CSC148 *Intro. to Computer Science*

Lecture 10: *BST Recursive Delete, Efficiency of Algorithms*

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Last week

- BST
  - Insert (and trace)
  - Iterative delete

- Today
  - More on BST
    - Recursive delete
  - Efficiency

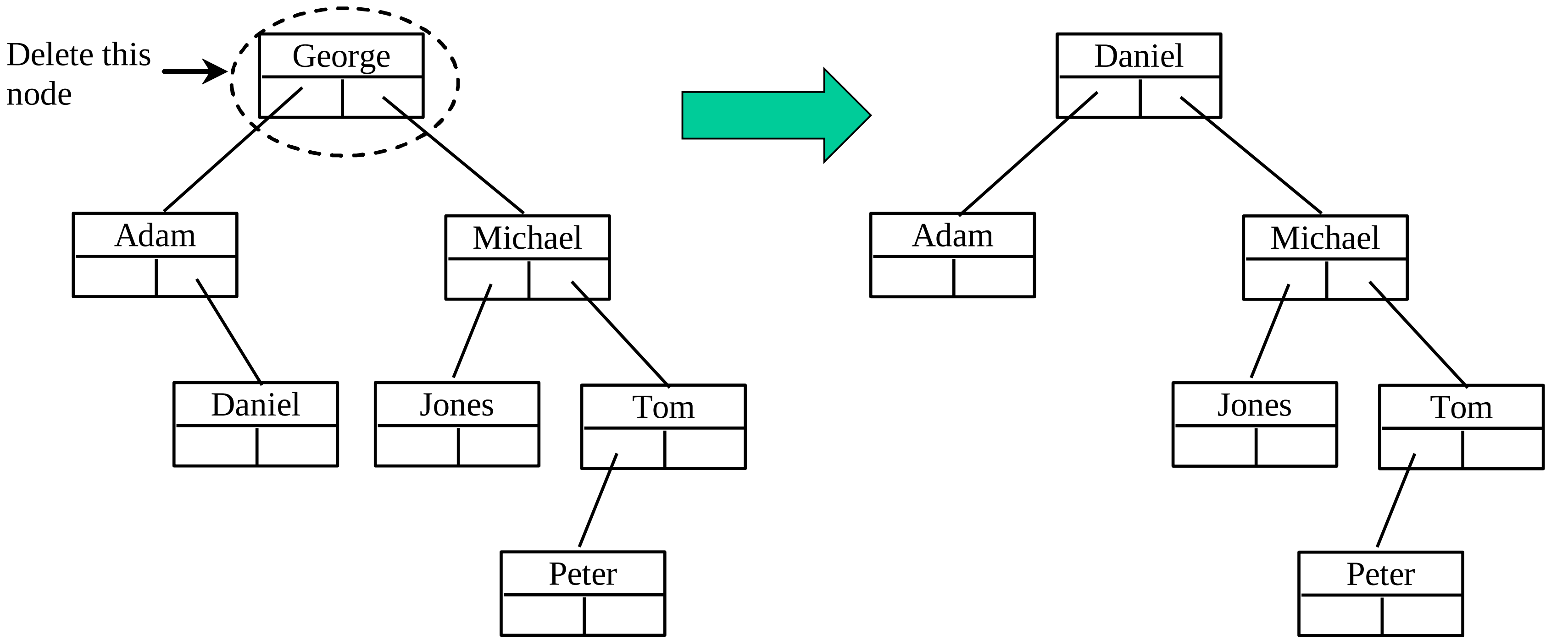
Last week

- **BST Delete**
  - Locate the node to be deleted and its parent
    - current and parent_of_current
  - **Case 1:** The current node has no left child:
    - Simply connect the parent with the right child of the current node.

Last week

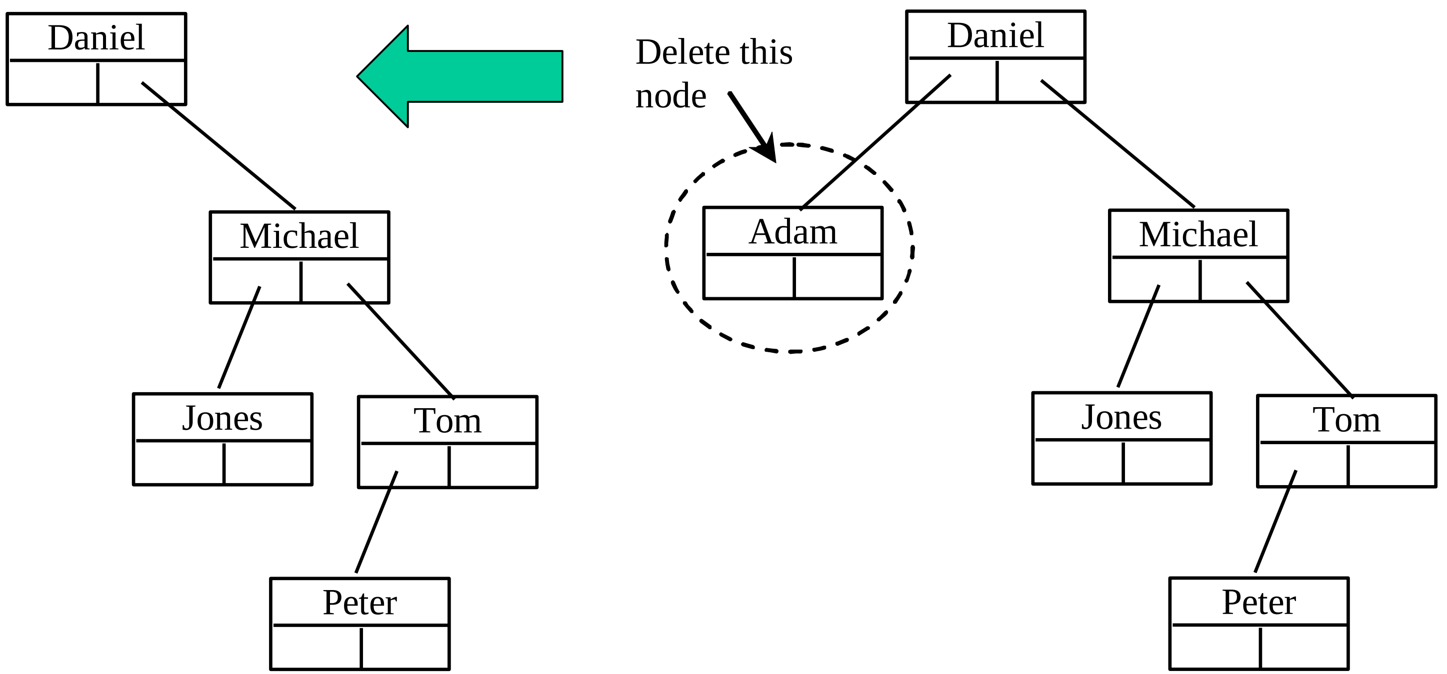
- **BST Delete**
  - **Case 2:** The current node has a left child:
    - Locate the right_most and parent_of_right_most
    - Replace the element value in the current node with the one in the right_most node,
    - Connect the parent_of_right_most node with the left child of the right_most node.

Examples



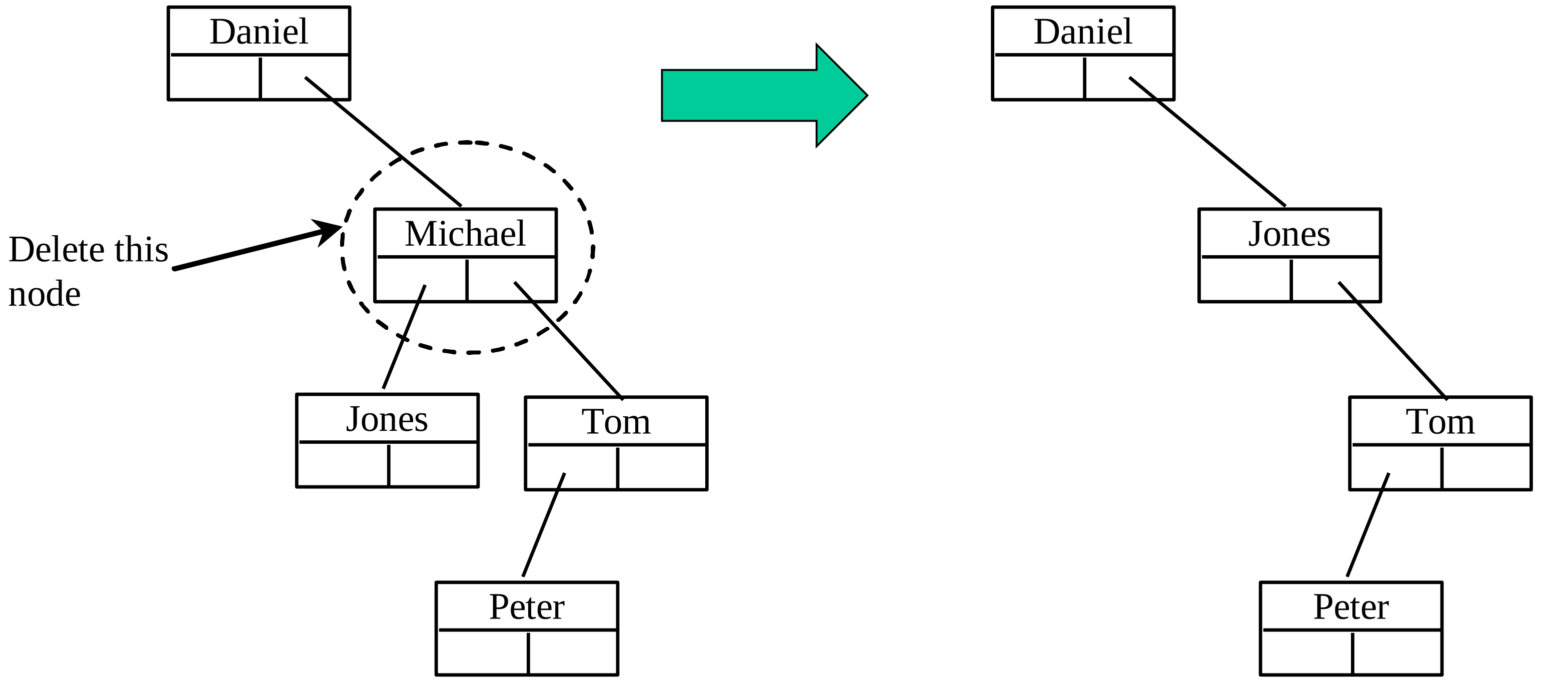
Case 1 or 2? 2

Examples



Case 1 or 2? 1

Examples



Case 1 or 2? 2

bst_delete

- First locate the nodes that contain the element and its parent. Call them current and parent.

```
parent = None
current = root

while current is not None and current.data != data:

    if data < current.data:
        parent = current
        current = current.left

    elif data > current.data:
        parent = current
        current = current.right

    else: pass # Element is in the tree pointed at by current

if current is None: return False # Element is not in the tree
```

Case 1: bst_delete

```
# Case 1: current has no left child
if current.left is None:
    # Connect the parent with the right child of the
      current node
    # Special case, assume the node being deleted is at
      root

    if parent is None:
        current = current.right
    else:
        # Identify if parent left or parent right should
          be connected
        if data < parent.data:
            parent.left = current.right
        else:
            parent.right = current.right
else:
    # Case 2: The current node has a left child
```

Case II: bst_delete

```
# Locate the rightmost node in the left subtree of
# the current node and also its parent
parent_of_right_most = current
right_most = current.left

while right_most.right is not None:
    parent_of_right_most = right_most
    right_most = right_most.right # Keep going to the right

# Replace the element in current by the element in rightMost
current.element = right_most.element

# Eliminate rightmost node
 if parent_of_right_most.right == right_most:
    parent_of_right_most.right = right_most.left
else:
    # Special case: parent_of_right_most == current
    parent_of_right_most.left = right_most.left
return True # Element deleted successfully
```

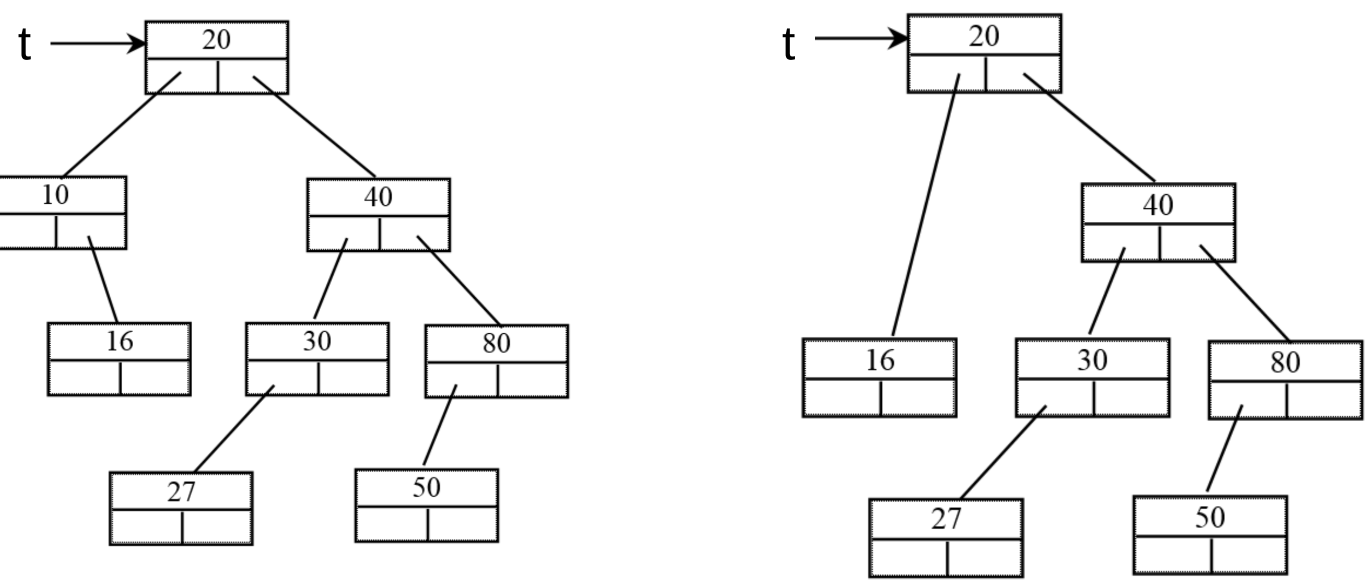
Exercise

- In Slides 3 and 4,
  - replace every *left* with *right*, every *right* with *left*, and also *largest* with *smallest*.
- and, implement the method.

- Next Topic:
  - A recursive method for BST delete.

bst_del_rec

- Let's define it as deleting a node (if exists) from the BST and returning the resulting BST
- Example:
  - t = bst_del_rec (t, 10)
  - deletes 10 from BST t and returns the reference to the tree



bst_del_rec(tree, data)

- **Base case**
  - If the tree is none return none

```
if not tree:
    return None
```

- **Recursive case I**
  - If data is less than tree data, delete it from left child

```
if data < tree.data:
    tree.left = bst_del_rec(tree.left, data)
```

- **Recursive case II**

```
if data > tree.data:
    tree.right = bst_del_rec(tree.right, data)
```

bst_del_rec(tree, data)

- What does it mean if none of the above if's have been true?
  - **We have located the tree node to be deleted**
- What next?
- There are two cases to consider …
- **Case I:**
  - If the tree node does not have a left child,
    - return the right child

```
if tree.left is None:
    return tree.right
```

bst_del_rec

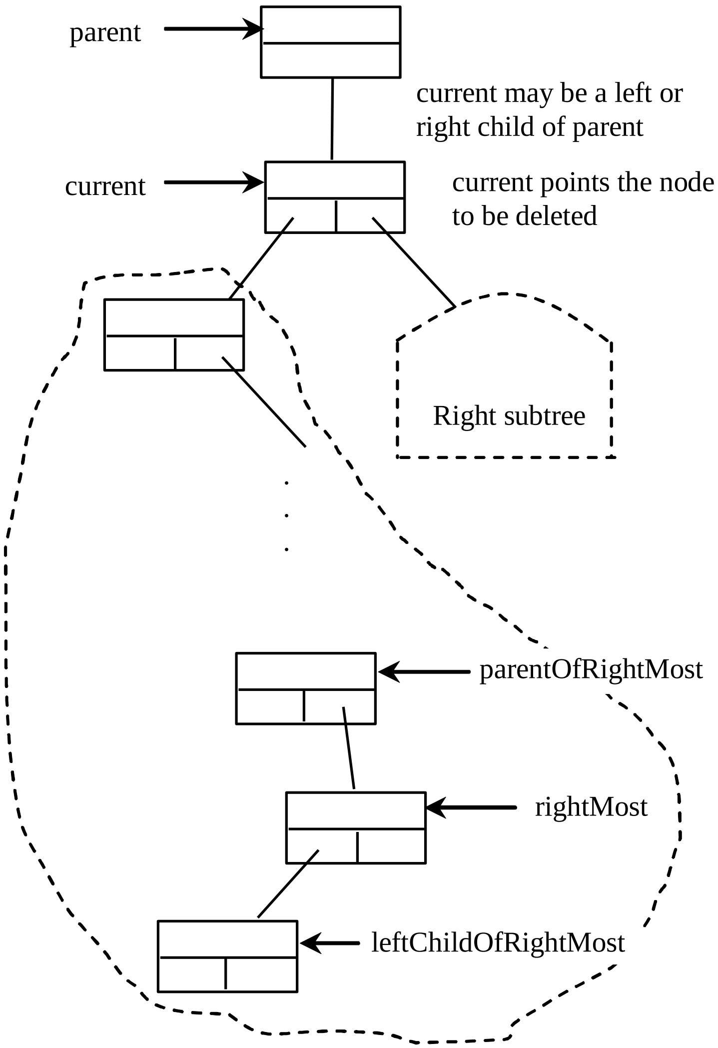
- Recall examples for case I:

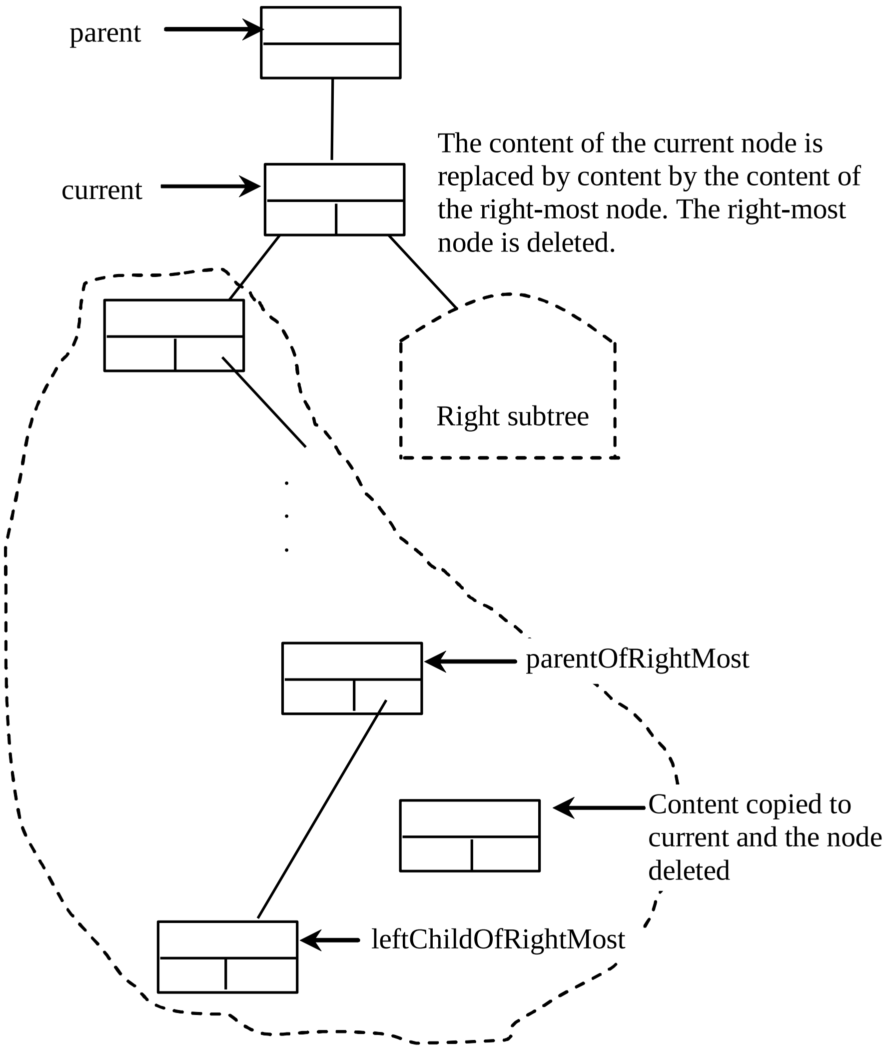
bst_del_rec(tree, data)

- **Case II:**
  - If the tree node does have a left child,
    - find the largest node of the left child
    - replace the tree node data with the largest just found
    - delete the largest

```
if tree.left is not None:
    largest = findmax(tree.left)
    tree.data = largest.data
    tree.left = bst_del_rec(tree.left,largest.data)
    return tree
```

Case 2 (diagram)





bst_del_rec

- Recall examples for case II:

bst_del_rec(tree, data)

```python
#putting everything together

# base case
if not tree:
    return None
# recursive case I
elif data < tree.data:
    tree.left = bst_del_rec(tree.left, data)
# recursive case II
elif data > tree.data:
    tree.right = bst_del_rec(tree.right, data)
# left child is empty
elif tree.left is None:
    return tree.right
# left child is not empty
else:
    largest = findmax(tree.left)
    tree.data = largest.data
    tree.left = bst_del_rec(tree.left,largest.data)
    return tree

# helper
def findmax(tree):
    return tree if not tree.right else findmax(tree.right)
```

Efficiency of algorithms

- BST: iterative delete vs. recursive delete?
  - Extra memory?
    - Constant vs. in order of height of tree
    - $O(1)$ vs. $O(\lg n)$ if balanced or $O(n)$ otherwise
  - Time?
    - Although both in order of height of tree, the latter requires more work
- **Fibonacci:** iteration vs. recursion?
  - Extra memory?
    - $O(1)$ vs. $O(n)$
  - Time?
    - $O(n)$ vs. $O(2^n)$!!

Efficiency of algorithms

n	$\log n$	n	$n \log n$	n^2	2^n	$n!$
10	3×10^{-11} s	10^{-10} s	3×10^{-10} s	10^{-9} s	10^{-8} s	3×10^{-7} s
10^2	7×10^{-11} s	10^{-9} s	7×10^{-9} s	10^{-7} s	4×10^{11} yr	*
10^3	1.0×10^{-10} s	10^{-8} s	1×10^{-7} s	10^{-5} s	*	*
10^4	1.3×10^{-10} s	10^{-7} s	1×10^{-6} s	10^{-3} s	*	*
10^5	1.7×10^{-10} s	10^{-6} s	2×10^{-5} s	0.1 s	*	*
10^6	2×10^{-10} s	10^{-5} s	2×10^{-4} s	0.17 min	*	*

Recursive vs iterative

- Recursive functions impose a loop
- The loop is implicit and the compiler/interpreter (here, Python) takes care of it
- This comes at a price: time & memory
- The price may be negligible in many cases

- After all, no recursive function is more efficient than its iterative equivalent

Recursive vs iterative cont'ed

- Every recursive function can be written iteratively (by explicit loops)
  - may require stacks too
- yet, when the nature of a problem is recursive, writing it iteratively can be
  - time consuming, and
  - less readable

- So, recursion is a very powerful technique for <u>problems that are naturally recursive</u>