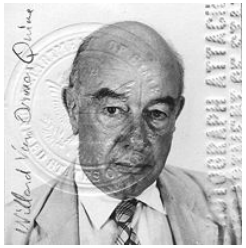
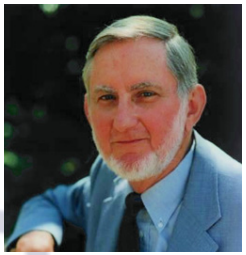
Logic Optimization:

Tabular Method (Quine-McCluskey)





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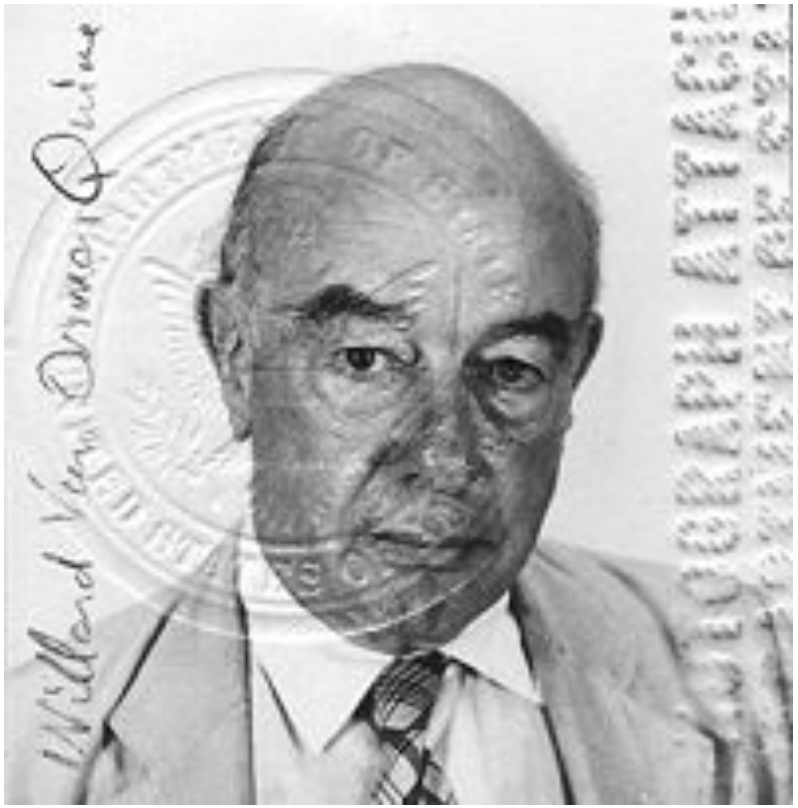
CS-226: Digital Logic Design

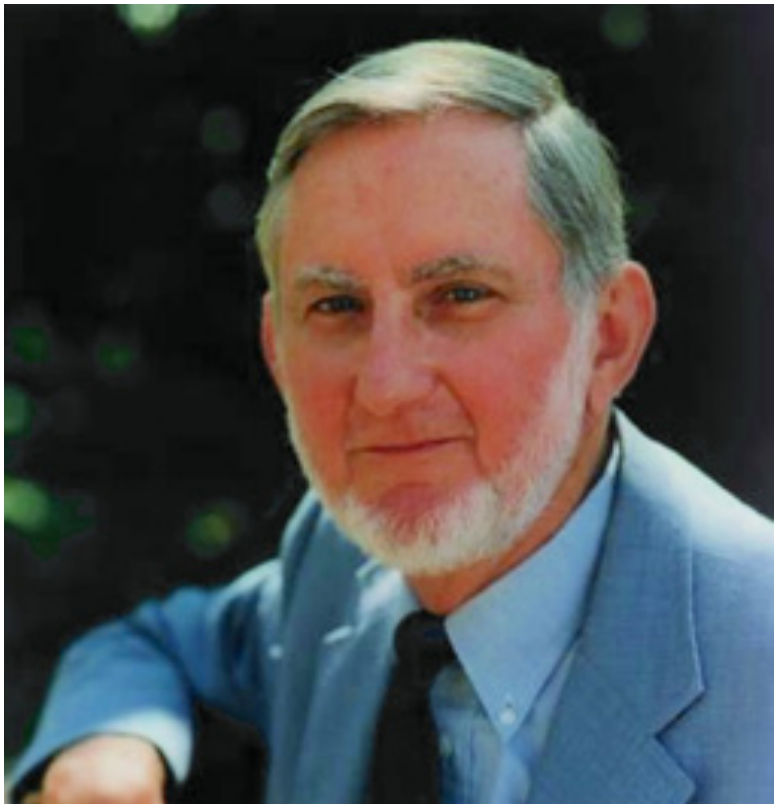
Lecture 16-A: 22 February 2021

CADSL

Quine-McCluskey



Willard V. O. Quine
1908 – 2000



Edward J. McCluskey
1929 -- 2016

Quine-McCluskey Tabular Minimization Method

- W. V. Quine, "The Problem of Simplifying Truth Functions," *American Mathematical Monthly*, vol. 59, no. 10, pp. 521-531, October 1952.

- E. J. McCluskey, "Minimization of Boolean Functions," *Bell System Technical Journal*, vol. 35, no. 11, pp. 1417-1444, November 1956.

$\overline{10}\ \overline{11}' = 1-$ a

$a\bar{b} + ab = a$

Q-M Tabular Minimization

- Minimizes functions with many variables.
- Begin with minterms:
  - Step 1: Tabulate minterms in groups of increasing number of true variables.
  - Step 2: Conduct linear searches to identify all prime implicants (PI).
  - Step 3: Tabulate PI's vs. minterms to identify EPI's.
  - Step 4: Tabulate non-essential PI's vs. minterms not covered by EPI's. *Select* minimum number of PI's to cover all minterms.
- MSOP contains all EPI's and *selected* non-EPI's.

$F(A,B,C,D) = \sum m(2,4,6,8,9,10,12,13,15)$

- Q-M Step 1: Group minterms with 1 true variable, 2 true variables, etc.

Minterm	ABCD	Groups
2	0010	1: single 1
4	0100	
8	1000	
6	0110	2: two 1's
9	1001	
10	1010	
12	1100	
13	1101	3: three 1's
15	1111	4: four 1's

Gp1 Gp2 Gp3 Gp4

(2,6) 0010 / 0110 → 0-10

(2,9) 0010 / 1001 ✗

(2,10) 0010 / 1010 → -010

(2,12) 0010 / 1100 ✗

(4,6) 0100 / 0110 → 01-0

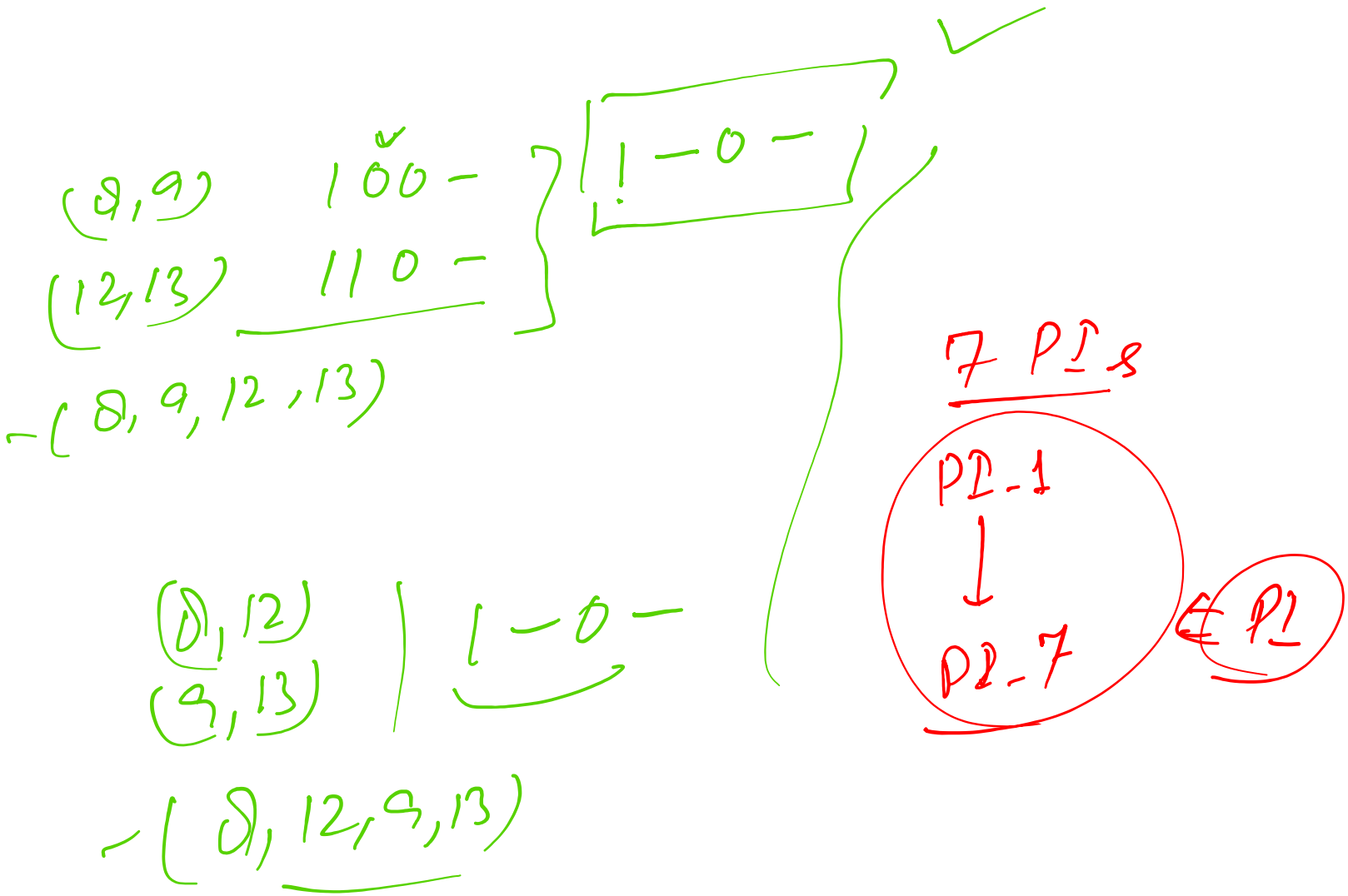
Q-M Step 2

- Find all implicants by combining minterms, and then combining products that differ in a single variable: For example,
  - 2 and 6, or $\bar{A}\ \bar{B}\ C\ \bar{D}$ and $\bar{A}\ B\ C\ \bar{D} \rightarrow \bar{A}\ C\ \bar{D}$, written as 0 – 1 0.
- Try combining a minterm (or product) with all minterms (or products) listed below in the table.
- Include resulting products in the next list.
- If minterm (or product) does not combine with any other, mark it as PI.
- Check the minterm (or product) and repeat for all other minterms (or products).

Step 2 Executed on Example

0-implicant 1-implicant 2-implicants

	List 1			List 2			List 3		
	Minterm	ABCD	PI?	Minterms	ABCD	PI?	Minterms	ABCD	PI?
Gp1	2 ✓	0010	X	2, 6 ✓	0-10	PI_2	8,9,12,13	1-0-	PI_1
	4 ✓	0100	X	2,10 ✓	-010	PI_3			
	8 ✓	1000	X	4,6 ✓	01-0	PI_4			
Gp2	6 ✓	0110	X	4,12	-100	PI_5			
	9 ✓	1001	X	8,9 ✓	100-	X			
	10 ✓	1010	X	8,10 ✓	10-0	PI_6			
	12 ✓	1100	X	8,12 ✓	1-00	X			
Gp3	13 ✓	1101	X	9,13 ✓	1-01	X			
Gp4	15 ✓	1111	X	12,13 ✓	110-	X			
				13,15	11-1	PI_7			


(9,9) 100-
(12,13) 110-
1-0-
-(8,9,12,13)
(8,12)
(9,13)
1-0-
-(8,12,9,13)
7 PIs
PI-1
PI-7
∈ PI
minimize (# PI)

Step 3: Identify EPI's

Covered by EPI →				x	x		x	x	x
Minterms →	2	4	6	8	9	10	12	13	15
PI_1 is EPI				x	x		x	x	
PI_2	x		x						
PI_3	x					x			
PI_4		x	x						
PI_5		x					x		
PI_6				x		x			
PI_7 is EPI								x	x

EPI = { PI-1, PI-7 }

UC = { 2, 4, 6, 10 }

select min PIs to cover UC

Step 4: Cover Remaining Minterms

	Remaining minterms →	2	4	6	10
x2 →	PI_2	x		x	
x3 →	PI_3	x			x
x4 →	PI_4		x	x	
x5 →	PI_5		x		
x6 →	PI_6				x

Integer linear program (ILP), available from MATLAB and other sources: Define integer {0,1} variables, xk = 1, select PI_k; xk = 0, do not select PI_k.

Minimize $\sum_k$ xk, subject to constraints:

$$x2 + x3 \geq 1$$
$$x4 + x5 \geq 1$$
$$x2 + x4 \geq 1$$
$$x3 + x6 \geq 1$$

A solution is x3 = x4 = 1, x2 = x5 = x6 = 0, or select PI_3, PI_4

Linear Programming (LP)

- A mathematical optimization method for problems where some “cost” depends on a large number of variables.
- An easy to understand introduction is:
  - S. I. Gass, *An Illustrated Guide to Linear Programming*, New York: Dover Publications, 1970.
- Very useful tool for a variety of engineering design problems.
- Available in software packages like MATLAB.

Step 4: Cover Remaining Minterms

	Remaining minterms →	2	4	6	10
α	PI_2	x		x	
β	PI_3	x			x
γ	PI_4		x	x	
δ	PI_5		x		
η	PI_6				x

Patrick's Method

(All min terms must be Covered)

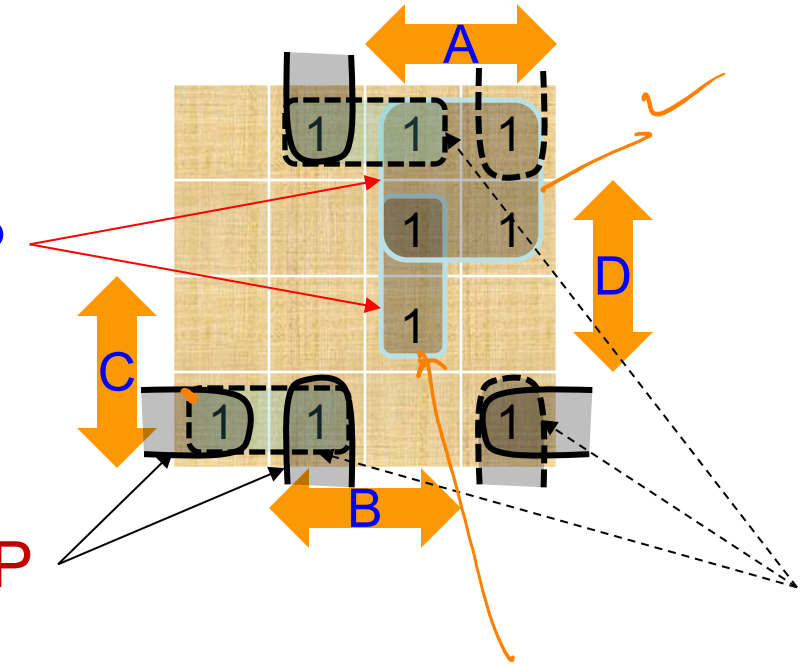
$(\alpha+\beta)\cdot(\gamma+\delta)\,(\alpha+\gamma)\,(\beta+\eta) = 1$

2 $\alpha+\beta$
4 $\gamma+\delta$
6 $\alpha+\gamma$
10 $\beta+\eta$

SOP terms

Q-M MSOP Solution and Verification

- F(A,B,C,D) = PI_1 + PI_3 + PI_4 + PI_7
  = 1-0- + -010 + 01-0 + 11-1
  = $A\,\bar{C} + \bar{B}\,C\,\bar{D} + \bar{A}\,B\,\bar{D} + A\,B\,D$
- See Karnaugh map.

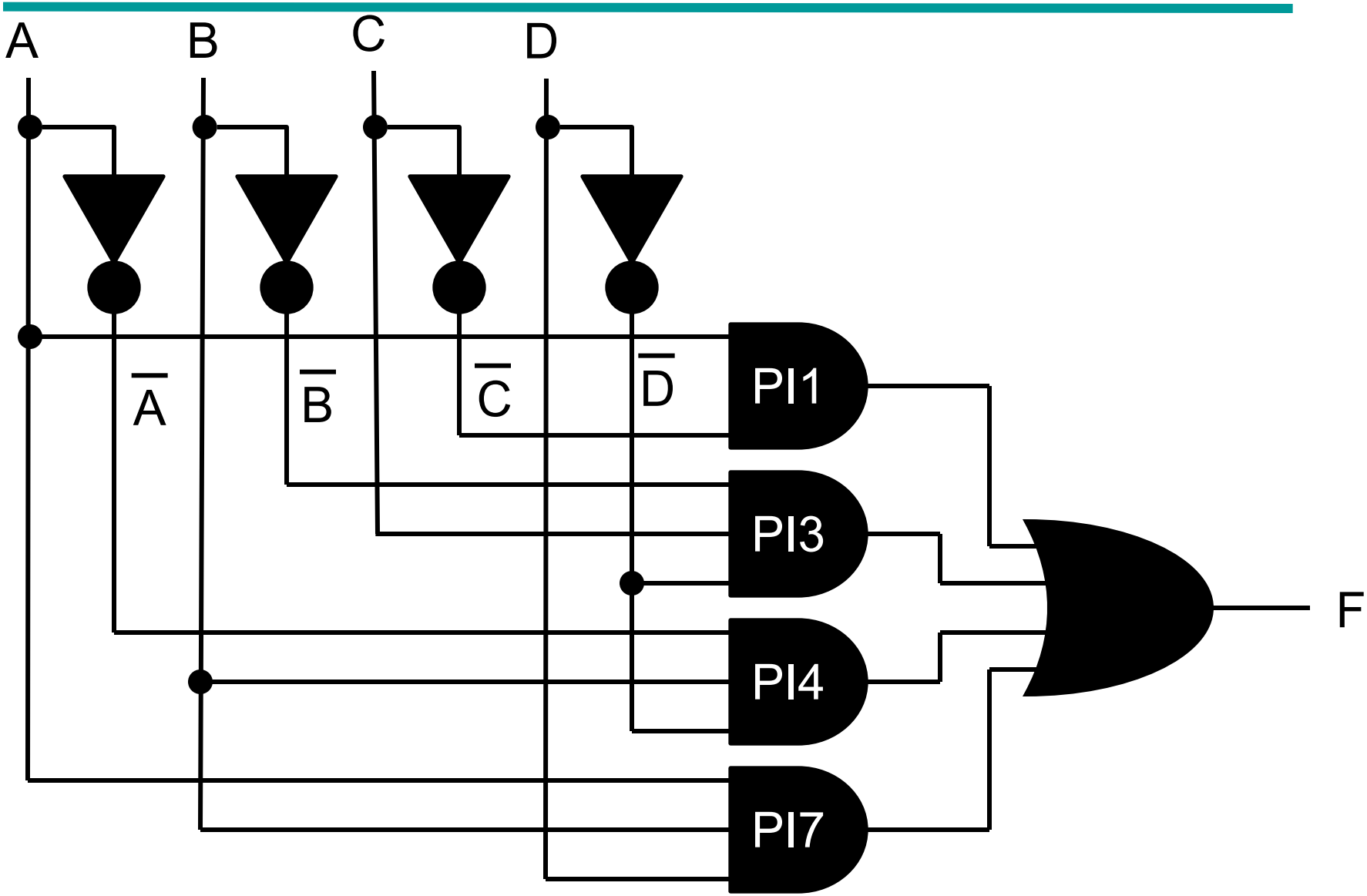


EPI's in MSOP

Non-EPI's in MSOP

Non-EPI's not in MSOP

Minimized Circuit



QM Minimizer on the Web

- http://quinemccluskey.com/

Thank You