac{\frac{1}{2} I_{cm} (\omega(t+\Delta t))^2 - \frac{1}{2} I_{cm} (\omega(t))^2}{\Delta t}$$

grenseverdien når $\Delta t \to 0$

$$\lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} = \omega$$

$$\lim_{\Delta t \to 0} \frac{(\omega(t+\Delta t))^2 - (\omega(t))^2}{\Delta t} = \frac{d\omega^2}{dt}$$

$$R \cdot F_T \cdot \omega = \frac{1}{2} I_{cm} \cdot \frac{d}{dt} \omega^2$$

$$\frac{d}{dt}\omega^2 = 2\omega \underbrace{\frac{d\omega}{dt}}_{\alpha}$$

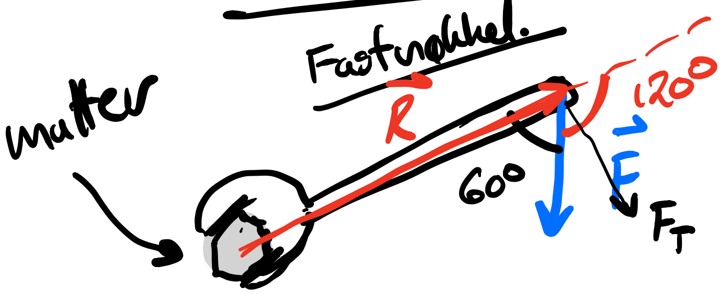
$$R \cdot F_T \cdot \cancel{\omega} = \frac{\cancel{1}}{\cancel{2}} I_{cm} \cdot \cancel{2}\cancel{\omega} \cdot \alpha$$

$$\boxed{R \cdot F_T = I_{cm}\,\alpha}$$ spinnsats

N. 2. lov for rotasjon

$F = ma$	$\tau = I_{cm}\,\alpha$

dreiemoment



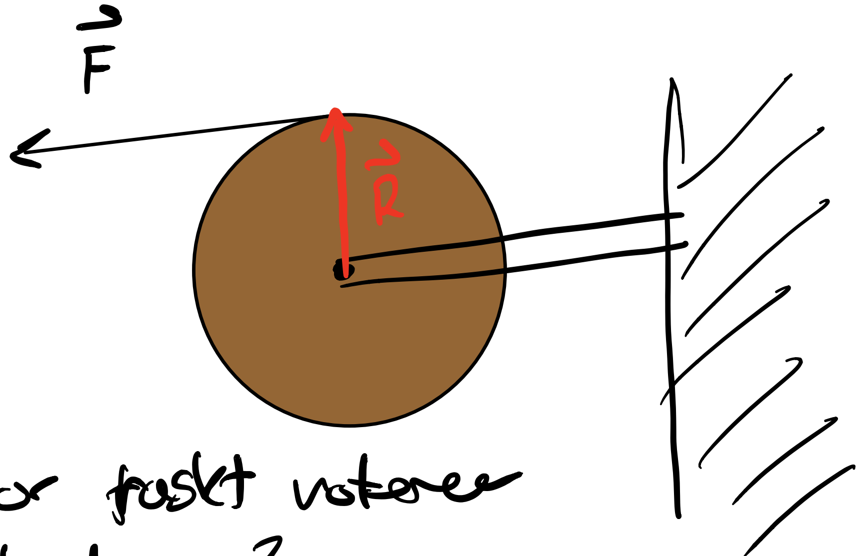
$R = 0{,}30\,m$

$F = 1000\,N$

Hvor stort er dreiemomentet?

$$F_T = F \cdot \sin 60^\circ$$

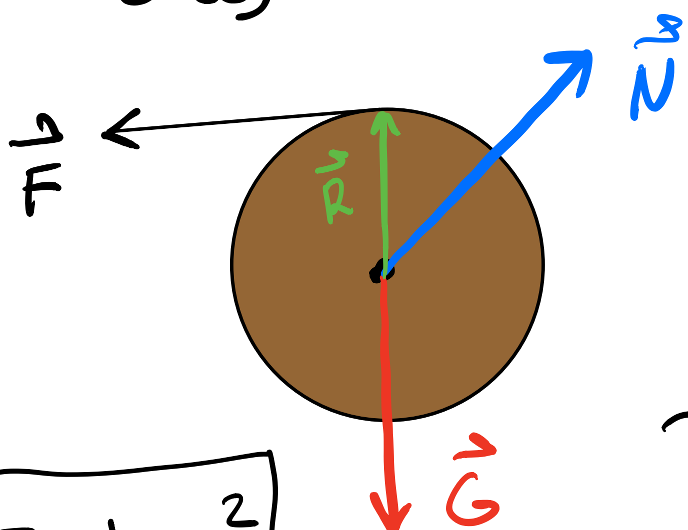
$$\tau = 0{,}30\,m \cdot 1000\,N \cdot \sin 60^\circ = \underline{260\,Nm}$$



Hvor fort roterer sylinderen?

$\omega(t)$

$\theta(t)$



$$\boxed{I_{cm} = \frac{1}{2} m R^2}$$

N. 2. lov:

$$\vec{F} + \vec{G} + \vec{N} = \underbrace{m\vec{a}}_{=0}$$

Spinnsats:

$$\tau_F + \tau_G + \tau_N = I_{cm} \alpha$$

$$\tau_F = R \cdot F$$

$$\tau_G = 0$$

$$\tau_N = 0$$

$$\cancel{R}\cdot F = \frac{1}{2} m R^{\cancel{2}} \cdot \alpha$$

$$\boxed{\alpha = \frac{2F}{mR}}$$

$$\omega = \int \alpha\, dt = \int \frac{2F}{mR}\, dt = \underline{\frac{2F}{mR} t}$$

$$\Theta = \int \omega\, dt = \int \frac{2F}{mR} t\, dt = \underline{\frac{F}{mR} t^2}$$