CSC343: Assignment 3

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Database Design

1A)

- Step One: Find all superkeys (F will not be in any superkey, as it is not on any FD LHS.)

$AG^+ = \{AG\}$
$BG^+ = \{BDG\}$
$CG^+ = \{CG\}$
$DG^+ = \{DG\}$
$EG^+ = \{EFG\}$

$ABG^+ = \{\textbf{ABCDEFG}\}$
$ACG^+ = \{\textbf{ABCDEFG}\}$
$ADG^+ = \{ADEFG\}$
$AEG^+ = \{AEFG\}$
$BCG^+ = \{\textbf{ABCDEFG}\}$
$BDG^+ = \{BDG\}$
$BEG^+ = \{BDEFG\}$
$CDG^+ = \{CDG\}$
$CEG^+ = \{CEFG\}$
$DEG^+ = \{DEFG\}$

$ABCG^+ = \{\textbf{ABCDEFG}\}$
$ABDG^+ = \{\textbf{ABCDEFG}\}$
$ABEG^+ = \{\textbf{ABCDEFG}\}$
$ACDG^+ = \{\textbf{ABCDEFG}\}$
$ACEG^+ = \{\textbf{ABCDEFG}\}$
$ADEG^+ = \{ADEFG\}$
$BCDG^+ = \{\textbf{ABCDEFG}\}$
$BCEG^+ = \{\textbf{ABCDEFG}\}$
$BDEG^+ = \{BDEFG\}$

$ABCDG^+ = \{\textbf{ABCDEFG}\}$
$ABCEG^+ = \{\textbf{ABCDEFG}\}$
$ABDEG^+ = \{\textbf{ABCDEFG}\}$
$ACDEG^+ = \{\textbf{ABCDEFG}\}$
$BCDEG^+ = \{\textbf{ABCDEFG}\}$

- Step Two: Remove superkeys with redundant attributes

$ABG^+ = \{ABCDEFG\}$
$ACG^+ = \{ABCDEFG\}$
$BCG^+ = \{ABCDEFG\}$
$\mathbf{ABG}CD^+ = \{ABCDEFG\}$
$\mathbf{ABG}CE^+ = \{ABCDEFG\}$
$\mathbf{ABG}DE^+ = \{ABCDEFG\}$
$\mathbf{ACG}DE^+ = \{ABCDEFG\}$
$\mathbf{BCG}DE^+ = \{ABCDEFG\}$

$\mathbf{ABG}C^+ = \{ABCDEFG\}$
$\mathbf{ABG}D^+ = \{ABCDEFG\}$
$\mathbf{ABG}E^+ = \{ABCDEFG\}$
$\mathbf{ACG}D^+ = \{ABCDEFG\}$
$\mathbf{ACG}E^+ = \{ABCDEFG\}$
$\mathbf{BCG}D^+ = \{ABCDEFG\}$
$\mathbf{BCG}E^+ = \{ABCDEFG\}$

- Answer: Remaining superkeys are minimal, and therefore are candidate keys

  ABG, ACG, BCG

1B)

- Step One: Rewrite all FDs so that each RHS only contains one attribute

$$
\begin{aligned}
B &\rightarrow D \\
BC &\rightarrow A \\
E &\rightarrow F \\
AB &\rightarrow C \\
AC &\rightarrow B \\
AD &\rightarrow E
\end{aligned}
$$

- Step Two: Check for possibly redundant LHS attributes

B $\rightarrow D$

BC $\rightarrow A$

$B^+ = \{B, D\}$ //So C cannot be removed from the LHS of this FD

$C^+ = \{C\}$ //So B cannot be removed from the LHS of this FD

E $\rightarrow F$

AB $\rightarrow C$

$A^+ = \{A\}$ //So B cannot be removed from the LHS of this FD

$B^+ = \{B, D\}$ //So A cannot be removed from the LHS of this FD

AC $\rightarrow B$

$A^+ = \{A\}$ //So C cannot be removed from the LHS of this FD

$C^+ = \{C\}$ //So A cannot be removed from the LHS of this FD

AD $\rightarrow E$

$A^+ = \{A\}$ //So D cannot be removed from the LHS of this FD

$D^+ = \{D\}$ //So A cannot be removed from the LHS of this FD

- Step Three: Check for redundant FDs

B → D

$B^+ = \{B\}$ //The closure of the LHS does not contain the RHS

BC → A

$BC^+ = \{BCD\}$ //The closure of the LHS does not contain the RHS

E → F

$E^+ = \{E\}$ //The closure of the LHS does not contain the RHS

AB → C

$AB^+ = \{ABDEF\}$ //The closure of the LHS does not contain the RHS

AC → B

$AC^+ = \{AC\}$ //The closure of the LHS does not contain the RHS

AD → E

$AD^+ = \{AD\}$ //The closure of the LHS does not contain the RHS

- Answer: The original set FD is already a minimal cover

1C)

- R is **NOT** in BCNF given set FD;
  $E^+ = \{E, F\}$, and therefore E is the LHS of a FD which is not a superkey.

- Perform a BCNF decomposition: // Where the bold subrelations and FD sets are in BCNF

R(ABCDEFG): $\{B \to D, BC \to A, E \to F, AB \to C, AC \to B, AD \to E\}$
$\diamond E \to F$:
  R(EF): {E → F}
  R(ABCDEG): $\{B \to D, BC \to A, AB \to C, AC \to B, AD \to E\}$
  $\diamond AD \to E$:
    R(ADE): {AD → E}
    R(ABCDG): $\{B \to D, BC \to A, AB \to C, AC \to B\}$
    $\diamond B \to D$:
      R(BD): {B → D}
      R(ABCG): $\{BC \to A, AB \to C, AC \to B\}$
      $\diamond BC \to A$:
        R(ABC): {AB → C, AC → B, BC → A}
        R(BCG): {}

- Therefore, the following subrelations are in BCNF:

R(EF)
R(ADE)
R(BD)
R(ABC)
R(BCG)

1D)

- R is **NOT** in 3NF given set FD;
  $E \rightarrow F$ violates 3NF because E is not a superkey ($E^+ = (E, F) \neq (ABCDEFG)$)
  and F is not in any candidate key (where the candidate keys are ABG, ACG, and BCG)

- Perform a 3NF decomposition:

  - 1) Set FD is already a minimal basis for itself

  - 2) For each FD $X \rightarrow Y$ in FD, create subrelation XY:

    BD
    ABC
    EF
    ABC
    ABC
    ADE

  - 3) Eliminate every relation whose schema is the subset of another:

    BD
    ABC
    EF
    ~~**ABC**~~
    ~~**ABC**~~
    ADE

  - 4) No relation is a superkey for $(ABCDEFG)$, so add the schema of some key:

    ABG

- Therefore, the following subrelations are in 3NF:

R(BD)
R(ABC)
R(EF)
R(ADE)
R(ABG)

2) *"Prove that if a relation S has only one-attribute keys, S is in BCNF if and only if it is in 3NF."*

- First, let BCNF be defined as:
  For every FD $X \to Y, X^+ =$ (all attributes in the relation)

- Next, let 3NF be defined as:
  For every FD $X \to Y, X^+ =$ (all attributes in the relation)
  OR
  Y is the member of any key

- However, in the case where all keys consist of one attribute, this can be rewritten as:
  For every FD $X \to Y, X^+ =$ (all attributes in the relation)
  OR
  Y is the member of any key $\Rightarrow$
  $Y^+ =$ (all attributes in the relation) $\Rightarrow$ // Y is the only member of the key $\Leftrightarrow$ Y **is** the key
  $X^+ =$ (all attributes in the relation) // Due to the fact that $X \to Y$

- Therefore, both BCNF and 3NF share the following definition and are thus equivalent:
  For every FD $X \to Y, X^+ =$ (all attributes in the relation) $\square$

Entity-Relationship Model