ENGS 23 Problem Set # 7 2016

Assigned: 4 May 2016
Due: 11 May 2016

•Staple all pages from each problem together, and put your name on the first page of each problem. Do not staple different problems together, but instead join them with a paper clip.
•Turn in a hardcopy of all of your Matlab code to receive credit for any problem set.
• To receive credit for any problem set request, be prepared upon request to send me an electronic copy of your Matlab code, to write your code from scratch under closed-book conditions, and to explain how your code works.
•For plots, use the grid on and legend() commands, but do not use the subplot() command.
•Paste relevant results from the Command window into another document and print it to document your work.
•No credit for an answer in comment lines of code; give answers in plots with text() command or in a hardcopy of the Command window.

1. (35 pts) Gradient and Divergence
The pcode, PS_7_1, to be blitzed to you will compute a matrix, T, similar to SFW, Fig. 5.4 or 5.19. Treat its scalar values as *temperature* in °C at points spaced 1 meter apart on a 2-D rectangular grid in the steady-state. Assume temperatures do not vary when planes above and below the given one are considered, so that the problem is fundamentally *two-dimensional*; i.e., there is no heat flow perpendicular to your given plane. Also assume the temperatures have been measured in a volume with a dimension, H, *perpendicular* to the given plane, for the sake of later calculations. Your value of H will appear in the title of your plot.
a) Sketch a control volume suited to the domain described above and indicate any surfaces that must be insulated to satisfy the description above. Your sketch is critical, because it will guide your thinking about this problem. Formally apply the Divergence Theorem to your sketch, in preparation for determining how much heat energy is being released within the volume.
b) Use the Matlab commands introduced in SFW Sec 5.2 first to make a contour plot of temperature to orient yourself to the temperature field.
Be sure to *think* about where heat must be flowing before taking the next steps:
c) Apply the concepts of Sec. 5.4 to compute and plot the local *heat fluxes*, Q(x,y), on the grid and depict the vector values with arrows, assuming thermal conductivity, κ, equals the value appearing in the title of your plot. Again, simply scale your calculation so as to compute total heat flux flowing in your domain of height, H, *not* the heat flux density, q(x,y).
d) Comment on how the largest and smallest heat fluxes are associated with spacing of the temperature contour lines in your figure from part b).
e) Further consider what powerful Sec. 5.5 concepts (and related Matlab operations) are relevant, then compute the value of total heat flux, Q_{out}, *out* of the control volume, realizing it might be zero or negative (i.e., net inflow). Again, this is a value in watts, not watts/m^2. Compute this by applying Fourier's Law explicitly to cells around the boundary, as was done in class; do not compute the sum of internal divergence.
f) Must there be heat sources or sinks located *within* your volume? Can you quantify their *sum*, as a number in watts? If so, what must be the sum?
g) Review the relevant Matlab commands, then compute

```
[qx, qy] = gradient(T);
sum(sum(divergence(qx, qy)));
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Does this value equal your result from the previous part? If not, explain why.

h) By inspection of your plots, can you specify the number of *distinct* sources and sinks within your volume? You know the total power put out by all sources, and now you are attempting to describe the sources more specifically.

i) Can you identify the locations of your sources by marking one of your plots?

j) Can you identify any *insulated* boundaries on your domain? Note that a given rectangle edge may not have a single boundary type enforced along its entire length; instead, the type of boundary may change midway along the edge. Indicate insulated boundaries or half-boundaries on a hardcopy.

k) Can you identify any Type I boundaries? Again, the type of boundary may change midway along an edge. If so, determine the boundary temperature value and mark such boundaries on a hardcopy to hand in.

2. (30 pts) Numerical Solution of Laplace's Equation

Consider a thermal system defined on the 2-D rectangular domain below, in which the height out of the plane, H, and the thermal conductivity, k, are values to be conveyed to you by pcode. Think of the domain as divided into 1 m x 1 m cells, as previously. There are no heat flux sources within the domain. The system output is the unknown temperature of each cell, as defined in the diagram.

a) Run the pcode, PS_7_2, to be sent to you, to learn what boundary conditions will apply to your domain, and what your values of M and N are. Review your class notes, then write Matlab code that will solve by iteration for the steady-state temperature distribution across your domain. Do not use the Matlab *del2()* command, but write nested loops instead. Appreciate that a *single* formula applies to *all* interior nodes, and that different formulas apply to boundary nodes. Let your iterations run until doubling the number of iterations changes the value of an interior node by only a few percent. A more specific convergence criterion will not be defined, because it is not the point of the problem.

b) Make a contour plot of your computed steady-state distribution (with y axis *reversed* to correspond with the matrix of temperatures), and annotate it by hand to show you have satisfied your boundary conditions. Strengthen your contour plot by labeling the contours with suitable, round values of temperature, not the default values (type "help contour" and "help clabel").

c) Choose the non-insulated boundary with the lowest temperature, then compute the total heat flux, Q, *out* of your domain along that boundary, as you have done previously.

d) Describe how to build a physical model of an *analogous electrical* system, apply analogous boundaries to it, and measure an analogous output from it.

3. (35 pts) Quantification of Nonideal Peltier Source

Run the pcode to be sent to you in the usual way to produce a plot of temperatures near the top and bottom of the bar from the Steady-State Heat Equation Lab exercise after hot water has been added to the dish in which the bottom of the bar is immersed. You made this intervention during the lab (or did the opposite, using ice water). The purpose was to change the temperature at the top of the bar, where the Peltier unit is, then answer the question: Does the Peltier unit put out *less* heat when it must operate at a higher temperature? The answer is yes, because the device is a non-ideal heat flux source. Your task in this problem is to *quantify* the known non-ideal behavior.

a) Draw the electrical circuit that is analogous to this lab apparatus, treating air temperature as circuit ground (not water temperature); the reason for this is that the Peltier unit is coupled to air temperature by the fan blowing room temperature air over the Peltier heat sink. The water dish is at some different temperature than air, and water is coupled to the bar through a resistor, R_III, whose value you measured during the exercise. Be sure to treat the Peltier an a non-ideal heat flux source, because this is the point of the exercise.

b) Review your plot of simulated data, then deduce the value of heat flux down the bar in the brief steady-state that follows each change in water temperature. Plot this value against the Peltier temperature, measured as the difference from air temperature. From this plotted data, deduce the value of the parallel resistor, $R_{Peltier}$, that makes the Peltier unit a non-ideal heat flux source.

c) Refer to your circuit diagram from part a), then explain why the Peltier unit would behave as an *ideal* heat flux source if the value of $R_{Peltier}$ approached infinity.

d) Annotate your circuit diagram from part a) with labeled current values through each resistor, for the case of the highest Peltier temperature that you plotted. Summarize your understanding of the effect of $R_{Peltier}$ by commenting on the effect that its finite value has on the current through the resistor representing the bar.

Definition of Temperature Matrix: