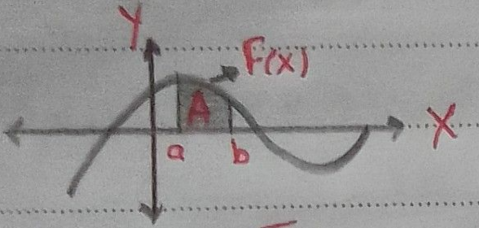
* التكامل المحدد *

التعريف الهندسي للتكامل المحدد

* هو المساحة المحصورة بين منحنى الدالة $F(x)$ والمحور الأساسي X



$$\int_a^b f(x)\,dx$$

بشرط $a < X < b$

$a, b \Rightarrow$ حدود التكامل $F(x) \Rightarrow$ الدالة $dx \Rightarrow$ تكامل بالنسبة لـ X

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if $\int_a^b f(x)\,dx \quad , \quad a < c < b$

$$= \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

$$\left| \int_a^b f(x)\,dx \right| = \text{Area}$$

هام ← [عشان لو المساحة تحت المنحنى سالب]

$$\int_{-a}^{+a} f(x)\,dx$$

- $f(x)$ odd $=$ zero
- $f(x)$ even ← متماثلة حول محور Y

$$\therefore = 2\int_{-a}^{a} f(x)\,dx$$

■ $\int_{-5}^{5} \frac{x}{\sqrt{1+x^4}}\,dx \quad \because x \Rightarrow \text{odd} \;,\; \sqrt{1+x^4} \Rightarrow \text{even} \;,\; \frac{\text{odd}}{\text{even}} = \text{odd}$

$= \text{zero}$

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1) $\int_1^2 (x^3 + 3x^2 + 1)\,dx$

$$\left[\frac{x^4}{4} + x^3 + x\right]_1^2$$

$$= (4 + 8 + 2) - \left(\frac{1}{4} + 1 + 1\right)$$

$$= 11.75$$

ملحوظة ← في التعويض نطرح حد التكامل الأكبر - حد التكامل الأصغر

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

ناتج تكامل a, b

* تعتبر المسألة خاطئة 100 % في حالة نسيان حتى لو الحل صح

a, b, dX

[2] $\int_0^3 \frac{2x-1}{(x+1)(x-2)}\,dx$, using Partial Fractions.

$\frac{2x-1}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}$ * بالضرب في $(x+1)(x-2)$

$2x-1 = A(x-2) + B(x+1)$

at $x = 2$ $3 = 3B$ $\therefore B = 1$

at $x = -1$ $-3 = -3A$ $\therefore A = 1$

$\therefore \int_0^3 \frac{2x-1}{(x+1)(x-2)}\,dx = \int_0^3 \frac{dx}{x+1} + \int_0^3 \frac{1}{x-2}\,dx = \left[\ln|x+1| + \ln|x-2|\right]_0^3$

$= \ln\left[|(x+1)(x-2)|\right]_0^3 = \ln\left[|x^2-x-2|\right]_0^3 = \ln(4) - \ln|2| = \ln\left(\frac{4}{2}\right) = \ln 2$

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[3] $\int_0^{\pi/2} \cos^3 x \sin^2 x\,dx \Rightarrow = \int_0^{\pi/2} (1-\sin^2 x)\sin^2 x\,d\sin x$

$= \int_0^{\pi/2} \sin^2 x - \sin^4 x\,d\sin x = \left(\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5}\right)_0^{\pi/2} = \left(\frac{\sin^3\frac{\pi}{2}}{3} - \frac{\sin^5\frac{\pi}{2}}{5}\right) - \text{zero}$

$= \frac{1}{3} - \frac{1}{5} = \frac{2}{15}$

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[4] $\int_0^{1/2} \frac{dx}{\sqrt{1-4x^2}}$ $\because \int \frac{1}{\sqrt{1-a^2}}\,dx = \sin^{-1} a$

$= \left[\frac{1}{2}\sin^{-1} 2x\right]_0^{\frac{1}{2}} = \frac{1}{2}\sin^{-1}(1) - \text{zero} = \frac{\pi}{4} - \text{zero} = \frac{\pi}{4}$

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5) $\int_0^{\frac{\pi}{2}} \sin^2 2x \, dx$

$\Rightarrow \sin^2 2x = \frac{1-\cos 4x}{2}$ $\quad \therefore \int_0^{\frac{\pi}{2}} \sin^2 2x \, dx = \frac{1}{2}\int_0^{\frac{\pi}{2}} (1-\cos 4x)\, dx$

$= \frac{1}{2}\int_0^{\frac{\pi}{2}} dx - \int_0^{\frac{\pi}{2}} \cos 4x \, dx = \frac{1}{2}\left| x - \frac{\sin 4x}{4} \right|_0^{\pi/2} = \frac{1}{2}\left(\frac{\pi}{2} - \text{zero}\right) = \frac{\pi}{4}$

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6) $\int_0^{\pi/2} x \sin^2 2x \, dx = \int_0^{\frac{\pi}{2}} x \cdot \frac{1-\cos 4x}{2} \, dx = \frac{1}{2}\int_0^{\frac{\pi}{2}} x(1-\cos 4x)\, dx$

$= \frac{1}{2}\left[\int_0^{\frac{\pi}{2}} x\, dx - \int_0^{\frac{\pi}{2}} x \cos 4x \, dx\right]$

$\frac{1}{2}\left[\left(\frac{x^2}{2}\right)\Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} x \cos 4x \, dx\right]$

• هنكامل الحد الثاني بالتكامل بالتجزئ

$\therefore u = x \Rightarrow du = dx$

$dv = \cos 4x \, dx \Rightarrow v = \frac{\sin 4x}{4}$

$\int u\,dv = uv - \int v\,du$

$\therefore \int x \cos 4x \, dx = \frac{x \sin 4x}{4} - \int \frac{\sin 4x}{4} dx = \frac{x \sin 4x}{4} + \frac{\cos 4x}{16}$

$\therefore \int_0^{\frac{\pi}{2}} x \sin^2 2x \, dx$

$= \frac{1}{2}\left[\left(\frac{x^2}{2}\right) - \frac{x \sin 4x}{4} + \frac{\cos 4x}{16}\right]_0^{\frac{\pi}{2}} = \frac{1}{2}\cdot\left(\frac{\pi^2}{8} - \text{zero}\right) = \frac{\pi^2}{16}$