

Tutorial 7

1. For a flow network $G = (V, E)$ and flow f let G_f denote the residual graph. For a parameter $\Delta \in \mathbb{N}$, we define $G_f(\Delta)$ to be the graph obtained from the residual graph G_f by retaining the edges with capacity at least Δ and deleting all the edges of capacity strictly smaller than Δ .

Now, consider the following algorithm for a graph with integral capacities.

```

Set  $f(e) = 0$  for all edges  $e \in E$ .
Let  $C$  be the largest capacity on any edge in  $G$ .
Let  $\Delta$  be the largest power of 2 such that  $\Delta \leq C$ .
while  $\Delta \geq 1$  do
  Compute  $G_f(\Delta)$ 
  while There exists an  $s$  to  $t$  path  $\pi$  in  $G_f(\Delta)$  do
    Compute  $\theta(\pi, f)$  in  $G_f(\Delta)$ 
    Set  $f' = \text{Aug}(\pi, \theta)$ 
    Compute  $G_{f'}(\Delta)$ , set  $f \leftarrow f'$ 
  end while
   $\Delta \leftarrow \Delta/2$ 
end while
Output  $f$ .
```

- (a) Prove that the above algorithm terminates.
 - (b) Prove that the algorithm correctly computes the maximum flow in the given graph.
 - (c) Analyse the running time of the above algorithm.
2. Prove the following statements about flows.
 - f_2 is a flow in G_{f_1} if and only if $f_1 + f_2$ is a flow in G .
 - $|f_1 + f_2| = |f_1| + |f_2|$.
 - f_2 is a max flow in G_{f_1} if and only if $f_1 + f_2$ is a max flow in G .
 - If f^* is a max flow in G and f is any flow then the max flow in G_f has value $|f^*| - |f|$.
 3. Consider the following algorithm for a graph with integral capacities.


```

Set  $f(e) = 0$  for all edges  $e \in E$ .
while there exists an  $s$  to  $t$  path do
  Compute  $G_f$ 
  Let  $\pi$  be a path obtained by using depth first search.
  Compute  $\theta(\pi, f)$  in  $G_f$ 
  Set  $f' = \text{Aug}(\pi, \theta)$ 
  Compute  $G_{f'}$ , set  $f \leftarrow f'$ 
end while
Output  $f$ .
```

 - (a) Prove that the above algorithm terminates.
 - (b) Prove that the algorithm correctly computes the maximum flow in the given graph.
 - (c) Analyse the running time of the above algorithm.
 4. Reading exercise. Read and understand this image segmentation application of the max-flow problem. http://srmanikandasriram.github.io/files/DSA/Term_Paper.pdf