Tutorial 4

1. Consider the following recursions. Solve them and order them in increasing order of their asymptotic growth rates.

   (a) $T(n) = 2T(n/4) + n^2$.
   (b) $T(n) = 3T(n/4) + n$.
   (c) $T(n) = T(n/2) + 5$.
   (d) $T(n) = 3T(n/3) + n^3$.
   (e) $T(n) = 2T(n-1) + 1$.
   (f) $T(n) = T(n/2) + T(n/3) + T(n/6) + n$.
   (g) $T(n) = 4T(n/2) + n \log n$.
   (h) $T(n) = \sqrt{n} \cdot T(\sqrt{n}) + n$.

2. Consider the following algorithm.

   I $i \leftarrow 0$, $j \leftarrow 0$;
   II while $(i \leq n)$ {
      i. $i \leftarrow i + j$;
      ii. $j \leftarrow j + 1$;
   }

   The above algorithm halts in _____ steps.

3. Given an array A of n distinct elements, an inversion in the array is a pair of elements $A[i]$ and $A[j]$ such that $i < j$ but $A[i] > A[j]$. Give an algorithm that runs in time $O(n \log n)$ to count the number of inversions in the given array.

4. Given a permutation of the numbers 1 to n, count the number of pairs (i, j) such that all numbers which occur between i and j in the permutation have value between i and j.