

Multiple-Level Optimization

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CADSL

Multiple-Level Optimization

- Multiple-level circuits are circuits that have more than two level (plus input and/or output inverters)
- For a given function, **multiple-level circuits can have reduced gate input cost compared to two-level (SOP and POS) circuits**
- Multiple-level optimization is performed by applying transformations to circuits represented by equations while evaluating cost



Transformations

- **Factoring** - finding a factored form from SOP or POS expression
 - Algebraic - No use of axioms specific to Boolean algebra such as complements or idempotence
 - Boolean - Uses axioms unique to Boolean algebra
- **Decomposition** - expression of a function as a set of new functions



Transformations (continued)

- Substitution of \check{G} into \check{F} - expression function \check{F} as a function of \check{G} and some or all of its original variables
- **Extraction** - decomposition applied to multiple functions simultaneously



Transformation Examples

- Algebraic Factoring ✓

$$F = \bar{A}\bar{C}\bar{D} + \bar{A}B\bar{C} + ABC + AC\bar{D} \quad G = \underline{16}$$

– Factoring:

$$F = \bar{A}(\bar{C}\bar{D} + B\bar{C}) + A(BC + C\bar{D}) \quad G = 16$$

– Factoring again:

$$F = \bar{A}\bar{C}(B + \bar{D}) + AC(B + \bar{D}) \quad G = \underline{12}$$

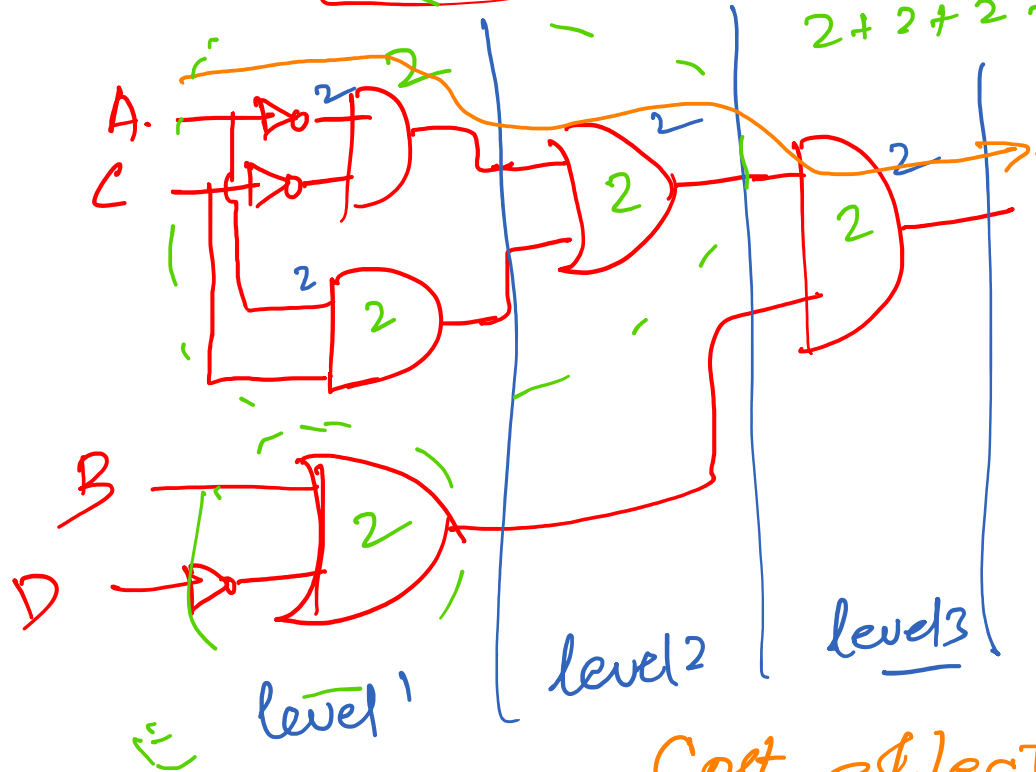
– Factoring again:

$$F = (\bar{A}\bar{C} + AC)(B + \bar{D}) \quad \underline{G} = 10$$



$$F = (\bar{A}\bar{C} + AC)(B + \bar{D})$$

$2 + 2 + 2 + 2 + 2 = 10$ Cost



delay
 $1 + 2 + 2 + 2 = 7$

Cost effective solution
 may not be optimal for
 performance.

Transformation Examples

- Decomposition

- $F = A'C'D' + A'BC' + ABC + ACD'$ $G = 16$

- The terms $A'C' + AC$ and $B + D'$ can be defined as new functions H and E respectively, decomposing

- $F = (A'C' + AC)(B + D')$:

- $F = H E, H = A'C' + AC, E = B + D' \quad G = 10$

- This series of transformations has reduced G from 16 to 10, a substantial savings.
- The resulting circuit has three levels plus input inverters.



Transformation Examples

- Substitution of E into F

- Returning to F just before the final factoring step:

$$F = \bar{A}\bar{C}(B + \bar{D}) + AC(B + \bar{D}) \quad G = 12 \quad \checkmark$$

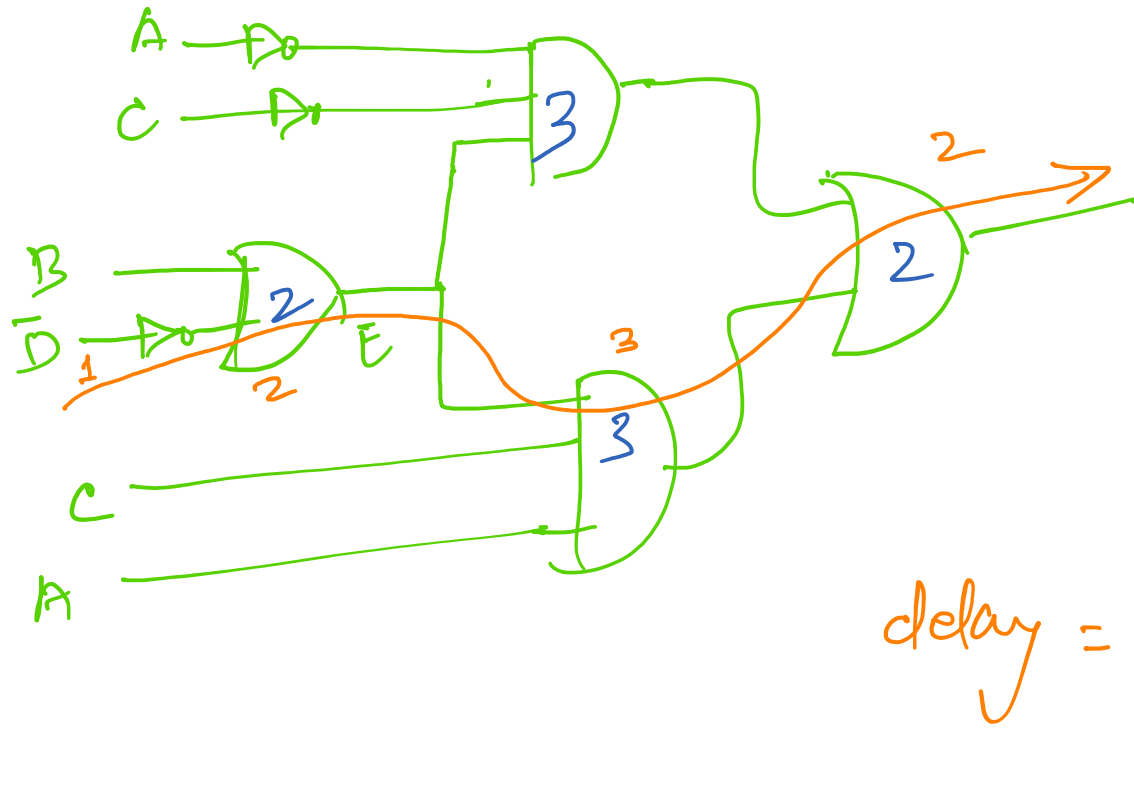
- Defining $E = B + \bar{D}$, and substituting in F:

$$F = \bar{A}\bar{C}E + ACE \quad G = 10$$

- This substitution has resulted in the same cost as the decomposition



Transformation Examples



Summary

- Multi-level Optimization
- Transformations
 - Factoring - find a factored form from SOP or POS expression
 - Decomposition - express a function as a set of new functions
 - Substitution - express function F as a function of G and some or all of its original variables



Thank You

