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Global exponential stability of nonresident computer virus models[☆]

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ABSTRACT

This paper is concerned with nonresident computer virus models which are defined on the nonnegative real vector space. By using differential inequality technique, we employ a novel argument to show that the virus-free equilibrium is globally exponentially stable, and the exponential convergent rate can be unveiled. Moreover, a numerical simulation is given to demonstrate our theoretical results.

1. Introduction

Network is an indispensable part of everyday life, it develops so fast and offers us more and more functionalities and facilities. At the same time, there is a rapid increase in computer viruses. In recent years, many dynamical models characterizing the spread of computer viruses over the internet were investigated, for example [1–9] and the cited references there in. Therefore, it is extremely important to analyze and protect computers against virus, particularly nonresident computer virus that does not store or execute itself from the computer memory.

Let $S(t), L(t)$ and $A(t)$ respectively denote percentage of susceptible computers, infected computers in which viruses have not been loaded in their memory, and infected computers in which viruses have been located in memory in the computers at time t. In various recent literature [5,6,10], the following system of differential equations is used to describe the dynamics of nonresident computer virus

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$$\begin{cases} \dfrac{dS(t)}{dt} = b - \mu_1 S(t) - \beta_1 S(t)L(t) - \beta_2 S(t)A(t) + \gamma_1 L(t) + \gamma_2 A(t) \\ \dfrac{dL(t)}{dt} = \beta_1 S(t)L(t) + \beta_2 S(t)A(t) + \alpha_2 A(t) - (\mu_2 + \alpha_1 + \gamma_1)L(t) \\ \dfrac{dA(t)}{dt} = \alpha_1 L(t) - (\mu_3 + \alpha_2 + \gamma_2)A(t). \end{cases} \tag{1.1}$$

It is assumed in the above model that all newly accessed computers are free of virus, and all viruses in the computers are nonresident. It is also assumed that the external computers are accessed to the internet at a constant rate b at each time t, the uninfected and latent computers as well as the infectious computers of the internal computers are disconnected from the internet respectively at the rates of μ_1, μ_2, and μ_3. The percentage of internal computers infected at time t increases by $\beta_1 SL + \beta_2 SA$, where β_1 and β_2 are positive constants. α_1 denotes the rate at which nonresident viruses within latent computers are loaded into memory, and α_2 is the rate at which nonresident viruses within infectious computers transfer control to the application program. γ_1 and γ_2 are the cure rates of latent and infectious computers. By the biological meanings, all parameters in model (1.1) are positive.

Under the following initial condition:

$$S(0) > 0, \qquad L(0) > 0 \quad \text{and} \quad A(0) > 0, \tag{1.2}$$

the authors in [10] proved the following results on the global asymptote stability of system (1.1) with initial value (1.2).

Theorem A (*See [10, Theorem 1.1]*)**.** *Let the basic reproduction number be*

$$R_0 = \frac{b\{\beta_1 + \beta_2\alpha_1/(\mu_3 + \alpha_2 + \gamma_2)\}}{\mu_1\{(\mu_2 + \alpha_1 + \gamma_1) - \alpha_1\alpha_2/(\mu_3 + \alpha_2 + \gamma_2)\}} < 1$$

and $\mu_1 \le \min(\mu_2, \mu_3)$. Then, the virus-free equilibrium $E^0 = (S^0, 0, 0) = (\frac{b}{\mu_1}, 0, 0)$ is globally asymptotically stable, and every solution $(S(t), L(t), A(t))$ of (1.1) with initial value (1.2) satisfies

$$\lim_{t\to+\infty} (S(t), L(t), A(t)) = (S^0, 0, 0).$$

But in fact, as a model of biological system, the initial condition:

$$S(0) \ge 0, L(0) \ge 0 \quad \textit{and} \quad A(0) \ge 0, \tag{1.3}$$

is more realistic than (1.2) in practical application of model (1.1). Set

$$IV_1 = \{(S(0), L(0), A(0)) | S(0) \ge 0, L(0) \ge 0 \ \ \textit{and} \ \ A(0) \ge 0\},$$
$$IV_2 = \{(S(0), L(0), A(0)) | S(0) \ge 0, \ \textit{and} \ \ L(0) = A(0) = 0\},$$

and

$$IV_3 = \{(S(0), L(0), A(0)) | S(0) \ge 0, L(0) \ge 0, A(0) \ge 0 \ \ \textit{and} \ \ L(0) + A(0) > 0\}.$$

It is obvious that

$$IV_3 = IV_1 \setminus IV_2,$$

and

$$(S(t), L(t), A(t))\Big|_{IV_2} = \left(\left[S(0) - \frac{b}{\mu_1}\right] e^{-\mu_1 t} + \frac{b}{\mu_1}, 0, 0\right), \tag{1.4}$$

exponentially converges to the virus-free equilibrium $E^0 = (S^0, 0, 0) = (\frac{b}{\mu_1}, 0, 0)$. Thus, to reveal the global dynamics of system (1.1) with initial value (1.3), we only need to study all solutions of system (1.1) with the

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following initial condition:

$$S(0) \geq 0, \qquad L(0) \geq 0, A(0) \geq 0 \quad \textit{and} \quad L(0) + A(0) > 0. \tag{1.5}$$

On the other hand, the convergence speed of $\lim_{t\to+\infty}(S(t), L(t), A(t)) = (S^0, 0, 0)$ is vitally important to the disease prevention and control in real world applications of the theoretical results on the nonresident computer virus models. In particular, since the exponential convergent rate can be unveiled, there have been extensive results on the problem of the exponential stability of epidemic model in the literature. We refer the reader to [11–17] and the references cited therein. Then a question naturally arises: *How is the exponential stability of the virus-free equilibrium* $(\frac{b}{\mu_1}, 0, 0)$ *of system* (1.1) *with initial value conditions* (1.5)?

Motivated by the above discussions, the main purpose of this paper is to give some sufficient conditions for the exponential stability of the virus-free equilibrium $(S^*, L^*, A^*) = (\frac{b}{\mu_1}, 0, 0)$ of system (1.1). We shall emphasize that our work has two new major contributions. Firstly, we successfully establish the conditions for the exponent stability in this paper, while the work in [10] establish conditions for the global asymptotic stability. Secondly, we deal with more general realistic initial conditions. An example together with its numerical simulations is also provided to illustrate the effectiveness of our results.

Similar to the Ref. [16], we give the definition of the exponential stability as follows:

Definition 1.1. Let $(S(t), L(t), A(t))$ be the solution of (1.1) with initial value condition (1.5). If there exist positive constants t_0, ζ and K such that

$$|S(t) - S^*| \leq Ke^{-\zeta t}, \qquad |L(t) - L^*| \leq Ke^{-\zeta t}, \qquad |A(t) - A^*| \leq Ke^{-\zeta t}, \quad \text{for all } t \geq t_0.$$

Then, the virus-free equilibrium $(S^*, L^*, A^*) = (\frac{b}{\mu_1}, 0, 0)$ is said to be *globally exponentially stable.*

2. Main results

Lemma 2.1. *Let* $\mu_1 \leq \min(\mu_2, \mu_3)$*. Then, every solution of* (1.1) *with initial value condition* (1.5) *is positive and bounded on* $(0, +\infty)$.

Proof. From Theorem 1.3.1 in [18], we can obtain that there exists a unique solution $(S(t, 0, x_0), L(t, 0, x_0), A(t, 0, x_0))$ of (1.1) passing through $(0, x_0)$ with $x_0 = (S(0), L(0), A(0))$ satisfying (1.5).

Let $[0, T)$ be the maximal right-interval of the existence of $(S(t, 0, x_0), L(t, 0, x_0), A(t, 0, x_0))$, and $(S(t), L(t), A(t)) = (S(t, 0, x_0), L(t, 0, x_0), A(t, 0, x_0))$ for all $t \in [0, T)$. We first prove that there exists $T_\eta > 0$ such that

$$S(t) > 0, L(t) > 0 \quad \text{and} \quad A(t) > 0, \quad \text{for all } t \in (0, T_\eta]. \tag{2.1}$$

According to (1.5), we shall consider the following two cases.

Case 1. If $L(0) > 0$, then, it follows from the continuity of $L(t)$ there exists $\eta_L > 0$ such that

$$L(t) > 0 \quad \text{for all } t \in (0, \eta_L] \subset (0, T). \tag{2.2}$$

Now, we prove that there exists $\eta_A > 0$ such that

$$A(t) > 0 \quad \text{for all } t \in (0, \eta_A] \subset (0, T). \tag{2.3}$$

When $A(0) > 0$, one can easily prove (2.3) by the continuity of $A(t)$. If $A(0) = 0$, then

$$A'(0) = \alpha_1 L(0) - (\mu_3 + \alpha_2 + \gamma_2)A(0) = \alpha_1 L(0) > 0,$$

which implies that (2.3) holds.

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Now, we claim that

$$S(t) > 0 \quad \text{for all } t \in (0,\ \min\{\eta_L, \eta_A\}]. \tag{2.4}$$

By way of contradiction, suppose that (2.4) does not hold. Then, there must exist $T_1 \in (0,\ \min\{\eta_L, \eta_A\})$ such that

$$S(T_1) = 0, \qquad S(s) > 0 \quad \text{for all} \quad s \in (0, T_1),$$

and

$$S'(T_1) = \lim_{s \to T_1{}^-} \frac{S(s) - S(T_1)}{s - T_1} \le 0.$$

But according to the first equation of system (1.1),

$$S'(T_1) = b - \mu_1 S(T_1) - \beta_1 S(T_1) L(T_1) - \beta_2 S(T_1) A(T_1) + \gamma_1 L(T_1) + \gamma_2 A(T_1) > b > 0,$$

which is a contradiction to the fact that $S'(T_1) \le 0$. Hence (2.4) holds. Together with (2.2) and (2.3), it implies that (2.1) holds for $T_\eta = \min\{\eta_L, \eta_A\}$.

Case 2. If $A(0) > 0$, then, it follows from the continuity of $A(t)$ there exists $\bar{\eta}_A > 0$ such that

$$A(t) > 0 \quad \text{for all} \quad t \in (0,\ \bar{\eta}_A]. \tag{2.5}$$

Now we turn to prove that there exists $\bar{\eta}_L > 0$ such that

$$L(t) > 0 \quad \text{for all} \quad t \in (0,\ \bar{\eta}_L] \subset (0,\ T). \tag{2.6}$$

One can easily prove (2.6) by the continuity of $L(t)$ when $L(0) > 0$. Here we consider the case of $L(0) = 0$. Then

$$L'(0) = \beta_1 S(0) L(0) + \beta_2 S(0) A(0) + \alpha_2 A(0) - (\mu_2 + \alpha_1 + \gamma_1) L(0) = \beta_2 S(0) A(0) + \alpha_2 A(0) > 0,$$

and (2.6) still holds.

Using a similar argument as that in the proof of (2.4) leads to

$$S(t) > 0 \quad \text{for all } t \in (0,\ \min\{\bar{\eta}_L, \bar{\eta}_A\}],$$

and it implies that (2.1) holds for $T_\eta = \min\{\bar{\eta}_L, \bar{\eta}_A\}$ with (2.5) and (2.6).

Next, we show that

$$S(t) > 0, L(t) > 0 \quad \text{and} \quad A(t) > 0, \quad \text{for all } t \in [T_\eta,\ T). \tag{2.7}$$

Otherwise, one of the following three cases must be hold.

(i) there exists $T_2 \in (T_\eta,\ T)$ such that

$$S(T_2) = 0, \qquad S(s) > 0, \qquad L(s) > 0 \quad \text{and} \quad A(s) > 0 \quad \text{for all} \quad s \in [T_\eta,\ T_2). \tag{2.8}$$

(ii) there exists $T_3 \in (T_\eta,\ T)$ such that

$$L(T_3) = 0, \qquad S(s) > 0, \qquad L(s) > 0 \quad \text{and} \quad A(s) > 0 \quad \text{for all} \quad s \in [T_\eta,\ T_3). \tag{2.9}$$

(iii) there exists $T_4 \in (T_\eta,\ T)$ such that

$$A(T_4) = 0, \qquad S(s) > 0, \qquad L(s) > 0 \quad \text{and} \quad A(s) > 0 \quad \text{for all} \quad s \in [T_\eta,\ T_4). \tag{2.10}$$

In view of (1.1) and (2.8)–(2.10), we obtain

$$\begin{cases} S(T_2) = e^{-\int_{T_\eta}^{T_2}(\mu_1+\beta_1 L(s)+\beta_2 A(s))ds} S(T_\eta) \\ \quad + \int_{T_\eta}^{T_2} e^{-\int_v^{T_2}(\mu_1+\beta_1 L(s)+\beta_2 A(s))ds}[b+\gamma_1 L(v)+\gamma_2 A(v)]dv > 0, \\ L(T_3) = e^{-(\mu_2+\alpha_1+\gamma_1)(T_3-T_\eta)} L(T_\eta) \\ \quad + e^{-(\mu_2+\alpha_1+\gamma_1)T_3} \int_{T_\eta}^{T_3} e^{(\mu_2+\alpha_1+\gamma_1)v}[\beta_1 S(v)L(v)+\beta_2 S(v)A(v)+\alpha_2 A(v)]dv > 0, \\ A(T_4) = e^{-(\mu_3+\alpha_2+\gamma_2)(T_4-T_\eta)} A(T_\eta) + e^{-(\mu_3+\alpha_2+\gamma_2)T_4} \int_{T_\eta}^{T_4} e^{(\mu_2+\alpha_1+\gamma_1)v}\alpha_1 L(v)dv > 0, \end{cases}$$

which is a contradiction to the above claim. Thus, (2.7) holds. This proves that

$$S(t) > 0, \qquad L(t) > 0, \quad \text{and} \quad A(t) > 0, \quad \text{for all} \quad t \in (0,\ T). \tag{2.11}$$

Let $N(t) = S(t) + L(t) + A(t)$. From the fact that $\mu_1 \le \min(\mu_2, \mu_3)$, (1.1) and (2.11) yield

$$\begin{cases} N'(t) = b - \mu_1 N(t) - (\mu_2 - \mu_1)L(t) - (\mu_3 - \mu_1)A(t) \le b - \mu_1 N(t), \\ N(t) \le N(0)\dfrac{1}{e^{\mu_1 t}} + \dfrac{b}{\mu_1}\dfrac{(e^{\mu_1 t}-1)}{e^{\mu_1 t}} \le N(0) + \dfrac{b}{\mu_1} := M_1, \end{cases} \tag{2.12}$$

for all $t \in (0,\ T)$. It follows that $S(t), L(t)$ and $A(t)$ are bounded on $(0,\ T)$. According to Theorem 1.2.1 in [18], we easily obtain $T = +\infty$. This completes the proof of Lemma 2.1. □

Remark 2.1. All the results on boundedness and positivity of solutions for (1.1) obtained in Reference [10] are under the initial condition (1.2). Since the initial set in (1.2) is a proper subset of the set in (1.5), Lemma 2.1 supplements and improves the corresponding results in [10].

Theorem 2.1. *In addition to the assumptions in Theorem A, further assume that*

$$-\mu_1 + \left(\beta_1 \frac{b}{\mu_1} + \beta_2 \frac{b}{\mu_1} + \gamma_1 + \gamma_2\right) < 0, -(\mu_2 + \alpha_1 + \gamma_1) + (\beta_1 + \beta_2)\frac{b}{\mu_1} + \alpha_2 < 0, \tag{2.13}$$

and

$$-(\mu_3 + \alpha_2 + \gamma_2) + \alpha_1 < 0. \tag{2.14}$$

Then, there exist positive constants t_0, ζ and K such that

$$|S(t) - S^*| \le K e^{-\zeta t}, L(t) \le K e^{-\zeta t} \quad \text{and} \quad A(t) \le K e^{-\zeta t}, \quad \text{for all } t \ge t_0. \tag{2.15}$$

Proof. From (2.12), we have

$$N(t) \le N(0)\frac{1}{e^{\mu_1 t}} + \frac{b}{\mu_1}\frac{(e^{\mu_1 t}-1)}{e^{\mu_1 t}} = \left[N(0) - \frac{b}{\mu_1}\right] e^{-\mu_1 t} + \frac{b}{\mu_1}, \quad \text{for all} \quad t \in (0,\ +\infty). \tag{2.16}$$

Let $t_0 > 0$ and $\varepsilon_0 > 0$ such that

$$S(t) + L(t) + A(t) = N(t) \le \varepsilon_0 + \frac{b}{\mu_1} \quad \text{for all} \quad t \in (t_0,\ +\infty), \tag{2.17}$$

and

$$-(\mu_2 + \alpha_1 + \gamma_1) + (\beta_1 + \beta_2)\left(\frac{b}{\mu_1} + \varepsilon_0\right) + \alpha_2 < 0. \tag{2.18}$$

Let

$$x(t) = (S(t) - S^*, L(t) - L^*, \qquad A(t) - A^*) = (\bar{S}(t), L(t), A(t)) \quad \text{for all} \quad t \in [0, \ +\infty).$$

Then, (1.1) gives

$$\begin{cases} \dfrac{d\bar{S}(t)}{dt} = -(\mu_1 + \beta_1 L(t) + \beta_2 A(t))\bar{S}(t) - \beta_1 \dfrac{b}{\mu_1} L(t) - \beta_2 \dfrac{b}{\mu_1} A(t) + \gamma_1 L(t) + \gamma_2 A(t) \\ \dfrac{dL(t)}{dt} = \beta_1 S(t)L(t) + \beta_2 S(t)A(t) + \alpha_2 A(t) - (\mu_2 + \alpha_1 + \gamma_1)L(t) \\ \dfrac{dA(t)}{dt} = \alpha_1 L(t) - (\mu_3 + \alpha_2 + \gamma_2)A(t) \end{cases}$$

which implies

$$\begin{cases} \bar{S}(t) = e^{-\int_{t_0}^{t} [\mu_1 + \beta_1 L(\theta) + \beta_2 A(\theta)] d\theta} \bar{S}(t_0) + \displaystyle\int_{t_0}^{t} e^{-\int_{v}^{t} [\mu_1 + \beta_1 L(\theta) + \beta_2 A(\theta)] d\theta} \\ \qquad \times \left[-\beta_1 \dfrac{b}{\mu_1} L(v) - \beta_2 \dfrac{b}{\mu_1} A(v) + \gamma_1 L(v) + \gamma_2 A(v) \right] dv, \\ L(t) = e^{-\int_{t_0}^{t} (\mu_2 + \alpha_1 + \gamma_1) d\theta} L(t_0) + \displaystyle\int_{t_0}^{t} e^{-\int_{v}^{t} (\mu_2 + \alpha_1 + \gamma_1) d\theta} \\ \qquad \times [\beta_1 S(v) L(v) + \beta_2 S(v) A(v) + \alpha_2 A(v)] dv, \\ A(t) = e^{-\int_{t_0}^{t} (\mu_3 + \alpha_2 + \gamma_2) d\theta} A(t_0) + \displaystyle\int_{t_0}^{t} e^{-\int_{v}^{t} (\mu_3 + \alpha_2 + \gamma_2) d\theta} \alpha_1 L(v) dv, \end{cases} \tag{2.19}$$

for all $t \geq t_0$.

From (2.13), (2.14) and (2.18), for any $\varepsilon \in (0, \ \varepsilon_0]$, we can choose two positive constants ζ and τ such that

$$\begin{cases} \zeta - \mu_1 + \left(\beta_1 \dfrac{b}{\mu_1} + \beta_2 \dfrac{b}{\mu_1} + \gamma_1 + \gamma_2 \right) < -\tau < 0, \\ \zeta - (\mu_2 + \alpha_1 + \gamma_1) + (\beta_1 + \beta_2) \left(\dfrac{b}{\mu_1} + \varepsilon \right) + \alpha_2 < -\tau < 0, \\ \zeta - (\mu_3 + \alpha_2 + \gamma_2) + \alpha_1 < -\tau < 0. \end{cases} \tag{2.20}$$

Let $\|x\|_0 = \max\{|\bar{S}(t_0)|, \ L(t_0), \ A(t_0)\}$, and $K_0 > 1$ be a constant. Consequently,

$$\|x(t_0)\| < \|x\|_0 + \varepsilon < K_0(\|x\|_0 + \varepsilon) = K_0(\|x\|_0 + \varepsilon) e^{\zeta t_0} e^{-\zeta t_0}.$$

In the following, we will show

$$\|x(t)\| = \max\{|\bar{S}(t)|, \ L(t), \ A(t)\} < K_0(\|x\|_0 + \varepsilon) e^{\zeta t_0} e^{-\zeta t} \quad \text{for all } t > t_0. \tag{2.21}$$

Contrarily, one of the following cases must occur.

Case I There exists $\theta_1 > 0$ such that

$$\begin{cases} |\bar{S}(\theta_1)| \ = \ K_0(\|x\|_0 + \varepsilon) e^{\zeta t_0} e^{-\zeta \theta_1}, \\ \ \|x(t)\| \ < \ K_0(\|x\|_0 + \varepsilon) e^{\zeta t_0} e^{-\zeta t} \quad \text{for all} \quad t \in [t_0, \ \theta_1). \end{cases} \tag{2.22}$$

Case II There exists $\theta_2 > 0$ such that

$$\begin{cases} L(\theta_2) = |L(\theta_2)| \ = \ K_0(\|x\|_0 + \varepsilon) e^{\zeta t_0} e^{-\zeta \theta_2}, \\ \qquad\qquad \|x(t)\| \ < \ K_0(\|x\|_0 + \varepsilon) e^{\zeta t_0} e^{-\zeta t} \quad \text{for all} \quad t \in [t_0, \ \theta_2). \end{cases} \tag{2.23}$$

Case III There exists $\theta_3 > 0$ such that

$$\begin{cases} A(\theta_3) = |A(\theta_3)| \ = \ K_0(\|x\|_0 + \varepsilon)e^{\zeta t_0}e^{-\zeta\theta_3}, \\ \qquad\qquad \|x(t)\| \ < \ K_0(\|x\|_0 + \varepsilon)e^{\zeta t_0}e^{-\zeta t} \quad \text{for all} \quad t \in [t_0, \ \theta_3). \end{cases} \tag{2.24}$$

If Case I holds, in view of (2.19), (2.20) and (2.22), we have

$$\begin{aligned}
|\bar{S}(\theta_1)| &= \left| e^{-\int_{t_0}^{\theta_1}[\mu_1+\beta_1 L(\theta)+\beta_2 A(\theta)]d\theta}\bar{S}(t_0) + \int_{t_0}^{\theta_1} e^{-\int_v^{\theta_1}[\mu_1+\beta_1 L(\theta)+\beta_2 A(\theta)]d\theta} \right. \\
&\quad \left. \times \left[-\beta_1\frac{b}{\mu_1}L(v) - \beta_2\frac{b}{\mu_1}A(v) + \gamma_1 L(v) + \gamma_2 A(v)\right] dv \right| \\
&\le e^{-\mu_1(\theta_1-t_0)}|\bar{S}(t_0)| + \int_{t_0}^{\theta_1} e^{-\mu_1(\theta_1-v)} \left[\beta_1\frac{b}{\mu_1}L(v) + \beta_2\frac{b}{\mu_1}A(v) + \gamma_1 L(v) + \gamma_2 A(v)\right] dv \\
&\le e^{-\mu_1(\theta_1-t_0)}(\|x\|_0+\varepsilon) + \int_{t_0}^{\theta_1} e^{-\mu_1(\theta_1-v)} \left(\beta_1\frac{b}{\mu_1} + \beta_2\frac{b}{\mu_1} + \gamma_1 + \gamma_2\right) K_0(\|x\|_0+\varepsilon)e^{\zeta t_0}e^{-\zeta v}dv \\
&= K_0(\|x\|_0+\varepsilon)e^{\zeta t_0}e^{-\zeta\theta_1}\left\{\frac{1}{K_0}e^{-(\theta_1-t_0)(\mu_1-\zeta)} + \int_{t_0}^{\theta_1} e^{-(\theta_1-v)(\mu_1-\zeta)}\left(\beta_1\frac{b}{\mu_1} + \beta_2\frac{b}{\mu_1} + \gamma_1 + \gamma_2\right) dv\right\} \\
&\le K_0(\|x\|_0+\varepsilon)e^{\zeta t_0}e^{-\zeta\theta_1}\left\{\frac{1}{K_0}e^{-(\theta_1-t_0)(\mu_1-\zeta)} + \int_{t_0}^{\theta_1} e^{-(\theta_1-v)(\mu_1-\zeta)}(\mu_1-\zeta)dv\right\} \\
&= K_0(\|x\|_0+\varepsilon)e^{\zeta t_0}e^{-\zeta\theta_1}\left\{1 - \left(1-\frac{1}{K_0}\right)e^{-(\theta_1-t_0)(\mu_1-\zeta)}\right\} \\
&< K_0(\|x\|_0+\varepsilon)e^{\zeta t_0}e^{-\zeta\theta_1},
\end{aligned}$$

which contradicts with the first equation in (2.22). Hence, Case I does not exist.

In Case II, together with (2.17), (2.20) and (2.23), (2.19), it implies that

$$\begin{aligned}
L(\theta_2) &= e^{-\int_{t_0}^{\theta_2}(\mu_2+\alpha_1+\gamma_1)d\theta}L(t_0) + \int_{t_0}^{\theta_2} e^{-\int_v^{\theta_2}(\mu_2+\alpha_1+\gamma_1)d\theta} \times [\beta_1 S(v)L(v) + \beta_2 S(v)A(v) + \alpha_2 A(v)]dv \\
&\le e^{-(\mu_2+\alpha_1+\gamma_1)(\theta_2-t_0)}L(t_0) + \int_{t_0}^{\theta_2} e^{-(\mu_2+\alpha_1+\gamma_1)(\theta_2-v)} \times [\beta_1 N(v)L(v) + (\beta_2 N(v) + \alpha_2)A(v)]dv \\
&\le e^{-(\mu_2+\alpha_1+\gamma_1)(\theta_2-t_0)}(\|x\|_0+\varepsilon) + \int_{t_0}^{\theta_2} e^{-(\mu_2+\alpha_1+\gamma_1)(\theta_2-v)} \\
&\quad \times \left[(\beta_1+\beta_2)\left(\frac{b}{\mu_1}+\varepsilon_0\right) + \alpha_2\right] K_0(\|x\|_0+\varepsilon)e^{\zeta t_0}e^{-\zeta v}dv \\
&= K_0(\|x\|_0+\varepsilon)e^{\zeta t_0}e^{-\zeta\theta_2}\left\{\frac{1}{K_0}e^{-(\theta_2-t_0)[(\mu_2+\alpha_1+\gamma_1)-\zeta]}\right. \\
&\quad \left. + \int_{t_0}^{\theta_2} e^{-(\theta_2-v)[(\mu_2+\alpha_1+\gamma_1)-\zeta]}[(\beta_1+\beta_2)\left(\frac{b}{\mu_1}+\varepsilon_0\right) + \alpha_2]dv\right\} \\
&\le K_0(\|x\|_0+\varepsilon)e^{\zeta t_0}e^{-\zeta\theta_2}\left\{\frac{1}{K_0}e^{-(\theta_2-t_0)[(\mu_2+\alpha_1+\gamma_1)-\zeta]}\right. \\
&\quad \left. + \int_{t_0}^{\theta_2} e^{-(\theta_2-v)[(\mu_2+\alpha_1+\gamma_1)-\zeta]}[(\mu_2+\alpha_1+\gamma_1)-\zeta]dv\right\} \\
&= K_0(\|x\|_0+\varepsilon)e^{\zeta t_0}e^{-\zeta\theta_2}\left\{1 - \left(1-\frac{1}{K_0}\right)e^{-(\theta_2-t_0)[(\mu_2+\alpha_1+\gamma_1)-\zeta]}\right\} \\
&< K_0(\|x\|_0+\varepsilon)e^{\zeta t_0}e^{-\zeta\theta_2},
\end{aligned}$$

which contradicts with the first equation in (2.23). Thus, Case II is not true.

Now we discuss the final case of what may happen. Making use of (2.19), (2.20) and (2.24), we can deduce that

$$\begin{aligned}
A(\theta_3) &= e^{-\int_{t_0}^{\theta_3}(\mu_3+\alpha_2+\gamma_2)d\theta}A(t_0) + \int_{t_0}^{\theta_3} e^{-\int_v^t(\mu_3+\alpha_2+\gamma_2)d\theta}\alpha_1 L(v)dv \\
&\le e^{-(\mu_3+\alpha_2+\gamma_2)(\theta_3-t_0)}A(t_0) + \int_{t_0}^{\theta_3} e^{-(\mu_3+\alpha_2+\gamma_2)(\theta_3-v)}\alpha_1 L(v)dv \\
&\le e^{-(\mu_3+\alpha_2+\gamma_2)(\theta_3-t_0)}(\|x\|_0+\epsilon) + \int_{t_0}^{\theta_3} e^{-(\mu_3+\alpha_2+\gamma_2)(\theta_3-v)} \times \alpha_1 K_0(\|x\|_0+\epsilon)e^{\zeta t_0}e^{-\zeta v}dv \\
&= K_0(\|x\|_0+\varepsilon)e^{\zeta t_0}e^{-\zeta\theta_3}\left\{\frac{1}{K_0}e^{-(\theta_3-t_0)[(\mu_3+\alpha_2+\gamma_2)-\zeta]} + \int_{t_0}^{\theta_3} e^{-(\theta_3-v)[(\mu_3+\alpha_2+\gamma_2)-\zeta]}\alpha_1 dv\right\} \\
&\le K_0(\|x\|_0+\varepsilon)e^{\zeta t_0}e^{-\zeta\theta_3}\left\{\frac{1}{K_0}e^{-(\theta_3-t_0)[(\mu_3+\alpha_2+\gamma_2)-\zeta]}\right. \\
&\quad \left. + \int_{t_0}^{\theta_3} e^{-(\theta_3-v)[(\mu_3+\alpha_2+\gamma_2)-\zeta]}[(\mu_3+\alpha_2+\gamma_2)-\zeta]dv\right\} \\
&= K_0(\|x\|_0+\varepsilon)e^{\zeta t_0}e^{-\zeta\theta_3}\left\{1-\left(1-\frac{1}{K_0}\right)e^{-(\theta_3-t_0)[(\mu_3+\alpha_2+\gamma_2)-\zeta]}\right\} \\
&< K_0(\|x\|_0+\varepsilon)e^{\zeta t_0}e^{-\zeta\theta_3},
\end{aligned}$$

which contradicts with the first equation in (2.24). Then, Case III will never happen.

From the above discussion, we obtain that (2.21) holds. Letting $\varepsilon \longrightarrow 0^+$, it follows

$$\|x(t)\| \le K_0\|x\|_0 e^{\zeta t_0}e^{-\zeta t} \quad \text{for all } t > t_0,$$

and this completes the proof of Theorem 2.1. □

3. An example

In this section, we give an example to show the existence and global exponential stability of the disease-free equilibrium of the system (1.1) by our theoretical results and also simulations.

Consider the following nonresident computer virus model:

$$\begin{cases}
\frac{dS(t)}{dt} = 11.2 - 0.8S(t) - 0.01S(t)L(t) - 0.02S(t)A(t) + 0.1L(t) + 0.1A(t), \\
\frac{dL(t)}{dt} = 0.01S(t)L(t) + 0.02S(t)A(t) + 0.1A(t) - (0.85+0.05+0.1)L(t), \\
\frac{dA(t)}{dt} = 0.05L(t) - (0.8+0.1+0.1)A(t),
\end{cases} \tag{3.1}$$

where $b = 11.2, \mu_1 = 0.8, \mu_2 = 0.85, \mu_3 = 0.8, \beta_1 = 0.01, \beta_2 = 0.02, \gamma_1 = \gamma_2 = 0.1, \alpha_1 = 0.05, \alpha_2 = 0.1$. Then

$$\begin{aligned}
&R_0 = \frac{b\{\beta_1 + \beta_2\alpha_1/(\mu_3+\alpha_2+\gamma_2)\}}{\mu_1\{(\mu_2+\alpha_1+\gamma_1) - \alpha_1\alpha_2/(\mu_3+\alpha_2+\gamma_2)\}} < \frac{33}{199} < 1, \\
&-\mu_1 + \left(\beta_1\frac{b}{\mu_1} + \beta_2\frac{b}{\mu_1} + \gamma_1 + \gamma_2\right) < -0.15 < 0, \\
&-(\mu_2+\alpha_1+\gamma_1) + (\beta_1+\beta_2)\frac{b}{\mu_1} + \alpha_2 < -0.45 < 0, \\
&-(\mu_3+\alpha_2+\gamma_2) + \alpha_1 = -0.95 < 0,
\end{aligned}$$

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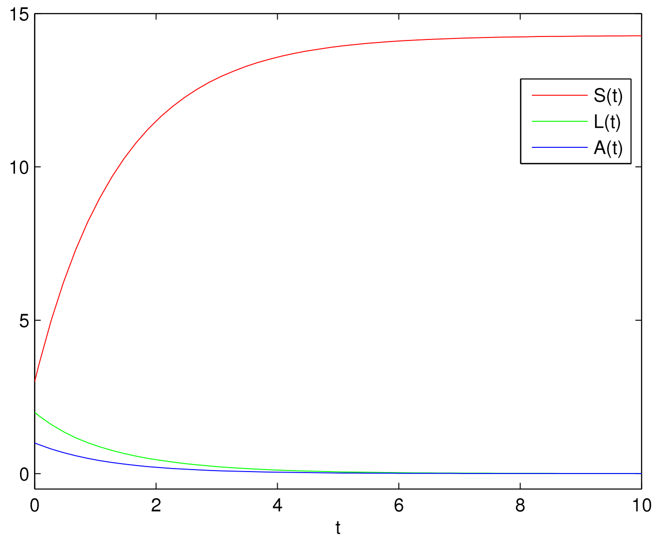

Fig. 1. Numerical solution $(S(t), L(t), A(t))$ of (3.1) with initial values $(3, 2, 1)$.

and it satisfies all the conditions in Theorem 2.1. Hence, system (3.1) is globally exponentially stable, and all solutions of the system converge exponentially to the virus-free equilibrium $(14, 0, 0)$ with the exponential convergent rate $\zeta \approx 0.002$. This fact is verified by the numerical simulation in Fig. 1.

Remark 3.1. Previous results in [1–10] do not concern about the global exponential stability of nonresident computer virus models, and the methods in [1–10] are not applicable for proving the global exponential stability of the virus-free equilibrium for system (3.1). Further work is to study the global exponential stability of the infected equilibrium of the nonresident computer virus models.

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