Coupled Oscillators

The coupled differential equations we need to solve are: $m\ddot{x}_1 + K_0 x_1 + K(x_1 - x_2) = 0$ and $m\ddot{x}_2 + K_0 x_2 - K(x_1 - x_2) = 0$.

When initial amplitudes are $x_{01} = 2$ and $x_{02} = 0$:

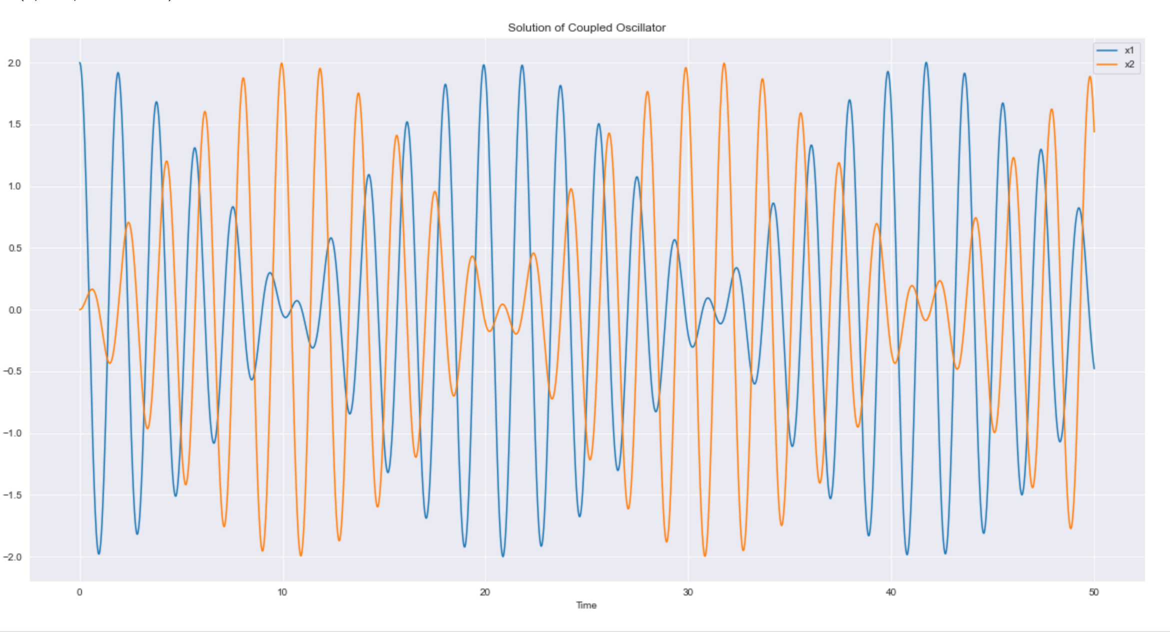
In [38]:

```
import matplotlib.pyplot as plt
import seaborn as sns
import math as mth
import numpy as np
sns.set_style('darkgrid')
```

In [39]:

```
def f1(t,x1,x2,v1):
    return -k0*x1-k*(x1-x2)

def f2(t,x1,x2,v2):
    return -k0*x2+k*(x1-x2)

k0 = 10
k = 1
ti = 0
x10 = 2
x20 = 0
v10 = 0
v20 = 0
tf = 50
n = 3000
h = (tf - ti) / n
t = ti
x1 = x10
x2 = x20
v1 = v10
v2 = v20

x1_list = [x10]
v1_list = [v10]
x2_list = [x20]
v2_list = [v20]
t_list = [ti]

for i in range(n):
    #print(t,x1,v1,x2,v2)

    k1x1 = v1
    j1x1 = f1(t,x1,x2,v1)
    k1x2 = v2
    j1x2 = f2(t,x1,x2,v2)

    k2x1 = v1 + (h*j1x1)/2.0
    j2x1 = f1(t+ h/2.0, x1+ (k1x1*h)/2.0,x2, v1 + (j1x1*h)/ 2.0)
    k2x2 = v2 + (h*j1x2)/2.0
    j2x2 = f2(t+ h/2.0,x1+ (k1x1*h)/2.0, x2+ (k1x2*h)/2.0, v2 + (j1x2*h)/ 2.0)

    k3x1 = v1 + (h*j2x1)/2.0
    j3x1 = f1(t+ h/2.0, x1+ (k2x1*h)/2.0, x2+ (k1x2*h)/2.0, v1 + (j2x1*h)/ 2.0)
    k3x2 = v2 + (h*j2x2)/2.0
    j3x2 = f2(t+ h/2.0,x1+ (k2x1*h)/2.0, x2+ (k2x2*h)/2.0, v2 + (j2x2*h)/ 2.0)

    k4x1 = v1 + (h*j3x1)
    j4x1 = f1(t+ h, x1+ (k3x1*h), x2+ (k2x2*h)/2.0, v1 + (j3x1*h))
    k4x2 = v2 + (h*j3x2)
    j4x2 = f2(t+ h, x1+ (k3x1*h), x2+ (k3x2*h), v2 + (j3x2*h))

    x1 = x1 + h*(k1x1 + 2.0* k2x1 + 2.0* k3x1 + k4x1)/6.0
    v1 = v1 + h*(j1x1 + 2.0* j2x1 + 2.0* j3x1 + j4x1)/6.0
    x2 = x2 + h*(k1x2 + 2.0* k2x2 + 2.0* k3x2 + k4x2)/6.0
    v2 = v2 + h*(j1x2 + 2.0* j2x2 + 2.0* j3x2 + j4x2)/6.0
    t = t+h

    x1_list.append(x1)
    v1_list.append(v1)
    x2_list.append(x2)
    v2_list.append(v2)
    t_list.append(t)

plt.plot(t_list, x1_list, label = "x1")
plt.plot(t_list, x2_list, label = "x2")
plt.legend()
plt.rcParams["figure.figsize"] = (20, 10)
plt.title("Solution of Coupled Oscillator")
plt.xlabel("Time")
plt.ylabel("Position")
```

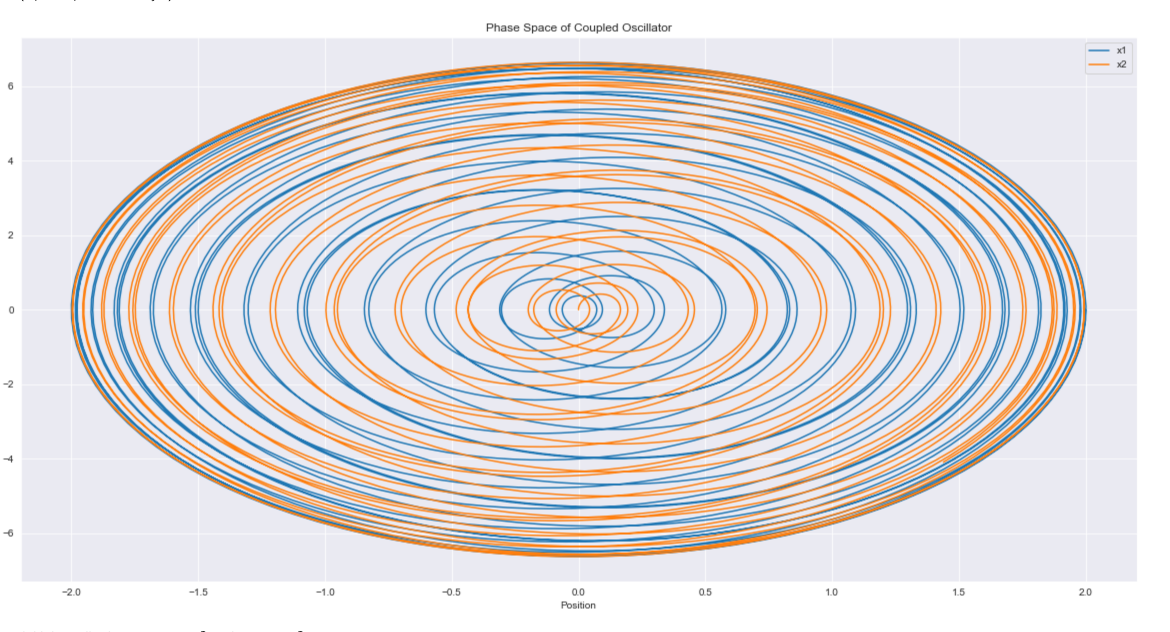
Out[39]: `Text(0, 0.5, 'Position')`



In [40]:

```
plt.plot(x1_list, v1_list, label = "x1")
plt.plot(x2_list, v2_list, label = "x2")
plt.legend()
plt.rcParams["figure.figsize"] = (20, 10)
plt.title("Phase Space of Coupled Oscillator")
plt.xlabel("Position")
plt.ylabel("Velocity")
```

Out[40]: `Text(0, 0.5, 'Velocity')`

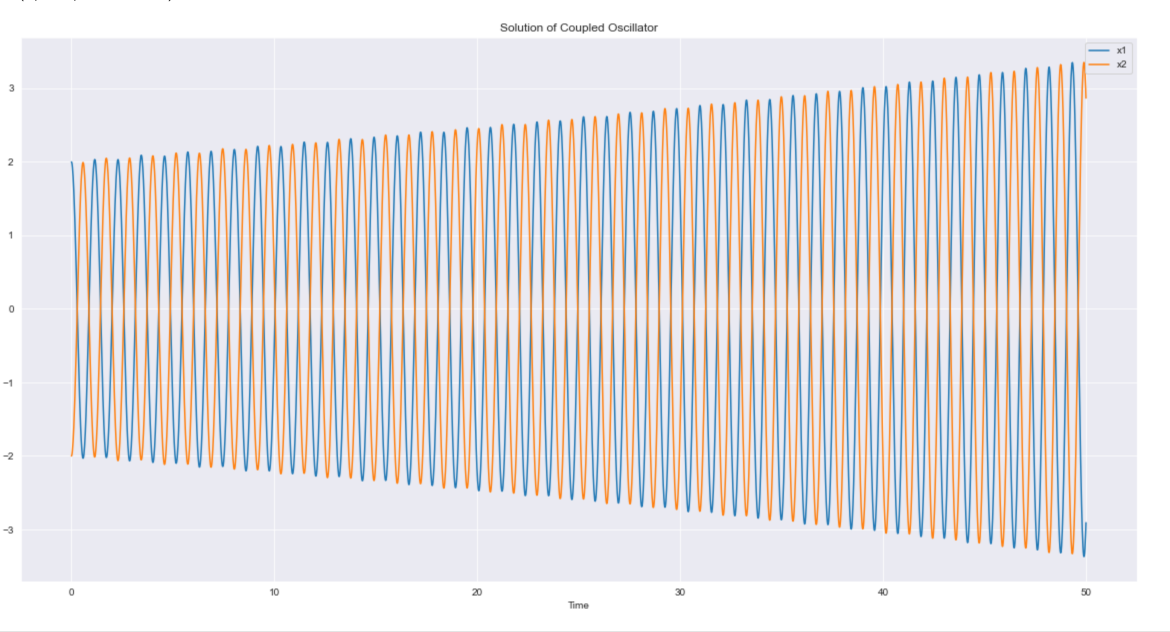


When initial amplitudes are $x_{01} = 2$ and $x_{02} = -2$:

In [41]:

```
def f1(t,x1,x2,v1):
    return -k0*x1-k*(x1-x2)

def f2(t,x1,x2,v2):
    return -k0*x2+k*(x1-x2)


k = 10
ti = 0
x10 = 2
x20 = -2
v10 = 0
v20 = 0
tf = 50
n = 3000
h = (tf - ti) / n
t = ti
x1 = x10
x2 = x20
v1 = v10
v2 = v20

x1_list = [x10]
v1_list = [v10]
x2_list = [x20]
v2_list = [v20]
t_list = [ti]

for i in range(n):
    #print(t,x1,v1,x2,v2)

    k1x1 = v1
    j1x1 = f1(t,x1,x2,v1)
    k1x2 = v2
    j1x2 = f2(t,x1,x2,v2)

    k2x1 = v1 + (h*j1x1)/2.0
    j2x1 = f1(t+ h/2.0, x1+ (k1x1*h)/2.0,x2, v1 + (j1x1*h)/ 2.0)
    k2x2 = v2 + (h*j1x2)/2.0
    j2x2 = f2(t+ h/2.0,x1+ (k1x1*h)/2.0, x2+ (k1x2*h)/2.0, v2 + (j1x2*h)/ 2.0)

    k3x1 = v1 + (h*j2x1)/2.0
    j3x1 = f1(t+ h/2.0, x1+ (k2x1*h)/2.0, x2+ (k1x2*h)/2.0, v1 + (j2x1*h)/ 2.0)
    k3x2 = v2 + (h*j2x2)/2.0
    j3x2 = f2(t+ h/2.0,x1+ (k2x1*h)/2.0, x2+ (k2x2*h)/2.0, v2 + (j2x2*h)/ 2.0)

    k4x1 = v1 + (h*j3x1)
    j4x1 = f1(t+ h, x1+ (k3x1*h), x2+ (k2x2*h)/2.0, v1 + (j3x1*h))
    k4x2 = v2 + (h*j3x2)
    j4x2 = f2(t+ h, x1+ (k3x1*h), x2+ (k3x2*h), v2 + (j3x2*h))

    x1 = x1 + h*(k1x1 + 2.0* k2x1 + 2.0* k3x1 + k4x1)/6.0
    v1 = v1 + h*(j1x1 + 2.0* j2x1 + 2.0* j3x1 + j4x1)/6.0
    x2 = x2 + h*(k1x2 + 2.0* k2x2 + 2.0* k3x2 + k4x2)/6.0
    v2 = v2 + h*(j1x2 + 2.0* j2x2 + 2.0* j3x2 + j4x2)/6.0
    t = t+h

    x1_list.append(x1)
    v1_list.append(v1)
    x2_list.append(x2)
    v2_list.append(v2)
    t_list.append(t)

plt.plot(t_list, x1_list, label = "x1")
plt.plot(t_list, x2_list, label = "x2")
plt.legend()
plt.rcParams["figure.figsize"] = (20, 10)
plt.title("Solution of Coupled Oscillator")
plt.xlabel("Time")
plt.ylabel("Position")
```

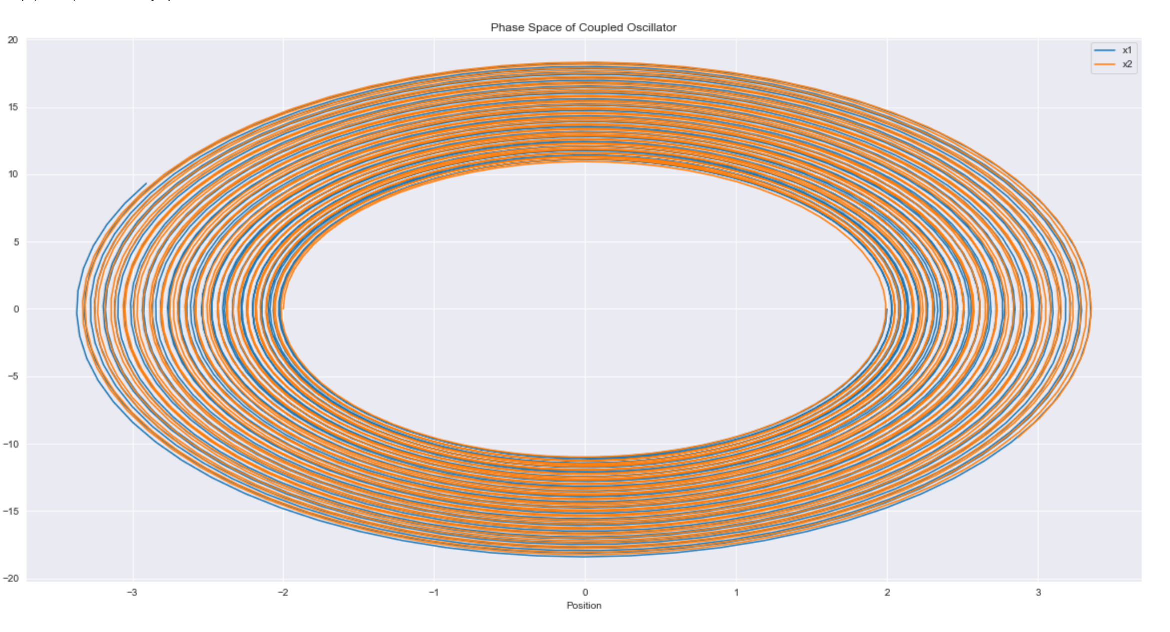
Out[41]: `Text(0, 0.5, 'Position')`



In [42]:

```
plt.plot(x1_list, v1_list, label = "x1")
plt.plot(x2_list, v2_list, label = "x2")
plt.legend()
plt.rcParams["figure.figsize"] = (20, 10)
plt.title("Phase Space of Coupled Oscillator")
plt.xlabel("Position")
plt.ylabel("Velocity")
```

Out[42]: `Text(0, 0.5, 'Velocity')`



Similarly we can plot for any initial amplitude.

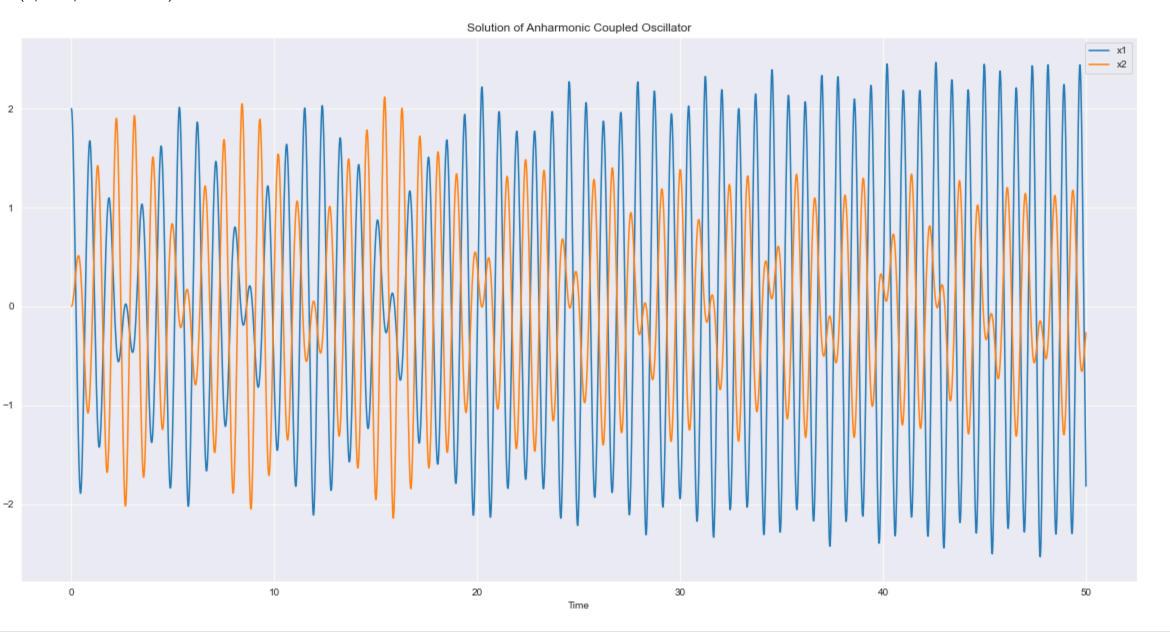
Anharmonic Coupled Oscillator:

For 2 and 0 initial amplitude:

In [49]:

```
def f1(t,x1,x2,v1):
    return -k0*x1-k*(x1-x2)- b*x1**3

def f2(t,x1,x2,v2):
    return -k0*x2+k*(x1-x2)- b*x2**3


b = 10
k = 10
ti = 0
x10 = 2
x20 = 0
v10 = 0
v20 = 0
tf = 50
n = 3000
h = (tf - ti) / n
t = ti
x1 = x10
x2 = x20
v1 = v10
v2 = v20

x1_list = [x10]
v1_list = [v10]
x2_list = [x20]
v2_list = [v20]
t_list = [ti]

for i in range(n):
    #print(t,x1,v1,x2,v2)

    k1x1 = v1
    j1x1 = f1(t,x1,x2,v1)
    k1x2 = v2
    j1x2 = f2(t,x1,x2,v2)

    k2x1 = v1 + (h*j1x1)/2.0
    j2x1 = f1(t+ h/2.0, x1+ (k1x1*h)/2.0,x2, v1 + (j1x1*h)/ 2.0)
    k2x2 = v2 + (h*j1x2)/2.0
    j2x2 = f2(t+ h/2.0,x1+ (k1x1*h)/2.0, x2+ (k1x2*h)/2.0, v2 + (j1x2*h)/ 2.0)

    k3x1 = v1 + (h*j2x1)/2.0
    j3x1 = f1(t+ h/2.0, x1+ (k2x1*h)/2.0, x2+ (k1x2*h)/2.0, v1 + (j2x1*h)/ 2.0)
    k3x2 = v2 + (h*j2x2)/2.0
    j3x2 = f2(t+ h/2.0,x1+ (k2x1*h)/2.0, x2+ (k2x2*h)/2.0, v2 + (j2x2*h)/ 2.0)

    k4x1 = v1 + (h*j3x1)
    j4x1 = f1(t+ h, x1+ (k3x1*h), x2+ (k2x2*h)/2.0, v1 + (j3x1*h))
    k4x2 = v2 + (h*j3x2)
    j4x2 = f2(t+ h, x1+ (k3x1*h), x2+ (k3x2*h), v2 + (j3x2*h))

    x1 = x1 + h*(k1x1 + 2.0* k2x1 + 2.0* k3x1 + k4x1)/6.0
    v1 = v1 + h*(j1x1 + 2.0* j2x1 + 2.0* j3x1 + j4x1)/6.0
    x2 = x2 + h*(k1x2 + 2.0* k2x2 + 2.0* k3x2 + k4x2)/6.0
    v2 = v2 + h*(j1x2 + 2.0* j2x2 + 2.0* j3x2 + j4x2)/6.0
    t = t+h

    x1_list.append(x1)
    v1_list.append(v1)
    x2_list.append(x2)
    v2_list.append(v2)
    t_list.append(t)

plt.plot(t_list, x1_list, label = "x1")
plt.plot(t_list, x2_list, label = "x2")
plt.legend()
plt.rcParams["figure.figsize"] = (20, 10)
plt.title("Solution of Anharmonic Coupled Oscillator")
plt.xlabel("Time")
plt.ylabel("Position")
```

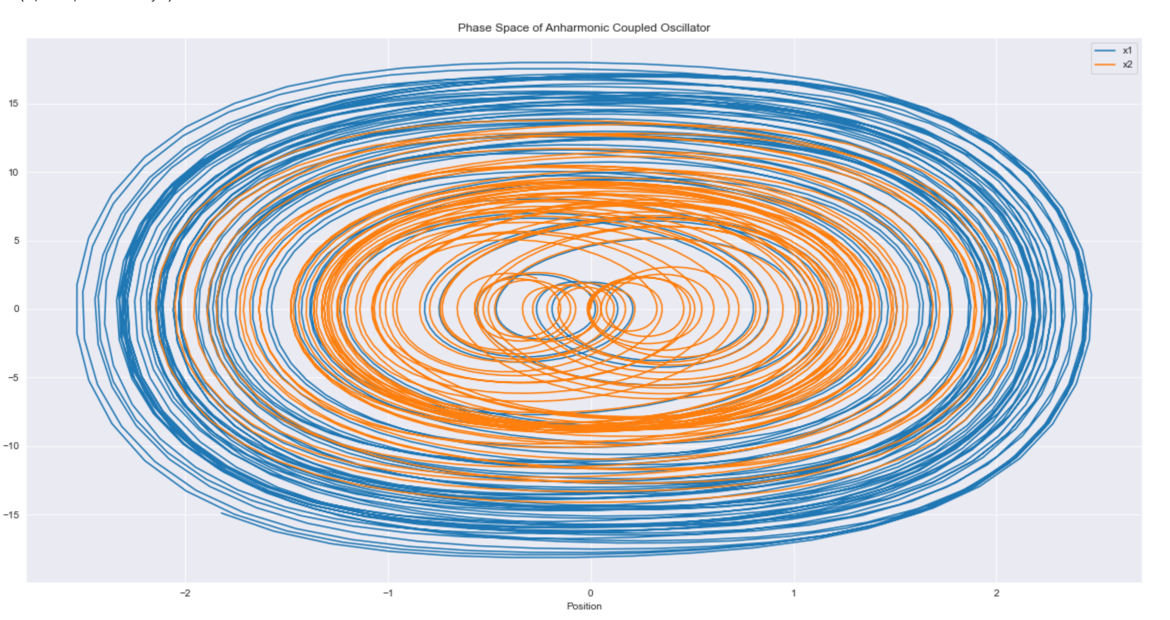
Out[49]: `Text(0, 0.5, 'Position')`



In [50]:

```
plt.plot(x1_list, v1_list, label = "x1")
plt.plot(x2_list, v2_list, label = "x2")
plt.legend()
plt.rcParams["figure.figsize"] = (20, 10)
plt.title("Phase Space of Anharmonic Coupled Oscillator")
plt.xlabel("Position")
plt.ylabel("Velocity")
```

Out[50]: `Text(0, 0.5, 'Velocity')`

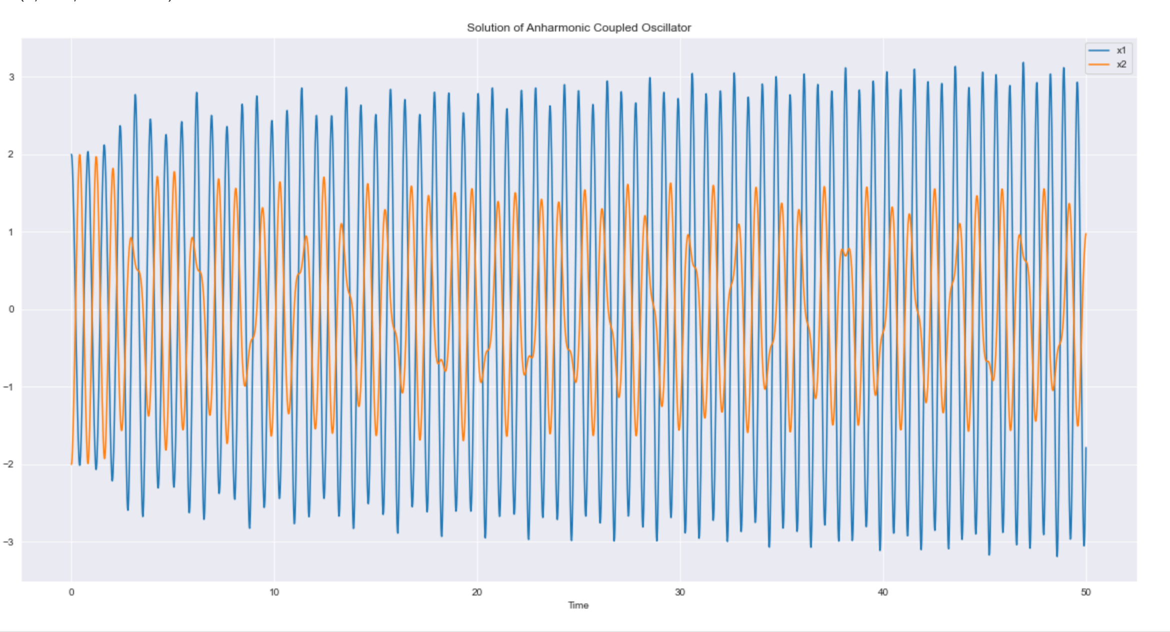


For 2 and 2 initial amplitude:

In [51]:

```
def f1(t,x1,x2,v1):
    return -k0*x1-k*(x1-x2)- b*x1**3

def f2(t,x1,x2,v2):
    return -k0*x2+k*(x1-x2)- b*x2**3

b = 10
k = 10
ti = 0
x10 = 2
x20 = -2
v10 = 0
v20 = 0
tf = 50
n = 3000
h = (tf - ti) / n
t = ti
x1 = x10
x2 = x20
v1 = v10
v2 = v20

x1_list = [x10]
v1_list = [v10]
x2_list = [x20]
v2_list = [v20]
t_list = [ti]

for i in range(n):
    #print(t,x1,v1,x2,v2)

    k1x1 = v1
    j1x1 = f1(t,x1,x2,v1)
    k1x2 = v2
    j1x2 = f2(t,x1,x2,v2)

    k2x1 = v1 + (h*j1x1)/2.0
    j2x1 = f1(t+ h/2.0, x1+ (k1x1*h)/2.0,x2, v1 + (j1x1*h)/ 2.0)
    k2x2 = v2 + (h*j1x2)/2.0
    j2x2 = f2(t+ h/2.0,x1+ (k1x1*h)/2.0, x2+ (k1x2*h)/2.0, v2 + (j1x2*h)/ 2.0)

    k3x1 = v1 + (h*j2x1)/2.0
    j3x1 = f1(t+ h/2.0, x1+ (k2x1*h)/2.0, x2+ (k1x2*h)/2.0, v1 + (j2x1*h)/ 2.0)
    k3x2 = v2 + (h*j2x2)/2.0
    j3x2 = f2(t+ h/2.0,x1+ (k2x1*h)/2.0, x2+ (k2x2*h)/2.0, v2 + (j2x2*h)/ 2.0)

    k4x1 = v1 + (h*j3x1)
    j4x1 = f1(t+ h, x1+ (k3x1*h), x2+ (k2x2*h)/2.0, v1 + (j3x1*h))
    k4x2 = v2 + (h*j3x2)
    j4x2 = f2(t+ h, x1+ (k3x1*h), x2+ (k3x2*h), v2 + (j3x2*h))

    x1 = x1 + h*(k1x1 + 2.0* k2x1 + 2.0* k3x1 + k4x1)/6.0
    v1 = v1 + h*(j1x1 + 2.0* j2x1 + 2.0* j3x1 + j4x1)/6.0
    x2 = x2 + h*(k1x2 + 2.0* k2x2 + 2.0* k3x2 + k4x2)/6.0
    v2 = v2 + h*(j1x2 + 2.0* j2x2 + 2.0* j3x2 + j4x2)/6.0
    t = t+h

    x1_list.append(x1)
    v1_list.append(v1)
    x2_list.append(x2)
    v2_list.append(v2)
    t_list.append(t)

plt.plot(t_list, x1_list, label = "x1")
plt.plot(t_list, x2_list, label = "x2")
plt.legend()
plt.rcParams["figure.figsize"] = (20, 10)
plt.title("Solution of Anharmonic Coupled Oscillator")
plt.xlabel("Time")
plt.ylabel("Position")
```

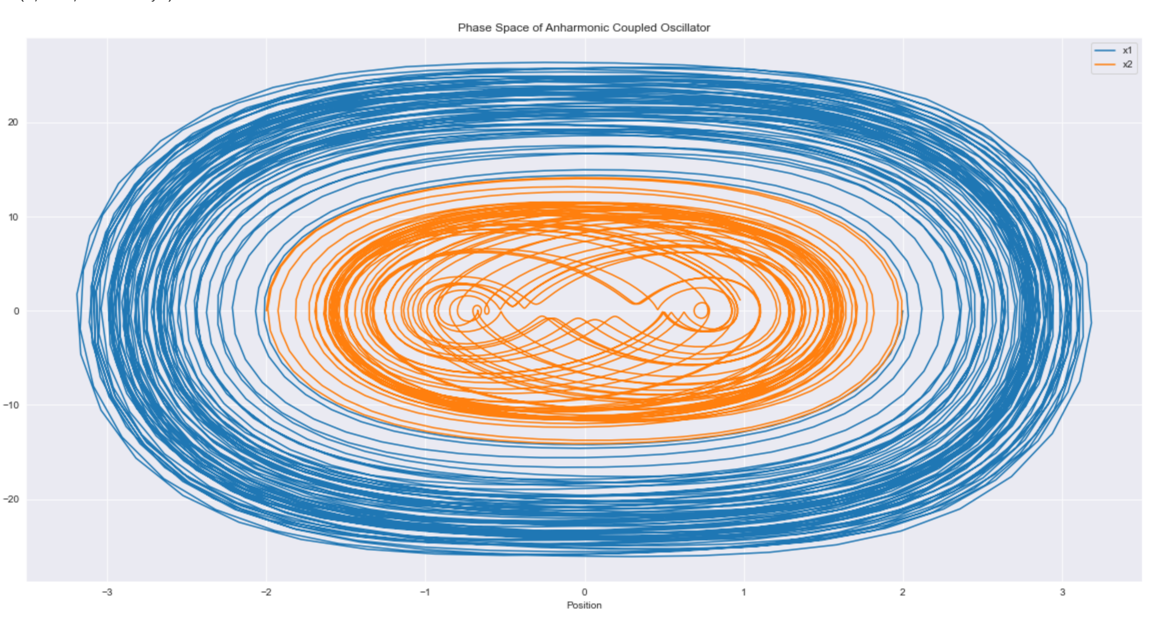
Out[51]: `Text(0, 0.5, 'Position')`



In [52]:

```
plt.plot(x1_list, v1_list, label = "x1")
plt.plot(x2_list, v2_list, label = "x2")
plt.legend()
plt.rcParams["figure.figsize"] = (20, 10)
plt.title("Phase Space of Anharmonic Coupled Oscillator")
plt.xlabel("Position")
plt.ylabel("Velocity")
```

Out[52]: `Text(0, 0.5, 'Velocity')`



Similarly we can plot for any initial condition.