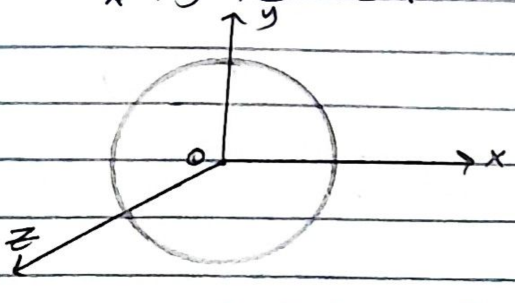
Date 15/4/2025 No Lecture (4)

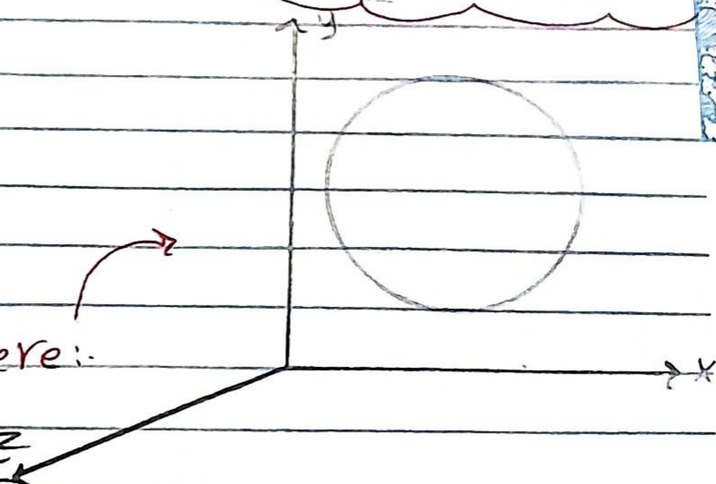
« equation of sphere »

* Equation of sphere that its center is origin point and its radius is r

$$x^2+y^2+z^2=r^2$$

← نفس معادلة الدائرة مع زيادة حد واحد فقط





* ~~general~~ standard form of equation of sphere:

$$(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2=r^2$$

center → (α, β, γ)

radius → r

$$x^2-2\alpha x+\alpha^2+y^2-2y\beta+\beta^2+z^2-2z\gamma+\gamma^2=r^2$$

ملاحظ:

← في معادلة الكرة يمكن أن يختفي الحد الذى من الدرجة الأولى ولكن لا يمكن أن يختفي الحد الذى من الدرجة الثانية وإذا إختفى حد من الحدود الثلاثة (x^2, y^2, z^2) يصبح الشكل قطع (ناقص أو زائد أو ---) بدلاً من الكرة.

* general form of Equation:

$$x^2+y^2+z^2+2uX+2vy+2wz+d=\text{zero}$$

مجموع الحدود المطلقة ← d

← في هذه الحالة من شكل المعادلة يصبح لدى حلين:

① ضبط القوانين. ② إكمال مربع.

② حفظ القوانين .. "نفرده الحل"

$$\text{Center} = (\alpha, \beta, \gamma) \Rightarrow \begin{cases} \alpha = -\frac{1}{2} \text{ Factor of } (x) \rightarrow u \\ \beta = -\frac{1}{2} \text{ Factor of } (y) \rightarrow v \\ \gamma = -\frac{1}{2} \text{ Factor of } (z) \rightarrow w \end{cases}$$

مجموع الحدود المطلقة → d

$$\text{radius } (r) = \sqrt{u^2 + v^2 + w^2 - d}$$

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ex (1):

- Find the equation of the sphere of center $(11, 8, -5)$ and radius 3 in standard form.

* Solution:

$r = 3$, $\alpha = 11$, $\beta = 8$, $\gamma = -5$

$$(x - \alpha)^2 + (y - \beta)^2 + (z - \gamma)^2 = r^2$$

$$(x - 11)^2 + (y - 8)^2 + (z - (-5))^2 = (3)^2$$

$$(x - 11)^2 + (y - 8)^2 + (z + 5)^2 = 9$$

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ex (2):

- Given that a sphere's equation is $(x+5)^2 + (y-12)^2 + (z-2)^2 - 289 =$ zero. determine its center and radius.

* Solution:

$$(x+5)^2 + (y-12)^2 + (z-2)^2 = 289$$

Compar with $\rightarrow (x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2 = r^2$

$\therefore \alpha = -5$

$\therefore \beta = 12$

$\therefore \gamma = 2$

$\therefore$ Center is $(-5, 12, 2)$

$\therefore r^2 = 289 \rightarrow \therefore r = \sqrt{289} =$ ٧ (m) ex(3):-

- Given $A = (0,4,4)$ and that $\overline{AB}$ is a diameter of the sphere $(x+2)^2 + (y+1)^2 + (z-1)^2 = 38$, what is the point B?

* Solution:-

$$(x+2)^2 + (y+1)^2 + (z-1)^2 = 38$$

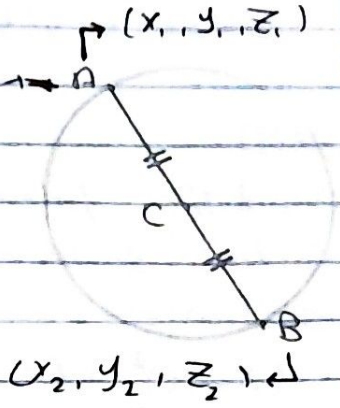
compar with $\rightarrow (x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2 = r^2$

$\therefore \alpha = -2$

$\therefore \beta = -1$

$\therefore \gamma = 1$

$\therefore$ center $= (-2,-1,1) \rightarrow ②$



$$\therefore C = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$$

$$C = \left(\frac{0+x_2}{2}, \frac{y_2+4}{2}, \frac{z_2+2}{2}\right) \rightarrow ①$$

* From ② in ①:-

$$\therefore C = (-2,-1,1) = \left(\frac{0+x_2}{2}, \frac{y_2+4}{2}, \frac{z_2+2}{2}\right)$$

① $-2 = \frac{0+x_2}{2} \Rightarrow x_2 = -4$

② $-1 = \frac{y_2+4}{2} \Rightarrow y_2 = -6$

③ $1 = \frac{z_2+2}{2} \Rightarrow z_2 = 1$

$\therefore$ point $B = (-4,-6,1)$

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ex(4):

- Find the center and radius of the sphere $x^2 + y^2 + z^2 - 8x + 8y + 10z + 8 = $ zero.

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* Solution:-

$$x^2+y^2+z^2-8x+8y+10z+8= \text{zero}$$

Compar with $x^2+y^2+z^2+2ux+2vy+2wz+d=\text{zero}$

$$\left.\begin{array}{l} \therefore \alpha = u = -4 \\ \therefore \beta = v = 4 \\ \therefore \gamma = w = 5 \end{array}\right\} \therefore \text{Center } (c) = (-4,4,5)$$

$$\therefore r=\sqrt{u^2+v^2+w^2-d} = \sqrt{(-4)^2+(4)^2+(5)^2-8} = \checkmark$$

Other solution:- ● بطريقة إكمال المربع :-

① $x^2-8x=(x+(-4))^2-(4)^2 \rightarrow (x-4)^2-16$

② $y^2+8y=(y+4)^2-(4)^2 \rightarrow (y+4)^2-16$

③ $z^2+10z=(z+5)^2-(5)^2 \rightarrow (z+5)^2-25$

$$\therefore (x-4)^2+(y+4)^2+(z+5)^2-16-16-25+8=\text{zero}$$

$$(x-4)^2+(y+4)^2+(z+5)^2=49$$

$$\left.\begin{array}{l} \therefore \alpha = u = -4 \\ \therefore \beta = v = 4 \\ \therefore \gamma = w = 5 \end{array}\right\} \therefore \text{Center } (c) = (-4,4,5)$$

$$\therefore r=\sqrt{u^2+v^2+w^2-d} = \sqrt{(-4)^2+(4)^2+(5)^2-8} = \checkmark$$

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ex(5):-

- Find the intercept made by the sphere $x^2+y^2+z^2=9$ on the line $x-3=y=z$.

* Solution:-

$x-3=y=z=t \quad \rightarrow x-3=t \quad \therefore x=t+3$

$\rightarrow y=t \quad \therefore y=t$

$\rightarrow z=t \quad \therefore z=t$

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• بالتعويض في معادلة الكرة :

$$x^2+y^2+z^2=9$$

$$(t+3)^2+(t)^2+(t)^2=9 \Rightarrow t^2+6t+9+t^2+t^2=9$$

$$\therefore 3t^2+6t=\text{zero}$$

$$3t(t+2)=\text{zero}$$

$3t=\text{zero}$ | $t+2=\text{zero}$

$\therefore t=\text{zero}$ | $\therefore t=-2$

$\therefore$ The intercept points is $((t+3),t,t)$

• when $t=\text{zero}$
The point is $(3,0,0)$

• when $t=-2$
The point is $(1,-2,-2)$

• Special Case:-

← نقطة التماس بين الخط المستقيم والكرة .

$$\frac{x-a}{\alpha}=\frac{y-b}{\beta}=\frac{z-c}{\gamma}$$

$(\alpha,\beta,\gamma) \Rightarrow$ نسب الإتجاه الخط المستقيم (l,m,n)
$(a,b,c) \Rightarrow$ نقطة على الخط المستقيم

The equation is $\boxed{x^2+y^2+z^2+2ux+2vy+2wz+d=\text{zero}}$

$$\begin{array}{ccc} a & b & c \\ \alpha & \beta & \gamma \\ u & v & w \end{array}$$

$(a\alpha+u\alpha+b\beta+v\beta+c\gamma+w\gamma)^2$ → The left side.

$(\alpha^2+\beta^2+\gamma^2)(a^2+b^2+c^2+2ua+2vb+2wc+d)$ → The Right side.

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If Right side = left side

يبقى فيه نقطة تماس بين الخط المستقيم والكرة .

ex:

- Check if $\frac{x+3}{4} = \frac{y+4}{3} = \frac{z}{5}$ is a Tangent To The sphere $x^2+y^2+z^2+4x+6y+10z = $ Zero

$a = -3$ $\quad b = -4$ $\quad c = $ Zero $\quad d = $ Zero

$\alpha = 4$ $\quad \beta = 3$ $\quad \gamma = 5$

$u = 2$ $\quad v = 3$ $\quad w = 5$

$[(-3\times4) + (2\times4) + (-4\times3) + (3\times3) + (0\times5) + (5\times5)]^2$

$= (-12+8-12+9+0+25)^2 = (28)^2 \longrightarrow$ left side

$= 324$

$((4)^2+(3)^2+(5)^2)((-3)^2+(-4)^2+(0)^2+(2\times2\times-3)+(2\times-4\times3)+(2\times0\times5)+0)$

$[16+9+25][9+16+0+(-12)+(-24)+0+0]$

$= (50)(-11) = -550$

$\because (a\alpha+b\beta+c\gamma+u\alpha+v\beta+w\gamma)^2 \neq (\alpha^2+\beta^2+\gamma^2)(a^2+b^2+c^2+2ua+2vb+2wc+d)$

$\therefore$ The given line intersects the sphere.