

# Exam

January 9th, 2018  
Västerås

## DVA414 – Industrial Robotics

(Till tentamensvakten: engelsk information behövs)

Teacher	Alessandro Papadopoulos, tel: 021-1073 23	
Exam duration	14:10-19:30	
Help allowed	Calculator, language dictionary, ruler, and APPENDIX attached to this exam.	
Points	48 p	
Grading	Swedish grades:	ECTS grades
	< 26p → failed	< 26 → failed
	26 – 34p → grade 3	26 – 29p → D
	35 – 41p → grade 4	30 – 36p → C
	42 – 48p → grade 5	37 – 41p → B
		42 – 48p → A

### Instructions

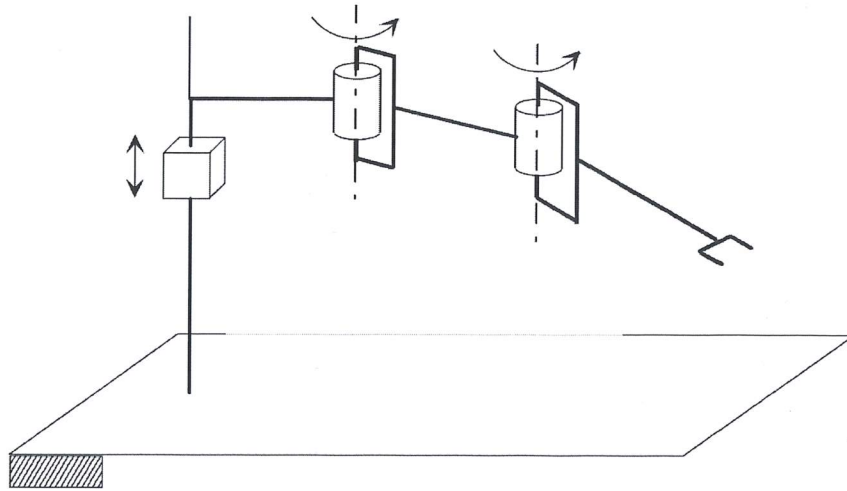
- Answers **MUST** be written in **English**.
- **Short and precise** answers are preferred. Do not write more than necessary.
- Use a **new sheet** for each of the assignments.
- If some assumptions are missing, or if you think the assumptions are unclear, **write down what do you assume** to solve the problem.
- Write **clearly**. If I cannot read it, you get zero points.

**Good luck!!!**

**Turn the page**

**EXERCISE 1 (DIRECT AND INVERSE KINEMATICS)****8 POINTS**

Given the PRR robot below:



1. Place reference frames for each link according to the DH (Denavit-Hartenberg) convention
2. Write a table with the values of the DH parameters for each link
3. Compute the homogeneous transformation matrix that represents the manipulator forward kinematics
4. Outline the inverse kinematics problem (without addressing hand orientation)

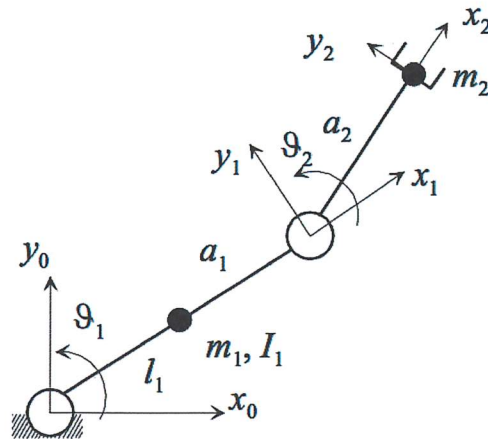
**EXERCISE 2 (TRAJECTORY PLANNING)****8 POINTS**

Derive a quintic trajectory which takes  $q(t)$  from the initial value  $q_i = 0\text{rad}$  to the final value  $q_f = 2\text{rad}$  within 1s, with initial and final joint velocity equal to 0, and initial and final acceleration equal to 0. Plan another trajectory with the same requirements on initial and final positions and velocities, but using a trapezoidal velocity profile where the maximum speed is 3rad/s.

$$S = \frac{\text{rad} - \text{rad} \cdot \text{s} \cdot \text{s}}{\text{rad} \cdot \text{s}}$$

**EXERCISE 3 (DYNAMICS)****8 POINTS**

Consider the planar manipulator in the vertical plane (gravity along the  $y$  axis) sketched in the picture, where **the mass of the second link is assumed to be concentrated at the end-effector**.



1. Find the expression of the inertia matrix of the manipulator, knowing that

$$p_1 = \begin{bmatrix} a_1 c_1 \\ a_1 s_1 \\ 0 \end{bmatrix}, \quad p_2 = \begin{bmatrix} a_1 c_1 + a_2 c_{12} \\ a_1 s_1 + a_2 s_{12} \\ 0 \end{bmatrix}.$$

2. Find the expression of the gravitational torques for this specific manipulator.

**EXERCISE 4 (HOMOGENEOUS TRANSFORMATION)****8 POINTS**

1. Find a representation of the rotation  $R_3^0$  defined by the following ordered sequence of basic rotations:
  - a. Rotation Frame0  $\rightarrow$  Frame1, by  $\theta$  about the current axis  $x$ , i.e.,  $x_0$
  - b. Rotation Frame1  $\rightarrow$  Frame2, by  $\phi$  about the **fixed** axis  $z$ , i.e.,  $z_0$
  - c. Rotation Frame2  $\rightarrow$  Frame3, by  $\alpha$  about the current axis  $x$ , i.e.,  $x_2$
2. Write the expression of the unit vectors  $x_3^0$ ,  $y_3^0$ , and  $z_3^0$ , when  $\theta = \frac{\pi}{2}$  rad,  $\phi = \frac{\pi}{3}$  rad, and  $\alpha = \frac{\pi}{6}$  rad.

**EXERCISE 5 (CONTROL)**

**8 POINTS**

1. List the closed loop control strategies in joint space.
2. Describe the motion control strategy “PD plus gravity compensation” in joint space, including
  - a. The control law
  - b. The control scheme
  - c. Limitations of the control strategy

**EXERCISE 6 (SAFETY AND SECURITY)**

**8 POINTS**

- Describe the difference between safety and security.
- Describe the relation between threat, vulnerability and risk.
- Describe at least two robot specific attack.

## Appendix – Formulas

### Kinematics

- The cross product between vectors  $\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$  and  $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$  is  $\mathbf{c} = \mathbf{a} \times \mathbf{b} = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$
- Tangent of an angle  $\theta$  given  $x$  and  $y$

$$\tan \theta = \frac{y}{x}$$

- Trigonometric formulas

$$\sin(-\alpha) = -\sin(\alpha)$$

$$\cos(-\alpha) = \cos(\alpha)$$

$$\tan(-\alpha) = -\tan(\alpha)$$

$$\sin(\alpha \pm \beta) = \sin(\alpha) \cos(\beta) \pm \cos(\alpha) \sin(\beta)$$

$$\cos(\alpha \pm \beta) = \cos(\alpha) \cos(\beta) \mp \sin(\alpha) \sin(\beta)$$

- Cosine theorem ( $a$  being the side opposite to  $\alpha$ )

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

### Differential Kinematics

- Derivative of a rotation matrix

$$\dot{\mathbf{R}} = \mathbf{S} \mathbf{R}$$

- Link velocities (general formula)

$$\boldsymbol{\omega}_i = \boldsymbol{\omega}_{i-1} + \boldsymbol{\omega}_{i-1,i}$$

$$\dot{\mathbf{p}}_i = \dot{\mathbf{p}}_{i-1} + \mathbf{v}_{i-1,i} + \boldsymbol{\omega}_{i-1} \times \mathbf{r}_{i-1,i}$$

- Jacobian computation

$$\begin{bmatrix} J_{P_i} \\ J_{O_i} \end{bmatrix} = \begin{cases} \begin{bmatrix} \mathbf{z}_{i-1} \\ \mathbf{0} \end{bmatrix}, & \text{for a prismatic joint} \\ \begin{bmatrix} \mathbf{z}_{i-1} \times (\mathbf{p}_e - \mathbf{p}_{i-1}) \\ \mathbf{z}_{i-1} \end{bmatrix}, & \text{for a revolute joint} \end{cases}$$

### Motion planning

- Harmonic trajectory

$$q(t) = \frac{q_f - q_i}{2} \left( 1 - \cos \left( \frac{\pi(t - t_i)}{t_f - t_i} \right) \right) + q_i, \quad q(t_i) = q_i, q(t_f) = q_f$$

$$\dot{q}(t) = \frac{\pi(q_f - q_i)}{2(t_f - t_i)} \sin \left( \frac{\pi(t - t_i)}{t_f - t_i} \right), \quad \dot{q}(t_i) = 0, \dot{q}(t_f) = 0$$

$$\ddot{q}(t) = \frac{\pi^2(q_f - q_i)}{2(t_f - t_i)^2} \cos \left( \frac{\pi(t - t_i)}{t_f - t_i} \right),$$