محاضرة 6

Date ..... / ..... / ..... No .....

chapter 3

Simple Harmonic Motion
الحركة التوافقية البسيطة

مراجعة القوانين

* $x'' = -\omega^2 x$
* $v^2 = \omega^2 (a^2 - x^2)$
∴ $x = a\cos(\omega t + \varepsilon)$ → زاوية الطور

$-1 \leq \cos \leq 1$

$-a \leq x \leq a$

Center ممكن يتغير وميبقاش (0,0)

* $V_{max} = \omega a$
* $F_{max} = \omega^2 a$

الزمن الدوري

- Periodic time $= "\tau" = \frac{2\pi}{\omega}$

- Frequency $= "N" = \frac{1}{\tau} = \frac{\omega}{2\pi}$

ex. Find the Periodic time for the simple harmonic motion which defind by $x'' = -25x$.

Solution

∵ $x'' = -\omega^2 x$

∴ $\omega = 5$

$\tau = \frac{2\pi}{5}$ #

ex₂ - Calculate the maximum velocity and the maximum acceleration for a partical moves a simple harmonic motion with a periodic time $\frac{\pi}{4}$ sec and an amplitude equal to 25 cm.

Solution

$\tau = \frac{\pi}{4}$, $a = 25$

$\tau = \frac{2\pi}{\omega}$ ∴ $\frac{\pi}{4} = \frac{2\pi}{\omega}$

$\omega = 8$

∴ $V_{max} = \omega a = 8 \times 25 = 200$

$F_{max} = \omega^2 a = 8^2 \times 25 = 1600$

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ex₃ IF the position x for a moving particle can be determind by $x = 0.45 \cos\frac{\pi t}{4} - 0.28 \sin\frac{\pi t}{4}$ the end of three consecutive seconds prove that the totel periodic time is:

$$2\pi / \cos^{-1}\left(\frac{x_1 + x_2}{x_2}\right)$$

Solution

∵ $x = 0.45 \cos\frac{\pi t}{4} - 0.28 \sin\frac{\pi t}{4}$

x نفاضل ← $x' = V = -\frac{0.45\pi}{4} \sin\frac{\pi t}{4} - \frac{0.28\pi}{4} \cos\frac{\pi t}{4}$

v نفاضل ← $x'' = F = (\frac{\pi}{4})^2 x$ ⟶ *

$A = 0.45 \quad , B = -0.28$

$a = \sqrt{A^2 + B^2} = \sqrt{(0.45)^2 + (0.28)^2} = 0.53$ #

$\tau = \frac{2\pi}{\omega} = \frac{2\pi \times 4}{\pi} = 8$ #

$V_{max} = \omega a = \frac{\pi \times 0.53}{4}$ #

$F_{max} = \omega^2 a = \left(\frac{\pi}{4}\right)^2 \times 0.53$ #

$\varepsilon = \tan^{-1}\left(\frac{-B}{A}\right) = \tan^{-1}\left(\frac{0.28}{0.45}\right) = 0.557$

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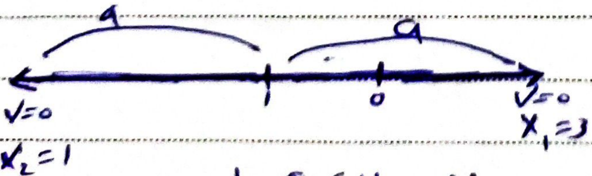
ex_6 if the velocity of a moving Particle is obtained From the relation $V^2 = -2x^2 + 4x + 6$ Prove that the Motion represents a Simple harmonic motion Calculate its Center, the maximum acceleration and the Frequency.

Solution

$x'' = \frac{1}{2} \frac{dV^2}{dx} = -(4x+4) \times \frac{1}{2} = -2(x-1)$

$\therefore$ Center $(1, 0)$, $\omega = \sqrt{2}$

$N = \frac{1}{t} = \frac{\omega}{2\pi} = \frac{\sqrt{2} \times \sqrt{2}}{2\pi \times \sqrt{2}} = \frac{1}{\sqrt{2}\,\pi}$



نحسب المسافة بين X_1 , X_2 والقسمة على 2

$= \frac{-2x^2}{-2} + \frac{4}{2}x + \frac{6}{2}$

$= x^2 - 2x - 3 = (x+1)(x-3)$

$\therefore x = -1 \quad , \quad x = 3$

$a = 2$