Satisfiability Problem

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CS-226: Digital Logic Design

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CADSL

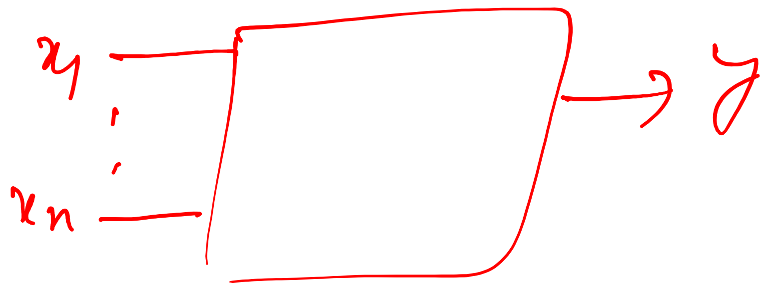
Function Representation

- Truth Table
- SOM
- POM
- ROBDD
- RM

AIG

SOP

POS



$f(x_1, x \cdots, y)$

POS

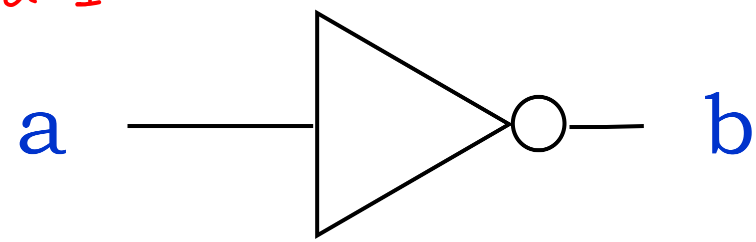
Function Representation

Truth Table

a	b
0	1
1	0

$a=0 \rightarrow b=1 \Leftrightarrow \bar{a} \rightarrow b$ ✓

$a=1 \rightarrow b=0 \Leftrightarrow a \rightarrow \bar{b}$ ✓



$(a \cancel{\rightarrow} b \quad (\bar{a} \rightarrow b) \cdot (a \rightarrow \bar{b})$ ✓ ✓

AND

$(\bar{a} + b) \cdot (\bar{a} + \bar{b})$

$f(a,b) = \underbrace{(a+b) \cdot (\bar{a}+\bar{b})}$ ✓

$f(a,b) = (a+b)(\bar{a}+\bar{b})$

Function Representation

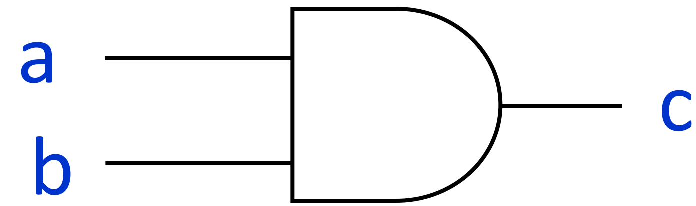
TRUTH TABLE

Bool

a	b	c
0	0	0
0	1	0
1	0	0
1	1	1

$a=0 \; \& \; b=0 \rightarrow c=0 \iff \bar{a}\cdot\bar{b} \rightarrow \bar{c}$

$a=0 \; \& \; b=1 \rightarrow c=0 \iff \bar{a}\cdot b \rightarrow \bar{c}$



$a=1 \; \& \; b=0 \rightarrow c=0 \iff a\cdot\bar{b} \rightarrow \bar{c}$

$a=1 \; \& \; b=1 \rightarrow c=1 \iff a\cdot b \rightarrow c$

$$(\bar{a}\cdot\bar{b}\rightarrow\bar{c})\cdot(\bar{a}\cdot b\rightarrow\bar{c})(a\cdot\bar{b}\rightarrow\bar{c})\cdot(a\cdot b\rightarrow c)$$

$$=(\overline{\bar{a}\cdot\bar{b}}+\bar{c})\cdot(\overline{\bar{a}\cdot b}+\bar{c})(\overline{a\cdot\bar{b}}+\bar{c})(\overline{a\cdot b}+c)$$

$$=(a+b+\bar{c})(a+\bar{b}+\bar{c})(\bar{a}+b+\bar{c})(\bar{a}+\bar{b}+c)$$

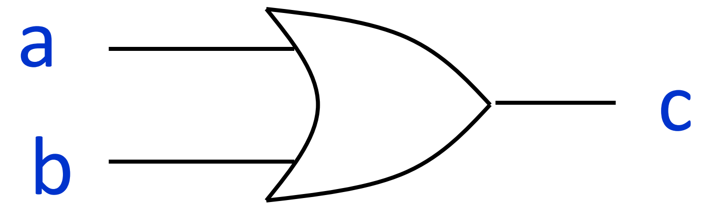
$$=(a+b+\bar{c})(a+\bar{b}+\bar{c})(a+b+\bar{c})(\bar{a}+b+\bar{c})(\bar{a}+\bar{b}+c)$$

$$f(a,b,c) = \underbrace{(a+\bar{c})(b+\bar{c})(\bar{a}+\bar{b}+c)}$$

POS

Function Representation

a	b	c
0	0	0
0	1	1
1	0	1
1	1	1



$\bar{a} \cdot \bar{b} \rightarrow \bar{c}$ ✓

$\bar{a} \cdot b \rightarrow c$ ✓

$a \cdot \bar{b} \rightarrow c$ ✓

$a \cdot b \rightarrow c$ ✓

$(\bar{a} \cdot \bar{b} \rightarrow \bar{c}) \cdot (\bar{a} \cdot b \rightarrow c)(a \cdot \bar{b} \rightarrow c)(a \cdot b \rightarrow c)$

$= (\overline{\bar{a} \cdot \bar{b}} + \bar{c})(\overline{\bar{a} \cdot b} + c)(\overline{a \cdot \bar{b}} + c)(\overline{a \cdot b} + c)$

$= (a+b+\bar{c})(a+\bar{b}+c)(\bar{a}+b+c)(\bar{a}+\bar{b}+c)$

$= (a+b+\bar{c})(a+\bar{b}+c)(\bar{a}+\bar{b}+c)(\bar{a}+b+c)(\bar{a}+\bar{b}+c)$

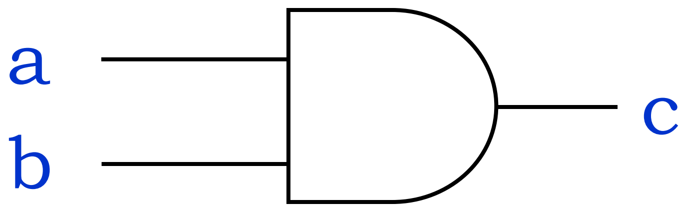
$f(a,b,c) = (a+b+\bar{c})(\bar{b}+c)(\bar{a}+c)$

clause

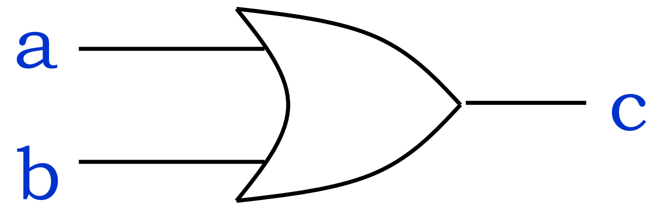
3 clauses

SOP

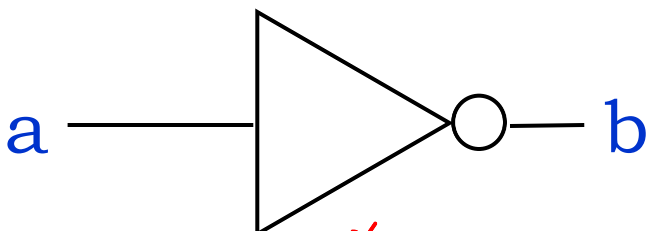
SAT Formulas for Simple Gates



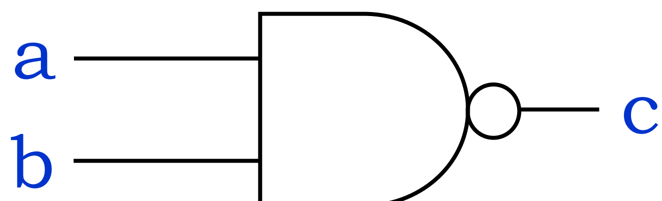
$(\bar{c}+a)(\bar{c}+b)(c+\bar{a}+\bar{b})$



$(c+\bar{a})(c+\bar{b})(\bar{c}+a+b)$



$(a+b)(\bar{a}+\bar{b})$



$(c+a)(c+b)(\bar{c}+\bar{a}+\bar{b})$

Function Representation

TRUTH TABLE

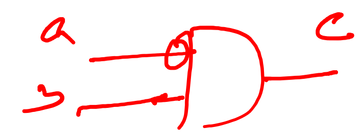
a	b	c
0	0	0
0	1	1
1	0	~~1~~ 0
1	1	0

$\bar{a}\cdot\bar{b} \to \bar{c}$

$\bar{a}\cdot b \to c$

$a\cdot\bar{b} \to \bar{c}$

$a\cdot b \to \bar{c}$



$(\bar{a}\cdot\bar{b}\to\bar{c})\cdot(\bar{a}\cdot b\to c)(a\cdot\bar{b}\to\bar{c})(a\cdot b\to\bar{c})$

$= (\bar{a}\cdot\bar{b}\to\bar{c})\cdot(\bar{a}\cdot b\to c)(\overline{a\cdot\bar{b}}+\bar{c})(\overline{a\cdot b}+\bar{c})$

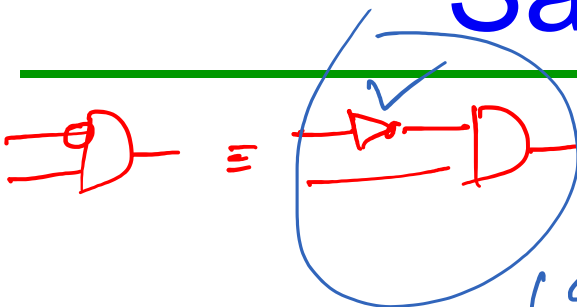
$(\overline{\bar{a}\cdot\bar{b}}+c)(\overline{\bar{a}\cdot b}+c)(\overline{a\cdot\bar{b}}+\bar{c})(\overline{a\cdot b}+\bar{c})$

$= (a+b+\bar{c})(a+\bar{b}+c)(\bar{a}+b+\bar{c})(\bar{a}+\bar{b}+\bar{c})$

$(a+b+\bar{c})(a+\bar{b}+c)(\bar{a}+c)$

$f(a,b,c) = (a+b+\bar{c})(a+\bar{b}+c)(\bar{a}+c)$

Satisfiability



$e = \bar{a} \cdot b \iff d = \bar{a},\; c = d \cdot b$

$(a+d)(\bar{a}+\bar{d})$

$(\bar{b}+\bar{d}+c)(b+\bar{c})(d+\bar{c})$

~~SOP~~ POS

$f(a,b,c,d) = \underbrace{(a+d)(\bar{a}+\bar{d})(\bar{b}+\bar{d}+c)(b+\bar{c})(d+\bar{c})}$

POS 5 clauses

what can be done using this representation

$f = (a+\bar{c})(b+\bar{c})(\bar{a}+\bar{b}+c) \Leftarrow$ AND oper.

$f = ?$

$a=0,\ b=0,\ c=0$, can satisfy or not (SAT.)

$a=0,\ b=0,\ c=1 \rightarrow$ Not satisfied.

Satisfiability

$f(a,b,c) = (a+\bar{b})(b+\bar{c})(\bar{a}+\bar{b}+c)$

if $a = 0,\ b = 0 \quad c = ?$

$\therefore \bar{c} \cdot \bar{c} \cdot 1 = \bar{c} = 1$

Hence $\because$ $\boxed{c = 0}$

For what value of a & b $c = 1$,

$c = 1 \quad a = ? \quad b = ?$

$= a \cdot b \cdot 1 = a \cdot b$

satisfiability

$a \cdot b = 1 \Rightarrow \begin{array}{l} a = 1 \\ b = 1 \end{array}$

SAT Problem Definition

Given a CNF formula, f :

- A set of variables, *V*
- Conjunction of clauses
- Each clause: disjunction of literals over *V*

AND

OR.

(a,b,c)

(C_1, C_2, C_3)

$C_1 \cdot C_2 \cdot C_3 = 1$

$C_1 = 1$

$C_2 = 1$

$C_3 = 1$

Does there exist an assignment of Boolean values to the variables, V which sets at least one literal in each clause to '1' ?

Example : $\underbrace{(a + b + \bar{c})}_{C_1}\underbrace{(\bar{a} + c)}_{C_2}\underbrace{(a + \bar{b} + c)}_{C_3}$

a = b = c = 1

SAT Problem

$f(a, b, c \cdots)$ POS

Thank You