ENGS 23 Problem Set # 4 2016

Assigned: 13 April 2016
Due: 20 April 2016

- Turn in a hardcopy of all of your Matlab code to receive credit for any problem set.
- Put your name in the title of all plots.
- Add copious comment lines to your code.
- Use the grid on command to make your plots useful.
- Upon request, be prepared to send me an electronic copy of your Matlab code.

- Use the legend() command to distinguish among several plotted output signals.
- Do not combine multiple plots with subplot() command.
- Staple all pages from each problem together. Do not staple different problem together, but instead secure them with a paper clip.

1. (35 pts) Deduce Thermal Diffusivity from a Model

You will receive a Matlab p-code file, PS_4_1, that will simulate the thermal response of a system, T(x,t), as described in SFW, Sec. 3.4.1 – 3.4.3. The system is responding to Type I homogeneous boundary conditions, and a nonzero initial condition, h(x), has been imposed along the length of the bar; however, it will not be as simple as some *constant* value from 0 to L. Nonetheless, your system will respond according to the fundamental series solution near the end of Sec. 3.4.2. Review the details of the expression for the solution in order to do this problem.

a) Run the code according to the instructions attached at the end of the problem set, study the temperature plots, $\phi(x)$ and $\psi(t)$, then deduce the value of *thermal diffusivity*, D, for your system. Note that the fourth plot (semi-log temperature) will not be useful, unless you define a sufficiently large value of time in 'xt'.

b) Assume k = 100 watt/(m °C), then compute the maximum *magnitude* of heat flux density, |q|, to be found *anywhere* along the bar at t = 100 seconds, by applying Fourier's Law. If you wish, use the Paintbrush tool above each figure to extract x and T data, and use polyfit() to compute slopes.

c) Determine the *number of terms, n,* in the fundamental series solution that was used to compute temperature T(x,t), using the fact that the number of terms has been set to equal *a power of 2*, to avoid ambiguity. Hint: Study Fig. 3.16 to confirm how n is related to the number of *extrema* in each mode: Each plot in the right column of the figure is one of the terms of your Sine Series from last week.

d) Type the commands immediately below in the Command window to overlay the plots of figure(2) for two different but *very* small values of time; make your choice of the two small values of time to illustrate the fact that *high-order modes disappear very rapidly*, while lower-order modes hardly decay at all for small values of time. Iterate your choice of time values as necessary to give a clear illustration. This fact of very rapid decay of high-order modes is fundamental to the solution to this problem derived near the end of Sec. 3.4.2.

```
xt = [ <position value> <small time value>];
PS_4_1
figure(2)
hold on
xt = [ <same position value> <another smaller time value>];
PS_4_1
figure(2)
print
hold off
< annotate hardcopy of figure(2) by hand to make point described above>
```

2. (30 pts) Steady-State Heat Conduction in Cylindrical Coördinates

Study SFW, Sec 3.2.4 in order to understand how heat conduction in cylindrical coördinates differs from rectangular. The derivation is familiar, but a distinctive feature of cylindrical geometry intrudes itself throughout, as you will see. Assume: that *Type I boundary conditions* have been applied to a cylindrical domain at R_a and R_b, exactly as shown in Fig. 3.5 of SFW, that you have waited for the *steady-state* to develop, and that a *partial* measurement of T(R) has been made for you to study.

a) Run the pcode PS_4_2 in the usual way to be given your *partial* measurement of the T(R) data and the unknown value that you are to deduce from the given information, e.g., find T_a or find T_b. Apply your study in the previous part to find the *numerical* value of the unknown variable specified by the pcode.

b) Now study Sec. 3.2.4 further, make note of the numerical value for thermal conductivity, k, that the pcode gave you above, then compute the *numerical* values of the following variables, *if possible*:

Heat flux density, q, in radial direction at $R = R_a$
Heat flux density, q, in radial direction at $R = R_b$
Heat flux density, q, in axial direction (out of page) at $R = R_a$
Heat flux density, q, in axial direction (out of page) at $R = R_b$
Heat flux per unit length in radial direction (no symbol defined) at $R = R_a$
Heat flux per unit length in radial direction (no symbol defined) at $R = R_b$
Total heat flux, Q, from R_a to R_b

Explain how you compute each variable, if it is possible; if not, explain why not.

Matlab Note: Recall that Matlab has *three* logarithm functions, so be sure to double-check which one you intend to use in analyzing your data.

3. (35 pts) Numerical Solution to Transient, 1-D Heat Equation

a) Write your own Matlab code to solve the transient 1-D Heat Equation, according to the method of SFW, Sec. 3.3, as discussed in class. The thermal properties of the bar you will analyze will be defined by running the pcode PS_4_3 in the usual way, as noted below. Other specifications for the problem as defined as follows:

Heat Equation
1-dimensional analysis along x-axis, no variations in y- or z-directions
Transient analysis, not steady-state
Unknown variable is T(x,t)
Boundary condition at x=0 (left end of bar) is Type I inhomogeneous: $T(x=0,t) = T_0 \sin \omega t$, where $T_0 = 10$ °C
T, period of input waveform, specified by pcode
Boundary condition at x = L (right end of bar) is Type I homogeneous: $T(x=L, t) = 0$
Initial condition is zero: $T(x,0) = 0$
L, length of bar, specified by pcode
k, thermal conductivity, specified by pcode
ρ, mass density, specified by pcode
c_p, specific heat at constant pressure, specified by pcode
A, cross-sectional area, not specified
Δx, spatial step, = 5 mm, or a smaller value if you see fit
γ, numerical parameter of Sec. 3.3 much smaller than stated stability criterion, your choice
(Be mindful of long run times with small γ.)

b) Why does the value of A not matter in this problem?
c) Compute the response of the signal T(x,t) at the location x = 5 cm, and wait for the *sinusoidal steady-state*; this means that you compute for a sufficiently long time that successive peaks have virtually the same value. Hand in a hardcopy of your plot of T(x=5 cm, t) against time, as well as the input temperature signal at x = 0, and be sure your plot shows t = 0, as well as the sinusoidal steady-state. Use the *text()* command to print your value of γ within the plot itself.
d) What is the zero-to-peak temperature excursion (mathematical amplitude) of T(x=5 cm, t)?
e) Annotate your hardcopy from part c) to show that the *period* of the signal in the sinusoidal steady-state equals the value given to you by the pcode.
f) Use your code to compute the amplitude of the sinusoidal signal in the steady state at two other locations of your choice along the bar. Plot those two amplitudes along with that from d) on semilog axes (position along bar on the linear scale). State mathematically how the amplitude drops off along the length of the bar: Write a specific numerical expression for the amplitude of $T_{sss}(x)$ as a function of x.
g) Inspect your plot from part c), then find the *phase shift* of the temperature signal at x = 5 cm with respect to the input temperature signal at x = 0. Is the phase shift a *lag* or a *lead*? What is its value in seconds, radians, and degrees?

Instructions for running your pcode simulations for ES 23 PS #4, Problem 1:

Type in the Matlab Command window, "code = ", then copy and paste your code number from the blitz message into the space to the right of the equal sign, and hit Return, as usual.

In the Command Window, define the global variable that you will use to control calculations by typing "xt = [0.05   1]". Here, 0.05 is the position of interest along the bar (in meters), and 1 is the time of interest (in seconds).

In the Command Window, type "PS_4_1", and hit return.

The code computes the response of temperature T(x,t) along the bar with:
  rectangular coordinates in one dimension (insulated sides),
  no internal heating,
  both ends fixed at temperature = 0,
  your own initial condition, h(x),
  your own bar length, L,
  and your own value of thermal diffusivity, D;
  everyone has thermal conductivity, k = 100 W/(m °C).
No noise has been added to the computed data.

Four plots will appear:
two of temperature vs position, x, along the bar (one at the time of interest that you have defined), and
two of temperature vs time at the position of interest that you have defined (one linear, one semi-log).

Spread them out on your screen, so as to be visible as you make repeated runs with PS_4.

Redefine the variable "xt" in the Command window to explore other positions along the bar and larger values of time, as you see fit; be sure to increment time in *powers of ten*, i.e., 10, 100, 1000,...in order to explore a suitably wide range of values. If you like, save typing by hitting the up arrow twice, editing the old command that appears, then hitting Return.

Rerun PS_4_1 with your new parameters defined in "xt" (again, use the up arrow if you like).

Zoom in on any figure, if it suits your purposes.

Exploit your control over these simulations of your system so as to answer the questions posed in the homework assignment. It goes without saying that a review of SFW, Sections 3.4.1-3.4.3 will be valuable!