MÄLARDALEN UNIVERSITY
School of Education, Culture and Communication
Department of Applied Mathematics
Examiner: Lars-Göran Larsson

EXAMINATION IN MATHEMATICS
MAA151 Single Variable Calculus, TEN2
Date: 2016-03-24 *Write time:* 3 hours
Aid: Writing materials, ruler

This examination is intended for the examination part TEN2. The examination consists of five RANDOMLY ORDERED problems each of which is worth at maximum 4 points. The PASS-marks 3, 4 and 5 require a minimum of 9, 13 and 17 points respectively. The minimum points for the ECTS-marks E, D, C, B and A are 9, 10, 13, 16 and 20 respectively. If the obtained sum of points is denoted S_2, and that obtained at examination TEN1 S_1, the mark for a completed course is according to the following:

$S_1 \geq 11,\ S_2 \geq 9$	AND	$S_1 + 2S_2 \leq 41$	$\rightarrow$	3
$S_1 \geq 11,\ S_2 \geq 9$	AND	$42 \leq S_1 + 2S_2 \leq 53$	$\rightarrow$	4
		$54 \leq S_1 + 2S_2$	$\rightarrow$	5
$S_1 \geq 11,\ S_2 \geq 9$	AND	$S_1 + 2S_2 \leq 32$	$\rightarrow$	E
$S_1 \geq 11,\ S_2 \geq 9$	AND	$33 \leq S_1 + 2S_2 \leq 41$	$\rightarrow$	D
$S_1 \geq 11,\ S_2 \geq 9$	AND	$42 \leq S_1 + 2S_2 \leq 51$	$\rightarrow$	C
		$52 \leq S_1 + 2S_2 \leq 60$	$\rightarrow$	B
		$61 \leq S_1 + 2S_2$	$\rightarrow$	A

Solutions are supposed to include rigorous justifications and clear answers. All sheets of solutions must be sorted in the order the problems are given in.

1. Is the series

$$\sum_{n=1}^{\infty} \frac{2^n}{n!}$$

convergent or not? If your answer is YES: Give an explanation of why and find the sum of the series! If your answer is NO: Give an explanation of why!

2. Find the Taylor polynomial of order 2 about the point -1 for the function f whose function curve $y = f(x)$ with $f(-1) = -2$ is a solution of the equation

$$2x^2y + xy^3 = 4$$

in a neighbourhood of $P : (-1, -2)$.

3. Find the range of the function f defined by

$$f(x) = \arctan(x) + \arctan(1/x) \quad \text{and} \quad D_f = (0, \infty).$$

4. Find the volume of the solid generated by rotating about the x-axis the square region Ω with its vertices at the points $P : (0, 1)$, $Q : (1, 2)$, $R : (0, 3)$ and $S : (-1, 2)$.

5. Evaluate the integral

$$\int_1^{\sqrt{2}} x^7 e^{x^4}\, dx\,,$$

and write the result in as simple form as possible.

Om du föredrar uppgifterna formulerade på svenska, var god vänd på bladet.

MÄLARDALENS HÖGSKOLA
Akademin för utbildning, kultur och kommunikation
Avdelningen för tillämpad matematik
Examinator: Lars-Göran Larsson

TENTAMEN I MATEMATIK
MAA151 Envariabelkalkyl, TEN2
Datum: 2016-03-24 *Skrivtid:* 3 timmar
Hjälpmedel: Skrivdon, linjal

Denna tentamen är avsedd för examinationsmomentet TEN2. Provet består av fem stycken om varannat SLUMPMÄSSIGT ORDNADE uppgifter som vardera kan ge maximalt 4 poäng. För GODKÄND-betygen 3, 4 och 5 krävs erhållna poängsummor om minst 9, 13 respektive 17 poäng. Om den erhållna poängen benämns S_2, och den vid tentamen TEN1 erhållna S_1, bestäms graden av sammanfattningsbetyg på en slutförd kurs enligt följande:

$$\begin{array}{lclcl} S_1 \geq 11,\ S_2 \geq 9 & \text{OCH} & S_1 + 2S_2 \leq 41 & \rightarrow & 3 \\ S_1 \geq 11,\ S_2 \geq 9 & \text{OCH} & 42 \leq S_1 + 2S_2 \leq 53 & \rightarrow & 4 \\ & & 54 \leq S_1 + 2S_2 & \rightarrow & 5 \end{array}$$

Lösningar förutsätts innefatta ordentliga motiveringar och tydliga svar. Samtliga lösningsblad skall vid inlämning vara sorterade i den ordning som uppgifterna är givna i.

1. Är serien

$$\sum_{n=1}^{\infty} \frac{2^n}{n!}$$

konvergent eller ej? Om ditt svar är JA: Ge en förklaring till varför och bestäm summan av serien! Om ditt svar är NEJ: Ge en förklaring till varför!

2. Bestäm Taylorpolynomet av ordning 2 kring punkten -1 för den funktion f vars funktionskurva $y = f(x)$ med $f(-1) = -2$ är en lösning till ekvationen

$$2x^2y + xy^3 = 4$$

i en omgivning till $P : (-1, -2)$.

3. Bestäm värdemängden för funktionen f definierad genom

$$f(x) = \arctan(x) + \arctan(1/x) \quad \text{och} \quad D_f = (0, \infty).$$

4. Bestäm volymen av den kropp som genereras genom att kring x-axeln rotera det kvadratiska område Ω som har sina hörn i punkterna $P : (0, 1)$, $Q : (1, 2)$, $R : (0, 3)$ och $S : (-1, 2)$.

5. Beräkna integralen

$$\int_1^{\sqrt{2}} x^7 e^{x^4}\, dx\,,$$

och skriv resultatet på en så enkel form som möjligt.

If you prefer the problems formulated in English, please turn the page.

MAA151 / Solutions to the final exam TEN2 2016-03-24

① $\sum_{n=1}^{\infty} \frac{2^n}{n!}$ Let $a_n = \frac{2^n}{n!}$

Then $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{2^{n+1}}{(n+1)!} \frac{n!}{2^n} = \lim_{n\to\infty} \frac{2}{n+1} = 0 < 1$

i.e. the series $\sum_{n=1}^{\infty} a_n$ is (absolutely) convergent according to the ratio test

The sum of the series is $\sum_{n=1}^{\infty} \frac{2^n}{n!} = \underbrace{\left(\sum_{n=0}^{\infty} \frac{2^n}{n!}\right)}_{\text{The Maclaurin series for } x\mapsto e^x \text{ at the point } 2} - \frac{2^0}{0!} = e^2 - 1$

② Let $\begin{cases} \gamma: 2x^2y + xy^3 = 4 \\ P: (-1,-2) \end{cases} \iff$ $y = f(x)$ in a neighbourhood of P where $f(-1) = -2$

Differentiation with respect of x once and twice gives

$\tilde{\gamma}: \quad 4xy + 2x^2\frac{dy}{dx} + 1\cdot y^3 + x\,3y^2\frac{dy}{dx} = 0 \iff (4xy + y^3) + (2x^2 + 3xy^2)\frac{dy}{dx} = 0$

$\tilde{\tilde{\gamma}}: \quad \left(4y + 4x\frac{dy}{dx} + 3y^2\frac{dy}{dx}\right) + \left(4x + 3y^2 + 6xy\frac{dy}{dx}\right)\frac{dy}{dx} + (2x^2 + 3xy^2)\frac{d^2y}{dx^2} = 0$

At P, $\tilde{\gamma}$ gives $4(-1)(-2) + (-2)^3 + [2(-1)^2 + 3(-1)(-2)^2]f'(-1) = 0$

i.e. $8 - 8 + (2-12)f'(-1) = 0$ i.e. $f'(-1) = 0$

and then $\tilde{\tilde{\gamma}}$ gives $(4(-2) + 0 + 0) + 0 + (2(-1)^2 + 3(-1)(-2)^2)f''(-1) = 0$

i.e. $-8 + (2-12)f''(-1) = 0$ i.e. $f''(-1) = -\frac{4}{5}$

The Taylor polynomial of order 2 for f about the point -1 is
$f(-1) + f'(-1)\cdot(x-(-1)) + \frac{1}{2}f''(-1)\cdot(x-(-1))^2 = -2 - \frac{2}{5}(x+1)^2$

③ $f(x) = \arctan(x) + \arctan\left(\frac{1}{x}\right)$ where $D_f = (0,\infty)$

Differentiation gives: $f'(x) = \frac{1}{1+x^2} + \frac{1}{1+\left(\frac{1}{x}\right)^2}\left(-\frac{1}{x^2}\right)$

$= \frac{1}{1+x^2} - \frac{1}{x^2+1} = 0$

Thus $f(x) = C$

Since e.g. $f(1) = \arctan(1) + \arctan(1) = 2\cdot\frac{\pi}{4} = \frac{\pi}{2}$,
the constant C is equal to $\frac{\pi}{2}$.

Summary: $f(x) = \frac{\pi}{2}$ for $x \in D_f$

i.e. $R_f = \left\{\frac{\pi}{2}\right\}$

Cont. MAA151
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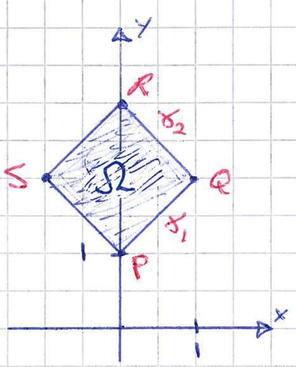

P: (0,1), Q: (1,2), R: (0,3), S: (-1,2)

$$\begin{cases} \gamma_1: \ y = x+1, & 0 \le x \le 1 \\ \gamma_2: \ y = -x+3, & 0 \le x \le 1 \end{cases}$$

The volume of the solid generated by rotating the region Ω about the x-axis is

$$V_x = 2\int_0^1 \pi\left[(-x+3)^2-(x+1)^2\right]dx$$

(where the factor 2 comes from the fact that the right half of Ω generates as much volume as the left half)

$$= 2\pi\int_0^1\left[(x^2-6x+9)-(x^2+2x+1)\right]dx$$

$$= 2\pi\int_0^1(-8x+8)\,dx = 8\pi\int_0^1\left[-2(x-1)\right]dx$$

$$= 8\pi\left[-(x-1)^2\right]_0^1 = 8\pi(-0+1) = 8\pi$$

Answer 8π v.u.

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$$\int_1^{\sqrt{2}} x^7 e^{x^4}dx = \int_1^{\sqrt{2}} x^4 e^{x^4} x^3 dx \quad \begin{bmatrix} x^4 = u \\ 4x^3dx = du \end{bmatrix}$$

$$= \int_1^4 u e^u \tfrac{1}{4}du$$

(Integration by parts)

$$= \frac{1}{4}\left(\left[u e^u\right]_1^4 - \int_1^4 1\cdot e^u du\right)$$

$$= \frac{1}{4}\left[(u-1)e^u\right]_1^4$$

$$= \frac{1}{4}\left(3e^4 - 0\right)$$

$$= \frac{3}{4}e^4$$

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EXAMINATION IN MATHEMATICS
MAA151 Single Variable Calculus
EVALUATION PRINCIPLES with POINT RANGES
Academic Year: 2015/16

Examination TEN2 – 2016-03-24 Maximum points for subparts of the problems in the final examination

1.	The series is convergent and the sum of the series is $e^2 - 1$	**2p**: Correctly, by the ratio test, found that the series is convergent **1p**: Correctly identified the series as, except for the first term, the Maclaurin series for the exponential function at the point 2 **1p**: Correctly found the sum of the series
2.	$-2-\frac{2}{5}(x+1)^2$	**1p**: Correctly differentiated with respect to x in the LHS and RHS of the equation **1p**: Correctly found the value of $f'(-1)$ **1p**: Correctly differentiated once more with respect to x in the LHS and RHS of the equation and correctly found the value of $f''(-1)$ **1p**: Correctly from the found values of the derivatives formulated the Taylor polynomial of order 2 about the point -1 for the function f
3.	$R_f = \left\{\frac{\pi}{2}\right\}$	**1p**: Correctly found that the derivative of the function is equal to zero **1p**: Correctly interpreted the result as that the function is constant in its domain $(0,\infty)$ **2p**: Correctly found the constant in the interval $(0,\infty)$ to be equal to $\pi/2$, and correctly summarized the range
4.	8π v.u.	**2p**: Correctly formulated an integral (or a sum of integrals) for the volume obtained by rotating the region about the x-axis (irrespective whether the method of cylindrical shells or the method of slicing have been applied) **1p**: Correctly found an antiderivative of the integrand **1p**: Correctly worked out the final step (inserting the limits) in finding the integral
5.	$\frac{3}{4}e^4$	**1p**: Correctly translated the integrand as working out the suitable substitution $x^4 = u$ **1p**: Correctly translated the limits as working out the suitable substitution $x^4 = u$ **1p**: Correctly worked out the antiderivative by parts **1p**: Correctly worked out the final step (inserting the limits) in finding the integral