الموضوع: ________ التاريخ:

بسم الله الرحمن الرحيم

* $\int \sin^{\frac{1}{3}} x \cos^3 x \, dx$

«Solu» $u = \sin x \qquad du = \cos x \, dx \qquad dx = \frac{du}{\cos x}$

$= \int \sin^{\frac{1}{3}} x \cos^2 x \, du = \int \sin^{\frac{1}{3}} x \cos^2 x \cos x \, dx = \int \sin^{\frac{1}{3}} x \, (1 - \sin^2 x) \, d(\sin x)$

$= \int \sin^{\frac{1}{3}} x \, d \sin x - \int \sin^{\frac{4}{3}} x \, d \sin x = \frac{3}{4} \sin^{\frac{4}{3}} x - \frac{3 \sin^{\frac{10}{3}} x}{10} + C$ #

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* $\int \operatorname{cosec} x \, dx$ (* بالضرب بسطاً ومقاماً في $(\operatorname{cosec} x + \cot x)$)

«Solu» $\int \frac{\operatorname{cosec} x \, [\operatorname{cosec} x + \cot x]}{\operatorname{cosec} x + \cot x} \, dx$

$= -\int \frac{-(\operatorname{cosec}^2 x + \operatorname{cosec} x \cot x)}{\operatorname{cosec} x + \cot x} = -\int \frac{-\operatorname{cosec}^2 x - \operatorname{cosec} x \cot x)}{\operatorname{cosec} x + \cot x}$

$= -\ln |\operatorname{cosec} x + \cot x| + C$

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* $\int \frac{1}{x} \sin^6 (\ln x) \cos (\ln x) \, dx$

«solu» $= \frac{1}{7} \sin^7 (\ln x)$

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* $\int \frac{dx}{\sqrt{1 - x^2}}$ «Solu» $\sin^{-1} x + C$

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* $\int \frac{x + \sin^{-1} x}{\sqrt{1 - x^2}} \, dx$ «Solu»

$= \int \frac{x}{\sqrt{1 - x^2}} dx + \int \frac{\sin^{-1} x}{\sqrt{1 - x^2}} dx = \frac{-1}{2} \int \frac{-2x}{\sqrt{1 - x^2}} dx + \int \frac{1 \cdot \sin^{-1} x}{\sqrt{1 - x^2}} dx$

$= -\sqrt{1 - x^2} + \frac{1}{2} (\sin^{-1})^2 + C$

$$\frac{dV_{(x)}V_{(x)}}{dx} \rightsquigarrow V\frac{du}{dx} + u\frac{dv}{dx}$$

$$\int d(uV) = \int V\,du + \int u\,dv \qquad uV = \int V\,du + \int u\,dV$$

$$\Rightarrow \int u(dv) = uV - \int V\,du$$

① $\int x e^{x} dx$ <Solu> $u = x \Rightarrow du = dx$ / $dV = e^{x} dx \Rightarrow V = e^{x}$

$$\therefore x e^{x} - \int e^{x} dx = x e^{x} - e^{x} + C$$

② $\int x^3 e^{x^2} dx$ <Solu> $\boxed{u = x^3 \quad V = e^{x^2} dx}$ حل خطأ (x)

$u = x^2$ $du = 2x\,dx$ / $V = e^{x^2}$ $dV = 2x e^{x^2}$

$$\frac{1}{2}\int x^2 (2x)\,dx = \frac{1}{2}\left[x^2 (e^{x^2}) - \int 2x e^{x^2} dx\right]$$

$$= \frac{1}{2}\left[x^2 e^{x^2} - e^{x^2} + c\right]$$

③ $\int \ln x\,dx$ <Solu> $u = \ln(x) \to du = \frac{1}{x} dx$ $V = x$

$$= x\ln x - \int \frac{x}{x} dx = x(\ln x - 1) + C$$

④ $\int \frac{\ln x\,dx}{\sqrt{x}}$ * الدكتور محلهاش

⑤ $\int \sin^{-1} x\,dx$ $u = \sin^{-1} x \to du = \frac{1}{\sqrt{1-x^2}} dx$ $dv = dx \therefore v = x$

$$= x\sin^{-1} x + \frac{1}{2}\int \frac{-2x}{\sqrt{1-x^2}} dx = x\sin^{-1} x + \sqrt{1-x^2} + C$$

⑥ $\int x \tan^{-1} x\,dx$ $V = x$ $u = \tan^{-1} x\,dx$

$$du = \frac{1}{1+x^2} \Rightarrow \frac{1}{2}\int \frac{(x^2+1)-1}{1+x^2} dx$$

$$= \frac{1}{2}\int \left| dx - \int \frac{1}{1+x^2} dx \right|$$

* $\int \frac{2x^2+3}{x\sqrt{9x^2-4}}\,dx$

بتوزيع البسط ع المقام

$$\int \frac{2x^2+3}{x\sqrt{9x^2-4}} = \int \left[\frac{2x^2}{x\sqrt{9x^2-4}} + \frac{3}{x\sqrt{9x^2-4}}\right]dx$$

$$= \int \frac{2x}{\sqrt{9x^2-4}}\,dx + \int \frac{3}{x\sqrt{9x^2-4}}\,dx \qquad \boxed{u = 9x^2-4} \qquad \boxed{du = 18x\,dx}$$

$$= \int \frac{2x}{\sqrt{9x^2-4}}\,dx = \frac{1}{9}\int \frac{du}{\sqrt{u}} = \frac{1}{9}\cdot 2\sqrt{u} + c = \frac{2\sqrt{9x^2-4}}{9} + c$$

$u = 3x \qquad du = 3dx$

$$\int \frac{3dx}{x\sqrt{9x^2-4}} = 3\int \frac{du}{u\sqrt{u^2-4}} = 3\sec^{-1}\left|\frac{u}{2}\right| + c = 3\sec^{-1}\left|\frac{3x}{2}\right| + c$$

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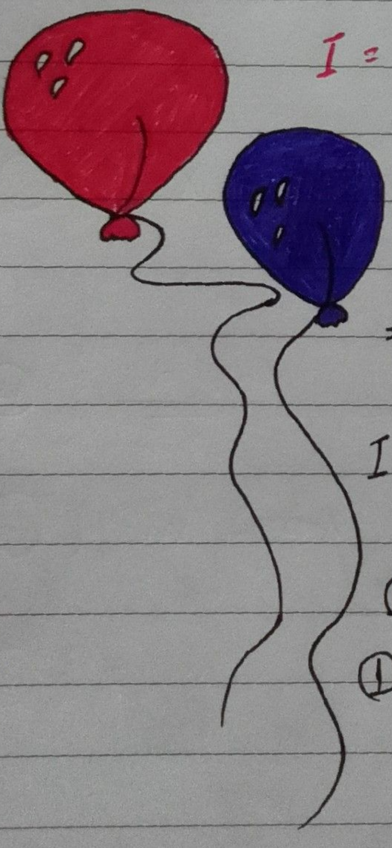
* شيت محاضرة !



$$I = \int \frac{\cos x}{\cos x + \sin x}\,dx$$

$$J = \int \frac{\sin x}{\cos x + \sin x}$$

<Solution>

$$I + J = \int \frac{\cos x}{\cos x + \sin x}\,dx + \int \frac{\sin x}{\cos x + \sin x}\,dx$$

$$= \int \frac{\cos x + \sin x}{\cos x + \sin x}\,dx = x + C \quad ①$$

$$I - J = \int \frac{\cos x - \sin x}{\cos x + \sin x}\,dx = \ln|\cos x + \sin x| + C \quad ②$$

①+② $I = \frac{1}{2}(x + \ln|\cos x + \sin x| + c)$

①-② $J = \frac{1}{2}(x - \ln|\cos x + \sin x| + c)$