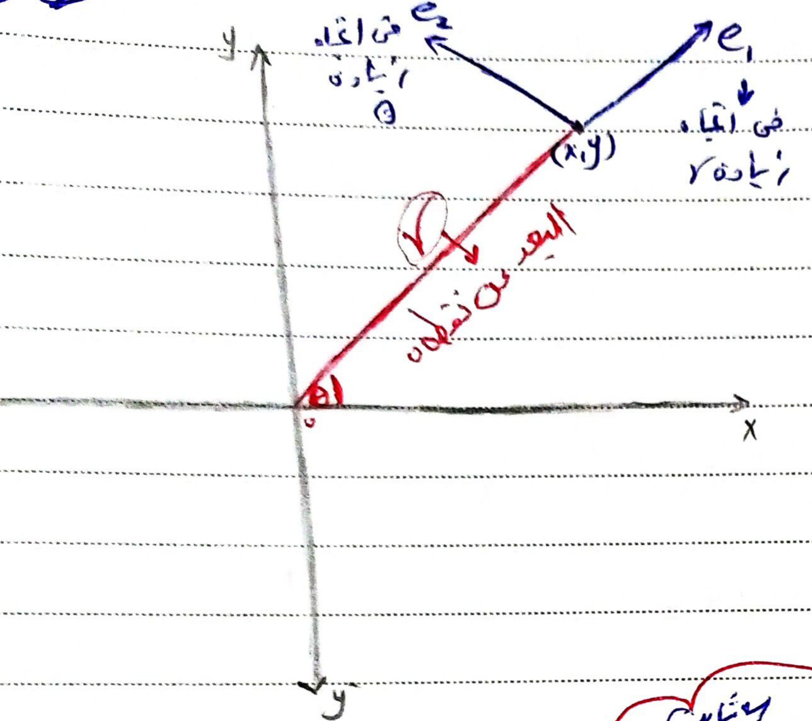
محاضرة ٤

الاحداثيات القطبية

Date ....../....../...... No ....



θ ← الزاوية فى اتجاه x-axis
لسه من قيمتها نعرف النقطة فى اى ربع

* $\dot{\theta}$ ← السرعة الزاوية

* $\ddot{\theta}$ ← العجلة الزاوية

ثابت $\omega = \dot{\theta}$
$zero = \dot{\omega}$

* $r = r e_1$

* $\vec{V} = \dot{r} e_1 + r \dot{\theta} e_2$ ← تفاضل r

* $\vec{F} = \ddot{r} e_1 + \dot{r}\dot{\theta} e_2 + \dot{r}\dot{\theta} e_2 + r\ddot{\theta} e_2 - r\dot{\theta}^2 e_1$

$\therefore \vec{F} = F_r e_1 + F_\theta e_2$

$\therefore \vec{F_r} = \ddot{r} - r\dot{\theta}^2$

$\vec{F_\theta} = 2\dot{r}\dot{\theta} + r\ddot{\theta}$ أو $\frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})$

$= \frac{1}{r}[2r\dot{r}\dot{\theta} + r^2\ddot{\theta}]$

$|F| = \sqrt{F_r^2 + F_\theta^2}$

ex,

- A Particle moves on the curve $r = a + b\sin\theta$ with a constant angular velocity ω, where a and b are constants. Find the acceleration for this Particle.

Solution

$$\dot{r} = b\omega\cos\theta$$

$$\ddot{r} = -b\omega^2\sin\theta$$

$$F_r = -b\omega^2\sin\theta - a\omega^2 - b\omega^2\sin\theta$$

$$= -2b\omega^2\sin\theta - a\omega^2$$ ← عوزين نطلع بدلاله r

$$= -2\omega^2[r - a] - a\omega^2$$ ← ممكن تبسط الثمن هكذا

$$F_\theta = 2b\omega^2\cos\theta$$

$$\because b^2\sin^2\theta + b^2\cos^2\theta = b^2$$

$$\therefore b^2\cos^2\theta = b^2 - b^2\sin^2\theta$$

$$\therefore b\cos\theta = \sqrt{b^2 - (b\sin\theta)^2}$$

$$F = 2\omega^2[b^2 - (r-a)^2]^{\frac{1}{2}}$$

ex$_2$

- Considering an illustration the ~~mothion~~ mation of a Particle in a circular trajectory having angular Velocity $\omega = \dot{\theta}$, and angular acceleration $\alpha = \dot{\omega}$. Calculate the velocity and the acceleration components.

Solution

$r = R$ ← نصف القطر
ثابت

$\dot{r} = zero$

$\ddot{r} = zero$

$\dot{\theta} = \alpha t + c$

علشان اجيب قيمة c نعوض بالشروط الابتدائية

$\therefore c = \omega$

$\therefore \dot{\theta} = \alpha t + \omega$

ex3

- A Particle moves with $\dot{\theta} = \omega = constant$ and $r = r_o e^{\beta t}$, where r_o and β are constants. Prove that for certain values of β, the particle moves with $a_r = 0$

Solution

$\dot{r} = r_o \beta^2 e^{\beta t}$

ملاحظة
r_o, β ثوابت

$a_r = r_o \beta^2 e^{\beta t} - r_o e^{\beta t} \omega^2 = zero$

$= r_o e^{\beta t} (\beta^2 - \omega^2) = zero$

$\beta^2 - \omega^2 = 0$

$\beta = \pm \omega$