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سكن دینامیک

Motion with constant acceleration

$$\Rightarrow F = \frac{dv}{dt} = a$$

$$\int dv = \int a dt$$

$$dv = a dt$$

$$t=0$$

$$v = at + C_1$$

$$v = v_0$$

$$\therefore C_1 = v_0$$

$$\Rightarrow v = v_0 + at$$

$$\frac{dx}{dt} = v_0 + at$$

$$*) \int dx = \int (v_0 + at) dt$$

$$x = v_0 t + \frac{1}{2} at^2 + C_2$$

$$x_0 = 0 \quad t = 0$$

$$\therefore C_2 = 0$$

$$\Rightarrow x = x_0 + v_0 t + \frac{1}{2} at^2$$

$$*) F = \frac{v dv}{dx} = a$$

$$\int v dv = \int a dx$$

$$\frac{1}{2} v^2 = ax + C_3$$

$$v^2 = 2ax + C_3 \quad \therefore C_3 = v_0^2$$

$$\Rightarrow v^2 = v_0^2 + 2ax$$

EX]1 $v_0 = 12 \text{ m/s}$ $x_0 = 3 \text{ m}$ $x = -5$ $t = 2$

Find acceleration (a).

Solu₂

$$x = x_0 + v_0 t + \frac{1}{2} at^2$$

$$-5 = 3 + (12 \times 2) + \frac{1}{2} a (2)^2$$

$$-5 = 3 + 24 + 2a \quad |a = -16|$$

EX]2 $m = 200 \text{ g}$ $h = 50 \text{ m}$ $x = 30 \text{ m}$ $g = 10 \text{ m/s}^2$

$v_0 = 0$ $x_0 = 0$ Find the time.

Solu₂

$$x = x_0 + v_0 t + \frac{1}{2} at^2$$

$$30 = 0 + 0 + \frac{1}{2} 10 t^2$$

$$t^2 = \frac{70}{5}$$

$$t = \sqrt{14} \text{ s}$$

Ex]3 Prove that the acceleration

$$x = a(2t + \sin 2t) \quad y = a(1 - \cos 2t)$$

« Solu₂ \Rightarrow

$$\dot{x} = a(2 + 2 \cos 2t) \Rightarrow \ddot{x} = -4a \sin 2t$$

$$\dot{y} = 2a \sin 2t \Rightarrow \ddot{y} = 4a \cos 2t$$

$$F = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{(-4a \sin 2t)^2 + (4a \cos 2t)^2}$$