

Satisfiability Problem

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CS-226: Digital Logic Design



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CADSL

SAT Problem Definition

Given a CNF formula, f :

- A set of variables, V (a, b, c)
- Conjunction of clauses (C₁, C₂, C₃)
- Each clause: disjunction of literals over V
OR

$$\checkmark \quad \checkmark \quad \checkmark \\ C_1 \cdot C_2 \cdot C_3$$

Does there exist an assignment of Boolean values to the variables, V which sets at least one literal in each clause to '1' ?

Example :

$$(a + b + \bar{c}) \quad (\bar{a} + c) \quad (a + \bar{b} + \bar{c}) \\ C_1 \qquad \qquad \qquad C_2 \qquad \qquad \qquad C_3$$

$$\boxed{a = b = c = 1} \\ \checkmark \quad a = 1 \quad \& \quad b = 1 \quad \& \quad c = 1$$



DPLL Algorithm for SAT

[Davis, Putnam, Logemann, Loveland 1960,62]

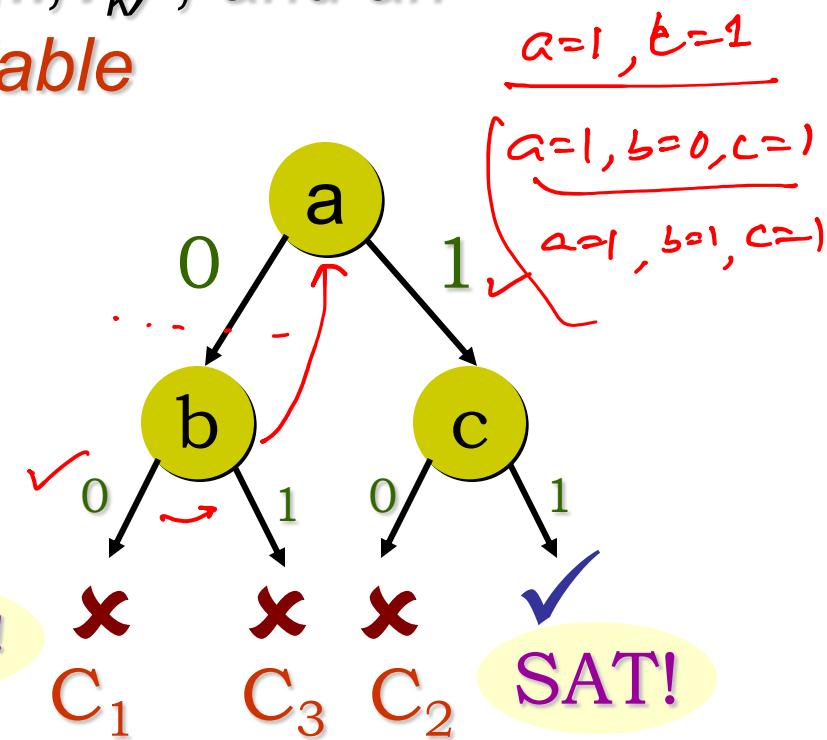
Given : CNF formula $f(v_1, v_2, \dots, v_k)$, and an ordering function Next Variable

Example :

$\varphi = (a + b)(\bar{a} + c)(a + \bar{b})$

The formula $\varphi = (a + b)(\bar{a} + c)(a + \bar{b})$ is shown in yellow boxes labeled C₁, C₂, and C₃. Each clause has a red checkmark above it. Below each clause is a red bracket underlining the variables, and below each bracket is a red label C₁, C₂, or C₃. A yellow oval labeled "CONFLICT!" is positioned below the clauses.

2^{n} possibilities



DPLL Algorithm

Simple algorithm

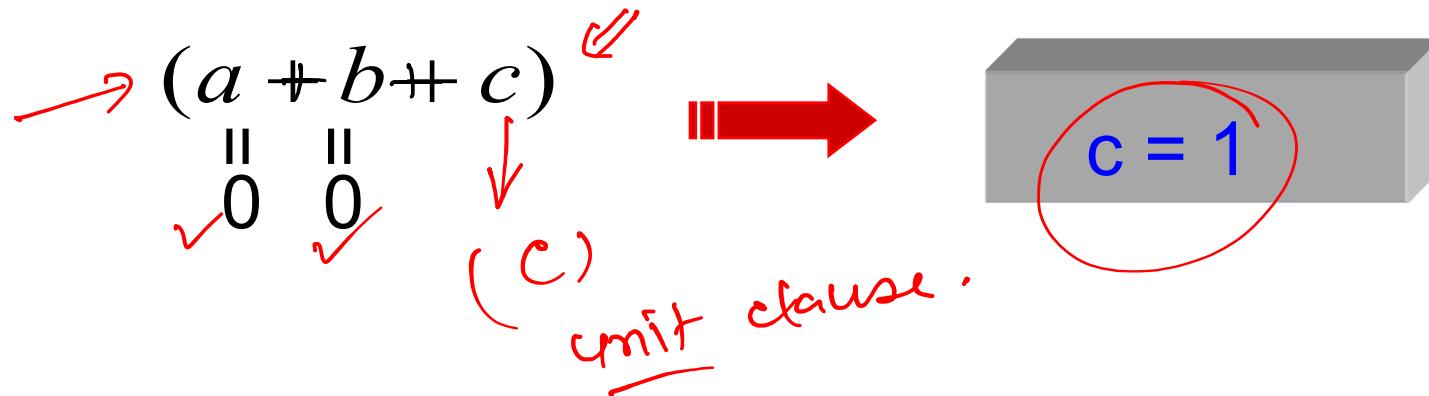
Not scalable

1960



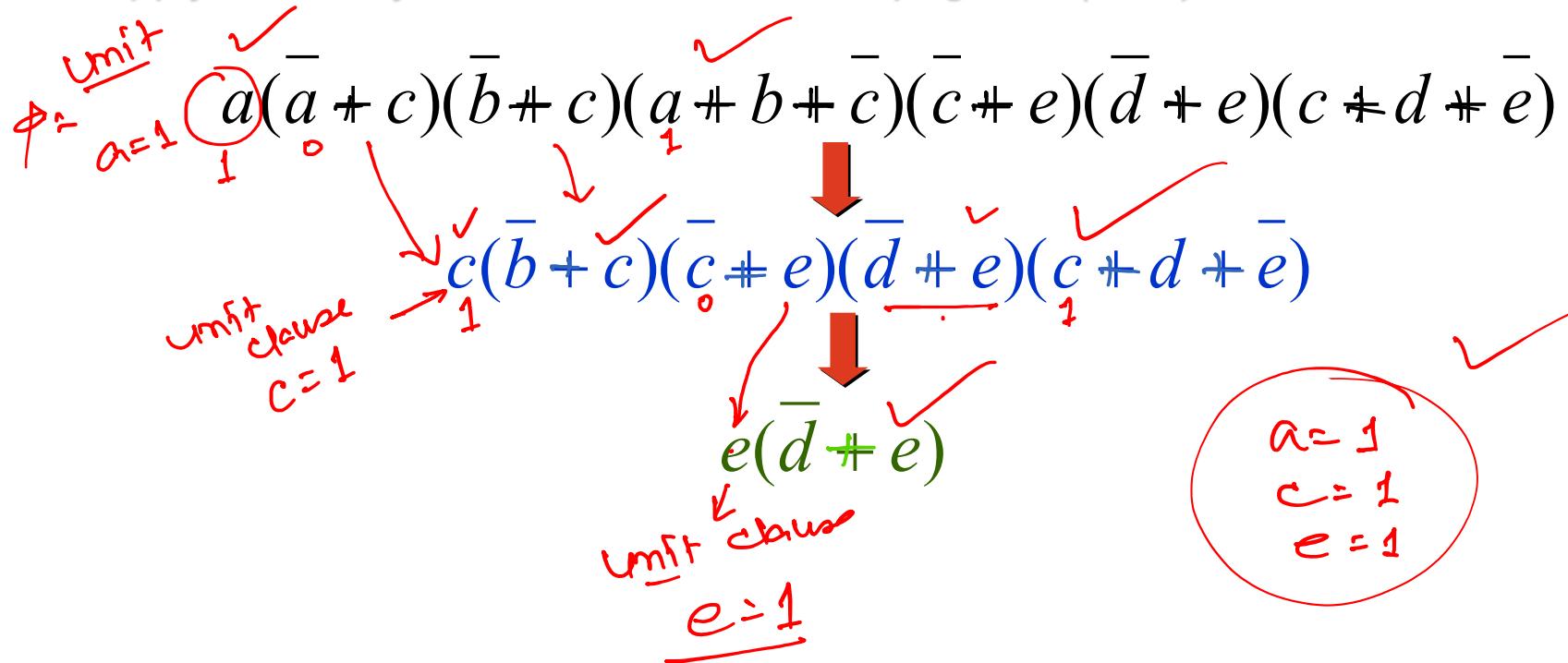
DPLL Algorithm: Unit Clause Rule

Rule: Assign to *true* any single literal clauses.



DPLL Algorithm: BCP

Apply Iteratively: *Boolean Constraint Propagation (BCP)*



Pure Literal Rule ✓

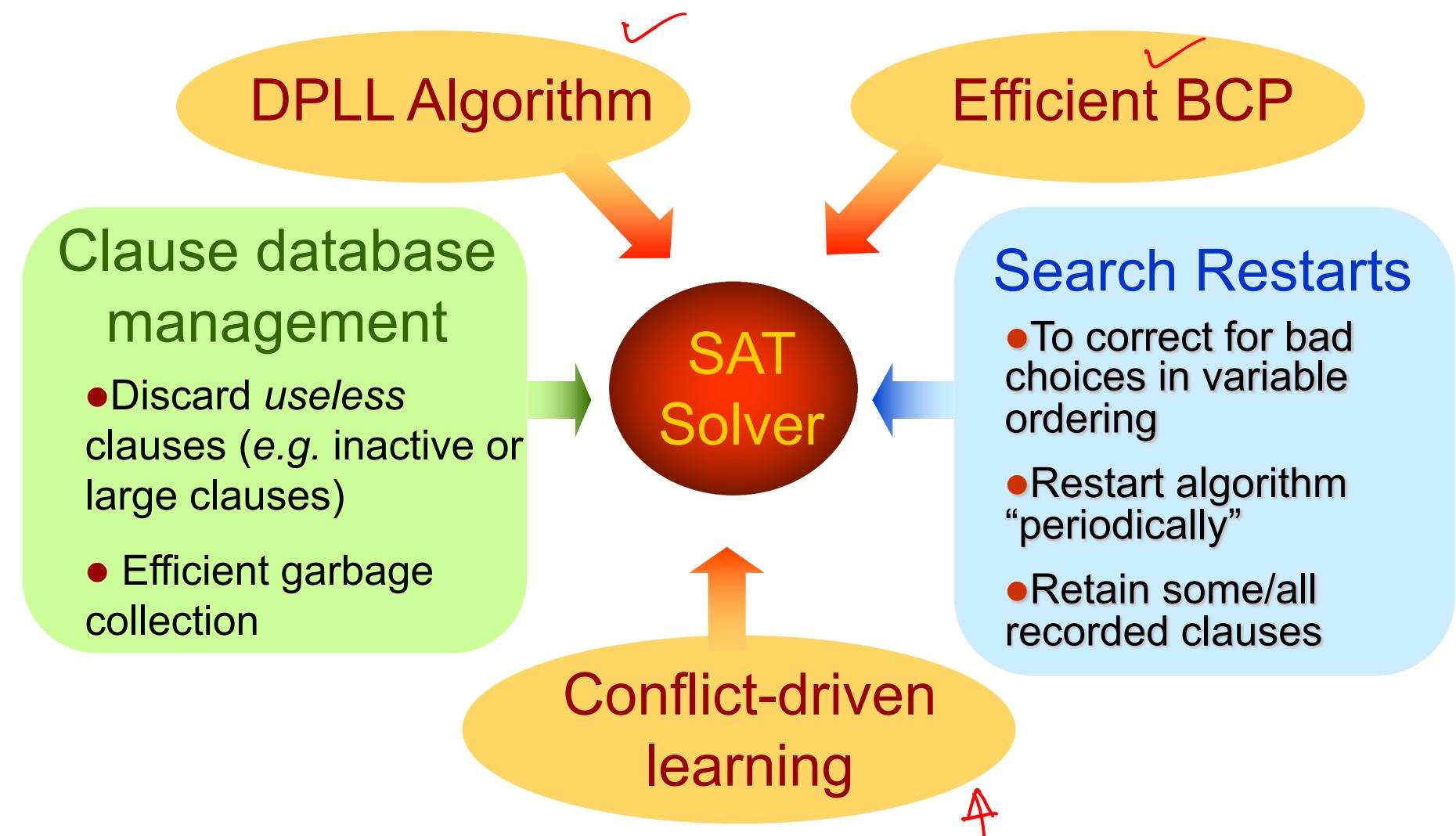
- A variable is pure if its literals are either all positive or all negative✓
- Satisfiability of a formula is unaffected by assigning pure variables the values that satisfy all the clauses containing them

$$\varphi = (\underline{a} + c)(b + \underline{c})(b + \neg d)(\neg a + \neg b + d)$$

- Set c to 1; if φ becomes unsatisfiable, then it is also unsatisfiable when c is set to 0.



Anatomy of a Modern SAT Solver



Modern SAT Solvers

$$\text{SAT} \quad \phi - f(a, b, c) \leftarrow \text{POS}$$

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Thank You



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