MÄLARDALEN UNIVERSITY
School of Education, Culture and Communication
Department of Applied Mathematics
Examiner: Lars-Göran Larsson

EXAMINATION IN MATHEMATICS
MAA151 Single Variable Calculus, TEN1
Date: 2017-08-14 *Write time:* 3 hours
Aid: Writing materials, ruler

This examination is intended for the examination part TEN1. The examination consists of eight RANDOMLY ORDERED problems each of which is worth at maximum 3 points. The PASS-marks 3, 4 and 5 require a minimum of 11, 16 and 21 points respectively. The minimum points for the ECTS-marks E, D, C, B and A are 11, 13, 16, 20 and 23 respectively. If the obtained sum of points is denoted S_1, and that obtained at examination TEN2 S_2, the mark for a completed course is according to the following

$$\begin{array}{lcrcl} S_1 \geq 11,\ S_2 \geq 9 & \text{AND} & S_1 + 2S_2 \leq 41 & \rightarrow & 3 \\ S_1 \geq 11,\ S_2 \geq 9 & \text{AND} & 42 \leq S_1 + 2S_2 \leq 53 & \rightarrow & 4 \\ & & 54 \leq S_1 + 2S_2 & \rightarrow & 5 \\ \\ S_1 \geq 11,\ S_2 \geq 9 & \text{AND} & S_1 + 2S_2 \leq 32 & \rightarrow & \text{E} \\ S_1 \geq 11,\ S_2 \geq 9 & \text{AND} & 33 \leq S_1 + 2S_2 \leq 41 & \rightarrow & \text{D} \\ S_1 \geq 11,\ S_2 \geq 9 & \text{AND} & 42 \leq S_1 + 2S_2 \leq 51 & \rightarrow & \text{C} \\ & & 52 \leq S_1 + 2S_2 \leq 60 & \rightarrow & \text{B} \\ & & 61 \leq S_1 + 2S_2 & \rightarrow & \text{A} \end{array}$$

Solutions are supposed to include rigorous justifications and clear answers. All sheets of solutions must be sorted in the order the problems are given in.

1. Find to the differential equation $y'' = -9y$ the solution that satisfies the initial conditions $y(0) = 1$, $y'(0) = -12$.

2. The function f is differentiable, and it is known that

$$\begin{array}{llllll} f(-2)=0\,, & f(-1)=3\,, & f(0)=2\,, & f(1)=-1\,, & f(2)=-1\,, & f(3)=-2\,, \\ f'(-2)=5\,, & f'(-1)=4\,, & f'(0)=-2\,, & f'(1)=3\,, & f'(2)=6\,, & f'(3)=-4\,. \end{array}$$

Find an equation for the tangent line τ to the curve $\gamma : y = f \circ f(x)$ at the point P whose x-coordinate is equal to 2.

3. Find out whether the function f definied by $f(x) = x^2 \ln(x)$ is bounded, bounded above, bounded below, or unbounded both above and below.

4. Find the function f such that $f'(x) = \tan(2x)$ and $f(0) = 0$.

5. Sketch in the same figure the function curves given by the three equations $y = \arcsin(x)$, $y = \arcsin(x/2)$ and $y = \arcsin(x-2)$.

6. Find out whether
$$\lim_{x \to 4^+} \frac{|\,8 - 2x|}{32 - 2x^2}$$
exists or not. If the answer is NO: Give an explanation of why! If the answer is YES: Give an explanation of why and find the limit!

7. Evaluate the integral $\displaystyle\int_{-8}^{8} \sqrt{64 - x^2}\, dx$ by interpreting it as a certain area measure.

8. Which of the series $\left\{ \begin{array}{l} \sqrt{3} + \frac{\sqrt{5}}{\sqrt{3}} + \frac{5}{3\sqrt{3}} + \ldots \\ \frac{32}{7\sqrt{7}} - \frac{8}{\sqrt{7}} + \frac{\sqrt{7}}{2} - \ldots \\ \frac{2}{\sqrt{e}} - \sqrt{e} + \frac{e\sqrt{e}}{2} - \ldots \end{array} \right.$ are geometric, and which of the geometric are convergent? Find the sums of the convergent geometric series.

Om du föredrar uppgifterna formulerade på svenska, var god vänd på bladet.

MÄLARDALENS HÖGSKOLA
Akademin för utbildning, kultur och kommunikation
Avdelningen för tillämpad matematik
Examinator: Lars-Göran Larsson

TENTAMEN I MATEMATIK
MAA151 Envariabelkalkyl, TEN1
Datum: 2017-08-14 *Skrivtid:* 3 timmar
Hjälpmedel: Skrivdon, linjal

Denna tentamen är avsedd för examinationsmomentet TEN1. Provet består av åtta stycken om varannat SLUMPMÄSSIGT ORDNADE uppgifter som vardera kan ge maximalt 3 poäng. För GODKÄND-betygen 3, 4 och 5 krävs erhållna poängsummor om minst 11, 16 respektive 21 poäng. Om den erhållna poängen benämns S_1, och den vid tentamen TEN2 erhållna S_2, bestäms graden av sammanfattningsbetyg på en slutförd kurs enligt

$$\begin{array}{lcrcl} S_1 \geq 11,\ S_2 \geq 9 & \text{OCH} & S_1 + 2S_2 \leq 41 & \rightarrow & 3 \\ S_1 \geq 11,\ S_2 \geq 9 & \text{OCH} & 42 \leq S_1 + 2S_2 \leq 53 & \rightarrow & 4 \\ & & 54 \leq S_1 + 2S_2 & \rightarrow & 5 \end{array}$$

Lösningar förutsätts innefatta ordentliga motiveringar och tydliga svar. Samtliga lösningsblad skall vid inlämning vara sorterade i den ordning som uppgifterna är givna i.

1. Bestäm till differentialekvationen $y'' = -9y$ den lösning som satisfierar begynnelsevillkoren $y(0) = 1$, $y'(0) = -12$.

2. Funktionen f är deriverbar, och det är bekant att

$$\begin{array}{llllll} f(-2)=0\,, & f(-1)=3\,, & f(0)=2\,, & f(1)=-1\,, & f(2)=-1\,, & f(3)=-2\,, \\ f'(-2)=5\,, & f'(-1)=4\,, & f'(0)=-2\,, & f'(1)=3\,, & f'(2)=6\,, & f'(3)=-4\,. \end{array}$$

Bestäm en ekvation för tangenten τ till kurvan $\gamma : y = f \circ f(x)$ i punkten P vars x-koordinat är lika med 2.

3. Utred om funktionen f definierad av $f(x) = x^2 \ln(x)$ är begränsad, uppåt begränsad, nedåt begränsad, eller obegränsad både uppåt och nedåt.

4. Bestäm funktionen f så att $f'(x) = \tan(2x)$ och $f(0) = 0$.

5. Skissa i en och samma figur funktionskurvorna givna av de tre ekvationerna $y = \arcsin(x)$, $y = \arcsin(x/2)$ och $y = \arcsin(x-2)$.

6. Utred om
$$\lim_{x \to 4^+} \frac{|\,8 - 2x|}{32 - 2x^2}$$
existerar eller ej. Om svaret är NEJ: Ge en förklaring till varför! Om svaret är JA: Ge en förklaring till varför och bestäm gränsvärdet!

7. Beräkna integralen $\displaystyle\int_{-8}^{8} \sqrt{64 - x^2}\, dx$ genom att tolka den som ett visst areamått.

8. Vilka av serierna
$$\begin{cases} \sqrt{3} + \frac{\sqrt{5}}{\sqrt{3}} + \frac{5}{3\sqrt{3}} + \ldots \\ \frac{32}{7\sqrt{7}} - \frac{8}{\sqrt{7}} + \frac{\sqrt{7}}{2} - \cdots \\ \frac{2}{\sqrt{e}} - \sqrt{e} + \frac{e\sqrt{e}}{2} - \ldots \end{cases}$$
är geometriska, och vilka av de geometriska är konvergenta? Bestäm summorna av de konvergenta geometriska serierna.

If you prefer the problems formulated in English, please turn the page.

MAA151 / Solutions to the final exam TEN1 2017-08-14

1 DE: $y'' + 9y = 0$ IV: $y(0) = 1$, $y'(0) = -12$

The auxiliary eq. is $r^2 + 9 = 0$ and give $y = A\cos(3x) + B\sin(3x)$ as the general solution of the DE. The IV:s give

$$\begin{cases} 1 = y(0) = A\cdot 1 + B\cdot 0 \\ -12 = y'(0) = -3A\cdot 0 + 3B\cdot 1 \end{cases} \Leftrightarrow \begin{cases} A = 1 \\ B = -4 \end{cases}$$

Thus $y = \cos(3x) - 4\sin(3x)$ is the solution of the IVP

2 $\gamma : y = f\circ f(x)$, $x_p = 2$

from the table of values

We get $y_p = f\circ f(x_p) = f(f(2)) = f(-1) = 3$

and $\frac{d}{dx} f\circ f(x)\Big|_{x=x_p} = f'(f(2))\cdot f'(2) = f'(-1)\cdot f'(2) = 4\cdot 6 = 24$

Thus $\tau : y - 3 = 24(x-2)$

3 $f(x) = x^2\ln(x)$ Diff. gives $f'(x) = 2x\ln(x) + x^2\cdot\frac{1}{x}$

$= 2x\left(\ln(x) + \frac{1}{2}\right) = 2x\ln(x\sqrt{e})$

A first-derivative test for f

x		$\frac{1}{\sqrt{e}}$	
$f'(x)$	$-$	0	$+$
$f(x)$	$\searrow$	loc. min	$\nearrow$

and $\begin{cases} f \text{ i continuous in } D_f = (0,\infty) \\ f(x) \to +\infty \text{ as } x \to +\infty \end{cases}$

give that f is bounded below

4 $f'(x) = \tan(2x)$, $f(0) = 0$

We get

$$f(x) = \int \tan(2x)\,dx = \int \frac{\sin(2x)}{\cos(2x)}\,dx \left[\begin{array}{l} \cos(2x) = u \\ -2\sin(2x)\,dx = du \end{array}\right]$$

$$= \int\left(-\frac{1}{2}\right)\frac{du}{u} = -\frac{1}{2}\ln|u| + C = -\frac{1}{2}\ln|\cos(2x)| + C$$

where C is a constant.

The IV $f(0) = 0$ gives the value of C but also that the solution is in an interval where $|\cos(2x)| = \cos(2x)$ (since $\cos(2\cdot 0) = 1 > 0$).

Thus $0 = f(0) = -\frac{1}{2}\ln(\cos(0)) + C = -\frac{1}{2}\ln(1) + C = 0 + C$

i.e. $f(x) = -\frac{1}{2}\ln(\cos(2x))$

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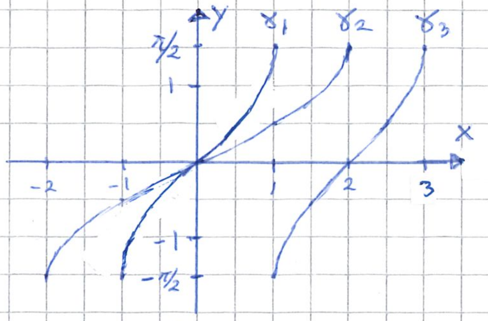
$$\begin{cases} \gamma_1: \ y = \arcsin(x) \\ \gamma_2: \ y = \arcsin(x/2) \\ \gamma_3: \ y = \arcsin(x-2) \end{cases}$$



where $\begin{cases} D_{f_1} = [-1, 1] \\ D_{f_2} = [-2, 2] \\ D_{f_3} = [1, 3] \end{cases}$ and $R_{f_1} = R_{f_2} = R_{f_3} = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

6

$$\lim_{x \to 4^+} \frac{|8-2x|}{32-2x^2} = \lim_{x \to 4^+} \frac{2 \cdot |4-x|}{2(4+x)(4-x)} = \lim_{x \to 4^+} \frac{-(4-x)}{(4+x)(4-x)}$$

$$= -\lim_{x \to 4^+} \frac{1}{4+x} = -\frac{1}{4+4} = -\frac{1}{8}$$

Since $x \to \frac{1}{4+x}$ is cont. at 4

Thus the limit exists and equals $-\frac{1}{8}$

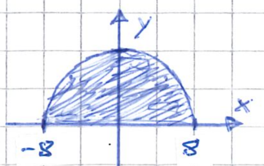
7

$$\begin{cases} \sqrt{64-x^2} = y \\ -8 \le x \le 8 \end{cases} \iff \begin{cases} x^2 + y^2 = 8^2 \\ y \ge 0 \\ -8 \le x \le 8 \end{cases}$$

i.e. the upper half of a circle with the mean-value point at the origin and a radius of length 8.

Thus $\int_{-8}^{8} \sqrt{64-x^2}\, dx = \frac{1}{2}\pi \cdot 8^2 = 32\pi$

The area measure of a half circle disc of radius 8



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$$\begin{cases} S_A: \ \sqrt{3} + \sqrt{\frac{5}{3}} + \frac{5}{3\sqrt{3}} + \dots \\ S_B: \ \frac{32}{7\sqrt{7}} - \frac{8}{\sqrt{7}} + \frac{\sqrt{7}}{2} - \dots \\ S_C: \ \frac{2}{\sqrt{e}} - \sqrt{e} + \frac{e\sqrt{e}}{2} - \dots \end{cases}$$

A series is geometric iff the series is on the form $\alpha + \alpha\beta + \alpha\beta^2 + \dots$

Series S_A: $\beta_1 = \frac{\sqrt{5/3}}{\sqrt{3}} = \frac{\sqrt{5}}{3}$, $\beta_2 = \frac{\frac{5}{3\sqrt{3}}}{\sqrt{\frac{5}{3}}} = \frac{\sqrt{5}}{3} = \beta_1$

i.e. the series is geometric. Since $|\beta| = \frac{\sqrt{5}}{3} < 1$ it is also convergent. Its sum equals $\frac{\sqrt{3}}{1-\frac{\sqrt{5}}{3}} = \frac{3\sqrt{3}}{3-\sqrt{5}}$.

Series S_B: $\beta_1 = \frac{-\frac{8}{\sqrt{7}}}{\frac{32}{7\sqrt{7}}} = -\frac{7}{4}$, $\beta_2 = \frac{\frac{\sqrt{7}}{2}}{-\frac{8}{\sqrt{7}}} = -\frac{7}{16} \neq \beta_1$

i.e. the series is not geometric.

Series S_C: $\beta_1 = \frac{-\sqrt{e}}{\frac{2}{\sqrt{e}}} = -\frac{e}{2}$, $\beta_2 = \frac{\frac{e\sqrt{e}}{2}}{-\sqrt{e}} = -\frac{e}{2} = \beta_1$

i.e. the series is geometric. Since $|\beta| = \frac{e}{2} > 1$, the series is not convergent.

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EXAMINATION IN MATHEMATICS
MAA151 Single Variable Calculus
EVALUATION PRINCIPLES with POINT RANGES
Academic Year: 2016/17

Examination TEN1 – 2017-08-14 Maximum points for subparts of the problems in the final examination

1. $y = \cos(3x) - 4\sin(3x)$

Note: The student who, for the general solution, has found anything else but a linear combination of $\cos(3x)$ and $\sin(3x)$, obtains **0p**.

1p: Correctly found the general solution of the DE
1p: Correctly differentiated the general solution in preparing for the adaption to the initial values
1p: Correctly adapted the general solution to the initial values, and correctly summarized the solution of the IVP

2. $y = 24x - 45$

1p: Correctly found the derivative of the function $f \circ f$, all with the purpose of finding the slope at the point P
1p: Correctly evaluated the derivative of $f \circ f$ at x_P
1p: Correctly evaluated the function at x_P, and correctly found an equation for the tangent line asked for

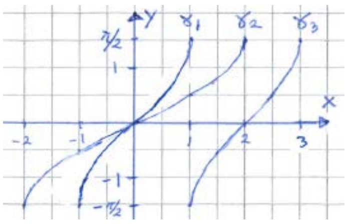
3. f is bounded below (but not above)

1p: Correctly worked out the details of a first derivative test
1p: Correctly concluded that f is bounded below
1p: Correctly concluded that f is not bounded above

4. $f(x) = -\frac{1}{2}\ln\big(2\cos(x)\big)$

1p: Correctly applied a substitution which simplifies the expression for the general antiderivative of f
1p: Correctly found the general antiderivative of f
1p: Correctly adapted the antiderivative to the value at 0

5.



1p: Correctly sketched the function curve $y = \arcsin(x)$
1p: Correctly sketched the function curve $y = \arcsin(x/2)$
1p: Correctly sketched the function curve $y = \arcsin(x-2)$

6. The limit exists and is equal to $-1/8$

Note: The student who have argued that the limit does not exist based on the fact that the fraction at the limit point is of the type "0/0" obtains **0p**. The student who have claimed that a fraction of the type "0/0" is equal to 0 obtains **0p**, especially if the succeeding conclusion is of the kind "*the limit does not exist since the value is equal to 0*".

1p: Correctly factorized the expression
1p: Correctly taken account of the absolute value bars
1p: Correctly concluded that the limit exists, and correctly found the limit

7. 32π

1p: Correctly identified the integrand as the function whose curve is the upper half of a circle with the mean-value point at the origin and with a radius of length 8
1p: Correctly interpreted the integral as a measure of the area of a half-circle disk
1p: Correctly evaluated the integral

8. The first series is a convergent, geometric series with the sum $\frac{3\sqrt{3}}{3-\sqrt{5}}$.

The second series is not geometric.

The third series is geometric but is not convergent.

1p: Correctly concluded that the first and the third series are geometric but not the second one
1p: Correctly concluded that the first of the two geometric series is convergent, and correctly found its sum
1p: Correctly concluded that the second of the two geometric series is divergent (and correctly not found any sum)