SAT Solvers

Virendra Singh
Professor
Computer Architecture and Dependable Systems Lab
Department of Electrical Engineering
Indian Institute of Technology Bombay
http://www.ee.iitb.ac.in/~viren/
E-mail: viren@ee.iitb.ac.in

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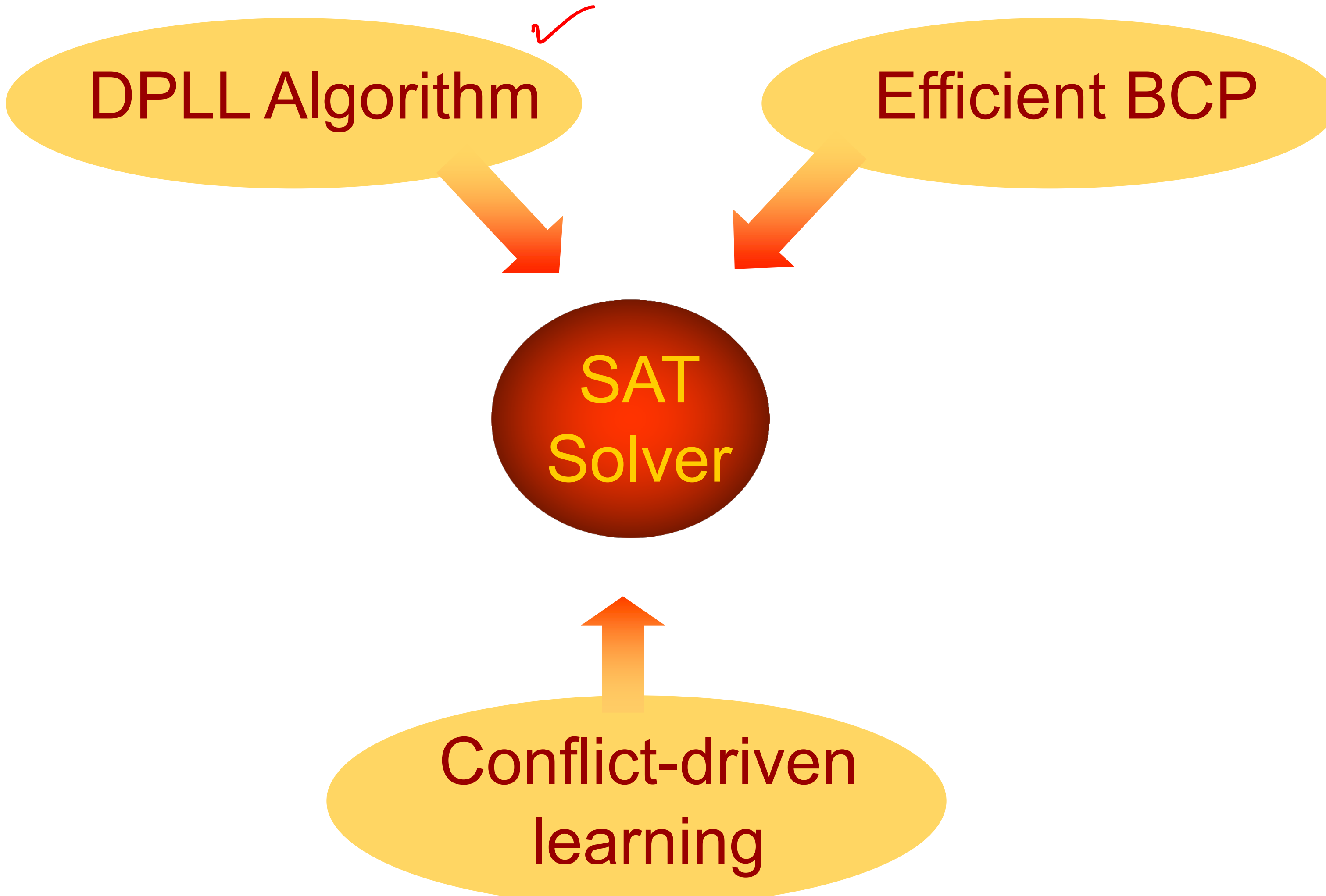
CADSL

Anatomy of a Modern SAT Solver



Stallmarck's Method (SM) in CNF

- Recursive application of the branch-merge rule to each variable with the goal of identifying common conclusions

$$\varphi = (a + b)(\neg a + c)\,(\neg b + d)(\neg c + d)$$

Try $a = 0$: $(a = 0) \Rightarrow (b = 1) \Rightarrow (d = 1)$ $C(a = 0) = \{a = 0, b = 1, d = 1\}$

Try $a = 1$: $(a = 1) \Rightarrow (c = 1) \Rightarrow (d = 1)$ $C(a = 1) = \{a = 1, c = 1, d = 1\}$

$C(a = 0) \cap C(a = 1) = \{d = 1\}$

Any assignment to variable a implies $d = 1$.
Hence, $d = 1$ is a necessary assignment !

Recursion can be of arbitrary depth

Recursive Learning (RL) in CNF

- Recursive evaluation of clause satisfiability requirements for identifying common assignments

$$\varphi = (a + b)(\neg a + d)(\neg b + d)$$

Try $a = 1$: $(a = 1) \Rightarrow (d = 1)$ $C(a = 1) = \{a = 1, d = 1\}$

Try $b = 1$: $(b = 1) \Rightarrow (d = 1)$ $C(b = 1) = \{b = 1, d = 1\}$

$C(a = 1) \cap C(b = 1) = \{d = 1\}$

Every way of satisfying $(a + b)$ implies $d = 1$.
Hence, $d = 1$ is a necessary assignment **!**

Recursion can be of arbitrary depth

Recursive Learning within DP

$\varphi = (a + b + c)(\neg a + d + e)\ (\neg b + d + c)$

$c = 0 \quad e = 0$

$d = 1$

Implications:

$(a = 1) \wedge (e = 0) \Rightarrow (d = 1)$

$(b = 1) \wedge (c = 0) \Rightarrow (d = 1)$

consensus

$(b + c + e + d)$

consensus

$(c + e + d)$

$(c = 0) \wedge ((e = 0) \wedge (c = 0)) \Rightarrow (d = 1)$

$\bar{c} \cdot \bar{e} \rightarrow d$

$(\overline{\bar{c} \cdot \bar{e}} + d) = (c + e + d)$

Clausal form:

$(c + e + d)$	Unit clause !

Clause provides explanation for necessary assignment $d = 1$

SM vs. RL

- Both complete procedures for SAT
- Stallmarck's method:
  - hypothetic reasoning based on variables
- Recursive learning:
  - hypothetic reasoning based on clauses
- Both can be integrated into backtrack search algorithms

Local Search

- Repeat *M* times:
  - Randomly pick complete assignment
  - Repeat *K* times (and while exist unsatisfied clauses):
    - Flip variable that will satisfy largest number of unsat clauses

$\varphi = (a + b)(\neg a + c)\ (\neg b + d)(\neg c + d)$ Pick random assignment

a=1, b=1 c=1 d=0

$\varphi = (a + b)(\neg a + c)\ (\neg b + d)(\neg c + d)$ Flip assignment on *d*

d=1

$\varphi = (a + b)(\neg a + c)\ (\neg b + d)(\neg c + d)$ Instance is satisfied !

Comparison

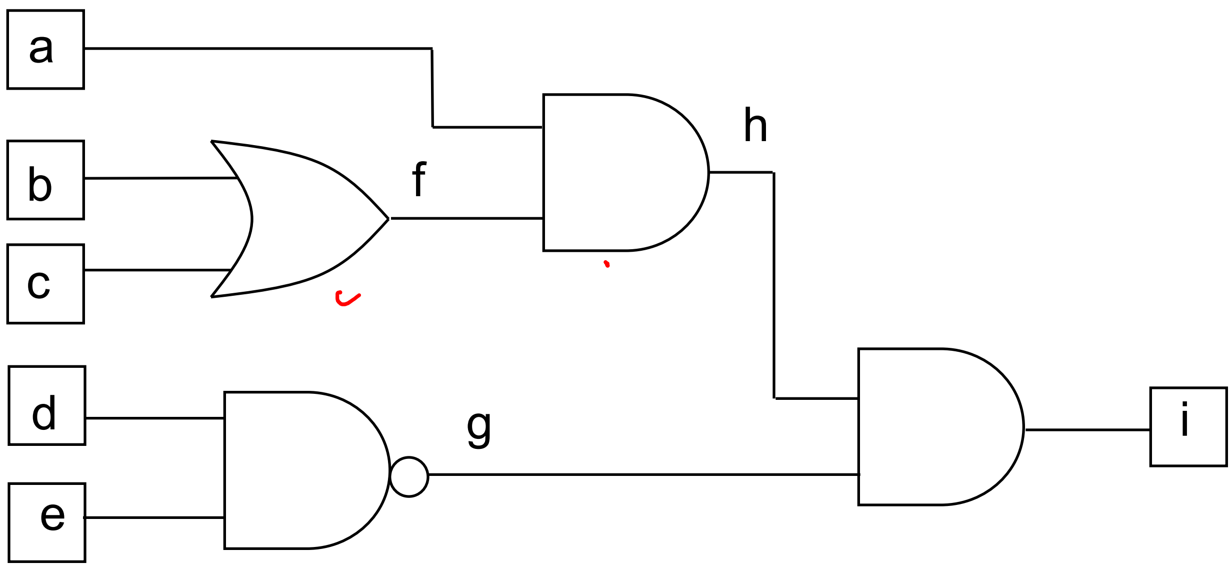
- Local search is incomplete
  - If instances are known to be SAT, local search can be competitive
- Stallmarck's Method (SM) and Recursive Learning (RL) are in general slow, though robust
  - SM and RL can derive too much unnecessary information
- For most EDA applications backtrack search (DP) is currently the most promising approach **!**
  - **Augmented with techniques for inferring new clauses/implicates (i.e. learning) !**

SAT Solvers Today

- Capacity:
  - Formulas upto a *million variables* and *3-4 million clauses* can be solved in *few hours*
  - Only for *structured instances e.g.* derived from real-world circuits & systems

- Tool offerings:
  - Public domain
    - GRASP : Univ. of Michigan
    - SATO: Univ. of Iowa
    - zChaff: Princeton University
    - BerkMin: Cadence Berkeley Labs.
  - Commercial
    - PROVER: Prover Technologies

Solving Circuit Problems as SAT

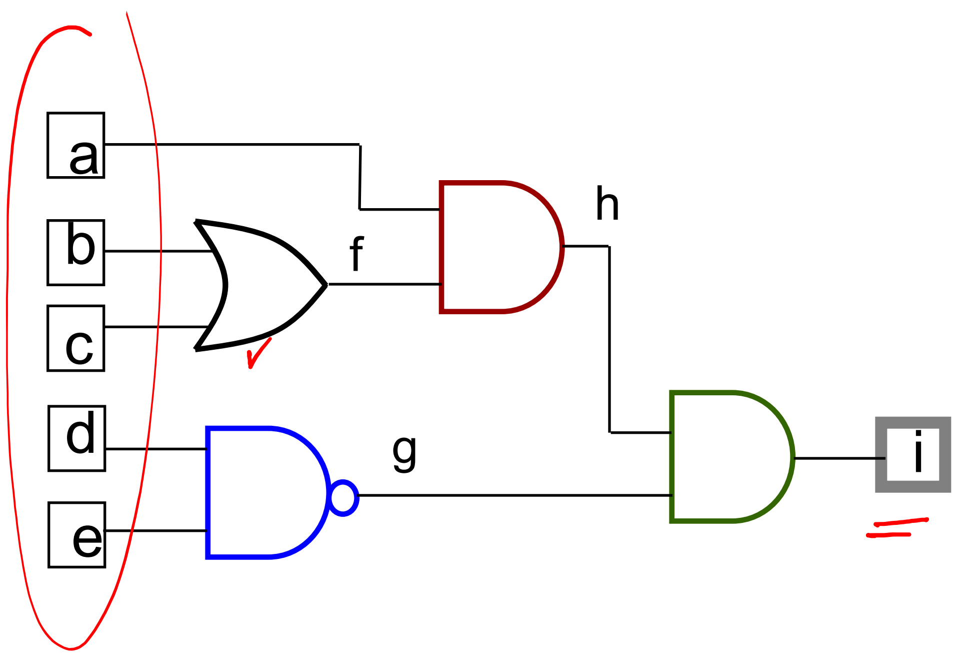


Input Vector Assignment ?

Primary Output ‘i’ to *1* ?

Solving circuit problems as SAT

- Set of clauses representing function of each gate
- Unit literal clause asserting output to *'1'*



$$(\bar{b} + f)(\bar{c} + f)(b + c + \bar{f})$$

$$(d + g)(e + g)(\bar{d} + \bar{e} + \bar{g})$$

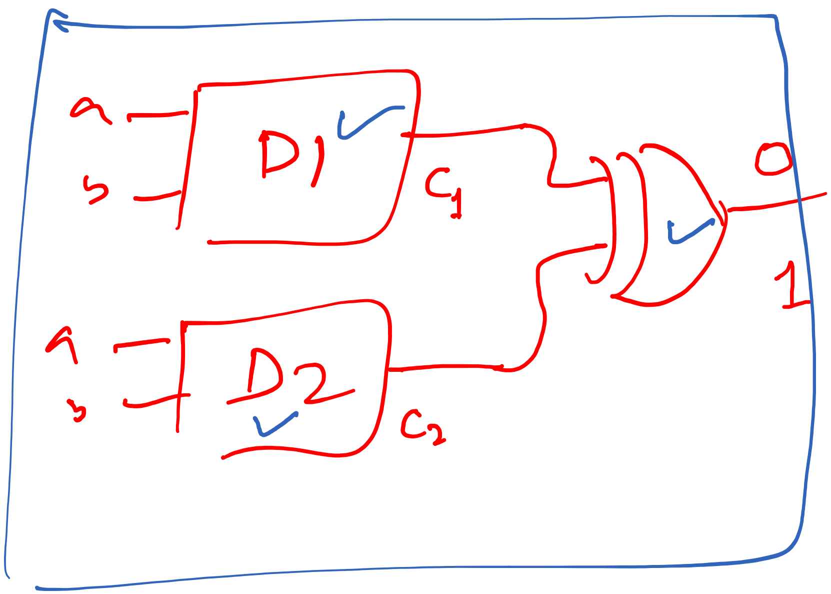
$$(a + \bar{h})(f + \bar{h})(\bar{a} + \bar{f} + h)$$

$$(h + \bar{i})(g + \bar{i})(\bar{h} + \bar{g} + i)$$

$$(i)$$

Application of SAT

VERIFICATION / EQUIVALENCE CHECKING.



$$D1 \overset{?}{\equiv} D2$$

$(\text{SAT exp of } D1) \cdot (\text{SAT exp } D2) \cdot (\text{SAT exp XOR}) = 1$

If input assignment exist

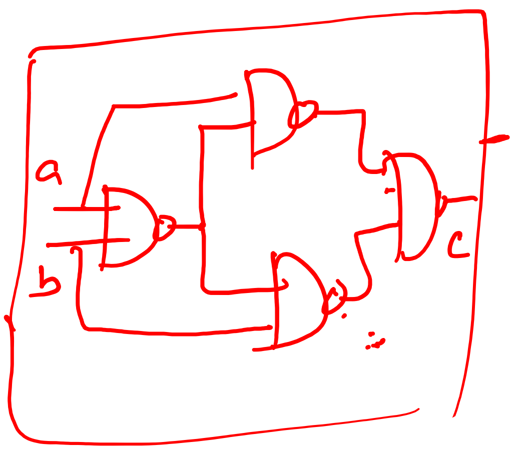
$D1 \not\equiv D2$

If does not exist $\boxed{D1 \equiv D2}$

Application of SAT

$C = a\bar{b}+\bar{a}b$

$\equiv$?



Thank You