CS 218 Design and Analysis of Algorithms

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Module 4: Coping with NP-hardness

Approximation Algorithm for Vertex Cover

We will present an approximation algorithm for the vertex cover problem.

Decision version.

Given: undirected graph $G = (V, E)$
Find: $C \subseteq V$ such that for every $e = (u, v) \in E$, either $u \in C$ of $v \in C$.

Optimization version.

Given: undirected graph $G = (V, E)$
Find: the smallest sized $C \subseteq V$ such that for every $e = (u, v) \in E$ either $u \in C$ of $v \in C$.

A greedy strategy for Vertex Cover

A possible greedy algorithm

Set $C \leftarrow \varnothing$ and $E' \leftarrow E$.
while $E' \neq \varnothing$ **do**
    Consider $e = (u, v) \in E'$
    if $u \notin V$ and $v \notin V$ **then**
        Set $C \leftarrow C \cup \{u, v\}$
        Remove all the edges incident on either u or v from E'.
    end if
end while
Output C.

A greedy strategy for Vertex Cover

A possible greedy algorithm

Set $C \leftarrow \emptyset$, $A \leftarrow \emptyset$ and $E' \leftarrow E$.

while $E' \neq \emptyset$ **do**

Consider any arbitrary edge $e = (u, v) \in E'$.

Set $A \leftarrow A \cup \{e\}$.

if $u \notin V$ and $v \notin V$ **then**

Note that this if condition is always true.

Set $C \leftarrow C \cup \{u, v\}$.

Remove all the edges incident on either u or v from E'.

end if

end while

Output C.

The greedy algorithm gives a 2-approximation

We will use a similar strategy as before.

Let C^* be the optimal vertex cover.

Lower-bounding $|C^*|$.

$|C^*| \geq |A|$.

As A is a matching.

Upper-bounding $|C|$ using $|A|$.

$|C| = 2|A|$.

This finishes the proof.