CSC148 *Intro. to Computer Science*

Lecture 9: *BST (insert, delete)*

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Last week

- Binary trees (branch factor =2)
- Depth-first traversal
  - inorder, preorder, and postorder
- Breadth-first traversal
  - level-order
- Binary Search Trees

- Today
  - More on BST
    - insert
    - delete

Binary Search Trees

- Add ordering conditions to a binary tree:
  - data are comparable
  - data in left subtree are less than node.data
  - data in right subtree are more than node.data

Binary Search Trees

Binary Search Trees

- a BST with 1 node has height 1
- a BST with 3 nodes may have height 2
- a BST with 7 nodes may have height 3
- a BST with 15 nodes may have height 4
- a BST with n nodes may have height $\lceil \lg n \rceil$

- if the BST is "balanced", then we can check whether an element is present in about $\lg n$ node accesses
  - This is significantly faster than a linear search: O(n)

bst_contains

```
def bst_contains(node, value):
    """
    Return whether tree rooted at node contains value.

    Assume node is the root of a Binary Search Tree

    :param node: node of a Binary Search Tree
    :type node: BinaryTree|None
    :param value: value to search for
    :type value: object
    :rtype: bool

    >>> bst_contains(None, 5)
    False
    >>> bst_contains(BinaryTree(7, BinaryTree(5), BinaryTree(9)), 5)
    True
    """
```

bst_contains

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    >>> bst_contains(None, 5)
    False
    >>> bst_contains(BinaryTree(7, BinaryTree(5), BinaryTree(9)), 5)
    True
    """
    if node is None:
        return False
    elif value < node.data:
        return bst_contains(node.left, value)
    elif value > node.data:
        return bst_contains(node.right, value)
    else:
        return True
```

bst_insert

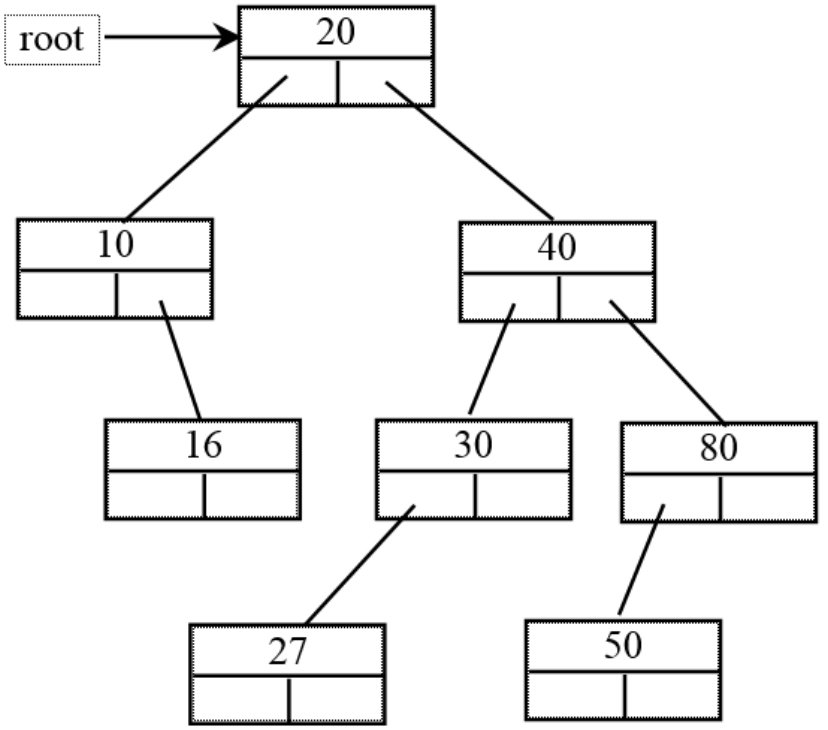
```
def insert(node, data):
    """Insert data in BST rooted at node if necessary, and return new root.
    Assume node is the root of a Binary Search Tree.
    :param node: root of a binary search tree.
    :type node: BinaryTree
    :param data: data to insert into BST, if necessary.
    :type data: object

    >>> b = BinaryTree(5)
    >>> b1 = insert(b, 3)
    >>> print(b1)
    5
        3
    <BLANKLINE>
    """
    return_node = node
    if not node:
        return_node = BinaryTree(data)
    elif data < node.data:
        node.left = insert(node.left, data)
    elif data > node.data:
        node.right = insert(node.right, data)
    else:  # nothing to do
        pass
    return return_node
```

bst_insert

- **Let's trace it for a few examples:**

bst_delete

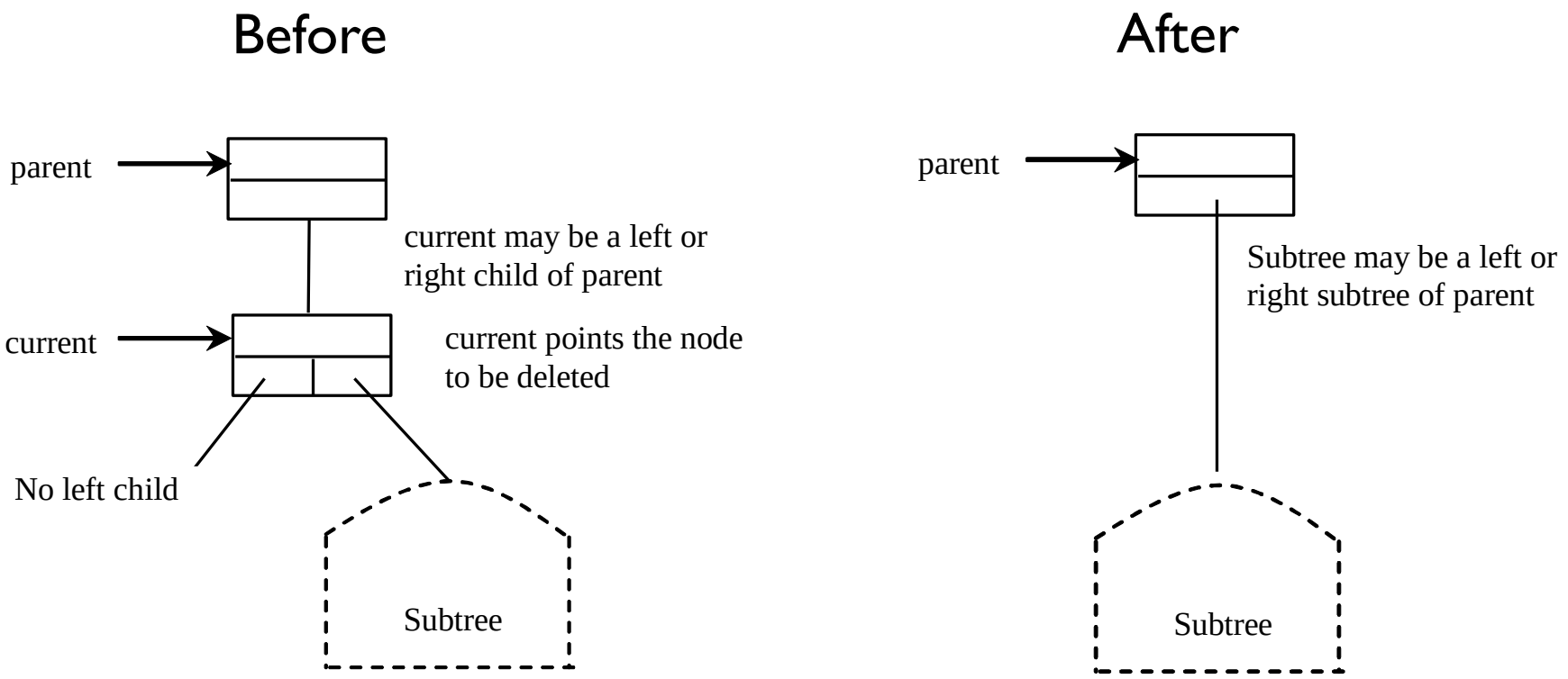


bst_delete

- First locate the node that contains the element and also its parent node.
- Let current point to the node that contains the element in the tree and parent point to the parent of the current node.
- There are two cases to consider …

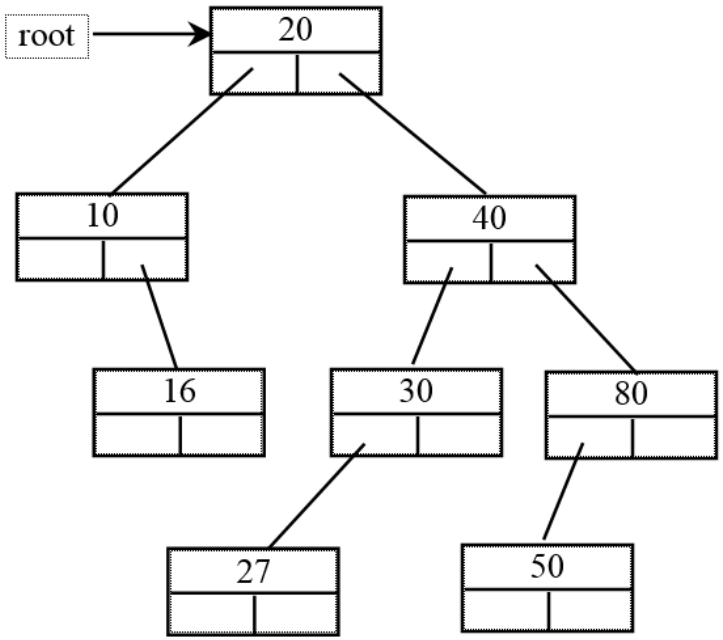
Case 1: The current node has no left child

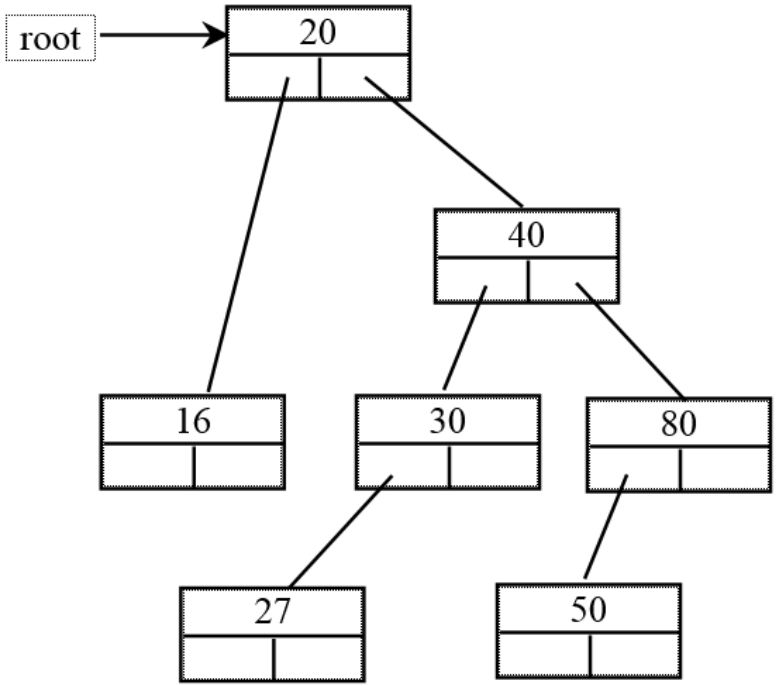
- Simply connect the parent with the right child of the current node.



Example for **Case 1**. Deleting node 10

Connect the parent of node 10 with the right child of node 10.





Case 2: The current node has a left child.

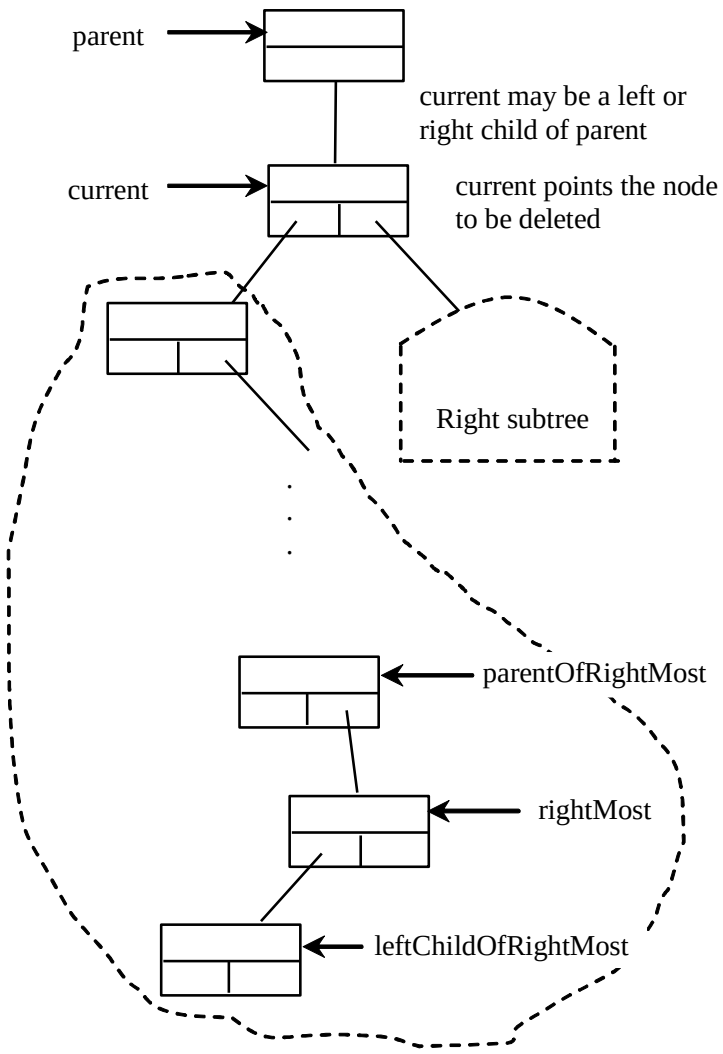
- Let right_most point to the node that contains the largest element in the left subtree of the current node.

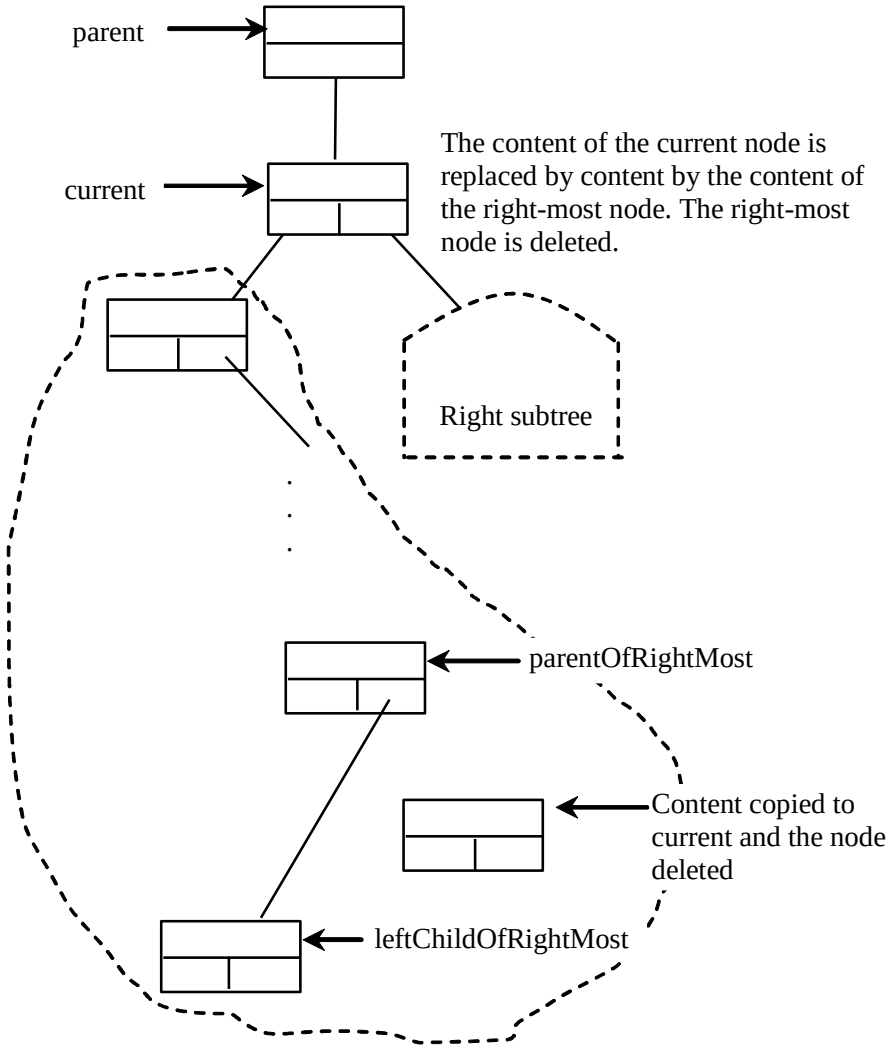
- Let parent_of_right_most point to the parent node of the right_most node.

Then:

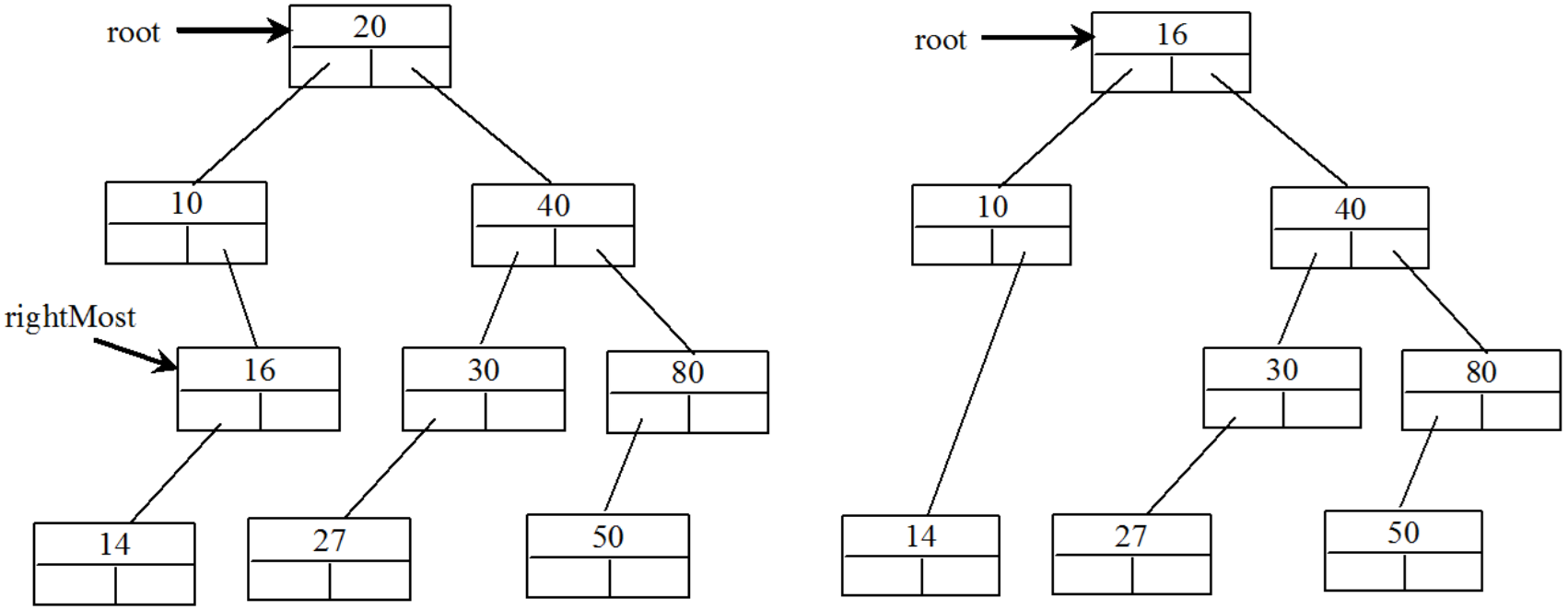
1. Replace the element value in the current node with the one in the right_most node,
2. Connect the parent_of_right_most node with the left child of the right_most node.

Case 2 (diagram)

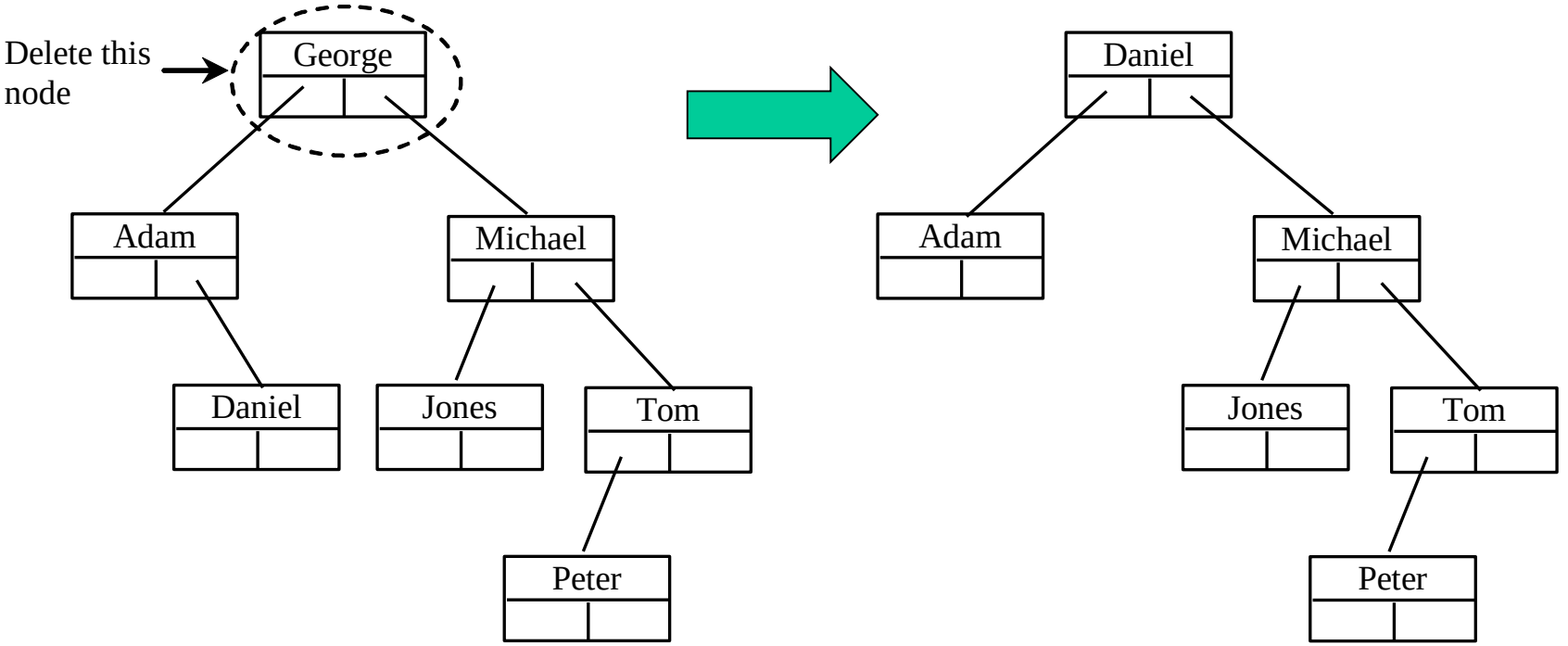




Example for **Case 2**. Deleting node 20

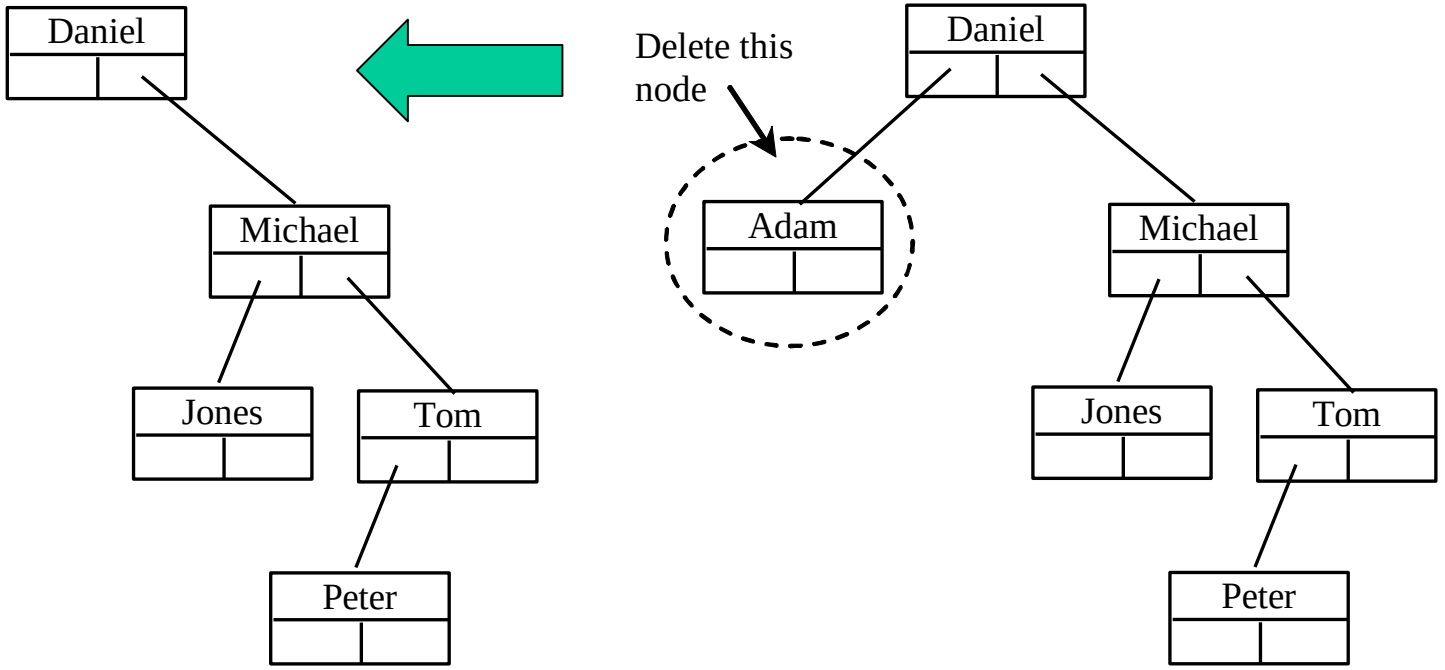


More Examples



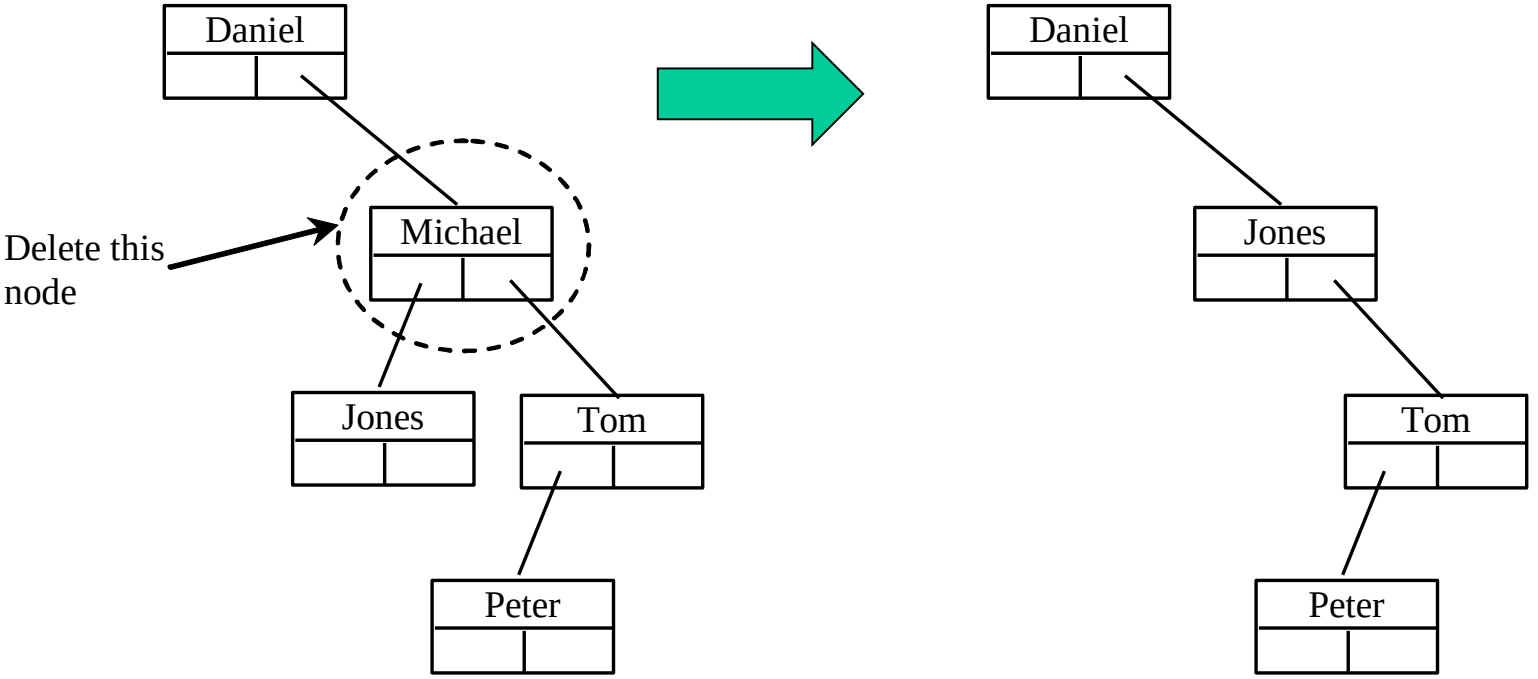
Case 1 or 2? 2

More Examples



Case 1 or 2?

More Examples



Case 1 or 2? 2

bst_delete

- First locate the nodes that contain the element and its parent. Call them current and parent.

```
parent = None
current = root

while current is not None and current.data != data:

    if data < current.data:
        parent = current
        current = current.left

    elif data > current.data:
        parent = current
        current = current.right

    else: pass # Element is in the tree pointed at by current

if current is None: return False # Element is not in the tree
```

Case 1: bst_delete

```
# Case 1: current has no left child
if current.left is None:
    # Connect the parent with the right child of the
      current node
    # Special case, assume the node being deleted is at
      root

    if parent is None:
        current = current.right
    else:
        # Identify if parent left or parent right should
          be connected
        if data < parent.data:
            parent.left = current.right
        else:
            parent.right = current.right
else:
    # Case 2: The current node has a left child
```

Case 11: bst_delete

```
# Locate the rightmost node in the left subtree of
# the current node and also its parent
parent_of_right_most = current
right_most = current.left

while right_most.right is not None:
    parent_of_right_most = right_most
    right_most = right_most.right # Keep going to the right

# Replace the element in current by the element in rightMost
current.element = right_most.element

# Eliminate rightmost node
 if parent_of_right_most.right == right_most:
    parent_of_right_most.right = right_most.left
else:
    # Special case: parent_of_right_most == current
    parent_of_right_most.left = right_most.left
return True # Element deleted successfully
```

Summary

- Homework:
  - In Slides 12 and 14,
    - replace every *left* with *right*, every *right* with *left*, and also *largest* with *smallest*.
  - And, implement the method.

- Next Week:
  - How bst_delete can be written recursively?