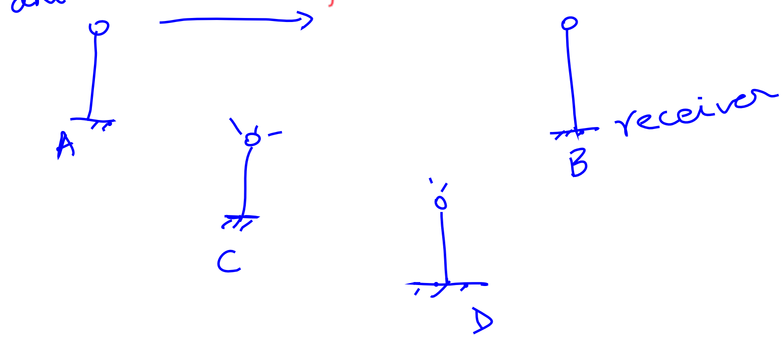
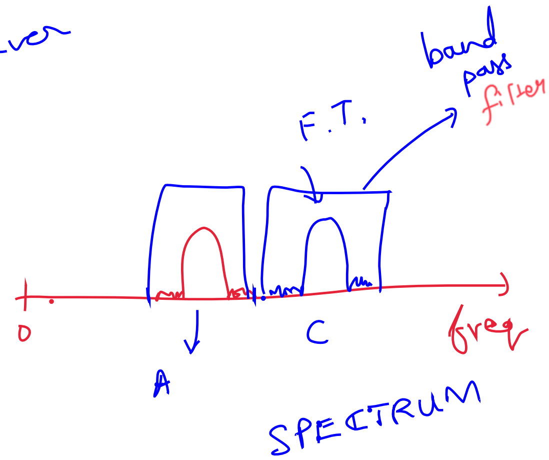
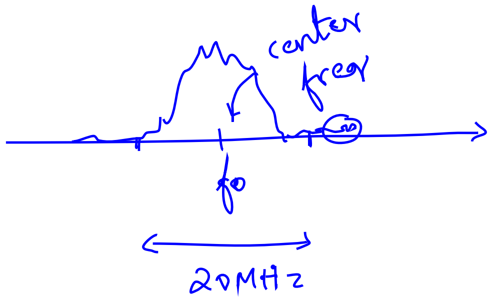
MODULATION – WIRELESS

LPF
HPF

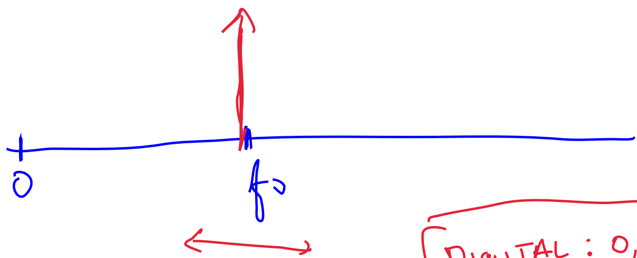






$\cos(2\pi f_0 t)$ —F.T.→ Fourier Transform

freq. time



DIGITAL : 0, 1

ANALOG : SIGNAL

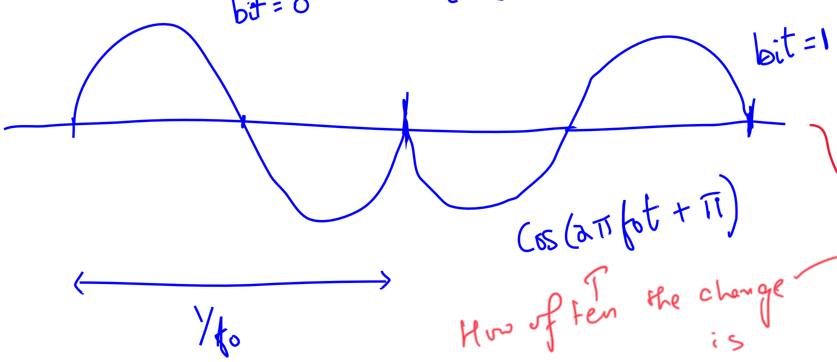
MODULATION

Carrier: $\cos(2\pi f_0 t)$

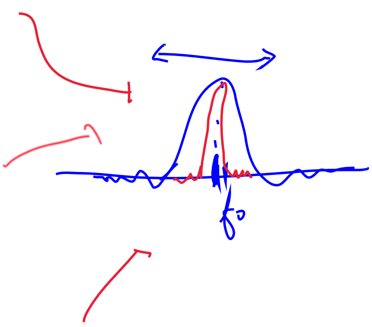
$A\cos(2\pi f_0 t + \theta)$ ..

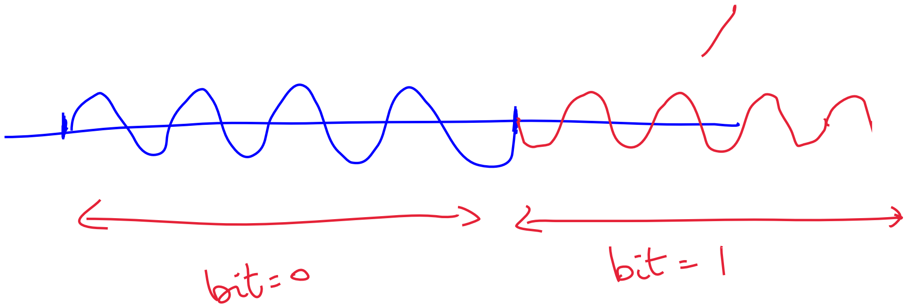
amplitude, frequency (F.M.) → Freq. Modulation, phase

$\cos 2\pi f_0 t$ bit = 0 $\cos(2\pi f_0 t + \pi)$



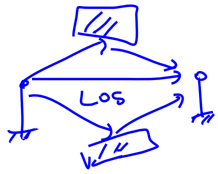
How often the change is





BIT ERROR RATE

$s(t) \rightarrow$ transmitted



Most general

$$r(t) = \sum_i a_i\, s(t - \tau_i) + n(t)$$

$r(t)$ → Received

τ_i → Delay → delay

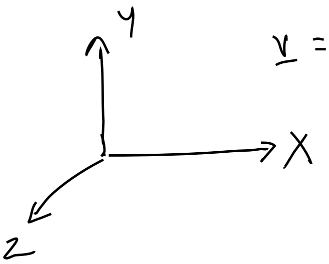
$+$ → added

$n(t)$ ↳ white noise

Additive white (A W Gaussian Noise) → AWGN

BER: Fraction of transmitted bits received in error

VECTOR SPACES



$\underline{v} = (v_x, v_y, v_z)$

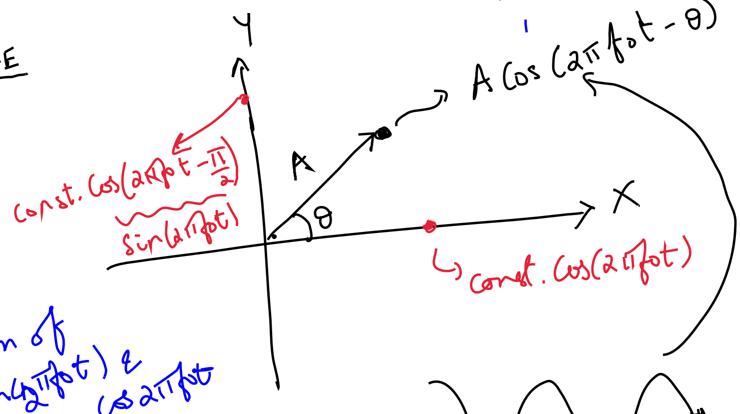
$v_x = \langle \underline{v}, \hat{e}_x \rangle$ → dot product

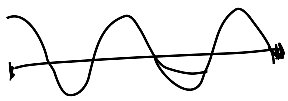
$v_y = \langle \underline{v}, \hat{e}_y \rangle$

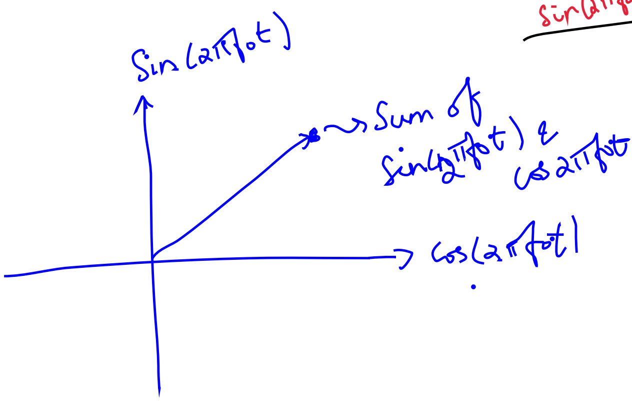
$v_z = \dots$

SIGNALS IN VECTOR SPACE

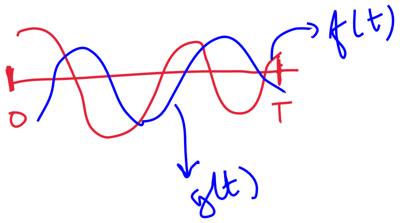
Focus on FREQ. f_0







$f(t)\,,\; g(t)$



INNER / DOT PRODUCT

$$\langle f(t), g(t)\rangle = \int_0^T f(t)\, g(t)\, dt$$

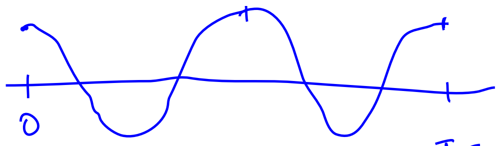
$r_x = \langle r(t), \hat{e}_x \rangle$; $r_y = \langle r(t), \hat{e}_y \rangle$

$r(t)$ ↓ received signal

$$\hat{e}_x = \overset{C}{\text{const.}}\ \cos(2\pi f_0 t)$$

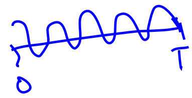
$$\hat{e}_y = \text{const.}\ \sin(2\pi f_0 t)$$

$\langle \hat{e}_x, \hat{e}_x \rangle = 1$



$\cancel{\int_0^T \cos 4\pi f_0 t\, dt} = 0$

$T = \dfrac{N}{f_0}$



$$\int_0^T \underbrace{c \cdot \cos(2\pi f_0 t)}_{\hat{e}_x} \cdot c \cdot \cos(2\pi f_0 t)\, dt$$

$$= c^2 \int_0^T \cos^2(2\pi f_0 t)\, dt = c^2 \underbrace{\int_0^T \frac{1 + \cos 4\pi f_0 t}{2}\, dt}\ ?$$

$$= c^2 \cdot \frac{1}{2} \cdot T = 1$$

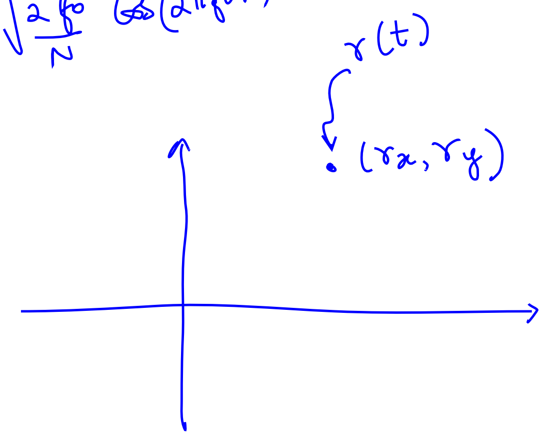
$$c = \sqrt{\frac{2}{T}} = \sqrt{\frac{2 f_0}{N}}$$

$$\hat{e}_x = c \cdot \cos(2\pi f_0 t) = \sqrt{\frac{2 f_0}{N}}\, \cos(2\pi f_0 t)$$

$$\hat{e}_y = c \cdot \sin(2\pi f_0 t)$$

$r_x = \langle r(t), \hat{e}_x \rangle$

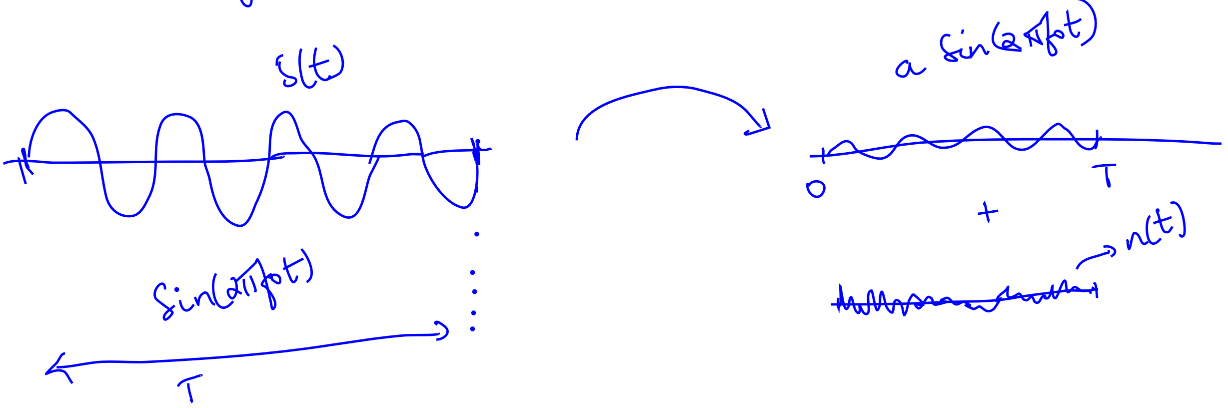
$r_y = \langle r(t), \hat{e}_y \rangle$



$$r(t) = \underbrace{a}_{\text{attenuation}} \cdot s(t) + \underbrace{n(t)}_{\text{AWGN}}$$

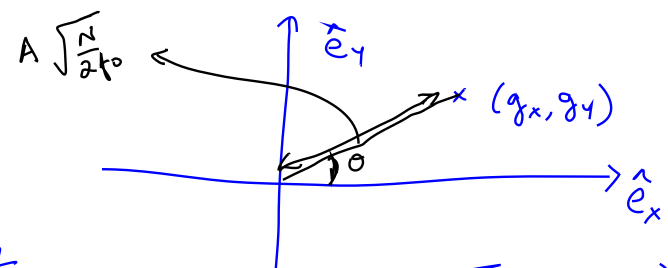
$$r_x = a \cdot \langle s(t), \hat{e}_x \rangle + \underbrace{\langle n(t), \hat{e}_x \rangle}_{n_x}$$

$$r_y = a \langle s(t), \hat{e}_y \rangle + \underbrace{\langle n(t), \hat{e}_y \rangle}_{n_y}$$



$$g(t) = A \cos(2\pi f_0 t - \theta)$$

$$\langle \bar{e}_x, \bar{e}_y \rangle \overset{?}{=} 0 \; ; \; T = \frac{N}{f_0}$$



$$\int_0^T \frac{2f_0}{N} \cos 2\pi f_0 t \, \sin 2\pi f_0 t \, dt$$

$$= \int_0^T \frac{2f_0}{N_0} \cdot \cancelto{0}{\frac{\sin 4\pi f_0 t}{2}} \, dt = 0$$

$$\hat{e}_x = \sqrt{\frac{2f_0}{N}} \cos(2\pi f_0 t)$$

$$\hat{e}_y = \sqrt{\frac{2f_0}{N}} \sin(2\pi f_0 t)$$

$g_x = ?$, $g_y = ?$

$$g(t) = A\left[\cos\theta \cos 2\pi f_0 t + \sin\theta \sin 2\pi f_0 t\right]$$

$$= A\left[\frac{\cos\theta \, \hat{e}_x + \sin\theta \, \hat{e}_y}{\sqrt{\frac{2f_0}{N}}} \right]$$

$$g_x = \langle g(t), \hat{e}_x \rangle = A\sqrt{\frac{N}{2f_0}} \left[\cos\theta \cancelto{1}{\langle \hat{e}_x, \hat{e}_x \rangle} + \sin\theta \cancelto{0}{\langle \hat{e}_y, \hat{e}_x \rangle} \right]$$

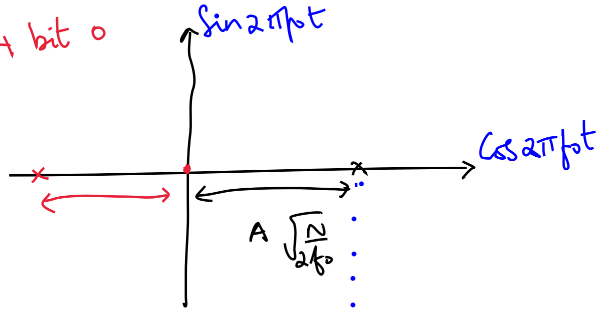
$$= A\sqrt{\frac{N}{2f_0}} \cos\theta$$

$$g_y = \langle g(t), \hat{e}_y \rangle = A\sqrt{\frac{N}{2f_0}} \sin\theta$$

Suppose we transmit $s_0(t) = A\cos 2\pi f_0 t \to$ bit 0

$s_1(t) = -A\cos 2\pi f_0 t$ ↳ bit 1

$0 \le t \le T = \frac{N}{f_0}$



CONSTELLATION DIAGRAM

$s(t) \curvearrowright \overbrace{\underbrace{a\, s(t)}\, + n(t)}^{r(t)}$

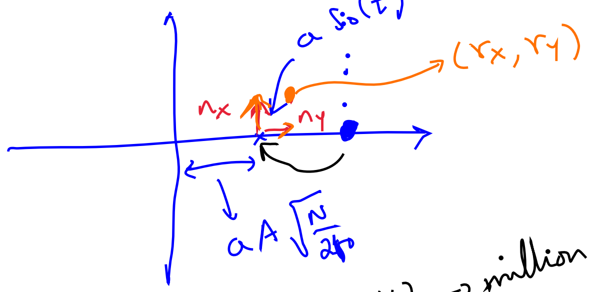
$s(t) \to (s_x, s_y)$

$n_x, n_y \sim$ Gaussian distribution

$r_x = a s_x + n_x$

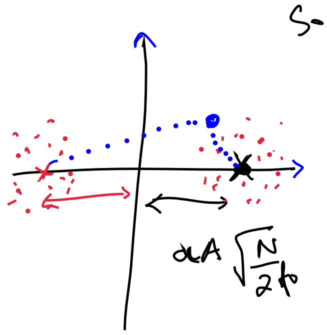
$r_y = a s_y + n_y$

n_x, n_y are i.i.d. Gaussian



$s_0(t) \to$ million times

$s_1(t)$



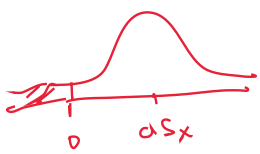
DETECTION AT RECEIVER

$r(t) = a\, s(t) + n(t)$

(i) Calculate r_x, r_y ; $r_x = \langle r(t), \hat{e}_x \rangle$; $r_y = \langle r(t), \hat{e}_y \rangle$

(ii) Find which constellation point is closest to (r_x, r_y)

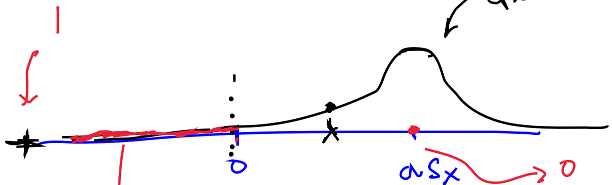
IN EXAMPLE



$r_x > 0 \longrightarrow$ bit 0

$r_x \le 0 \longrightarrow$ bit 1

$r_x = \underset{\text{SIGNAL}}{a s_x} + \underset{\text{NOISE}}{n_x}$

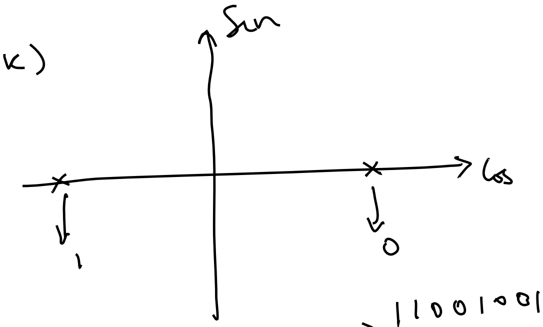


tail prob. of Gaussian gives prob of detecting bit wrongly

SIGNAL TO NOISE RATIO (SNR) $= \dfrac{\text{SIGNAL POWER}}{\text{NOISE POWER}}$

MODULATION EXAMPLES

1. Binary Phase Shift Keying (BPSK)

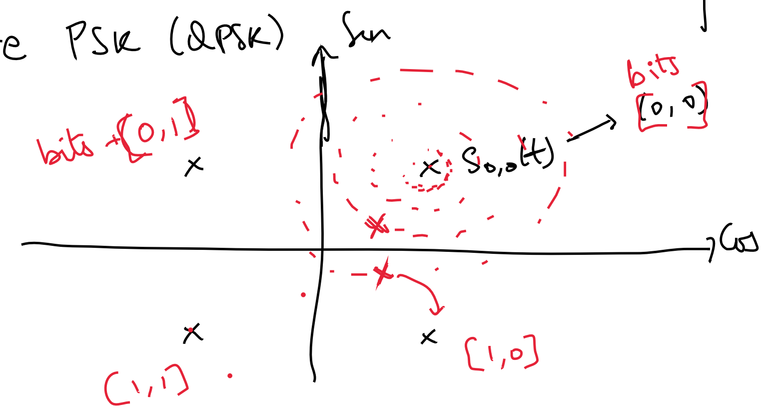


→ 11001001

11

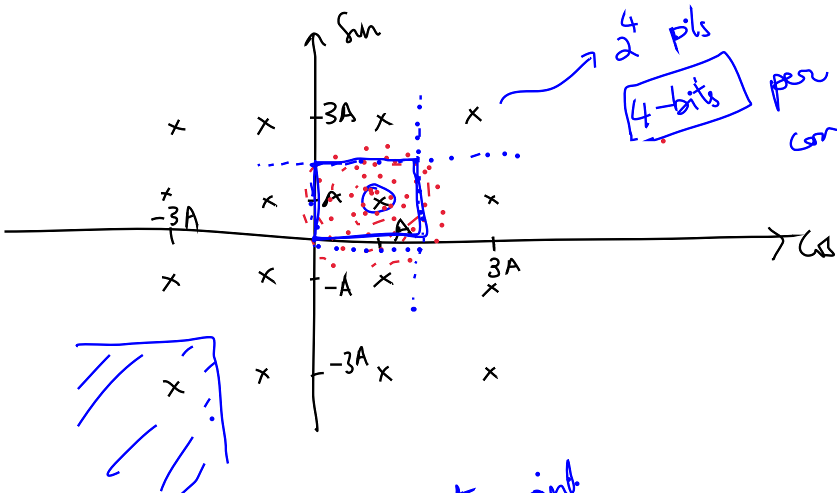
r_x, r_y

2. Quadrature PSK (QPSK)



- Take pairs of data bit
- Transmit corresponding constel. point
- Detect bits as before

3 QAM-16



$\to 2^4$ pts

$\boxed{4\text{-bits}}$ per constell. pt.

QAM-256 ⟶ 8 bits per const. point

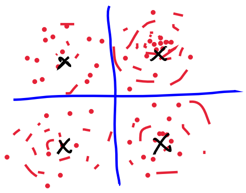
What constellation to use?

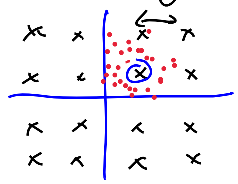
1) Allowed transmit power

2) Received Signal Power (attenuation, noise)

BER → function of SNR

QPSK vs. QAM-16





closer than QPSK, BER goes up

RECEIVED SIGNAL POWER SAME

BPSK → QPSK

↓ slowest

↓ QAM-16