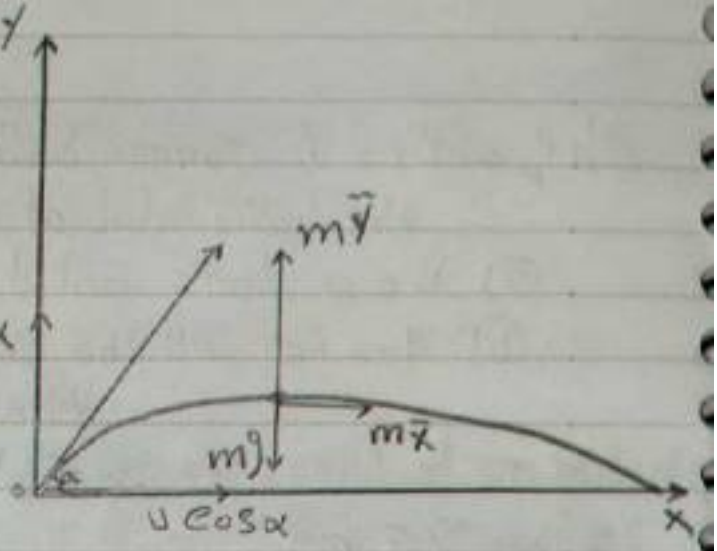
« Projectile motion »
[حركة المقذوفات]



v ⇒ initial velocity

α ⇒ direction of initial velocity above the horizontal ox.

مقاومة الهواء مهملة !! = zero

$$m\ddot{x} = 0 \quad (1)$$

$$m\ddot{y} = -mg \quad (2)$$

From equation (1) we find that

$$\frac{d\dot{x}}{dt} = 0 \Rightarrow \dot{x} = Const = v\cos\alpha \quad (3)$$

From equation (2) we find that

$$\frac{d\dot{y}}{dt} = -g$$

$$\dot{y} = v\sin\alpha - gt$$

$$x = vt\cos\alpha \quad *$$

$$y = vt\sin\alpha - \frac{1}{2}gt^2 \quad *$$

Flight time ⇒ $t = \left(\frac{2v}{g}\right)\sin\alpha$ $\quad \{g = g_8 = g_{80}\}$

Horizontal Range ⇒ $R = v\left(\frac{2v}{g}\sin\alpha\right)\cos\alpha$

$$= \frac{v^2}{g}\sin 2\alpha$$

$R_{max} = v^2/g$

aximum Height ⇒ $h = y_{max} = \frac{u^2}{2g} \sin^2\alpha$

$t_h = (\frac{u}{g}) \sin\alpha$ ★ زمن الطيران = ½ زمن أقصى ارتفاع

Ch ½ ⇒ EX 3 : A Cannon ball is fired at an angle of 30° to the horizontal at a speed of 25 ms^{-1}

(a) How long will it be before the impact?

(b) How far will the Cannon ball travel before hitting the ground?

« Solu »

⇒ $u = 25\ ms^{-1}$

⇒ $\alpha = 30°$ $\quad y = ut\sin\alpha - \frac{1}{2}gt^2 \quad y = 0$

$0 = 25T\sin 30 - \frac{1}{2}gT^2 \quad T(25\sin 30 - \frac{1}{2}gT) = 0$

$50T = 0$ or $T = (50\sin 30)/g = 50\sin 30/9.8$

$= 2.55\ s$ #

$X = uT\cos\alpha$

$R = 25 \times 2.55 \times \cos 30 = 55.231\ m$ #

Ch ½ ⇒ EX 7 , A ball is kicked at an angle of 35° with the ground : (a) what should be the initial velocity of the ball so that it hits a target that is 30 meters away at height of 1.8 meters ?

(b) what is the time for the ball to reach the target?

« Solu »

$(x, y) = (b, 1.8)$

$X = V_0 \cos(35°) t$

$30 = V_0 \cos(35°) t \qquad t = 30 / V_0 \cos(35°)$

$Y = y_0 \sin\alpha - \frac{1}{2} g t^2$

$1.8 = V_0 \sin(35°)(30/V_0 \cos(35°)) - \frac{1}{2} \times 9.8 (30/V_0 \cos(35°))^2$

$V_0 \cos(35°) = 30\sqrt{9.8/2(30\tan(35°) - 1.8)}$

$V_0 = 18.3 \text{ m/s}$ #

$\therefore t = X / V_0(\cos(35)) = 30/18.3\cos(35) = 2.0\,s$ #

Ch 4 ⇒ Ex 8 : A ball kicked From ground Level at an initial velocity of 60 m/s and an angle θ with ground reaches a horizontal distance of 200 meters.

[a] what is the size of angle θ?

[b] what is time of Flight of the ball?

« Solu »

$X = V_0 t \cos\theta \qquad t = \frac{2V_0}{g}\sin\theta$ [1]

$\hookrightarrow t = X / V_0 \cos\theta$ [2]

$X/V_0\cos\theta = \frac{2V_0}{g}\sin\theta$

$200/V_0\cos\theta = 2V_0\sin\theta/g$

$2V_0^2 \cos\theta\sin\theta = 200g$

$V_0^2 \sin 2\theta = 200g$

$\sin(2\theta) = 200g/V_0^2 = 200(9.8)/60^2$

$\boxed{\theta = 16.5°}$ #

time of Flight $= 200/V_0\cos(16.5) = 3.48\,s$ #

Ch 4 ⇒ Ex 9: $E = \frac{1}{2} m v_x^2 = \frac{1}{2} m [v_0 \cos\alpha]^2$

$$22 = \frac{1}{2}(0.6)(v_0 \cos(35))^2$$

$$v_0 = (1/\cos(35))\sqrt{(44/0.6)} = 10.4 \text{ m/s}$$ #

المسافة رأسية بين البداية والنهاية

$E_f - E_i = mgH$ ← عكس الاتجاه علشان H سالبة تطلع موجبة

$32.4 - 22 = mgH$

$\therefore H = 10.4/(0.6 \times 9.8) = 1.8 \text{ m}$ #

Ch 4 ⇒ Ex 10: $v_x = v_0 \cos\theta = 1000/40 = 25 \text{ m/s}$ (1)

time of flight $= 2v_0 \sin\theta / g = 40 \text{ s}$

$v_0 \sin\theta = 20 \times 9.8 = 196$ (2)

From 1, 2 $\frac{v_0 \sin\theta}{v_0 \cos\theta} = \frac{196}{25}$ $\tan\theta = 196/25$

$\therefore \theta = 82.7°$ $v_0 \cos\theta = 25 \text{ m/s}$

$v_0 = 25/\cos(82.7°) = 196.8 \text{ m/s} \simeq 197 \text{ m/s}$ #

طريقة أخرى ← (2) × (1) ← جمع وتربيع

$v_0^2[\cos\theta]^2 + v_0^2[\sin\theta]^2 = 25^2 + 196^2$

$v_0^2[\cos^2\theta + \sin^2\theta] = 25^2 + 196^2$

$v_0^2 = 39041$

$v_0 = 197 \text{ m/s}$ #

ch 4 ⇒ Ex 11 : $y = -0.025x^2 + 0.5x$ معادلة المسار

$$y = x\tan\alpha - \frac{g}{2v^2}x^2\sec^2\alpha$$

$\tan\alpha = 0.5 \Rightarrow \alpha = 26.5°$ #

$-0.025 = -0.5\left(9.8/[V_0\cos(26.5°)]^2\right)$

$\therefore V_0 = 15.6$ m/s #

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ch 3 ⇒ Ex 12 :

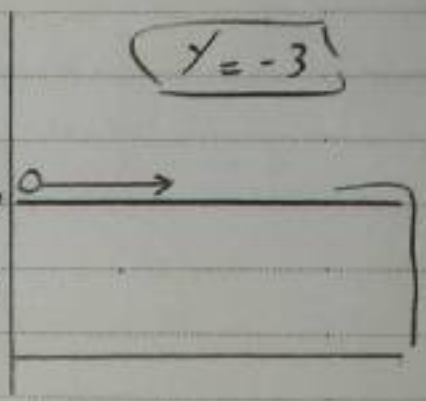
$y = ut\sin\alpha - \frac{1}{2}gt^2$ ($y = -3$)

$-3 = -\frac{1}{2}gt^2$

$t = \sqrt{3/\frac{1}{2}\times 9.8}$

$= 0.78$ s #

الكرتان وصلوا في نفس الزمن لان لهم نفس تعجيل الجاذبية الارضية ونفس الوزن



$X_A = ut\cos\alpha$

$= 10 \times 0.78 = 7.8$m

لان اتجاه $\alpha = 0$ قذف الكرة افقي

$X_B = v_B t\cos\alpha$

$= 15 \times 0.78 = 11.7$

$|X_B - X_A| = |11.7 - 7.8| = 3.9$ m # المسافة بين الكرة A , الكرة B