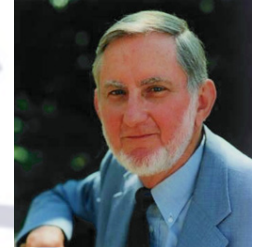
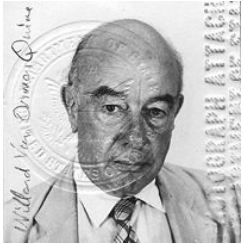


# Logic Optimization: Tabular Method (Quine-McCluskey)



---

Virendra Singh

Professor

Computer Architecture and Dependable Systems Lab

Department of Electrical Engineering

Indian Institute of Technology Bombay

<http://www.ee.iitb.ac.in/~viren/>

E-mail: [viren@ee.iitb.ac.in](mailto:viren@ee.iitb.ac.in)

*CS-226: Digital Logic Design*



---

*Lecture 16-B: 22 February 2021* **CADSL**

# Q-M Tabular Minimization

---

- Minimizes functions with many variables.
- Begin with minterms:
  - Step 1: **Tabulate minterms** in groups of increasing number of true variables.
  - Step 2: Conduct **linear searches** to identify all prime implicants (PI).
  - Step 3: Tabulate PI's vs. minterms to identify EPI's.
  - Step 4: Tabulate non-essential PI's vs. minterms not covered by EPI's. *Select* minimum number of PI's to cover all minterms.
- MSOP contains all EPI's and *selected* non-EPI's.



# Function with Don't Cares

$$F(A,B,C,D) = \sum m(4,6,8,9,10,12,13) + \sum d(2, 15)$$

- Q-M Step 1: Group “all” minterms with 1 true variable, 2 true variables, etc.

	Minterm	ABCD	Groups
Gp 1	2	0010	1: single 1
	4	0100	
	8	1000	
Gp 2	6	0110	2: two 1's
	9	1001	
	10	1010	
	12	1100	
Gp 3	13	1101	3: three 1's
Gp 4	15	1111	4: four 1's

$\checkmark$   
 $(2,6)$   $\begin{array}{r} 0010 \\ 0110 \\ \hline 0-10 \end{array}$   
 $(2,9)$   $\begin{array}{r} 0010 \\ 1001 \\ \hline 1001 \end{array} \times$




# Step 2: Same As Before on “All” Minterms

List 1 <i>0-implicant</i>			List 2 <i>1-implicant</i>			List 3		
Minterm	ABCD	PI?	Minterms	ABCD	PI?	Minterms	ABCD	PI?
✓ 2	0010	X	2, 6	0-10 ✓	PI2	8,9,12,13	1-0- ✓	PI1 ✓
✓ 4	0100	X	2,10	-010 ✓	PI3			
✓ 8	1000	X	4,6	01-0 ✓	PI4			
✓ 6	0110	X	4,12	-100 ✓	PI5			
✓ 9	1001	X	✓ 8,9	100-	X			
✓ 10	1010	X	8,10	10-0 ✓	PI6			
✓ 12	1100	X	✓ 8,12	1-00	X			
✓ 13	1101	X	✓ 9,13	1-01	X			
✓ 15	1111	X	✓ 12,13	110-	X			
			13,15	11-1 ✓	PI7			



# Step 3: Identify EPI's Ignoring Don't Cares



Covered by EPI →			x	x		x	x
Minterms →	4	6	8	9	10	12	13
PI1 is EPI			x	x		x	x
PI2		x					
PI3					x		
PI4	x	x					
PI5	x					x	
PI6			x		x		
PI7							x

Handwritten orange vertical lines are drawn through columns 3, 4, 6, and 7. A green checkmark is above column 4.

$$EPI = \{PI1\}$$

# Step 4: Cover Remaining Minterms

Remaining minterms →	4	6	10
PI_2		x	
PI_3			x
PI_4	x	x	
PI_5	x		
PI_6			x

to select min no. A  
PIs to cover  
F  
SPT  
all the min  
terms  
{2, 6, 10}

Integer linear program (ILP), available from Matlab and other sources: Define integer  $\{0,1\}$  variables,  $x_k = 1$ , select PI\_k;  $x_k = 0$ , do not select PI\_k.

Minimize  $\sum_k x_k$ , subject to constraints:

$$x_4 + x_5 \geq 1$$

$$x_2 + x_4 \geq 1$$

$$x_3 + x_6 \geq 1$$

at least once

A solution is  $x_3 = x_4 = 1$ ,  $x_2 = x_5 = x_6 = 0$ , or select PI\_3, PI\_4



# Step 4: Cover Remaining Minterms

Remaining minterms →		4	6	10
$p_2$ —	PI_2		x	
$p_3$ —	PI_3			x
$p_4$ —	PI_4	x	x	
$p_5$ —	PI_5	x		
$p_6$ —	PI_6			x

Patrick's Method (POS)

convert to SOP

$$\begin{aligned}
 & (p_4 + p_5) (p_2 + p_4) (p_3 + p_6) = 1 \\
 & = (p_2 p_4 + p_4 + p_5 p_2 + p_4 p_5) (p_3 + p_6) \\
 & = p_2 p_3 p_4 + \textcircled{p_3 p_4} + p_2 p_3 p_5 + p_3 p_4 p_5 + p_2 p_4 p_6 + \textcircled{p_4 p_6} + p_2 p_5 p_6 + p_4 p_5 p_6 = 1
 \end{aligned}$$

$$p_3 = 1 \quad p_4 = 1$$

EPI = PI<sub>1</sub>

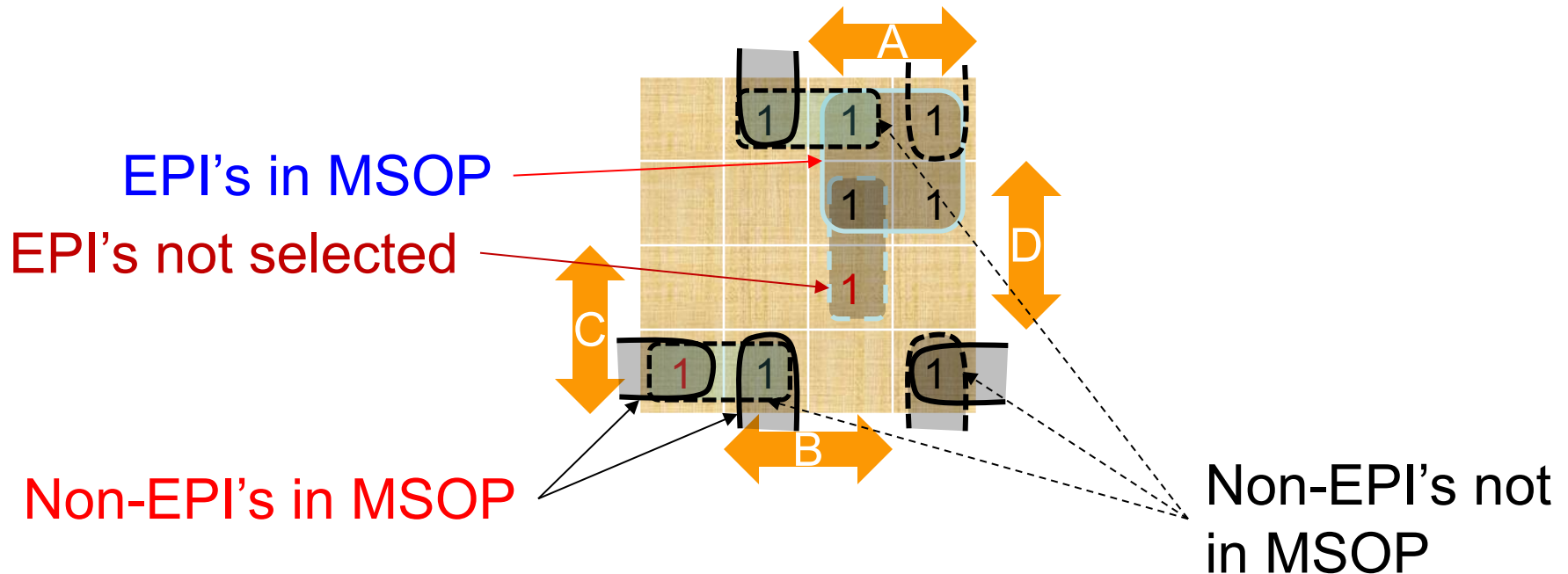
SPI = {PI<sub>3</sub>, PI<sub>4</sub>}



# Q-M MSOP Solution and Verification

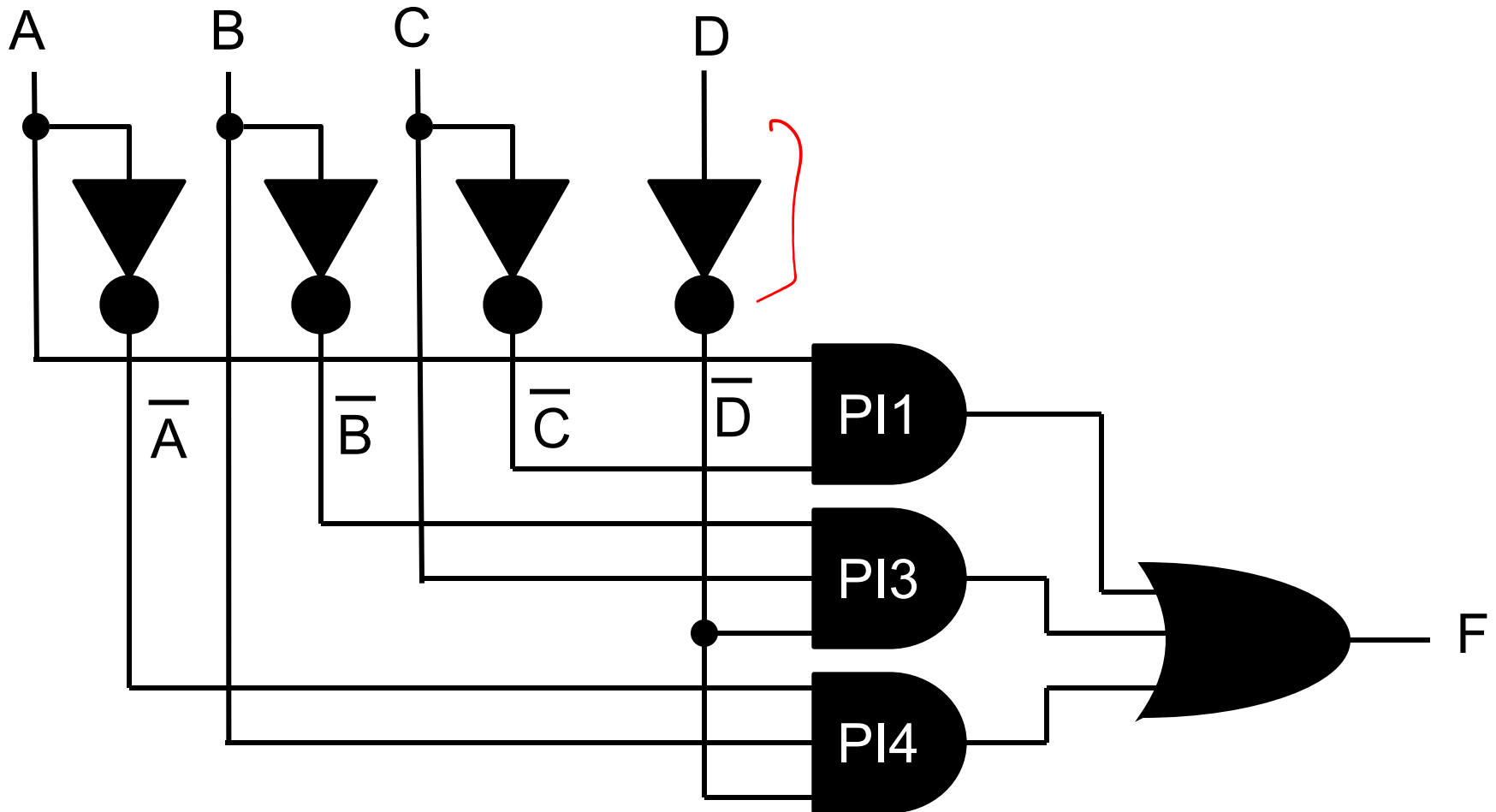
$$\begin{aligned} \blacksquare \quad F(A,B,C,D) &= \overset{\checkmark}{\text{PI\_1}} + \overset{\checkmark}{\text{PI\_3}} + \overset{\checkmark}{\text{PI\_4}} \\ &= \overset{\checkmark}{1-0-} + -010 + 01-0 \\ &= \overset{\checkmark}{A} \overset{\checkmark}{\bar{C}} + \bar{B} C \bar{D} + \bar{A} B \bar{D} \end{aligned}$$

- See Karnaugh map.





# Minimized Circuit



# Thank You

