Tutorial 5

1. We saw five steps which are crucial in designing dynamic programming algorithms. Give dynamic programming solutions to the following string related problems by giving these five steps for each.

   (a) Given two strings $u, v \in \{0,1,2,3\}^n$, find the longest common subsequence between these two strings.

   (b) Given a string $u \in [M]^n$, where $M \in \mathbb{N}$, find the longest increasing subsequence in u.

   (c) Let $\mathcal{C} : \Sigma \to \{0,1\}^*$ be a not necessarily prefix-free encoding of English text, where Σ represents the alphabet of English language. Suppose this encoding table is given to you and to look up the code for any letter in the table takes time $O(1)$.

   Given a string $u \in \{0,1\}^n$, find the number of possible valid decoded words of u.

   (d) Given two strings $u, v \in \{0,1\}^n$ compute the minimum number of changes needed to be made to u so that it becomes v. (The kind of changes allowed are only flip a bit from 0 to 1 or vice-a-versa.)

   (e) Given two strings $u, v \in [M]^n$, where $M \in \mathbb{N}$, compute the minimum cost of converting u to v. The kind of change allowed is updating a letter u_i with some other letter from the alphabet $[M]$. Suppose $u_i = m$ and it is made m' then the cost of this change is $|m - m'|$, i.e. the absolute value of the difference between m and m'.

   (f) Suppose you are given two strings $u \in [M]^n$ and $v \in [M]^N$, where $M, n, N \in \mathbb{N}$ and $n \leq N$. Compute the minimum cost of converting u to v. The kind of changes allowed are: updating a letter u_i with some other letter or dropping a letter. The cost of any change is 1.