Problem 4:

Distinct keys:

original Caesar cipher: 26 keys, a1p1: 52 keys, a1p2: 52 keys, a1p3: (n^52) keys (n is the length of key word) Here should be 52^n

Yes, problem 2 cipher is stronger than original, since problem 2 cipher has dynamic keys to encrypt text.

Weakness:

a1p1: Capital and small letter key is distinct to show the message. If the text has more lowercase, we can easily guess the key is uppercase. Otherwise, the key is lowercase. So we only need maximum 26 guesses to get the key to transform the message.

a1p2: It has the similar problem as a1p1. It also has a problem that, since we used the previous letter to encrypt the message. If we find the key for the first letter, we can easily get all the keys.

a1p3: The key only has lowercase and uppercase letters.

Addressed weaknesses:

a1p1: Increasing the number of keys, for example we can use the ASCII table as the keys. Mixed uppercase and lowercase like "AabcBC" instead of "AaBb"

a1p2: Same as the a1p1 to increasing and mixed keys. When encrypting the message we can use 2 letters previous to encrypt the current letter. Or we can invert the order of message first, then use 2 letters previous to encrypt.

a1p3: Increasing the keys.