Bevegelsesmengde
Impuls
Kollisjoner

Kap 12

Newtons 2. lov

$$\vec{F} = m\vec{a}$$

$$\int_{t_0}^{t_1} \vec{F}\,dt = \int_{t_0}^{t_1} m\vec{a}\,dt = m\vec{v}\Big|_{t_0}^{t_1}$$

$$\int_{t_0}^{t_1} \vec{F}\,dt = m\,\vec{v}(t_1) - m\,\vec{v}(t_0)$$

$$m\vec{v}_1 - m\vec{v}_0 = \Delta\vec{p}$$

$$\vec{p}_1 - \vec{p}_0$$

$\vec{p} = m\vec{v}$ bevegelsemengde [kgm/s]

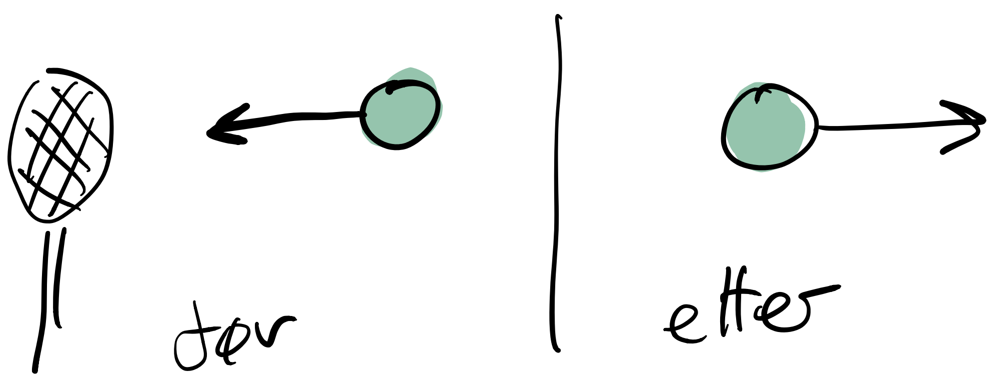
(momentum)

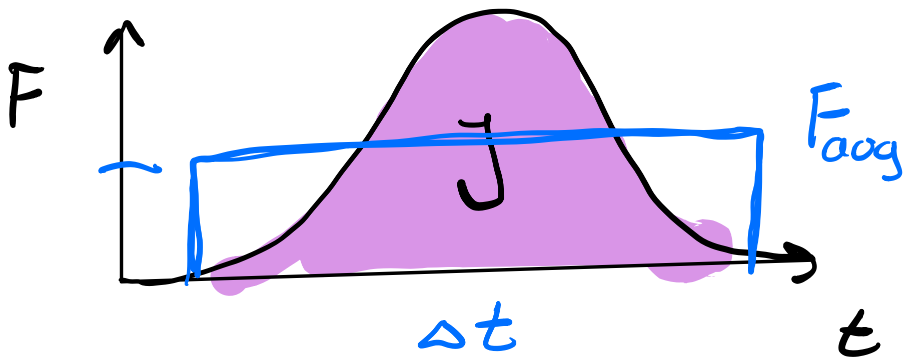
Newtons versjon av Newtons 2. lov

$$\boxed{\vec{F} = \frac{d}{dt}\vec{p}} = \frac{d}{dt}(m\vec{v})$$

$$\frac{d}{dt}(m\vec{v}) = \underbrace{\frac{dm}{dt}}_{=0}\vec{v} + m\underbrace{\frac{d\vec{v}}{dt}}_{\vec{a}}$$

Impuls $\quad J = \int_{t_0}^{t_1} \vec{F}\,dt$

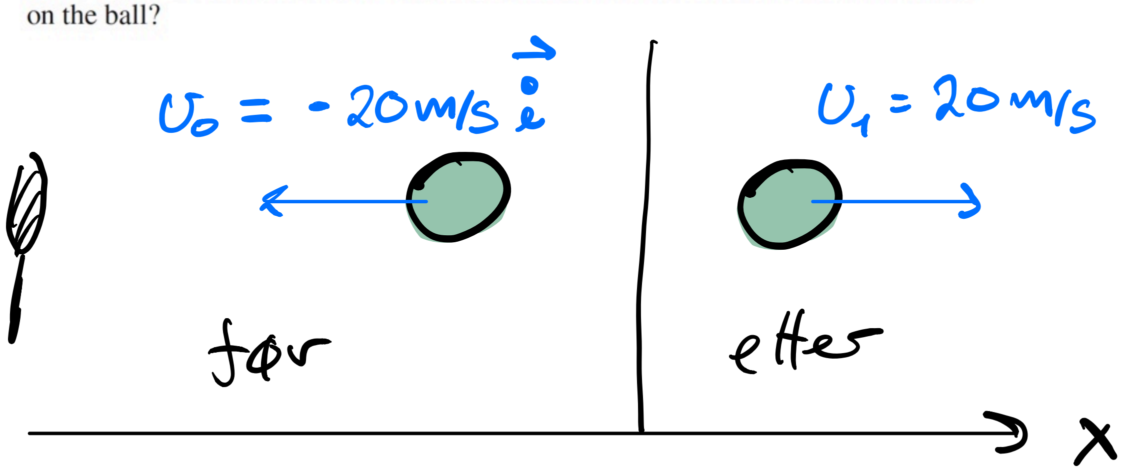




$$J = \int_{t_0}^{t_1} \vec{F}\,dt = \vec{F}_{avg} \cdot \Delta t$$

Eks

Problem: A tennis ball of mass 57 g is approaching you with a horizontal velocity $v_0 = 20\,\text{m/s}$. You hit the ball, returning it with a horizontal velocity $v_1 = 20\,\text{m/s}$, now in the opposite direction. (a) What is the impulse **J** on the ball while it is in contact with the racket during the collision? (b) The ball and racket are in contact for 2.0 ms. What is the average net force on the racket during the collision? (c) You want to return the ball as a high lob and give the ball a velocity $v_1 = 15\,\text{m/s}$ at angle of 45° upward. What is now the impulse on the ball and the net force from the racket on the ball?



$$\vec{J} = \int_{t_0}^{t_1} \vec{F}\,dt = \vec{P_1} - \vec{P_0}$$

$$\vec{P_0} = 57g \cdot (-20\ m/s\ \vec{i})$$

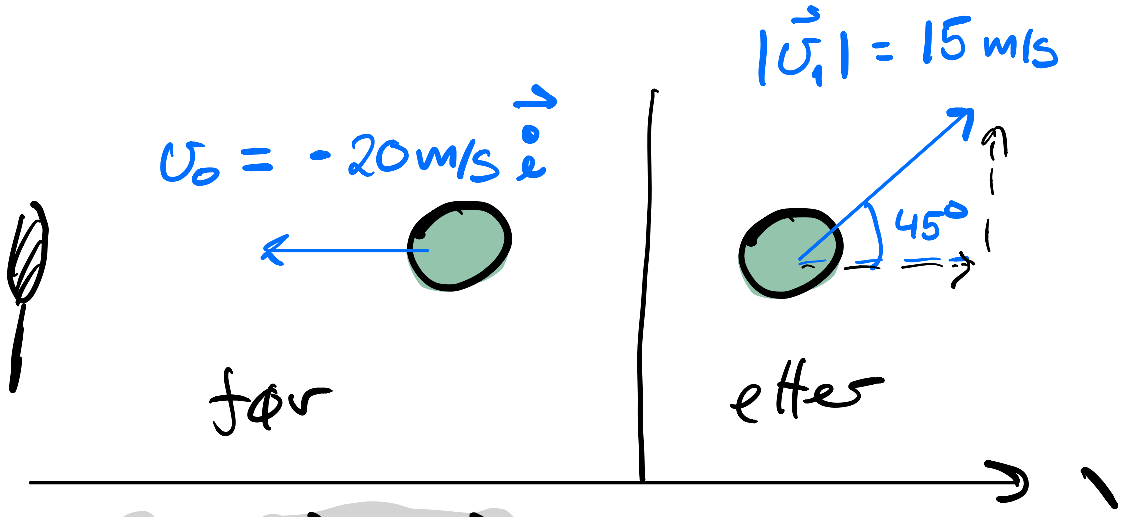
$$\vec{P_1} = 57g \cdot (20\ m/s\ \vec{i})$$

$$\vec{J} = 57g \cdot 20\ m/s\ \vec{i} - -57g \cdot 20\ m/s\ \vec{i}$$

$$\underline{\underline{\vec{J} = 2{,}3\ kg\ m/s\ \vec{i}}}$$

$$\vec{J} = \vec{F}_{avg} \cdot \Delta t$$

$$\vec{F}_{avg} = \frac{\vec{J}}{\Delta t} = \frac{2{,}3\ kg\ m/s\ \vec{i}}{0{,}002\ s} = \underline{\underline{1{,}1\,kN}}$$



$$\vec{J} = \vec{P}_1 - \vec{P}_0$$

$$\vec{P}_0 = 57g \cdot (-20\,m/s)\,\vec{i}$$

$$\vec{P}_1 = 57g \cdot 15\,m/s \cdot \left(\cos 45°\,\vec{i} + \sin 45°\,\vec{j}\right)$$

$$J_x = \underbrace{57g \cdot 15\,m/s \cdot \cos 45°}_{P_{1x}} - \underbrace{57g \cdot (-20\,m/s)}_{P_{0x}}$$

$$J_x = 1{,}74\,kg\,m/s$$

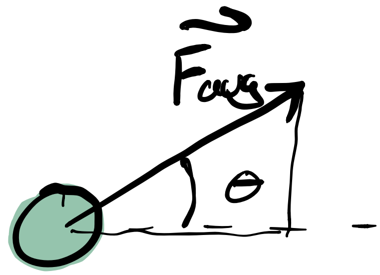
$$J_y = 57g \cdot 15\,m/s \cdot \sin 45°$$

$$J_y = 0{,}60\,kg\,m/s$$

$$\vec{J} = 1{,}74\ \text{kgm/s}\ \vec{i} + 0{,}60\ \text{kgm/s}\ \vec{j}$$

$$\vec{F}_{avg} = \frac{\vec{J}}{\Delta t} = \frac{1{,}74\ \text{kgm/s}\ \vec{i} + 0{,}60\ \text{kgm/s}\ \vec{j}}{0{,}002\ \text{s}}$$

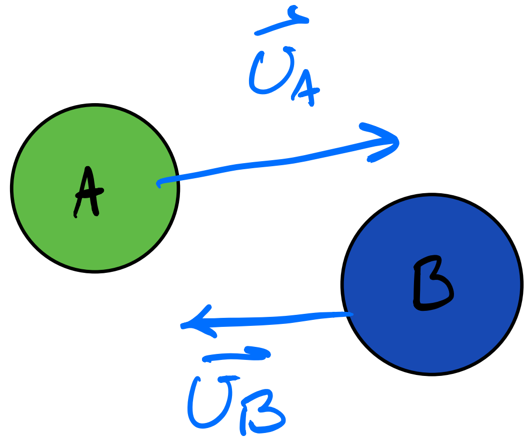
$$\vec{F}_{avg} = 0{,}87\ \text{kN}\ \vec{i} + 0{,}30\ \text{kN}\ \vec{j}$$



$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{0{,}30\ \text{kN}}{0{,}87\ \text{kN}}\right)$$

$$\underline{\theta = 19^0}$$

Bevegelsesmengden til to objekt



Kule A blir påvirket av

$\vec{F}_{B\text{ på }A}$

Kule B blir påvirket av

$\vec{F}_{A\text{ på }B}$

$$\boxed{\vec{F}_{B\text{ på }A} = -\vec{F}_{A\text{ på }B} = \vec{F}}$$

N. 3. lov

System består av A og B.

$$\vec{P} = \vec{P_A} + \vec{P_B}$$

Det virker ingen eksterne krefter på systemet.

$$\vec{F} = \vec{F_{B \text{ på } A}} = \frac{d}{dt}\vec{P_A}$$

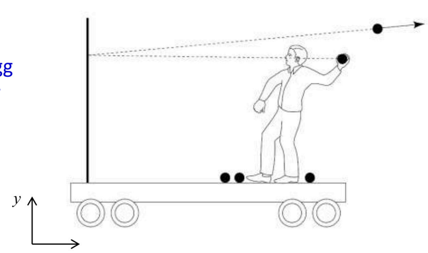
$$-\vec{F} = \vec{F_{A \text{ på } B}} = \frac{d}{dt}\vec{P_B}$$

$$\vec{F} - \vec{F} = \frac{d}{dt}\vec{P_A} + \frac{d}{dt}\vec{P}$$

$$0 = \frac{d}{dt}\left(\vec{P_A} + \vec{P_B}\right) = \frac{d}{dt}\vec{P}$$

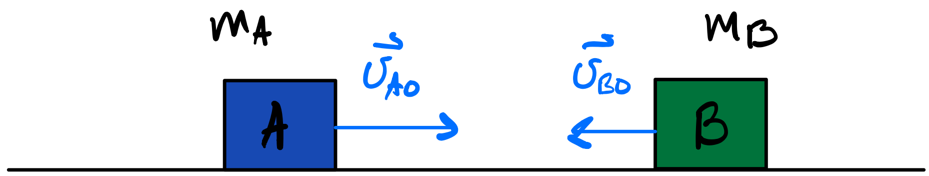
$\vec{P_A} + \vec{P_B} = \vec{P}$ er konstant

Du står på en vogn som er i ro på et friksjonsfritt spor. Du kaster en ball i en vegg som er festet i vognen. Hvis ballen spretter tilbake som vist på figuren blir da vognen satt i bevegelse?



1. Ja, den beveger seg mot høyre.
2. Ja, den beveger seg mot venstre.
3. Nei, den forblir i ro.

Kollisjoner



Bevegelsesmengden er bevart.

$$\vec{p}_{A0} + \vec{p}_{B0} = \vec{p}_{A1} + \vec{p}_{B1}$$

før kollisjon   etter kollisjon

(1) $$m_A \vec{v}_{A0} + m_B \vec{v}_{B0} = m_A \vec{v}_{A1} + m_B \vec{v}_{B1}$$

Antar energien er bevart i kollisjonen.

Elastisk kollisjon $K_{før} = K_{etter}$

(2) $$\frac{1}{2} m_A v_{A0}^2 + \frac{1}{2} m_B v_{B0}^2 = \frac{1}{2} m_A v_{A1}^2 + \frac{1}{2} m_B v_{B1}^2$$

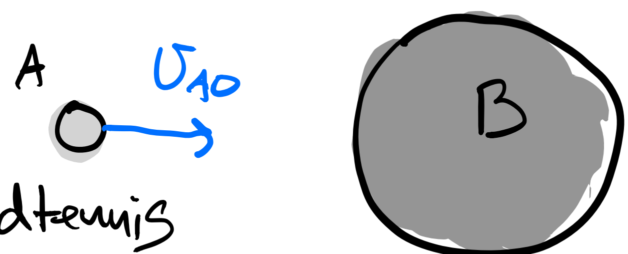
Kinetisk energi før   Kinetisk energi etter

Elastisk kollisjon:

$$v_{A1} = \frac{m_A - m_B}{m_A + m_B} v_{A0}$$

$$v_{B1} = \frac{2m_A}{m_A + m_B} v_{A0}$$

Etter mye algebra.



Bordtennis ball

Bowling-kule

$$m_A << m_B \qquad \frac{m_A}{m_B} \approx 0$$

$$v_{A1} = \frac{(m_A - m_B)\frac{1}{m_B}}{(m_A + m_B)\frac{1}{m_B}} v_{A0} = \frac{\cancelto{0}{\frac{m_A}{m_B}} - \overset{1}{\frac{m_B}{m_B}}}{\cancelto{0}{\frac{m_A}{m_B}} + \overset{1}{\frac{m_B}{m_B}}} v_{A0}$$

$$v_{A1} = -v_{A0}$$

$$v_{B1} = \frac{(2m_A)\frac{1}{m_B}}{(m_A + m_B)\frac{1}{m_B}} v_{A0} \qquad \frac{2 \cdot \cancelto{0}{\frac{m_A}{m_B}}}{\cancelto{0}{\frac{m_A}{m_B}} + \cancelto{1}{\frac{m_B}{m_B}}} v_{A0}$$

$$\boxed{v_{B1} = 0}$$

Elastisk kollisjon $\quad r = 1$

Uelastisk kollisjon $\quad 0 < r < 1$

Fullstendig uelastisk kollisjon $\quad r = 0$