problem 1:

$$52a + b = 6 \pmod{71}$$

$$20a + b = 51 \pmod{71}$$

$$4a + b = 38 \pmod{71}$$

$$[20a + b = 51 \pmod{71}] - [4a + b = 38 \pmod{71}]$$

$$16a = 13 \pmod{71}$$

$$\gcd(16, 71) = 1$$

$$71 = (4)16 + 7$$

$$16 = (2)7 + 2$$

$$7 = (3)2 + 1$$

$$\therefore \quad 1 = 7 - (3)2$$

$$= 7 - (3)(16 - (2)7)$$

$$= (7)7 - (3)16$$

$$= (7)(71 - (4)16) + (-3)16$$

$$= (7)(71) + (-31)16$$

$$\therefore \quad (-31)16 = 1 \pmod{71}$$

$$16a = 13 \pmod{71}$$

$$a = 13 \times (-31) \pmod{71}$$

$$= -403 \pmod{71}$$

$$= 23 \pmod{71}$$

$$(4)23 + b = 38 \pmod{71}$$

$$b = 38 - (4) \times 23 \pmod{71}$$

$$= -54 \pmod{71}$$

$$= 17 \pmod{71}$$

$$a = 23 \quad b = 17$$

problem 3:

$$20 = 23a + b \pmod{475}$$

$$436 = 20a + b \pmod{475}$$

$$[20 = 23a + b \pmod{475}] - [436 = 20a + b \pmod{475}]$$

$$-416 = 3a \pmod{475}$$

$$\gcd(3, 475) = 1$$

$$475 = (158)3 + 1$$

$$1 = 475 + (-158)3$$

$$(-158)3 = 1 \pmod{475}$$

$$3a = -416 \pmod{475}$$

$$a = (-416)\times(-158) \pmod{475}$$

$$= 178 \pmod{475}$$

$$436 = 20 \times 175 + b \pmod{475}$$

$$b = 201 \pmod{475}$$

$\therefore R_{i+1} = 178 R_i + 201 \pmod{475}$

$$R_1 = 178 R_0 + 201 \pmod{475}$$

$$23 = 178 R_0 + 201 \pmod{475}$$

$$-178 = 178 R_0 \pmod{475}$$

$$\gcd(178, 475) = 1$$

$$475 = (2)(78 + 119$$

$$178 = (1)\ 119 + 59$$

$$119 = (2)\ 59 + 1$$

$$1 = 119 + (-2)59$$

$$= 119 + (-2)[178 - (1)\ 119]$$

$= (3)119 + (-2)178$

$= (-2)178 + (3)(475 - (2)178)$

$= (3)475 + (-8)178$

$R_0 = (-178) \times (-8) \pmod{475}$

$= 474 \pmod{475}$

$R_4 = 178\, R_3 + 201 \pmod{475}$

$= 178 \times 436 + 201 \pmod{475}$

$= 384 \pmod{475}$

$\therefore \quad a = 178 \quad b = 201 \quad R_0 = 474$

$R_4 = 384$