الموضوع:

التاريخ: / /

« Half-Angle Substitution »

التكامل باستخدام نصف الزاوية

هو طريقة تستخدم في تكامل الدوال المثلثية، خاصة عندما تكون هناك دوال مثلثية مرفوعة لأسس زوجية أو عبارات تحتوي على جذور مربعة أو إذا كان كسر مقامه لا يحتوي على كثيرات حدود ولكن دوال مثلثية « $\sin$, $\cos$, $\tan$, ... »

$$\Rightarrow U = \tan\frac{x}{2}$$

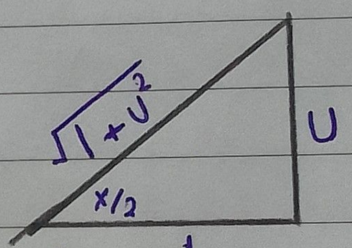
$$\therefore du = \frac{1}{2}\sec^2\frac{x}{2}\,dx = \frac{1}{2}\left[1+\tan^2\frac{x}{2}\right]dx$$

$$du = \frac{1}{2}\left[1+U^2\right]dx \Rightarrow \boxed{dx = \frac{2\,du}{1+U^2}}$$

حفظ #

$$\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$$

$$= 2\,\frac{U}{\sqrt{1+U^2}}\cdot\frac{1}{\sqrt{1+U^2}} = \frac{2U}{1+U^2}$$



$$\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$$

$$*\cos\frac{x}{2} = \frac{1}{\sqrt{1+U^2}} \quad \& \quad *\sin = \frac{U}{\sqrt{1+U^2}}$$

$$= \left(\frac{1}{\sqrt{1+U^2}}\right)^2 - \left(\frac{U}{\sqrt{1+U^2}}\right)^2 = \frac{1}{1+U^2} - \frac{U^2}{1+U^2} = \frac{1-U^2}{1+U^2}$$

$$\int\frac{dx}{2+\cos x} = \int\frac{2du/1+U^2}{2+(1-U^2/1+U^2)} = \int\frac{2du/1+U^2}{\cancel{2-2U^2/1+U^2}} = \int\frac{2du/\cancel{1+U^2}}{3+U^2/\cancel{1+U^2}} = \int\frac{2du}{3+U^2}$$

$$1-U^2+2+2U^2/1+U^2$$

$$= 2\int\frac{du}{(\sqrt{3})^2+U^2} = \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{U}{\sqrt{3}}\right)+C = \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\tan\frac{x}{2}\right)+C$$

تكاملات عكسية !!

$$\left.\int\frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)\right|$$

$$\Rightarrow \int \frac{dx}{\sin x + \tan x} = \int \frac{2du/1+u^2}{\frac{2u}{1+u^2} + \frac{2u}{1-u^2}}$$

$$\therefore \tan x = \frac{\sin x}{\cos x}$$

$$= \frac{2u}{1+u^2} \Big/ \frac{1-u^2}{1+u^2}$$

$$= \frac{2u}{1-u^2}$$

$$= \int \frac{2du/\cancel{1+u^2}}{\frac{2u(1\cancel{-u^2}+1\cancel{+u^2})}{\cancel{(1+u^2)}(1-u^2)}} = \frac{1}{2}\int \frac{(1-u^2)\,du}{u}$$

$$\tan\frac{x}{2} \leftarrow \quad = \frac{1}{2}\left[\ln|u| - \frac{u^2}{2}\right] + c \quad \rightarrow \tan^2\frac{x}{2}$$

⇐ تكامل حاصل ضرب الدوال المثلثية :-

1. $\int \sin mx \cdot \sin nx \, dx = \frac{1}{2}\left(\cos(m-n)x - \cos(m+n)x\right)$

2. $\int \cos mx \cdot \cos nx \, dx = \frac{1}{2}\left(\cos(m-n)x + \cos(m+n)x\right)$

3. $\int \sin mx \cdot \cos nx \, dx = \frac{1}{2}\left(\sin(m-n)x + \sin(m+n)x\right)$

$$\Rightarrow \int \sin 3x \cos 5x \, dx = \frac{1}{2}\int (\sin(3-5)x + \sin(3+5)x)$$

$$= \frac{1}{2}\int [-\sin 2x + \sin 8x] dx = \frac{1}{2}\left[\frac{1}{2}\cos 2x - \frac{1}{8}\cos 8x\right] + C$$

$$= \frac{1}{4}\cos 2x - \frac{1}{16}\cos 8x + C$$

$$\Rightarrow \int \cos\frac{x}{2} \sin\frac{x}{3} \, dx =$$

$$= \frac{1}{2}\int (\sin(\frac{1}{2}-\frac{1}{3})x + \sin(\frac{1}{3}+\frac{1}{2})x) = \frac{1}{2}\int [\sin\frac{x}{6} + \sin\frac{5x}{6}] dx$$

$$\int \sin -\frac{x}{6} dx = \int \sin\frac{x}{6} dx = -6 \cos\frac{x}{6}$$

$$\int \sin\frac{5x}{6} dx = -\frac{6}{5}\cos\frac{5x}{6}$$

$$= \frac{1}{2}\left(-6\cos\frac{x}{6} - \frac{6}{5}\cos\frac{5x}{6}\right) = -3\cos\frac{x}{6} - \frac{3}{5}\cos\frac{5x}{6} + C$$

$$\Rightarrow \int \sin 5x \sin 2x \, dx = \frac{1}{2}\int (\cos(5-2)x - \cos(5+2)x) dx$$

$$= \frac{1}{2}\int [\cos 3x - \cos 7x] dx = \frac{1}{2}\left(\frac{1}{3}\sin 3x - \frac{1}{7}\sin 7x\right) + C$$

$$= \frac{1}{6}\sin 3x - \frac{1}{14}\sin 7x + C$$

!! أخر اثنين
الدكتور
محلهمش