MÄLARDALEN UNIVERSITY
School of Education, Culture and Communication
Department of Applied Mathematics
Examiner: Lars-Göran Larsson

EXAMINATION IN MATHEMATICS
MAA150 Vector Algebra, TEN1
Date: 2018-04-23 *Write time:* 3 hours
Aid: Writing materials, ruler

This examination is intended for the examination part TEN1. The examination consists of five RANDOMLY ORDERED problems each of which is worth at maximum 5 points. The PASS-marks 3, 4 and 5 require a minimum of 12, 16 and 21 points respectively. The minimum points for the ECTS-marks E, D, C, B and A are 12, 13, 16, 20 and 24 respectively. If the obtained sum of points is denoted S_1, and that obtained at examination TEN1 S_2, the marks for a completed course are determined according to

$S_1, S_2 \geq 12$	AND	$S_1 + 2S_2 \leq 47$	$\rightarrow$	3
$S_1, S_2 \geq 12$	AND	$48 \leq S_1 + 2S_2 \leq 62$	$\rightarrow$	4
		$63 \leq S_1 + 2S_2$	$\rightarrow$	5

$S_1, S_2 \geq 12$	AND	$S_1 + 2S_2 \leq 38$	$\rightarrow$	E
$S_1, S_2 \geq 12$	AND	$39 \leq S_1 + 2S_2 \leq 47$	$\rightarrow$	D
$S_1, S_2 \geq 12$	AND	$48 \leq S_1 + 2S_2 \leq 59$	$\rightarrow$	C
$S_1, S_2 \geq 12$	AND	$60 \leq S_1 + 2S_2 \leq 71$	$\rightarrow$	B
		$72 \leq S_1 + 2S_2$	$\rightarrow$	A

Solutions are supposed to include rigorous justifications and clear answers. All sheets of solutions must be sorted in the order the problems are given in.

1. Find an equation for the line λ which is orthogonal to the plane

$$\pi : (x, y, z) = (15 + 2r - 3s, 4 - 5r - 2s, -7 - 4r + 5s), \quad r, s \in R,$$

and which includes the point $P : (6, -8, 3)$. It is assumed that the standard basis is a right-handed ON-basis.

2. Sketch the region

$$\Omega = \{z \in \mathbb{C} : \left|z - \tfrac{1-3i}{1-i}\right| \geq 1,\ \mathrm{Re}(z) \leq 2\}$$

where $\mathbb{C}$ denotes the set of all complex numbers.

3. The point $P : (3, 2, 2)$ is reflected in the plane $\pi : 2x + y - 4z + 7 = 0$. Find the coordinates of the mirror image of the point P. It is assumed that the standard basis is an ON-basis.

4. Compute the determinant $\begin{vmatrix} 3 & 3 & 3 & 7 \\ 3 & 3 & 7 & 3 \\ 3 & 7 & 3 & 3 \\ 7 & 3 & 3 & 3 \end{vmatrix}$.

5. Find, for every real value of the parameter a, the solution set of the system of linear equations

$$\begin{cases} x - 2y - az = a - 6\,, \\ -3x + 5y + 2az = 14 - 3a\,, \\ 2x + y + a^2 z = 4a + 2\,. \end{cases}$$

Om du föredrar uppgifterna formulerade på svenska, var god vänd på bladet.

MÄLARDALENS HÖGSKOLA
Akademin för utbildning, kultur och kommunikation
Avdelningen för tillämpad matematik
Examinator: Lars-Göran Larsson

TENTAMEN I MATEMATIK
MAA150 Vektoralgebra, TEN1
Datum: 2018-04-23 *Skrivtid:* 3 timmar
Hjälpmedel: Skrivdon, linjal

Denna tentamen är avsedd för examinationsmomentet TEN1. Provet består av fem stycken om varannat SLUMPMÄSSIGT ORDNADE uppgifter som vardera kan ge maximalt 5 poäng. För GODKÄND-betygen 3, 4 och 5 krävs erhållna poängsummor om minst 12, 16 respektive 21 poäng. Om den erhållna poängen benämns S_1, och den vid tentamen TEN2 erhållna S_2, bestäms graden av ett sammanfattningsbetyg på en slutförd kurs enligt

$S_1, S_2 \geq 12$	OCH	$S_1 + 2S_2 \leq 47$	$\rightarrow$	3
$S_1, S_2 \geq 12$	OCH	$48 \leq S_1 + 2S_2 \leq 62$	$\rightarrow$	4
		$63 \leq S_1 + 2S_2$	$\rightarrow$	5

Lösningar förutsätts innefatta ordentliga motiveringar och tydliga svar. Samtliga lösningsblad skall vid inlämning vara sorterade i den ordning som uppgifterna är givna i.

1. Bestäm en ekvation för den linje λ som är ortogonal mot planet

$$\pi : (x, y, z) = (15 + 2r - 3s, 4 - 5r - 2s, -7 - 4r + 5s), \quad r, s \in R,$$

och som inkluderar punkten $P : (6, -8, 3)$. Det antages att standardbasen är en högerorienterad ON-bas.

2. Skissa området

$$\Omega = \{z \in \mathbb{C} : \left|z - \tfrac{1-3i}{1-i}\right| \geq 1,\ \mathrm{Re}(z) \leq 2\}$$

där $\mathbb{C}$ betecknar mängden av alla komplexa tal.

3. Punkten $P : (3, 2, 2)$ speglas i planet $\pi : 2x + y - 4z + 7 = 0$. Bestäm koordinaterna för spegelbilden av P. Det antages att standardbasen är en ON-bas.

4. Beräkna determinanten $\begin{vmatrix} 3 & 3 & 3 & 7 \\ 3 & 3 & 7 & 3 \\ 3 & 7 & 3 & 3 \\ 7 & 3 & 3 & 3 \end{vmatrix}$.

5. Bestäm, för varje reellt värde på parametern a, lösningsmängden till det linjära ekvationssystemet

$$\begin{cases} x - 2y - az = a - 6\,, \\ -3x + 5y + 2az = 14 - 3a\,, \\ 2x + y + a^2 z = 4a + 2\,. \end{cases}$$

If you prefer the problems formulated in English, please turn the page.

MAA150 / Solutions to the final exam TEN1 2018-04-23

1

$$\begin{cases} \pi : (x,y,z) = (15+2r-3s,\ 4-5r-2s,\ -7-4r+5s), & r,s \in \mathbb{R} \\ P : (6,-8,3) \end{cases}$$

$\lambda \perp \pi$ gives that $\lambda // v$ where $v = u_1 \times u_2$ and $u_1 = (2,-5,-4)$, $u_2 = (-3,-2,5)$ are both parallel with π

We get $v = (2,-5,-4) \times (-3,-2,5) = (-25-8,\ 12-10,\ -4-15) = (-33,2,-19)$

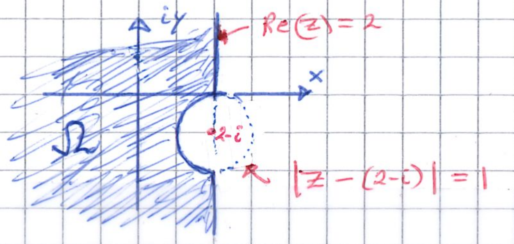
Thus $\lambda : (x,y,z) = (x_P, y_P, z_P) + t\,v$ i.e. $\lambda : (x,y,z) = (6-33t,\ -8+2t,\ 3-19t)$, $t \in \mathbb{R}$

2

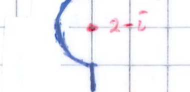
$$\Omega = \left\{ z \in \mathbb{C} : \left| z - \frac{1-3i}{1-i} \right| \geq 1,\ \operatorname{Re}(z) \leq 2 \right\}$$

Since $\frac{1-3i}{1-i} = \frac{(1-3i)\cdot(1+i)}{1+1} = \frac{1+3+i-3i}{2} = 2-i$,

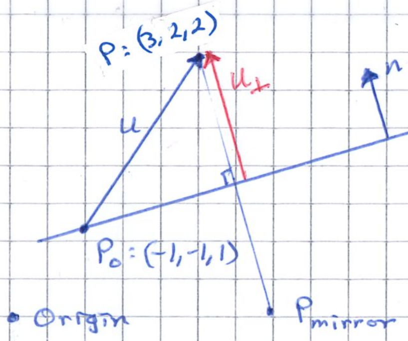
the eq. $\left| z - \frac{1-3i}{1-i} \right| \geq 1 \Leftrightarrow |z-(2-i)| \geq 1$ means all points in $\mathbb{C}$ which are on or outside a unit circle centered at the point $2-i$.



In the sketch, we notice that Ω includes all points on and to the left of the curve



3



$\pi : 2x+y-4z+7=0$

$n = (2,1,-4) \perp \pi$

$P_0 \in \pi$

$u = u_{\overrightarrow{OP}} - u_{\overrightarrow{OP_0}} = (3,2,2) - (-1,-1,1) = (4,3,1)$

We have that $u_\perp = \operatorname{proj}_n(u) = \frac{u \cdot n}{\|n\|^2} n = \frac{(4,3,1)\cdot(2,1,-4)}{\|(2,1,4)\|^2}(2,1,-4)$

$$= \frac{8+3-4}{2^2+1^2+4^2}(2,1,-4) = \frac{7}{21}(2,1,-4) = \frac{1}{3}(2,1,-4)$$

and then

$$u_{\overrightarrow{OP_{mirror}}} = u_{\overrightarrow{OP}} - 2u_\perp = (3,2,2) - 2\cdot\tfrac{1}{3}(2,1,-4) = \left(\tfrac{5}{3}, \tfrac{4}{3}, \tfrac{14}{3}\right)$$

The mirror image P_{mi} of P in π has thus the coordinates $\left(\frac{5}{3}, \frac{4}{3}, \frac{14}{3}\right)$

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④

$$\begin{vmatrix} 3 & 3 & 3 & 7 \\ 3 & 3 & 7 & 3 \\ 3 & 7 & 3 & 3 \\ 7 & 3 & 3 & 3 \end{vmatrix} \begin{matrix} (-1) \\ \\ \\ \\ \end{matrix} = \begin{vmatrix} 3 & 3 & 3 & 7 \\ 0 & 0 & 4 & -4 \\ 0 & 4 & 0 & -4 \\ 4 & 0 & 0 & -4 \end{vmatrix} = \begin{vmatrix} 3 & 3 & 3 & 16 \\ 0 & 0 & 4 & 0 \\ 0 & 4 & 0 & 0 \\ 4 & 0 & 0 & 0 \end{vmatrix}$$

expansion along the 4^{th} column

$$= 16(-1)^{4+1} \begin{vmatrix} 0 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 0 \end{vmatrix} + 0 + 0 + 0 = -16(-4^3) = \underline{4^5 = 2^{10} = 1024}$$

⑤

$$\begin{cases} x - 2y - az = a - 6 \\ -3x + 5y + 2az = 14 - 3a \\ 2x + y + a^2 z = 4a + 2 \end{cases} \Leftrightarrow \begin{cases} x - 2y - az = a - 6 \\ -y - az = -4 \\ 5y + (a^2 + 2a)z = 2a + 14 \end{cases}$$

$$\Leftrightarrow \begin{cases} x + az = a + 2 \\ y + az = 4 \\ a(a-3)z = 2(a-3) \end{cases}$$

<u>If</u> $a = 0$ <u>then</u> $\begin{cases} x = 2 \\ y = 4 \\ 0 = -6 \end{cases}$ <u>i.e.</u> no root

<u>If</u> $a = 3$ <u>then</u> $\begin{cases} x + 3z = 5 \\ y + 3z = 4 \\ 0 = 0 \end{cases} \Leftrightarrow (x, y, z) = (5 - 3t, 4 - 3t, t), \ t \in \mathbb{R}$

<u>If</u> $a \neq 0, 3$ <u>then</u> $\begin{cases} x + az = a + 2 \\ y + az = 4 \\ az = 2 \end{cases} \Leftrightarrow \begin{cases} x = a \\ y = 2 \\ z = 2/a \end{cases}$

<u>Answer:</u>

$$\boxed{(x, y, z) = \begin{cases} (5 - 3t, 4 - 3t, t), \ t \in \mathbb{R} & \text{if } a = 3 \\ \left(a, 2, \frac{2}{a}\right) & \text{if } a \neq 0, 3 \end{cases}}$$

No root exists if $a = 0$

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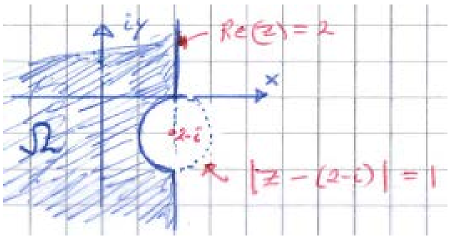
EXAMINATION IN MATHEMATICS
MAA150 Vector algebra
EVALUATION PRINCIPLES with POINT RANGES
Academic year: 2017/18

Final examination TEN1 – 2018-04-23 Maximum points for subparts of the problems in the final examination

1. $\lambda : (x,y,z) = (6,-8,3) + t\,(-33,2,-19),\ t \in R$

1p: Correctly interpreted the equation for the plane π concerning vectors which are parallel with the plane
2p: Correctly for the construction of an equation for the line λ found a vector which is orthogonal to the plane π
2p: Correctly formulated an equation for the line λ

2.



1p: Correctly for a final interpretation of the conditions for Ω, rewrited $(1-3i)/(1-i)$ as $2-i$
2p: Correctly interpreted the geometrical meaning of the condition $|z-(2-i)| \geq 1$
2p: Correctly sketched the region Ω

3. $P_{\text{mi}} : \left(\frac{5}{3}, \frac{4}{3}, \frac{14}{3}\right)$

2p: Correctly concluded that the mirror image P_{mi} of P can be addressed by adding to the coordinates of the point P, twice the negative coordinates of the ortogonal projection of a vector $u_{\overline{P_0P}}$ on a normal to the plane
2p: Correctly found the mentioned orthogonal projection
1p: Correctly found the coordinates of the mirror image of P

4. 1024

------------------------**One possible scenario** ------------------------

1p: Correctly added $-1 \cdot \text{row}_1$ to the other rows
2p: Correctly added the first three columns to the fourth
1p: Correctly expanded the determinant along the fourth column (or along any of the 2nd, 3rd or 4th column)
1p: Correctly found the value of the determinant

------------------------ **Other scenarios** ------------------------

For other scenarios, the criteria should in the proportions correspond as close as possible to those above

5. $(x,y,z) = \begin{cases} (5-3t, 4-3t, t) & \text{if } a = 3 \\ \left(a, 2, \frac{2}{a}\right) & \text{if } a \neq 0{,}3 \end{cases}$

No root exists if $a = 0$.

1p: Correctly concluded that the solving of the system of linear equations has to be divided into three cases, namely $a = 0$, $a = 3$ and $a \neq 0, 3$
1p: Correctly found that no root exists if $a = 0$
1p: Correctly found the (parametric) triples if $a = 3$
2p: Correctly found the unique triple for every $a \neq 0, 3$