Special Functions

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CADSL

Dual Function

- To obtain ~~dual of~~ Dual of Boolean Expression, exchange AND's and OR's; and exchange 0's and 1's. The functional definition is:

$$\text{Dual of } f(x_1, x_2, \ldots xn) = \text{Complement of } f(\bar{x_1}, \overline{x2}, \ldots, \bar{x_n})$$

$f(a,b,c) = ab + bc + ca$

$\text{dual of } f = \overline{(\bar{a}\cdot\bar{b} + \bar{b}\cdot\bar{c} + \bar{c}\cdot\bar{a})}$

$= \overline{(\bar{a}\cdot\bar{b})}\cdot\overline{(\bar{b}\cdot\bar{c})}\cdot\overline{(\bar{c}\cdot\bar{a})}$

$= (a+b)\cdot(b+c)\cdot(c+a)$

$f(a,b,c) = a\bar{b} + bc \qquad \text{dual of } f = (a+\bar{b})\cdot(b+c)$

Self Dual Function

- A function is dual of itself
- function f is *self-dual iff* when complementing its input variables, the output becomes complement of f.

$$\textit{Dual of } f(x_1, x_2, \ldots . xn) = f(x_1, x_2, \ldots . xn)$$

$$\underline{\text{dual}}\ f = \overline{f(\bar{x}_1, \bar{x}_2 \ldots \bar{x}_n)} = f(x_1, x_2 \ldots x_n)$$

$$f(\bar{x}_1, \bar{x}_2, \ldots \bar{x}_n) = \overline{f(x_1, x_2 \ldots x_n)}$$

Self Dual Function

$f = \boxed{ab + bc + ca}$

dual of $f = (a+b)\cdot(b+c)\cdot(c+a)$

$= (ab + b + ac + bc)(c + a)$

$= (abc + bc + ac + bc + ab + ab + ac + abc)$

$= \underbrace{abc + bc} + ac + ab$

$\boxed{= bc + ac + ab}$

Symmetrical Function

- A Boolean function that does not change under any permutation of its input variables is called a Totally Symmetric Function

$f(a,b,c) = ab + bc + ca$
$= ba + ac + cb$
$= ab + ac + bc$

symmetrical

$f(a,b) = a \cdot b$
$= a + b$
$= a \oplus b$

$f(a,b,c) = a \oplus b \oplus c$
$= b \oplus a \oplus c$
$= c \oplus b \oplus a$

Symmetrical Function

- When a function does not change under any permutation of a subset of its variables is called a <u>Partially Symmetric Function</u>

$f(a,b,c) = \bar{a}\bar{b}c + \bar{a}b\bar{c} + abc$

partially symmetrical in bc

$= \bar{a}\cdot\bar{c}b + \bar{a}c\bar{b} + acb$

$= \bar{a}\bar{b}c + \bar{a}b\bar{c} + abc$

Symmetry Theorem

- A function $f(x_1, x_2, \ldots, x_n)$ is totally symmetric *iff* it can be specified by a list of integers $A = \{a_1, a_2, \ldots, a_m\}$, $0 \leq a_i \leq n$ so that $f = 1$ *iff* exactly a_i of the n variables are 1.

$f = ab + bc + ca$ $\quad A = \{2, 3\}$

$f = a \oplus b \oplus c \oplus d$ $\quad A = \{1, 3\}$

$1 \oplus 0 \oplus 0 \oplus 0 = 1$

$1 \oplus 1 \oplus 1 \oplus 0 = 1$

$1 \oplus 1 \oplus 0 \oplus 1 = 1$

Symmetry Theorem

Unate Function

$f = ab + \underline{bc} + ca$

$f = \bar{a}\bar{b} + \bar{c}\bar{a}$

$f = \bar{a}b + b\bar{c}$

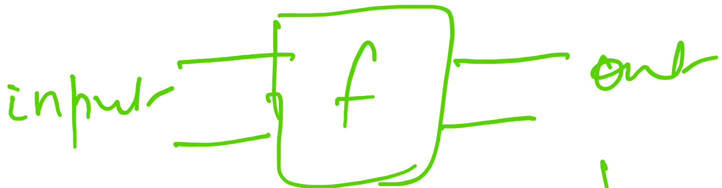
$f = \bar{a}b + \bar{b}\bar{c}$

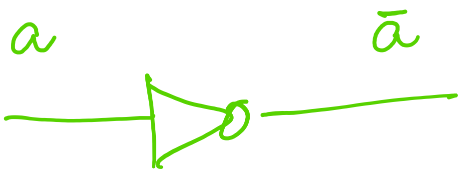
positive unate functio

-ve unate funct..

unate

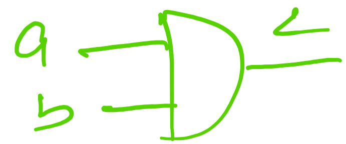
Reversible Function





0	1
1	0

Reversible.



0	0	0
0	1	0
1	0	0
1	1	1

Irreversible

Quantum circuits

Thank You