→ Chapter "2" «Equation of plane»

* Information about equation of plane:

(1) equation of plane is equation in three variables.

(١) معادلة المستوى تكون معادلة في ٣ متغيرات.

(2) variables are of the first degree.

(٢) المتغيرات تكون من الدرجة الثانية. (الأس = ١)

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* Equation of plane:

- There are three forms of the equation of plane.

[1] The vector form:

[1] شكل المعادلة المتجهة :

$$\Rightarrow \vec{n} \cdot (\vec{r} - \vec{r_0}) = zero$$

[2] The normal form:

[2] شكل المعادلة الطبيعي :

$$\Rightarrow \vec{n} \cdot \vec{r} = \vec{n} \cdot \vec{r_0} \quad or \Rightarrow a(x - x_0) + b(y - y_0) + c(z - z_0) = zero$$

[3] The Cartesian form (general form):

[3] شكل المعادلة الكارتيزي (الشكل العام) :

$$\Rightarrow ax + by + cz + d = zero \quad -d$$

$$ax + by + cz = -d$$

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* look:

ملاحظ :

- عندما يكون متجه أو نقطة (في الفراغ) موجودين في المستوى $yz \Leftarrow x = zero$:

- عندما يكون متجه أو نقطة (في الفراغ) موجودين في المستوى $xy \Leftarrow z = zero$:

- عندما يكون متجه أو نقطة (في الفراغ) موجودين في المستوى $xz \Leftarrow y = zero$:

وتسمى معادلة المستوى في هذه الحالات بـ (standared form)

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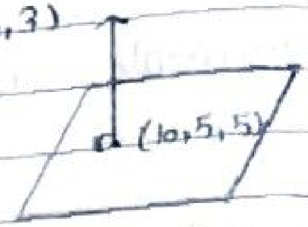
* How to find the position of plane:

[1] point in this plane and normal vector ($\vec{n}$).

[1] يجب وجود نقطة في هذا المستوى و متجه عمودي على هذا المستوى.

ex(1):-

- Give the equation of the plane with normal vector (10, 8, 3) that contains the point (10, 5, 5)



* solution:-

$\vec{n} \cdot (\vec{r} - \vec{r_0}) = $ zero "The vector form"

$(10, 8, 3) \cdot ((x, y, z) - (10, 5, 5)) = $ zero

$(10, 8, 3) \cdot (x - 10, y - 5, z - 5) = $ zero

$\therefore 10(x - 10) + 8(y - 5) + 3(z - 5) = $ zero "normal form" $\vec{r_0} = (10, 5, 5)$

$\vec{n} = (10, 8, 3)$

$10x - 100 + 8y - 40 + 3z - 15 = $ zero

$10x + 8y + 3z - 155 = $ zero "The cartesian form (general)"

$\vec{n} \cdot \vec{r} = \vec{n} \cdot \vec{r_0}$

$(10, 8, 3) \cdot \vec{r} = (10, 8, 3) \cdot (10, 5, 5)$

$(10, 8, 3) \cdot \vec{r} = 100 + 40 + 15 = 155$ "The normal form"

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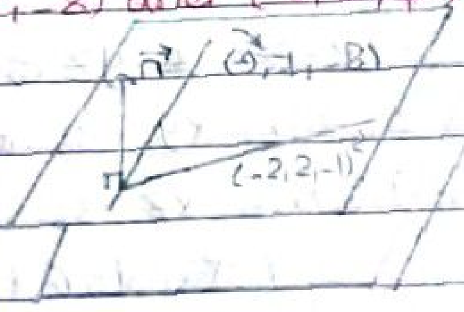
[2] 2 vectors and point in this plane.

[2] متجهين (في المستوى أو أي مستوى آخر له علاقة بالمستوى الأول) ونقطة به.

ex(2):-

- Find the general equation of the plane that passes through the point (5, -1, +1) and is parallel to the two vectors (9, 7, -8) and (-2, 2, +1)



* Solution:-

$$\vec{n} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 9 & 7 & -8 \\ -2 & 2 & 1 \end{vmatrix} = \underline{i} \begin{vmatrix} 7 & -8 \\ 2 & 1 \end{vmatrix} - \underline{j} \begin{vmatrix} 9 & -8 \\ -2 & 1 \end{vmatrix} + \underline{k} \begin{vmatrix} 9 & 7 \\ -2 & 2 \end{vmatrix}$$

$\therefore \vec{n} = +23\underline{i} + 7\underline{j} + 32\underline{k}$

$\vec{n} \cdot (\vec{r} - \vec{r_0}) = $ zero

$(23, 7, 32) \cdot ((x, y, z) - (5, -1, 1)) = $ zero "The vector form"

$(23, 7, 32) \cdot (x - 5, y + 1, z - 1) = $ zero

$23(x-5) + 7(y+1) + 32(z-1) = \text{zero}$ "normal form"

$23x - 115 + 7y + 7 + 32z - 32 = \text{zero}$

$23x + 7y + 32z = 115 - 7 + 32$

$23x + 7y + 32z = 140$ "The Cartesian form (general)"

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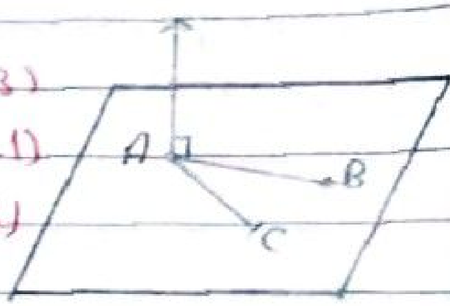
[3] 3 Points in This plane:- [3] 3 نقاط في هذا المستوى:-

ex(3):-

write, in normal form, The equation of the plane $(1,0,3)$, $(1,2,-1)$, and $(6,1,6)$.

* solution:-



$A = (1,0,3)$

$B = (1,2,-1)$

$C = (6,1,6)$

$\overrightarrow{AB} = (1,2,-1) - (1,0,3) = (0,2,-4)$

$\overrightarrow{AC} = (6,1,6) - (1,0,3) = (5,1,3)$

$$\therefore \vec{n} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 2 & -4 \\ 5 & 1 & 3 \end{vmatrix} = \underline{i}\begin{vmatrix} 2 & -4 \\ 1 & 3 \end{vmatrix} - \underline{j}\begin{vmatrix} 0 & -4 \\ 5 & 3 \end{vmatrix} + \underline{k}\begin{vmatrix} 0 & 2 \\ 5 & 1 \end{vmatrix}$$

$\vec{n} = 10\underline{i} - 20\underline{j} - 10\underline{k}$

$\vec{n} \cdot (\vec{r} - \vec{r_0}) = \text{zero}$

$(10, -20, -10) \cdot ((x,y,z) - (1,0,3)) = \text{zero}$ "The vector form"

$(10, -20, -10) \cdot (x-1, y-0, z-3) = \text{zero}$

$10(x-1) - 20(y) - 10(z-3) = \text{zero}$ "The normal form"

$10x - 10 - 20y - 10z + 30 = \text{zero}$

$10x - 20y - 10z = 10 - 30$

$10x - 20y - 10z = -20$ "The cartesian form (general)"

ex(4):- The equation of a plane has the general form $5x + 6y + 9z - 28 = 0$

$\therefore n = (5,6,9)$ "x, y, z معاملات" $\therefore d = 28$

$\therefore$ The vector equation is $\vec{n} \cdot \vec{r} = -d \Rightarrow (5,6,9) \cdot (x,y,z) = -28$