Logic Function Representation

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CS-226: Digital Logic Design

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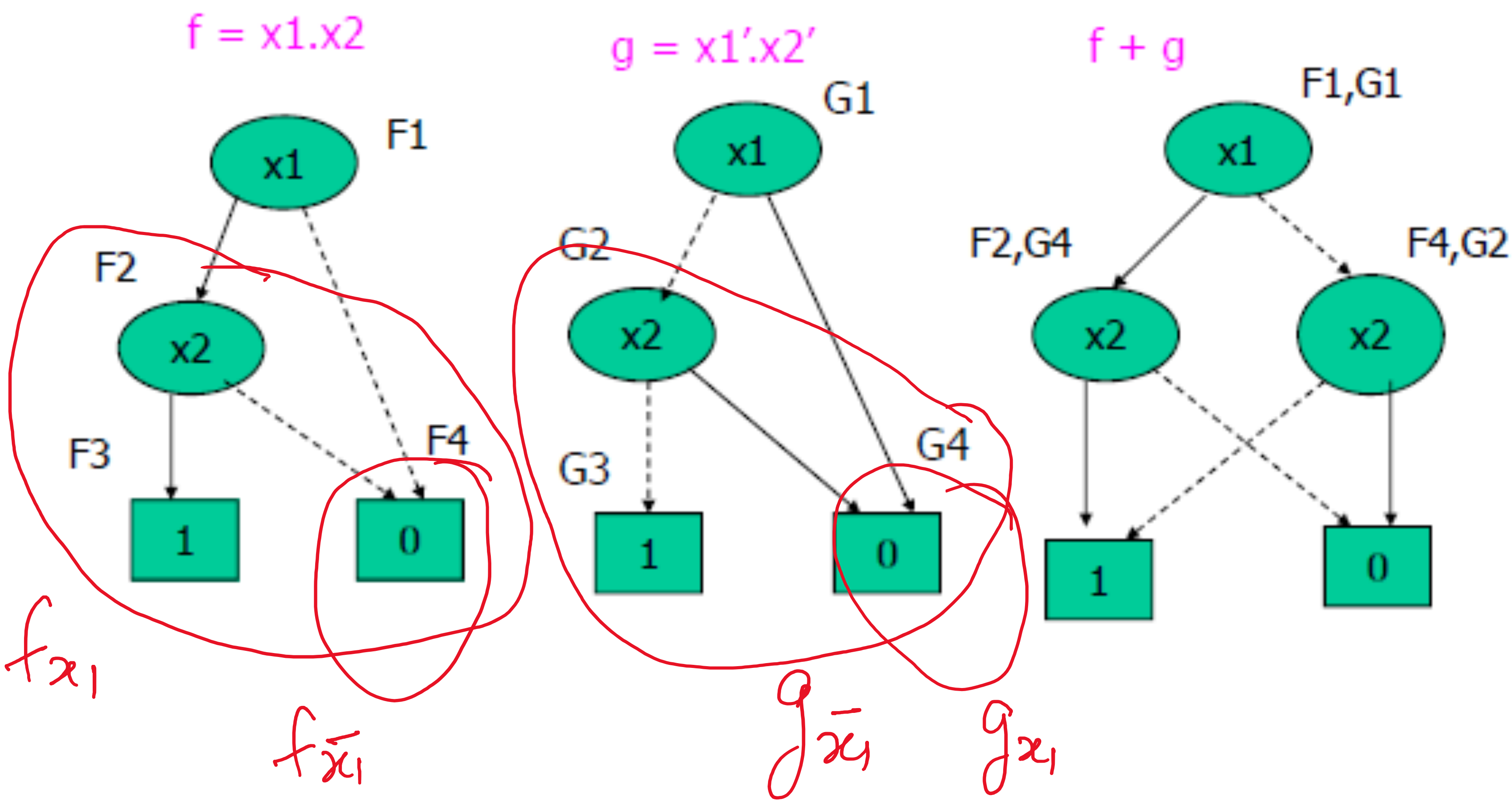
CADSL

Canonical Forms

- Canonical Forms in common usage:
  - Truth Table
  - Sum of Minterms (SOM)
  - Product of Maxterms (POM)
  - Binary Decision Diagram (BDD)
  - Reed Muller Representation

Operations with BDD: Example



BDD Construction

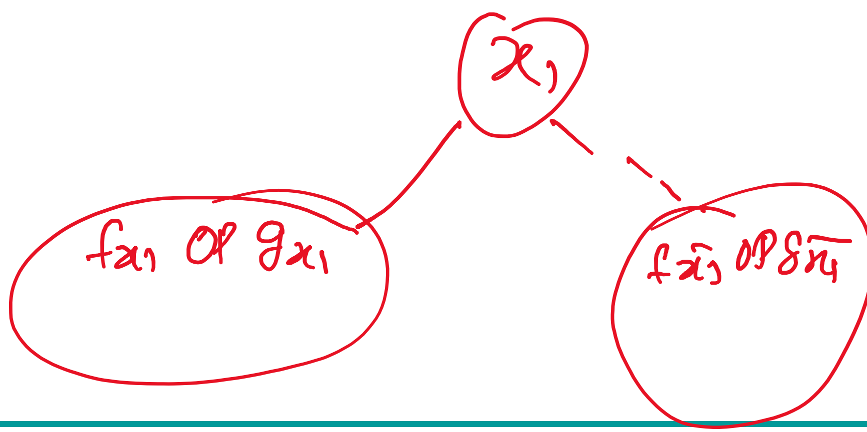
$$f(x_1, x_2 \cdots x_n) \cdot OP \cdot g(x_1, x_2 \cdots x_n)$$

$$\Downarrow$$

$$(x_1 f_{x_1} + \bar{x}_1 f_{\bar{x}_1}) \; OP \; (x_1 g_{x_1} + \bar{x}_1 g_{\bar{x}_1})$$

$$\Downarrow$$

$$x_1 (f_{x_1} \; OP \; g_{x_1}) + \bar{x}_1 (f_{\bar{x}_1} \; OP \; g_{\bar{x}_1})$$



BDD Construction

Construct ROBDD of f

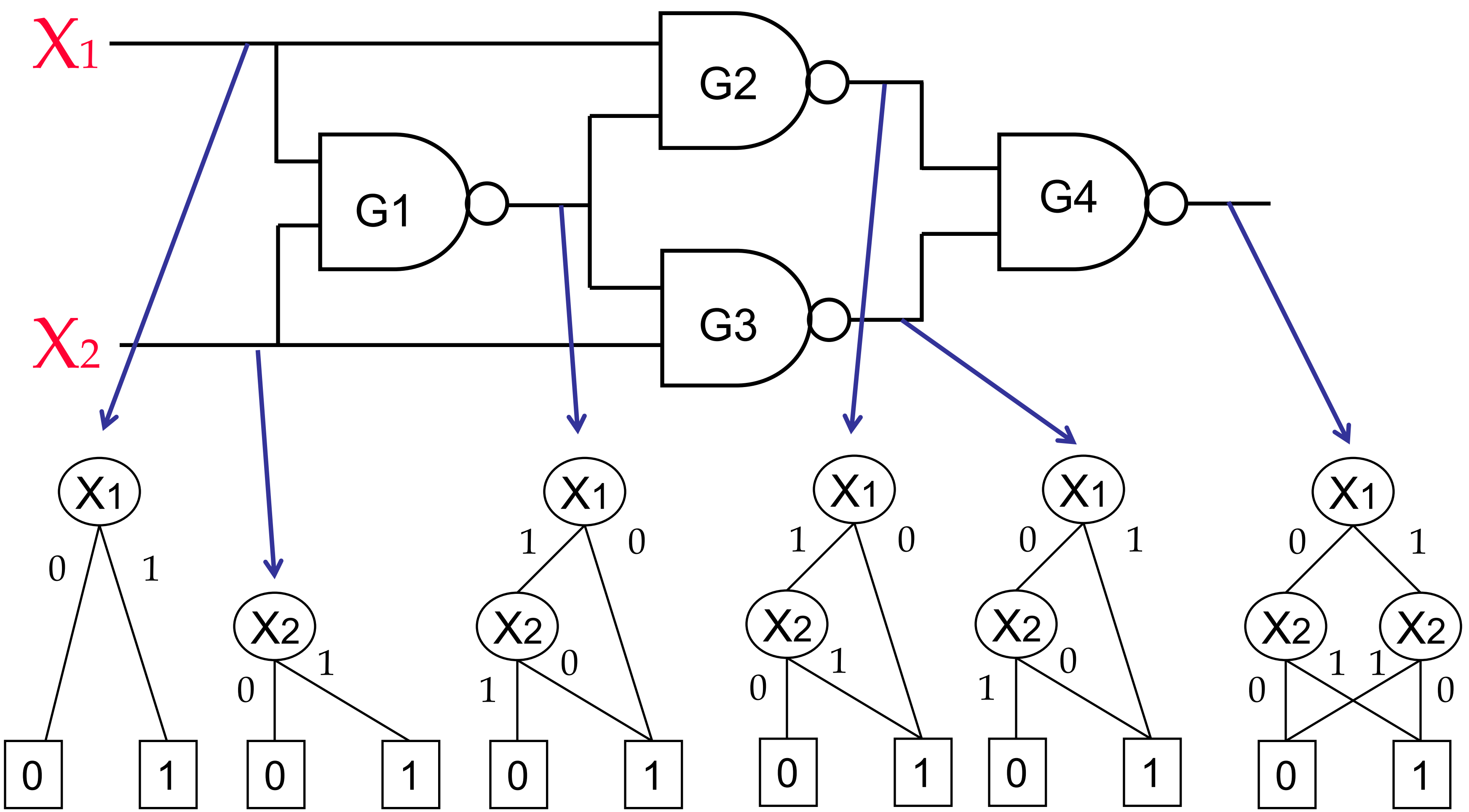
construct ROBDD of g

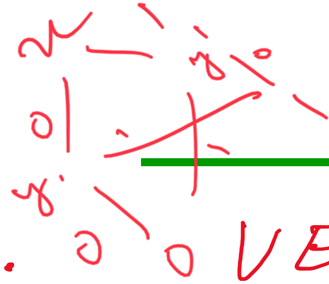
Apply operation f . OP. g

then ~~[illegible]~~ reduce it to construct

ROBDD of f. OP. g

From Circuits to BDD





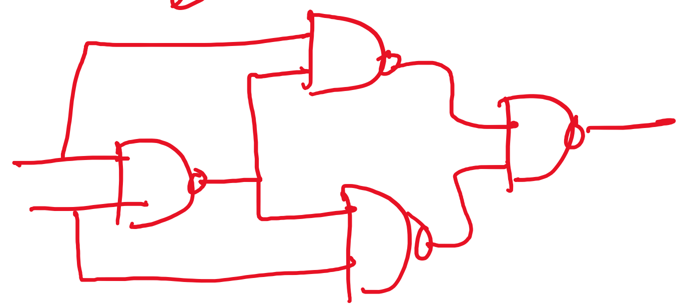
ROBDD

VERIFICATION

SPECIFICATION $\equiv$ IMPLEMENTATION

<u>XOR</u> operator

$$x\cdot\bar{y} + \bar{x}\cdot y \overset{?}{\equiv}$$



ROBDD

ROBDD

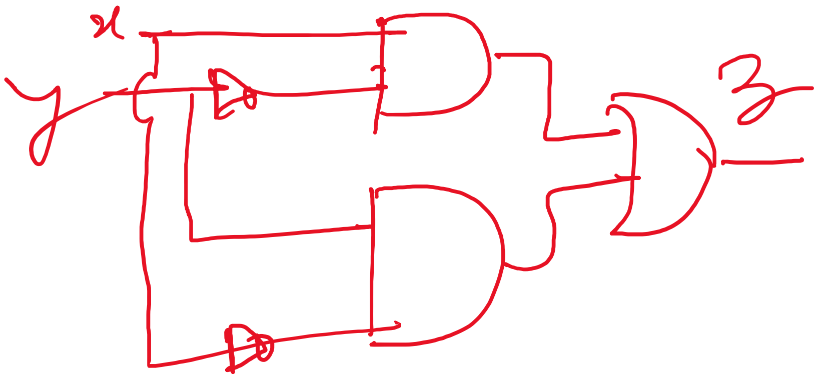
IMPLEMENTION1 $\equiv$ IMPLEMENTATION2

- <u>EQUIVALENCE</u> CHECKING

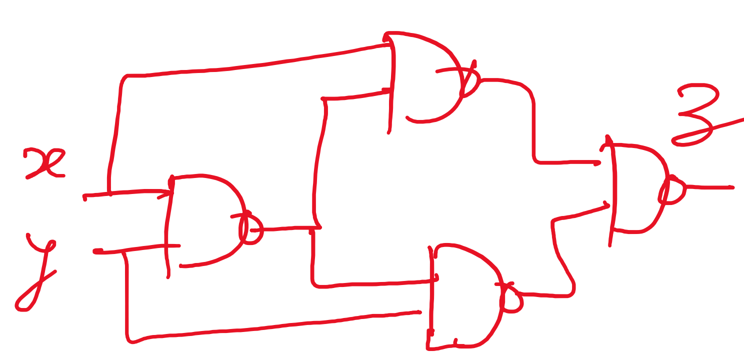
Match two ROBDD for isomorphism

ROBDD

IMPLEMENTATION 1 ≡ IMPLEMENTATION2

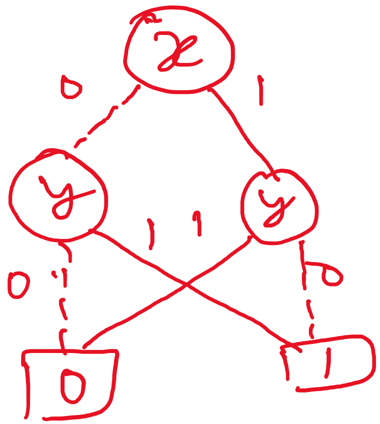


?
≡

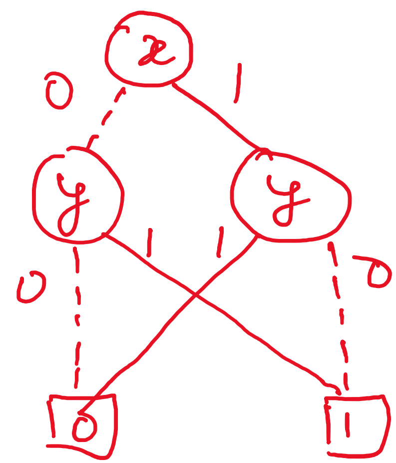


ROBDD

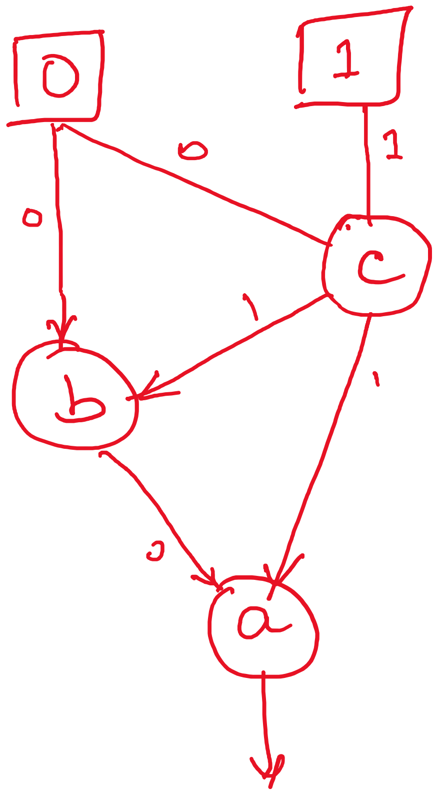
ROBDD



Isomorphic

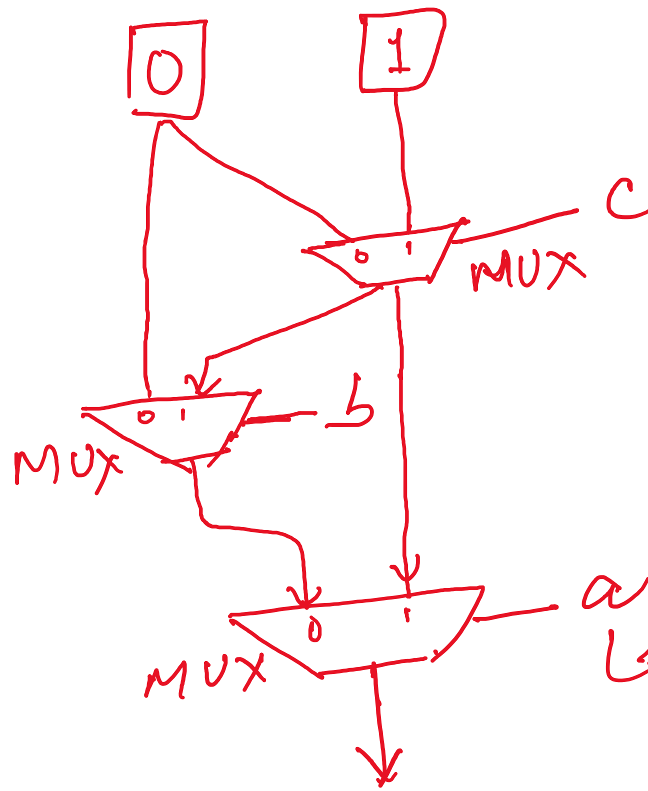


Synthesis from ROBDD

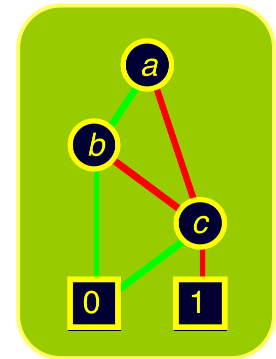


ROBDD → just upside down

f



a (select signal to mux)



Replace a node by a multiplexer.

Synthesis from ROBDD

- Replace all nodes by Multiplexers

N-level logic

Cost $\propto$ # Nodes | Cost of a Mux = c

Cost = c · x # Nodes

Performance.

Delay $\propto$ Max. no. of nodes in any path

Mux delay = τ

Max delay = N · τ → upper bound on delay

Max variables

Reed Muller Representation

Function Decomposition

- Shanon's decomposition

$$f(x_1, x_2, \quad ..xi, ... \, xn)$$
$$= xi.f(x_1, x_2, \quad .. \, x_i = 1, ... \, xn) \oplus \overline{x_i}.f(x_1, x_2, \quad .x_i, = 0, ... \, xn)$$

- Positive Davio decomposition

$$f(x_1, x_2, \; ..xi, ... \, xn) = f(x_1, x_2, \; ..xi = 0, ... \, xn) \oplus x_i.(f(x_1, x_2, \; ..xi = 1 ... \, xn) \oplus f(x_1, x_2, \; ..xi = 0 ... \, xn))$$

$$f = f_{\overline{x_i}} \oplus x_i \cdot (f_{x_i} \oplus f_{\overline{x_i}})$$

- Negative Davio decomposition

$$f(x_1, x_2, \; ..xi, ... \, xn) = f(x_1, x_2, \; ..xi = 1, ... \, xn) \oplus \overline{} xi.(f(x_1, x_2, \; ..xi = 1 ... \, xn) \oplus f(x_1, x_2, \; ..xi = 0 ... \, xn))$$

$$f = f_{x_i} \oplus \overline{x}_i \cdot (f_{x_i} \oplus f_{\overline{x}_i})$$

Reed Muller Representation

$$f = x_i f_{x_i} \oplus \bar{x}_i \cdot f_{\bar{x}_i}$$
$$= x_i f_{x_i} \oplus (1 \oplus x_i) f_{\bar{x}_i}$$
$$= x_i f_{x_i} \oplus f_{\bar{x}_i} \oplus x_i f_{\bar{x}_i}$$
$$= f_{\bar{x}_i} \oplus x_i f_{x_i} \oplus x_i f_{\bar{x}_i}$$
$$= f_{\bar{x}_i} \oplus x_i (f_{x_i} \oplus f_{\bar{x}_i})$$

Reed Muller expansion

Boolean Difference.

$$f = (1 \oplus \bar{x}_i) f_{x_i} \oplus \bar{x}_i \cdot f_{\bar{x}_i}$$
$$= f_{x_i} \oplus \bar{x}_i f_{x_i} \oplus \bar{x}_i f_{\bar{x}_i}$$
$$= f_{x_i} \oplus \bar{x}_i \cdot (f_{x_i} \oplus f_{\bar{x}_i})$$

Reed Muller Representation

- Positive Davio decomposition

$f(x_1, x_2, \ldots xi, \ldots xn) = f(x_1, x_2, \ldots xi = 0, \ldots xn) \oplus x_i.(f(x_1, x_2, \ldots, x_i = 1, \ldots xn) \oplus f(x_1, x_2, \ldots, x_i = 0, \ldots xn))$

- Generalization

$$f(x_1, x_2, \ldots xi, \ldots xn) = a_0 \oplus a_1 x_1 \oplus a_2 x_2 \ldots\ldots \oplus a_r x_1 x_2 . \oplus a_p x_1 x_2 x_3 \oplus \ldots\ldots \oplus a_m x_1 x_2 \ldots xn$$

Thank You