Tutorial 8

1. Show that the following problems are in NP.
   (a) The scheduling problem introduced in Lecture 24 (slide deck).
   (b) Hamiltonian path problem.
   Given: A directed graph $G = (V, E)$ and special vertices s, t.
   Check: Does there exists a simple path from s to t that visits every vertex exactly once.
   (c) Hamiltonian cycle problem.
   Given: A directed graph $G = (V, E)$ and a special vertex u.
   Check: Does there exists a simple cycle that starts and ends at u and visits every other vertex exactly once.
   (d) 3-SAT problem.
   Given: a CNF formula φ over variables $x_1, \ldots, x_n$ such that each clause has at most 3 variables.
   Check: does there exists an assignment $\tilde{a} \in \{0, 1\}^n$ to $x_1, \ldots, x_n$ such that $\tilde{a}$ satisfies φ.
   \+ Prove that 2-SAT is in P.
   (e) k-Independent set problem.
   Given: A graph $G = (V, E)$.
   Check: Does there exists subset $S \subseteq V$ such that $|S| = k$ and for every $u, v \in S$, $(u, v) \notin E$.

2. State true or false.
   (a) There exists problems that are not known to be in P and are also not known to be in NP.
   (b) If $\Pi \in$ NP then so is $\overline{\Pi} \in$ NP.
   (c) If $\Pi \in$ NP and $\Pi' \leq_m \Pi$ then $\Pi' \in$ NP.
   (d) If $\Pi \in$ NP and $\Pi \leq_m \Pi'$ then $\Pi' \in$ NP.
   (e) If $\Pi \leq_m \Pi'$ then $\Pi' \leq \Pi$.
   (f) If $\Pi \in$ P and $\Pi' \in$ NP then $\Pi \leq_m \Pi$.