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Application to a drinking water network of robust periodic MPC ☆

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ABSTRACT

In this paper the application of a novel robust predictive controller for tracking periodic references to a section of Barcelona's drinking water network is presented. The system is modeled using a large scale uncertain differential-algebraic discrete time linear model in which it is assumed that a prediction of the water demand is available and that it is affected by unknown and bounded uncertainties. The control objective is to satisfy the water demand while trying to follow a given reference of the level of the tanks of the network. The controller considered has been modified to account for algebraic equations and large scale models and it joins a dynamic trajectory planner and a robust predictive controller in a single layer to guarantee that the closed-loop system converges asymptotically to a neighborhood of optimal reachable periodic trajectory satisfying the constraints for all possible uncertainties even in the presence of sudden changes in the reference. To demonstrate these properties three different simulation scenarios have been considered.

1. Introduction

Drinking water networks (DWNs) are large-scale systems subject to a set of operating, safety and quality-of-service constraints. The dynamics of this type of systems is usually affected by stochastic disturbances as is shown in Ocampo-Martinez, Puig, Cembrano, and Quevedo (2013) and Grosso, Ocampo-Martinez, and Puig (2013). Thus, an interesting research focus is the improvement of the management of DWN guaranteeing the water supply when there are errors in the demand forecasting as Barcelli, Ocampo-Martinez, Puig, and Bemporad (2010) and Ocampo-Martinez, Barcelli, Puig, and Bemporad (2012) have pointed out. Examples of this interest are Sampathirao et al. (2014) where various methods for demand forecasting were studied, such as seasonal ARIMA, BATS and support vector machine presenting a set of statically validated time series models; Agudelo-Vera et al. (2014) where a methodology to determine the robustness of the water drinking distribution systems was developed testing the performance of three networks under three future demand scenarios using head loss and resident time as indicators; and Le Quiniou, Mandel, and Monier (2014) where off-line support tools to optimize the procurement management of water reducing the energy costs where developed and applied to a DWN in France.

Model predictive control (MPC) is one of the most important control techniques used in industry to operate multi-variable constrained systems. Recently, this modern control technique has been applied to improve the management of DWN. In Fiorelli, Schutz, and Meyers (2011) MPC was used to manage the water storage in a small DWN in Luxemburg. In Ocampo-Martínez, Bovo, and Puig (2011) a decentralized MPC over a partitioned model of the Barcelona's drinking water network was presented. In Grosso, Ocampo-Martínez, Puig, and Joseph (2014) a chance-constrained MPC strategy based on a finite horizon stochastic optimization problem with joint probabilistic constraints was proposed. In Pascual et al. (2013) some model predictive control techniques are applied to the supervisory flow management in large-scale DWNs. In this case, MPC was used to generate the set-points for the regulatory controller (low level layer). In Grosso, Ocampo-Martinez, and Puig (2012) a model predictive control strategy to assure reliability of the DWN given a costumer service level and a forecasting demand was presented. In Grosso, Ocampo-Martinez, Puig, Limon, and Pereira (2014) a multi-objective cost function using a economically oriented model predictive control strategies was studied.

The operation of DWNs is strongly conditioned by the uncertainties in the forecast water demand and the possibly time varying costs, see Quevedo et al. (2006). Therefore, the optimal operation from an economic point of view may not be to regulate the system to a steady state but to follow a time-varying trajectory. If the evolution of the reference is known a priori, the tracking error can be predicted and then MPC can be designed to achieve asymptotic stability, however when the reference may be changed

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without a predefined deterministic law or even randomly the tracking problem is considerably more difficult since MPC is naturally suited to deterministic control problems. In the case of arbitrary changes of the reference, the resulting reference trajectory may not be reachable, which is an important issue in this context. Examples can be found in Pannocchia (2004) and Pannocchia, Laacho, and Rawlings (2005) where the set-point change is considered equivalent to a disturbance to be rejected and asymptotic stability and offset-free is ensured by integrating a disturbance model in the prediction model. A different approach was proposed in the context of reference governors (Bemporad, Casavola, & Mosca, 1997; Gilbert, Kolmanovsky, & Tan, 1999) that guarantee robust tracking without considering the performance of the obtained controller nor the domain of attraction. In Ferramosca, Limon, Alvarado, Alamo, and Camacho (2009) and Limon, Alvarado, Alamo, and Camacho (2008) a different strategy based on a single layer that unites dynamic trajectory planning and control was proposed which has been extended to deal with periodic references and economic MPC in Limon et al. (2015, 2014) and Pereira, Limon, Muñoz de la Peña, Valverde, and Alamo (2015).

The aforementioned approaches do not take into account the periodic of the demand which leads to periodic references which appear naturally in important control problems such as repetitive control (Lee, Natarajan, & Lee, 2001), periodic systems (Gondhalekar, Oldewurtel, & Jones, 2013; Kern, Bhm, Findeisen, & Allgwer, 2009) or economic operation of complex systems (Huang, Harinath, & Biegler, 2011). In addition, although there exists prediction of the future demand, the real demand is uncertain which leads to the need to consider robust predictive formulations, which is a relevant problem that has received a lot of attention from the MPC community (see Mayne, Rawlings, Rao, & Scokaert, 2000). Worst-case approaches, for example, Campo and Morari (1987) and Scokaert, Rawlings, and Meadows (1997), can be found in the literature, but usually are characterized by a large computational burden. More recent approaches are based on minimizing a cost function that depends on nominal predictions while guarantee robust constraint satisfaction, see Chisci, Rossiter, and Zappa (2001), Mayne et al. (2000) and Langson, Chryssochoos, Rakovic, and Mayne (2004). Following this idea, in Alvarado, Limon, Muñoz de la Peña, Alamo, and Camacho (2010) a method that under certain assumptions avoids the computation of minimal robust positive invariant sets was proposed.

The main contribution of this paper is to propose a model predictive control technique capable of coping with the inherent issues of the management of the DWN: (i) efficient operation of the network fulfilling the operation limits of the tanks and actuators; (ii) satisfaction of the water demand at each time in spite of the mismatches of the forecasted values; (iii) guarantee of these properties in the presence of possible variations in management policy that may be derived, for instance, from changes in the unitary prices of the energy supply or the water supply. To this aim, the robust MPC for tracking presented in Pereira, Muñoz de la Peña, Limon, Alvarado, and Alamo (2016) has been extended to deal with this problem. The following extensions have been introduced: (i) capability to deal uncertain difference-algebraic models that describe the DWN; (ii) introduction of a feedforward control scheme to guarantee the satisfaction the real demand at each sampling time in spite on the uncertain variation; (iii) introduction of a design methodology appropriate for large-scale DWN and (iv) guarantee of robust closed-loop stability and constraint satisfaction of the controlled network.

In order to demonstrate these features, the proposed controller has been applied in simulation to a section of Barcelona's drinking water network presented in Sampathirao et al. (2014). Different scenarios have been considered to demonstrate that the proposed controller is capable of operating the DWN efficiently satisfying the operation limit and converging to the best possible trajectory in spite of the uncertainty on the demand and sudden changes in the trajectory to be tracked.

2. Description of the drinking water network

In order to demonstrate that robust MPC is a suitable control scheme to deal with the management of DWN, a section of Barcelona's drinking water network is chosen as case study. This case study has been proposed and used to develop novel control strategies by Quevedo and Puig's group, see Ocampo-Martínez et al. (2011), Ocampo-Martinez et al. (2013) and the references therein.

Barcelona's DWN is divided into two layers: the transport network, that links the water treatment plants with the reservoirs spread over the city, and the distribution network, that links the reservoirs with the consumers. As the distribution network is sectorised, each sector of the distribution network can be described as a demand for the appropriate reservoir which can be forecasted by means of time-series model (Ocampo-Martínez et al., 2011).

The water management system consists of a low-level control level that manipulates the actuators of the valves and the pumps to ensure a flow set-point provided by the upper-level control. This regulatory level accomplishes the fast and nonlinear behavior of the network and allows the upper level control to be designed by using a flow-based control-oriented model where the manipulable inputs are the setpoints of the flows of the valves and pumps provided to the regulatory level (Ocampo-Martínez et al., 2011; Sampathirao et al., 2014).

The case study of this paper is a part of the transport network that consists of 17 water tanks connected to 25 reservoirs or demand points from which water is consumed, and nine water supply points from which water is obtained.

The network is modeled from the water balance equations at each of the network nodes and tanks. The water balance equations of the tanks is read as the variation of the volume of water stored in each tank, $\mathbf{x}_i$ (in m^3), is equal to sum of the input flows (in m^3/s) minus the output flows (in m^3/s). This balance applied to each one of the 17 tanks of the network leads to the following set of differential equations:

$$\dot{\mathbf{x}}_1(t) = \mathbf{u}_3(t) + \mathbf{u}_4(t) - \mathbf{d}_1(t) \tag{1a}$$

$$\dot{\mathbf{x}}_2(t) = \mathbf{u}_5(t) - \mathbf{d}_3(t) \tag{1b}$$

$$\dot{\mathbf{x}}_3(t) = \mathbf{u}_7(t) - \mathbf{u}_8(t) + \mathbf{u}_{10}(t) + \mathbf{u}_{11}(t) - \mathbf{d}_4(t) \tag{1c}$$

$$\dot{\mathbf{x}}_4(t) = \mathbf{u}_8(t) + \mathbf{u}_9(t) - \mathbf{u}_{10}(t) - \mathbf{u}_{11}(t) + \mathbf{u}_{13}(t) - \mathbf{u}_{14}(t) + \mathbf{u}_{19}(t) \tag{1d}$$

$$\dot{\mathbf{x}}_5(t) = \mathbf{u}_{12}(t) - \mathbf{u}_{15}(t) + \mathbf{u}_{16}(t) - \mathbf{u}_{20}(t) - \mathbf{u}_{21}(k) \tag{1e}$$

$$\dot{\mathbf{x}}_6(t) = \mathbf{u}_6(t) + \mathbf{u}_{20}(t) - \mathbf{u}_{23}(t) + \mathbf{u}_{27}(t) \tag{1f}$$

$$\begin{aligned}\dot{\mathbf{x}}_7(t) = {} & \mathbf{u}_{17}(t) - \mathbf{u}_{18}(t) - \mathbf{u}_{22}(t) + \mathbf{u}_{23}(t) + \mathbf{u}_{24}(t) - \mathbf{u}_{31}(t) - \mathbf{u}_{32}(t) \\ & + \mathbf{u}_{37}(t) - \mathbf{u}_{38}(t) - \mathbf{d}_{13}(t)\end{aligned} \tag{1g}$$

$$\dot{\mathbf{x}}_8(t) = \mathbf{u}_{21}(t) - \mathbf{u}_{24}(t) - \mathbf{u}_{27}(t) - \mathbf{u}_{33}(t) - \mathbf{u}_{34}(t) - \mathbf{d}_{10}(t) \tag{1h}$$

$$\dot{\mathbf{x}}_9(t) = -\mathbf{u}_{28}(t) + \mathbf{u}_{29}(t) + \mathbf{u}_{33}(t) - \mathbf{d}_8(t) \tag{1i}$$

$$\dot{\mathbf{x}}_{10}(t) = -\mathbf{u}_{29}(t) + \mathbf{u}_{30}(t) - \mathbf{u}_{36}(t) - \mathbf{u}_{37}(t) + \mathbf{u}_{38}(t) - \mathbf{u}_{42}(k) + \mathbf{u}_{45}(t) + \mathbf{u}_{51}(t) + \mathbf{u}_{52}(t) - \mathbf{d}_{12}(t) \tag{1j}$$

$$\dot{\mathbf{x}}_{11}(t) = \mathbf{u}_{35}(t) + \mathbf{u}_{36}(t) - \mathbf{d}_{11}(t) \tag{1k}$$

$$\dot{\mathbf{x}}_{12}(t) = \mathbf{u}_{41}(t) + \mathbf{u}_{47}(t) - \mathbf{u}_{48}(t) + \mathbf{u}_{56}(t) - \mathbf{d}_{18}(t) \tag{1l}$$

$$\dot{\mathbf{x}}_{13}(t) = \mathbf{u}_{42}(t) - \mathbf{u}_{44}(t) - \mathbf{d}_{19}(t) \tag{1m}$$

$$\dot{\mathbf{x}}_{14}(t) = -\mathbf{u}_{46}(t) - \mathbf{u}_{53}(t) - \mathbf{u}_{54}(t) + \mathbf{u}_{55}(t) + \mathbf{u}_{57}(t) + \mathbf{u}_{58}(t) - \mathbf{d}_{21}(t) \tag{1n}$$

$$\dot{\mathbf{x}}_{15}(t) = -\mathbf{u}_{49}(t) + \mathbf{u}_{50}(t) + \mathbf{u}_{53}(t) - \mathbf{d}_{23}(t) \tag{1o}$$

$$\dot{\mathbf{x}}_{16}(t) = \mathbf{u}_{54}(t) + \mathbf{u}_{59}(t) - \mathbf{d}_{24}(t) \tag{1p}$$

$$\dot{\mathbf{x}}_{17}(t) = \mathbf{u}_{48}(t) + \mathbf{u}_{60}(t) - \mathbf{d}_{22}(t) \tag{1q}$$

where $\mathbf{x}_i(t)$ denotes the volume of water (in m^3) stored in the i-th tank, $\mathbf{u}_j(t)$ denotes water flow (in m^3/s) through j-th actuator given and $\mathbf{d}_k(t)$ denotes the water flow $\left(\text{in} \frac{m^3}{s}\right)$ k-th water demand. Denoting $\mathbf{x} \in \mathbb{R}^{61}$ as the vector of the water volume stored in the tanks, $\mathbf{u} \in \mathbb{R}^{61}$ as the vector of manipulable flows of the network and $\mathbf{d} \in \mathbb{R}^{25}$ as the vector of the water demands of the reservoirs, these equations can be written as follows:

$$\dot{\mathbf{x}}(t) = \mathbf{A_c}\mathbf{x}(t) + \mathbf{B_{cu}}\mathbf{u}(t) + \mathbf{B_{cd}}\mathbf{d}(t) \tag{2}$$

On the other hand, the water balance equations of the nodes is read as the sum of the input flows (in m^3/s) minus the sum of the output flows (in m^3/s) is zero. Thus, this balance applied to each one of the eleven nodes of the network results in the following set of algebraic equations:

$$\mathbf{u}_1(t) - \mathbf{u}_2(t) - \mathbf{u}_5(t) - \mathbf{u}_6(t) = 0 \tag{3a}$$

$$\mathbf{u}_2(t) - \mathbf{u}_3(t) = \mathbf{d}_2(t) \tag{3b}$$

$$\mathbf{u}_{18}(t) - \mathbf{u}_{13}(t) = \mathbf{d}_5(t) \tag{3c}$$

$$\mathbf{u}_{14}(t) + \mathbf{u}_{15}(t) - \mathbf{u}_{19}(t) - \mathbf{u}_{25}(t) + \mathbf{u}_{26}(t) = \mathbf{d}_7(t) \tag{3d}$$

$$\mathbf{u}_{22}(t) - \mathbf{u}_{30}(t) = \mathbf{d}_9(t) \tag{3e}$$

$$\mathbf{u}_{32}(t) - \mathbf{u}_{39}(t) - \mathbf{u}_{40}(t) = \mathbf{d}_{14}(t) \tag{3f}$$

$$\mathbf{u}_{25}(t) - \mathbf{u}_{26}(t) + \mathbf{u}_{32}(t) + \mathbf{u}_{34}(t) + \mathbf{u}_{40}(t) - \mathbf{u}_{41}(t) = \mathbf{d}_{15}(t) \tag{3g}$$

$$\mathbf{u}_{39}(t) - \mathbf{u}_{45}(t) + \mathbf{u}_{46}(t) - \mathbf{u}_{47}(t) = \mathbf{d}_{17}(t) \tag{3h}$$

$$\mathbf{u}_{28}(t) - \mathbf{u}_{35}(t) - \mathbf{u}_{43}(t) + \mathbf{u}_{49}(t) = \mathbf{d}_{16}(t) \tag{3i}$$

$$\mathbf{u}_{43}(t) + \mathbf{u}_{44}(t) = \mathbf{d}_{20}(t) \tag{3j}$$

$$\mathbf{u}_{61}(t) - \mathbf{u}_{50}(t) - \mathbf{u}_{51}(t) - \mathbf{u}_{52}(t) - \mathbf{u}_{56}(t) - \mathbf{u}_{57}(t) - \mathbf{u}_{58}(t) - \mathbf{u}_{59}(t) - \mathbf{u}_{60}(t) = \mathbf{d}_{25}(t) \tag{3k}$$

This set of equations can be rewritten as follows:

$$0 = \mathbf{E_u}\mathbf{u}(t) + \mathbf{E_d}\mathbf{d}(t) \tag{4}$$

Constraints which limit the maximum volume of each tank and the maximum water flow of each actuator (the flows of the network are one directional and cannot be reversed) are considered. In particular it is assumed that $0 \le \mathbf{x}_i(t) \le \mathbf{x}_i^{max}$ and $0 \le \mathbf{u}_i(t) \le \mathbf{u}_i^{max}$ for all tanks and actuators. The maximum values for each tank and actuator can be found in the supplementary material in Tables 1 and 2 respectively. There are large differences between the maximum flow values of each of the actuators. For example, actuator 50 has a maximum value of 15 m^3/s, while actuators 8 or 9 have minimum values of 0.03 m^3/s or 0.0056 m^3/s, respectively. It is worth to note that one of the actuators is assumed to be zero for operating reasons, in particular actuator 7, which has a maximum flow of 10^{-5} m^3/s. These differences must be taken into account in the controller design to avoid feasibility and constraint satisfaction issues in the presence of uncertainty in the demand.

It is important to note that the model of the tank depends on the demand, which is not precisely known, but forecasted. This implies that the controller might be able to incorporate this prediction to compensate this effect but this must account for the possible mismatches between the real demand and the expected one in order to ensure the behavior of the system and the fulfill the operation limits of the network. With a slight abuse of notation, in what follows the forecasted demand will be denoted as $d_i(t)$ and the uncertainty on the demand as $w_i(t)$.

In order to design the proposed controller, the differential-algebraic equations (2) and (4) that describe the network has been discretized using the Euler approximation and a sampling time of one hour. Taking into account also the uncertainty on the demand, the resulting uncertain discrete time algebraic-difference linear model that defines the network is as follows:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B_u}\mathbf{u}(k) + \mathbf{B_d}(\mathbf{d}(k) + \mathbf{w}(k)) \tag{5a}$$

$$0 = \mathbf{E_u}\mathbf{u}(k) + \mathbf{E_d}(\mathbf{d}(k) + \mathbf{w}(k)) \tag{5b}$$

where $\mathbf{x}(k) \in \mathbb{R}^{17}$ denotes the vector of the volume of the tanks in m^3 at $t = 3600$ k. $\mathbf{u}(k) \in \mathbb{R}^{61}$ denotes the vector of water flows through the sixty one actuators given in m^3/s at $t = 3600k$, vector $\mathbf{d}(k) \in \mathbb{R}^{25}$ denotes the known predicted demands in m^3/s at $t = 3600$ k and vector $\mathbf{w}(k) \in \mathbb{R}^{25}$ denotes the prediction error in these demands. The matrices of the model are calculated as follows: $\mathbf{A} = \mathbf{I_{17}} + 3600\mathbf{A_c}$, $\mathbf{B_u} = 3600\mathbf{B_{cu}}$ and $\mathbf{B_d} = 3600\mathbf{B_{cd}}$, where $\mathbf{I_{17}}$ is the identity matrix of order 17.

The states of the system and the manipulable inputs must be fulfill the constraints $\mathbf{x}(k) \in \mathcal{X}$, $\mathbf{u}(k) \in \mathcal{U}$, where sets $\mathcal{X}$ and $\mathcal{U}$ are defined from corresponding maximum and minimum values.

In this case study, the predicted demand $\mathbf{d}(k)$ that will be used in the simulations has been obtained from historic data (Ocampo-Martinez et al., 2012) and this can be found in the supplementary material in Tables 3–5. The prediction error is assumed to be bounded in the set $\mathcal{W}$ defined as follows:

$$\mathbf{w}(k) \in \mathcal{W} \triangleq \{\mathbf{w} \in \mathbb{R}^{25}: \quad |\mathbf{w}_i| \le \mathbf{w}_i^{max}, \quad \forall\, i = 1, \ldots, 25\} \tag{6}$$

where the maximum prediction error is equal to 5% of the maximum demand during the test, that is, $\mathbf{w}_i^{max} = 0.05\max_k \mathbf{d}_i(k)$.

3. Robust MPC design for the DWN

3.1. Controller design aspects

The control objective is to drive the system as close as possible to an arbitrary state and input periodic target reference. For this case study, the target trajectories of the tank levels $\mathbf{x}_i^t(k)$ are given in the supplementary material in Tables 6–8. These trajectories where obtained solving an optimization problem to optimize the economic operation cost taking into account the water and the electricity costs but not taking into account the constraints. The value of the actuator water flow references $\mathbf{u}_i^t(k)$ have been obtained from the solution in the least squares sense to the under determined system of equations obtained from the dynamic model and the predicted values of the demand. The target trajectories do not satisfy the constraints for all times, in particular the target level of the tanks is greater than the maximum allowed value during some time.

To design a robust controller, an auxiliary control input is introduced from the explicit solution of Eq. (5b) in order to satisfy the water balance equations for any demand prediction error. Note that it is not possible to formulate a robust model predictive control optimization problem based on a differential-algebraic equation using as optimization variables the actuator flows because it would not be possible to satisfy the water balance equality constraints for all possible uncertainties. The value of the water flows $\mathbf{u}(k)$ that satisfy the water balance equations are given by

$$\mathbf{u}(k) = \mathbf{M_1}\mathbf{d}(k) + \mathbf{M_2}\mathbf{v}(k) \tag{7}$$

where $\mathbf{v}(k) \in \mathbb{R}^{50}$ denotes the new set of control inputs which guarantee the water balance equations in the eleven nodes that do not have a tank. Matrices $\mathbf{M_1} \in \mathbb{R}^{61\times 25}$ and $\mathbf{M_2} \in \mathbb{R}^{61\times 50}$ are obtained from the solution of (5b). Matrix $\mathbf{M_2}$ is an orthogonal basis for the null space of $\mathbf{E_u}$ obtained from the singular value decomposition. Matrix $\mathbf{M_1}$ provides a particular solution to the equation that depends on $\mathbf{d}(k)$. There are infinite solutions which provide different ways of distributing the flow to satisfy the balance equations for a given demand. An inappropriate selection of matrix $\mathbf{M_1}$ may lead to optimization problems with a reduced feasibility region, in particular if matrix $\mathbf{M_1}$ distributes the demand in a way such that the actuators with a lower maximum value are saturated. In order to distribute the demand taking into account that each flow has a different maximum value, matrix $\mathbf{M_1}$ has been chosen in a way such that minimizes the norm of the matrix $\mathbf{M_1^T G M_1}$ where $\mathbf{G}$ is a diagonal matrix that weights each actuator $\mathbf{u}_i$ inversely to the square of its maximum value.

The real water demand must always be satisfied, to this end at each sampling time, the MPC controller will decide the optimal value for the auxiliary control input $\mathbf{v}^*(k)$, which is designed to satisfy the predicted demand $\mathbf{d}(k)$, however, the real value of the actuators $\mathbf{u}^*(k)$ are obtained both from $\mathbf{v}^*(k)$ and the real demand which is available instantaneously; that is, taking into account the prediction error $\mathbf{w}(k)$:

$$\mathbf{u}^*(k) = \mathbf{M_1}(\mathbf{d}(k) + \mathbf{w}(k)) + \mathbf{M_2}\mathbf{v}^*(k) \tag{8}$$

3.2. Tightening local control law

In order to design the proposed robust MPC for tracking periodic references, a local control law aimed at reducing the effect of the uncertainty in the predictions is needed. In the water distribution control problem considered, this local control law decides the value of the auxiliary control inputs $\mathbf{v}(k)$ on behalf of the deviation of the perturbed predictions from the nominal predictions obtained the previous sampling time $e(k)$. The objective of this control law is to reject the uncertainty. To this end, a linear control law, that is $\mathbf{v}(k) = \mathbf{K}\mathbf{e}(k)$ is designed for the system

$$\mathbf{e}(k+1) = \mathbf{A}\mathbf{e}(k) + \mathbf{B_u}\mathbf{M_1}\mathbf{v}(k) \tag{9}$$

where $\mathbf{e}(k)$ in the aforementioned deviation. This system is obtained taking into account the definition of the auxiliary control variable and ignoring the effect of the predicted demand in system (5).

The local controller is used to design the reduced set of constraints that guarantee both robust constraint satisfaction and recursive feasibility of the controller in closed-loop. In order to guarantee recursive feasibility and hence, closed-loop convergence to the optimal feasible periodic trajectory, the local control gain must satisfy

$$\max_{\mathbf{w}\in\mathcal{W}} \| (\mathbf{A} + \mathbf{B_u}\mathbf{M_1}\mathbf{K})^{N-1}\mathbf{w} \| \le \sigma \tag{10}$$

where σ is the tolerance of the optimization problem solver. This implies that the local controller is able to eliminate the effect of any uncertainty after $N-1$ time steps. Although in principle any stabilizing linear gain that guarantees disturbance rejection in N steps could be used, an inappropriate design of this controller may result in empty feasibility regions of the MPC optimization problems. To avoid this issue, the controller has to be designed taking into account the constraints on the tanks and the actuators, and more precisely, it has to take into account the difference in the state and actuators ranges. Design procedures to guarantee that the resulting optimization problem has a non-empty feasibility region are outside the scope of this work. The reader can refer to Alvarado et al. (2010) for an LMI based design procedure for this problem.

In the considered WDN, an LQR control law has been designed using weight matrices that depend on the maximum tank and water flow levels. In particular, the weighting matrices Q_K and R_K are defined as follows

$$\mathbf{Q_K} = diag(1/\mathbf{x}_i^{max}),\ \ \mathbf{R_K} = p\mathbf{M_2^T}diag(1/\mathbf{u}_i^{max})\mathbf{M_2} \tag{11}$$

where p weights the input cost with respect to the state cost. The value $p=10$ was chosen by trial and error.

The control gain obtained using these weights satisfies the uncertainty rejection assumption and yields an MPC optimization problem with a non-empty feasibility region. In particular, the maximum eigenvalue of the matrix $(\mathbf{A} + \mathbf{B_u}\mathbf{M_1}\mathbf{K}))^{23}$ is $1.6483 \cdot 10^{-34}$.

3.3. Proposed robust MPC controller

The cost function[1] of the proposed controller is defined as follows:

$$V_N\left(\mathbf{x}, \hat{\mathbf{x}}^t, \hat{\mathbf{u}}^t, \hat{\mathbf{d}}; \mathbf{x}_0^r, \hat{\mathbf{v}}^r, \hat{\mathbf{v}}\right) = V_t\left(\mathbf{x}, \hat{\mathbf{d}}; \mathbf{x}_0^r, \hat{\mathbf{v}}^r, \hat{\mathbf{v}}\right) + V_p\left(\hat{\mathbf{x}}^t, \hat{\mathbf{u}}^t, \hat{\mathbf{d}}; \mathbf{x}_0^r, \hat{\mathbf{v}}^r\right) \tag{12}$$

where

$$V_t\left(\mathbf{x}, \hat{\mathbf{d}}; \mathbf{x}_0^r, \hat{\mathbf{v}}^r, \hat{\mathbf{v}}\right) = \sum_{i=0}^{N-1} \|\mathbf{x}(i) - \mathbf{x}^r(i)\|_{\mathbf{Q}}^2 + \|\mathbf{u}(i) - \mathbf{u}^r(i)\|_{\mathbf{R}}^2$$
$$V_p\left(\hat{\mathbf{x}}^t, \hat{\mathbf{u}}^t, \hat{\mathbf{d}}; \mathbf{x}_0^r, \hat{\mathbf{v}}^r\right) = \sum_{i=0}^{T-1} \left\|\mathbf{x}^r(i) - \mathbf{x}^t(i)\right\|_{\mathbf{S}}^2 + \left\|\mathbf{u}^r(i) - \mathbf{u}^t(i)\right\|_{\mathbf{V}}^2 \tag{13}$$

The parameters that define the optimization problem at time step

[1] Hat bold letters denote trajectories of signals over the prediction horizon/period.

k are the current state $\mathbf{x}$, the future state and actuator trajectories given by vectors $\hat{\mathbf{x}}^t$, $\hat{\mathbf{u}}^t$ respectively and the predicted demand given by vector $\hat{\mathbf{d}}$. The optimization variables are the auxiliary reference defined by its initial state $\mathbf{x}_0^r$ and future T-step (one period) auxiliary control input trajectory $\hat{\mathbf{v}}^r$, and the predicted N-step auxiliary control input trajectory $\hat{\mathbf{v}}^r$.

The control scheme considered is a tracking controller and the cost considered has no physical interpretation (nor economic). The term $V_t\left(\mathbf{x}, \hat{\mathbf{d}}; x_0^r, \hat{\mathbf{v}}^r, \hat{\mathbf{v}}\right)$ penalizes the tracking error of the open-loop predicted trajectories with respect to the planned reachable reference along the prediction horizon N. This cost defines the transient behavior. The term $V_p\left(\hat{\mathbf{x}}^t, \hat{\mathbf{u}}^t, \hat{\mathbf{d}}; \mathbf{x}_0^r, \hat{\mathbf{v}}^r\right)$ penalizes the error between the artificial reference trajectory and the target reference trajectory along one period of length T time steps. This cost defines the steady state behavior.

It is important to note that the control effort needed to achieve the effective response (that is the tank level reference) is not optimized directly in this tracking controller. The controller tries to minimize the cost function which depends on two different terms, first, the deviation of the steady state periodic trajectory from the desired one (which defines precisely the distance criterion) and a tracking term that defines the transient behavior. Modifying the cost matrices of the tracking and the artificial reference cost, different transient trajectories and hence different control efforts would be achieved, however, the steady state control effort is defined by the desired reference, which is decided by a higher hierarchy system.

The optimal trajectories of the proposed robust MPC for tracking periodic signals can be obtained from the solution of the following finite horizon optimal control problem $\mathcal{P}_N(\mathbf{x}, \hat{\mathbf{x}}^t, \hat{\mathbf{u}}^t, \hat{\mathbf{d}})$[2]

$$\min_{\mathbf{x}_0^r, \hat{\mathbf{v}}^r, \hat{\mathbf{v}}} V_N\left(\mathbf{x}, \hat{\mathbf{x}}^t, \hat{\mathbf{u}}^t, \hat{\mathbf{d}}; \mathbf{x}_0^r, \hat{\mathbf{v}}^r, \hat{\mathbf{v}}\right) \text{s. t. } \mathbf{x}(0) = \mathbf{x} \tag{14a}$$

$$\mathbf{x}(i+1) = \mathbf{A}\mathbf{x}(i) + \mathbf{B_u}\mathbf{u}(i) + \mathbf{B_d}\mathbf{d}(i) \quad i \in \mathbb{Z}_N^1 \tag{14b}$$

$$\mathbf{u}(i) = \mathbf{M_1}\mathbf{d}(i) + \mathbf{M_2}\mathbf{v}(i) \quad i \in \mathbb{Z}_{N-1}^0 \tag{14c}$$

$$\mathbf{x}(i) \in \mathcal{X}_i \quad i \in \mathbb{Z}_N^1 \tag{14d}$$

$$\mathbf{u}(i) \in \mathcal{U}_i \quad i \in \mathbb{Z}_{N-1}^0 \tag{14e}$$

$$\mathbf{x}(N) = \mathbf{x}^r(N) \tag{14f}$$

$$\mathbf{x}^r(i+1) = \mathbf{A}\mathbf{x}^r(i) + \mathbf{B_u}\mathbf{u}^r(i) + \mathbf{B_d}\mathbf{d}(i) \quad i \in \mathbb{Z}_T^1 \tag{14g}$$

$$\mathbf{u}^r(i) = \mathbf{M_1}\mathbf{d}(i) + \mathbf{M_2}\mathbf{v}^r(i) \quad i \in \mathbb{Z}_{T-1}^0 \tag{14h}$$

$$\mathbf{x}^r(i) \in \mathcal{X}_N \quad i \in \mathbb{Z}_T^1 \tag{14i}$$

$$\mathbf{u}^r(i) \in \mathcal{U}_{N-1} \quad i \in \mathbb{Z}_{T-1}^0 \tag{14j}$$

$$\mathbf{x}^r(T) = \mathbf{x}^r(0) = \mathbf{x}_0^r \tag{14k}$$

where the sets $\mathcal{X}_i$ and $\mathcal{U}_i$ are defined as follows

$$\mathcal{X}_i = \mathcal{X} \ominus \oplus_{j=0}^{i-1} (\mathbf{A} + \mathbf{B_u}\mathbf{M_2}\mathbf{K})^j(\mathbf{B_d} + \mathbf{B_u}\mathbf{M_1})\mathcal{W}$$
$$\mathcal{U}_i = \mathcal{U} \ominus \mathbf{M_1}\mathcal{W} \ominus \mathbf{M_2}\mathbf{K} \oplus_{j=0}^{i-1} (\mathbf{A} + \mathbf{B_u}\mathbf{M_2}\mathbf{K})^j(\mathbf{B_d} + \mathbf{B_u}\mathbf{M_1})\mathcal{W} \tag{15}$$

It is important to remark that the calculation of these sets is trivial, even for the system of dimension 17 considered. The calculation, however, of a robust positive invariant set may be in general a difficult task, precluding the application of robust scheme to large scale systems.

The optimal solution of this optimization problem at time step k is denoted $\left(\mathbf{x}_0^{r*}(k), \hat{\mathbf{v}}^{r*}(k), \hat{\mathbf{v}}^*(k)\right)$. The value of the water flows of each actuator depends on the solution of this optimization problem and the real demand and is obtained as follows:

$$\mathbf{u}(k) = \mathbf{M_2}\mathbf{v}^*(0|k) + \mathbf{M_1}(\mathbf{d}(k) + \mathbf{w}(k)) \tag{16}$$

where $\mathbf{v}^*(0|k)$ is the optimum value for the first auxiliary input at time step k. This implies that the water flows are different from the ones predicted in the optimization problem because they have to be modified to account for the prediction errors. For this reason, in order to guarantee robust constraint satisfaction on the constraints of the water flows, the feasible set must be reduced taking into account the possible effect of the prediction error. Constraints (14e) and (14j) reduce the feasible set of water flows $\mathcal{U}$ by the set $\mathbf{M_1}\mathcal{W}$ to account for this issue.

The constraints of the optimization variables, which define maximum and minimum values of the tank levels and the network water flows, are contracted with every step of the prediction horizon. Constraints (14e) and (14j) show that as the prediction step i increases, the sets are reduced taking into account the possible effect of a perturbation on the predicted system in closed-loop with the auxiliary controller. This contraction is time invariant and can be calculated off-line. Constraints (14b)-(14c) are defined by the nominal model, that is, assuming that the prediction error is zero, and provides the predicted state and input trajectories. Constraint (14b) imposes that the initial state of the predicted trajectory is equal to the state of the system at time step k. Constraint (14f) states that the predicted state must reach the artificial reference in T steps. These constraints are used to guarantee recursive feasibility using an appropriately defined shifted solution. Constraints (14g)–(14h) are defined by the nominal model, that is, assuming that the prediction error is zero, and provides the artificial references state and input trajectories. Note that the initial state of the artificial reference is a free variable, however, it is constrained to be a periodic trajectory in constraint (14k). The artificial references must satisfy the state and input constraints, but because in order to guarantee recursive feasibility, the artificial reference is used to define the shifted input trajectory at prediction time $N-1$, the constraint set is contracted by the same set for all steps which depends on the prediction horizon N. In particular, the artificial references must satisfy (14d) and (14e) for $i = N-1$.

If the controller is not designed appropriately, the admissible tank levels and actuators flow sets for the predicted trajectories may be empty for some time step i. In this case, the optimization problem is unfeasible for all states. The chosen matrix $\mathbf{M_1}$ and the control law gain $\mathbf{K}$ for the simulations guarantee the cancelation of the effect of an uncertainty in $N-1$ time steps and that the feasibility set is not empty.

It is important to note that the constraints of the optimization

[2] $\mathbb{Z}_j^i$ denotes the sequence $\{i, \ldots, j\}$.

problem do not depend on the target trajectories. This implies that a sudden change in these trajectories cannot cause a loss of feasibility of the optimization problem. This will be shown in the simulation example.

For this case study, the prediction horizon is chosen equal to the period, that is $N = T = 24$. The cost matrices $\mathbf{Q}$, $\mathbf{R}$, $\mathbf{S}$ and $\mathbf{V}$ are defined as follows

$$\mathbf{Q} = 100\cdot\mathcal{I}_{17},\ \mathbf{R} = 10\cdot\mathcal{I}_{61},\ \mathbf{S} = 700\cdot\mathcal{I}_{17},\ \mathbf{V} = 700\cdot\mathcal{I}_{61} \tag{17}$$

where $\mathcal{I}_n$ is the identity matrix of dimension n. These matrices define the optimal trajectories but do not affect the closed-loop properties of the controller.

The provided target trajectory may not be coherent with the dynamic model or the constraints. As proved in Pereira et al. (2016), the system in closed-loop converges asymptotically to a neighborhood of the trajectory obtained minimizing $V_p\left(\hat{\mathbf{x}}^t, \hat{\mathbf{u}}^t, \hat{\mathbf{d}}; \mathbf{x}_0^r, \hat{\mathbf{v}}^r\right)$ w.r.t. $\left(\mathbf{x}_0^r, \hat{\mathbf{v}}^r\right)$, subject to constraints (14g)–(14k). This optimization problem is denoted the robust planner. Because the demand and the target references are periodic, and the cost function is strictly convex, the optimal periodic trajectories do not depend on the time step k in which the optimization problem is formulated. The resulting trajectory takes into account the effect of the uncertainty in the constraints in order to guarantee robust constraint satisfaction. In this case, the robust planner is not independent of the prediction horizon of the corresponding robust MPC for tracking, because the reduction of the constraint sets depends on N. In the simulations the optimal trajectories of this optimization problem are denoted as the robust planner trajectories. If the prediction error is assumed to be zero, the nominal planner trajectories defined in Limon et al. (2015) are obtained. It is important to remark that it is not necessary to solve the planner optimization problems to define the MPC controller. The convergence property stems directly from the controller formulation.

3.4. Recursive feasibility

A traditional analysis of the closed-loop properties of the controller considered are beyond the scope of this work. This analysis can be found in Pereira et al. (2016), however, the implementation procedure presented takes into account the effect of the uncertainty in the decision of the flows through the valves and pumps and that in general, for large scale systems is difficult to obtain invariant sets. To this end the constraints of the MPC optimization problem have been modified taking into account these issues. The procedure proposed to take into account the algebraic constraints and the use of a terminal equality constraint in the presence of bounded uncertainties instead of a traditional robust invariant set leads to a different proof of recursive robust feasibility, which is necessary to inherit the closed-loop properties of the controller presented in Pereira et al. (2016).

In this section it is proved that closed-loop constraint satisfaction and recursive feasibility of the optimization problem is guaranteed if the initial state is inside the feasibility region, even in the presence of sudden changes in the reference. To this end, a feasible solution for $\mathbf{x}(k+1)$ denoted shifted solution is obtained from the optimal solution for $\mathbf{x}(k)$. The notation $|k$ is used to denote the time step to which a given variable is referred and bold letter to denote vectors or a sequence of variables. The shifted variables are denoted with the superscript s and $\Delta x^{*s} = \mathbf{x}^s(i|k+1) - \mathbf{x}^*(i+1|k)$. The shifted solution at time $k+1$ is obtained as follows:

$$\mathbf{x}^s(0|k+1) = \mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B_u}\mathbf{u}(0|k)^* + \mathbf{B_d}\mathbf{d}(k) + \mathbf{B_d}\mathbf{w}(k) \tag{18a}$$

$$\mathbf{v}^{rs}(i|k+1) = \mathbf{v}^{r*}(i+1|k), \quad i = 0, \ldots, N-2 \tag{18b}$$

$$\mathbf{v}^{rs}(N-1|k+1) = \mathbf{v}^{r*}(0|k) \tag{18c}$$

$$\mathbf{v}^s(i|k+1) = \mathbf{v}^*(i+1|k) + \mathbf{K}\Delta x^{*s} \quad i = 0, \ldots, N-2\mathbf{v}^s(N-1|k+1) = \mathbf{v}^{r*}(0|k) \tag{18d}$$

Taking into account constraints (14b), (14c), (14g) and (14h) it follows that:

$$\mathbf{u}^{rs}(i|k+1) = \mathbf{u}^{r*}(i+1|k), \quad i = 0, \ldots, N-2 \tag{19a}$$

$$\mathbf{u}^{rs}(N-1|k+1) = \mathbf{u}^{r*}(0|k) \tag{19b}$$

$$\mathbf{u}^s(i|k+1) = \mathbf{u}^*(i+1|k) + \mathbf{M}_2\mathbf{K}\Delta x^{*s} \quad i = 0, \ldots, N-2 \tag{19c}$$

$$\mathbf{u}^s(N-1|k+1) = \mathbf{u}^{r*}(0|k) \tag{19d}$$

Taking into account that the artificial reference is a periodic trajectory, the shifted artificial reference states are the following

$$\mathbf{x}^{rs}(i|k+1) = \mathbf{x}^{r*}(i+1|k), \quad i = 0, \ldots, N-1\mathbf{x}^{rs}(N|k+1) = \mathbf{x}^{r*}(1|k) \tag{20a}$$

The shifted predicted states are obtained using the following equation:

$$\mathbf{x}^s(i+1|k+1) = \mathbf{A}\mathbf{x}^s(i|k+1) + \mathbf{B_u}\mathbf{u}^s(i|k+1) + \mathbf{B_d}\mathbf{d}(i) \tag{21}$$

with $\mathbf{x}^s(0|k+1) = \mathbf{x}(k+1)$. By definition, these states satisfy

$$\mathbf{x}^s(i|k+1) = \mathbf{x}^*(i+1|k) + (\mathbf{A} + \mathbf{B_u}\mathbf{M_2}\mathbf{K})^i(\mathbf{B_d} - \mathbf{B_u}\mathbf{M_1})\mathbf{w}(k) \quad i = 0, \ldots, N-1 \tag{22}$$

providing a bound of the error between the state of the proposed feasible solution at time $k+1$ and the predicted state in k. In addition, if $\mathbf{K}$ is chosen as a $N-1$ dead-beat control law, that is, it satisfies that

$$(\mathbf{A} + \mathbf{B_u}\mathbf{M_2}\mathbf{K})^{N-1} = 0 \tag{23}$$

then it follows that

$$\mathbf{x}^s(N-1|k+1) = \mathbf{x}^*(N|k) = \mathbf{x}^{r*}(N|k) = \mathbf{x}^{rs}(N-1|k+1) \tag{24}$$

and taking into account that

$$\mathbf{u}^s(N-1|k+1) = \mathbf{u}_{0|k}^{r*} = \mathbf{u}^{rs}(N-1|k+1) \tag{25}$$

it follows that

$$\mathbf{x}^s(N, k+1) = \mathbf{x}^{rs}(N|k+1) \tag{26}$$

Constraints (14d) and (14e) are satisfied at time step $k+1$ for $i = N-1$ because the optimal artificial reference satisfies (14i) and (14j) for all future time steps and the optimal and shifted states satisfy (22). By definition the shifted trajectories satisfy the model equations so (14b), (14c), (14g) and (14h) are satisfied. Taking into account that $\mathbf{x}(i+1|k)$ satisfies (14d) for $i+1$ and that (26) holds, it follows that $\mathbf{x}(i|k+1)$ satisfies (14d) for i. The same holds true for constraint (14e). The terminal equality constraint (14f) is satisfied because the $N-1$ dead beat control law cancels the disturbance in the predicted states in $N-1$ states and the shifted trajectory follows the artificial optimal at time trajectory step k following (19d). Constraints (14i), (14j) and (14k) hold because the optimal artificial reference at time step k is periodic and the shifted reference trajectory is not modified following (18b) and (18c). Because at time step N the artificial reference state and actuators must satisfy (14d) and (14e) for $i = N-1$ because the shifted trajectory follows the artificial trajectory, see (19d), those constraints must be included along the whole prediction horizon as shown in (14i) and (14j).

4. Simulation results

To demonstrate the properties of the proposed controller different simulations scenarios have been considered. The simulations have been made with Matlab 2013a using the function *quadprog* to solve the resulting QP optimization problem. The number of decision variables is 6144 because a simultaneous formulation was used in which the tank levels, actuator water flows and auxiliary control input for both the predicted and the artificial reference trajectories were included as decision variables. For all the simulations, the initial volume of each tank is 60% of its corresponding maximum volume and the simulation length is three days.

First, the robust MPC for tracking periodic references is compared with the nominal MPC for tracking periodic references proposed in Limon et al. (2014, 2015). The nominal MPC controller is based on the same optimization problem, but assuming that the prediction error is zero for all times. Both controllers use the same design parameters. The main difference between both controllers is that the nominal controller does not take into account the uncertainties, which may lead to constraint violation and possible loss of feasibility. The objective of the first simulation is twofold, first to compare the behavior of the nominal controller and the proposed robust controller and second to show the proposed controller convergence properties. To this end, for these simulations the prediction error is assumed to be zero. This implies that for this simulation both the nominal and the robust controller closed-loop trajectories converge to their corresponding planner with zero error. The planner trajectories follow the target trajectories when possible. It can be seen that the target trajectories do not satisfy the constraints for all times.

Figs. 1 and 2 show the trajectories of the tank levels 10 and 11 of the nominal (blue discontinuous) and robust (blue) controller. In this case the nominal controller drives the closed-loop system as close as possible to the target reference (red discontinuous) without violating the constraints reaching the trajectory provided by the nominal planner (note that the reference is not always feasible). The robust controller converges to the trajectory provided by the robust planner (green discontinuous), which is the best trajectory that the disturbed system can follow when the closed loop system is tracking the proposed reference without violating the constraints. The trajectory provided by the robust planner does not reach the constraint limits in order to guarantee robust constraint satisfaction in the presence of disturbances. The water flow of the actuators show similar results. Fig. 3 shows the water flow trajectories of actuator 15. It can be seen that the nominal controller saturates the controller in certain times, while the robust controller converges to the robust planner trajectory, which has to take into account possible prediction errors and hence has to be more conservative.

The time needed to converge to the optimal cost is about 4 hours. The cost of the robust planner is about 8.9×10^{11} and the cost of the nominal controller is about 8.17×10^{11}. The conservativeness feature of the robust controller is the cause of this mismatch between the costs.

In the second scenario, the same simulations are carried out assuming that the demand was always 5% lower than the predicted value, that is, $\mathbf{d}(k) + \mathbf{w}(k) = 0.95\mathbf{d}(k)$ for all times. This is a worst case scenario that is included in the uncertainty bounds used to design the robust controller. This implies that even in this case, the controller guarantees robust constraint satisfaction and

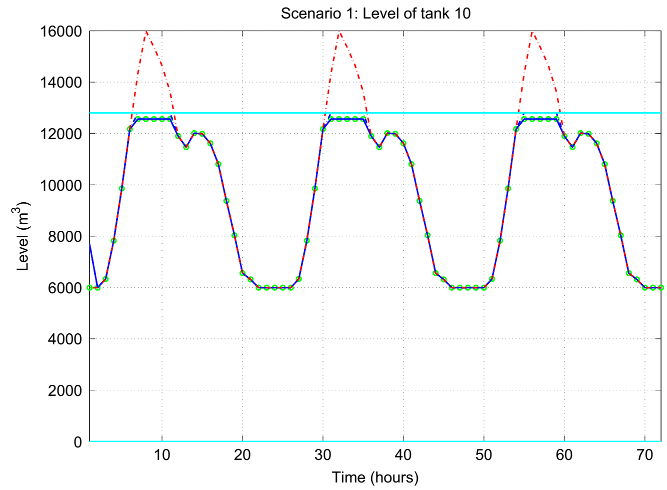

Fig. 1. Tank 10 (scenario 1): closed loop system with nominal controller (discontinuous blue), closed-loop system with robust controller (blue), robust planner trajectory (discontinuous green with circle), tank level constraint (cyan), target trajectory (discontinuous red). (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

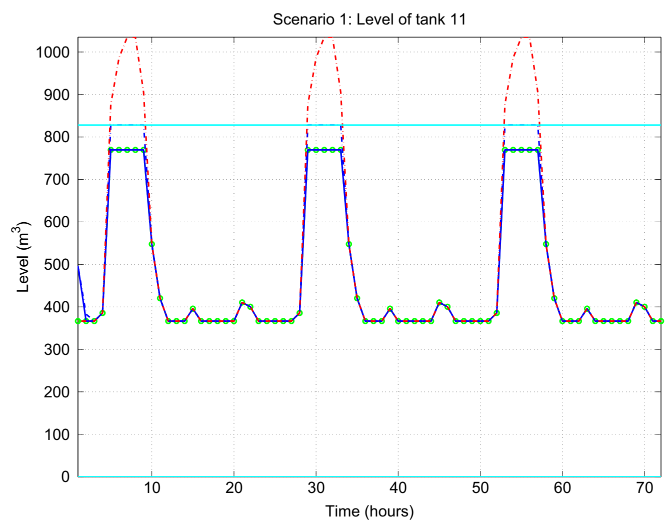

Fig. 2. Tank 11 (scenario 1): closed loop system with nominal controller (discontinuous blue), closed-loop system with robust controller (blue), robust planner trajectory (discontinuous green with circle), tank level constraint (cyan), target trajectory (discontinuous red). (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

recursive feasibility. On the other hand, the nominal controller does not guarantee constraint satisfaction. The simulations demonstrate this issue and show that often the level of the tanks and the water flow of the actuators of the trajectories of the nominal controller were higher than the maximum levels, in particular, when the nominal planner trajectory was saturated or close to the constraints. The robust controller closed-loop trajectories satisfied the constraints for all times. Figs. 4 and 5 show the trajectories of the tank levels 10 and 11 of the nominal (yellow discontinuous) and robust (blue) controller. In the case of the proposed disturbed close loop system the nominal controller violates the upper constraints becoming the closed loop system unfeasible but in the case of the robust controller the trajectory are always below the upper limit.

Fig. 6 shows the evolution of actuator 15 along the three days of simulation and it can be seen how the trajectory of this water flow is equal to the trajectory of the robust planner as in the case of the evolution of the tanks. When the drinking water network is

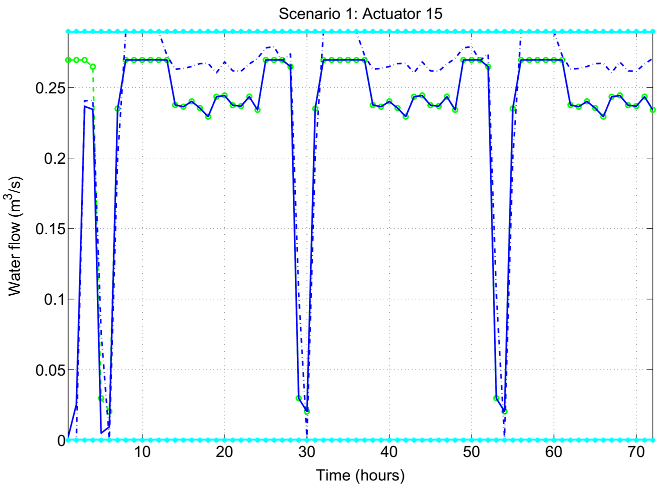

Fig. 3. Actuator 15 (scenario 1): closed loop system with nominal controller (discontinuous blue), closed-loop system with robust controller (blue), robust planner trajectory (discontinuous green with circle), constraints related to water flow in actuator 15 (cyan). (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

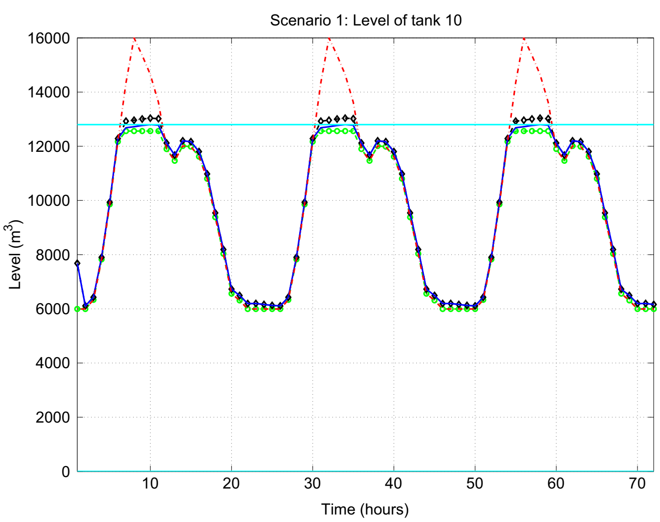

Fig. 4. Tank 10 (scenario 2): closed loop system with nominal controller (discontinuous black with diamond), closed-loop system with robust controller (blue), robust planner trajectory (discontinuous green with circle), tank level constraint (cyan), target trajectory (discontinuous red). (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

subject to prediction error in the demand, the trajectory of the closed-loop system does not converge the robust planner but it never breaks the constraints.

In the third scenario a sudden change in the target levels and water flow trajectories is shown. The prediction error in this simulation is 5% of the maximal demand along the period but its sign is random. In this simulation the target level for each tank changes after 38 h. The new target trajectory for each tank $\hat{x}_i^t(k)$ is obtained as $\hat{\mathbf{x}}_i^t(k) = \mathbf{x}_i^{max} - \mathbf{x}_i^t(k)$. The corresponding values of the actuator references $\mathbf{u}_i^t(k)$ have been obtained from the solution in the least squares sense to the under determined system of equations obtained from the dynamic model and the predicted values of the demand. The trajectory of the robust planner is different for both target references. Their corresponding costs are $8.8926 \cdot 10^{11}$ and $1.0854 \cdot 10^{12}$ respectively. The simulation shows that the robust MPC optimization cost converges to a neighborhood of the cost of the original trajectory, and then changes suddenly to the

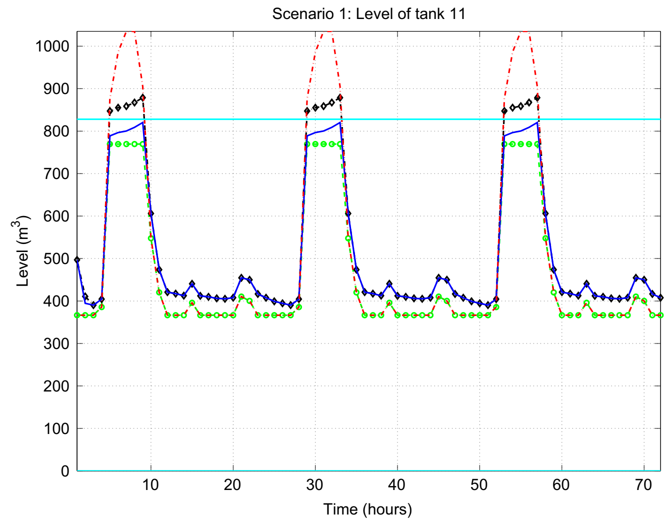

Fig. 5. Tank 11 (scenario 2): closed loop system with nominal controller (discontinuous black with diamond), closed-loop system with robust controller (blue), robust planner trajectory (discontinuous green with circle), tank level constraint (cyan), target trajectory (discontinuous red). (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

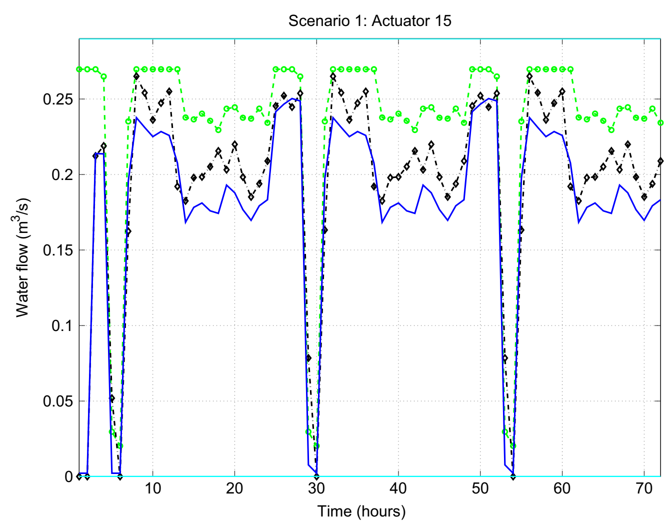

Fig. 6. Actuator 15 (scenario 2): closed loop system with nominal controller (discontinuous black with diamond), closed-loop system with robust controller (blue), robust planner trajectory (discontinuous green with circle), constraints related to water flow in actuator 15 (cyan). (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

cost of the modified trajectory without losing feasibility or violating any constraints.

Figs. 7 and 8 show the evolution of the closed-loop system and its behavior when the tracking reference suddenly changes. In these figures the trajectory of the robust planner for the first reference (green discontinuous with circle) and for the second reference (black discontinuous with x) are represented. The evolution of the closed-loop system modifies its evolution from following the first reference to follow the new reference without breaking the constraints even with disturbances. In the case of the behavior of actuator 18, the evolution of the water flow follow the same pattern of the tank level 10. The simulation demonstrates that the robust controller (under certain assumptions) maintains recursive feasibility even when the references are not periodically constant and without the necessity to calculate any robust invariant set.

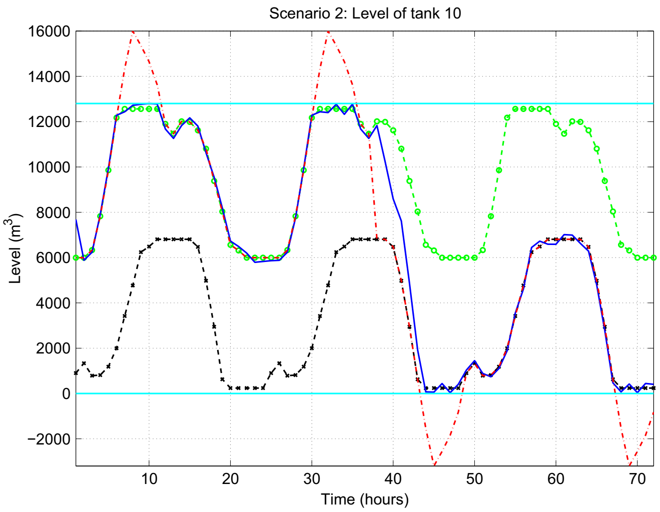

Fig. 7. Tank 10 (scenario 3): closed-loop system with robust controller (blue), robust planner for reference 1 (discontinuous green), robust planner for reference 2 (discontinuous black with x), tank level constraint (cyan), target trajectory (discontinuous red). (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

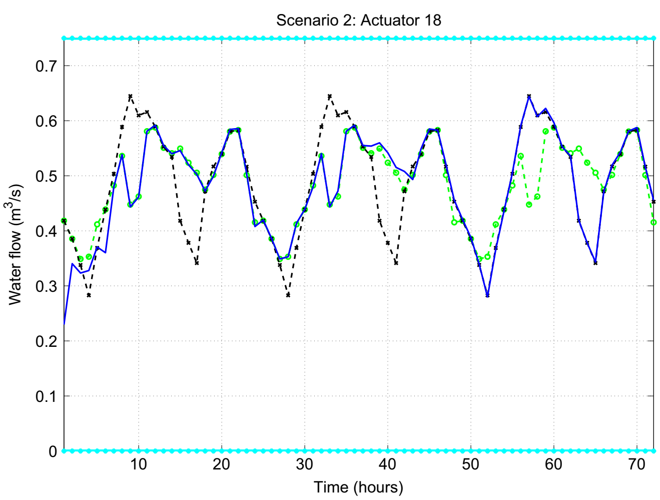

Fig. 8. Actuator 18 (scenario 3): closed-loop system with robust controller (blue), robust planner for reference 1 (discontinuous green), robust planner for reference 2 (discontinuous black with x), constraints related to water flow in actuator 18 (cyan). (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

Fig. 9 shows the evolution of the cost of the closed-loop system (blue discontinuous with circle) with the robust controller and how it changes at instant 38 from the previous cost value (green discontinuous with circle) to the new cost value (black discontinuous with x). It can be seen that the cost function for the second reference is higher and in addition, how there is sudden jump in the optimal cost when the reference changes. Note that the robust controller is not modified.

4.1. Evaluation and comparison of the proposed controller

As was commented before, three different scenarios have been tested to evaluate the benefits of the proposed controller. For these scenarios, the behavior of the controlled network is measured by a performance index that is the average optimal cost of the controller

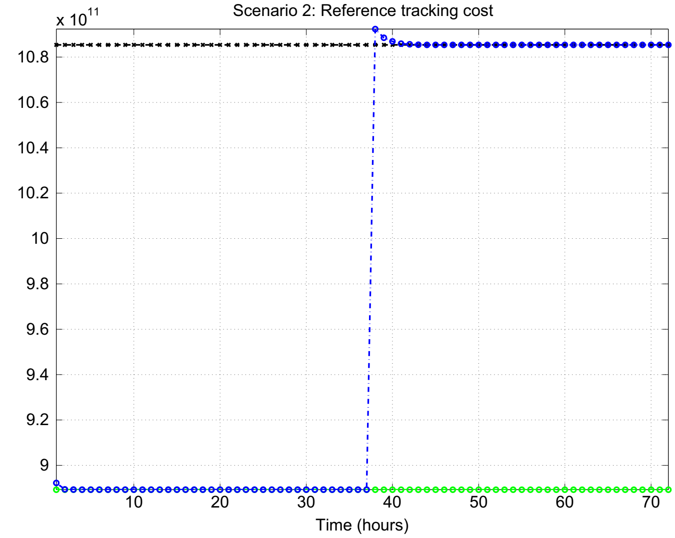

Fig. 9. Trajectory of the optimal cost (scenario 3) of the robust controller (blue discontinuous with circle) and the robust planers for references 1 and 2 (green discontinuous with circle and black discontinuous with x respectively). (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

$$\Phi = \frac{1}{T_s} \sum_{k=0}^{T_s - 1} V_N^*(k)$$

where $T_s = 72$ is the number of samples that takes the whole test.

In the first scenario, the convergence of the controller is demonstrated in the case that the real demands are equal to the forecasted ones. In this case, the nominal controller and the proposed robust controller have been applied and, as expected, in both cases, the controlled network converges asymptotically to the best possible evolution of the network, which is calculated by the corresponding planner. In the first column of Table 2 it is shown that the performance index Φ of the robust planner which is the best possible index that can be achieved by the robust controller since the trajectory of the planner is the one that minimizes cost $V_p(\cdot)$. The performance index of the robust controller is slightly larger than the one of the robust planner due to the tracking cost along the transient. On the other hand, the nominal cost achieves a lower performance index since the set of constraints considered in this controller includes the set of constraints of the robust one. This loss of performance is the price to pay to use a robust controller that is safe in presence of uncertainty.

The second scenario shows the effect of the uncertainty on the former controllers: the nominal one and the robust one. In this case it is considered that the demands are lower than forecasted leading to a potential overload of the tanks, which is the worst scenario. As expected, the nominal controller presents a worse behavior and is not capable of satisfying the constraints along the test. Thanks to the fact that the feasibility of the nominal controller is not lost along the test, the nominal control law has been applied for the whole test and the number of samples when any of the state variables are out of the operation limits have been counted. Table 1 shows the percentage of time in which the network controlled by the nominal and by the robust controller satisfy all the state and actuator constraints. It can be seen that the robust formulation satisfies the constraints for all times. The second column of Table 2 shows the performance index of the robust controller and the robust planner, the nominal one is not calculated since the constraints are not fulfilled. As expected, the performance of the robust planner is equal to the one of the scenario 1 since the forecasted demands are the same. The robust controller exhibits a

Table 1
Constraint satisfaction for Scenario 2.

Controller	Constraint satisfaction (%)
Nominal	68.71
Robust	100

Table 2
Performance index Φ measured for each scenario.

Controller	Scenario 1	Scenario 2	Scenario 3
Nominal	$8.1883659 \cdot 10^{11}$	–	–
Robust	$8.8930378 \cdot 10^{11}$	$8.8930708 \cdot 10^{11}$	$9.848186 \cdot 10^{11}$
Robust planner	$8.8925994 \cdot 10^{11}$	$8.8925994 \cdot 10^{11}$	$9.818756 \cdot 10^{11}$

slightly larger performance index due to effect of the transient and the uncertainty of the demand. This latter effect is the cause of getting a performance index worse than the one of the scenario 1.

The third scenario has been carried out to show the behavior of the proposed controller in the presence of random uncertainty under a sudden change of the target trajectory to track, which may be derived for instance from a change in the management policy of the overall DWN. As has been proved, the controlled network fulfills the operation limits of the tanks and the actuators in spite of the uncertainty and the change in the operation conditions. Besides, the controller steers the network to a neighborhood of the optimal trajectory after the change.

The third column of Table 2 shows the total accumulated cost for the different scenarios and control laws, including the cost of the planners for the two references considered. During the first 38 h, the first target trajectory is applied and the optimal cost of the robust planner is $V_p^* = 8.892599 \cdot 10^{11}$, while in the 34 remaining hours, the second target trajectory is provided yielding a optimal cost $V_p^* = 10.853873 \cdot 10^{11}$. The resulting performance index Φ of the optimal trajectory is then $9.818756 \cdot 10^{11}$. The performance index of the controlled network is $9.8481857 \cdot 10^{11}$ whose difference with the index of the optimal trajectory is due to the cost of the transient.

5. Conclusions

In this work, a novel robust MPC for tracking periodic references is applied to an uncertain discrete time algebraic-differential linear model of a large scale water distribution network obtained from the water balance equations of a section of Barcelona's water drinking network. To this end, a large scale model of the network and uncertain predictions of the demand have been considered. The control objective is to track a periodic arbitrary reference while guaranteeing robust constraint and water demand satisfaction. The proposed controller provides closed-loop robust constraint satisfaction even in the presence of sudden changes in the periodic target reference and asymptotic convergence to an optimal (in a sense) trajectory. The proposed controller is based on the solution of a single quadratic programming optimization problem and is defined without the necessity of the computation of a robust positive invariant set. These features are very important in practical applications and make this controller an appropriate approach to control large scale systems.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.conengprac.2016.08.017.

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