Reed Muller Representation

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CS-226: Digital Logic Design

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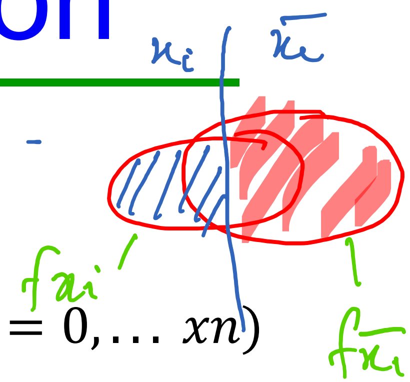
CADSL

Canonical Forms

- Canonical Forms in common usage:
  - Truth Table ✓
  - Sum of Minterms (SOM) ✓
  - Product of Maxterms (POM) ✓
  - Binary Decision Diagram (BDD) ✓ ROBDD
  - Reed Muller Representation

Reed Muller Representation

Function Decomposition



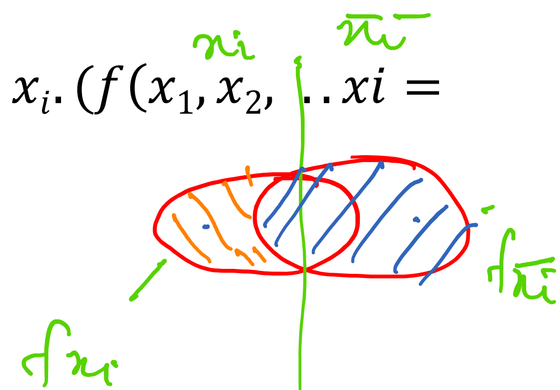
- Shanon's decomposition

$f = x_i \cdot f_{x_i} \oplus \overline{x_i}\, f_{\overline{x_i}}$

$= x_i\, f_{x_i} + \overline{x_i} \cdot f_{\overline{x_i}}$

$f(x_1, x_2, \quad ..xi, ...\ xn)$
$= xi.\, f(x_1, x_2, \quad ..\ x_i = 1, ...\ xn) \oplus \overline{x_i}.\, f(x_1, x_2, \quad .x_i, = 0, ...\ xn)$

- Positive Davio decomposition



$f(x_1, x_2, \ ..xi, ...\ xn) = f(x_1, x_2, \ ..xi = 0, ...\ xn) \oplus x_i.(f(x_1, x_2, \ ..xi = 1 ...\ xn) \oplus f(x_1, x_2, \ ..xi = 0 ...\ xn))$

$f = f_{\overline{x_i}} \oplus x_i (\underbrace{f_{\overline{x_i}} \oplus f_{x_i}}) = f_{\overline{x_i}} \oplus x_i \frac{\partial f}{\partial x_i}$

- Negative Davio decomposition

$f(x_1, x_2, \ ..xi, ...\ xn) = f(x_1, x_2, \ ..xi = 1, ...\ xn) \oplus \overline{x}i.(f(x_1, x_2, \ ..xi = 1 ...\ xn) \oplus f(x_1, x_2, \ ..xi = 0 ...\ xn))$

$f = f_{x_i} \oplus \overline{x_i} \cdot (f_{\overline{x_i}} \oplus f_{x_i}) = \underbrace{f_{x_i} \oplus \overline{x_i} \cdot \frac{\partial f}{\partial x_i}}$

Reed Muller Representation

- Positive Davio decomposition

$f(x_1, x_2, \ldots xi, \ldots\ xn) = f(x_1, x_2, \ldots xi = 0, \ldots\ xn) \oplus x_i.(f(x_1, x_2, \ldots, x_i = 1, \ldots\ xn) \oplus f(x_1, x_2, \ldots, x_i = 0, \ldots\ xn))$

$f = f_{\bar{x}_i} \oplus x_i \frac{\partial f}{\partial x_i}$

- Generalization

$f(x_1, x_2, \ldots xi, \ldots\ xn) =$

$\Rightarrow$ $a_0 \oplus a_1 x_1 \oplus a_2 x_2 \ldots\ldots \oplus a_r x_1 x_2 \oplus a_{r+1} x_1 x_3 \oplus$

$\oplus a_p x_1 x_2 x_3 \oplus \ldots\ldots \oplus a_m x_1 x_2 \ldots xn$

Reed Muller Representation

Canonical AND-XOR

Disjoint Sum of Products

Reed Muller Representation

Canonical → VERIFICATION

Spec = Implementation

RM ↗   ↑RM → || IMPL1 $\overset{?}{=}$ IMPL2

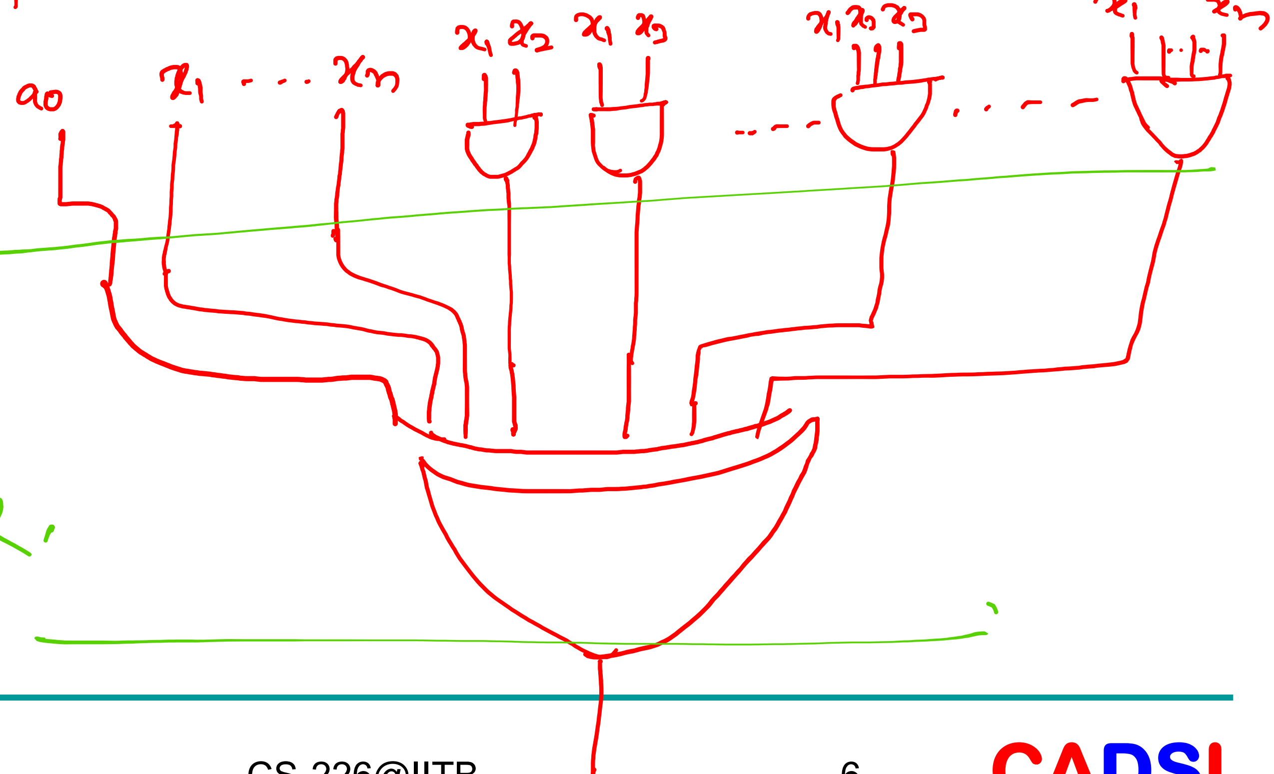
Synthesis

$f = a_0 \oplus a_1 x_1 \oplus \underline{a_2 x_2} \cdots \oplus \underline{a_r x_1 x_2} \cdots \oplus a_m x_1 x_2 \ldots x_n$

AND

2level circuit

XOR.



Reed Muller Representation

— SOP

- $f(a,b,c) = \overline{a}.\overline{b} + b.\overline{c} + a.b$

$f_a = f|_{a=1} = 0 + b.\bar{c} + b = b + b\bar{c} = b$

$f_{\bar{a}} = f|_{a=0} = \bar{b} + b.\bar{c} + 0 = \bar{b} + b\bar{c} = \bar{b} + \bar{c}$

$f = f_{\bar{a}} \oplus a.(f_{\bar{a}} \oplus f_a)$

$= (\bar{b} + \bar{c}) \oplus a.((\bar{b} + \bar{c}) \oplus b)$

$f_b = \bar{c} \oplus a.(\bar{c} \oplus 1) = \bar{c} \oplus a.c$

$f_{\bar{b}} = 1 \oplus a.(1 \oplus 0) = 1 \oplus a = \bar{a}$

$f = f_{\bar{b}} \oplus b.(f_{\bar{b}} \oplus f_b) = \bar{a} \oplus b.(\bar{a} \oplus \bar{c} \oplus ac)$

$= \bar{a} \oplus \bar{a}b \oplus b\bar{c} \oplus abc$

$(1 \oplus c)$

$1 \oplus c \oplus (1 \oplus a)b \oplus b\cancel{c} \oplus abc$

$= 1 \oplus a \oplus \cancel{b} \oplus ab \oplus \cancel{b} \oplus bc \oplus abc$

$\boxed{= 1 \oplus a \oplus ab \oplus bc \oplus abc}$

Reed Muller Representation

$f = \bar{a} \oplus \bar{a}\bar{b} \oplus b\bar{c} \oplus abc$

$f_c = \bar{a} \oplus \bar{a}b \oplus 0 \oplus ab \quad = \bar{a} \oplus \bar{a}b \oplus ab$

$f_{\bar{c}} = \bar{a} \oplus \bar{a}b \oplus b \oplus 0 \quad = \bar{a} \oplus \bar{a}b \oplus b$

$f = f_{\bar{c}} \oplus c \cdot (f_{\bar{c}} \oplus f_c)$

$= \bar{a} \oplus \bar{a}b \oplus b \oplus c \cdot (\cancel{\bar{a}} \oplus \cancel{\bar{a}b} \oplus b \oplus \cancel{\bar{a}} \oplus \cancel{\bar{a}b} \oplus ab)$

$= \bar{a} \oplus \bar{a}b \oplus b \oplus c \cdot (b \oplus ab)$

$= \bar{a} \oplus \bar{a}b \oplus b \oplus bc \oplus abc$

$= 1 \oplus a \oplus (1 \oplus a) \cdot b \oplus b \oplus bc \oplus abc$

$= 1 \oplus a \oplus \cancel{b} \oplus ab \oplus \cancel{b} \oplus bc \oplus abc$

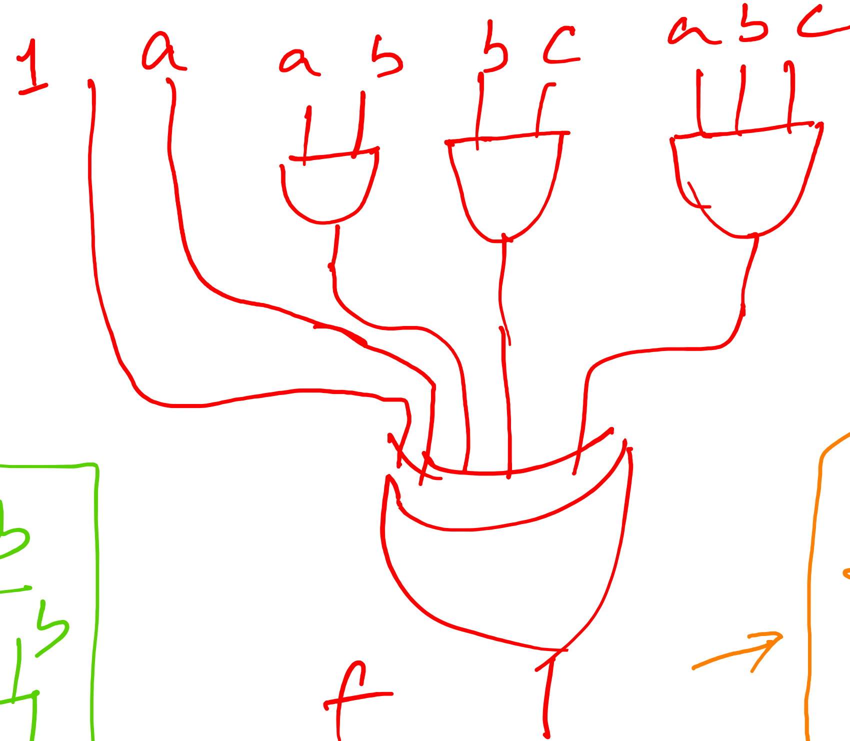
$\boxed{f = 1 \oplus a \oplus ab \oplus bc \oplus abc}$

RM form. ↖ Disjoint Sum of product

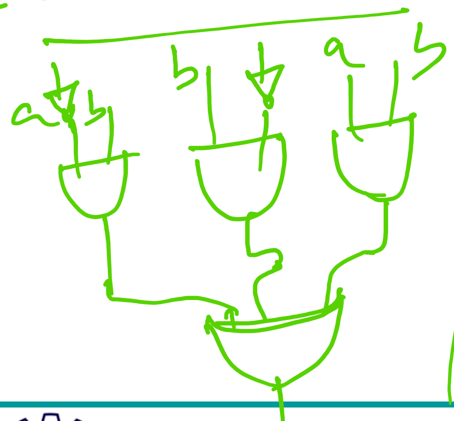
Reed Muller Representation

$f: 1 \oplus a \oplus ab \oplus bc \oplus abc$

Synthesis using AND XOR



$f = \bar{a}b + b\bar{c} \oplus ab$



TESTABILITY

100% Testability

Thank You