Sequential Circuits

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CS-226: Digital Logic Design

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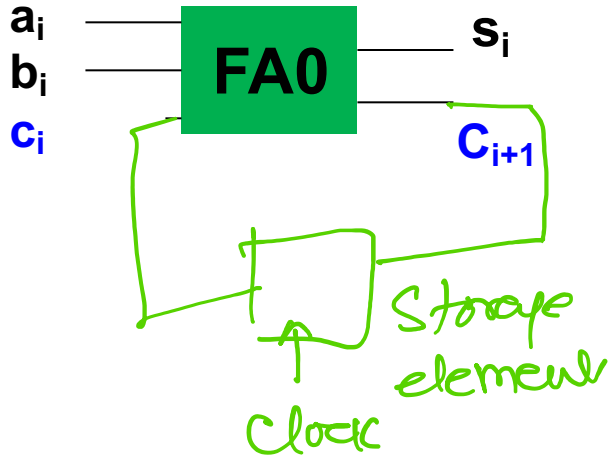
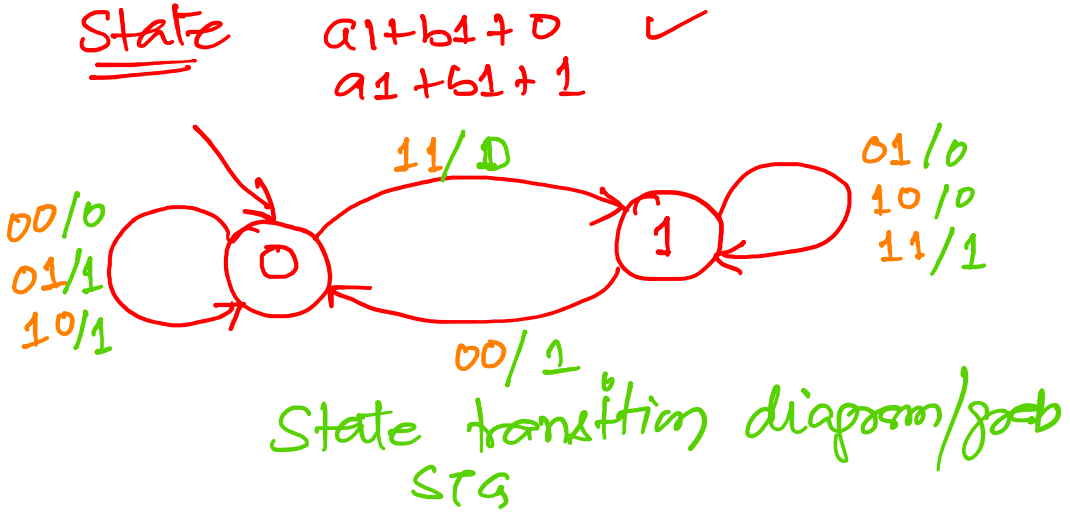
CADSL

Serial Adder

STATE MACHINE

```
c32  c31 . . .  c2  c1  0
      a31 . . . a2  a1  a0
    + b31 . . . b2  b1  b0
-------------------------
      s31 . . .  s2  s1  s0
                      c1
```

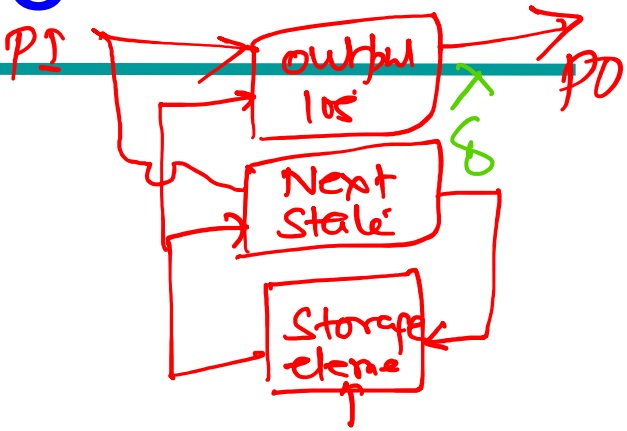




Stati	NS, output			
	00	01	10	11
0	0	0	0	1
1	0	1	1	1

State transition table.

FINITE State Machine



$M = (I, O, S, S_0, \delta, \lambda)$

I : & Input symbols $\{00, 01, 10, 11\}$

O : output symbol. $\{0, 1\}$

S : Set of states $\{0, 1\}$

S_0 : Initial state $\{0\}$

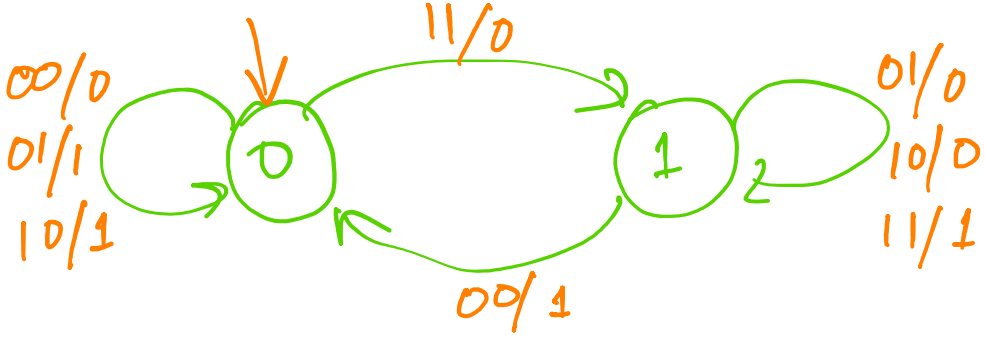
$\delta : S \times I \rightarrow S$ (State transition function)

$\lambda : S \times I \rightarrow O$ (Output function)

Design

1. How many states
2. How many storage elements $\log_2 |S|$
3. δ
4. λ

State Machine



only 2 states

$S = \{0, 1\}$ $|S| = 2$

one storage element

$I = \{00, 01, 10, 11\}$

$O = \{0, 1\}$

$\delta : S \times I \to S$

$\delta(a, b, s)$

s \ ab	00	01	11	10
0	0	0	1	0
1	0	1	1	1

$\delta(a,b,s) = a \cdot b + s \cdot b + s \cdot a$

$\lambda : S \times I \to O$

$\lambda(s, a, b)$

s \ ab	00	01	11	10
0	0	1	0	1
1	1	0	1	0

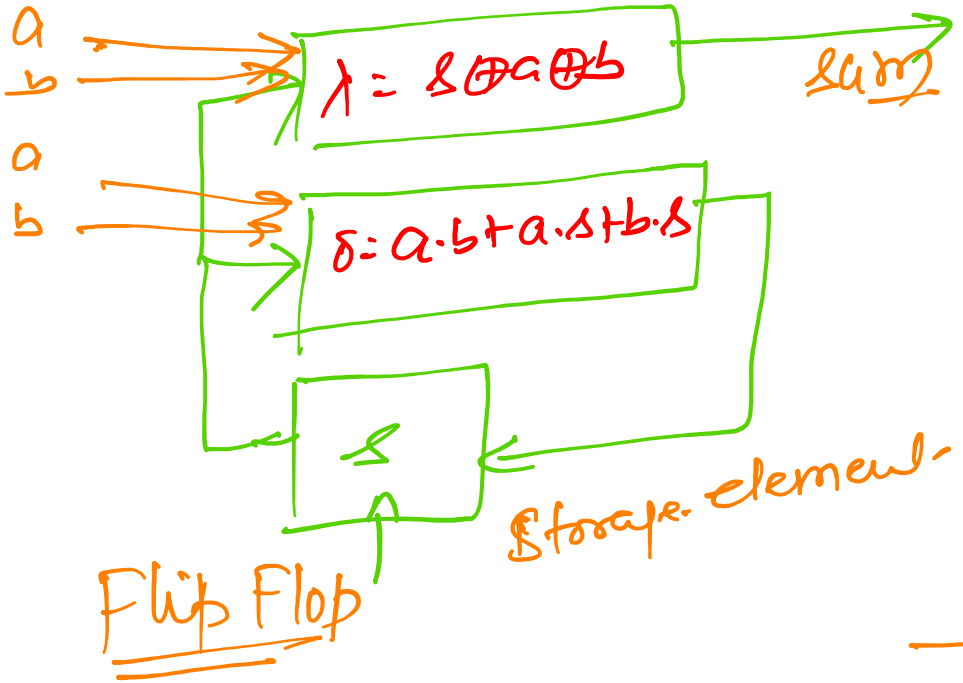
$s \oplus a \oplus b$

$\lambda = s \cdot \bar{a} \cdot \bar{b} + \bar{s} \cdot \bar{a} b + s \cdot a \cdot b + \bar{s} \cdot a \cdot \bar{b}$

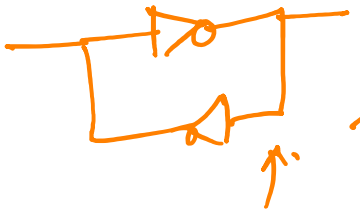
$= s(\bar{a} \cdot \bar{b} + a \cdot b) + \bar{s}(\bar{a} b + a \bar{b})$

$s \cdot \overline{(a \oplus b)} + \bar{s}(a \oplus b)$

Finite State Machine



Synthesize a finite state



1 Storage. (FF)

δ

λ

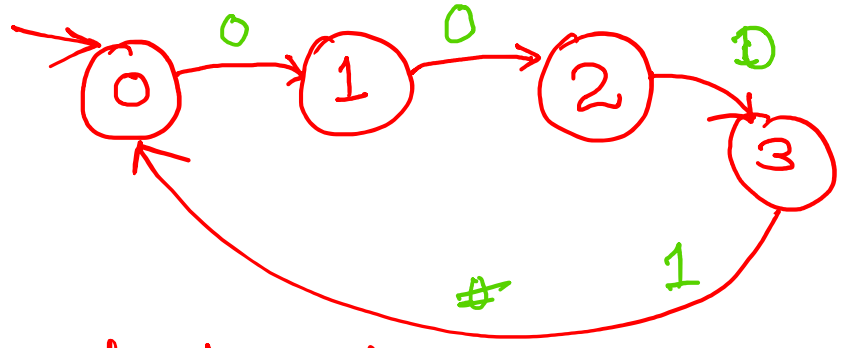
$\delta(t+1) = a(t)\cdot b(t) + a(t)\cdot\delta(t) + b(t)\cdot\delta(t)$

$\lambda(t+1) = \delta(t) \oplus a(t) \oplus b(t)$

Finite State Machine

0, 1, 2, 3, 0, 1, 2, 3, 0, 1, 2, 3, 0

0 0 0 1 0 0 0 1 0 0 ... 1



State transition diagram

states = |S| = 4

two state variable (S_1, S_0)

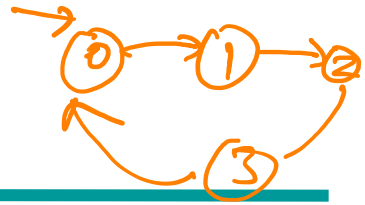
2 storage element (FF)

$O = \{0, 1\}$

$T = \#$

δ ? ↗ !

Finite State Machine



s_1, s_0

$s_1\ s_0$

0 → 00
1 → 01
2 → 10
3 → 11

$s_1\ s_0$: t_1, t_0 ↑ next

$t_1(t) = s_1(t+1)$

$\delta^1(s_1, s_0)$

s_1 \ s_0	0	1
0	0	1
1	1	0

$\delta^1 = s_1\bar{s}_0 + \bar{s}_1 s_0$

$= s_0 \oplus s_1$

$\delta^0(s_1, s_0)$

s_1 \ s_0	0	1
0	1	0
1	1	0

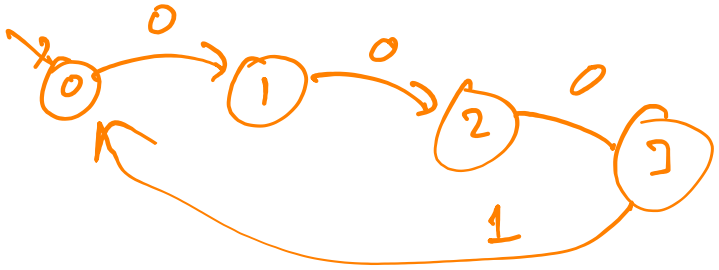
$\delta^0 = \bar{s}_0$

transition

s_0	s_1	t_1	t_0
0	0	0	1 ✓
0	1	1	0 ✓
1	0	1	1 ✓
1	1	0	0 ✓

δ^1 δ^0

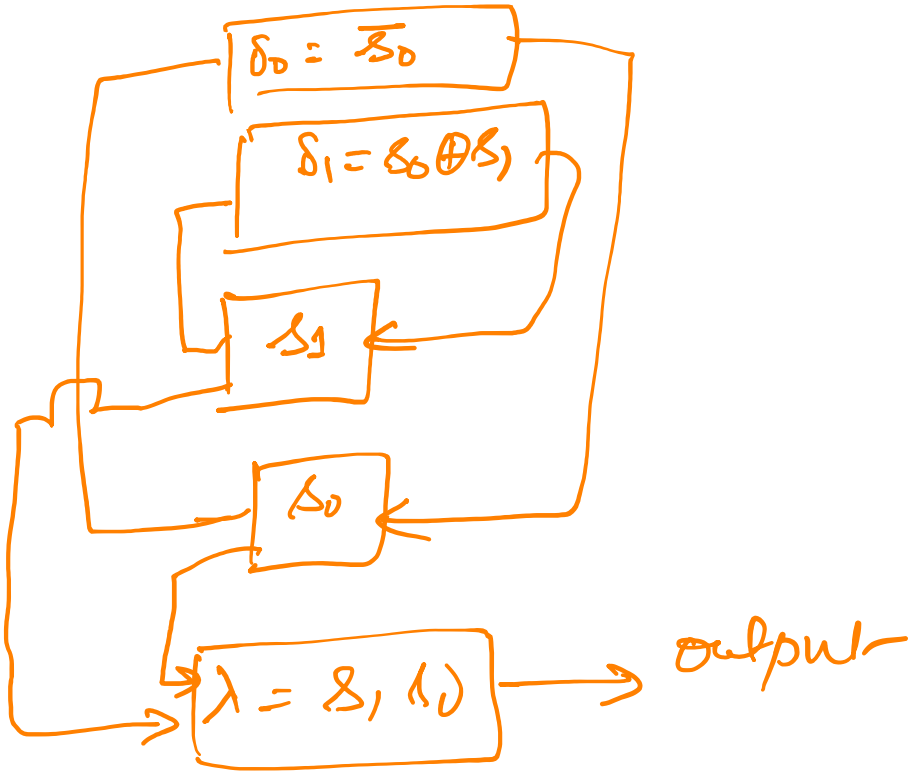
Finite State Machine



00 → 0
01 → 0
10 → 0
11 → 1

S_1 \ S_0	0	1
0	0	0
1	0	1

$\lambda = S_1 S_0$



Finite State machine

d Construct state tons

- Define states
- construct state transition graph
  or STT
  - # storage elements log |S|
  - define encoding of states
  - Compute δ, λ

Thank You