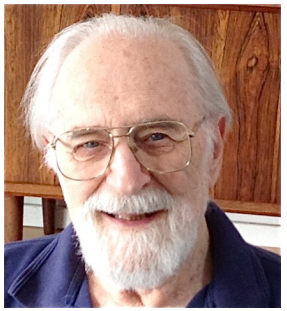
Logic Optimization: Karnaugh Map



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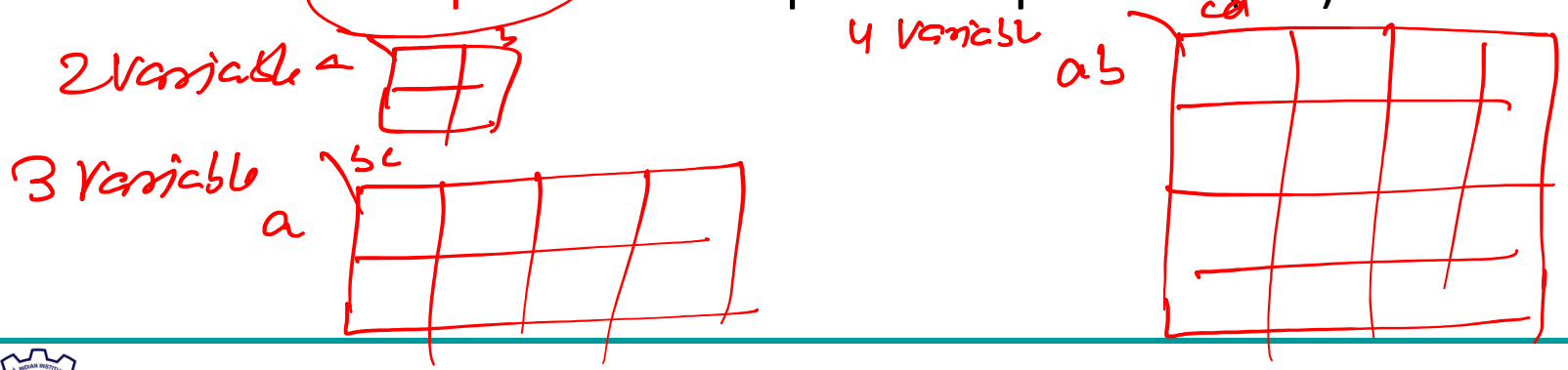
CS-226: Digital Logic Design

Lecture 15-B: 18 February 2021

CADSL

Minimum Sum of Products (MSOP)

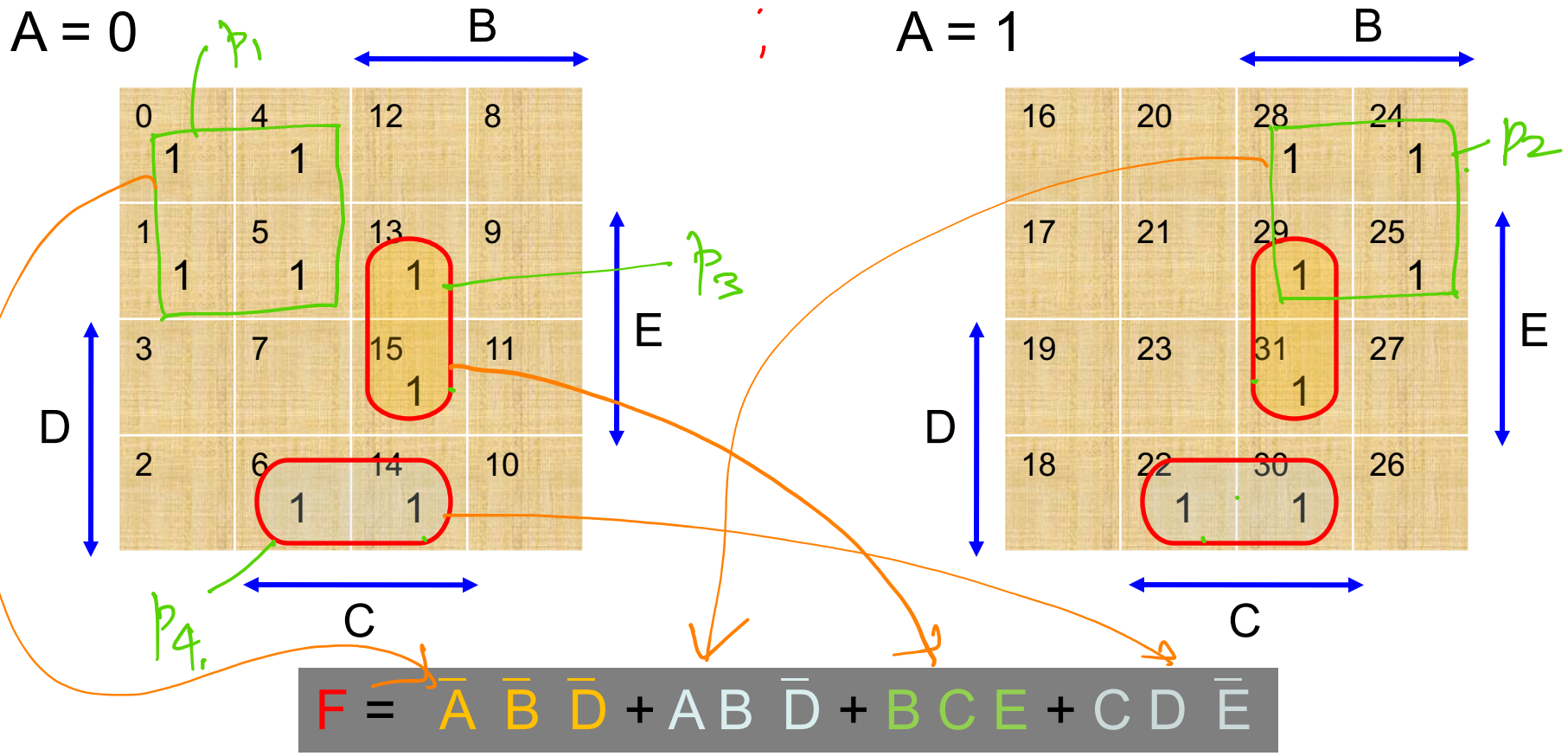
- Identify all prime implicants (PI) by letting minterms and implicants grow.
- Construct MSOP with PI only :
  - Cover all minterms
  - Use only essential prime implicants (EPI)
  - Use no redundant prime implicant (RPI)
  - Use cheaper selective prime implicants (SPI)



Five-Variable Function

- F(A,B,C,D,E)

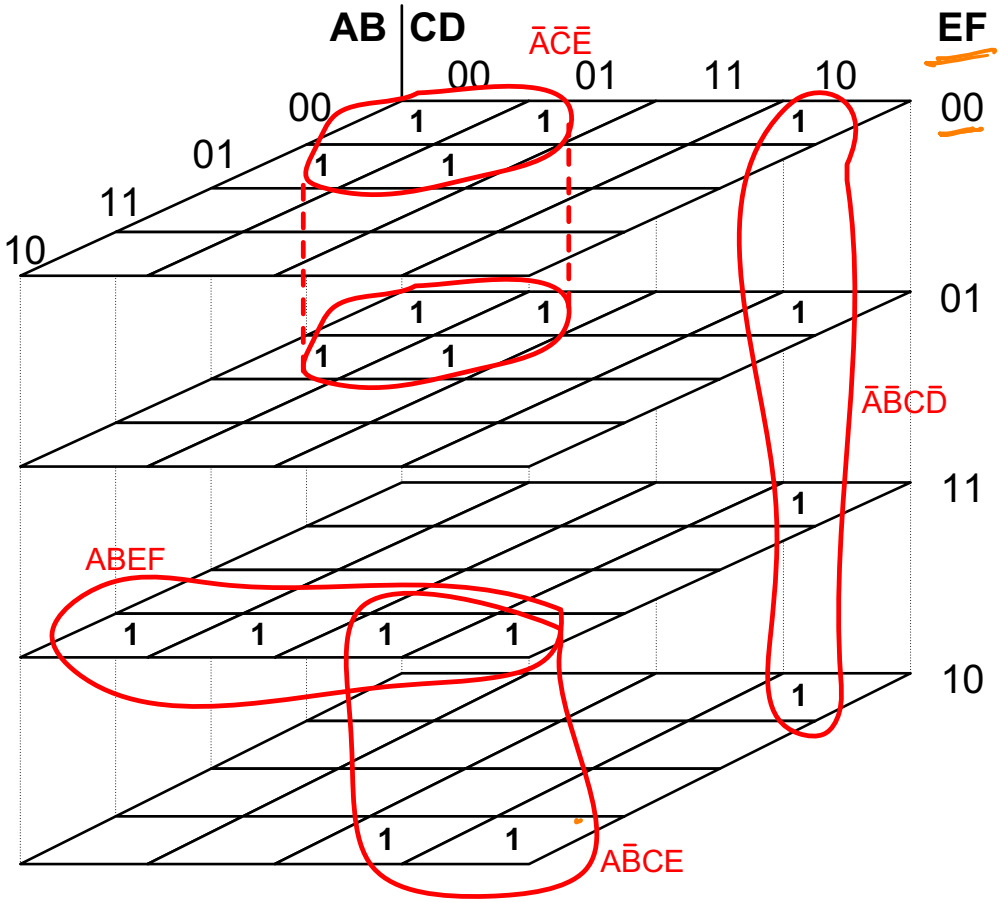
  $= \sum m(0,1,4,5,6,13,14,15,22,24,25,28,29,30,31)$



Why Not More Than Five Variables?

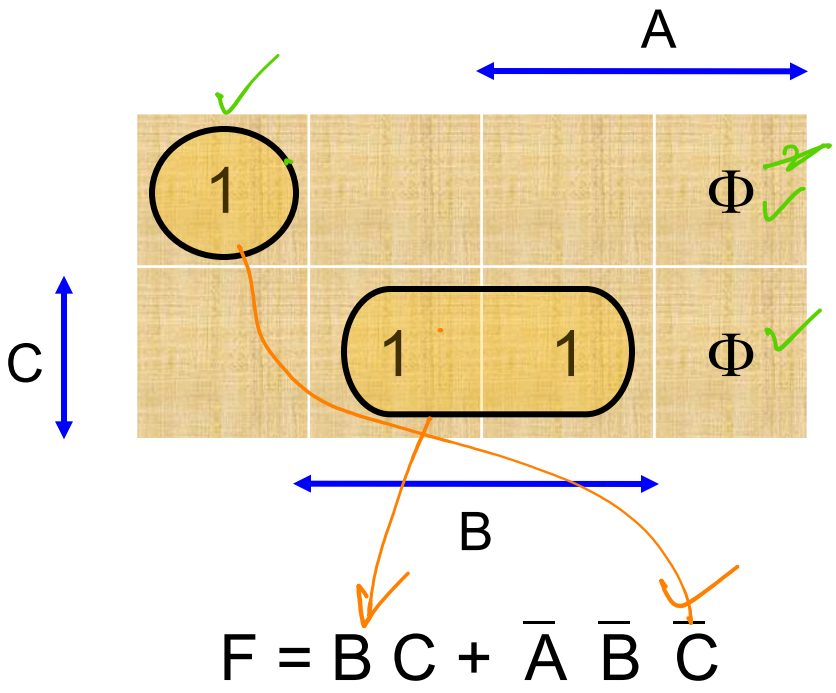
- Too hard to visualize in 3+ dimensions
- Systematic approach: "Tabular Methods"
  - Used in computer programs
- Why K-Maps at all?
  - Faster for quick optimizations
  - Understand Boolean logic and HW design
  - Design with simplification in mind

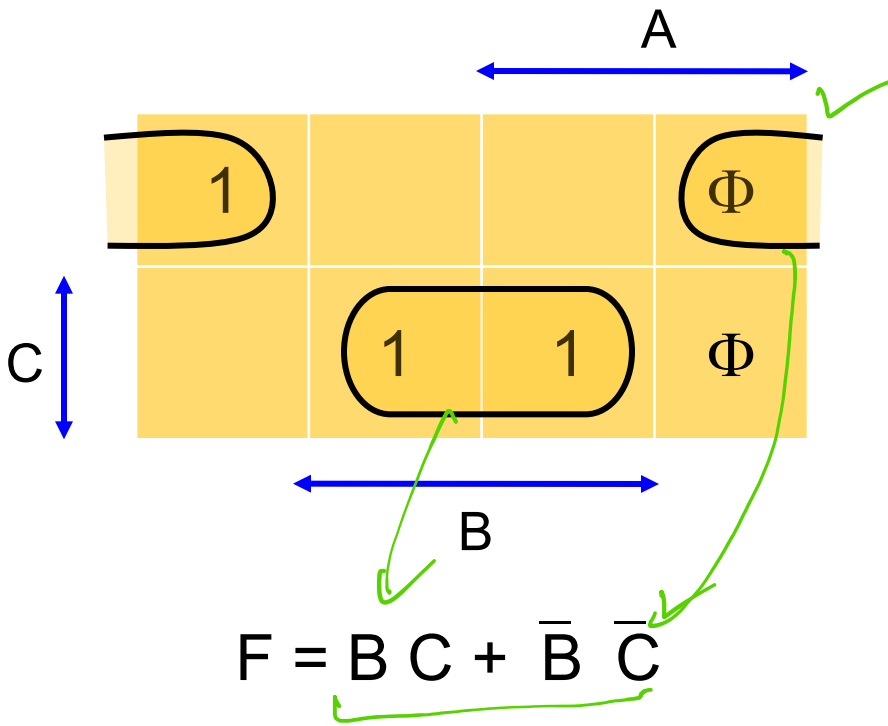
Six-Variable K-Map



Functions with Don't Care Minterms

- F(A,B,C) = ∑m(0,3,7) + d(4,5)
- Include don't care minterms when beneficial.

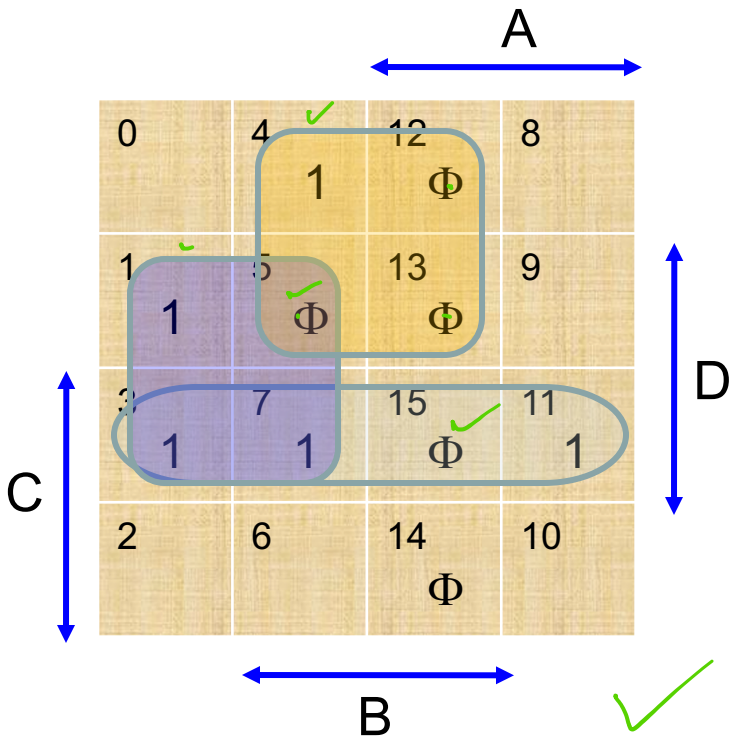




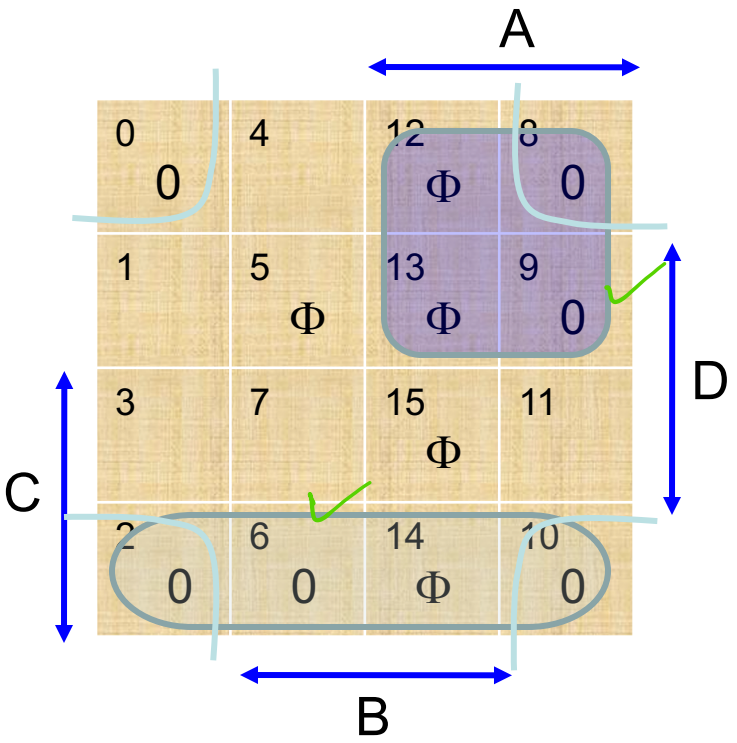
Minimized SOP and POS

- $F(A,B,C,D) = \sum m(1,3,4,7,11) + d(5,12,13,14,15)$

  $= \prod M(0,2,6,8,9,10)\ D(5,12,13,14,15)$



$F = B\bar{C} + \bar{A}D + CD$



$\bar{F} = \bar{B}\bar{D} + C\bar{D} + A\bar{C}$

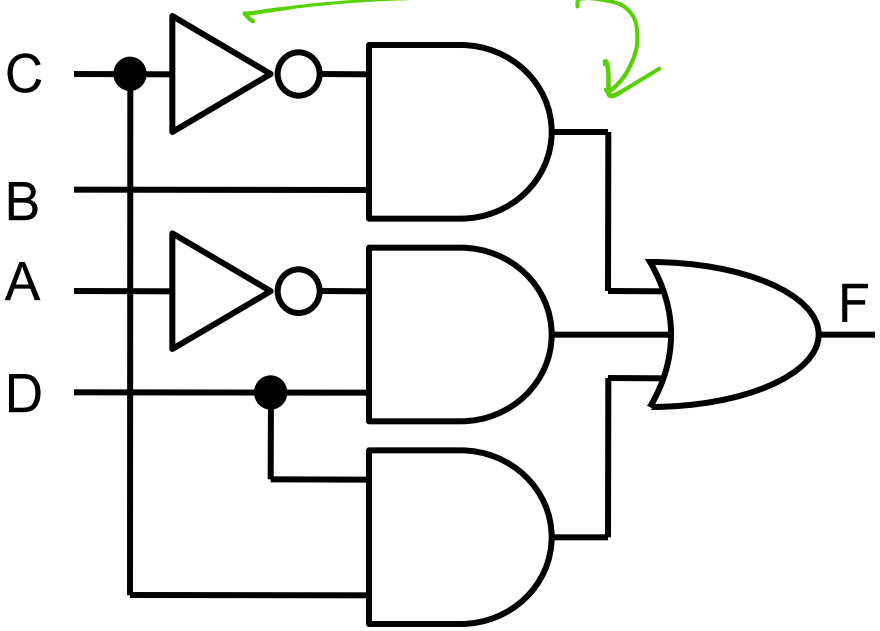
$F = (B + D)(\bar{C} + D)(\bar{A} + C)$

SOP and POS Circuits

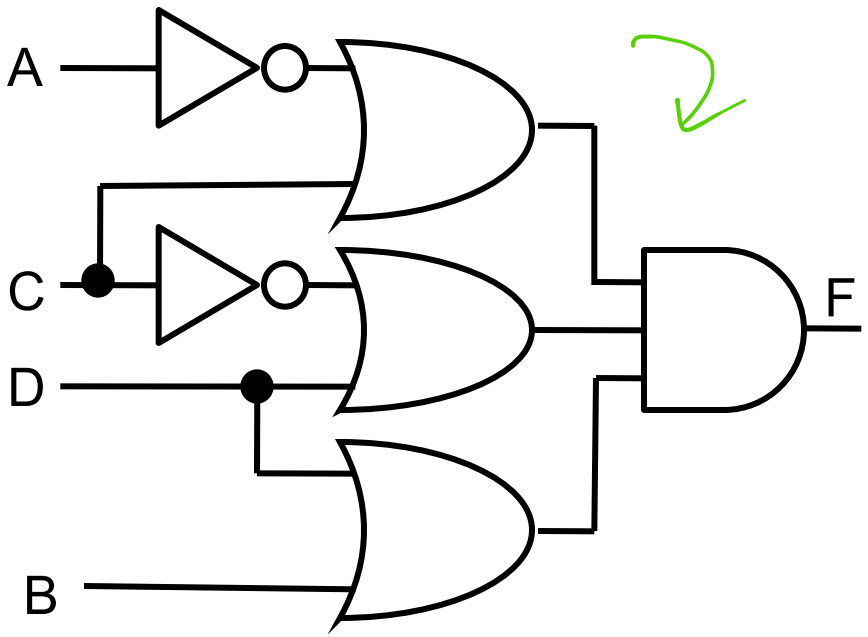
- F(A,B,C,D) = $\sum$ m(1,3,4,7,11)+d(5,12,13,14,15)
  = Π M(0,2,6,8,9,10) D(5,12,13,14,15)

MSOP min MPOS

F = B $\bar{C}$ + $\bar{A}$D + C D



F = (B + D)($\bar{C}$ + D)($\bar{A}$ + C)



Multiple-Output Minimization

Multiple-Output Minimization

Inputs				Outputs	
A	B	C	D	F1	F2
0	0	0	0	0	0
0	0	0	1	1	0
0	0	1	0	0	0
0	0	1	1	0	0
0	1	0	0	0	0
0	1	0	1	1	0
0	1	1	0	0	0
0	1	1	1	1	1
1	0	0	0	0	1
1	0	0	1	1	1
1	0	1	0	0	1
1	0	1	1	0	1
1	1	0	0	0	0
1	1	0	1	1	1
1	1	1	0	0	0
1	1	1	1	1	1

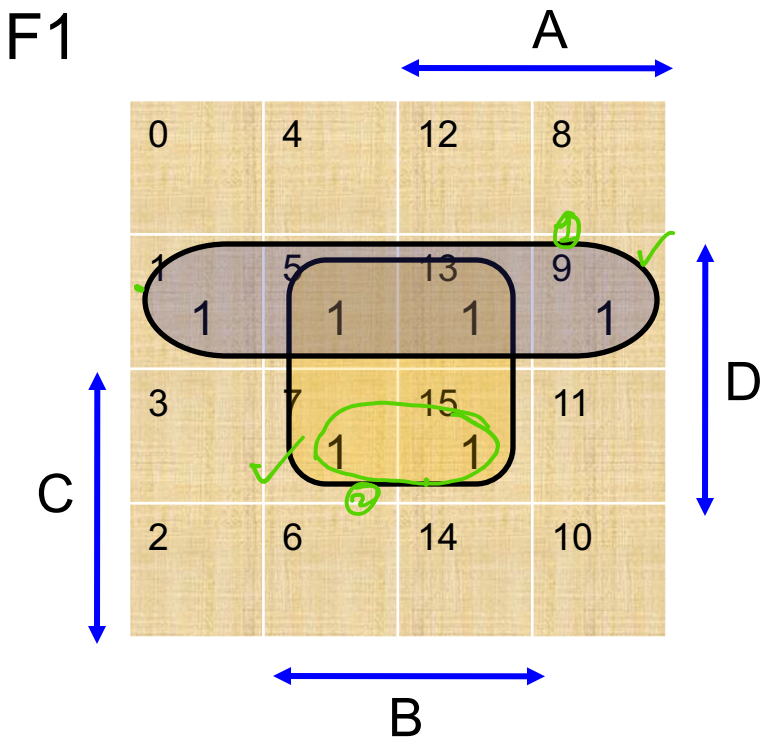
$F_1(A,B,C,D)$

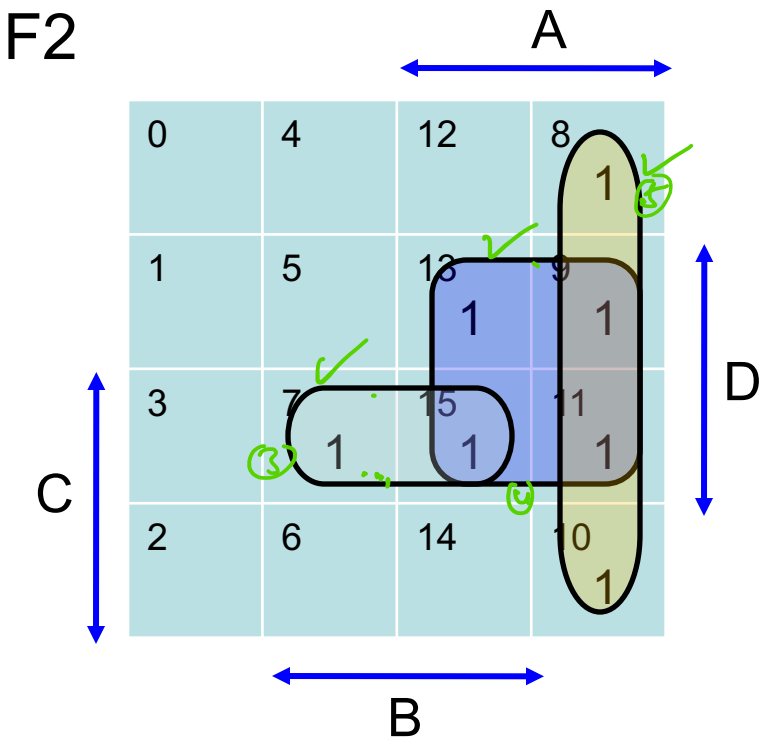
$F_2(A,B,C,D)$

Individual Output Minimization

Need five products.

11 literals

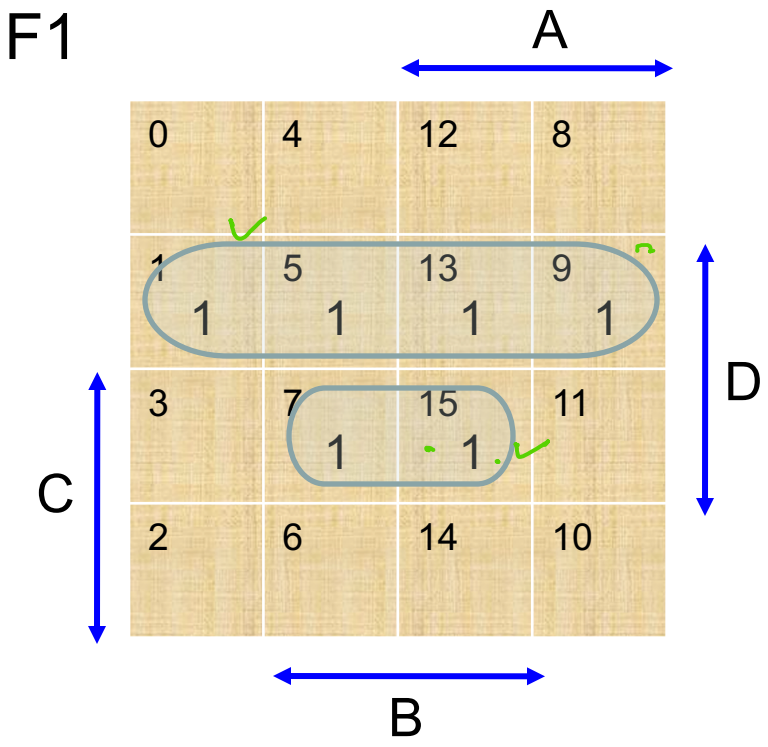


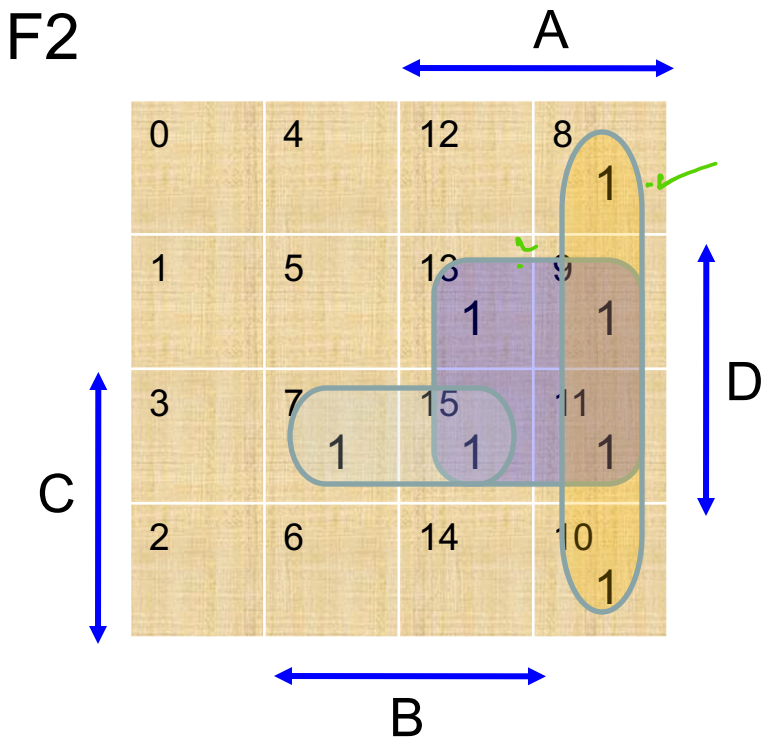


Global Minimization

Need four products.

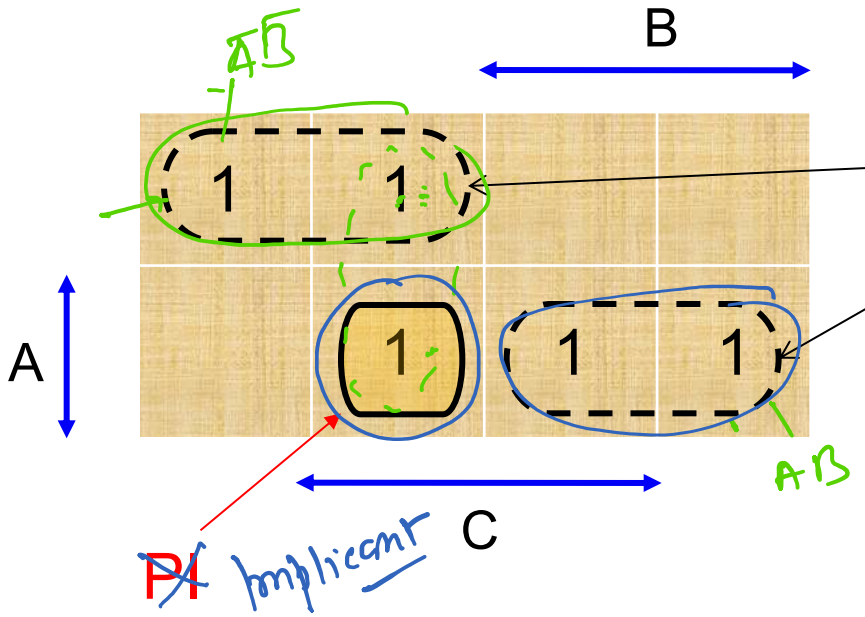
9 literals





Reed Muller Representation

Reed Muller Representation



MSOP:

$\bar{A}\bar{B} + AB + \bar{B}C$

$= \bar{A}\bar{B} + AB + A\bar{B}C$

disjoint SOP

$+ \rightarrow \oplus$

$F = \bar{A}\bar{B} \oplus AB \oplus A\bar{B}C$

$F: (1 \oplus A)(B \oplus 1) \oplus A \cdot B \oplus A \cdot (B \oplus 1) \cdot C$

$F = 1 \oplus A \oplus B \oplus AC \oplus ABC$

Reed Muller Representation

$(A \oplus 1)(B \oplus 1) \oplus AB \oplus AC(B \oplus 1)$

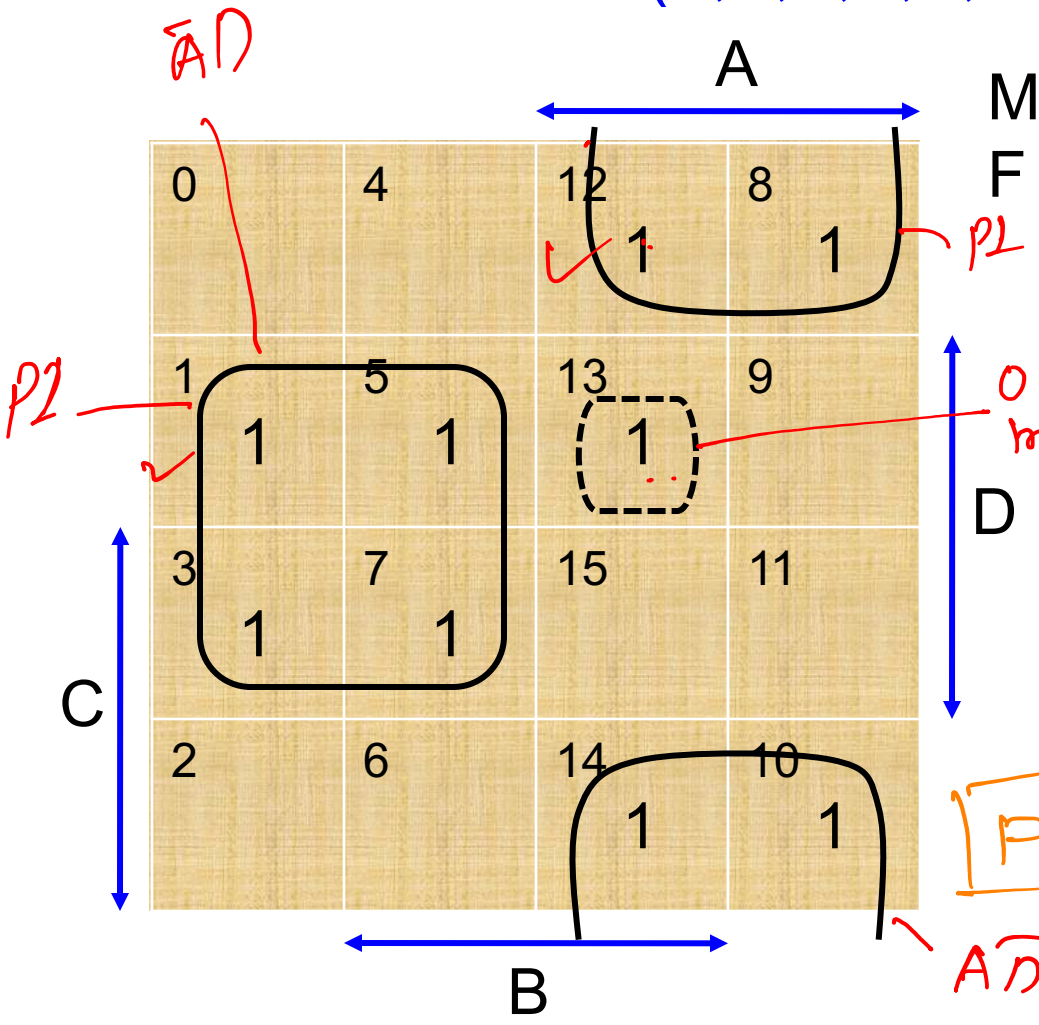
$= 1 \oplus A \oplus B \oplus \cancel{AB} \oplus \cancel{AB} \oplus ACB \oplus AC$

$F = 1 \oplus A \oplus B \oplus AC \oplus ABC$

RM Representation

Reed Muller Representation

$F=\sum m(1,3,5,7,8,10,12,13,14)$



MSOP:

$F = \bar{A}D + A\bar{D} + AB\bar{C}D$

$F = \bar{A}D \oplus A\bar{D} \oplus AB\bar{C}D$

$F = A \oplus D \oplus ABD \oplus ABCD$

R-M Canonical.

Reed Muller Representation

$F = \bar{A}D \oplus A\bar{D} \oplus ABCD$

$= (1 \oplus A) \cdot D \oplus A \cdot (1 \oplus D) \oplus ABD(1 \oplus C)$

$= D \oplus AD \oplus A \oplus AD \oplus ABD \oplus ABCD$

$= F = A \oplus D \oplus ABD \oplus ABCD$

RM expression

Thank You