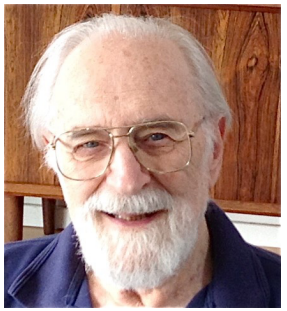
Logic Optimization: Karnaugh Map



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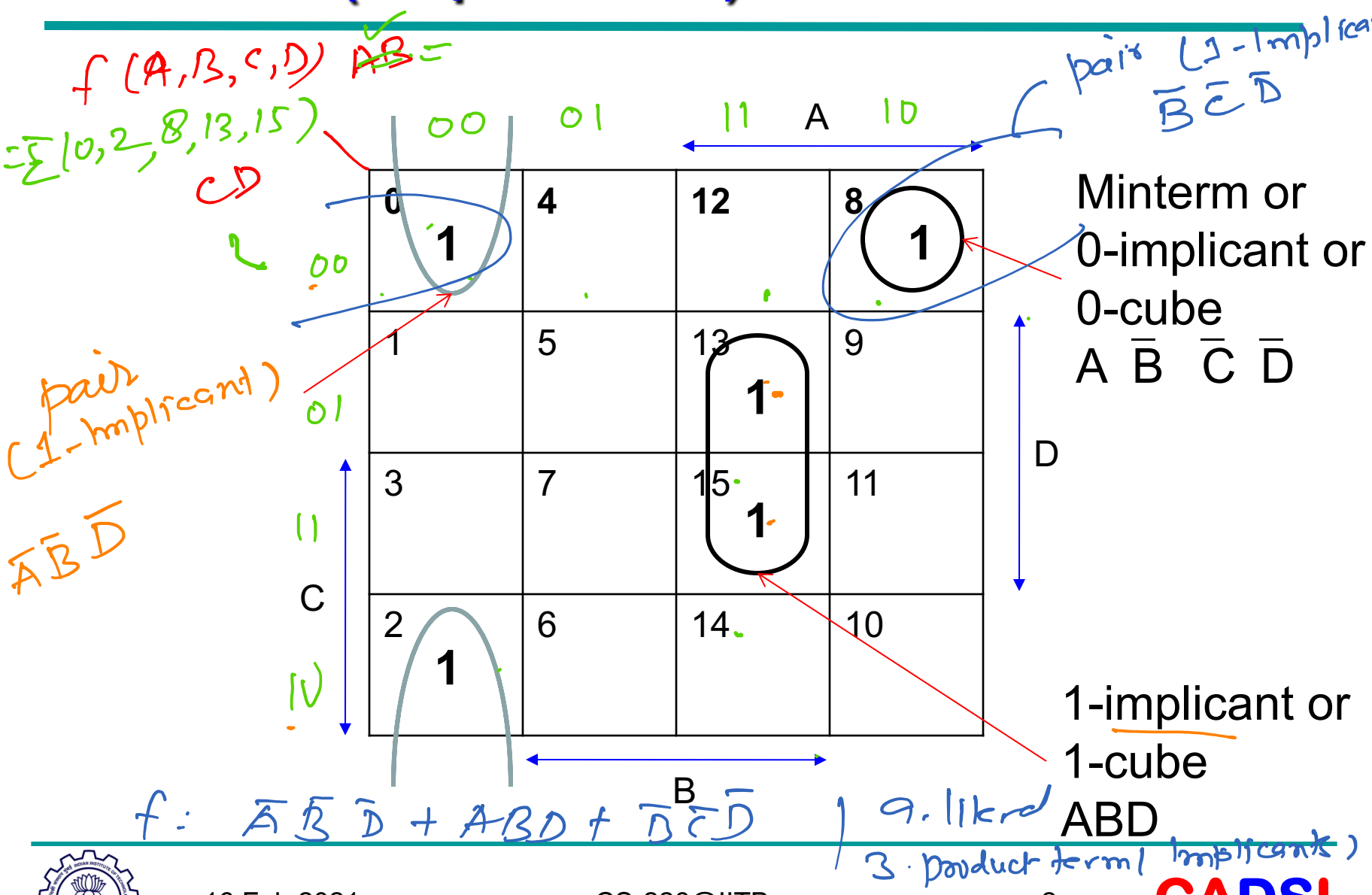
CS-226: Digital Logic Design

Lecture 14-B: 16 February 2021

CADSL

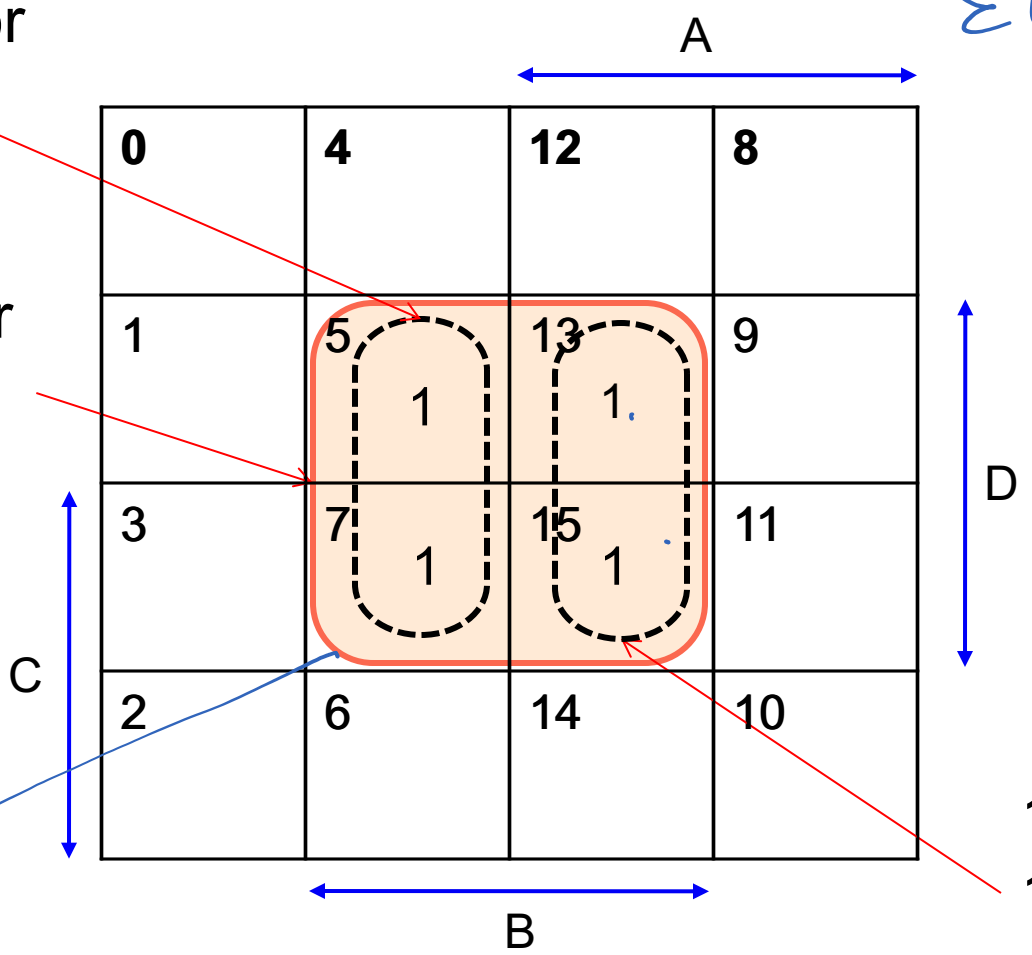
Cubes (Implicants) of 4 Variables



Growing Cubes, Reducing Products

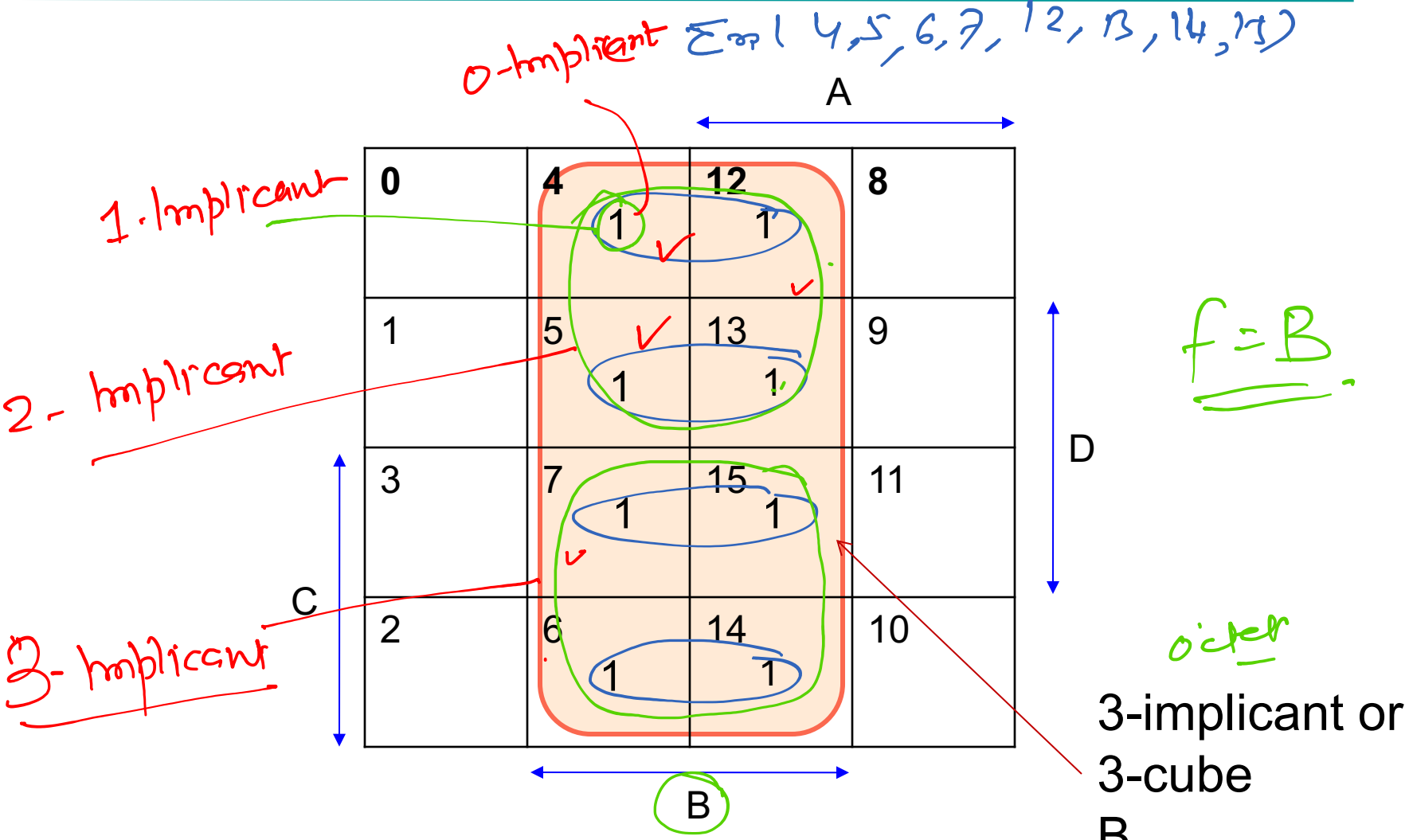
1-implicant or
1-cube
$\bar{A}BD$

2-implicant or
2-cube
BD



1-implicant or
1-cube
ABD

Largest Cubes or Smallest Products



Implication and Covering

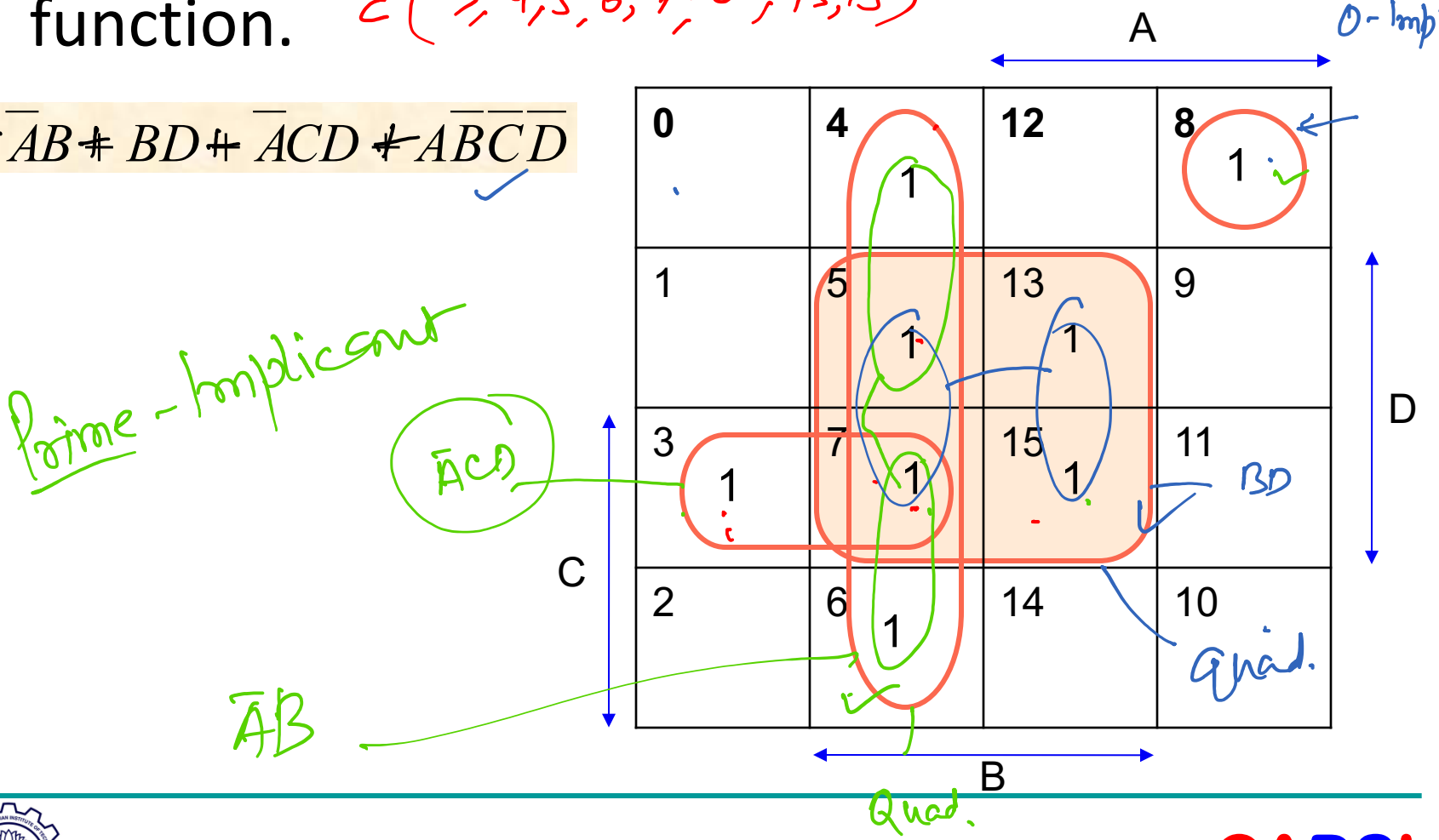
- A larger cube covers a smaller cube if all minterms of the smaller cube are included in the larger cube.
- A smaller cube implies (or subsumes) a larger cube if all minterms of the smaller cube are included in the larger cube.

Implicants of a Function

- Minterms, products, cubes that imply the function.

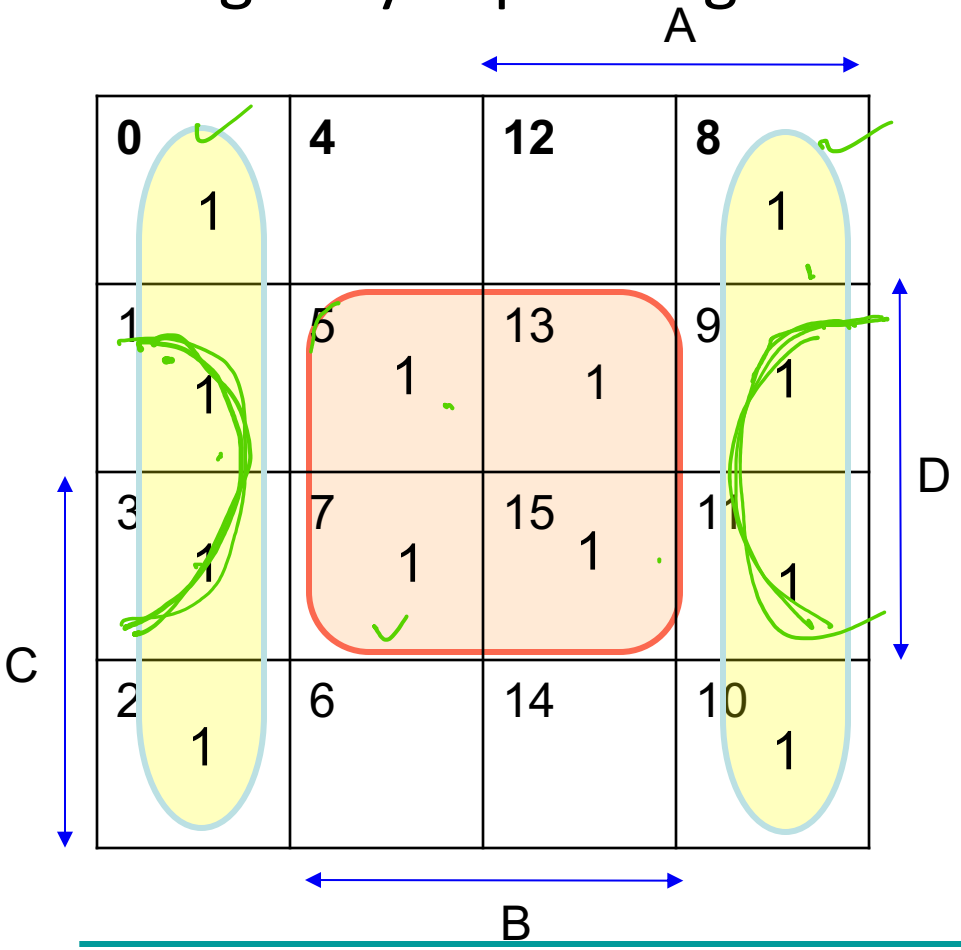
$\Sigma(3,4,5,6,7,8,13,15)$

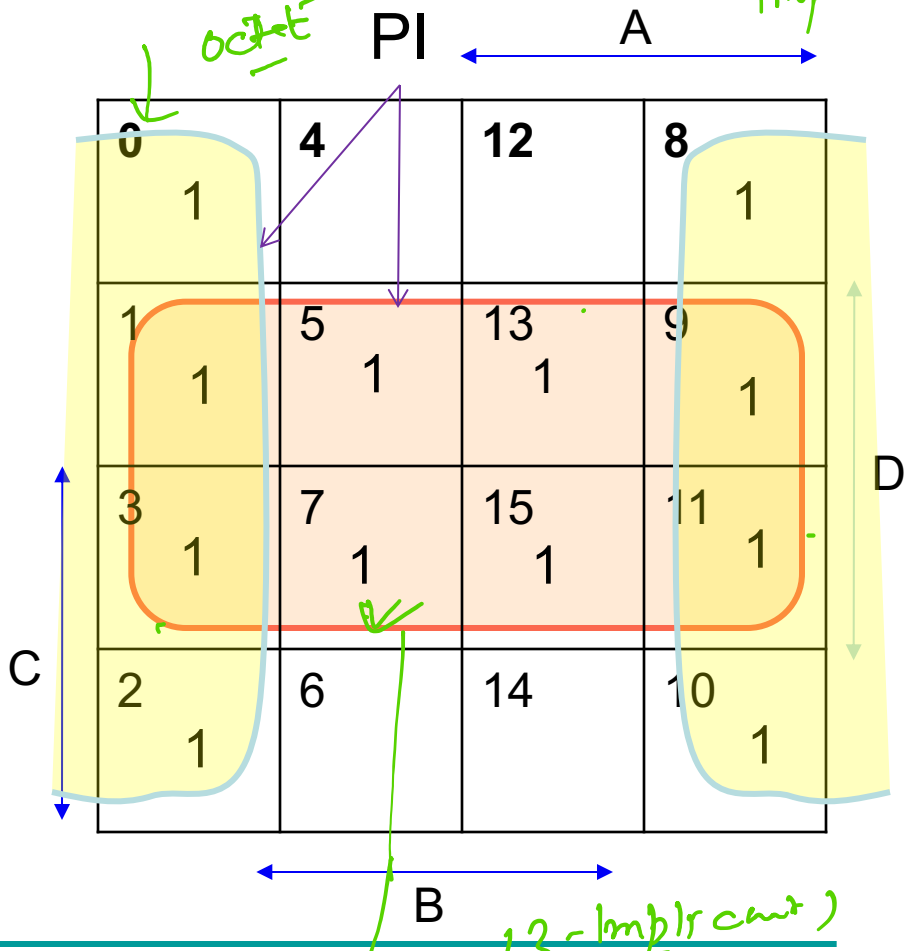
$$F = \bar{A}B + BD + \bar{A}CD + A\bar{B}\bar{C}\bar{D}$$



Prime Implicant (PI)

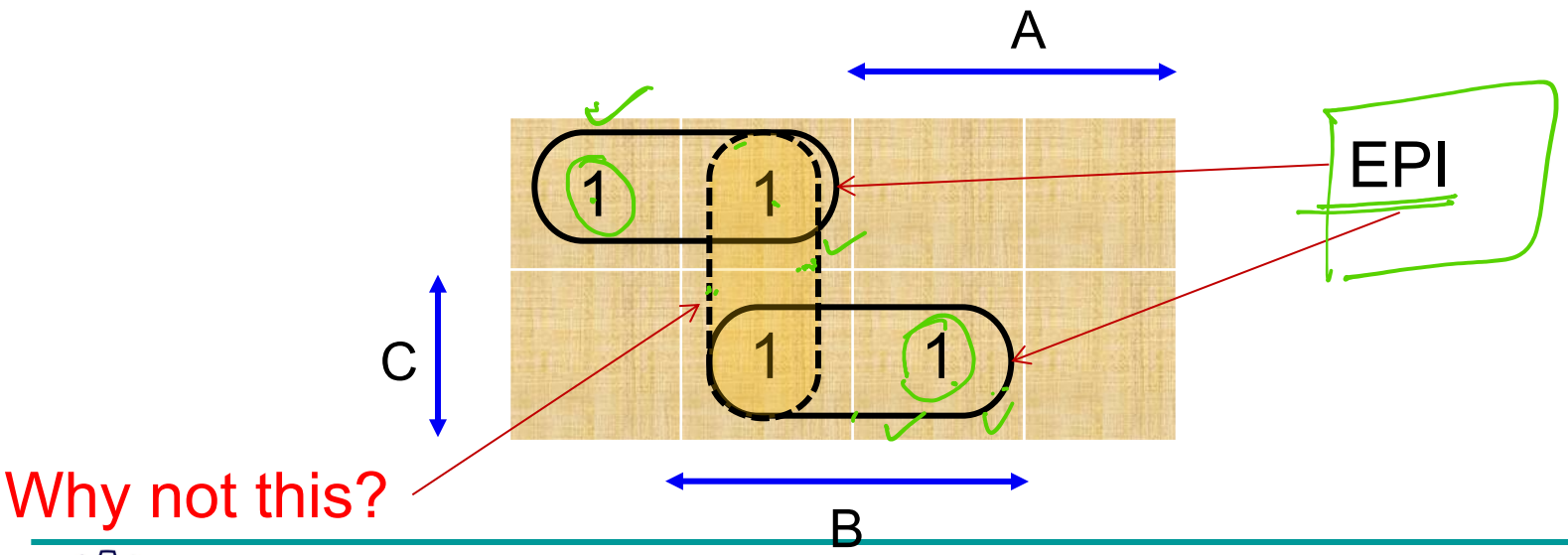
- A cube or implicant of a function that cannot grow larger by expanding into other cubes.





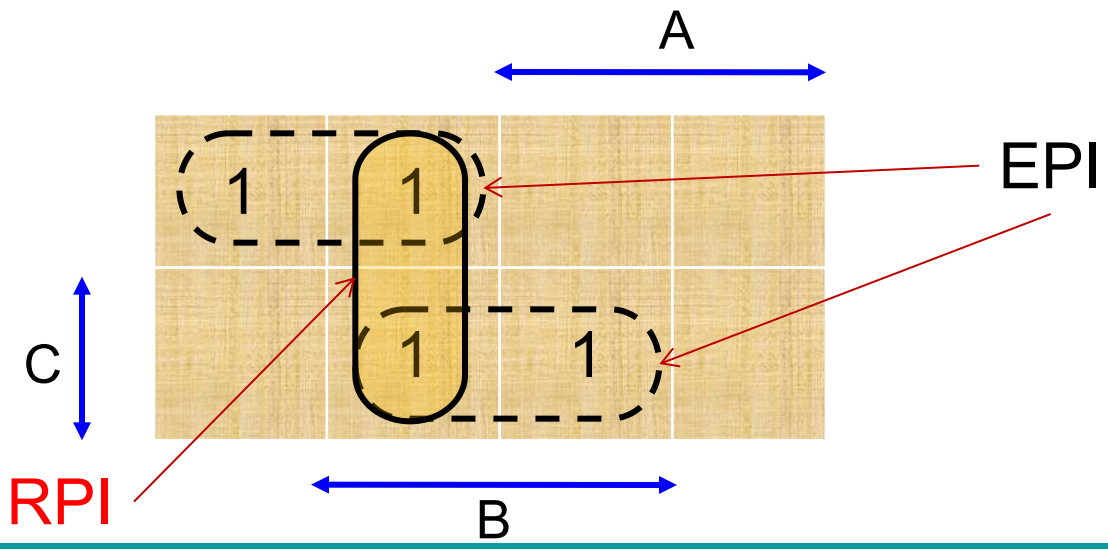
Essential Prime Implicant (EPI)

- If among the minterms subsuming a prime implicant (PI), there is at least one minterm that is covered by this and only this PI, then the PI is called an essential prime implicant (EPI).
- Also called essential prime cube (EPC).



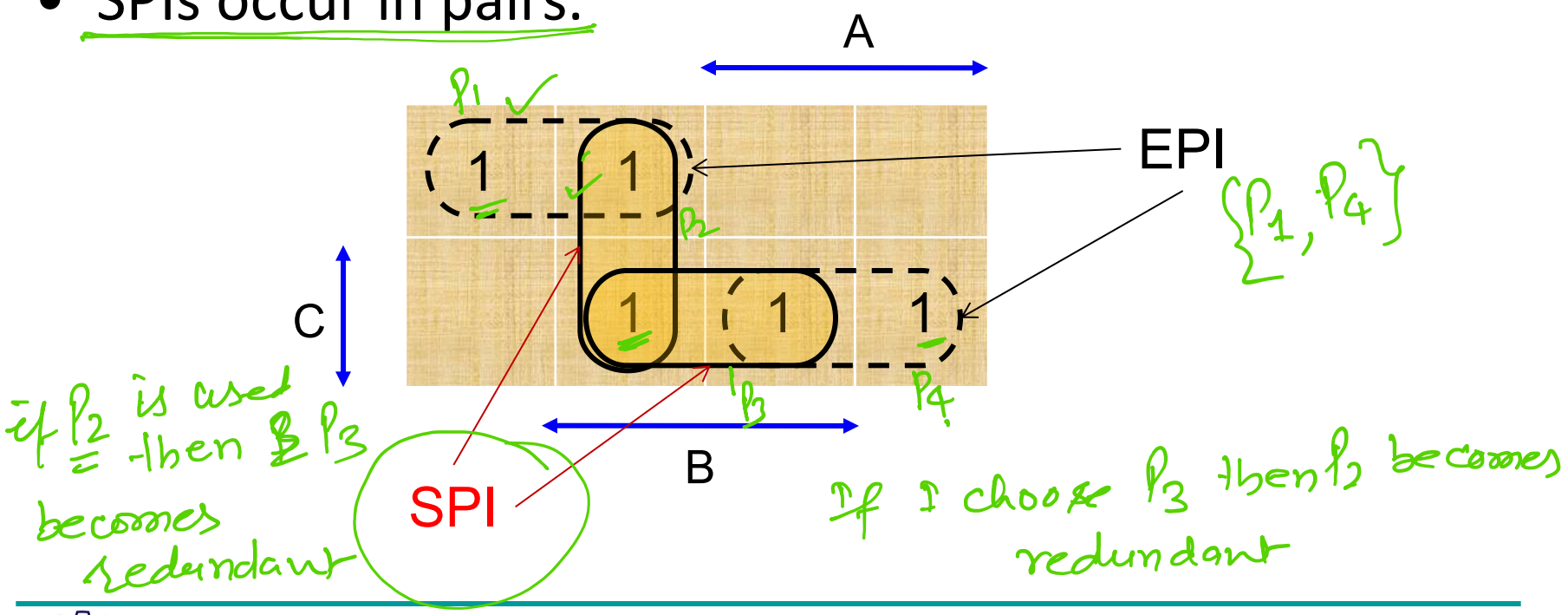
Redundant Prime Implicant (RPI)

- If each minterm subsuming a prime implicant (PI) is also covered by other essential prime implicants, then that PI is called a redundant prime implicant (RPI).
- Also called redundant prime cube (RPC).



Selective Prime Implicant (SPI)

- A prime implicant (PI) that is neither EPI nor RPI is called a selective prime implicant (SPI).
- Also called selective prime cube (SPC).
- SPIs occur in pairs.



Minimum Sum of Products (MSOP)

- Identify all prime implicants (PI) by letting minterms and implicants grow.
- Construct MSOP with PI only :
  - Cover all minterms
  - Use only essential prime implicants (EPI)
  - Use no redundant prime implicant (RPI)
  - Use cheaper selective prime implicants (SPI)
  - A good heuristic – Choose EPI in ascending order, starting from 0-implicant, then 1-implicant, 2-implicant, . . .

min(# prod terms)

min(# PI)

min(# EPI + # SPI) = min(# SPI) ✓

have to be there

must be there

or

min(# literals) = min(# literals in EPI + # literals in SPI)

min(# literals in SPI)

Thank You