Formal languages, automata, and theory of computation

Thursday, January 7, 14:10 - 18:30
Teacher: Daniel Hedin, phone 021-107052 (15:00 - 17:00)

The exam has a total of 40 points distributed over 14 questions. No aids are allowed. Answers must be given in English and should be clearly justified.

- Explain all solutions. A correctly explained solution with minor mistakes may render full points.
- Write clearly and *use the definitions and notation we have used in the course*. Unreadable solutions will not be graded.
- Start each question on a new page and only write on one side of the page.

Regular languages (14 p) The NFA in Figure 1 accepts a language L over $\Sigma = \{a, b\}$.

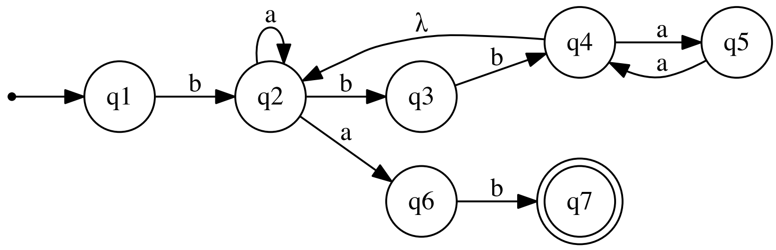

Figure 1: An NFA

1) Construct a regular expression for L. (2p)

2) Construct a regular grammar for L. (2p)

3) Construct a DFA for L. Note that the DFA does not have to be minimal. (3p)

4) Minimize the DFA in Figure 2. (3p)

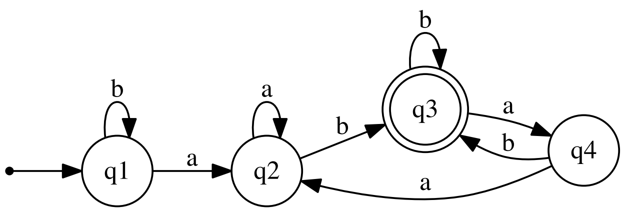

Figure 2: A DFA

5) Use the pumping lemma to prove that $L = \{a^n b^m \mid n = k * m \text{ for } n, m, k > 0\}$ is not regular. A detailed proof is needed for full points. (4p)

Context-free languages (14 p)

6) Construct a push-down automaton for the language $L = \{a^n b^m \mid n \leq m \leq 2n\}$. (3p)

7) What language does the NPDA in Figure 3 accept? (3p)

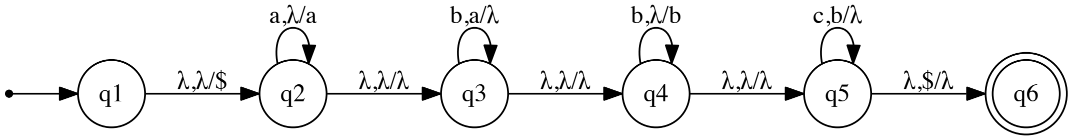

Figure 3: An NPDA

8) Construct a DPDA that is equivalent to the NPDA in Figure 3. (3p)

9) Construct a context-free grammar for the language $L = \{a^n b^n c^m d^m \mid m, n \geq 0\}$. (2p)

10) Explain the notion of ambiguity and show that the following grammar is ambiguous. (3p)

$$\begin{array}{lcl} S & \rightarrow & Ab \mid aaB \\ A & \rightarrow & a \mid Aa \\ B & \rightarrow & b \end{array}$$

Restriction-free languages and theory of computation (12 p) Consider the Turing machine M in Figure 4.

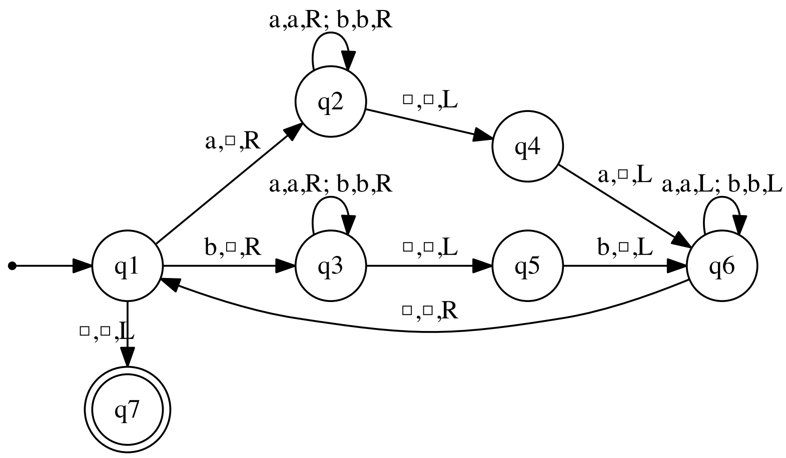

Figure 4: Turing machine M

11) Write down δ, the transition function of M. (2p)

12) What language does M accept? (2p)

13) Select a string $w_a \in \{a,b\}^4$ that is accepted by M and a string $w_r \in \{a,b\}^4$ and show the executions of M on the selected strings as two sequences of instantaneous descriptions. (4p)

14) Explain the concept of reduction proof. In particular, explain what is reduced to what (be careful with the direction of the reduction) and which conclusions that can be drawn from such reductions. (4p)