PERFORMANCE ESTIMATION OF AXIAL FLOW TURBINES

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A comprehensive method of estimating the performance of axial flow steam and gas turbines is presented, based on analysis of linear cascade tests on blading, on a number of turbine test results, and on air tests of model casings. The validity of the use of such data is briefly considered. Data are presented to allow performance estimation of actual machines over a wide range of Reynolds number, Mach number, aspect ratio and other relevant variables. The use of the method in connection with three-dimensional methods of flow estimation is considered, and data presented showing encouraging agreement between estimates and available test results. Finally 'carpets' are presented showing the trends in efficiencies that are attainable in turbines designed over a wide range of loading, axial velocity/blade speed ratio, Reynolds number and aspect ratio.

INTRODUCTION

TURBINE PERFORMANCE can only be satisfactorily determined by tests on full scale machines. Such tests, however, reflect the aggregate effect of a large number of features influencing efficiency, and for a basic understanding of turbine performance it is necessary to analyse such features. The procedure presented in this paper can be used in the preliminary performance estimation of new designs and will supplement estimation methods based on tests of previous turbines of similar design.

With an accurate theoretical method of performance prediction the designer can assess in the early design phase the performance merit or penalty that will result from any proposed departure from standard tested machines. While it is not suggested that the method could ever become a substitute for absolute test data, it should prove to be of direct help in assessing the changes in performance which occur at conditions other than those for which the basic tests were originally carried out. Finally, this method can also be used to indicate the lines along which future aerodynamic development should proceed.

A number of methods of performance estimation have been published in the past, such as that of Ainley and Mathieson (1)‡ and Traupel (2). The former found wide acceptance in the aircraft and industrial gas turbine fields, but in the manner in which it was often used—prior at least to modifications suggested by Dunham and Came (3)—little allowance was made for aspect ratio and blade height effects. Thus, use of the method produced unconvincing answers for typical steam turbine designs. Here a method is set out which has been developed for use with steam and gas turbines. It attempts to take into account the full range of Reynolds number and aspect ratio encountered in such machines, and in addition to deal with most of the auxiliary sources of loss which are sometimes omitted from published methods. It is a development of the method published by Craig and Janota at the 1965 CIMAC conference (4), extended to cover off-design conditions and other effects.

Examined against turbine test data, this analysis has generally given results correct to an accuracy of $\pm 1\frac{1}{4}$ per cent.

Notation

A	Fluid relative inlet angle.
Aa	Annulus area.
Ak	Total effective area of clearance.
At	Total blade throat area.
B	Fluid relative outlet angle.
b	Blade backbone length.
C_{cr}	Critical throat velocity.
C_{w1}	Guide outlet absolute tangential velocity.
C_{w2}	Runner outlet absolute tangential velocity.
C_z	Axial velocity.
C_1	Guide outlet absolute velocity.
Co	Guide inlet absolute velocity.
CR	Contraction ratio.
D	Diameter: D_h = hub diameter.

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e	Back surface radius.
F	Parameter used in evaluating losses, with various subscripts.
G	Mass flow through blades.
g	Gravitational constant.
h	Height.
i	Incidence.
i_{min}	Incidence for minimum loss.
i+stall	Positive stalling incidence.
i−stall	Negative stalling incidence.
$(i+\text{stall})_{basic}$	Value for standard value of s/b and CR.
$(i-\text{stall})_{basic}$	Value for standard value of s/b.
J	Mechanical equivalent of heat.
l	Length.
Ma	Mach number.
N	Loss ratio.
o	Blade throat opening.
p	Perimeter length.
Re	Reynolds number.
Re_o	Reynolds number based on blade opening.
s	Blade pitch.
T_w	Torque coefficient.
te	Trailing edge thickness.
U	Blade speed.
v	Specific volume.
W_1	Relative velocity at inlet to runner.
W_2	Relative velocity at outlet to runner.
X	Total loss factor.
x	Basic loss factor.
α	Blade inlet angle.
ΔC_w	$C_{w1}+C_{w2}$.
Δg	Leakage flow bypassing blades.
Δi+stall	Correction added to basic value with suffix denoting effect of s/b or CR.
Δi−stall	Correction added to basic value.
ΔL	Lap defined in Fig. 21.
Δx	Additional loss factor added to basic loss factor.
$\Delta\eta$	Efficiency debit, with subscript denoting component.
η_b	Blading efficiency.
η_p	Profile section efficiency.
η_t	Total stage efficiency.
λ	Loss coefficient.
ϕ	Guide blade velocity coefficient.
ψ	Runner blade velocity coefficient.

Subscripts for loss factors and ratios

a	Annulus.
b	Basic.
h/b	Aspect ratio.
i	Incidence.
m	Mach number.
p	Profile.
r	Reynolds number.
s	Secondary.
s/e	Back curvature.
t	Trailing edge thickness.

RELEVANCE OF CASCADE DATA

The method is based on a correlation of profile and secondary losses obtained from linear cascade tests, supplemented by information on losses (such as clearance losses) derived from specific turbine tests, and by data on casing losses, derived for the most part from air tests. It is relevant therefore to start by considering how far cascade tests provide a satisfactory basis for estimating the losses in an actual turbine.

A linear cascade differs from blading in a real turbine in two ways. First, differences occur when the cascade tests are carried out:

(1) with a different working fluid;
(2) with a different Reynolds number;
(3) with a different scale of blade;
(4) with a different surface roughness;
(5) with a different Mach number.

Differences of this sort, if they occur, are capable of being corrected, with the major proviso that the information is available as to how the correction should be made. Second, the following, more fundamental, differences also exist:

Cascade flow	*Turbine flow*
Uniform inlet conditions.	Inlet flow containing wakes, disturbances due to preceding secondary flow, and stage leakage.
Linear cascade.	Annular flow.
Walls stationary relative to blades.	For unshrouded stages walls may be moving relative to blades.

Differences of this type are an inherent limitation in the use of stationary linear cascade data, and no measurements, however extensive, obtained from such cascades can completely allow for such differences. The only check that can be made upon them is to compare the results of carefully interpreted cascade data with actual turbine performance, and to deduce from the overall result the magnitude and importance of the errors involved.

If it be thought that the effects of such differences are so extensive as to swamp the points of similarity, then clearly no useful purpose can be served by cascade testing or its analysis, and turbine development has to proceed by trial and error. If it be thought that the effects are negligible, then turbines can be designed wholly round cascade data, and the results predicted with some degree of precision. It is the experience of the writers that the truth lies between these two extremes, and that carefully used cascade data serve as a guide to trends in performance which will occur in actual machines; but that some care is needed in the interpretation of cascade data to avoid errors caused by the differences between the cascade and the turbine.

The model turbine occupies an intermediate position between that of the linear cascade analogy and a real turbine. Model turbines are generally designed to an exact scale of the actual machine and differences of the second type listed above are usually eliminated. On the other hand, differences of the first type may be present, especially those given by conditions (2)–(4). Extrapolation from such model turbine data to the real fluid condition is therefore subject to some of the same uncertainties as occur with cascade data, and indeed can only be properly carried out if the measured loss is broken down into its constituent parts and each constituent individually corrected.

Other methods of performance prediction based essentially on calculations of the potential flow round aerofoils are coming increasingly into use in the industry. Such methods are inherently more discriminating than the type of analysis described in this paper (taking into account the precise blade profile), but it should be remembered that such methods deal only with the profile loss in turbines, which contribute rather less than half the total losses in many real machines. Until the more difficult problem of theoretical secondary flow prediction has been resolved, cascade and turbine test data must remain the basic sources of information on secondary loss.

BREAKDOWN OF STAGE LOSS

Before proceeding to deal with the losses in detail, it is convenient to consider first the nature of the losses in a stage. The work done on the rotor blades is indicated by the change in tangential momentum, and the overall integrated value can be calculated from the velocity conditions for the mass actually passing through the rotor blades. The energy given up by the gas is more than this, owing to the friction on the blade profiles, and loss in blade wakes (profile loss); the friction on the walls at root and tip, and other end effects (secondary loss); and any losses due to sudden enlargements in the fluid path, or wall cavities (annulus loss).

However, not all the fluid passes through the rotor blades, because of leakage through diaphragm glands, balance holes, and over the rotor blade tips; so the actual work per unit total mass flow is less than the work done on the blades per unit blade mass flow as evaluated above. Further, windage and bearing losses reduce the coupling power below that produced at the blades. Losses resulting from partial admission are also (in part) similar to windage loss, and can conveniently all be treated as a difference between blade and coupling work, as can lacing wire and wetness losses with rather less theoretical justification.

The total breakdown in losses can therefore be subdivided into the following constituent parts, assumed to be non-interacting:

Group 1	*Group 2*
Guide profile loss.	Guide gland leakage loss.
Runner profile loss.	Balance hole loss.
Guide secondary loss.	Rotor tip leakage loss.
Runner secondary loss.	Lacing wire loss.
Guide annulus loss (lap and cavity).	Wetness loss (where two-phase flow occurs).
Runner annulus loss (lap, cavity and annulus).	Disc windage loss.
	Losses due to partial admission.

A blading efficiency can then be defined as

$$\eta_b = \frac{\text{Work done in blading}}{\text{Work done in blading} + \text{Group 1 losses}} \qquad (1)$$

this being essentially

$$\frac{\Delta H \text{ useful at blades}}{\Delta H \text{ isentropic}}$$

referred to the mass flow passing through the rotor blade.

The overall stage efficiency is then defined in a similar form as

$$\eta_t = \frac{\text{Work done in blading} - \text{Group 2 losses}}{\text{Work done in blading} + \text{Group 1 losses}} \qquad (2)$$

this being essentially

$$\frac{\Delta H \text{ useful at coupling}}{\Delta H \text{ isentropic}}$$

referred to the total mass flow at the stage.

It is convenient to evaluate the Group 1 losses as loss factors based on the relative blade outlet velocities in the case of profile, secondary and lap losses, and on the inlet velocity in the case of the annulus loss. The Group 2 losses are evaluated as a net deficit in stage efficiency, this being the simplest way in which they are derived from test data. In this form the overall stage total head efficiency becomes

$$\eta_t = \frac{\text{Work done in blading}}{\text{Work done in blading} + \text{Group 1 losses}} - \sum(\text{Group 2 efficiency debits}) \qquad (3)$$

in which the Group 1 loss term may be written approximately as

$$\text{Group 1 losses} = (X_p + X_s + X_a)_g \frac{C_1^2}{200gJ} + \left(X_p + X_s + X_a \frac{C_2^2}{W_2^2}\right)_r \frac{W_2^2}{200gJ} \qquad (4)$$

where the first term of the Group 1 loss equation refers to the guide blade losses and the second term to the runner blade.

It should be noted that the definition of stage efficiency is based directly on inlet and outlet total head conditions specified at stations corresponding to guide entry of the particular stage assessed and the downstream stage respectively. Total head conditions applicable to the downstream stage are thus specified by the net work done and the overall stage efficiency defined above, as shown typically in the enthalpy–entropy diagram in Fig. 2. This procedure modifies that normally used in steam turbine practice which is based on static conditions where the implied total head at guide entry is obtained through an estimated

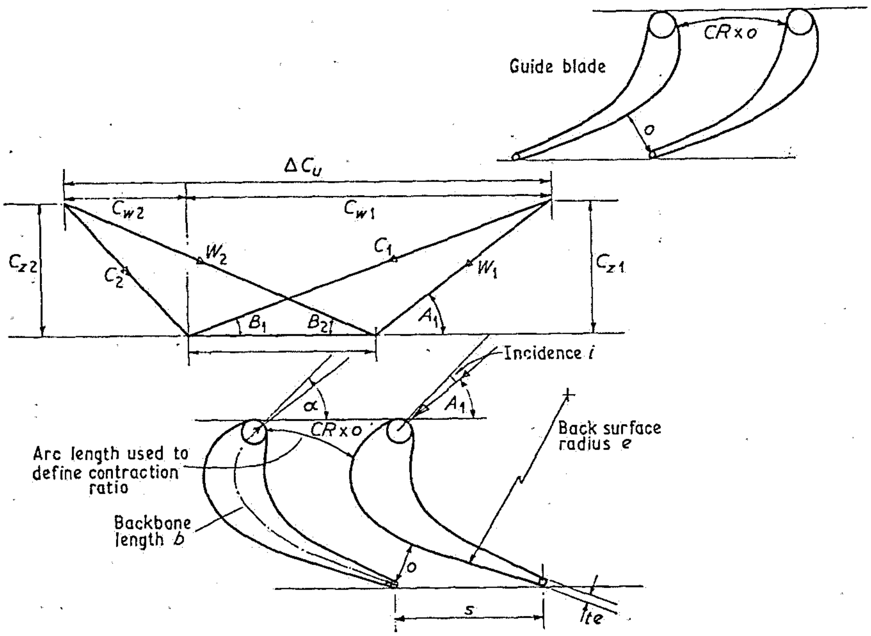

Fig. 1. Turbine blade and velocity triangle notation

'carry-in' kinetic energy. The latter is calculated as a fraction of the kinetic energy at runner outlet (stage leaving loss). This procedure is clumsy to apply where there are variations in velocity between runner outlet and guide entry, whereas the total head method is always direct.

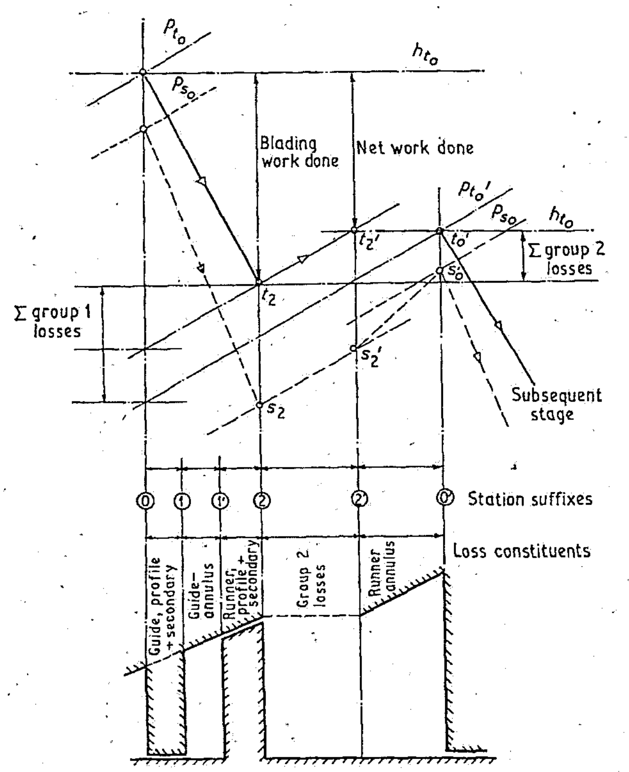

Fig. 2. Typical enthalpy–entropy diagram of stage

Once total head conditions at entry to the downstream stage are fixed the static conditions follow simply from mass flow continuity.

The four subsequent sections are concerned with the estimation of the loss coefficients and efficiency debits required to evaluate the overall stage total head efficiency defined by equation (3). These losses have to be evaluated consistently in conjunction with a three-dimensional flow solution, and to make this solution as realistic as possible the correct losses must be introduced into the flow analysis between specific axial stations within the stage with which each loss can be directly associated. Profile loss is essentially variable and calculable along the blade height, while secondary loss can be evaluated separately for root and tip conditions. Lap, annulus, and cavity losses occur between blades but these losses are essentially one-dimensional and should not be applied as a variable along the blade height.

The precise manner in which Group 2 losses are absorbed into the main flow field is uncertain. For convenience they—and any unshrouded rotor tip loss—are incorporated in an assumed mixing zone just downstream of the runner outlet station.

CORRELATION OF CASCADE DATA

The correlation of profile and secondary loss is based on an analysis of linear cascade data. One major problem in such correlation is the choice of independent variables, since cascade tests are not normally carried out with a variation of one parameter only. For instance, if the effect of Reynolds number is being measured, almost invariably the Mach number or the aspect ratio of the cascade is being simultaneously altered unless—and this is rarely so—the test is done on a variable density rig. If the wrong choice of independent variable is made, satisfactory correlation can still frequently be achieved within the range covered by the tests. However, errors, which may possibly be serious, will then arise when such a correlation is applied to values outside that range. Only by checking the final correlation against the range of variables encountered in practice can one ascertain whether the right choice of independent variable has been made.

The correlation evolved and presented in this paper is based on the analysis of over 100 specific cascade tests and on comparisons with a wide variety of published information. All losses in this section are related on a basis of velocity coefficients and are dependent on the following parameters:

(1) Reynolds number (*Re*) (based on outlet velocity and blade opening);
(2) aspect ratio (blade height/backbone length ratio);
(3) blade angles and passage geometry;
(4) pitch to backbone length ratio;
(5) Mach number (*Ma*);
(6) incidence.

The profile loss correlation is presented in the form of a basic loss correlation for incompressible flow conditions involving a variation in (3) and (4) only. To this basic value multiplying correction factors are presented which are to be applied where values of the other parameters differ from the standard values assumed in the basic condition. The basic correlation itself was derived originally from low speed tests where it could be assumed that the *Ma* effects could be ignored and that only items (1)–(4) were truly independent.

Effect of Reynolds number

The loss effect of *Re* on blade cascade performance is pronounced; typically in the range of *Re* between 2×10^4 and 2×10^5 the loss will be halved. A general prediction method for use in steam turbine analysis requires that the effect of *Re* should be predictable up to values of Re_o equal to about 4×10^6 which are now obtained at the inlet of modern high pressure (h.p.) cylinders. Thus any correlation of cascade data which neglects the Reynolds number of test is of little value.

In the method presented, the *Re* has been based on the blade opening, rather than on the chord or axial width, because it gives better correlation: and this correlation, derived from an analysis of *Re* effects in cascade, is shown by the standard finish curve of Fig. 3. At high *Re* values the surface roughness of the blade (or of the annulus walls with secondary loss) becomes important in conditions where it effectively controls the boundary layer thickness. To allow for this effect, data derived from Speidel (5) have been superimposed on the cascade *Re* correction to form the composite plot given in Fig. 3 covering a range of relative surface roughness.

When the *Re* effect defined in Fig. 3 was taken into account, no consistent effect of subsonic *Ma* on velocity coefficient was found for blade profiles designed with little profile curvature on the suction surface downstream of the throat. Thus the proposed correlation contains no Mach number effect on this type of profile at subsonic conditions.

Estimation of profile loss

The correlation of the profile loss for subsonic flow at or near the incidence at which the loss is a minimum (basic loss) is given in Figs 4 and 5. Fig. 4 defines a lift parameter F_L and Fig. 5 gives the basic loss parameter $x_p(s/b)\sin B$ as a function of two variables: $F_L s/b$ and the blade passage contraction ratio (*CR*). The losses on

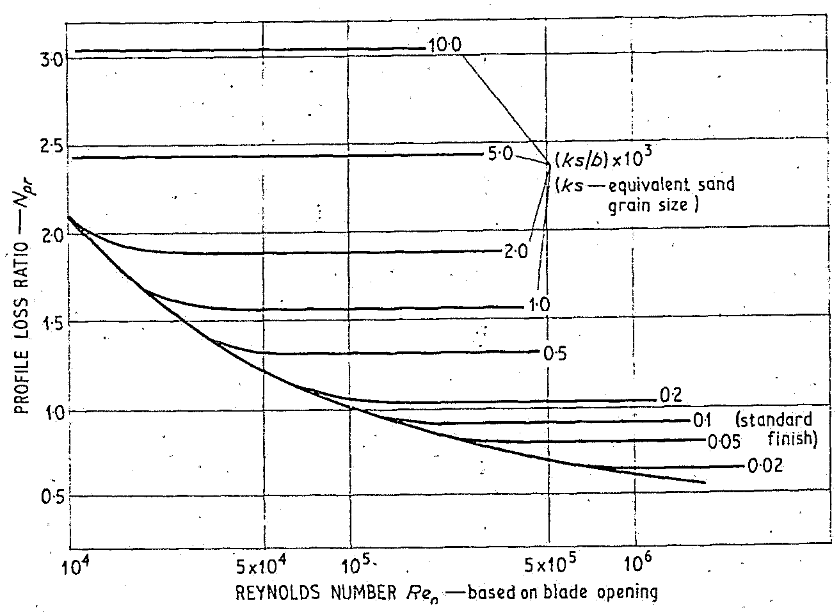

Fig. 3. Profile loss ratio against Reynolds number effect

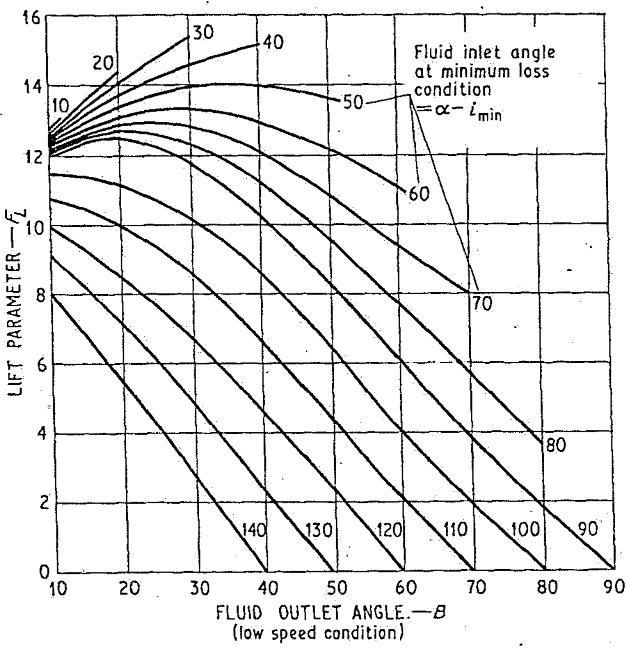

Fig. 4. Lift parameter, F_L

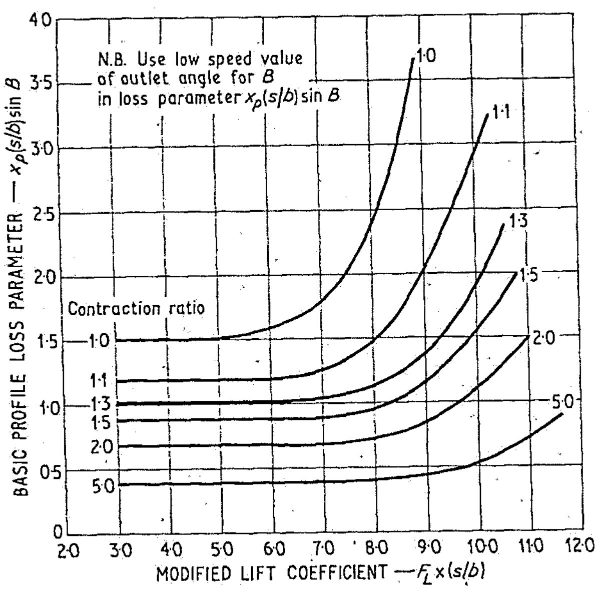

Fig. 5. Basic profile loss

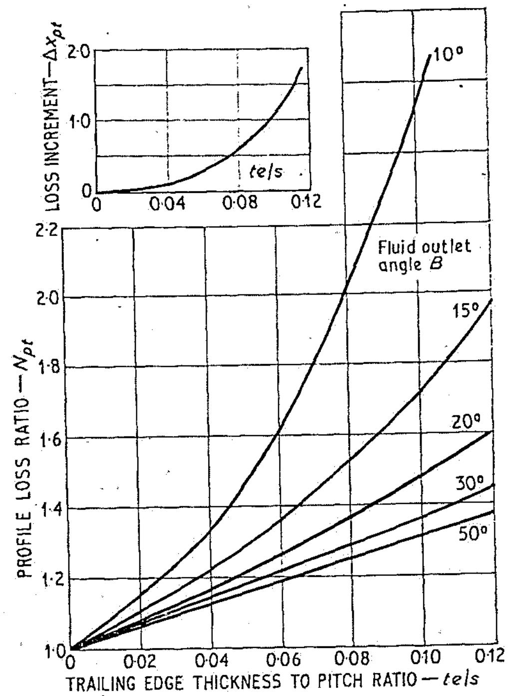

Fig. 6. Trailing edge thickness losses

which Fig. 5 is based have been adjusted to a zero trailing edge thickness condition, using the correction given in Fig. 6, this being derived by using a theoretical approach similar to that of Stewart (**6**). This correlation satisfactorily predicts the increase of losses associated with separation from the blade surface as the pitch to chord ratio is increased.

The basic profile loss correlation defined above relies upon a geometric cascade property referred to as the *CR*. This term denotes the internal blade passage width ratio of the particular cascade considered based on inlet to throat. The inlet internal passage width is not easy to define uniquely for any arbitrary cascade, but may be taken as the length of the maximum circular arc which can be drawn wholly within the passage at inlet and which is normal to both profile surfaces. For application to design analysis where the particular blade profiles have not been specified, a typical value of contraction ratio may be obtained from the data in Fig. 7 which cover a range of profile geometry.

Where the blade outlet *Ma* exceeds unity, a fundamental additive correction is made to the basic profile loss defined above. The correlation of this additive loss, referred specifically to convergent profiles designed with a straight suction surface downstream of the throat, is given in Fig. 8. For profiles designed with a pronounced convex suction surface curvature downstream of the throat, a further additive loss is required over and above that given for straight backed blades. The correlation of this second correction is shown in Fig. 9 where the additive loss factor is given as a function of outlet *Ma* and the ratio of the blade pitch to the mean suction surface radius s/e. As can be seen from the data in Fig. 9, *Ma* effects on curved suction surface profiles can exist at subsonic Mach numbers as low as 0·7 and occur when sonic conditions are attained locally on the suction surface.

While the above loss factor analysis is adequate for design calculations where the incidences are held close to optimum values, it is also necessary to consider off-design application where incidence losses may become appreciable. A correction for incidence is given in Fig. 10 in the

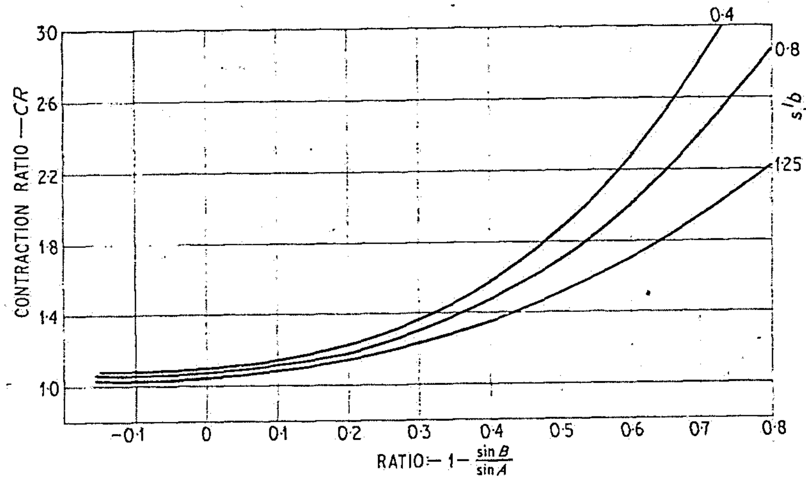

Fig. 7. Contraction ratio for average profiles

form of a loss ratio plotted against the incidence parameter $i-i_{\min}/i_{\text{stall}}-i_{\min}$. The form of correction is similar to that given by Ainley and Mathieson (1) except that in the present correlation the negative stalling incidence and minimum loss incidence have been correlated independently of the positive stalling value. Throughout the incidence analysis the stalling incidence is defined arbitrarily, as in (1), as the incidence at which the profile loss is twice the minimum value.

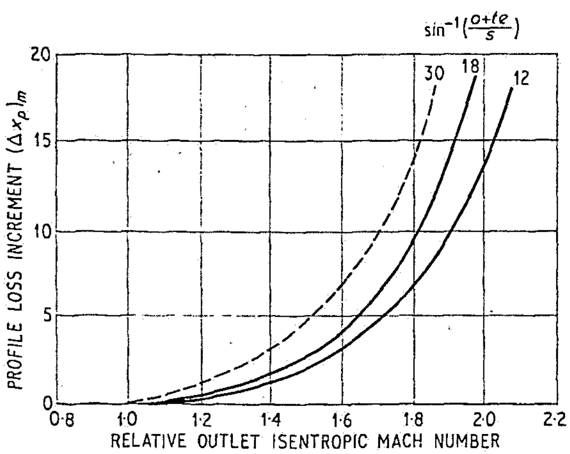

Fig. 8. Mach number loss for convergent blading

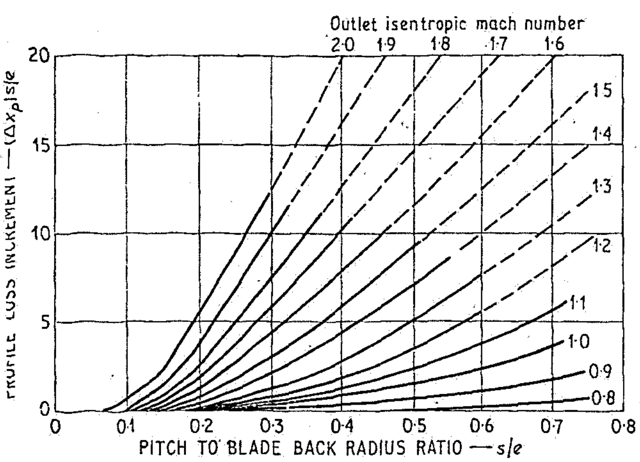

Fig. 9. Blade back radius losses

Figs 11–14 present data from which the positive and negative stalling incidences can be estimated. The minimum loss incidence is calculated from the data in Fig. 15 and equation (9) using the estimated values of the stalling incidences. A loss curve for each blade section can then be evaluated, using these values and the basic curve in Fig. 10. For initial design studies it is of course generally permissible to ignore incidence effects in the $-10°$ to $+5°$ range.

The incidence parameters are then evaluated from the equations, when $\alpha \leqslant 90°$:

$$i+\text{stall} = (i+\text{stall})_{\text{basic}}+(\Delta i+\text{stall})_{s/b}+(\Delta i+\text{stall})_{CR} \quad . \quad . \quad . \quad (5)$$

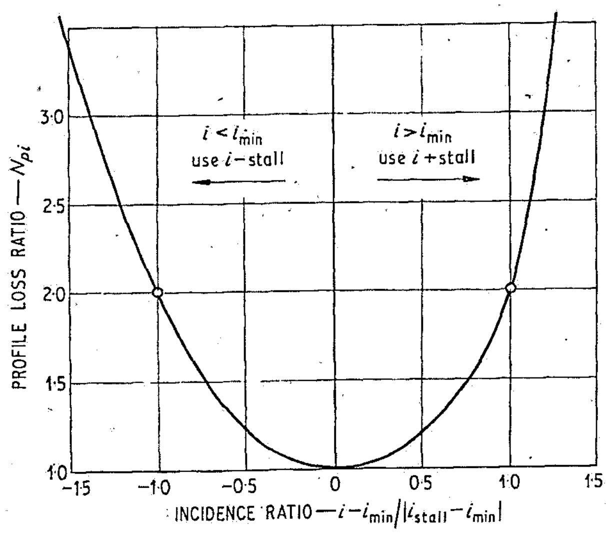

Fig. 10. Incidence losses

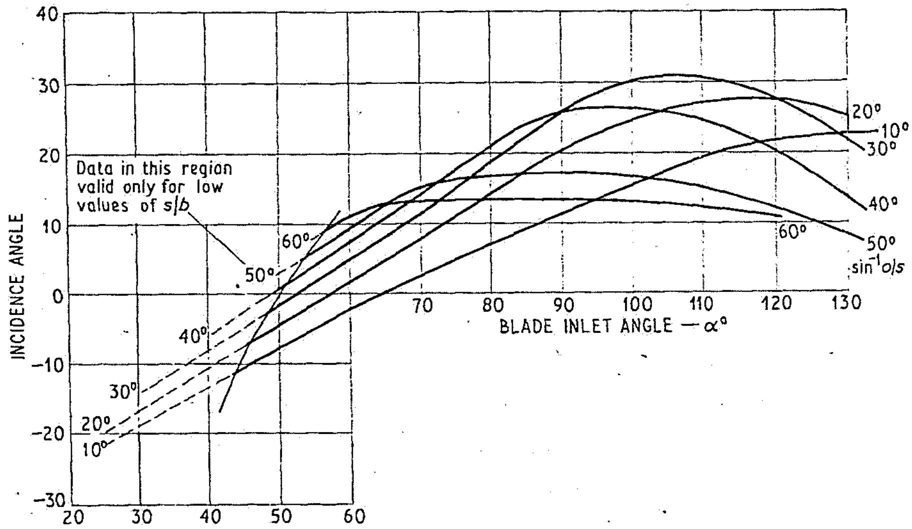

Fig. 11. Basic positive stalling incidence $(i+\text{stall})_{\text{basic}}$

where $(i+\text{stall})_{\text{basic}}$ is given by Fig. 11, $(\Delta i+\text{stall})_{s/b}$ and $(\Delta i+\text{stall})_{CR}$ are given in Fig. 12.

$$i-\text{stall} = (i-\text{stall})_{\text{basic}}+(\Delta i-\text{stall})_{s/b} \quad . \quad (6)$$

where $(i-\text{stall})_{\text{basic}}$ and $(\Delta-i_{\text{stall}})_{s/b}$ are given in Fig. 13.

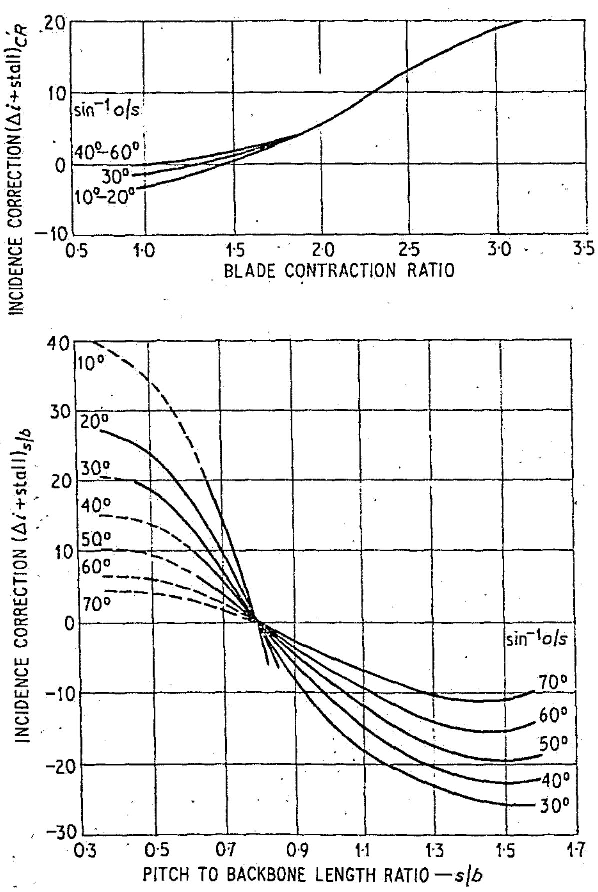

Fig. 12. Incidence corrections for positive stalling incidence

When $\alpha > 90°$:

$$i+\text{stall} = (i+\text{stall})_{\text{basic}}+\left(1-\frac{\alpha-90}{90-\sin^{-1} o/s}\right) \times[(\Delta i+\text{stall})_{s/b}+(\Delta i+\text{stall})_{CR}] \quad (7)$$

where $(i+\text{stall})_{\text{basic}}$ is given in Fig. 14, $(\Delta i+\text{stall})_{s/b}$ and $(\Delta i+\text{stall})_{CR}$ are given in Fig. 12.

$$i-\text{stall} = (i-\text{stall})_{\text{basic}}+\left(1-\frac{\alpha-90}{90-\sin^{-1} o/s}\right) \times(\Delta i-\text{stall})_{s/b} \quad (8)$$

where $(i-\text{stall})_{\text{basic}}$ is given in Fig. 14, $(\Delta i-\text{stall})_{s/b}$ is given in Fig. 13.

The minimum loss incidence may then be evaluated from the equation

$$i_{\text{min}} = \frac{(i+\text{stall})+F_i(i-\text{stall})}{1+F_i} \quad . \quad . \quad (9)$$

where the incidence parameter F_i is given in Fig. 15.

To summarize, the profile loss factor in the proposed correlation is obtained using the equation

$$X_p = x_{pb}N_{pr}N_{pi}N_{pt}+(\Delta x_p)_t+(\Delta x_p)_{s/e}+(\Delta x_p)_m \quad (10)$$

A further correction is required to allow for three-dimensional flow effects where the meridional streamlines are not parallel, and this is discussed in a later section.

When the relative isentropic outlet Ma exceeds about 1·4, the losses can be reduced if a blade profile giving a convergent–divergent passage is used. The profile loss of such a blade is best derived from a cascade test, but may be estimated from theoretical considerations for the purposes of a generalized correlation. Detailed specification of the design and performance prediction of such blades is outside the scope of this paper, but briefly the loss can be calculated at the limit loading condition where the blade is just fully loaded. The trailing shock system at this condition is estimated from the theoretical surface velocity

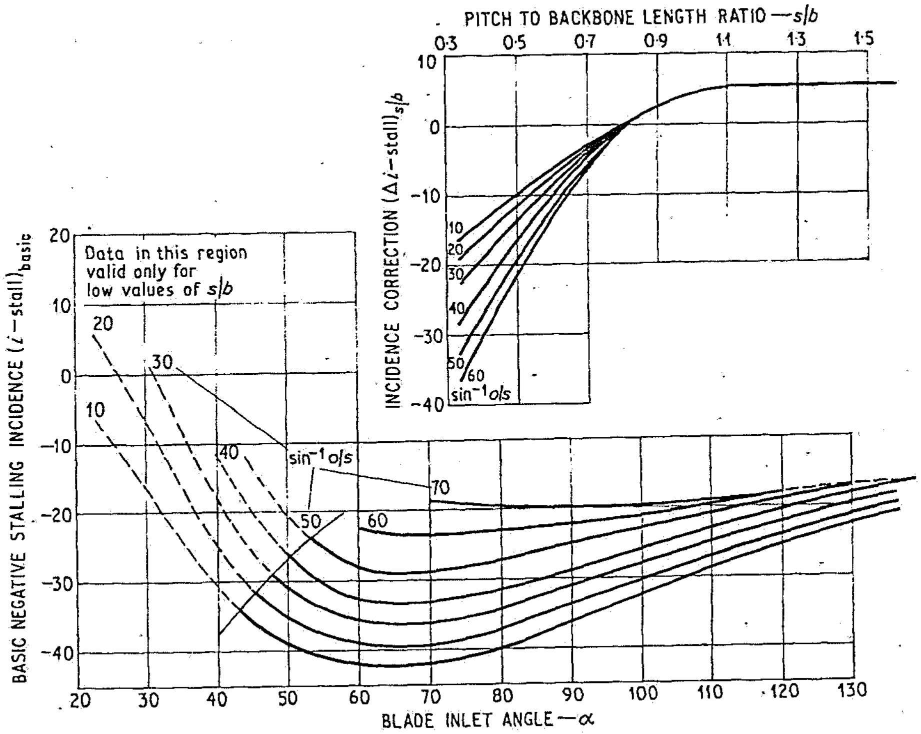

Fig. 13. Negative stalling incidence

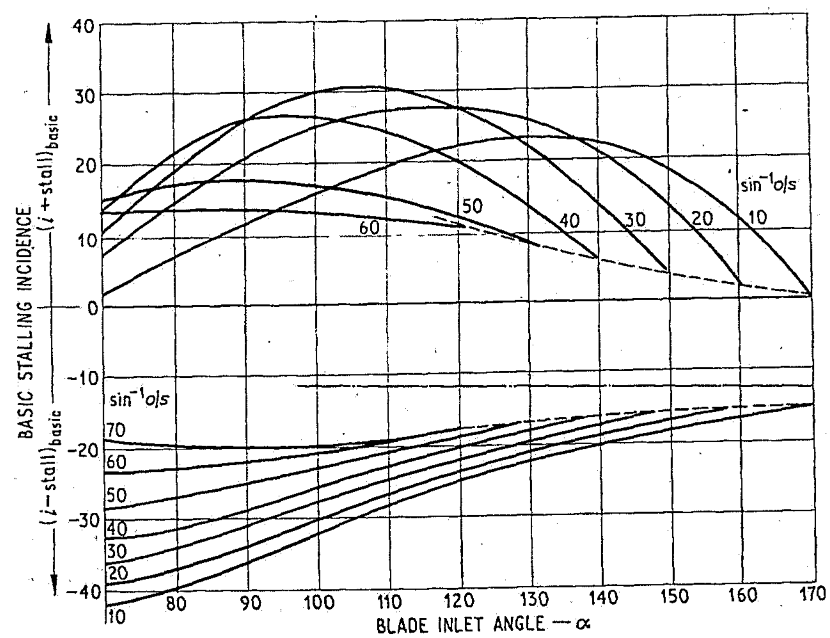

Fig. 14. Basic stalling incidences—for values of blade angle greater than 90°

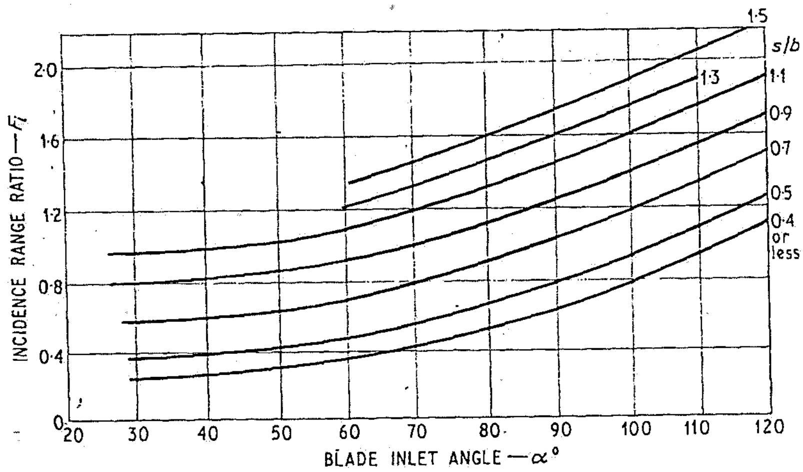

Fig. 15. Minimum loss incidence—range ratio F_i

condition just upstream of the trailing edge, by using shock reattachment criteria derived from data given by Nash (7); the wake thickness is estimated from the surface boundary layer properties. At conditions other than at limit load the variation of profile loss with Ma. can be obtained from a generalized correlation of test data. An example of this procedure is illustrated in Fig. 16 where this general method has been applied to a particular blade profile for which test data are available for comparison.

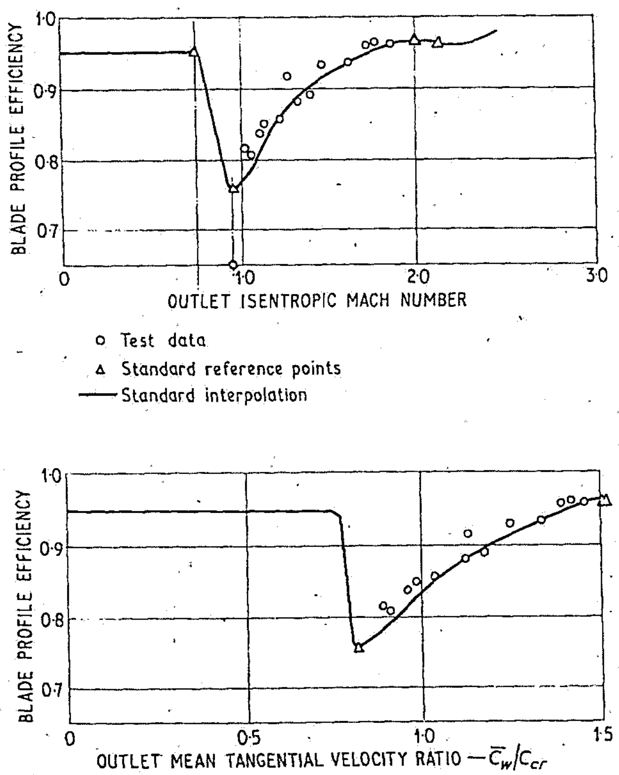

Fig. 16. Predicted and measured profile efficiencies for a typical convergent–divergent tip profile

Estimation of secondary loss

Secondary loss in a turbine consists partly of a true aerodynamic secondary loss and partly also of wall friction, both effects being complicated by any irregularities in the wall shape that may exist and by interaction with clearance flows. Precise prediction cannot therefore be expected and since the relative velocity between fluid and wall is of some importance, distinction must be made between shrouded and unshrouded stages. The correlation proposed here refers specifically to shrouded blade rows, but it is capable of adaptation for application to unshrouded blade rows.

The correlation is based on the assumption that the secondary loss is approximately inversely proportional to the aspect ratio of the blading and in addition shows a Reynolds number effect similar to that exhibited by the basic profile loss. Consideration of the Reynolds number effect serves to explain, qualitatively at least, the anomaly existing in other correlations where differing aspect ratio effects are quoted, depending on whether the blade height or the chord is being varied. For a change in aspect ratio obtained by varying the chord only, the Re effect will partly offset the fundamental aspect ratio effect, resulting in a reduced change of loss compared with that predicted for the same change in aspect ratio obtained by varying the height. In this correlation the effect depends strongly upon the absolute level of Reynolds number; at very high values the Re effect will become negligible and aspect ratio losses of blades of equivalent roughness will be equally affected by height and chord changes.

The proposed correlation is given in Figs 17 and 18 where a blade loading parameter and the relative velocity ratio are used as the independent variables. As the aspect ratio decreases the tip and root secondary loss concentrations tend to merge together and the rate of increase in loss is less than would be anticipated from a strict inverse law. The overall secondary loss factor is then estimated from the equation

$$X_s = (N_s)_r (N_s)_{h/b} (x_s)_b \quad . \quad . \quad . \quad (11)$$

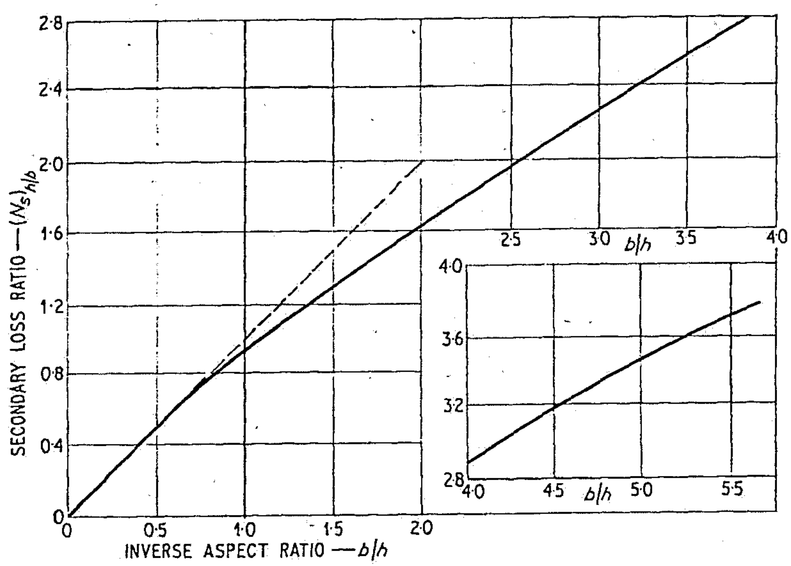

Fig. 17. Secondary loss–aspect ratio factor

and the correlation approximately holds over a range of blade incidences provided that the correct velocities are used in evaluating $(x_s)_b$.

DIFFUSING PASSAGES BETWEEN STAGES AND WALL CAVITIES

Additional losses occur where there is an appreciable amount of diffusion between two adjacent stages or where wall cavities occur between the guide and runner blades, and a further loss will be incurred where lap is introduced. The effect of lap on clearance losses is treated separately later in the paper.

The annulus loss factor X_a is given by the sum of the following three individual loss factors:

annulus loss factor (X_{a1})—given in Fig. 19;
cavity loss factor (X_{a2})—given in Fig. 20;
cavity loss factor (X_{a3})—calculated as a sudden expansion loss.

In all cases the equivalent non-dimensional loss factor is based on the inlet dynamic head. Distinction in the application of the data in Fig. 19 has to be made, depending upon whether the expansion is controlled or uncontrolled. The controlled expansion data are given by the full lines as a function of the equivalent diffuser cone angle, while the uncontrolled expansion data are represented by the broken lines as a function of distance ratio. Both sets of data are dependent upon the overall area ratio (inlet to outlet) while the effect of flow extraction is best simulated by considering the area ratio defined by the outer streamline of the flow passing into the downstream stage.

Typical data on cavity losses have been taken directly from Yablonik, Markovich and Al'tshuler (8) and presented in Fig. 20 in the form of a loss parameter dependent upon two variables largely defined by the geometric dimensions of the cavity and the stage. Differences between the loss through cavities situated between the guide and runner blades or between the runner and the downstream guide blade are said to be largely due to blade interaction effects which are much greater in the former instance. The data presented here are given mainly in order to emphasize that a comprehensive method of performance evaluation must allow for such losses. However, it should be noted that losses arising from cavities fundamentally

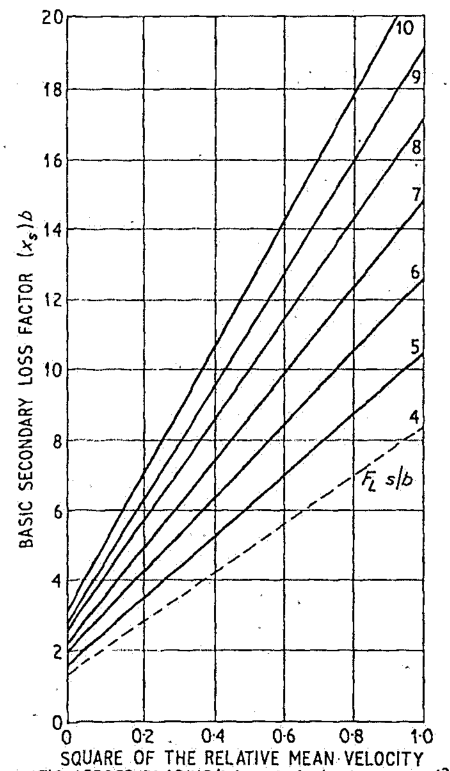

Fig. 18. Secondary loss–basic loss factor

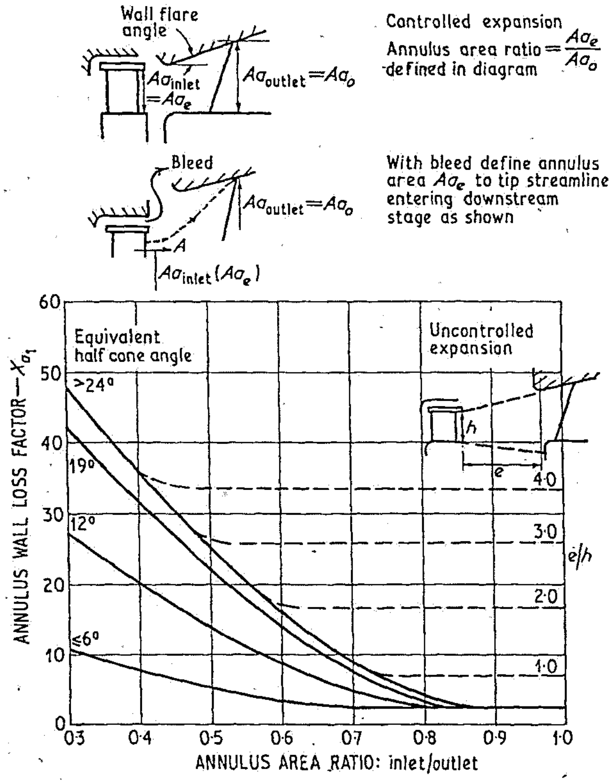

Fig. 19. Annulus wall loss

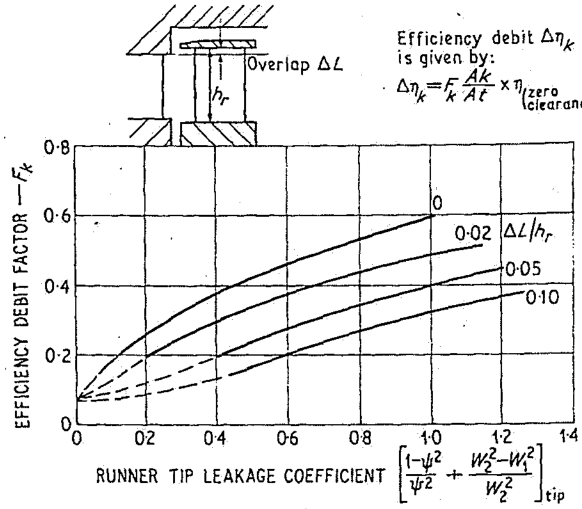

Fig. 21. Shrouded efficiency loss

different in shape from that considered in (8) should be obtained from actual test data.

LAP, CLEARANCE, BALANCE HOLES AND GLANDS

The third main source of loss in a turbine is that from leakage, either over blade tips, around shrouding, or—with disc and diaphragm designs of steam turbine—through disc balance holes and diaphragm glands. Such flows may be modified by the presence of lap (a sudden enlargement between stationary and moving blade rows frequently found in steam turbine designs).

Positive lap appears to have two effects: to give a loss which closely approximates to a sudden enlargement calculated by standard formula; and to influence the static pressure immediately after the lap in a manner which will reduce leakage flow.

This static pressure reduction may be the equivalent of perhaps a 10 per cent reduction in reaction. The existence of positive lap therefore tends to reduce leakage effects, and for any given clearance an optimum lap exists.

Typical clearance loss correlation is given in Fig. 21 for shrouded blading. At zero reaction this loss does not become zero, as some windage loss from the shroud band remains, and this strictly requires separate calculation. Where

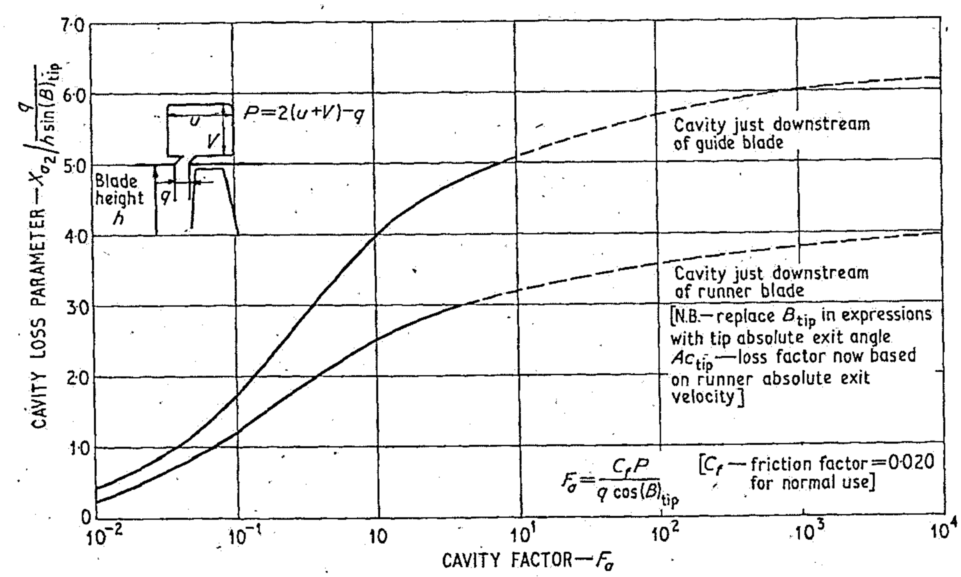

Fig. 20. Cavity loss

multiple seals are used to control leakage, an effective leakage area can be calculated by using an inverse square law. The data presented here refer specifically to seals operating just downstream of an abrupt flow disturbance. In cases where more uniform entry conditions to a seal are likely, the area of that seal should be multiplied by 1·5, to allow for the increase in its effective discharge coefficient, before using it in conjunction with Fig. 21. Additional correction may be made to take into account the detail of shroud overhang design. Lap is given in Fig. 21 as a function of blade height, which is a reasonable approximation in high and intermediate pressure cylinders, but requires modification for stages with long blades.

For unshrouded blades, the authors believe that the data in (1) are reasonably representative, provided that axial velocity remains approximately constant across the blade row and provided that the relative velocities are well below the sonic value. In steam turbine designs, unshrouded blading normally occurs only at the rear of the low pressure (l.p.) cylinder where neither of the above conditions will apply. For these conditions, it is suggested that a fair approximation to the tip loss is given by a value equal to 1·5 times that derived from use of the data in Fig. 21.

The leakage flow through glands can be calculated by standard formulae. It is convenient to represent the effective loss and other flow bypass losses, including that caused by balance holes, by an equivalent efficiency debit $\Delta\eta$, where

$$(\Delta\eta)\ \text{leakage} = \left(\frac{\Delta g}{G+\Delta g}\right)\eta_b \quad . \quad . \quad (12)$$

and $\Delta g/(G+\Delta g)$ denotes the leakage fraction.

Balance holes are fitted, mainly in high and intermediate pressure steam turbine stages, to reduce or control the axial thrust by allowing a small radial flow down the face of the disc. The effect of this flow is to maintain an inwardly decreasing pressure across the upstream face of the disc which materially reduces the axial thrust, accurate estimates of which can then be made from the predictable pressure gradients. Calculation of the leakage flow can be carried out using the data on flow coefficients for balance holes given in (9), by considering a net flow balance across the rotor disc, though our own tests do not wholly confirm the values in (9).

Correctly designed balance holes may provide some benefit to the overall stage performance since they will automatically swallow the gland leakage flow. In this event the leakage flow will not pass up the upstream face of the disc, and spillage effects into the main flow which could induce an early separation of the flow along the inner casing wall will be avoided.

MISCELLANEOUS LOSSES

Lacing wire

For wires of circular cross section the mean blade loss is increased by approximately 1 per cent of the local relative velocity head at the wire section for each 1 per cent of passage area blocked by the wire. Then, in terms of an equivalent efficiency debit required to calculate the overall stage efficiency, the wire loss is given by

$$\Delta\eta_l = \frac{\left\{\dfrac{\text{Wire area}\times C_d}{\text{Passage area}}\dfrac{W^2_{\text{local}}}{2gJ}\right\}\eta_b}{\text{W.D. blading}} \quad . \quad (13)$$

where a wire drag coefficient is added to account for non-circular wires. For wires of elliptical section of fineness ratio of $\frac{1}{4}$ the loss will be decreased by 70 per cent compared with a circular wire ($C_d = 1{\cdot}0$). Efficiency analysis on particular stages shows considerable scatter because of a reactive effect of the wire on the flow through the blading, as distinct from the wire drag loss itself. It should be noted that in the form given above the wire efficiency debit will depend quite strongly on the stage reaction.

Wetness loss

Various corrections have been proposed to allow for the additional losses suffered in stages operating with wet steam and these have been reviewed by Wood (10). No absolute evidence has been published which completely substantiates one method in preference to the others, most of which in any case give somewhat similar predictions of loss. In order to complete the turbine efficiency correlations given in this paper, it is suggested that (at present) the simplest procedure, given by Baumann (11), be used. This proposes a loss of 1 per cent in stage efficiency per 1 per cent of mean stage wetness.

However, it should be pointed out that most of the data on which such approximations are based are obtained from tests on low pressure wet steam. Where the machine operates on high pressure wet steam, as for instance on turbines for water cooled nuclear reactors, there is evidence that the loss may be appreciably less. High pressure wet steam has its liquid phase in droplets whose maximum size (determined on stability grounds) is much smaller. The droplets thus tend, for a given wetness, to be more numerous and better dispersed. They may be typically only about seven diameters apart. It is not difficult, therefore, to believe that they may behave less independently of the vapour phase than at low pressure.

Disc windage

The windage efficiency debit is derived from a power loss analysis, the actual debit being calculated by the equation

$$\Delta\eta_{\text{disc windage}} = \frac{\Delta P_w}{\text{W.D. blading}}\,\eta_b \quad . \quad (14)$$

where the power loss term (in W.D. units) is given by

$$\Delta P_w = T_w\,\frac{\left(\dfrac{\text{rev/min}}{100}\right)^3 (D_h)^5}{Gv}\,\frac{10}{3{\cdot}471^{-10}}\ (\text{B.t.u./lb}) \quad . \quad . \quad . \quad (15)$$

It is recommended that the values of torque coefficient, T_w, be taken from the results given by Daily and Nece

(12). The torque coefficient varies with disc Re and disc–casing spacing ratio.

Partial admission

No new correlation is proposed here, the best available being that given by Suter and Traupel (13).

CASINGS

Any performance method must make allowance for losses in inlet and exhaust casings as these are very important, particularly on cylinders with few stages or where high axial velocities are employed. A comprehensive treatment of the subject, however, is outside the scope of a paper of this length.

In general it is always desirable, and sometimes essential, that casings should be tested on models using the correct Reynolds number and Mach number, and closely simulating the flow distribution at the boundary planes. Interactions between blading and casing, and in certain instances between casing and condenser, can be of considerable significance.

For general performance prediction, where air model tests are not available, one can deduce approximate casing losses based on model tests. Comprehensive data for exhaust diffusers are given by Sovran and Klomp (14); for casings with restricted space, typical of some steam turbine low pressure exhausts, correlations of the type given in Fig. 22 can be used. Where casing geometry is restricted to a particular design concept, more accurate but less general correlations can be produced.

Using the data given in Fig. 22 the exhaust loss coefficient can be estimated from the equation

$$\lambda = \frac{F_{mo}}{4}\left[1-\left(\frac{Aa_0}{Aa_1}\right)^2\right]+F_fF_{m_1}\left[\left(\frac{Aa_0}{Aa_1}\right)^2+\left(\frac{Aa_0}{Aa_2}\right)^2+\left(\frac{Aa_0}{Aa_3}\right)^2\right] \quad (16)$$

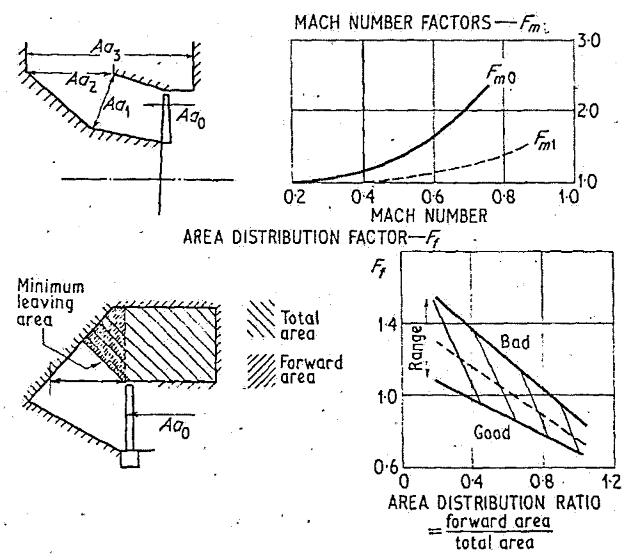

Fig. 22. Exhaust casing loss

where parameters F_{mo}, F_f and F_m are given in Fig. 22 and the area terms defined in the diagram.

APPLICATION TO TURBINE EFFICIENCY EVALUATION

In order to evaluate the performance of a complete machine, the radial velocity distributions at blade inlet and outlet stations are required in addition to the component loss data. One-dimensional mean diameter methods, i.e. using average velocity values at each station together with mean section loss data, may not introduce significant errors in very short blading, but this procedure should be discouraged since it fails to discriminate between good and bad root or tip conditions. For shorter gas turbine blading and steam turbine blading used in the h.p. and intermediate pressure (i.p.) cylinders, it is usually adequate to derive the radial velocity distributions from calculations using only the simple radial equilibrium condition. For longer blading, used typically in the rear stages of the l.p. cylinders, the radial flow solution must take into account both the streamline curvature and radial velocity effects, together with the influence of the upstream and downstream stages.

The general procedure used to evaluate stage efficiency follows simply from the subdivision of loss constituents given in the earlier part of the paper, where equation (3) defines the total head stage efficiency in terms of Group 1 losses and Group 2 efficiency debits. Whereas the Group 2 constituents are simply overall stage efficiency debits which each have a unique value for the particular stage considered, the losses included in Group 1 are essentially variable along the blade length. Thus if the efficiency in equation (3) is taken to represent the overall mean stage efficiency, the losses of Group 1 in that equation must be interpreted as the integrated value over the blade length. For short blades it is recommended that the Group 1 loss term should be evaluated from at least three separate sections along each blade length corresponding to root, mean and tip radii. The local value of the sum of Group 1 losses for each blade can then be obtained by applying the respective part of equation (4) individually to each section, using local computed values of the loss factors and velocity conditions. An overall value of loss can then be established by a simple averaging process which, assuming a parabolic distribution of loss between the three section values for each blade, is given by

$$(\sum \text{losses}_1)_{\text{average}} = \tfrac{1}{6}[(\sum \text{losses}_1)_{\text{root}} + (\sum \text{losses}_1)_{\text{tip}} + 4(\sum \text{losses}_1)_{\text{mean}}] \quad (17)$$

where $(\sum \text{losses}_1)$ refers to Group 1 losses evaluated from equation (4) for both guide and runner blade, at local conditions indicated by the subscripts. For stage designs showing some variation of stage work from root to tip the value used in equation (3) must similarly be derived from averaging techniques.

For very long blades the accuracy of the above procedure will be improved by weighting the individual section losses and work done by the local mass flow. For stages in which streamline curvature procedures are used to establish the velocity conditions, the Group 1 losses can be introduced locally along each streamline within the calculation procedure. Thus the work done by the blading and the isentropic heat drop are evaluated locally across the stage; overall mean values can then be obtained by integration. Substitution of the identical calculation for the sum of the efficiency debits will give the net work done, and the overall total head stage efficiency can be obtained from the basic definition of total head stage efficiency as

$$\eta_T = \frac{\text{integrated net work done}}{\text{integrated overall isentropic total heat drop}}$$

This process is exact and, for very large heat drops, can show up to 0·5 per cent discrepancy from that derived from the approximate relation given in equations (3) and (4). It should be noted that the annulus loss has been included in Group 1 losses for convenience, but it should, in fact, be evaluated using the average exit velocity and held constant over the blade length.

While a detailed consideration of streamline curvature techniques is outside the scope of this paper, the following relevant comments can be made. There is substantial evidence that the multi-stage streamline curvature programmes do in fact compute a flow solution similar to that actually occurring within a model turbine stage, but it is also clear that considerable care must be exercised over the assumptions which have to be made in order to effect a solution. For example, the form of the tip streamline has a considerable influence on the overall flow solution, and in wide flared steam turbine stages it is not immediately obvious where the effective inviscid flow boundary should be positioned. Local reaction values are considerably influenced by the values of loss coefficient used in the solution; thus correct allowance for the radial variation of profile and secondary loss should be introduced automatically within the calculation procedure. Care must be taken, however, to ensure that the secondary flow loss concentrations near the tip and root are not allowed to accumulate stage by stage, as these losses are redistributed by internal shear action in a real fluid.

In the rear stages of a l.p. cylinder, considerable streamline displacements occur in the regions where there is a large annulus flare. Under certain conditions this effect can result in a difference of specific mass flow between inlet and outlet stations across a blade row. This flow condition, of course, will considerably modify the loss of each section compared with its equivalent cascade value, which has been evaluated from conditions where the specific mass flow is constant across the section. A simple procedure to employ in these circumstances is to modify the one-dimensional contraction ratio definition used in Fig. 5 to a two-dimensional value obtained by multiplying the basic section value by the streamline contraction ratio from inlet to outlet. Partial confirmation of this procedure is provided by the tests reported by Deich *et al.* on flared guide blade losses (15). Finally, it should be noted that the geometric data on which the section loss is based should correspond to the cross section defined by the intersection of each meridional stream surface with the blade. In conditions where the velocity has a strong radial flow component this procedure will greatly modify the backbone length of the blade section.

In diverging streamline flow situations, the method proposed above is adequate only for blade sections which have a reasonably high value of the basic one-dimensional contraction ratio. Where this is not so, it is suggested that the following procedure be used. From some specific relation between total surface diffusion and loss, a value of the diffusion parameter can be evaluated which will correspond to the predicted (cascade equivalent) value of loss coefficient. This value of surface diffusion can then be increased in proportion to the relative decrease in specific mass flow across the blade section; a corresponding increased loss can be obtained from the loss–surface diffusion correlation. It is further suggested that an adequate loss–diffusion correlation may be derived from the correlation of blade outlet momentum thickness with total blade diffusion, described by Stewart, Whitney and Wong (16) in conjunction with the standard relation between total outlet momentum thickness and blade loss.

It should be noted that this correction is only strictly applicable to convergent blades operating with subsonic outlet velocities. If diverging streamline flow occurs across a blade section of small contraction ratio operating with supersonic outlet velocities, it is quite possible that the physical throat is effectively at the blade inlet and that the blade passage is of a convergent–divergent form. If this effect is pronounced, severe losses can result in the outlet *Ma* range 0·8–1·3.

ACCURACY OF THE METHOD

The overall validity of the method has been checked against a large number of turbines for which adequate test information was available. The purpose of the assessment, while obviously checking the reliability and accuracy of the overall method, was to establish whether consistent errors

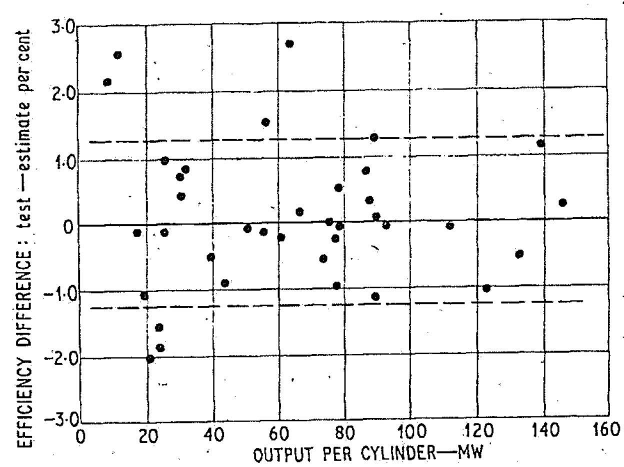

Fig. 23. Comparison of predicted and measured test efficiencies

:ould be detected from the comparison of calculated to neasured performance. For instance, if the method inrolved optimistic or pessimistic assumptions about econdary loss, then it might be expected that this would iave appeared as a consistent error in the comparison with lata obtained from measurements on turbines operating it h.p. levels. In fact no systematic or major discrepancies iave been found in an analysis of over fifty machines, nost calculated values of overall efficiency being subitantially within $\pm 1\frac{1}{4}$ per cent of the measured values. A plot of the data obtained is given in Fig. 23 where the efficiency error defined as the difference between the measured and calculated values of efficiency is shown, the output of each machine examined being used as a reference parameter. The tests with greater errors at small output (Fig. 23) refer largely to certain l.p. cylinder data where the measured values have been deduced from overall heat balances rather than from direct measurements.

It is claimed that these results give some grounds for confidence in the accuracy of the method and suggest that cascade test data, which form the foundation of the method, are a more reliable guide than some would have us believe

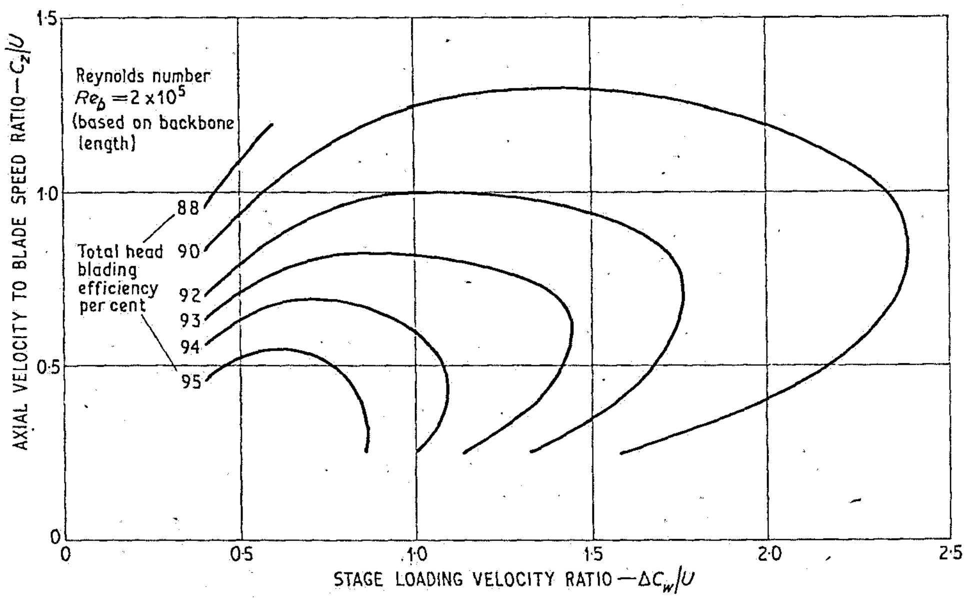

Fig. 24. Calculated total head efficiency for symmetric velocity triangle designs with guide and runner aspect ratios equal to 4·0

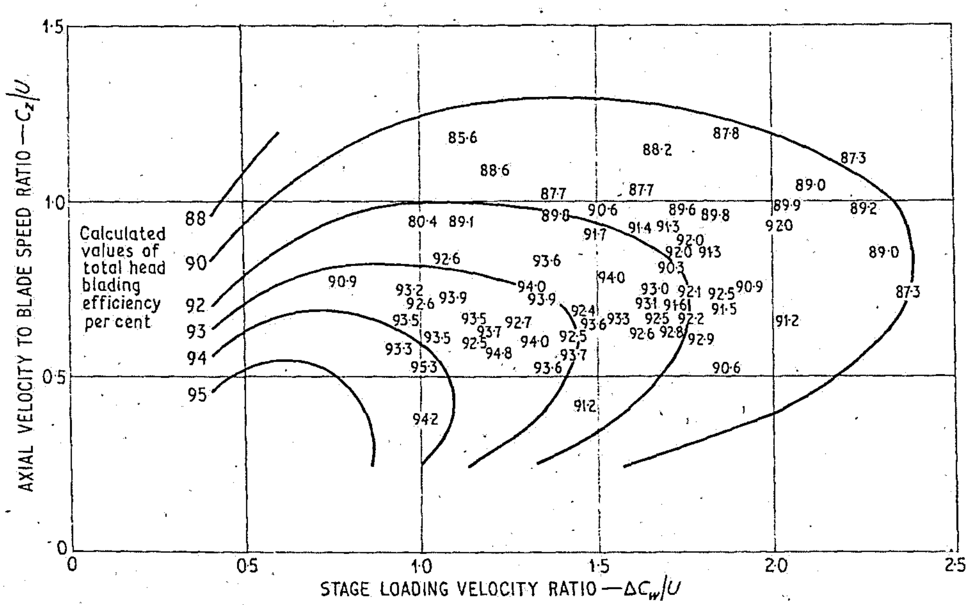

Fig. 25. Overlay of calculated total head efficiency contours given in Fig. 24 with test data published in (18)

when they are carefully interpreted. It is recognized that the efficiency comparison shown in Fig. 23 does not necessarily imply that the error on each loss constituent is small, but simply that the algebraic sum of the errors is small, i.e. some errors could be self-cancelling. However, the comparison of measured to calculated efficiencies has been extended over such a wide variety of designs that if any large constituent error existed in practice, one could expect it to have shown up as a major discrepancy in the overall efficiency.

Application to general turbine design

A theoretical method of performance prediction has the advantage that it gives the designer a method by which the relative performance merits of varying arrangements can be assessed in the early design phase. For preliminary design studies it is useful to construct generalized performance carpets which predict the stage efficiency changes that are implied by the variation of certain basic aerodynamic parameters. Three such carpets are presented in Figs 24, 26 and 27. In each instance blade data representing good design practice is used to compute the performance.

The data in Fig. 24 have been based on conditions typical of normal gas turbine practice, assuming a stage design based on symmetrical velocity triangles with a blade aspect ratio $h/b = 4{\cdot}0$ and $Re_b = 2\times10^5$. In Fig. 25 this carpet has been superimposed on the test results for gas turbines published by Smith (17), and the agreement is good. Data in Figs 26 and 27 are based on velocity triangles more typical of h.p. impulse steam turbine practice assuming a zero outlet swirl. The data in Fig. 26

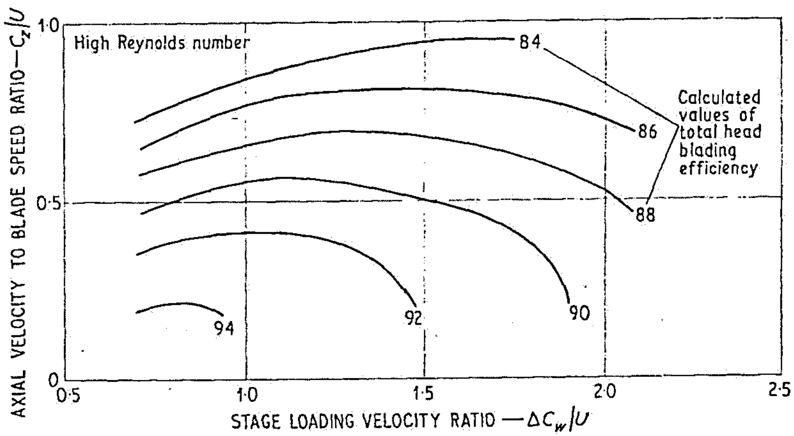

Fig. 26. Calculated total head blading efficiency for zero outlet swirl designs with guide and runner aspect ratios equal to 0·5 and 1·0 respectively

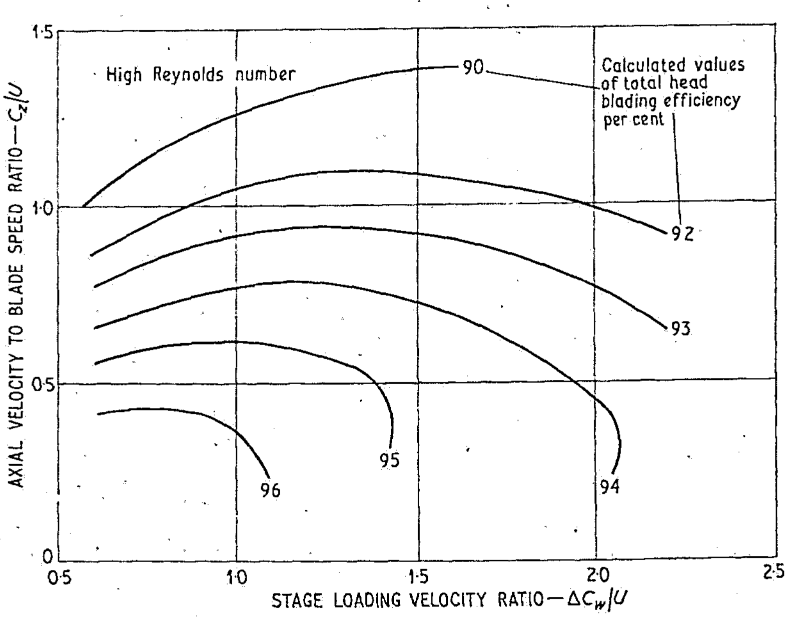

Fig. 27. Calculated total head blading effiency for zero outlet swirl designs with guide and runner aspect ratios equal to 4·0

assume aspect ratios of 0·5 and 1·0 on the guide and runner blade respectively, in contrast to the data in Fig. 27 which assume the uniformly high value of 4·0 for both blades. Comparison of Figs 26 and 27 indicates the magnitude of the aspect ratio effect predicted by the method at high *Re*. The *Re* effect can be deduced from a comparison of Figs 24 and 27 at the value of the stage loading parameter $\Delta C_w/U = 1{\cdot}0$, a condition where both velocity triangle configurations are identical.

In all calculations used to construct the carpet diagrams in Figs 24, 26 and 27 the method presented has been used in conjunction with the following simplifying assumptions:

(1) efficiency is based on the sum of profile and secondary losses only, i.e. excluding the tip clearance losses, etc.;

(2) the value of efficiency applies to a single section only, i.e. that corresponding to the specified velocity triangles;

(3) subsonic flow.

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APPENDIX

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