University of Alberta
Computing Science Department
CMPUT 340 - Fall 2020

Assignment 3
Due date: October 23 at 23.59
5 Marks

Introduction to Numerical Methods (CMPUT 340)

Although this assignment requires you to write Python code, you don't need to submit your code to eClass. All you need to submit is a pdf with what is being asked in the questions below.

1. (2 Marks) (Heath 2018) How many zeros does the function $f(x) = \sin(10x) - x$ have? Write Python code to find all zeros of the function. Plot the graph of the function to have an idea of the range of x-values you will need to search over. You can use either Newton's or the Interval Bisection algorithm to solve the problem (you can implement the algorithm or use a library routine). You need to initialize the algorithm with different x_0-values or a, b-intervals to be able to retrieve all zeros. Plot the function and highlight the roots you encountered (e.g., plot the function and add a bullet on each root).

2. (3 Marks) You will implement methods for finding the square root of a number.

   a) (1 Mark) Implement the following iterative rule, where z is the number you want to compute the square root for and x_0 is a guess of the square root of z.

   $$x_{k+1} = (z + x_k)/(1 + x_k)$$

   Test the algorithm with $z = 81$ and $x_0 = 70$. Plot the errors $x_k - \sqrt{z}$ for the iterations of the algorithm and explain what is happening with the even and odd iterations.

   b) (1 Mark) Implement the following iterative rule and test it with $z = 81$ and $x_0 = 70$; plot the errors $x_k - \sqrt{z}$ for the iterations of the algorithm.

   $$x_{k+1} = 0.5(x_k + z/x_k)$$

   Which algorithm is converging faster? What is the convergence rate of the second algorithm?

   c) (1 Marks) The second algorithm was obtained by applying Newton's formula to the equation $z = x^2$. Derive the update rule to find the n-th root of z. You can test the formula on a few examples to ensure your derivation is correct.