Quartic Oscillator :

A quartic oscillator is an anharmonic oscillator, where the potential is $V = \frac{1}{2}\omega_0^2 x^2 + \frac{1}{3}\alpha x^3 + \frac{1}{4}\beta x^4$

So, the equation of motion becomes $\ddot{x} = -\omega_0^2 x - \alpha x^2 - \beta x^3$

Now, let's write a program to solve the above differential equation by using RK-4 method.

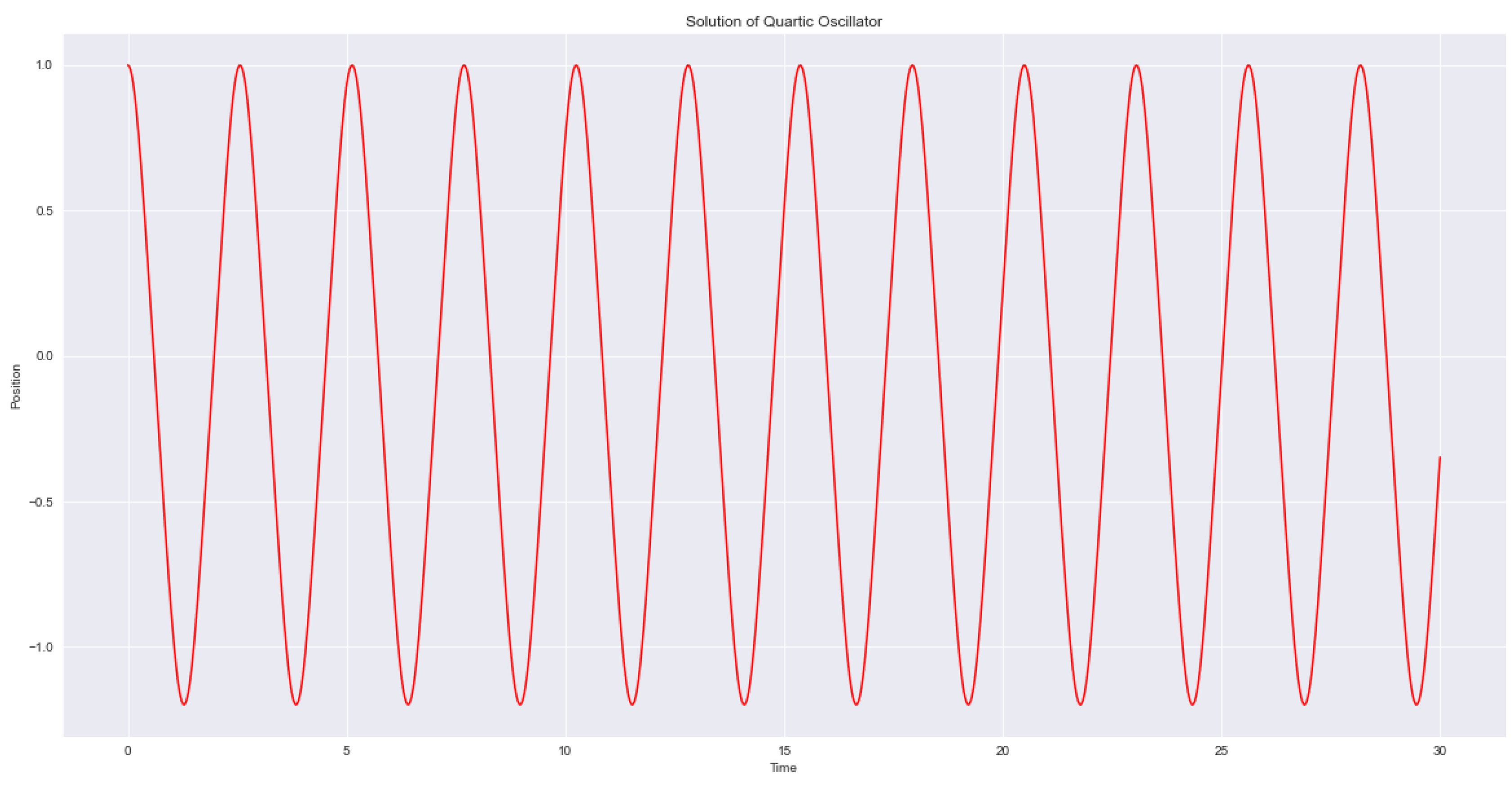
Program:

In [1]:
```
import matplotlib.pyplot as plt
import seaborn as sns
import math as mth
sns.set_style('darkgrid')
```

In [ ]:
```
def f(t, x, v):
    return -w0**2*x - a* x**2 - b* x**3

w0 = 1
a = 2
b = 3
ti = 0
x0 = 1
v0 = 0
tf = 30
n = 1000
h = (tf - ti) / n
t = ti
x = x0
v = v0

x_list = [x0]
v_list = [v0]
t_list = [ti]

for i in range(n):
    print(t,x,v)

    k1 = v
    j1 = f(t,x,v)

    k2 = v + (h*j1)/2.0
    j2 = f(t+ h/2.0, x+ (k1*h)/2.0, v + (j1*h)/ 2.0)

    k3 = v + (h*j2)/2.0
    j3 = f(t+ h/2.0, x+ (k2*h)/2.0, v + (j2*h)/ 2.0)

    k4 = v + (h*j3)
    j4 = f(t+ h, x+ (k3*h), v + (j3*h))

    x = x + h*(k1 + 2.0* k2 + 2.0* k3 + k4)/6.0
    v = v + h*(j1 + 2.0* j2 + 2.0* j3 + j4)/6.0
    t = t+h

    x_list.append(x)
    v_list.append(v)
    t_list.append(t)
```

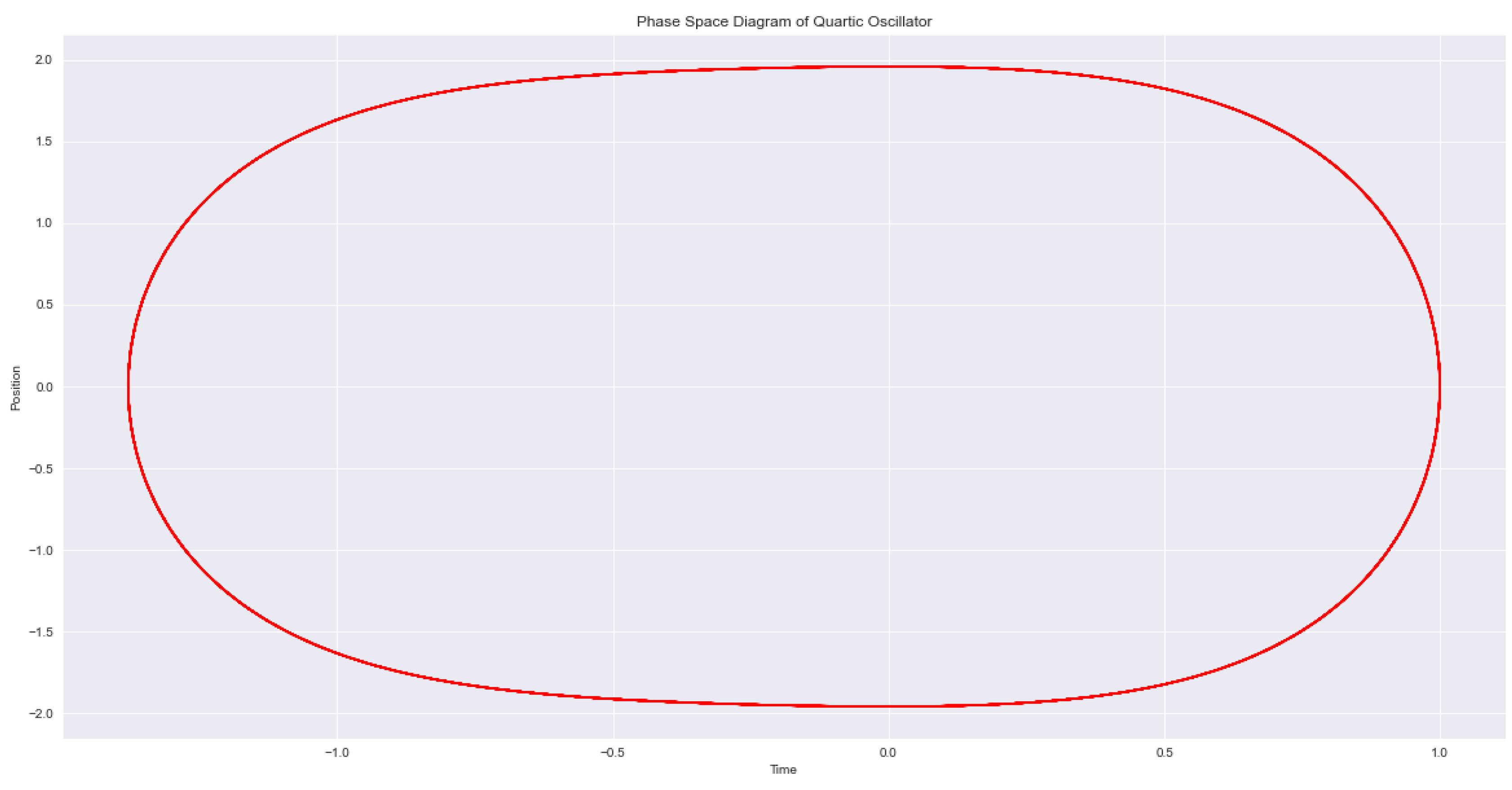
In [7]:
```
plt.plot(t_list, x_list, color='r')
plt.xlabel("Time")
plt.ylabel("Position")
plt.rcParams["figure.figsize"] = (20, 10)
plt.title("Solution of Quartic Oscillator")
```

Out[7]:
```
Text(0.5, 1.0, 'Solution of Quartic Oscillator')
```



In [4]:
```
plt.plot(x_list, v_list, color='r')
plt.xlabel("Time")
plt.ylabel("Position")
plt.rcParams["figure.figsize"] = (20, 10)
plt.title("Phase Space Diagram of Quartic Oscillator")
```

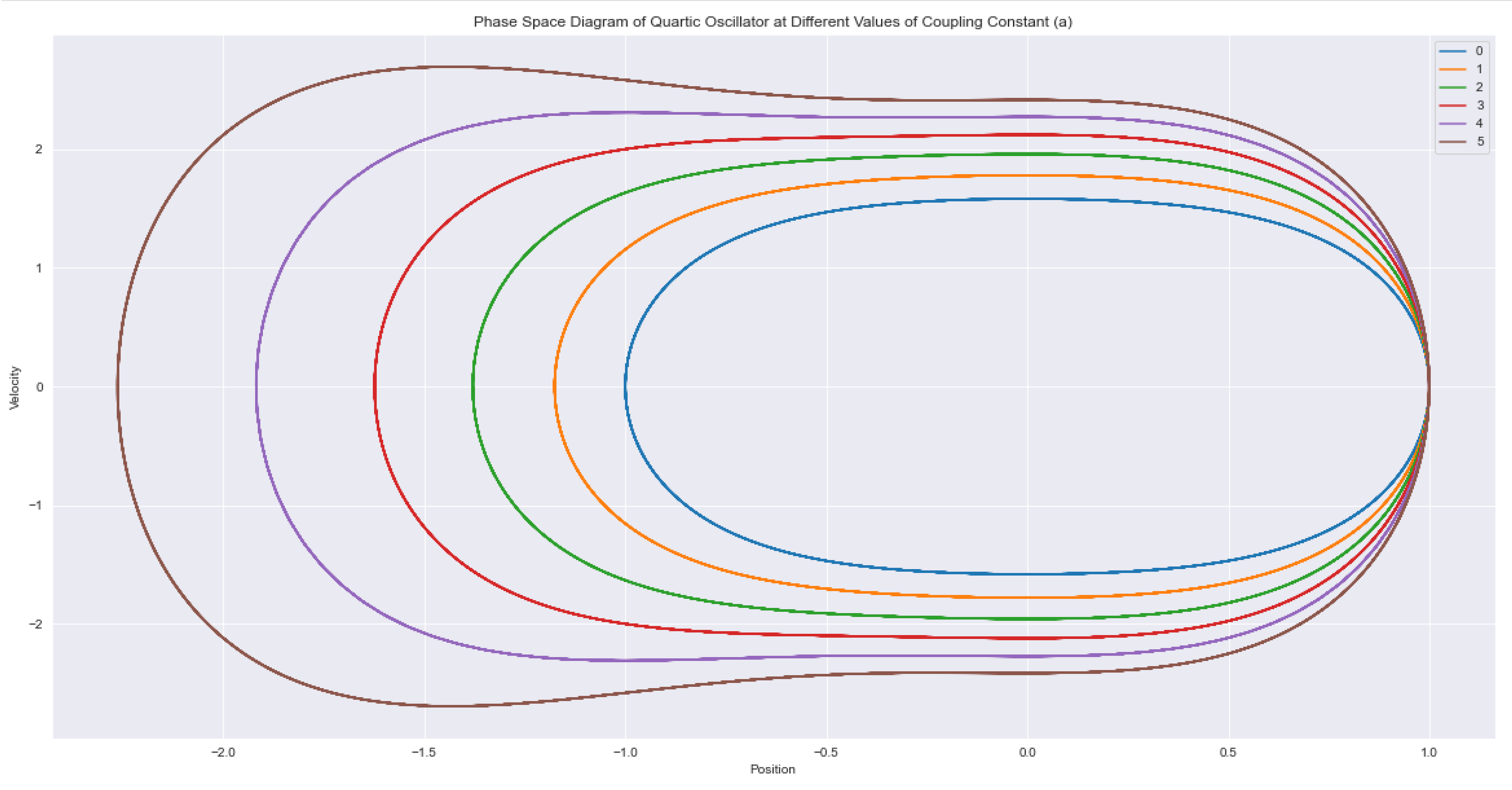
Out[4]:
```
Text(0.5, 1.0, 'Phase Space Diagram of Quartic Oscillator')
```



Phase Space Diagram of Quartic Oscilltor for Different Values of Coupling Constants :

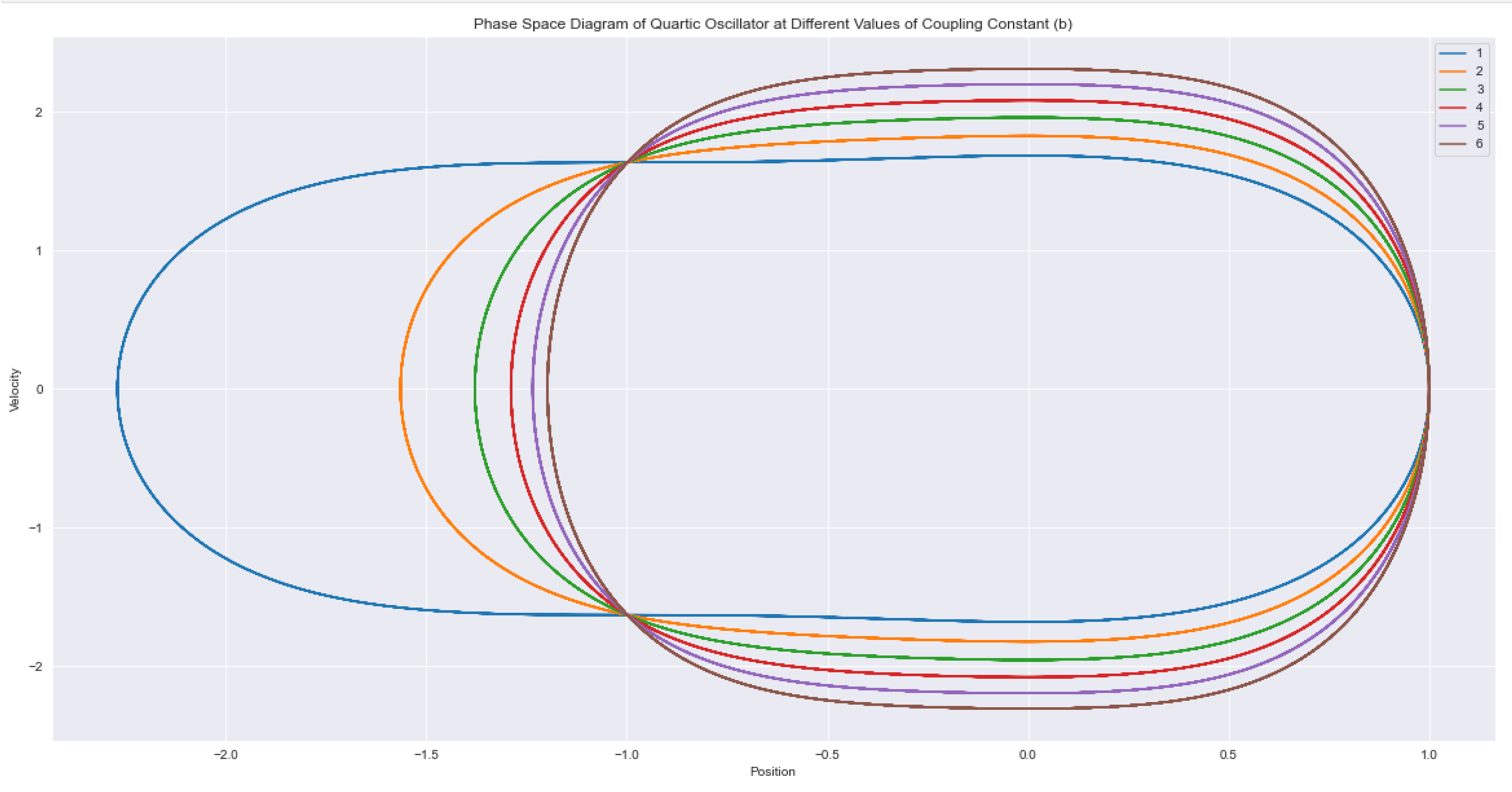
Let's plot the phase space diagrams for different values of α.

In [5]:
```
def f(a, t, x, v):
    return -w0**2*x - a* x**2 - b* x**3

w0 = 1
b = 3
ti = 0
x0 = 1
v0 = 0
tf = 30
n = 1000
h = (tf - ti) / n
t = ti
x = x0
v = v0

x_list = [x0]
v_list = [v0]
t_list = [ti]

for a in range(0,6):

    t = ti
    x = x0
    v = v0

    x_list = [x0]
    v_list = [v0]
    t_list = [ti]

    for i in range(n):
        #print(a,t,x,v)

        k1 = v
        j1 = f(a,t,x,v)

        k2 = v + (h*j1)/2.0
        j2 = f(a, t+ h/2.0, x+ (k1*h)/2.0, v + (j1*h)/ 2.0)

        k3 = v + (h*j2)/2.0
        j3 = f(a, t+ h/2.0, x+ (k2*h)/2.0, v + (j2*h)/ 2.0)

        k4 = v + (h*j3)
        j4 = f(a, t+ h, x+ (k3*h), v + (j3*h))

        x = x + h*(k1 + 2.0* k2 + 2.0* k3 + k4)/6.0
        v = v + h*(j1 + 2.0* j2 + 2.0* j3 + j4)/6.0
        t = t+h

        x_list.append(x)
        v_list.append(v)
        t_list.append(t)

    plt.plot(x_list, v_list, label = a)
    plt.legend()
    plt.rcParams["figure.figsize"] = (20, 10)
    plt.title("Phase Space Diagram of Quartic Oscillator at Different Values of Coupling Constant (a)")
    plt.xlabel("Position")
    plt.ylabel("Velocity")
```



Now, Let's plot for different values of β.

In [6]:
```
def f(b, t, x, v):
    return -w0**2*x - a* x**2 - b* x**3

w0 = 1
a = 2
ti = 0
x0 = 1
v0 = 0
tf = 30
n = 1000
h = (tf - ti) / n
t = ti
x = x0
v = v0

x_list = [x0]
v_list = [v0]
t_list = [ti]

for b in range(1,7):

    t = ti
    x = x0
    v = v0

    x_list = [x0]
    v_list = [v0]
    t_list = [ti]

    for i in range(n):
        #print(b,t,x,v)

        k1 = v
        j1 = f(b,t,x,v)

        k2 = v + (h*j1)/2.0
        j2 = f(b, t+ h/2.0, x+ (k1*h)/2.0, v + (j1*h)/ 2.0)

        k3 = v + (h*j2)/2.0
        j3 = f(b, t+ h/2.0, x+ (k2*h)/2.0, v + (j2*h)/ 2.0)

        k4 = v + (h*j3)
        j4 = f(b, t+ h, x+ (k3*h), v + (j3*h))

        x = x + h*(k1 + 2.0* k2 + 2.0* k3 + k4)/6.0
        v = v + h*(j1 + 2.0* j2 + 2.0* j3 + j4)/6.0
        t = t+h

        x_list.append(x)
        v_list.append(v)
        t_list.append(t)

    plt.plot(x_list, v_list, label = b)
    plt.legend()
    plt.rcParams["figure.figsize"] = (20, 10)
    plt.title("Phase Space Diagram of Quartic Oscillator at Different Values of Coupling Constant (b)")
    plt.xlabel("Position")
    plt.ylabel("Velocity")
```



From the above plots it is clear that the anhormocity is increasing with increase in coupling constant α and β. In the 1st case, when we increased the α, the curves appear to be shifted towards left and in the 2nd case, when we increased the β, the curves appear to be shifted towards right.