محاضرة 6

chapter 3

Date ..../..../.... No ....

Simple Harmonic Motion

الحركة التوافقية البسيطة

مراجعه القوانين

$* \; x'' = -w^2 x$

$* \; v^2 = w^2 (a^2 - x^2)$

$\therefore \; x = a\cos(wt + \varepsilon)$ → زاوية الطور

$-1 \le \cos \le 1$

$-a \le x \le a$

ممكن تتغير و ميبقاش center $(0,0)$

$* \; V_{max} = wa$

$* \; F_{max} = w^2 a$

الزمن الدوري

- Periodic time $= "\tau" = \dfrac{2\pi}{w}$

- Frequency $= "N" = \dfrac{1}{\tau} = \dfrac{w}{2\pi}$

ex. Find the Periodic time for the simple harmonic motion which defind by $x'' = -25x$.

Solution

$\because \; x'' = -w^2 x$

$\therefore \; w = 5$

$\tau = \dfrac{2\pi}{5}$ ##

ex_2 - Calculate the maximum velocity and the Maxigmum acceleration For a Partical moves a Simple harmonic motion with a Perrodic time $\frac{\pi}{4}$ sec and an amplitude equal to 25 cm.

Solution

$$t = \frac{\pi}{4} \quad , \quad a = 25$$

$$t = \frac{2\pi}{\omega} \quad \therefore \quad \frac{\pi}{4} = \frac{2\pi}{\omega}$$

$$\omega = 8$$

$$\therefore V_{max} = \omega a = 8 \times 25 = 200$$

$$F_{max} = \omega^2 a = 8^2 \times 25 = 1600$$

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ex_3 IF the Position x For a moving Particle Can be determind by $x = 0.45 \cos \frac{\pi t}{4} - 0.28 \sin \frac{\pi t}{4}$ the end of three Consecutive Seconds Prove that the totel Periodic time is:

$$2\pi / \cos^{-1}\left(\frac{x_1 + x_2}{x_2}\right)$$

Solution

$$\therefore x = 0.45 \cos \frac{\pi t}{4} - 0.28 \sin \frac{\pi t}{4}$$

x تفاضل $\longleftarrow$ $x' = V = -\frac{0.45\pi}{4} \sin \frac{\pi t}{4} - \frac{0.28\pi}{4} \cos \frac{\pi t}{4}$

V تفاضل $\longleftarrow$ $x'' = F = (\frac{\pi}{4})^2 x \longrightarrow *$

$A = 0.45 \quad , B = -0.28$

$a = \sqrt{A^2 + B^2} = \sqrt{(0.45)^2 + (0.28)^2} = 0.53$ #

$\tau = \frac{2\pi}{\omega} = \frac{2\pi \cdot 4}{\pi} = 8$ #

$V_{max} = \omega a = \frac{\pi \times 0.53}{4}$ #

$F_{max} = \omega^2 a = (\frac{\pi}{4})^2 \times 0.53$ #

$\varepsilon = \tan^{-1}(\frac{-B}{A}) = \tan^{-1}(\frac{0.28}{0.45}) = 0.557$

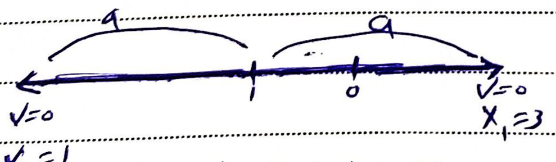
ε

ex$_6$ if the velocity of a moving Particle is obtained From the relation $v^2 = -2x^2 + 4x + 6$
Prove that the motion represents a simple harmonic motion Calculate its Center the maximum acceleration and the Frequency.

Solution

$x'' = \frac{1}{2}\frac{dv^2}{dx} = -(4x+4) \times \frac{1}{2} = -2(x-1)$

$\therefore$ Center $(1, 0)$ $\quad , \omega = \sqrt{2}$



$N = \frac{1}{t} = \frac{\omega}{2\pi} = \frac{\sqrt{2} \times \sqrt{2}}{2\pi \times \sqrt{2}} = \frac{1}{\sqrt{2}\,\pi}$

هنجيب المسافة بين X_1 , X_2 , القسمة على 2

$= \frac{-2x^2}{2} + \frac{4}{2}x + \frac{6}{2}$

$= x^2 - 2x - 3 = (x+1)(x-3)$

$\therefore x = -1 \quad , x = 3$

$a = 2$

Dar Kareem

$F_{max} = \omega^2 a = 2 \times 2 = 4$