CS 228 : Logic in Computer Science

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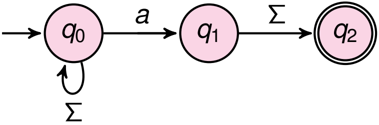
Recap

- Discussed determinism of DFAs : every word has a unique path in the DFA, starting from any state
- In particular, every word has a unique path in the DFA starting from the start state
- If this path leads to a good state, the word is accepted, else it is rejected.
- Looked at closure properties : complementation, intersection, union.
- Looked at proof techniques for correctness of a constructed DFA.

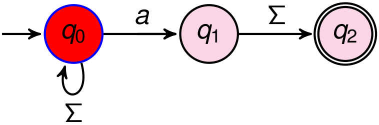
Moving on to Non-determinism

- We looked at DFA
- Showed closure under union, intersection and complementation
- Now we look at a more relaxed model, which is as good as a DFA

The Comfort of Non-determinism

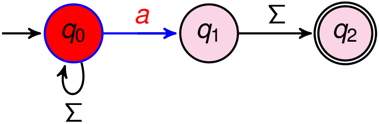


- Assume we relax the condition on transitions, and allow
  - $\delta : Q \times \Sigma \rightarrow 2^Q$
  - $\delta(q_0, a) = \{q_0, q_1\}, \delta(q_2, a) = \emptyset$
  - Is *aabb* accepted?



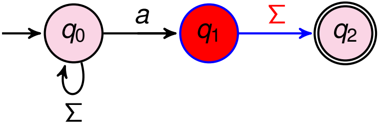
One run of *aabb*

Is *aabb* accepted?



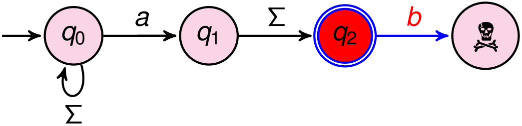
One run of *aabb*

Is *aabb* accepted?



One run of *aabb*

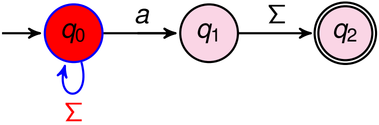
Is *aabb* accepted?



- A non-accepting run for *aabb*

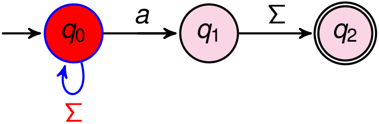
Another run of *aaab*

Is *aaab* accepted?



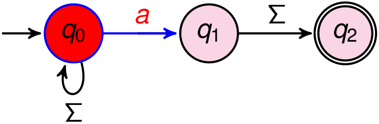
Another run of *aaab*

Is *aaab* accepted?



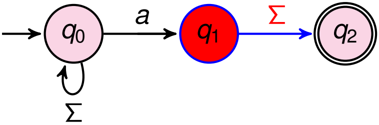
Another run of *aaab*

Is *aaab* accepted?



Another run of *aaab*

Is *aaab* accepted?

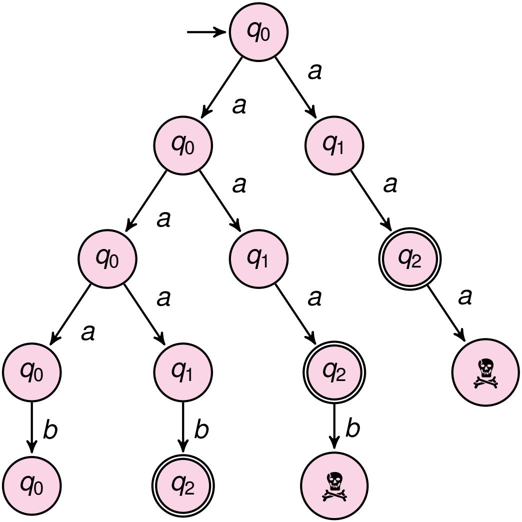


- An accepting run for *aaab*

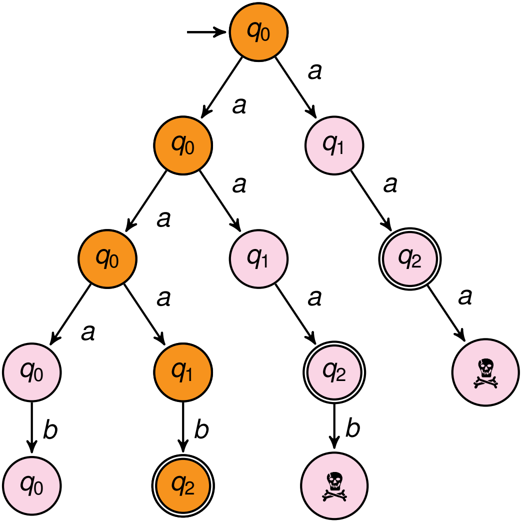
Nondeterministic Finite Automata(NFA)

- $N = (Q, \Sigma, \delta, Q_0, F)$
  - Q is a finite set of states
  - $Q_0 \subseteq Q$ is the set of initial states
  - $\delta : Q \times \Sigma \to 2^Q$ is the transition function
  - $F \subseteq Q$ is the set of final states
- Acceptance condition : A word w is accepted iff it has atleast one accepting path

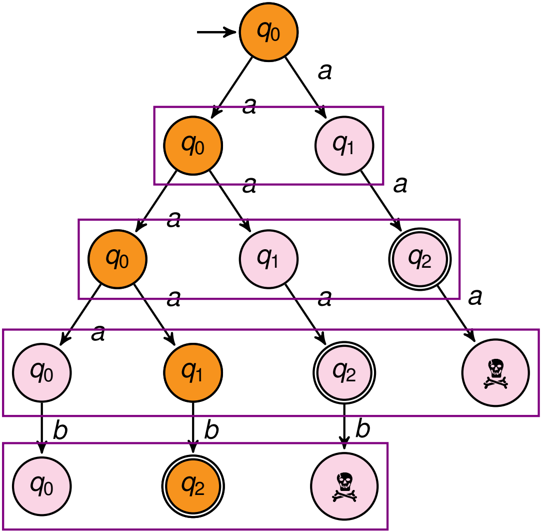
Run Tree of *aaab*



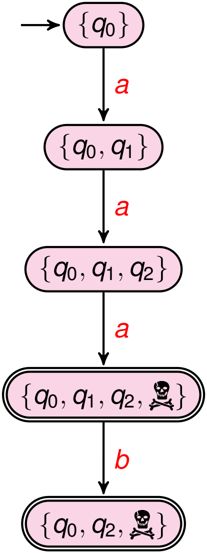
Run Tree of *aaab*



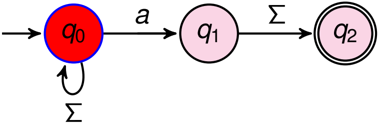
Run Tree of *aaab*

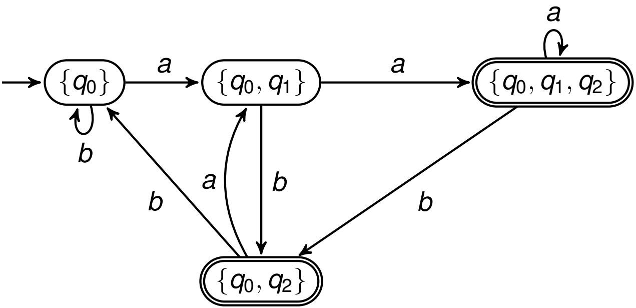


The Single Run



An Example





NFA and DFA

- Any DFA is also an NFA
- Any NFA can be converted into a language equivalent DFA
  - Combine all the runs of w in the NFA into a single run in the DFA
  - Combine states occurring in various runs to obtain a set of states
  - A set of states evolves into another set of states
  - Use $\delta : Q \times \Sigma \rightarrow 2^Q$, obtain $\Delta : 2^Q \times \Sigma \rightarrow 2^Q$
  - Δ is an extension of δ
  - Accept if the obtained set of states contains a final state

NFA and DFA

Given NFA $N = (Q, \Sigma, Q_0, \delta, F)$, obtain the DFA $D = (2^Q, \Sigma, Q_0, \Delta, F')$

- $\Delta : 2^Q \times \Sigma \to 2^Q$ is defined by $\Delta(A, a) = \bigcup_{q \in A} \delta(q, a)$
- $F' = \{S \in 2^Q \mid S \cap F \neq \emptyset\}$

Note that $\hat{\delta}(A, a) = \bigcup_{q \in A} \delta(q, a) = \Delta(A, a)$
Show that

- $\hat{\Delta} : 2^Q \times \Sigma^* \to 2^Q$ is same as $\hat{\delta} : 2^Q \times \Sigma^* \to 2^Q$ (recall $\delta : Q \times \Sigma \to 2^Q$)
- $\hat{\Delta}(A, xa) = \Delta(\hat{\Delta}(A, x), a) = \bigcup_{q \in \hat{\Delta}(A,x)} \delta(q, a)$
- $\hat{\delta}(A, xa) = \bigcup_{q \in \hat{\delta}(A,x)} \delta(q, a)$

NFA = DFA

$$x \in L(D) \leftrightarrow \hat{\Delta}(Q_0, x) \in F'$$

$$\leftrightarrow$$

$$\hat{\delta}(Q_0, x) \in F'$$

$$\leftrightarrow$$

$$\hat{\delta}(Q_0, x) \cap F \neq \emptyset$$

$$\leftrightarrow$$

$$x \in L(N)$$

Regularity

A language L is regular iff there exists an NFA A such that $L = L(A)$