CSC343 Assignment 3

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November 2021

1 Database Design

1. (a) Since G is not on the right side of all FDs, G has to be a component for all super keys and thus part of the candidate keys. Thus, we only need to find combinations that can form $ABCDEF$.

$ABCEDF+ = \{ABCDEF\}$

$ABCEF+ = \{ABCDEF\}, BCDEF+ = \{ABCDEF\}, ABCDE+ = \{ABCDEF\}, ABDEF+ = \{ABCDEF\}, ACDEF+ = \{ABCDEF\}, ABCDF+ = \{ABCDEF\}$

Each of the above can be further trimmed with FD provided:

$BCEF+ = \{ABCEF\}, ABCE+ = \{ABCEF\}, ABEF+ = \{ABCEF\}, ACEF+ = \{ABCEF\}$
$BCEF+ = \{BCDEF\}, BCDE+ = \{BCDEF\}$
$ABCE+ = \{ABCDE\}, BCDE+ = \{ABCDE\}, ABDE+ = \{ABCDE\}, ACDE+ = \{ABCDE\}, ABCD+ = \{ABCDE\}$
$ABEF+ = \{ABDEF\}, ABDE+ = \{ABDEF\}, ABDF+ = \{ABDEF\}$
$ACDE+ = \{ACDEF\}, ACDF+ = \{ACDEF\}$
$ABCF+ = \{ABCDF\}, BCDF+ = \{ABCDF\}, ABDF+ = \{ABCDF\}, ACDF+ = \{ABCDF\}$

The unique ones are:
$ABCD, ABCE, ABCF, ABDE, ABDF, ABEF, ACDE, ACDF, ACEF, BCDE, BCDF, BCEF$

Each of the above can be further trimmed down:

$ABC+ = \{ABCD\}, BCD+ = \{ABCD\}, ABD+ = \{ABCD\}, ACD+ = \{ABCD\}$ $BCE+ = \{ABCE\}, ABE+ = \{ABCE\}, ACE+ = \{ABCE\}$
$BCF+ = \{ABCF\}, ABF+ = \{ABCF\}, ACF+ = \{ABCF\}$
$ABE+ = \{ABDE\}, ABD+ = \{ABDE\}$
$ABF+ = \{ABDF\}$
$ABE+ = \{ABEF\}$
$ACD+ = \{ACDE\}$
$ACDF$ cannot be trimmed down further.
$ACE+ = \{ACEF\}$
$BCE+ = \{BCDE\}$
$BCF+ = \{BCDF\}$
$BCE+ = \{BCEF\}$

The unique ones are:
$ABC, ABD, ABE, ABF, ACD, ACE, ACF, BCD, BCE, BCF$

Each of the above can be further trimmed down:
$BC+ = \{ABC\}, AB+ = \{ABC\}, AC+ = \{ABC\}$ $AB+ = \{ABD\}$
ABE cannot be trimmed down further.
ABF cannot be trimmed down further.
ACD cannot be trimmed down further.
ACE cannot be trimmed down further.
ACF cannot be trimmed down further.
$BC+ = \{BCD\}$ BCE cannot be trimmed down further.
BCF cannot be trimmed down further.

The unique ones are:
AB, BC, AC
As we have discussed above, G has to be in the super key. Thus, the answer is: ABG, BCG, and ACG

(b) To calculate the minimal cover for FD:

Step 1: Convert RHS into single attributes $\{B \to D, BC \to A, E \to F, AB \to C, AC \to B, AD \to E\}$
No changes made.

Step 2: Remove extra LHS Attributes Only $\{BC \to A, AB \to C, AC \to B, AD \to E\}$ have more than 1 attributes in LHS.
For $BC \to A$:
$B+ = \{B, D\}, C+ = \{C\}$
For $AB \to C$:
$A+ = \{A\}, B+ = \{B, D\}$
For $AC \to B$:
$A+ = \{A\}, C+ = \{C\}$
For $AD \to E\}$:
$A+ = \{A\}, D+ = \{D\}$
Thus, none of the LHS are extra.

Step 3: Remove redundant FD.
$\{B \to D, BC \to A, E \to F, AB \to C, AC \to B, AD \to E\}$
None of the FDs were repeated.
Thus, the set of FD provided is already the minimal cover.

(c) For R to be in BCNF given FD, each F $X \to Y$ in FD has to have a determinant X which is a super key of R. However, none of the FD has G in the LHS, which means that none are the super key of R. Thus, R is not in BCNF with given FD.

$B+ = \{BD\}, BC+ = \{ABCDEF\}, E+ = \{EF\}, AB+ = \{ABCDEF\}, AC+ = \{ABCDEF\}, AD+ = \{ADEF\}$
Thus, all of them violates BCNF.
To do a BCNF decomposition:
$S = R = \{ABCDEFG\}$
$S = \{ABCDEFG, BD\}$ pick $B \to D$ which violates BCNF.
$S = \{ABCEFG, BD\}$ remove RHS of the picked FD in the original set.
$S = \{ABCEFG, BD, BCA\}$ pick $BC \to A$ which violates BCNF.
$S = \{BCEFG, BD, BCA\}$ remove RHS of the picked FD in the original set.

$S = \{BCEFG, BD, BCA, EF\}$ pick $E \rightarrow F$ which violates BCNF.
$S = \{BCEG, BD, BCA, EF\}$ remove RHS of the picked FD in the original set.
$AB \rightarrow C$ and $AC \rightarrow B$ have not violated S in the subset BCA $AD \rightarrow E$ does not correspond to any of the sub-relations in S.

Thus, one way of BCNF decomposition shows $S = \{BCEG, BD, BCA, EF\}$.

(d) For R to be in 3NF, it has to meet the followings:
For each F: $X \rightarrow A$ in FD, X is a super key or A is a prime attribute.

For $B \rightarrow D$, B is not a super key nor is D is prime attribute as D is not part of the key.

Thus, R is not in 3NF.

Step 1: Find minimal cover of FD, which is solved previously.
$\{B \rightarrow D, BC \rightarrow A, E \rightarrow F, AB \rightarrow C, AC \rightarrow B, AD \rightarrow E\}$

Step 2: For each FD $X \rightarrow Y$, create $R_i = XY$
$R_1 = \{BD\}, R_2 = \{BCA\}, R_3 = \{EF\}, R_4 = \{ABC\}, R_5 = \{ACB\}, R_6 = \{ADE\}$

Step 3: If the key K of R does not occur in any relation R_i, create 1 more relation $R_i = K$.
The candidate keys of R are ABG, BCG, and ACG, and 1 more relation from any one of them would be sufficient.

Duplicate relations will also be removed.

Thus, the answer is: $R = \{BD, BCA, EF, ADE, ABG\}$

2. Prove that if a relation s has only one-attribute keys, S is in BCNF if and only if it is in 3NF.

For S to be in BCNF, each non-trivial F in FD where $X \rightarrow A$ has to be that X is a super key.

For S to be in 3NF, each F in FD where $X \rightarrow A$ has to be either that X is a super key or A s a prime attribute which means that A is part of a key.

However, S only has one-attribute keys. If S meets 3NF purely through the condition that, for each $X \rightarrow A$, A is a prime attribute, A is also the prime key, which results in the closure of X being the entire relation S, making X a super key of S, and also a redundant FD at the same time.

If there are no redundant FD in relation S, S can only meet definition of 3NF through the condition that X is a super key while A is not a prime attribute. If there are redundant FD in relation S, under both conditions for 3NF, X will be a super key regardless.

Thus, if S does not satisfy 3NF, based on the above analysis, it indicates that there exists one F where $X \rightarrow A$ in FD that X is not a super key, which contradicts with S satisfying BCNF at the same time.

Therefore, in this situation, S can satisfy BCNF if and only if S can satisfy 3NF.

2 Entity-Relationship model