University of Alberta
Computing Science Department
CMPUT 340 - Fall 2020

Assignment 2
Due date: October 2 at 23.59
5 Marks

Introduction to Numerical Methods (CMPUT 340)

Although this assignment requires you to write Python code, you don't need to submit your code on eClass. All you need to submit is a pdf with your answers to the questions below. Thus, feel free to use Scipy's to solve the least squares problems that arise in the problems.[1]

1. (2 Marks) Different animals emit different amount of heat. The table below shows the mass in (kg) and metabolism (Watts) for different animals. If you plot the log-log plot of the data below you will

Species	Mass (kg)	Metabolism (W)
Cow	400	270
Human	70	82
Sheep	45	50
Hen	2	4.8
Rat	0.3	1.45
Dove	0.16	0.97

   observe an almost straight line, which means that mass and metabolism can be described by a function $y = kx^n$. Find the values of k and n that best describe the data above. In addition to the values of k and n, present a log-log plot with the points of the table above as well as the line of your model.

2. (3 Marks) (Heath, 2018) A planet follows an elliptical orbit, which can be represented in a Cartesian (x, y) coordinate system by the equation

$$ay^2 + bxy + cx + dy + e = x^2 \,.$$

   a) (1 Mark) Use Scipy to determine the orbital parameters a, b, c, d, e given the following observations of the planer's position:

x	1.02	0.95	0.87	0.77	0.67	0.56	0.44	0.30	0.16	0.01
y	0.39	0.32	0.27	0.22	0.18	0.15	0.13	0.12	0.13	0.15

   In addition to printing the values for the orbital parameters, plot the resulting orbit and the given data points in the (x, y) plane. One approach to plot the orbit is to create a mesh of x, y-values and use them to compute the value $z = ay^2 + bxy + cx + dy + e - x^2$. Then you can plot the (x, y, z) data as a 3D plot. The orbit that has values of $z \approx 0$ is the orbit that is given by the a, b, c, d, e values.[2]

[1] `https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.lsq_linear.html`
[2] See `https://jakevdp.github.io/PythonDataScienceHandbook/04.12-three-dimensional-plotting.html`.

b) (2 Marks) This least squares problem is nearly rank-deficient. To see what effect this has on the solution, perturb the input data slightly by adding to each coordinate of each data point a random number uniformly distributed on the interval $[-0.005, 0.005]$ and solve the least squares problem with the perturbed data. The data can be perturbed as follows, assuming your data is in `xdata` and `ydata`.

```
for i in range(len(xdata)):
    xdata[i] = xdata[i] + random.uniform(-0.005, 0.005)
    ydata[i] = ydata[i] + random.uniform(-0.005, 0.005)
```

Compare the new values for the parameters with those previously computed. What effect does this difference have on the plot of the orbit? Can you explain this behavior?