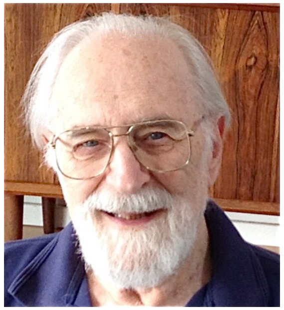
Logic Optimization: Karnaugh Map



Virendra Singh

Professor

Computer Architecture and Dependable Systems Lab

Department of Electrical Engineering

Indian Institute of Technology Bombay

http://www.ee.iitb.ac.in/~viren/

E-mail: viren@ee.iitb.ac.in

EE-224: Digital Systems

Lecture 18-A: 20 October 2020

CADSL

Minimum Sum of Products (MSOP)

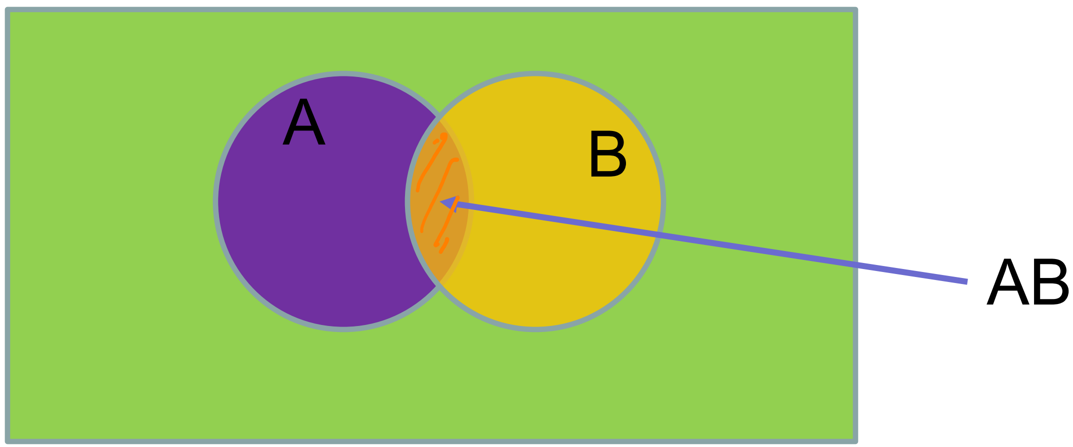
- Identify all prime implicants (PI) by letting minterms and implicants grow.
- Construct MSOP with PI only :
  - Cover all minterms ✓
  - Use only essential prime implicants (EPI) — MUST
  - Use no redundant prime implicant (RPI) ×
  - Use cheaper selective prime implicants (SPI)

min(# SPI) — delay & cost

min(# literals in SPI) — cost

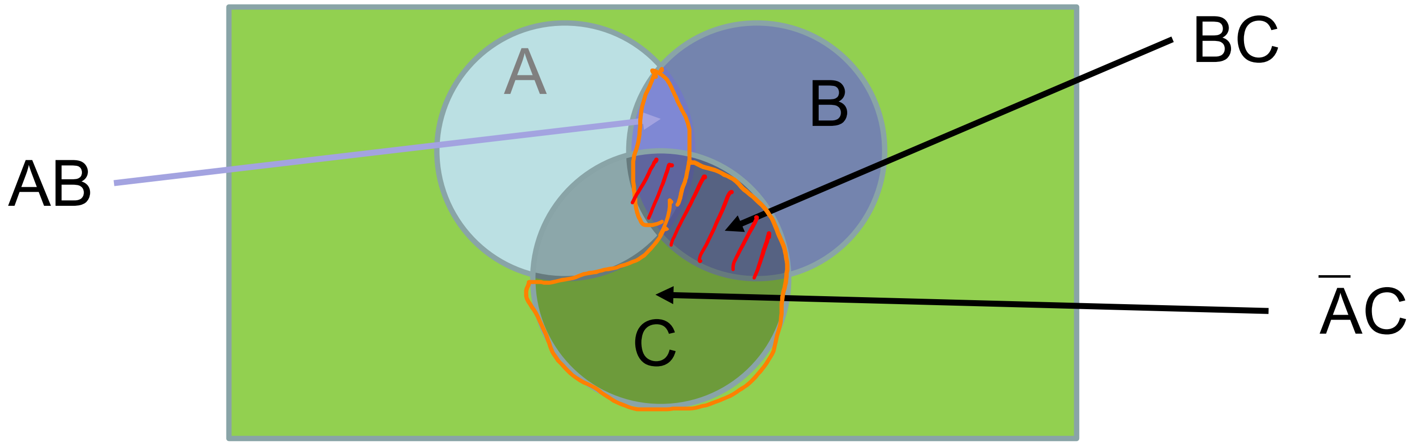
Absorption Theorem

- For two Boolean variables: A, B
- $A + AB = A$
- Proof:



Consensus Theorem

- For three Boolean variables: A, B, C
- $A\,B + \bar{A}\,C + B\,C = A\,B + \bar{A}\,C$
- Proof:



Growing Implicants to PI

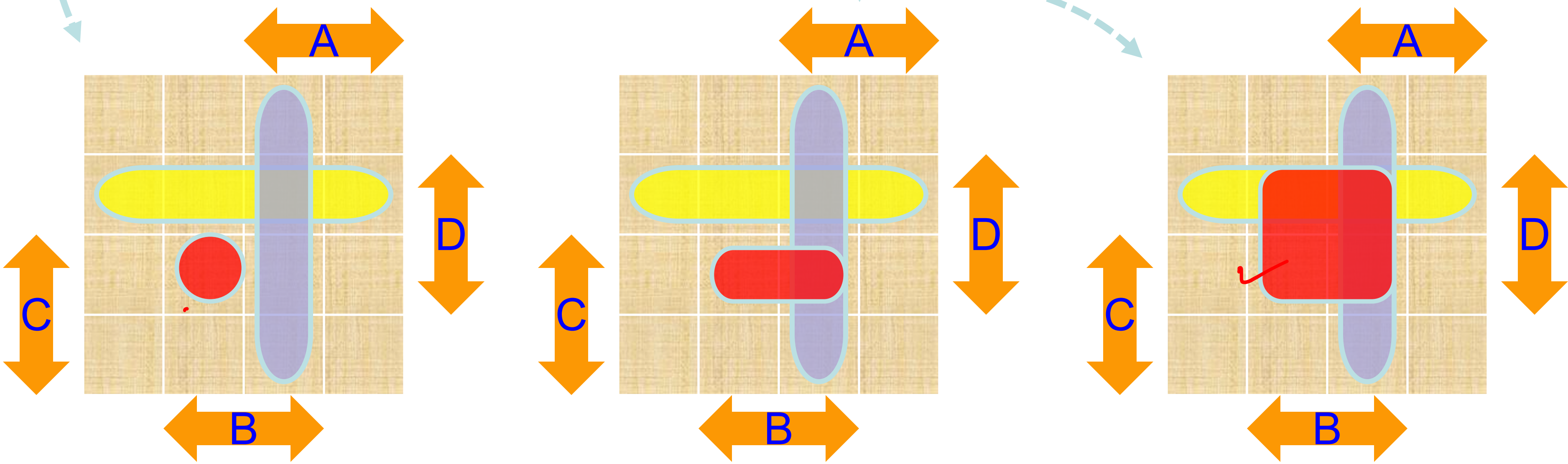
- $F = AB + \bar{C}D + \bar{A}BCD$ *initial implicants*

  $= AB + \bar{A}BCD + BCD + \bar{C}D$ *consensus th.*

  $= AB + BCD + \bar{C}D$ *absorption th.*

  $= AB + BCD + \bar{C}D + BD$ *consensus th.*

  $= AB + \bar{C}D + BD$ *absorption th.*



Identifying EPI

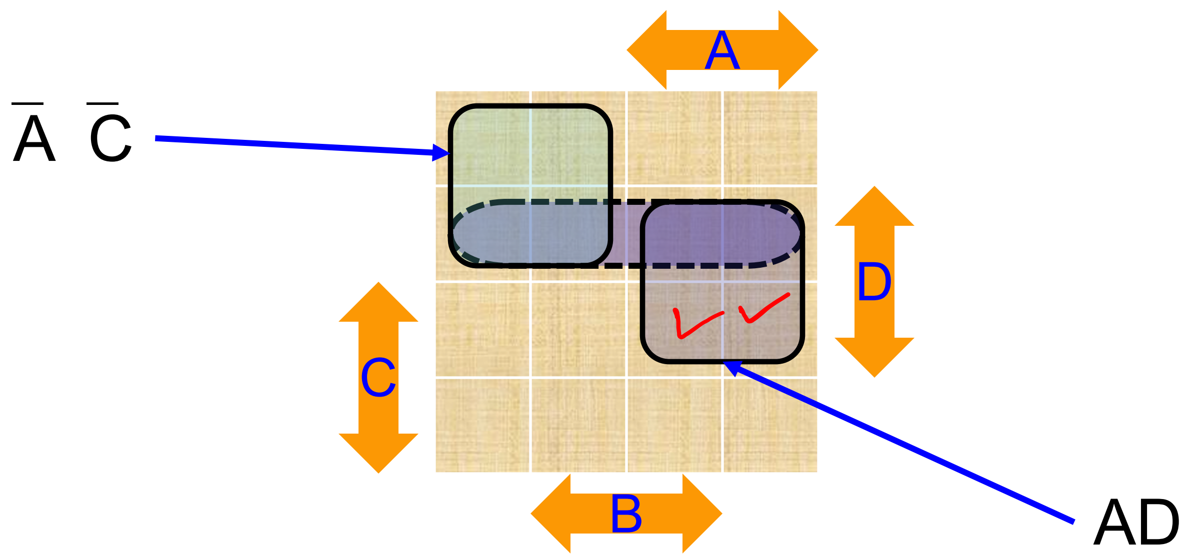
- Find all prime implicants.
- From prime implicant SOP, remove a PI. Whether this PI is EPI or not?
- Apply consensus theorem to the remaining SOP.
- If the removed PI is generated, then it is either an RPI or an SPI.
- If the removed PI is not generated, then it is an EPI

Example

- PI SOP: $F = AD + \bar{A}\bar{C} + \bar{C}D$
- Is AD an EPI?

  $F - \{AD\} = \bar{A}\bar{C} + \bar{C}D$, no new PI can be generated

  Hence, AD is an EPI. *Similarly,* $A\bar{C}$ *is an EPI.*



Example (Cont.)

- PI SOP: $F = AD + \bar{A}\bar{C} + \bar{C}D$
- Is $\bar{C}D$ an EPI?

$$F - \{\bar{C}D\} = AD + \bar{A}\bar{C}$$

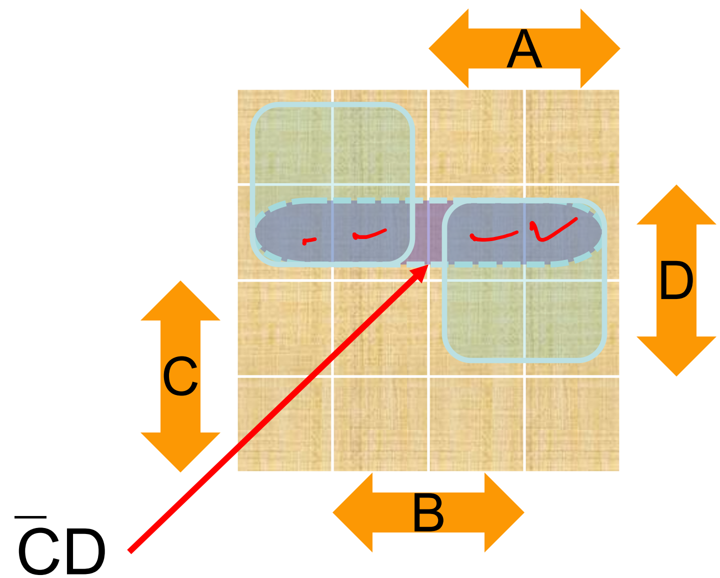
$$= AD + \bar{A}\bar{C} + \bar{C}D \quad \text{(Consensus theorem)}$$

Hence $\bar{C}D$ is not an EPI

(it is an RPI)

Minimum SOP:

$$F = AD + \bar{A}\bar{C}$$



Finding MSOP

min (#SPI)
min(# literals in SPI)

1. Start with minterm or cube SOP representation of Boolean function.
2. Find all prime implicants (PI).

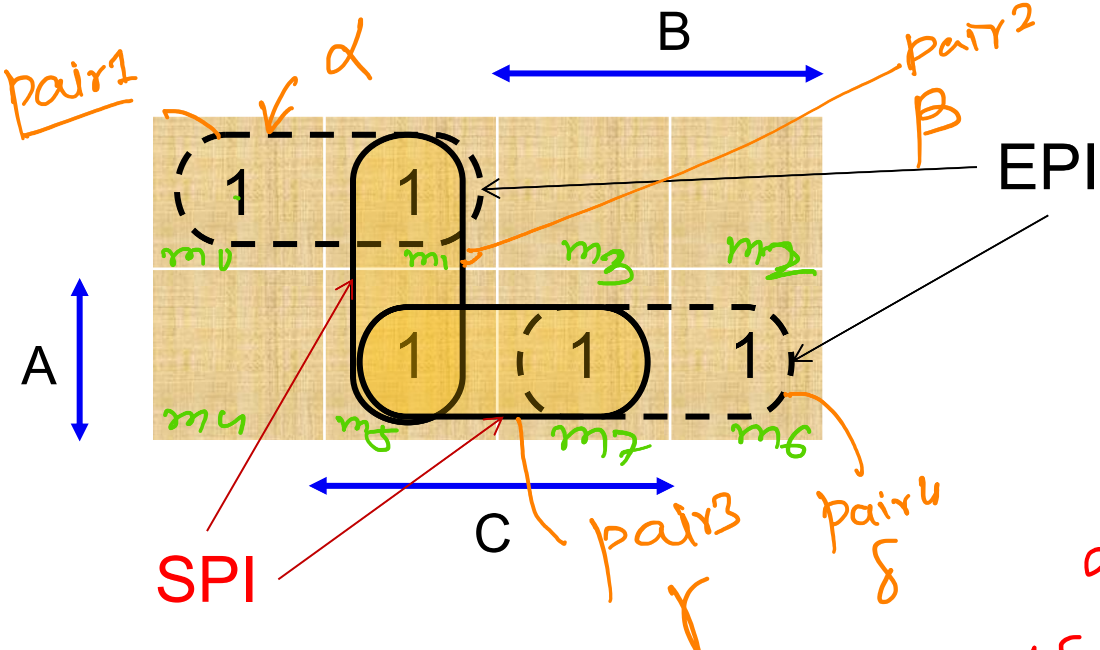
   MSOP = { EPI }

3. Include all EPI's in MSOP.
4. Find the set of uncovered minterms, {UC}.
5. MSOP is minimum if {UC} is empty. *DONE.*
6. For a minterm in {UC}, include the largest PI from remaining PI's (non-EPI's) in MSOP.
7. Go to step 4.

Iterative algorithm

Selection of SPI: Patrick's Method

$\Sigma m(0, 1, 5, 6, 7)$



0 - α ✓ EPI

1 α+β

5 β+γ

6 δ ✓ EPI

7 γ+δ

logical function

$\alpha \cdot (\alpha+\beta)(\beta+\gamma)\delta(\delta+\gamma) = 1$

$\underline{\alpha\delta(\alpha+\beta)(\beta+\gamma)(\gamma+\delta) = 1}$

$\alpha = 1 \quad \delta = 1$

$\boxed{\beta+\gamma = 1}$

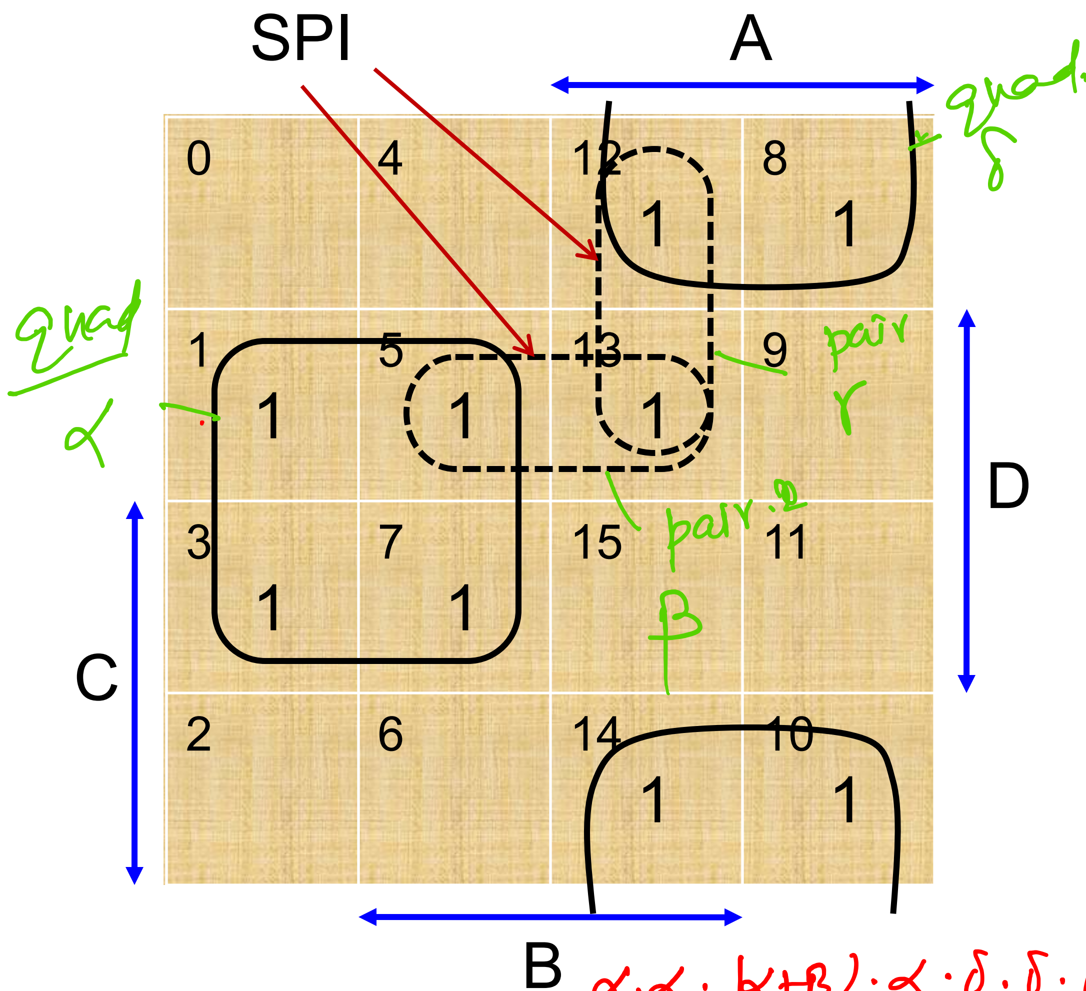
$\beta = 1$ or $\gamma = 1$

all min terms must be covered by PIs in MSOP

min(#SPI)

β or γ.

Example: $F=\sum m(1,3,5,7,8,10,12,13,14)$



MSOP:

$F = \bar{A}D + A\bar{D} + AB\bar{C}$

$1 = \alpha$
$3 = \alpha$
$5 = \alpha + \beta$
$7 = \alpha$
$8 = \delta$
$10 = \delta$
$12 = \delta + \gamma$
$13 = \beta + \gamma$
$14 = \delta$

$\alpha\,\delta\,(\alpha+\beta)(\delta+\gamma)$

$(\beta+\gamma) = 1$

$\alpha = 1$, $\delta = 1$ EPI

$\boxed{\beta + \gamma = 1}$

$\beta = 1$ or $\gamma = 1$

$\alpha\cdot\alpha\cdot(\alpha+\beta)\cdot\alpha\cdot\delta\cdot\delta\cdot(\delta+\gamma)(\beta+\gamma)\,\delta = 1$

$\rightarrow \underline{\alpha \beta \delta}$

or

$\rightarrow \underline{\alpha \gamma \delta}$

Minimize # SPI

min # literals

Minterms Covered by a Product

k-implicant

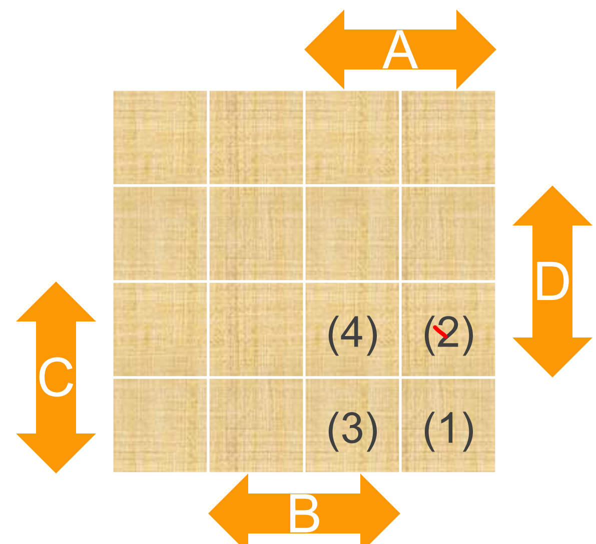
- A product from which k variables have been eliminated, covers 2^k minterms.

n-k

- Example: For four variables, A, B, C, D

  Product AC covers $2^2 = 4$ minterms:

  1) $A\ \bar{B}\ C\ \bar{D}$
  2) $A\ \bar{B}\ C\ D$
  3) $A\ B\ C\ \bar{D}$
  4) $A\ B\ C\ D$



Obtained by inserting the eliminated variables in all possible ways.

Thank You