Assignment - 1

CS224, Max. Marks: 40

NOTE: This assignment is to be done **individually**. You can discuss the assignment, but the final submitted solution has to be written by each student individually. You can upload your solutions in PDF format (scan of handwritten solution is okay) to Moodle before the deadline.

Consider a communication system that consists of a single transmitter and a single receiver. Assume that the transmitted signal is attenuated and then corrupted by additive white Gaussian noise at the receiver. Denote the point in the two-dimensional constellation diagram (with cosine on X-axis and sine on Y-axis) corresponding to the *transmitted signal* $s(t)$ is $\underline{s} = (s_x, s_y)$. The unit vector on the x-axis is $\sqrt{\frac{2}{T}}\cos(2\pi f_0 t)$ (that is point (1,0)) and the unit vector on the y-axis (that is (0,1)) is $\sqrt{\frac{2}{T}}\sin(2\pi f_0 t)$, where T is the symbol duration (which is a multiple of $1/f_0$). Let us define the dot-product between two signals $g(t)$ and $h(t)$ as

$$< g(t), h(t) >= \int_0^T g(t)h(t)$$

We will assume that the *transmitted energy of any symbol* $s(t)$ is given by $||\underline{s}||^2 :=< s(t), s(t) >= s_x^2 + s_y^2$.

The signal is attenuated by factor α. Hence the received constellation point is $\underline{r} = (r_x, r_y)$ where $r_x = \alpha s_x + n_x$ and $r_y = \alpha s_y + n_y$ where n_x, n_y are i.i.d. Gaussian random variables with zero mean and variance $\frac{N_0}{2}$, where N_0 is the noise energy per symbol. Note that half the noise energy is in the X-axis direction and the other half in the Y-axis direction, which is why variance is $N_0/2$ in each direction.

We define the signal-to-noise ratio (SNR) per symbol at the receiver as the ratio of the following two quantities: (i) $\alpha^2\times$(average energy per transmitted symbol), and (ii) noise energy per symbol. Average energy per transmitted symbol is just the expected value (mean) of energy of a transmitted symbol.

In the following, derive the required probabilites in terms of the $Q(.)$ function which is defined as

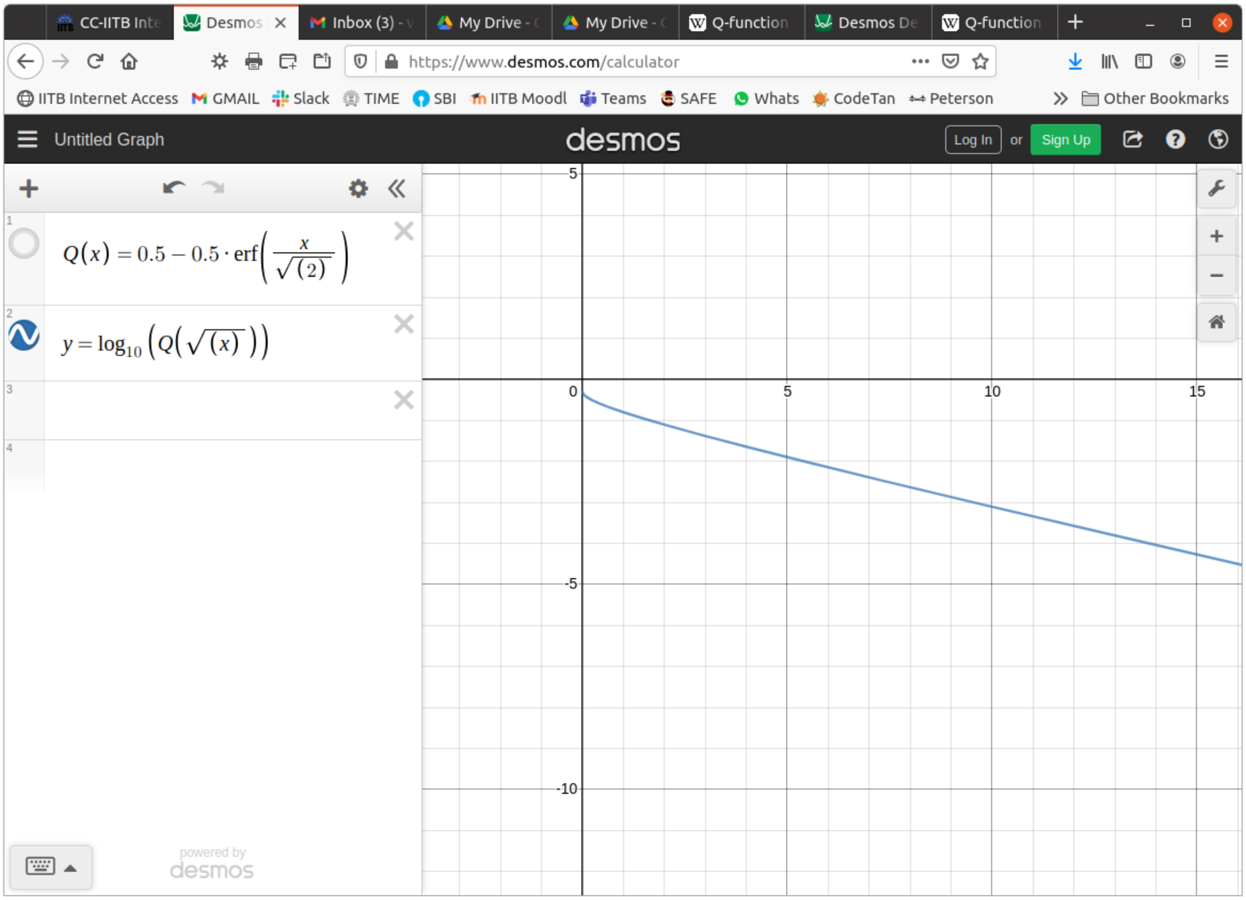
$$Q(z) = \frac{1}{\sqrt{2\pi}} \int_z^\infty \exp(-x^2/2)\ dx. \tag{1}$$

Write your final answers for each probability as a function of SNR per symbol. Show your working for all questions.

1. (5 marks) (BPSK) Suppose the transmitter uses constellation diagram $(-A, 0)$ and $(A, 0)$ to convey bit information 1 and 0 respectively. This means that $\underline{s}$ is chosen as one of these constellation points depending on the value of the bit to be transferred. Derive an expression for the probability of incorrectly detecting the transmitted bit in terms of SNR per symbol. Assume that bits 1 and 0 are transmitted with equal probability.

2. (10 marks) (QPSK) Suppose the transmitter uses constellation diagram $(A/\sqrt{2}, A/\sqrt{2})$, $(-A/\sqrt{2}, A/\sqrt{2})$, $(-A/\sqrt{2}, -A/\sqrt{2})$ and $(A/\sqrt{2}, -A/\sqrt{2})$, and assigns bits 00, 01, 11, 10 to these points respectively. Assume that all constellation points are transmitted with equal probability. Calculate the probability of the first bit being received in error. Calculate the probability of the second bit being received in error. Are these two probabilities equal? Are they greater than or less than the probability calculated for BPSK above (assuming the same SNR per symbol for BPSK and QPSK)?

3. (15 marks) Suppose constellation points $(-3A/\sqrt{5}, 0)$, $(-A/\sqrt{5}, 0)$, $(A/\sqrt{5}, 0)$, and $(3A/\sqrt{5}, 0)$, to transmit data across a wireless link. The data bits assigned to the above mentioned constellation points are 11, 00, 10, and 01. Assume that all constellation points are transmitted with equal probability. Calculate the probability of the first bit being received in error. Calculate the probability of the second bit being received in error. Are these two probabilities



equal? Are they greater than or less than the two probabilities calculated for QPSK above (assuming same SNR for QPSK and this scheme)?

4. (10 marks) Draw a single graph with SNR on the x-axis and $\log_{10}$ of the various probabilities in (1)-(3). You can draw it by hand (or take a screen shot of a scientific calculator application) and mark which line corresponds to which probability. To know what the graph looks like, you can use an online graphing tool: https://www.desmos.com/calculator . You can try typing in the left panel $Q(x) = 0.5 - 0.5 * erf(x/\sqrt(2))$ etc. as shown in the figure. You can add more plots just by entering more functions (e.g. $y = x^2$) in the rows in the left panel.