

Problem Set 2

1. Which of the following are FO definable? Which are not FO-definable (without a proof), but regular?
 - (a) The set of words over $\{a, b\}$ which has equal number of occurrences of ab and ba . For example, aba is in the language, while $abab$ is not.
 - (b) The set of words over $\{a, b, \#\}$ with a single occurrence of $\#$, and every symbol before the $\#$ is an a , and all symbols after the $\#$ are b 's.
 - (c) The set of strings over $\{a, b\}$ which does not contain any occurrence of ba .
 - (d) The set of strings over $\{0, 1\}$ such that the second symbol from both ends is 0.
 - (e) Let $\Sigma = \left\{ \begin{pmatrix} a \\ b \end{pmatrix} \mid a, b \in \{0, 1\} \right\}$. A string over Σ gives two rows of 0's and 1's. Treat each row as a binary number. The set of words

$$\{w \in \Sigma^* \mid \text{the top row is larger than the bottom row} \}$$

2. Consider the following FO formulae. In each case,
 - (a) what is $L(\varphi)$? (b) what is $\overline{L(\varphi)}$? (c) Is $L(\varphi)$ regular? (d) Is $\overline{L(\varphi)}$ regular?
 - (1) $\forall x(x \neq x)$
 - (2) $\exists x \exists y [x < y \wedge Q_b(x) \wedge Q_a(y) \wedge \forall x [(x < z < y) \rightarrow Q_a(z)]]$
 - (3) $\exists x [Q_a(x) \wedge \exists y [S(x, y) \wedge \forall z [z \leq y]]]$
 - (4) $\exists x \forall y [x \leq y \wedge Q_a(x)] \wedge \exists x \forall y [y \leq x \wedge Q_b(x)] \wedge \forall x \forall y [Q_a(x) \wedge S(x, y) \rightarrow Q_b(y)] \wedge \forall x \forall y [Q_b(x) \wedge S(x, y) \rightarrow Q_a(y)]$
3. Consider the following automaton. What is the language L accepted? Can you write an FO formula φ such that $L = L(\varphi)$?

