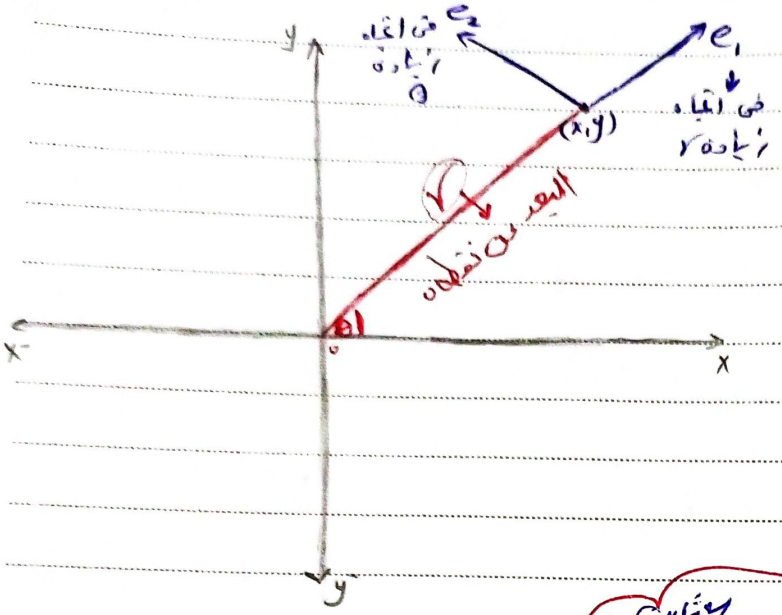
محاضرة 3

الاحداثيات القطبية

Date / / No



* θ ← الزاوية في اتجاه x-axis لسه من قيمتها نعرف النقطة في اي ربع

* $\dot{\theta}$ ← السرعة الزاوية

* $\ddot{\theta}$ ← العجلة الزاوية

بثبات $\omega = \dot{\theta}$

Zero $= \omega^{\cdot\cdot}$

* $r = r e_1$

* $\vec{V} = \dot{r} e_1 + r \dot{\theta} e_2$ ← تفاضل r

* $\vec{F} = \ddot{r} e_1 + \dot{r}\dot{\theta} e_2 + \dot{r}\dot{\theta} e_2 + r\ddot{\theta} e_2 - r\dot{\theta}^2 e_1$

$\therefore \vec{F} = F_r e_1 + F_\theta e_2$

$\therefore \vec{F}_r = \ddot{r} - r\dot{\theta}^2$

$\vec{F}_\theta = 2\dot{r}\dot{\theta} + r\ddot{\theta} \Rightarrow \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})$

$= \frac{1}{r}[2r\dot{r}\dot{\theta} + r^2\ddot{\theta}]$

$|F| = \sqrt{F_r^2 + F_\theta^2}$

ex,

- A Particle moves on the curve $r = a + b\sin\theta$ with a constant angular velocity ω, where a and b are constants. Find the acceleration for this Particle.

Solution

$\dot{r} = b\omega\cos\theta$

$\ddot{r} = -b\omega^2\sin\theta$

$f_r = -b\omega^2\sin\theta - a\omega^2\sin\theta$

$= -2b\omega^2\sin\theta - a\omega^2$ ← لازم نطلع بدلالة r

$= -2\omega^2[r - a] - a\omega^2$ ← ممكن تسبط الناتج هكذا

$f_\theta = 2b\omega^2\cos\theta$

$\because b^2\sin^2\theta + b^2\cos^2\theta = b^2$

$\therefore b^2\cos^2\theta = b^2 - b^2\sin^2\theta$

$\therefore b\cos\theta = \sqrt{b^2 - (b\sin\theta)^2}$

$f_\theta = 2\omega^2[b^2 - (r-a)^2]^{\frac{1}{2}}$

ex2

- Considering an illustration the ~~motion~~ motion of a particle in a circular trajectory having angular velocity $w = \dot{\theta}$, and angular acceleration $\alpha = \dot{w}$. Calculate the velocity and the acceleration components.

Solution

$r = R$ ← نصف القطر

ل ثابت

$\dot{r} = 2\dot{r}_0$

$\ddot{r} = 2\ddot{r}_0$

$\dot{\theta} = \alpha t + c$

علشان اجيب قيمة c انعوض بالشروط الابتدائية

$\therefore c = w$

$\therefore \dot{\theta} = \alpha t + w$

ex3

- A particle moves with $\dot{\theta} = w = \text{constant}$ and $r = r_0 e^{\beta t}$, where r_0 and β are constants. Prove that for certain values of β, the particle moves with $a_r = 0$.

Solution

$\dot{r} = r_0 \beta e^{\beta t}$

$\ddot{r} = r_0 \beta^2 e^{\beta t}$

$a_r = \ddot{r} - r\dot{\theta}^2 = 2\ddot{r}_0$

$= r_0 e^{\beta t}(\beta^2 - w^2) = 2\ddot{r}_0$

ثوابت r_0, β

$\beta^2 - w^2 = 0$

$\beta = \pm w$