University of Alberta
Computing Science Department
CMPUT 340 - Fall 2020

Assignment 1
Due date: September 17

Introduction to Numerical Methods (CMPUT 340)

How to submit your assignment: You can either answer the questions on a paper, take a photo of the paper, generate a pdf, and then submit the file to eClass. An alternative is to answer the questions on a tablet, generate a pdf of your solutions, and then submit the file to eClass.

1. Answer the following questions about $\sin(x)$. Recall that $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$ and $\lim_{x\to 2\pi} \frac{\sin(x)}{x} = 0$

   a) Compute the condition number of $\sin(x)$.

   b) For which values of x the function $\sin(x)$ is well-conditioned?

   c) For which values of x the function $\sin(x)$ is ill-conditioned?

2. List the numerical problems one could have by solving the quadratic formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ in a floating-point system.

3. (Heath, 2018)

   a) Using a four-digit decimal arithmetic compute the formula of the area of the planet using the formula $A = 4\pi r^2$ for $r = 6370$ km.

   b) Using the same formula and precision, compute the difference in surface area if the value for the radius is increased by 1 km.

   c) Since $dA/dr = 8\pi r$, the change in surface area is approximated by $8\pi rh$, where h is the change in radius. Use this formula, still with four-digit arithmetic, to compute the difference in surface area due to an increase of 1 km in radius. How does the value obtained using this approximate formula compare with that obtained from the "exact" formula in part b?

   d) Determine which of the previous two answers is more nearly correct by repeating both computations using heigher precision, say, six-digit decimal arithmetic.

   e) Explain the results you obtained in parts a-d.

4. The 2-norm requires one to square all values of a given vector before taking the square root of their sum.

$$||x||_2 = \left(\sum_{i=1}^{n} |x_i|^2 \right)^{1/2}$$

   A problem with this norm is that the squared value of an entry x_i can overflow. Derive a procedure that prevents overflow from happening. As a hint, think of how you can modify the values x_i while not changing the value of the norm.

5. (Heath, 2018) Write the LU factorization for the following matrix. Show both L and U explicitly.

$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$