Integral Solutions with Detailed Steps

1) $\int \left(1 - x + x^{1/2} - \frac{1}{x^3} + \frac{1}{x}\right) dx$

Step 1: Break the integral into separate terms:

$$\int 1\,dx - \int x\,dx + \int x^{1/2}\,dx - \int \frac{1}{x^3}\,dx + \int \frac{1}{x}\,dx$$

Step 2: Integrate each term individually:

$$x - \frac{x^2}{2} + \frac{2x^{3/2}}{3} + \frac{1}{2x^2} + \ln|x| + C$$

Final Answer:

$$\boxed{x - \frac{x^2}{2} + \frac{2x^{3/2}}{3} + \frac{1}{2x^2} + \ln|x| + C}$$

2) $\int x^3\sqrt{1 - x^4} dx$

Step 1: Use substitution. Let $u = 1 - x^4$, then $du = -4x^3 dx$.

Step 2: Rewrite the integral in terms of u:

$$\int x^3\sqrt{u} \cdot \frac{du}{-4x^3} = -\frac{1}{4}\int \sqrt{u}\,du$$

Step 3: Integrate with respect to u:

$$-\frac{1}{4} \cdot \frac{2}{3}u^{3/2} + C = -\frac{1}{6}u^{3/2} + C$$

Step 4: Substitute back $u = 1 - x^4$:

$$-\frac{1}{6}(1 - x^4)^{3/2} + C$$

Final Answer:

$$\boxed{-\frac{1}{6}(1 - x^4)^{3/2} + C}$$

3) $\int \frac{\sin x}{\sqrt{3+2\cos x}} dx$

Step 1: Use substitution. Let $u = 3 + 2\cos x$, then $du = -2\sin x\,dx$.

Step 2: Rewrite the integral in terms of u:

$$\int \frac{\sin x}{\sqrt{u}} \cdot \frac{du}{-2\sin x} = -\frac{1}{2}\int u^{-1/2}\,du$$

Step 3: Integrate with respect to u:

$$-\frac{1}{2} \cdot 2u^{1/2} + C = -\sqrt{u} + C$$

Step 4: Substitute back $u = 3 + 2\cos x$:

$$-\sqrt{3 + 2\cos x} + C$$

Final Answer:

$$\boxed{-\sqrt{3 + 2\cos x} + C}$$

4) $\int \frac{x\,dx}{\sqrt{3+x^2}}$

Step 1: Use substitution. Let $u = 3 + x^2$, then $du = 2x\,dx$.

Step 2: Rewrite the integral in terms of u:

$$\int \frac{x}{\sqrt{u}} \cdot \frac{du}{2x} = \frac{1}{2}\int u^{-1/2}\,du$$

Step 3: Integrate with respect to u:

$$\frac{1}{2} \cdot 2u^{1/2} + C = \sqrt{u} + C$$

Step 4: Substitute back $u = 3 + x^2$:

$$\sqrt{3 + x^2} + C$$

Final Answer:

$$\boxed{\sqrt{3 + x^2} + C}$$

5) $\int (2 + 3x^2 - \cos x)dx$

Step 1: Break the integral into separate terms:

$$\int 2\,dx + \int 3x^2\,dx - \int \cos x\,dx$$

Step 2: Integrate each term individually:

$$2x + x^3 - \sin x + C$$

Final Answer:

$$\boxed{2x + x^3 - \sin x + C}$$

6) $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

Step 1: Use substitution. Let $u = \sqrt{x}$, then $du = \frac{1}{2\sqrt{x}}dx$.

Step 2: Rewrite the integral in terms of u:

$$\int \sin u \cdot 2\,du = 2\int \sin u\,du$$

Step 3: Integrate with respect to u:

$$2(-\cos u) + C = -2\cos u + C$$

Step 4: Substitute back $u = \sqrt{x}$:

$$-2\cos\sqrt{x} + C$$

Final Answer:

$$\boxed{-2\cos\sqrt{x} + C}$$

7) $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)dx$

Step 1: Break the integral into separate terms:

$$\int \sqrt{x}\,dx + \int \frac{1}{\sqrt{x}}\,dx$$

Step 2: Integrate each term individually:

$$\frac{2x^{3/2}}{3} + 2\sqrt{x} + C$$

Final Answer:

$$\boxed{\frac{2x^{3/2}}{3} + 2\sqrt{x} + C}$$

8) $\int \frac{\ln x}{x}dx$

Step 1: Use substitution. Let $u = \ln x$, then $du = \frac{1}{x}dx$.

Step 2: Rewrite the integral in terms of u:

$$\int u\,du$$

Step 3: Integrate with respect to u:

$$\frac{u^2}{2} + C$$

Step 4: Substitute back $u = \ln x$:

$$\frac{(\ln x)^2}{2} + C$$

Final Answer:

$$\boxed{\frac{(\ln x)^2}{2} + C}$$

9) $\int \cos ec(\sin x) \cos x \, dx$

Step 1: This integral requires clarification of the function. The notation is unclear.

Final Answer:

$$\boxed{\text{This integral requires clarification of the function.}}$$

10) $\int x(2 - 3x)dx$

Step 1: Expand the integrand:

$$\int (2x - 3x^2) \, dx$$

Step 2: Integrate each term individually:

$$x^2 - x^3 + C$$

Final Answer:

$$\boxed{x^2 - x^3 + C}$$

11) $\int \frac{1}{x}(3 - 2\ln x)^{1/4}dx$

Step 1: Use substitution. Let $u = 3 - 2\ln x$, then $du = -\frac{2}{x}dx$.

Step 2: Rewrite the integral in terms of u:

$$\int u^{1/4} \cdot \frac{du}{-2} = -\frac{1}{2} \int u^{1/4} \, du$$

Step 3: Integrate with respect to u:

$$-\frac{1}{2} \cdot \frac{4}{5} u^{5/4} + C = -\frac{2}{5} u^{5/4} + C$$

Step 4: Substitute back $u = 3 - 2\ln x$:

$$-\frac{2}{5}(3-2\ln x)^{5/4}+C$$

Final Answer:

$$\boxed{-\frac{2}{5}(3-2\ln x)^{5/4}+C}$$

12) $\int \frac{x}{\tan x^2} dx$

Step 1: This integral requires clarification of the function. The notation is unclear.

Final Answer:

$$\boxed{\text{This integral requires clarification of the function.}}$$

13) $\int \cos ec^7 \frac{x}{3} \cot \frac{x}{3} dx$

Step 1: This integral requires clarification of the function. The notation is unclear.

Final Answer:

$$\boxed{\text{This integral requires clarification of the function.}}$$

14) $\int \frac{x}{\sqrt{1-x^2}} dx$

Step 1: Use substitution. Let $u = 1 - x^2$, then $du = -2x\,dx$.

Step 2: Rewrite the integral in terms of u:

$$\int \frac{x}{\sqrt{u}} \cdot \frac{du}{-2x} = -\frac{1}{2}\int u^{-1/2}\,du$$

Step 3: Integrate with respect to u:

$$-\frac{1}{2} \cdot 2u^{1/2} + C = -\sqrt{u} + C$$

Step 4: Substitute back $u = 1 - x^2$:

$$-\sqrt{1-x^2}+C$$

Final Answer:

$$\boxed{-\sqrt{1-x^2}+C}$$

15) $\int \sqrt[4]{1+\cos 3x}\,\sin 3x\,dx$

Step 1: Use substitution. Let $u = 1 + \cos 3x$, then $du = -3\sin 3x\,dx$.

Step 2: Rewrite the integral in terms of u:

$$\int u^{1/4} \cdot \frac{du}{-3} = -\frac{1}{3} \int u^{1/4}\, du$$

Step 3: Integrate with respect to u:

$$-\frac{1}{3} \cdot \frac{4}{5} u^{5/4} + C = -\frac{4}{15} u^{5/4} + C$$

Step 4: Substitute back $u = 1 + \cos 3x$:

$$-\frac{4}{15}(1 + \cos 3x)^{5/4} + C$$

Final Answer:

$$\boxed{-\frac{4}{15}(1 + \cos 3x)^{5/4} + C}$$

16) $\int \frac{\sec^2 2x}{1+\tan 2x}\, dx$

Step 1: Use substitution. Let $u = 1 + \tan 2x$, then $du = 2\sec^2 2x\, dx$.

Step 2: Rewrite the integral in terms of u:

$$\int \frac{\sec^2 2x}{u} \cdot \frac{du}{2\sec^2 2x} = \frac{1}{2} \int \frac{1}{u}\, du$$

Step 3: Integrate with respect to u:

$$\frac{1}{2} \ln |u| + C$$

Step 4: Substitute back $u = 1 + \tan 2x$:

$$\frac{1}{2} \ln |1 + \tan 2x| + C$$

Final Answer:

$$\boxed{\frac{1}{2} \ln |1 + \tan 2x| + C}$$

17) $\int \left(\frac{3}{\sqrt{x}} - \frac{x\sqrt{x}}{4} \right) dx$

Step 1: Break the integral into separate terms:

$$\int \frac{3}{\sqrt{x}}\, dx - \int \frac{x\sqrt{x}}{4}\, dx$$

Step 2: Integrate each term individually:

$$6\sqrt{x} - \frac{x^{5/2}}{10} + C$$

Final Answer:

$$\boxed{6\sqrt{x} - \frac{x^{5/2}}{10} + C}$$

18) $\int \frac{dx}{x\sqrt{2+\ln x}}$

Step 1: Use substitution. Let $u = 2 + \ln x$, then $du = \frac{1}{x}dx$.

Step 2: Rewrite the integral in terms of u:

$$\int \frac{1}{\sqrt{u}}\, du$$

Step 3: Integrate with respect to u:

$$2\sqrt{u} + C$$

Step 4: Substitute back $u = 2 + \ln x$:

$$2\sqrt{2 + \ln x} + C$$

Final Answer:

$$\boxed{2\sqrt{2 + \ln x} + C}$$

19) $\int \left(\frac{1}{x^2} + \frac{4}{x\sqrt{x}} + 2\right) dx$

Step 1: Break the integral into separate terms:

$$\int \frac{1}{x^2}\, dx + \int \frac{4}{x\sqrt{x}}\, dx + \int 2\, dx$$

Step 2: Integrate each term individually:

$$-\frac{1}{x} - \frac{8}{\sqrt{x}} + 2x + C$$

Final Answer:

$$\boxed{-\frac{1}{x} - \frac{8}{\sqrt{x}} + 2x + C}$$

20) $\int \sin 5x\, dx$

Step 1: Integrate $\sin 5x$ directly:

$$-\frac{\cos 5x}{5} + C$$

Final Answer:

$$\boxed{-\frac{\cos 5x}{5} + C}$$

21) $\int \sin^{\frac{1}{2}} 2x \cos 2x \, dx$

Step 1: Use substitution. Let $u = \sin 2x$, then $du = 2\cos 2x \, dx$.

Step 2: Rewrite the integral in terms of u:

$$\int u^{1/2} \cdot \frac{du}{2} = \frac{1}{2} \int u^{1/2} \, du$$

Step 3: Integrate with respect to u:

$$\frac{1}{2} \cdot \frac{2}{3} u^{3/2} + C = \frac{1}{3} u^{3/2} + C$$

Step 4: Substitute back $u = \sin 2x$:

$$\frac{1}{3} \sin^{3/2} 2x + C$$

Final Answer:

$$\boxed{\frac{1}{3} \sin^{3/2} 2x + C}$$

22) $\int \frac{x^3+x^2-x}{x^{3/2}} \, dx$

Step 1: Simplify the integrand:

$$\int \left(x^{3/2} + x^{1/2} - x^{-1/2} \right) dx$$

Step 2: Integrate each term individually:

$$\frac{2x^{5/2}}{5} + \frac{2x^{3/2}}{3} - 2\sqrt{x} + C$$

Final Answer:

$$\boxed{\frac{2x^{5/2}}{5} + \frac{2x^{3/2}}{3} - 2\sqrt{x} + C}$$

23) $\int \tan^4 5x \sec^2 5x \, dx$

Step 1: Use substitution. Let $u = \tan 5x$, then $du = 5 \sec^2 5x \, dx$.

Step 2: Rewrite the integral in terms of u:

$$\int u^4 \cdot \frac{du}{5} = \frac{1}{5} \int u^4 \, du$$

Step 3: Integrate with respect to u:

$$\frac{1}{5} \cdot \frac{u^5}{5} + C = \frac{u^5}{25} + C$$

Step 4: Substitute back $u = \tan 5x$:

$$\frac{\tan^5 5x}{25} + C$$

Final Answer:

$$\boxed{\frac{\tan^5 5x}{25} + C}$$

24) $\int x(3 + x^2)^5 \, dx$

Step 1: Use substitution. Let $u = 3 + x^2$, then $du = 2x \, dx$.

Step 2: Rewrite the integral in terms of u:

$$\int u^5 \cdot \frac{du}{2} = \frac{1}{2} \int u^5 \, du$$

Step 3: Integrate with respect to u:

$$\frac{1}{2} \cdot \frac{u^6}{6} + C = \frac{u^6}{12} + C$$

Step 4: Substitute back $u = 3 + x^2$:

$$\frac{(3 + x^2)^6}{12} + C$$

Final Answer:

$$\boxed{\frac{(3 + x^2)^6}{12} + C}$$

التاريخ: / /

الموضوع:

بسم الله الرحمن الرحيم

trigonometric function :-

$\sec^2 x - \tan^2 x = 1 \rightsquigarrow \sec^2 x = 1 + \tan^2 x$

$a^2 - x^2 \Rightarrow \sin x$ or $\cos x$	$x = a\sin\theta \quad dx = a\cos\theta\, d\theta$
$a^2 + x^2 \Rightarrow \tan x$	$x = a\tan\theta \quad dx = a\sec^2\theta\, d\theta$
$x^2 - a^2 \Rightarrow \sec x$	$x = a\sec\theta \quad dx = a\sec\theta\tan\theta\, d\theta$

ex1) $\int \sqrt{(a^2 - x^2)}\, dx$

$$= \int \sqrt{(a^2 - a^2\sin^2\theta} \; a\cos\theta\, d\theta = \int a^2 \sqrt{1 - \sin^2\theta}\, \cos\theta\, d\theta$$

$$= a^2 \int \cos^2\theta\, d\theta$$

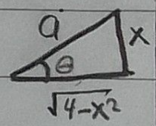
$\Rightarrow \cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$

$\Rightarrow \sin 2\theta = 2\sin\theta\cos\theta$

$$\therefore \frac{a^2}{2} \int (1 + \cos 2\theta)\, d\theta = \frac{a^2}{2}\theta + \frac{a^2}{4}\sin 2\theta + C$$

$$= \frac{a^2}{2}\sin^{-1}\frac{x}{a} + \frac{x\sqrt{a^2 - x^2}}{2} + C$$

ex2 $\int \frac{dx}{(4 - x^2)^{3/2}}$

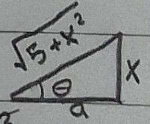


$$= \int \frac{2\cos\theta\, d\theta}{(2^2 - 2^2\sin^2\theta)^{3/2}} = \int \frac{\cos\theta\, d\theta}{(2\sqrt{1 - \sin^2\theta})^3} = \frac{1}{4}\int \frac{d\theta}{\cos^2\theta}$$

$$= \frac{1}{4}\int \sec^2\theta\, d\theta = \frac{1}{4}\tan\theta + C$$

$$= \frac{1}{4}\left(\frac{1}{\sqrt{4 - x^2}}\right) + C$$

$\int \frac{dx}{x^2\sqrt{5 + x^2}}$

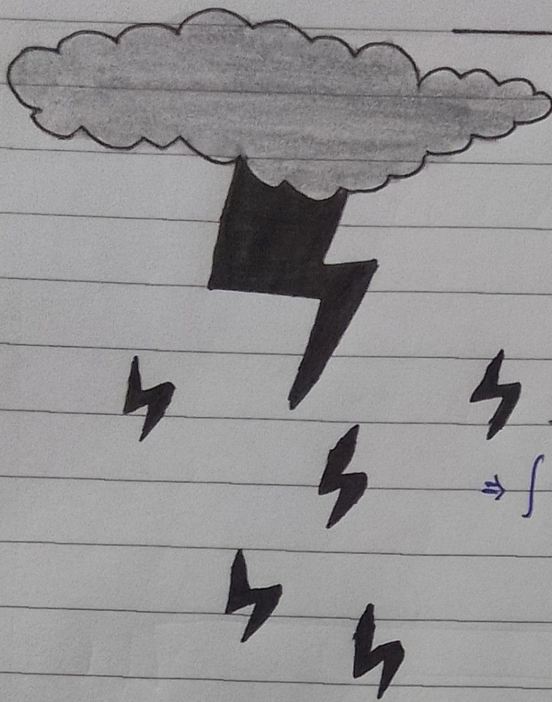


$$= \int \frac{\sqrt{5}\sec^2\theta\, d\theta}{5\tan^2\theta\,(5 + 5\tan^2\theta)^{\frac{1}{2}}} = \frac{1}{5}\int \frac{\sec^2\theta\, d\theta}{\tan^2\theta\,\sec\theta}$$

$$= \frac{1}{5}\int \frac{\sec\theta\, d\theta}{\tan^2\theta}$$

$$= \frac{1}{5}\int \sec\theta\, \cot^2\theta\, d\theta = \frac{1}{5}\int \frac{1}{\cos\theta} * \frac{\cos^2\theta}{\sin^2\theta}\, d\theta$$

$$= \frac{1}{5}\int \cos\theta \sin^{-2}\theta\, d\theta = \frac{-1}{5}\operatorname{cosec}\theta + C = \frac{-\sqrt{5 + x^2}}{5x} + C$$



$$\Rightarrow \int \frac{dx}{x^2\sqrt{x^2-25}} = 5\int \frac{\sec\theta \tan\theta \, d\theta}{25\sec^2\theta\sqrt{25\sec^2\theta-25}}$$

$$= \frac{1}{25}\int \frac{d\theta}{\sec\theta \tan\theta}$$

أكمل

ملحوظة!!

$x = 5\sec\theta$

$dx = 5\sec\theta\tan\theta\, d\theta$

$$\Rightarrow \int \sin^n\theta \cos^m\theta \, d\theta$$

① n, m odd

② n odd, m even

③ n, m even

!! ② في الحالة الثانية بأخذ واحد من العدد الفردي وادخله لـ $d\theta$

$$\int \sin^3\theta \cos^2\theta \, d\theta$$

$$= \underbrace{\int \sin\theta}_{\text{تكامل}} \sin^2\theta \cos^2\theta \, d\theta$$

$$= -\int \sin^2\theta \cos^2\theta \, d\cos\theta$$

$$= -\int (1-\cos^2\theta)\cos^2\theta \, d(\cos\theta)$$

$$= -\int \cos^2\theta \, d\cos\theta + \int \cos^4\theta \, d\cos\theta$$

$$= -\frac{\cos^3\theta}{3} + \frac{\cos^5\theta}{5} + C$$

!! ③ في الحالة الثالثة بأخذ واحدة بدلالة الثانية واستخدام قانون ضعف الزاوية

$$\int \sin^2\theta \cos^2\theta \, d\theta$$

$$= \frac{1}{4}\int (1-\cos 2\theta)(1+\cos 2\theta)\, d\theta$$

$$= \frac{1}{4}\int (1-\cos^2 2\theta)\, d\theta$$

$$= \frac{1}{4}\theta - \frac{1}{4^2}\int (1+\cos 4\theta)\, d\theta$$

$$= \frac{1}{4}\theta - \frac{\theta}{8} - \frac{\sin 4\theta}{32} + C$$