Q2

1.

$$\begin{aligned}\exists Closure_s\ ([&Closure_s(s)\\ &\wedge \forall x \forall y[(Closure_s(x) \wedge E(x,y)) \implies Closure_s(y)]\\ &\wedge \forall X([X(s) \wedge \forall x \forall y[(X(x) \wedge E(x,y)) \implies X(y)] \implies (Closure_s \subseteq X))]\\ &\wedge Closure_s(t))\end{aligned}$$

where $A \subseteq B \equiv \forall x(A(x) \implies B(x))$
Essentially, we are saying that there exists some set($Closure_s$) that follows certain properties:
Line1: It contains s
Line2: If it contains x, then it contains all states directly adjacent to x
Line3: It is the minimal such set
and then, in the graph, it must also contain t. This $Closure_s$ is what one would call the transitive closure of s. It contains all states reachable from s.

2.

$$\forall X(([\exists y X(y)] \wedge [\exists x \forall y(X(y) \implies y \leq x)]) \implies \exists z \forall x[\forall y(X(y) \implies y \leq x)] \implies z \leq x)$$

Q3

They both have the same expressiveness, i.e. any language represented by an MSO formula can be represented by an MSO_0 formula and vice versa. We show how to construct these formulae. Given an MSO_0 formula, perform the following replacements:

MSO_0	MSO
$Sing(X)$	$\exists x X(x) \wedge \forall y X(y) \implies x = y$
$X \subseteq Y$	$\forall x X(x) \implies Y(x)$
$X < Y$	$Sing(X) \wedge Sing(Y) \wedge \exists x \exists y[X(x) \wedge Y(y) \wedge x < y]$
$S(X,Y)$	$Sing(X) \wedge Sing(Y) \wedge \exists x \exists y[X(x) \wedge Y(y) \wedge S(x,y)]$
$Q_a(X)$	$\forall x X(x) \implies Q_a(x)$

To convert MSO to MSO_0, we consider $X = \{x\}$ and $Y = \{y\}$. Essentially, we replace as following:

MSO	MSO_0
$\forall x \phi(x)$	$\forall X Sing(X) \implies \phi'(X)$
$\exists x \phi(x)$	$\exists X Sing(X) \wedge \phi'(X)$
$x = y$	$Sing(X) \wedge Sing(Y) \wedge X = Y$
$x < y$	$Sing(X) \wedge Sing(Y) \wedge X < Y$
$S(x, y)$	$Sing(X) \wedge Sing(Y) \wedge S(X, Y)$
$Q_a(x)$	$Sing(X) \wedge Q_a(X)$

where ϕ' is formed by appropriate replacements in ϕ. The $Sing$ in the last 4 rows is rather redundant, you may omit it.

Q4

$$\begin{aligned}
&\exists X(Sing(X) \wedge \forall Y[Sing(Y) \implies (X < Y \implies Q_a(Y))]) \\
\equiv&\exists X(Sing(X) \wedge \forall Y(X < Y \implies Q_a(Y)))
\end{aligned}$$

Q5

Language is $b(aa^*bbb)^*$

$$\begin{aligned}
&\exists x[first(x) \wedge Q_b(x)] \wedge \exists x[last(x) \wedge Q_b(y)] \\
\exists U \exists V[\forall x U(x) &\implies (Q_b(x) \wedge \exists y[S(y,x) \wedge Q_a(y)] \wedge \exists y[S(x,y) \wedge V(y)]) \\
\wedge(\forall x V(x) &\implies (Q_b(x) \wedge \exists y[S(y,x) \wedge U(y)] \wedge \exists y[S(x,y) \wedge \neg U(y) \wedge \neg V(y) \wedge Q_b(y)]) \\
\wedge \forall x[Q_a(x) &\implies \exists y S(x,y) \wedge (Q_a(y) \vee U(y))] \\
\wedge \forall x[Q_b(x) &\implies ([U(x) \wedge \neg V(x)] \vee [V(x) \wedge \neg U(x)] \vee \\
&\quad [\neg U(x) \wedge \neg V(x) \wedge (\forall y S(y,x) \implies V(y)) \wedge (\forall y S(x,y) \implies Q_a(y))])])]
\end{aligned}$$

Language is $b(aa^*b_U b_V b)^*$.
Line1: First and Last letter is b
Line2: U is the set of all b's that are preceeded by an a and succeeded by a letter in V
Line3: V is the set of all b's that are preceeded by a letter in U and succeeded by a letter that is neither in U, nor in V, but a b
Line4: Every a is followed by another a or a letter in U
Line5: Every b is either only in U, or only in V or,
Line6: neither in U, nor in V, in which case, if it has a predecessor, that predecessor must be in V, and if it has a successor, that successor must be an a.