Tutorial 1

Notation: We use $\log n$ to denote log of n to the base 2 and $\ln n$ to denote log of n to the base e.

1. For each list of five functions, arrange the functions in the list in the ascending order of their growth rate. That is, if function $g(n)$ immediately follows function $f(n)$ in your list, then it should be the case that $f(n)$ is $O(g(n))$. Moreover, in case for two functions, $f(n), g(n)$, if $f(n) = \Theta(g(n))$ then indicate that clearly.

   (a) $n, n \log n, n^2, 2^n, n!$.

   (b) $\log n, 2^{\sqrt{\log n}}, \frac{n}{\log n}, \log(\log n - \log \log n), (\log n)^{1/10}$.

   (c) $n^{\log n}, 2^{n^{10/\log n}}, 20, n/\log n, 2^{\log^2 n}$.

   (d) $\log^* n, \log n^{(\log n)}, (\log n)!, \log \log n, 2^{\sqrt{2 \log n}}$.

2. State true or false. Justify your answer.

   (a) $n^{1/\log n} = \Theta(1)$.

   (b) Say $n < m$. $m^2 = \Omega(n^2)$.

   (c) Say $n < m$. m^2 can never be equal to $\Theta(n^2)$.

   (d) $\sqrt{n} = \Theta(n^{0.4})$.

3. Let A be an array of n distinct numbers. A number at location $1 < i < n$ is said to be a maxima in the array if $A[i-1] < A[i]$ and $A[i] > A[i+1]$. Also, $A[1]$ is a maxima if $A[2] < A[1]$ and $A[n]$ is the maxima if $A[n-1] < A[n]$. Find the maxima in the array in time $O(\log n)$.

4. Let A be a sorted array of length n in which a number repeats $\lceil n/2 \rceil$ times. All other numbers in the array are distinct. Give an algorithm that reads only $O(1)$ locations of the array and finds the number that repeats.

5. You have m contiguous memory cells. There is a location $1 \leq i \leq m$ in it such that all the memory cells that occur after this cell are corrupt. If the computer reads a corrupt location then it crashes. Given an algorithm to find i which results in at most 2 computer crashes.

   Suppose each memory probe has a cost of k. Can you design an algorithm that results in at most 2 crashes and has $O(k\sqrt{m})$ cost?