Logic Testing:

Test Generation

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CS-226: Digital Logic Design

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CADSL

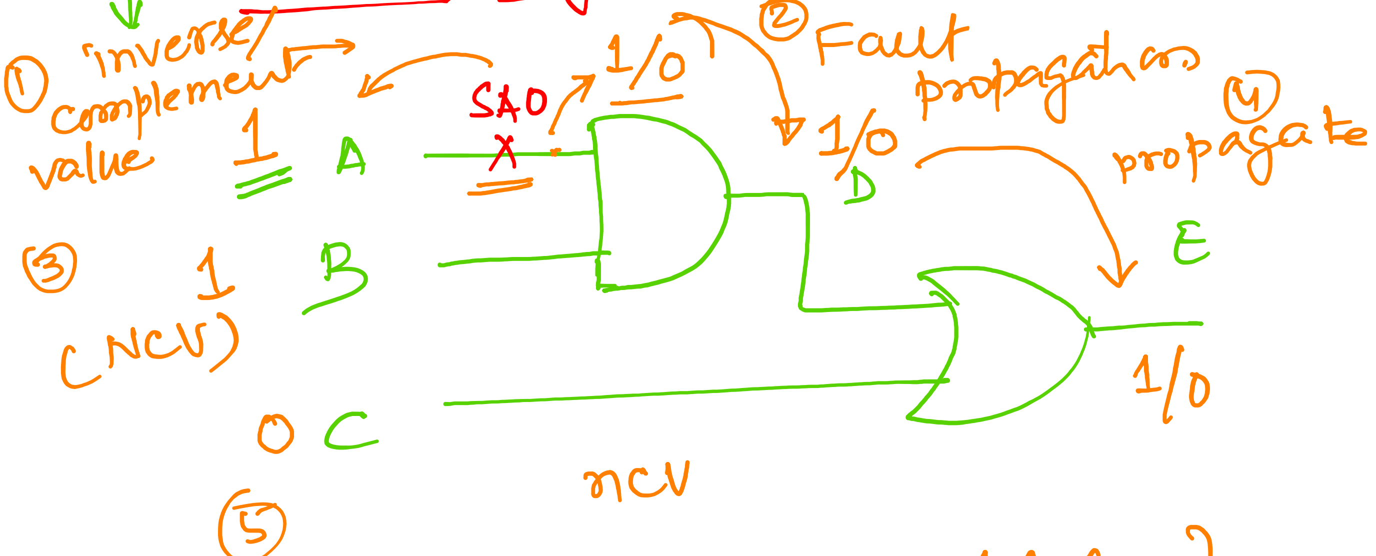
Test Generation

Objective

Find the minimum no. of test vectors to test all modeled faults of a circuit

Fault sensitization

How to generate a test

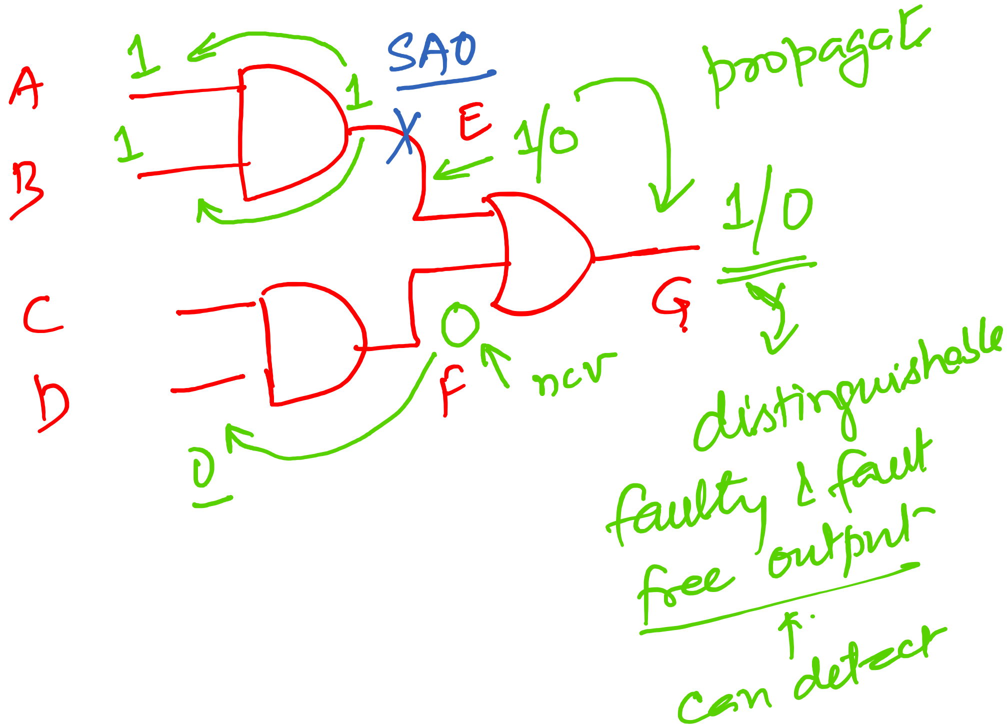


TV = 1 1 0

Test Generation

1. Define the fault location (net & type of fault)
2. Activate/Sensitize the fault (having complementary value just before fault site)
3. Propagate the fault effect towards PO (primary output)
4. need ncv assignment at the gates which are in propagation path
5. If fault effect reaches to PO, - done
   Input Assignment is Test Vector (TV)

Test Generation



A = 1 & B = 1

Sensitize the fault

Propagation

ncv at F

F = 0

TV = 1 1 X 0

$$\begin{bmatrix} 1110 \\ 1100 \end{bmatrix}$$

1. ALGORITHMIC ✓
2. ALGEBRAIC ✓

Boolean Difference

$F(x_1, x_2 \cdots x_i \cdots x_n) = \underbrace{x_i f_{x_i} \oplus \bar{x}_i f_{\bar{x}_i}}$

- **Shannon's Expansion Theorem:**

$$F(X_1, X_2, \ldots, X_n) = X_2 \; F(X_1, 1, \ldots, X_n) + \overline{X_2} \; F(X_1, 0, \ldots, X_n)$$

- **Boolean Difference (partial derivative):**

$$\frac{\partial F_j}{\partial g} = F_j(1, X_1, X_2, \ldots, X_n) \oplus F_j(0, X_1, \ldots, X_n)$$

Boolean Difference

$\frac{\partial f}{\partial x_i} = \underbrace{f_{x_i} \oplus f_{\bar{x}_i}}$

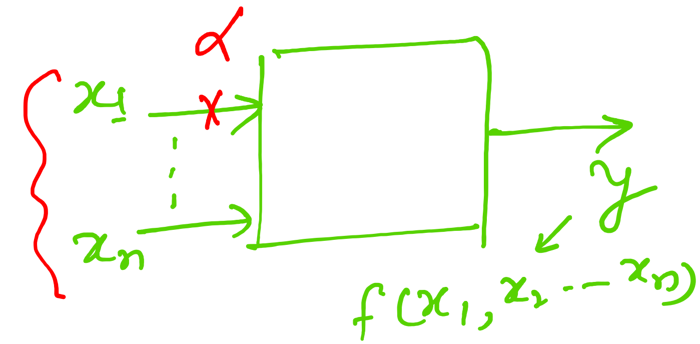
- **Fault Detection Requirements:**

$$G(X_1, X_2, \ldots, X_n) = 1$$

$$\frac{\partial F_j}{\partial g} = F_j(1, X_1, X_2, \ldots, X_n) \oplus F_j(0, X_1, \ldots, X_n) = 1$$

Test Generation

An input assignment can be a test vector if it produces distinguishable faulty & fault free response.



$$y = f(x_1, x_2 \cdots x_i \cdots x_n)$$

$$\boxed{f = x_i f_{x_i} \oplus \bar{x}_i f_{\bar{x}_i}}$$ (Shannon's expansion)

Assume there is a fault α at x_i

faulty output f_α

Fault - x_i SA0 $\longrightarrow$ $f_\alpha = f_{\bar{x}_i}$ ✓

x_i SA1 $\longrightarrow$ $f_\alpha = f_{x_i}$ ✓

Test Generation

$$\underbrace{\underline{f}(x) \oplus \underline{f_\alpha}(x)}_{\text{distinguishable}} = 1$$

$x = \underline{x_1, x_2, x_3 \quad x_i \cdots x_n}$

$\rightarrow$ if x is a Test Vector

$f(x) \rightarrow$ distinguishable $\leftarrow f_\alpha(x)$

$$x_i f_{x_i}(x) \oplus \bar{x}_i f_{\bar{x}_i}(x) \oplus \underline{\underline{f_\alpha}}(x) = 1$$

$\underline{\text{Let}}$ x_i $\underline{SA0}$, $\quad f_\alpha(x) = f_{\bar{x}_i}(x)$

$$x_i f_{x_i} \oplus \bar{x}_i f_{\bar{x}_i} \oplus f_{\bar{x}_i} = 1$$

$$x_i f_{x_i} \oplus \bar{x}_i \cdot f_{\bar{x}_i} \oplus (x_i \oplus \bar{x}_i) \cdot f_{\bar{x}_i} = 1 \qquad (A \oplus A = 0)$$

$$\underbrace{x_i f_{x_i}} \oplus \underbrace{\bar{x}_i \cancel{f_{\bar{x}_i}}} \oplus x_i f_{\bar{x}_i} \oplus \underbrace{\bar{x}_i \cancel{f_{\bar{x}_i}}} = 1$$

Test Generation

$$x_i f_{x_i} \oplus x_i f_{\bar{x}_i} = 1$$

$$x_i \left(f_{x_i} \oplus f_{\bar{x}_i} \right) = 1$$

$$\boxed{x_i \cdot \frac{\partial f}{\partial x_i} = 1}$$

✓ $x_i = 1$ ← Fault sensitization [x_i SA0]

$$\frac{\partial f}{\partial x_i} = \underbrace{f_{x_i} \oplus f_{\bar{x}_i}}_{} = 1$$ ← Fault propagation condition

Test Generation

x_i $\underline{s\text{-}a\text{-}1}$ $\qquad f_\alpha = f_{x_i}$

$$f(X) \oplus f_\alpha(X) = 1$$

$$f(X) \oplus f_{x_i}(X) = 1$$

$$x_i f_{x_i} \oplus \overline{x_i} f_{\overline{x_i}} \oplus f_{x_i} = 1$$

$$x_i f_{x_i} \oplus \overline{x_i} \cdot f_{\overline{x_i}} \oplus (x_i \oplus \overline{x_i}) \cdot f_{x_i} = 1$$

$$\underbrace{x_i \cancel{f_{x_i}}} \oplus \overline{x_i} \cdot f_{\overline{x_i}} \oplus \underbrace{x_i \cancel{f_{x_i}}} \oplus \overline{x_i} f_{x_i} = 1 \quad \checkmark$$

$$\overline{x_i} f_{\overline{x_i}} \oplus \overline{x_i} \cdot f_{x_i} = 1 \Rightarrow \overline{x_i}\,(f_{x_i} \oplus f_{\overline{x_i}}) = 1$$

$$\overline{x_i} = 1 \Rightarrow x_i = 0 \rightarrow \text{Fault sensitization}$$

& $\frac{\partial f}{\partial x_i} = f_{x_i} \oplus f_{\overline{x_i}} = 1 \rightarrow$ Fault propagation

$X = x_1 x_2 \cdot x_i \cdots x_n \rightarrow TV$

Boolean Difference

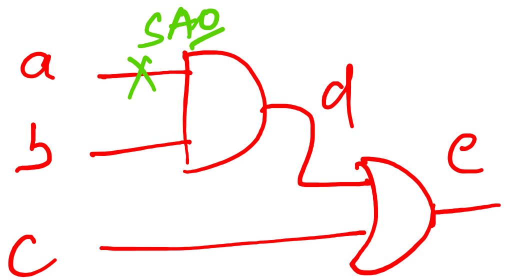
$$\boxed{f(x_1,\ldots,x_i=0,\ldots,x_n)\oplus f(x_1,\ldots,x_i=1,\ldots,x_n)=1}$$

- Represented by the symbol *df(x)/dx*
- *df*(*x*)/*dx*$_{i \text{ for } x=0}$ fn and *df(x)/dx*$_{i\ for\ x=1}$ fn' are called *the* *residues/co-factors* of the function for *x = xi*
- One of the residue is the good-circuit value and the other is the faulty-circuit value for x_i
- To detect the fault, the two residues should be complementary

Fault propagation

- Solving the equation yield the values of the primary inputs to detect a stuck-at fault on x_i
- The test pattern is: $x_i\, df(x)/dx_i = 1$ & $x_i'\, df(x)/dx_i = 1$

Test Generation



$e = f(a,b,c) = ab+c$

TV

$a \cdot \frac{\partial f}{\partial a} = 1$

$a=1$ and $f_a \oplus f_{\bar{a}} = 1$

$f_a = b+c$

$f_{\bar{a}} = c$

$a \cdot [(b+c) \oplus c] = 1$

$a \cdot [(b+c)\bar{c} + \overline{(b+c)} \cdot c] = 1$

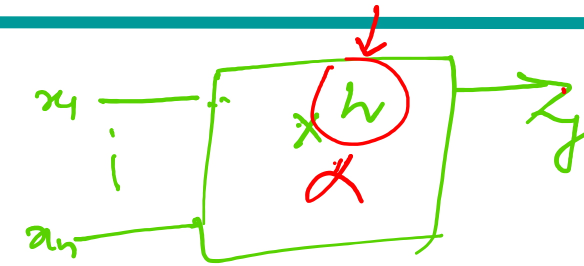
$\Rightarrow a \cdot [b\bar{c} + 0 + \underbrace{\bar{b} \cdot \bar{c} \cdot c}_{0}] = 1 \Rightarrow \boxed{a \cdot b \cdot \bar{c} = 1}$

$a=1$, $b=1$, $\bar{c}=1$

$a=1$, $b=1$, $c=0$ TV

Test Generation

$f(X) = \underline{f}(x_1 x_2 \cdots x_i \cdots \underline{x_n, h}) \rightarrow$



$f_\alpha \rightarrow$ faulty function

(fault at h)

$\underline{h} = f(x_1, x_2 \cdots \underline{x_n})$

$$\underline{f(X,h)} \oplus f_\alpha(X,h) = 1$$

f_α let h $\underline{sa0}$ $\qquad f_\alpha(X,h) = f_{\bar h}(X,h)$

$$\underline{f}(X,h) \oplus f_{\bar h}(\underline{X,h}) = 1$$

$h = 1$ ✓

$\frac{\partial f}{\partial h} = 1$ ✓

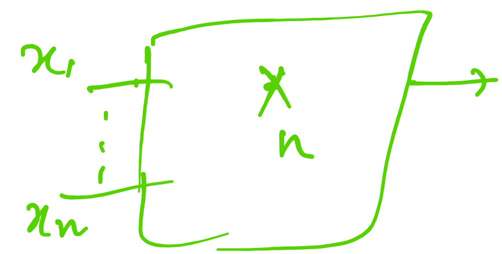
$$h f_h \oplus \bar h f_{\bar h} \oplus f_{\bar h} = 1$$

$$h f_h \oplus \bar h f_{\bar h} \oplus (h \oplus \bar h) f_{\bar h} = 1$$

$$h f_h \oplus \cancel{\bar h f_{\bar h}} \oplus h f_{\bar h} \oplus \cancel{\bar h f_{\bar h}} = 1 \Rightarrow \boxed{h(f_h \oplus f_{\bar h}) = 1}$$

Test Generation

h sa0



$$h \cdot (f_h \oplus f_{\bar{h}}) = 1$$

if h sa1 $\quad f_x = f_h$

then

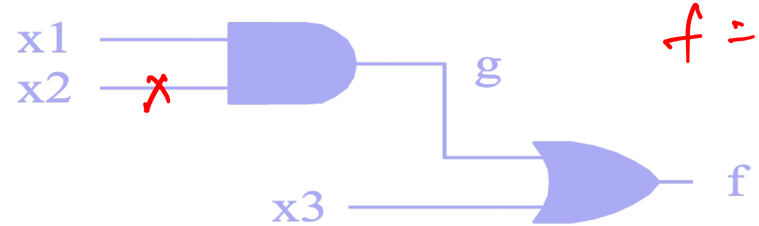
$$\bar{h}\,(f_h \oplus f_{\bar{h}}) = \bar{h}\,\frac{\partial f}{\partial h} = 1$$

$$\boxed{\begin{array}{c} \bar{h} = 1 \\ \dfrac{\partial f}{\partial h} = 1 \end{array}}$$

Fault Detection

- $x_i\, df(x)/dx_i = 1$ for s-a-0 at x_i
- $x_i'\, df(x)/dx_i = 1$ for s-a-1 at x_i
- As an example, let us consider the function

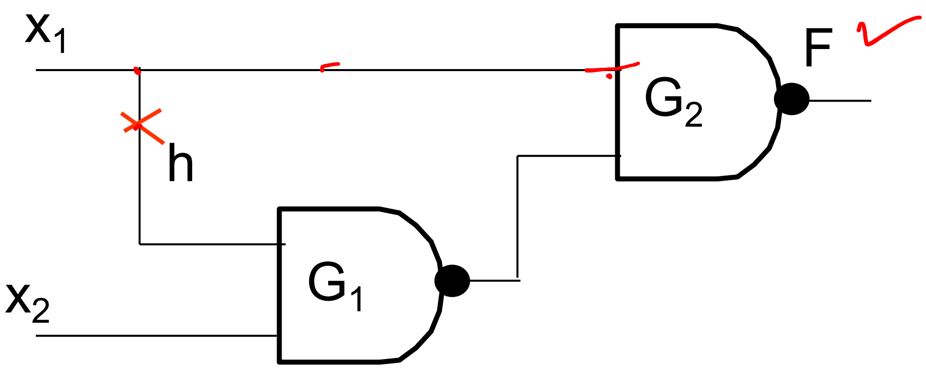
$f = x_1x_2 + x_3$



- $f(x) = x_1x_2 + x_3$
- Thus $df(x)/dx_2 = x_3 \oplus (x_1 + x_3) = x_3'x_1 = 1$. Then
- $x_1 = 1$ and $x_3 = 0$.
- For the SA1 and SA0 faults on x_2, the patterns are then $x_1x_2x_3$ = (100) and (110), respectively.

Fault Detection

$h(x) \cdot \frac{\partial f}{\partial h} = 1$



S-a-0 fault at h

Test Vector

$h(X)\ dF^*(X,h)/dh = 1$

$F(X,h) = x_1' + hx_2$

$x_1 \cdot x_1x_2 = x_1x_2 = 1$

$h(X) = x_1'$

$x_1 = 1$

$x_2 = 1$

TV

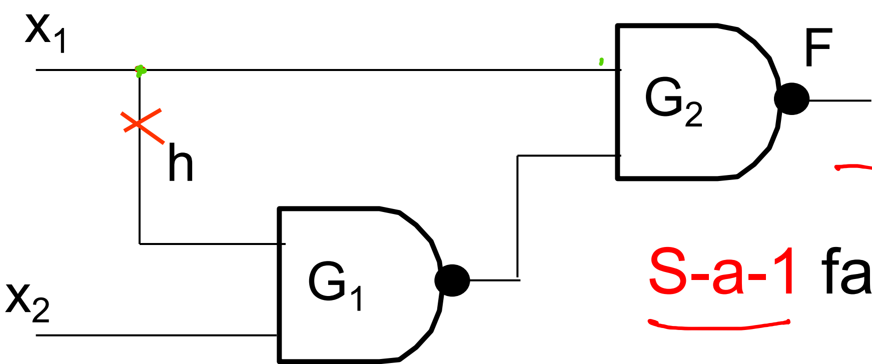
$dF^*(X,h)/dh = x'_1 \oplus (x_1' + x_2)$

$= x_1x_2$

Fault Detection

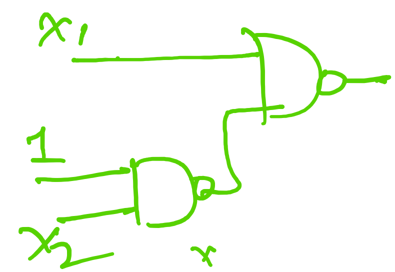
$\bar{h} \cdot \frac{\partial f}{\partial h} = 1$

$\bar{x}_1$ $x_1 x_2$



S-a-1 fault at h

≡



Test Vector

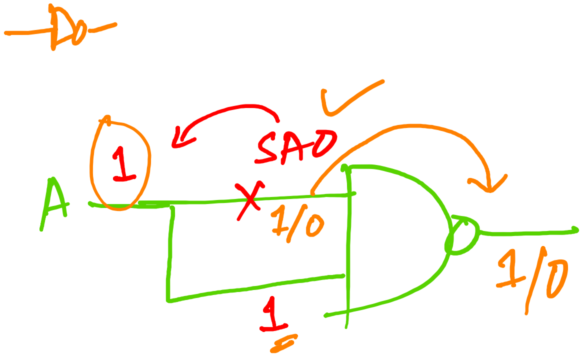
$h(X)'\, dF^*(X,h)/dh = 1$

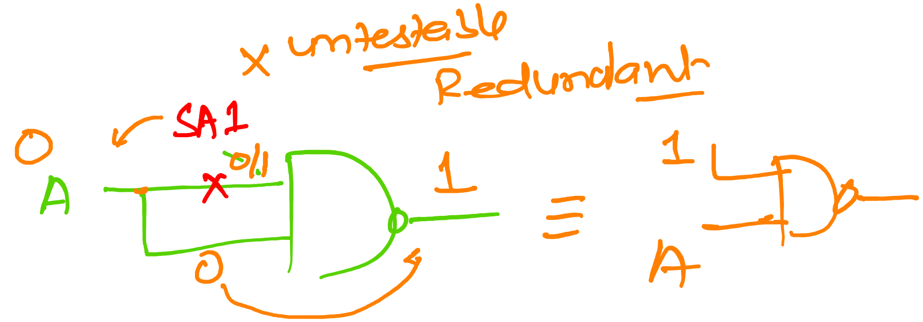
$x'_1 x_1 x_2 = x_1 x_2 = 0$ ✓

It cannot satisfy required condition

Fault - Redundant

due to redundancy in circuit





Redundant Faults

FC   Fault coverage $= \dfrac{\#\text{ detectable faults}}{\#\text{ faults}}$

Fault Efficiency $= \dfrac{\#\text{ detectable faults}}{\#\text{ faults} - \#\text{ Redundant faults}}$

faults: 1000

990 - detectable

10 Redundant

$FC = \dfrac{990}{1000} \times 100 = 99\%$

$FE = \dfrac{990}{1000-10} = 100\%$

Boolean Difference → TV or Redundant

Test / Fault Efficiency = 100 %

Algorithmic method — Backtracking

$\boxed{\text{\# Test vectors must be small}}$

TE/FE ≃ 100%

$\boxed{\text{Fanout free circuit}}$

Reconvergent fanout.

Thank You