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Cooperative interconnection settlement among ISPs through NAP

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ABSTRACT

This paper studies the settlement problem among Internet Service Providers (ISPs) who interconnect with each other through Network Access Point (NAP). A cooperative game framework is adopted for the analysis. Two commonly adopted allocations, i.e., the non-settlement profit allocation and the Shapley-value based profit allocation, are analyzed and compared. We check whether these two allocations can encourage ISPs to interconnect with each other (i.e., in the core of the game) and, at the same time, demonstrate fairness in settlement. Our results show that the non-settlement allocation is not in the core and does not preserve fairness, and the Shapley-value based profit allocation is in the core and demonstrates fairness. However, the complex structure of Shapley-value makes it difficult to understand for ISPs and hard to be implemented at NAP especially when ISPs can only make their pricing decisions independently. Therefore, we propose a Characterized Profit Allocation (CPA) which is in the core, preserves fairness and is easy to interpret. We further propose a settlement rule based on CPA which enables the ISPs to act independently but achieve global optimality. We also extend our basic model to incorporate interconnection quality decisions and market competition, and show that the proposed settlement rule and its extended form work well in these scenarios. Numerical experiments confirm that CPA and its corresponding settlement rule can effectively encourage interconnection among ISPs and motivate ISPs to expand their networks.

1. Introduction

Internet users perceive the Internet as seamless and global, while behind the scene there exist many individual networks connected with each other instead of a sole massive network. Internet Service Providers (ISPs) connect their networks to each other by interconnection arrangements in order to enable communication among end-users from different networks. There are three interconnection modes: NAP (Network Access Point) peering, private peering and transit. When an ISP decides to connect itself to other networks, it has to consider the impact of the interconnection arrangements on its cost and profit and decide its interconnection strategy.

In China, ISPs, such as China Netcom and China Telecom, interconnect with each other mainly through private peering and NAP peering. Currently there are only ten national NAPs located at major cities such as Beijing, Shanghai and Guangzhou etc., and the quality of communication between networks and the data transmission efficiency across different regions are relatively low due to the limited number of NAPs. To facilitate data exchange and extenuate traffic congestion at national NAP, local NAPs have been built successively in several cities since 2004. This kind of local Internet exchange point has been proven beneficial to local Internet ecosystems, especially in developing countries and regions such as Latin America (Galperín, 2016; Weller & Woodcock, 2013). Similarly, the construction of these local NAPs in China has greatly reduced the interconnection costs and improved the network response speed, data exchange quality and safety.

However, new problems arose as local NAPs were put into use. For example, Shanghai Network Access Point SHNAP (SHNAP) initially adopted a non-settlement rule for the data exchange between its member ISPs. Under the non-settlement rule, ISPs do not charge each other for data exchange. Since SHNAP can save great money for ISPs for interconnection, it attracted 16 ISPs to connect to it and the data exchange volume increased very fast in the first few years. The non-settlement rule, however, began to show its inefficiency after several years of implementation, as it can hardly reflect the cost and revenue to peer in SHNAP for each ISP. As a result, it caused an imbalance of profit allocation among member ISPs, and ISPs, especially those large ones, had no incentive to connect to SHNAP and would rather peer with other ISPs privately. Similar problem in Argentina has also been documented by Galperín, (2016). Furthermore, ISPs who have already connected to SHNAP were reluctant to invest in capacity to improve the interconnection quality. At this stage, the introduction of a rational settlement rule, which allows fair profit allocation and stimulates ISPs

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to exchange traffic in local NAP, becomes the key to promote local NAP peering and enhance the value of local network.

Our study is developed against the general background of SHNAP. SHNAP wants to find a settlement rule that can make ISPs' selfish behavior results in globally optimal interconnecting decisions, i.e., all the local ISPs choose to interconnect with each other through SHNAP and make jointly optimal decisions. To achieve this goal, the profit allocation induced by this settlement rule must be fair in profit distribution and can bring more profit to each ISP. As NAPs are continually being set up all over the world for both regional interconnection and global interconnection (such as London Internet Exchange which has members from 40 countries), this study can provide valuable managerial insights on how to improve NAP peering efficiency not only for SHNAP, but also for practitioners worldwide.

In this study, we first build an analytical model of individual ISP's demand and profit and develop a cooperative game framework to analyze the different profit allocation rules. We show that interconnection with other ISPs can bring extra profit to ISPs. We further check whether the allocation rules, i.e., non-settlement profit allocation, the Shapley-value based profit allocation and the Characterized Profit Allocation we propose, can encourage ISPs to interconnect with each other and, at the same time, demonstrate fairness in profit distribution. Results show that the non-settlement allocation, which exerts no side payments on ISPs, is not in the core of the game, and does not preserve fairness either. The Shapley-value based profit allocation, which is known as fair allocation rule, on the other hand, is in the core. As Shapley-value based allocation requires complex computation and is difficult for ISPs to understand, we propose a new allocation rule, i.e., the Characterized Profit Allocation, which is easy to interpret. Our analysis shows that CPA is in the core and preserves fairness, and that it performs well in different situations. Numerical experiments are conducted to support our conclusions from theoretical analysis.

The rest of the paper is organized as follows. Section 2 reviews the related literature studying Internet interconnection, especially those who endeavored to approach a cooperative settlement rule. In Section 3, we set up the baseline model, including demand function and profit function. Section 4 builds the cooperative game model to analyze the impact of different profit allocation rules on ISPs' profit, and proposes a settlement rule to implement the Characterized Profit Allocation. Section 5 extends the basic model in two directions: consider the interconnection quality choice of ISPs and introduce competition in the market, and examine if the proposed settlement rule can encourage ISPs to connect to NAP under these two scenarios. In this section we also analyze a model with linear network externality. Numerical experiments are conducted in Section 6. Section 7 summarizes the findings and discusses future research directions.

2. Related literature

Under non-cooperative game analysis framework, there is a large body of literature studying ISP interconnection strategies, including determining compatibility and access charges. Cremer, Rey, and Tirole (2000) develop their research on basis of Katz and Shapiro's model of network externalities (Katz & Shapiro, 1985) and study the strategies of Internet backbone providers. They use a Cournot-cum-installed-bases model and show that compared with small Internet Backbone Provider (IBP), larger IBP prefers a lower interconnection quality. This result is robust even if customer can connect to several IBPs (multi-homing). Foros and Hansen (2001) follow their work by modeling network externalities in a two-stage game where the two ISPs choose compatibility level at stage 1 and compete over market shares a la Hotelling at stage 2. They find that ISPs can reduce competitive pressure in stage 2 by increasing compatibility, so that a higher compatibility in stage 1 will be achieved. While previous studies (Cremer et al., 2000; Foros & Hansen, 2001; Foros, Kind, & Sand, 2005; Matsubayashi & Yamada, 2008) all model network externality as a linear function of the number of customers, in this study we model network externality as a quadratic function of network size according to Metcalfe's law. Some more recent studies of Jahn and Prüfer (2008) and Badasyan and Chakrabarti (2008) also use simple game-theoretic model to analyze ISPs' interconnection choices. These models vary in complexity, with the former dealing with asymmetric networks and the latter dealing with different cost structures, which is essential to evaluating potential peering arrangements (Motiwala, Dhamdhere, Feamster, & Lakhina, 2012). In terms of pricing, He and Walrand (2005) show that non-cooperative pricing strategies may result in unfair profit distribution. They propose a fair allocation policy based on the weighted proportional fairness criterion. López (2011) extends Laffont, Marcus, Rey, and Tirole (2003) analysis to asymmetric but reciprocal access pricing in the presence of an arbitrary number of network operators. He shows that the configuration of interconnection charges has important implications for the market structure: If the reciprocal access charge of a pair of networks departs away from a given symmetric access charge, then the two networks are driven out of one side of the market (consumers/websites).

Several studies address the profit allocation among interconnected ISPs under non-cooperative game analysis framework. Huston (1998) suggests that ISP interconnection settlement can base on inbound traffic volume, on outbound traffic volume, on a hybrid of inbound and outbound traffic volume, or on the line capacity regardless of volume. Weiss and Shin (2004) believe that the cost of ISP interconnection is a function of traffic, and the traffic volume is a function of a market share. Thus, they address the interconnection settlement problem with knowledge of inbound and outbound traffic flows. Tan, Chiang, and Mookerjee (2006) propose a more complex pricing scheme that considers network utilization, link capacity, and the cost structure of the interconnecting ISPs. They show that a usage-based, utilization-adjusted interconnection agreement could align the costs and revenues of the providers while allowing them to achieve higher service levels.

Cooperative game theory is currently a hot topic in operational research (Chen & Chen, 2013; Hu, Caldentey, & Vulcano, 2013; Lozano, Moreno, Adenso-Díaz, & Algaba, 2013; Borkotokey, Kumar, & Sarangi, 2015; Karsten & Basten, 2014; Kimms & Kozeletskyi, 2016), and there is also a growing trend of using cooperative game theory to approach a profit allocation that encourages network-wide interconnection. Cheung, Chiu, and Huang (2008) find that with information of global topology and traffic information for each ISP tier, there exist prices that can make the revenue division under bilateral settlement equal to that calculated by Shapley value. Shapley value, which is known to be in the core of a convex cooperative game (Roth, 1988), indicates the marginal contribution each agent makes to the coalition. Ma, Chiu, Lui, Misra, and Rubenstein (2010) also adopt a Shapley-value approach and show that profit model based on Shapley value can make ISPs' selfish behavior result in global optimal routing and interconnecting decisions. Following this study, Ma et al. make several attempts to implement this Shapley-value rule. They first use a content-eyeball model to show that Shapley-value revenue distribution can be implemented by bilateral payment between eyeball and content ISPs (Ma, Chiu, Lui, Misra, & Rubenstein, 2008), and then extend their model to include transit ISPs (Ma, Chiu, Lui, Misra, & Rubenstein, 2011). In those studies, Shapley value is calculated as the cost of handling traffic, and it is strongly affected by ISP topology structure and routing strategies. Mycek et al. (2009) extend the concept in Ma et al. (2008) with a fair income distribution policy, coupling a routing decomposition optimization framework that deals with multiple connections. They use decomposition result parameters to solve the complex issue of computing Shapley values, while Misra, Ioannidis, Chaintreau, and Massoulié (2010) present another method based on fluid approximation, which can also effectively reduce the complexity of computing Shapley values. Furthermore, Singh, Sarkar, Aram, and Kumar (2012) study the similar problem of cooperative profit sharing in wireless network markets. They model such cooperation using the theory of transferable payoff cooperative game and propose a set of payoffs that are commensurate with the resource the network providers invest and the wealth they generate. Also they develop an algorithm to obtain the optimal resource allocation and corresponding profit sharing rule. The authors numerically show that cooperation can tremendously enhance network providers' profits. Indeed, cooperative game theory is a powerful tool in analyzing the behavior and interaction of the individual nodes in various communication networks (Liu, Li, Huang, Ying, & Xiao, 2013; Matsubayashi, Umezawa, Masuda, & Nishino, 2005; Saad, Han, Debbah, Hjørungnes, & Başar, 2009). Most of the previous researches pay attention to sharing of the physical resources, such as base stations, and propose the profit allocation based on the Shapley value. In our study, we extract the fact that interconnection can bring positive network externality and focus on profit allocation rules other than the Shapley value.

Our study adds to interconnection literature in two ways. First, previous researches focus on private peering and transit, and little attention is spared to NAP peering. However, as more and more global or local traffic are exchanged at NAPs, the operational model of these NAPs needs more exploration, and our study takes a step forward with a discussion of a rational settlement rule. Second, most of existing literature model network externality as a linear function, which we think is inadequate to appropriately reflect the network value according to our analysis in Section 5.3. Our study adopts the quadratic network externality function by an appeal to Metcalfe's law, and analyzes profit allocations under cooperative game framework. We believe that our analytical approach and the proposed Characterized Profit Allocation can be used in other networks with network externality, such as telecommunication networks.

3. The model

In this section, we describe our basic model and assumptions, and analyze the benchmark system where each ISP does not interconnect with others.

We consider a set of ISPs $N = \{ISP_i, i = 1, \dots, n\}, (n \geq 2)$. Each ISP is characterized by two parameters. The first is the intrinsic demand potential D_i, which is related to the coverage area of ISP_i and is considered to be exogenous. The second parameter is the installed network size e_i, which can be composed of the number of installed end-users and the richness of contents (Xu, 2007). The e_i in our model mainly reflects the richness of contents, and decision makers can use the number of ISP_i's installed websites weighted by popularity factors to measure e_i (Ma et al., 2008).

A consumer makes purchase decision based on the price and associated network size of the ISP and can only subscribe to one ISP. Let y_i denote the associated network size of ISP_i, which is the total size of network interconnected with ISP_i. The associated network size of ISP_i equals to its installed network size e_i if ISP_i does not interconnect with any other ISPs. Otherwise, y_i equals $\sum_{ISP_j \in S} e_j$, where S is the set of ISPs interconnected with ISP_i including ISP_i itself. We write $\sum_{ISP_j \in S} e_j$ as E_S in the remainder for simplification.

Consistent with previous studies (Foros & Hansen, 2001; Foros et al., 2005; Matsubayashi & Yamada, 2008), we model the realized demand of ISP_i, denoted as $d_i(S)$, as a function of its associated network size y_i and price $p_i(S)$ given that ISP_i interconnect with all other ISPs in set S. Specifically, when ISP_i does not interconnect with any other ISP, its pricing decision and the corresponding realized demand is denoted as p_i and d_i. We assume that the realized demand of ISP_i is linear to its price. We further assume that there is no direct competition among ISPs in the market. This assumption simplifies the analysis, but it still makes sense if different ISPs' networks do not overlap with each other or they target at different groups of users. The China Education and Research Network (CERN), for example, provides Internet service only to educational and research institutes. Additionally, regional ISP monopoly is not a rare phenomenon in both China and United States. According to Federal Communications Commission's report (2014), 35 percent of American households have two or less options of ISP that provides at least 10 Mbps downstream and at least 1.5 Mbps upstream connectivity service in their residential locations. Later, we relax this assumption in a special case where two ISPs interconnect and also compete with each other.

Despite that previous studies assume a linear network externality, we assume the network effect to be βy_i^2 ($\beta > 0$), i.e., the realized demand increases quadratically with the associate network size (linear network externality is also discussed in Section 5.3). This assumption is made based on the Metcalfe's Law (Metcalfe, 1995) which states that the value of a network is proportional to the square of the number of connected users of the system. The Metcalfe's law characterizes the network effects of various communication technologies and networks such as the Internet, social networking, and the World Wide Web. As more and more contents in Internet are generated by Internet users in the era of "we media", we believe the weighted number of installed websites can also carry partial information on the size of installed end-users of an ISP, so we adopt the Metcalfe's law and model the network externalities in quadratic form. Based on above assumptions, we construct the demand function consisting of intrinsic demand potential, price and associated network size as follows:

$$d_i(S) = D_i - \alpha p_i(S) + \beta y_i^2 \quad (1)$$

where α, β are positive constant coefficients denoting the demand variation responsive to the price and network size, respectively. The actual demand decreases with price and increases with the associated network size.

An ISP's cost consists of three parts, i.e., the fixed set up cost or maintenance cost of the network, the variable cost to serve a consumer, and the interconnection cost. As long as the fixed cost is smaller than the ISP's equilibrium revenue minus variable cost, the fixed cost has no effect on the equilibrium. To simplify the exposition, we assume that the fixed cost equals zero. Without loss of generality, we also take the variable cost to be zero. Following previous researches (Badasyan & Chakrabarti, 2008; Weiss & Shin, 2004) that consider interconnection cost as a function of traffic, we model the interconnection cost as the data transmission cost between users and resources in the associated network. For simplification, we assume that each customer has one unit demand for each resource in the associated network. Let c_o and c_t denote the unit cost of data transmission for originating network and terminating network respectively, and $c_o + c_t = t$. Typically, the terminating network bears most of the transmission cost, i.e., $c_t > c_o$, due to the 'hot potato' routing strategy (Laffont, Marcus, Rey, & Tirole, 2001, 2003). Therefore, the profit function of ISP_i is formulated as:

$$\pi_i(S) = p_i(S) d_i(S) - c_o e_i \sum_{ISP_j \in S} d_j(S) - c_t d_i(S) E_S \quad (2)$$

Each ISP makes the pricing decision to maximize its own profit.

We present the results of a non-interconnection system, where ISPs do not interconnect with each other, as a benchmark. It is

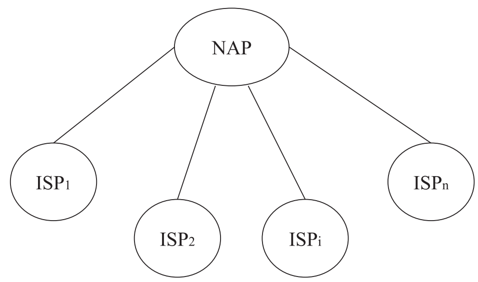


Fig. 1. Interconnection through NAP.

straightforward to verify that $\pi_i(p_i)$ is concave. Therefore, we have the following theorem.

Theorem 1. *In a non-interconnection system, for ISP_i, the optimal price p_i^*, the corresponding demand d_i^* and the optimal profit π_i^* are given by following equations:*

$$p_i^* = \frac{D_i + \beta e_i^2 + \alpha t e_i}{2\alpha}, \quad d_i^* = \frac{D_i + \beta e_i^2 - \alpha t e_i}{2},$$
$$\pi_i^* = \frac{\left(D_i + \beta e_i^2 - \alpha t e_i\right)^2}{4\alpha}.$$

Here we assume that $D_i + \beta e_i^2 - \alpha t e_i > 0$ for any $ISP_i \in N$ such that each ISP has positive demand. We also assume that $2\beta e_i - \alpha t > 0$ holds for any $ISP_i \in N$. This assumption makes sense as the demand of an ISP should be increasing in its network size.

4. Settlement analysis in a cooperative game framework

When an ISP interconnects with other ISPs (see Fig. 1), its subscribers can visit networks interconnected with it. On one hand, more consumers will be attracted to the ISP since they can visit more resources, and the ISP can generate more revenue by serving more end-users. On the other hand, interconnection will also incur an extra cost, as the data transmission cost increases with the increasing number of end-users and the expanded associated network size.

With the demand function defined in Section 3, the total profit of coalition S, denoted as $\Pi(S)$, can be formulated as

$$\Pi(S) = \sum_{ISP_i \in S} [p_i(S) d_i(S) - t d_i(S) E_S]$$

It is straightforward to check that $\Pi(S)$ is separable in p_i, and is concave in each p_i. Therefore, we have the following theorem.

Theorem 2. *In an ISP coalition S, for $ISP_i \in S$, the jointly optimal price $p_i^*(S)$, the corresponding demand $d_i^*(S)$ and the optimal profit $\Pi^*(S)$ are given by following equations:*

$$p_i^*(S) = \frac{D_i + \beta E_S^2 + \alpha t E_S}{2\alpha}, \quad d_i^*(S) = \frac{D_i + \beta E_S^2 - \alpha t E_S}{2},$$
$$\Pi^*(S) = \frac{\sum_{ISP_i \in S} \left(D_i + \beta E_S^2 - \alpha t E_S\right)^2}{4\alpha}.$$

Comparing the optimal total profit above with the results in Theorem 1, it is easy to see that interconnection among ISPs elevates the total profit from $\frac{\sum_{ISP_i \in S} (D_i + \beta e_i^2 - \alpha t e_i)^2}{4\alpha}$ to $\frac{\sum_{ISP_i \in S} (D_i + \beta E_S^2 - \alpha t E_S)^2}{4\alpha}$. This increase in profit due to interconnection affirms the viability of implementing profit allocation rules to encourage interconnection.

We then use cooperative game theory to discuss the profit allocation problem among all ISPs. We refer to the subset S ($S \subseteq N$) of ISPs who interconnected with each other as coalition S and to the set N as the grand coalition. Thus, we formulate our problem as a cooperative game $(N, \Pi^*(S))$, in which $\Pi^*(S)$ is the characteristic function specifying the optimal total profit associated with coalition S. A vector $r = (r_1, \ldots, r_n)$ is called an allocation and each element r_i corresponds to the portion of total profit of grand coalition that ISP_i should get. If $\sum_{i=1}^n r_i = \Pi^*(N)$, then the allocation is said to be *efficient*. An allocation is said to be *individually rational* if $r_i \geq \pi_i^*$ and to be *stable* for a coalition S if $\sum_{ISP_i \in S} r_i \geq \Pi^*(S)$. Altogether, an allocation is said to be in the core if it satisfies the following two conditions:

(i) Efficiency: $\sum_{ISP_i \in N} r_i = \Pi^*(N)$;
(ii) Coalitional rationality: $\sum_{ISP_i \in S} r_i \geq \Pi^*(S)$ for any $S \subseteq N$.

When an allocation is in the core, no subset of ISPs would secede from the grand coalition to form smaller coalitions, including being on their own. In addition to the requirement of being in the core, it is desirable for an allocation to be perceived as *fair* (for a more elaborate discussion of fairness in cost allocation rules see Moulin, 1995). To be clear, the allocation of the total profit should reflect the value of each ISP's network: ISPs with larger network size should earn a larger proportion of total profit. Thus the benefit of every ISP can be guaranteed, and ISPs should have incentive to expand their networks to increase profit. As a result, ISP interconnection is encouraged, and the development of Internet market is boosted.

In what follows, we analyze three different profit allocations. The first is the **non-settlement profit allocation**, in which each ISP invoices its user for the services, but no financial settlement is made across ISPs. The non-settlement profit allocation was adopted by SHNAP. The second allocation is the widely researched **Shapley-value based profit allocation**. The third is the **Characterized Profit Allocation** we proposed, in which each ISP has to pay the other ISPs for accessing their networks.

4.1. Non-settlement profit allocation

The non-settlement profit allocation implies that there is no side-payments among ISPs. That is, an ISP does not pay or charge other ISPs for network accessing. Non-settlement allocation can be regarded as a special profit allocation rule, where each ISP simply connects to other networks but no financial settlement is payable.

Let $r_A = (r_{A1}, \ldots, r_{An})$ denote the profit allocation vector under non-settlement allocation. Since there is no settlement, we have r_{Ai} as follows:

$$\begin{aligned} r_{Ai} &= \pi_i^*(N) = p_i^*(N) d_i^*(N) - c_o e_i \sum_{ISP_j \in N} d_j^*(N) - c_t d_i^*(N) E_N \\ &= \frac{\left[D_i + \beta E_N^2 - \alpha(c_o + c_t) E_N\right]\left[D_i + \beta E_N^2 + \alpha(c_o - c_t) E_N\right]}{4\alpha} \\ &\quad - \frac{c_o e_i}{2} \sum_{ISP_j \in N} \left(D_j + \beta E_N^2 - \alpha t E_N\right) \end{aligned}$$

Theorem 3. *Non-settlement profit allocation is not in the core of the game $(N, \Pi^*(S))$.*

Proof. See A.1 for the proof. □

When a set of ISPs possess networks of relatively large size but small intrinsic demand potentials, they may find joining in the NAP less profitable than forming a coalition by themselves through private peering.

Furthermore, non-settlement does not make a fair allocation. According to the Metcalfe's Law, the ISP with larger network size adds more value to the interconnected network than the ISP with smaller network size. But under non-settlement allocation, an ISP with larger network size receives less profit allocation from the grand coalition given that all ISPs have the same intrinsic demand potential. So, r_A cannot truly reflect how much contribution each ISP has made to the grand coalition. If an ISP decides to expand its network, it may benefit other ISPs more than itself. Thus, ISPs would be reluctant to invest in their networks or interconnect with other ISPs if the non-settlement allocation was implemented. Thus, the non-settlement profit allocation may restrain the development of internet interconnection.

4.2. Shapley-value based profit allocation

Shapley value is an important concept in cooperative game, and it is well-known for its fairness property. The payoff to a player under Shapley-value based profit allocation is calculated as the marginal contribution of a player averaged over joining orders of the coalition. To be specific, it can be formulated as

$$\varphi_i(N, \Pi^*) = \frac{1}{|N|!} \sum_{\zeta \in Z} \Delta_i(\Pi^*, P(\zeta, i)), \forall ISP_i \in N.$$

where $\Delta_i(\Pi^*, S) = \Pi^*(S \cup \{ISP_i\}) - \Pi^*(S)$, Z is the set of all $|N|!$ orderings of N, and $P(\zeta, i)$ is the set of ISPs preceding ISP_i in the ordering ζ.

Let $r_B = (r_{B1}, \ldots, r_{Bn}) = (\varphi_1, \ldots, \varphi_n)$ denote the profit allocation vector under Shapley-value based allocation. As $(N, \Pi^*(S))$ is a convex cooperative game, Shapely value is consequently in the core.

Though in the core and satisfying desirable properties such as symmetry, linearity, additivity, and most importantly, efficiency and fairness, Shapley value is difficult to calculate when the number of players in the coalition becomes large. In addition, the complex structure of Shapley value may thwart ISPs from understanding the rules properly and make this profit allocation difficult to be implemented at NAP.

4.3. Characterized profit allocation

Network size is a critical factor for an interconnected network to attract more end-users and make greater profit, so it is important for an allocation to reward ISPs who invest more in network construction and make bigger contributions to the grand coalition with larger size of network. Thus, the Characterized Profit Allocation (CPA) is developed to meet this end. Under this proposed allocation, the total profit of all interconnected ISPs is redistributed in a way that reflects their specific characteristics, including the network size and intrinsic demand potential.

Let $r_C = (r_{C1}, \ldots, r_{Cn})$ denote the profit allocation vector under the CPA. Based on the structure of the optimal total profit, we design r_{Ci} as the following:

$$r_{Ci} = \frac{1}{4\alpha}\left[D_i^2 + e_i(\beta E_N - \alpha t) \sum_{ISP_j \in N} \left(2D_j + \beta E_N^2 - \alpha t E_N\right)\right].$$

Theorem 4. *The Characterized Profit Allocation is in the core of the game $(N, \Pi^*(S))$ and is a fair allocation.*

Proof. See A.2 for the proof. □

As we can see, ISP_i's investment in network expansion will bring extra profit to other ISPs who make no effort as well as to itself and we are particularly interested in how the total profit is allocated among ISP_i and other ISPs. To check that, the marginal profit of each ISP as e_i increases is calculated as follows:

$$\frac{\partial r_{Ci}}{\partial e_i} = \frac{1}{4\alpha}\left[(\beta E_N - \alpha t) \sum_{ISP_k \in N} (2D_k + \beta E_N^2 - \alpha t E_N) + n e_i(\beta E_N - \alpha t)\right.$$
$$\left.(2\beta E_N - \alpha t) + \beta e_i \sum_{ISP_j \in N} \left(2D_j + \beta E_N^2 - \alpha t E_N\right)\right] \quad (3)$$

$$\frac{\partial r_{Cj}}{\partial e_i} = \frac{1}{4\alpha}\left[n e_j(\beta E_N - \alpha t)(2\beta E_N - \alpha t)\right.$$
$$\left. + \beta e_j \sum_{ISP_k \in N} \left(2D_k + \beta E_N^2 - \alpha t E_N\right)\right] \quad (4)$$

Comparing (3) and (4), we find that, despite that all ISPs benefit from ISP_i's investment in the network size, ISP_i can get an extra part of profit compared to other ISPs. This property ensures that the ISP who invests in its own network benefits more than other ISPs from the expansion of the entire interconnected network, which again demonstrates the fairness of CPA.

4.4. Settlement rule for independent ISPs

From the analysis above, we conclude that the non-settlement allocation is not in the core of the cooperative game $(N, \Pi^*(S))$, and shows no fairness in profit allocation either. The Shapley-value based profit allocation is a traditional fair allocation rule and is in the core, but it's complicated and obscure formulation cripples its practical implementation. The CPA preserves fairness and is also in the core, so it is desirable that ISPs interconnected through NAP can achieve the profit allocation under this allocation rule.

However, the cooperative game framework is somewhat unsatisfactory in that it does not explicitly describe the ISPs' equilibrium pricing action. This is crucial to our problem because the profit allocations we discuss above make sense only when optimal total profit can be achieved. In practice, ISPs are unlikely to make pricing decisions together to achieve global optimum, and they usually make their pricing decisions independently. Thus, the introduction of a settlement rule under which ISPs will independently make the jointly optimal pricing decisions is very essential. It can effectively motivate ISPs to cooperate, i.e., connecting to the NAP to exchange traffic with each other.

Based on CPA, we propose a settlement rule under which each ISP connecting to NAP receives a subsidy (or pay a side-payment) while it makes its pricing decision independently. For ISP_i, the subsidy $s_i(N)$ is composed as follows:

$$s_i(N) = c_o\left(e_i \sum_{ISP_j \in N} d_j - d_i E_N\right)$$
$$+ \frac{1}{4\alpha}\left[e_i(\beta E_N - \alpha t) \sum_{ISP_j \in N} \left(2D_j + \beta E_N^2 - \alpha t E_N\right)\right.$$
$$\left. - \left(2D_i + \beta E_N^2 - \alpha t E_N\right)\left(\beta E_N^2 - \alpha t E_N\right)\right].$$

In the following, we use the subsidiary vector **s** to denote the settlement rule we proposed. Then each ISP will set out to optimize its profit π_i^c at NAP:

$$\pi_i^c = \pi_i(N) + s_i(N)$$
$$= p_i d_i - (c_o + c_t) d_i E_N$$

$$+\frac{1}{4\alpha}\left[e_i(\beta E_N-\alpha t)\sum_{ISP_j\epsilon N}\left(2D_j+\beta E_N^2-\alpha tE_N\right)\right.$$
$$\left.-\left(2D_i+\beta E_N^2-\alpha tE_N\right)\left(\beta E_N^2-\alpha tE_N\right)\right]$$

The equilibrium pricing decision of each ISP coincides with the jointly optimal pricing, and the profit π_i^c each ISP earns after receiving subsidiary (paying side-payment) is exactly the same as r_{Ci}. Under settlement rule $\boldsymbol{s}$, each ISP gets more profit in the grand coalition than in any other coalitions, so they will not split from the grand coalition, and the aim of encouraging network interconnection through NAP can be achieved. An ISP can get extra profit than others under CPA when it expands its network, so the proposed settlement rule $\boldsymbol{s}$ can also encourage ISPs to constantly invest in their network construction. Thus, the NAP interconnection market will be pulled onto the track of quick development.

In Section 6, we conduct numerical experiments to demonstrate the effectiveness of CPA.

5. Extensions

5.1. Incorporating quality decision in interconnection settlement

In practice, ISPs with asymmetric network sizes usually have asymmetric incentives to provide interconnection quality, which will determine how useful and efficient the interconnection is. To address this issue, we extend our model to consider ISPs' interconnection quality decision along with pricing decision. Some scholars have looked into the similar price and quality-based competition problem in the interconnection market. Matsubayashi and Yamada (2008) study how the asymmetry in consumer loyalty affect firms' price and quality competition. Le Cadre, Barth, and Pouyllau (2011) analyze the price and quality choice of each player in a vertically integrated autonomous system under four types of contract, and their numerical illustration suggests that grand coalition cooperation contract is efficient when consumers' QoS sensitivity is relatively low.

The basic model is extended as follows. When an ISP connect to an NAP to exchange traffic, it has to decide its access bandwidth, and it can take some technical measures such as uplink and downlink bandwidth limits to control the transmission quality for inbound and outbound traffics. We extract two decision variables, γ_i^U and γ_i^D, denoting the uplink quality and downlink quality respectively. Here $\gamma_i^U,\ \gamma_i^D\in[\underline{\gamma},1]$, $\underline{\gamma}\in(0,1)$ is the lowest interconnection quality provided at the NAP, and 1 represents perfect interconnection quality. An ISP with large network and rich resources may tend to limit the number of routing paths available and decrease the access bandwidth, i.e., choosing a low level of interconnection quality, to avoid superabundant traffic load. As a result of imperfect interconnection, end-users cannot equally access to resources that belong to different ISPs in Internet, so end-users subscribing to different ISPs perceive the size of the interconnected network differently. In accordance with Cremer et al. (2000), the resulting perceived network size (i.e., QoS level of ISP_i) is constructed as a function of its own network size, the network size of other ISPs and the quality of interconnection decisions of all ISPs connected to the NAP. For example, if there are three ISPs connecting to NAP, as shown in Fig. 2, then ISP_1*'s* subscribers can visit its own resources with perfect quality and visit the other two ISPs' resources with quality determined by ISP_1's downlink quality and ISP_2 and ISP_3's uplink quality, so the QoS level of ISP_1 is $\gamma_1^D(\gamma_2^U e_2+\gamma_3^U e_3)+e_1$.

In general, after introducing the interconnection quality decisions, the demand function changes to $d_i(S)=D_i-\alpha p_i(S)+\beta\hat{y}_i^2$, where $\hat{y}_i=\gamma_i^D\sum_{j\neq i}\gamma_j^U e_j+e_i$ denotes the QoS level of ISP_i. The

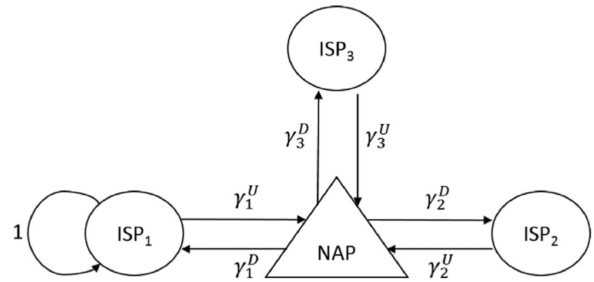


Fig. 2. Factors determining ISPs' QoS level in a market of three ISPs.

profit function remains to be $\pi_i(S)=p_i(S)d_i(S)-c_o e_i\sum_{ISP_j\in S}d_j-c_t d_i E_S$. For ISP_i, its uplink quality does not affect its own QoS level, but has a positive effect on other ISPs' QoS level. When ISP_i improves its uplink quality, other ISPs' end-users will have a better experience visiting resources in ISP_i's network, and thus a better perception of the interconnected network, so more users will be attracted to subscribe to these ISPs while the number of ISP_i's subscribers remains the same. The increase in the number of other ISPs' end-users raises the total data transmission cost ISP_i bears, and leads to a lower level of ISP_i's uplink quality choice. Indeed, ISP_i's equilibrium uplink quality decision is $\underline{\gamma}$, suggesting that ISPs will choose the lowest level of uplink quality when making decisions independently in order to optimize their own profit. On the other hand, ISPs will choose perfect downlink quality at equilibrium, and its pricing decision is given in Theorem 5.

Theorem 5. *When ISPs make decisions independently, the equilibrium uplink quality, downlink quality and price are* $\gamma_i^{U^*}=\underline{\gamma}$, $\gamma_i^{D^*}=1$, $p_i^*(S)=\frac{D_i+\beta(\sum_{j\neq i}\underline{\gamma}e_j+e_i)^2+\alpha c_o e_i+\alpha c_t E_S}{2\alpha}$ *respectively.*

Proof. See A.3 for the proof. □

Theorem 6. *The jointly optimal uplink and downlink quality decisions and pricing decision are* $\gamma_i^U=\gamma_i^D=1$ *and* $p_i^*(S)=\frac{D_i+\beta E_S^2+\alpha tE_S}{2\alpha}$ *respectively.*

Proof. See A.4 for the proof. □

When interconnected ISPs cooperate with each other and make jointly optimized decisions, they will all set their downlink and uplink quality levels simultaneously at 1, to maximize the network externality effect and to attract as more end-users as they can. As the optimal choice of γ_i^D and γ_i^U are both 1, this problem reduces to the original model we have discussed in section 3 and 4, so the optimal pricing decision is the same as in Theorem 2. The analysis of the three profit distribution rules also holds, and the CPA still works the best among the three allocations. We just have to do a slight modification to the settlement rule **s** to induce ISPs to make jointly optimal decisions.

The subsidiary $s_i^Q(N)$ an ISP receives (pays) is:

$$s_i^Q(N)=c_o\left(e_i\sum_{ISP_j\epsilon N}d_j-d_iE_N\right)$$
$$+\frac{1}{4\alpha}\left[\gamma_i^U e_i(\beta E_N-\alpha t)\sum_{ISP_j\epsilon N}\left(2D_j+\beta E_N^2-\alpha tE_N\right)\right.$$
$$\left.-\left(2D_i+\beta E_N^2-\alpha tE_N\right)\left(\beta E_N^2-\alpha tE_N\right)\right]$$

Then each ISP will set out to optimize its profit π_i^{Qc} at NAP:

$$\pi_i^{Qc}=\pi_i(N)+s_i^Q(N)$$
$$=p_id_i-(c_o+c_t)d_iE_N$$

$$+\frac{1}{4\alpha}\left[\gamma_i^{U}e_i(\beta E_N-\alpha t)\sum_{ISP_j\in N}\left(2D_j+\beta E_N^2-\alpha tE_N\right)\right.$$
$$\left.-\left(2D_i+\beta E_N^2-\alpha tE_N\right)\left(\beta E_N^2-\alpha tE_N\right)\right]$$

Under this settlement rule $\mathbf{s^Q}$, a high level of uplink quality can bring an ISP more profit, so each ISP will be incentivized to spontaneously choose the highest uplink and downlink quality and set the price at global optimal level. However, the efficiency of settlement rule $\mathbf{s^Q}$ is not robust, i.e., when an ISP is not fully rational, or has other considerations and make decisions deviating from optimal, $\mathbf{s^Q}$ cannot efficiently allocate all profit to ISPs. Luckily, $\sum_{ISP_i\in N}\pi_i^{Qc}$ will not exceed $\sum_{ISP_i\in N}\pi_i(N)$ causing profit imbalance, and we believe ISPs will converge to optimal decisions after periods of strategy adjustments.

As ISPs obtain higher profits which fairly reflect their network values and contributions to the network when they exchange traffic with other ISPs through NAP, they have no incentive switching to private peering. In the meantime, the proposed settlement rule $\mathbf{s^Q}$ can encourage ISPs to choose high interconnection quality. The efficiency of an NAP will be improved by the increased number of ISPs connecting to it and the enhanced interconnection quality, and internet users will have a better experience in terms of the availability of resources and the access speed when browsing the Internet.

5.2. Competition while cooperation

We consider two ISPs in a specific region and relax the assumption that there is no competition between the two ISP. As in the basic model, each ISP has a certain coverage area and corresponding intrinsic demand potential, but end-users located in ISP_i's coverage area may be attracted to subscribe to ISP_j if ISP_j has a price advantage over ISP_i. To account for the price competition, we extend the demand function to be $d_i=D_i-\alpha_1p_i+\alpha_2p_j+\beta y_i^2$, $i,j\in\{1,2\},\ \ i\neq j$. The composition of the demand in competitive market is similar to that of Matsubayashi and Yamada (2008) except that there is no quality competition in our model.

We derive the pricing decision of each ISP in four scenarios respectively:

(1) Non-interconnection: ISPs do not interconnect with each other and decide on prices independently;
(2) Private peering: ISPs interconnect with each other via private peering and decide on prices independently, and there is no payment between the two ISPs;
(3) NAP peering: ISPs interconnect with each other via NAP and make pricing decisions independently according to the settlement rule **s** implemented at the NAP.
(4) Cooperative peering: ISPs interconnect with each other and decide on prices cooperatively to achieve optimal total profit. This scenario serves as a benchmark.

Theorem 7. *(a) Unique Nash equilibria exist for first three scenarios respectively. Equilibrium pricing decisions are presented as follows:*

(1) Non-interconnection

$$p_i^*=\frac{2\alpha_1D_i+\alpha_2D_j+2\alpha_1^2te_i+\alpha_1\alpha_2te_j+2\alpha_1\beta e_i^2+\alpha_2\beta e_j^2}{(2\alpha_1+\alpha_2)(2\alpha_1-\alpha_2)}$$

(2) Private peering

$$p_1^*=\frac{2\alpha_1D_1+\alpha_2D_2+\alpha_2c_o(\alpha_2-\alpha_1)(e_1-e_2)}{(2\alpha_1+\alpha_2)(2\alpha_1-\alpha_2)}+\frac{\beta(e_1+e_2)^2+\alpha_1c_t(e_1+e_2)-(\alpha_2-\alpha_1)c_oe_1}{2\alpha_1-\alpha_2}$$

$$p_2^*=\frac{2\alpha_1D_2+\alpha_2D_1+2\alpha_1c_o(\alpha_2-\alpha_1)(e_1-e_2)}{(2\alpha_1+\alpha_2)(2\alpha_1-\alpha_2)}+\frac{\beta(e_1+e_2)^2+\alpha_1c_t(e_1+e_2)-(\alpha_2-\alpha_1)c_oe_1}{2\alpha_1-\alpha_2}$$

(3) NAP peering

$$p_i^*=\frac{\alpha_2D_j+2\alpha_1D_i}{(2\alpha_1+\alpha_2)(2\alpha_1-\alpha_2)}+\frac{\beta(e_1+e_2)^2+\alpha_1t(e_1+e_2)}{2\alpha_1-\alpha_2}$$

(b) The optimal pricing decision p_i^* under cooperative peering is $\frac{\alpha_j(D_2-D_1)}{2(\alpha_1+\alpha_2)(2\alpha_1-\alpha_2)}+\frac{D_1+\beta(e_1+e_2)^2+t(\alpha_1-\alpha_2)(e_1+e_2)}{2(\alpha_1-\alpha_2)}$.

Proof. For part (a), by simultaneously solving the first order condition of each ISP's profit function in each scenario, we can derive the optimal pricing decision for each ISP. For part (b), as the Hessian matrix of the total profit function $\Pi=d_1[p_1-t(e_1+e_2)]+d_2[p_2-t(e_1+e_2)]$ is negative definite, Π reaches a global maximum at the critical point, which is solved as p_i^* in part (b). □

By comparing the jointly optimal pricing decision and the optimal pricing decision under NAP peering, we find that the proposed settlement rule **s** fails to induce ISPs to make jointly optimal pricing decisions. However, numerical experiments (see Section 6.4) suggest that the total profit of two ISPs under NAP peering is close to the jointly optimal total profit.

We also conduct another numerical experiment (see Section 6.4) to illustrate the impact of settlement rule **s** on each ISP's profit in a competitive market. Results show that our proposed settlement rule can serve the competitive market as well by fairly apportioning the total profit to ISPs.

5.3. Linear network externality

Though one of the theoretical contribution of this study is to suggest that the network externality should be modeled as a quadratic function of network size in order to capture the characteristic of the internet, previous studies in interconnection field modeled the network externality as a linear term. To be consistent with these studies, we also adopt the linear network externality assumption and reformulate the demand function in basic model as $d_i(S)=D_i-\alpha p_i(S)+\beta y_i$. Under this linear network externality assumption, we derive the optimal price $p_i^*(S)$, the corresponding demand $d_i^*(S)$ and the optimal profit $\Pi^*(S)$ of ISPs in coalition S as follows:

$$p_i^*(S)=\frac{D_i+\beta E_S+\alpha tE_S}{2\alpha},\quad d_i^*(S)=\frac{D_i+\beta E_S-\alpha tE_S}{2},$$
$$\Pi^*(S)=\frac{\sum_{ISP_i\in S}(D_i+\beta E_S-\alpha tE_S)^2}{4\alpha}$$

Then the three profit allocations are:

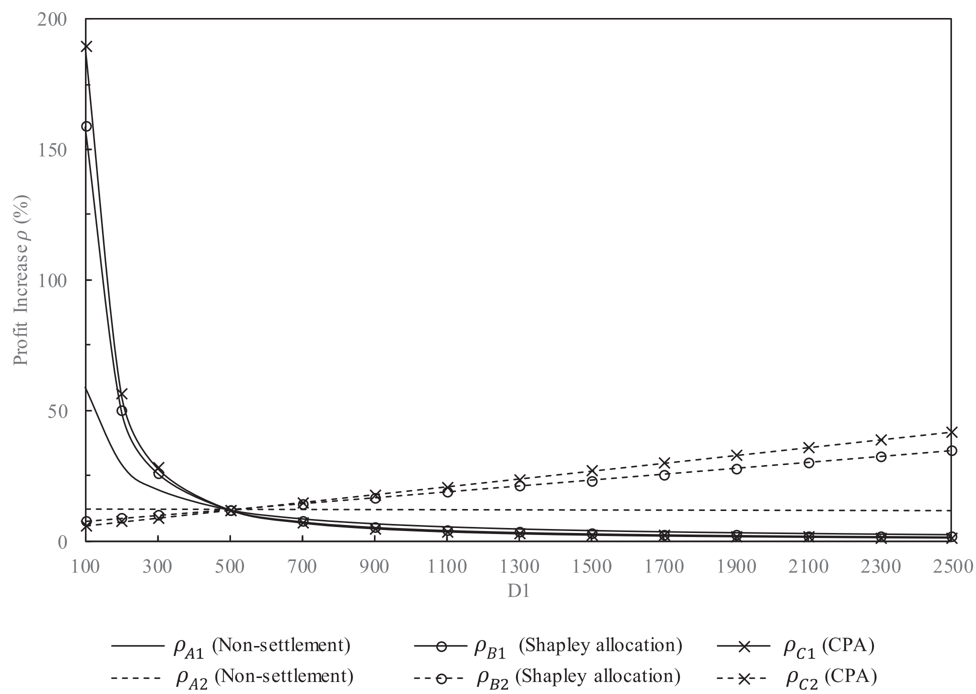
(1) Non-settlement profit allocation:

$$r_{Ai}=\frac{[D_i+\beta E_N-\alpha(c_o+c_t)E_N][D_i+\beta E_N+\alpha(c_o-c_t)E_N]}{4\alpha}-\frac{c_oe_i}{2}\sum_{ISP_j\in N}\left(D_j+\beta E_N-\alpha tE_N\right)$$

(2) Shapley-value based profit allocation:

$$r_{Bi}=\frac{1}{|N|!}\sum_{\zeta\in Z}\Delta_i(\Pi^*,P(\zeta,i)),\ \ \forall ISP_i\in N,$$

where $\Delta_i(\Pi^*,S)=\Pi^*(S\cup\{ISP_i\})-\Pi^*(S)$, Z is the set of all $|N|!$ orderings of ISPs, and $P(\zeta,i)$ is the set of ISPs preceding ISP_i in the ordering ζ.



$*\ N = \{ISP_1, ISP_2\}, e_1 = e_2 = 10, D_2 = 500, \alpha = 0.8, \beta = 0.1, t = 0.1, c_0 = \frac{1}{3}t, c_t = \frac{2}{3}t.$

Fig. 3. Sensitivity analysis on intrinsic demand potential.

(3) Characterized Profit Allocation (CPA):

$$r_{Ci} = \frac{1}{4\alpha}\left[D_i^2 + e_i(\beta - \alpha t)\sum_{ISP_j \in N}\left(2D_j + \beta E_N - \alpha t E_N\right)\right].$$

As for the core analysis, the conclusions are similar to that of quadratic-network-externality model. Non-settlement allocation is not in the core when there exist ISPs with relatively large network size and small intrinsic demand potential, while Shapley-value based profit allocation and CPA are in the core. As for the fairness property analysis, on the other hand, we present a numerical experiment in Section 6.2, and results show that linear network externality is inadequate to reflect the contribution of ISPs made to the interconnected network. This conclusion makes the quadratic-network-externality assumption essential.

6. Numerical experiments

From Sections 6.1 to 6.3 numerical experiments are conducted to illustrate the effectiveness of CPA in basic model. In Section 6.4, we provide numerical evidence of the effectiveness of the proposed settlement rule **s** in a competitive market with two players.

6.1. The benefit of interconnection

We have learned that interconnecting with other ISPs through NAP can bring more profit. In this section, we explore how the profit increase under three allocations (non-settlement allocation, Shapley-value based allocation and characterized allocation) through interconnection are affected by different parameters. We use ρ_{ji} to indicate the percentage increase of profit of ISP_i under allocation j ($j = A, B,$ or C) compared with the non-interconnection case. According to the definition, we have:

$$\rho_{ji} = \frac{r_{ji} - \pi_i^*}{\pi_i^*} * 100\%$$

where π_i^* is the optimal profit of ISP_i in the case of non-interconnection.

Numerical experiments are carried out in the case of two ISPs.

First, we check how the benefit of interconnection for ISP_1 and ISP_2 are affected by ISP_1's intrinsic demand potential D_1. As shown in Fig. 3, it is clear that interconnection can always bring profit improvement for both ISPs, no matter which allocation is implemented.

It is interesting to notice that when D_1 is small, ISP_1's profit increase is much more significant. The reason is twofold. First, the effect of network externality on profit increase measured in percentage is more evident for ISP with lower independent optimal profit π_i^*, despite that both ISPs enjoy the same amount of network externality. Second, under CPA and Shapley allocation an ISP enjoys part of the other ISP's profit for providing resources to end-users of the other network. Thus, when D_1 is small, ISP_1 can receive more compensation from ISP_2 than it has to pay to ISP_2. This rationale also explains why ρ_{B1} and ρ_{C1} is higher than ρ_{A1} when D_1 is smaller than D_2.

Conclusively, all three allocations favor ISP with relatively smaller intrinsic demand potential, and the gap between two ISPs' percentage increase of profit decreases following the sequence of CPA, Shapley allocation, and non-settlement allocation.

Second, we check how benefits of interconnection for ISP_1 and ISP_2 are affected by ISP_1's network size e_1. As shown in Fig. 4, under all three allocations, ISP_2's percentage of profit increase climbs up high as e_1 increases, while ISP_1's percentage of profit increase is concave but not monotone increasing in e_1.

Actually, increase in e_1 can benefit ISPs in two aspects. First, it can enlarge the total network size of the grand coalition, which in turn can generate higher total revenue by attracting more end-users. Second, larger e_1 assures ISP_1 a bigger proportion of the total profit under CPA and Shapley allocation. However, as e_1 increases further to a fairly large extent compared to ISP_2, ISP_1 can make a considerable amount of profit operating by itself without any interconnection. So when e_1 is fairly large, the profit increase of ISP_1

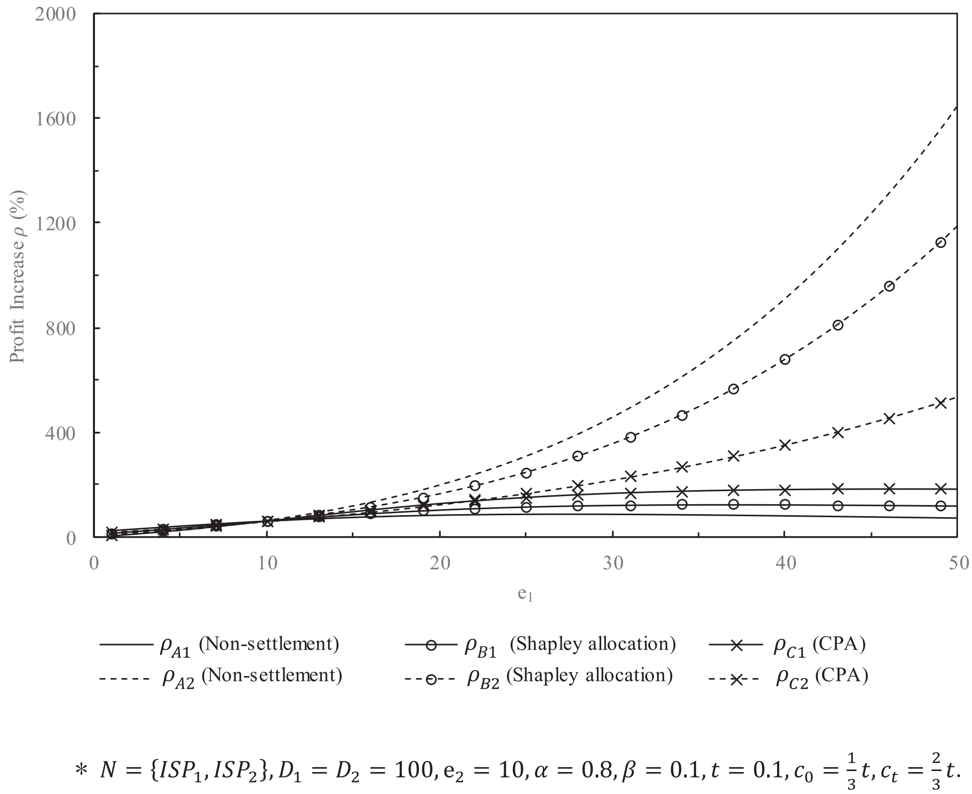


Fig. 4. Sensitivity analysis on network size.

brought by interconnection starts to decrease in percentage, while ISP_2's percentage of profit increase becomes fairly large because π_2^* stays constant.

Another interesting observation is that, in terms of percentage of profit increase, non-settlement allocation and Shapley allocation always favor the ISP with relatively smaller network size, while CPA favors ISP with relatively larger network size except when there is too large a difference between the sizes of two ISPs' networks. This fact to some extent demonstrates that CPA can effectively compensate for an ISP's network investment.

6.2. Fairness of different allocations

This subsection compares the degree of fairness of these three allocations. In the case of two ISPs, we plot the profit ratio ($\frac{r_{j1}}{r_{j2}}$, $j = A, B,$ *or* C) curves against network size ratio ($\frac{e_1}{e_2}$) to see the degree of fairness of different allocations. We conduct this numerical experiment for the model with quadratic network externality (the basic model) and the model with linear network externality (described in Section 5.3) respectively.

The result for the basic model is presented in Fig. 5a. The profit ratio curve for non-settlement allocation ($\frac{r_{A1}}{r_{A_2}}$) is a near-horizontal line with a downward slope, indicating that non-settlement allocation does not reward ISPs for bringing more resources to the interconnected network. The profit ratio of CPA and Shapley allocation both increase with the network size ratio, indicating that these two allocations reward the ISP with larger network size more. It is clear to tell from Fig. 5a that the slope of the curve corresponding to CPA is the closest to 1, suggesting that it is the fairest allocation among these three.

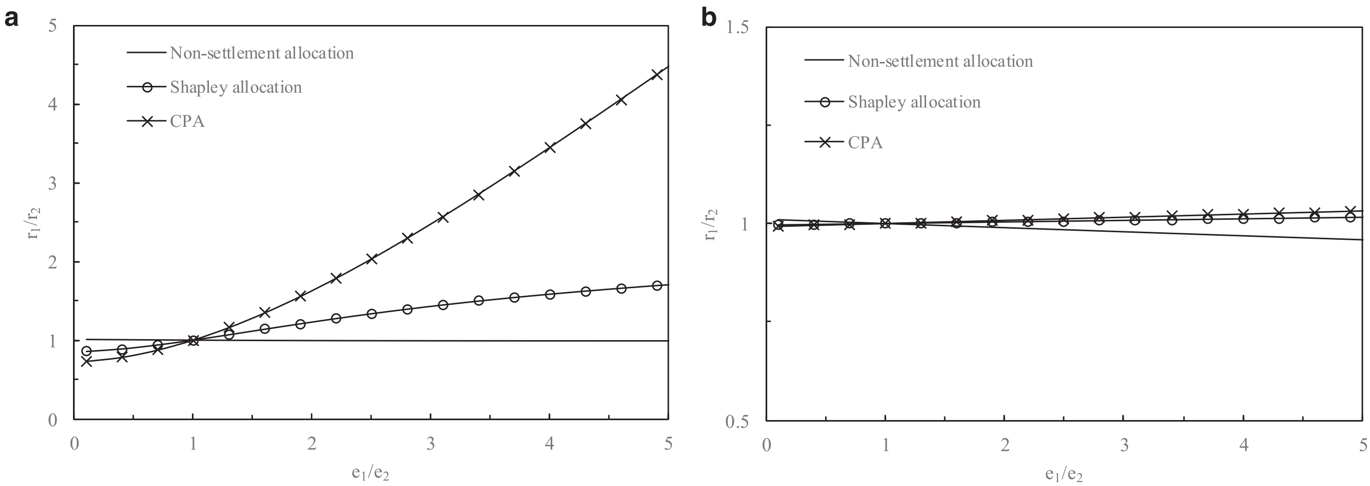
We conduct the same experiment to see the performance of different allocations in the model with linear network externality. In Fig. 5b, we can see that as the network size ratio of ISP_1 to ISP_2 increases, the profit ratios under all three allocations almost remain unchanged, with Shapley allocation and CPA showing an increasing trend and non-settlement allocation showing a decreasing trend. As the change in profit ratio is highly disproportionate to the change in network size ratio, and the difference between the profits of two ISPs is very small, we conclude the linear network externality is inadequate to reflect the network value. If we cannot properly recognize the value of network externality, it will be hard to devise any settlement rule that can appropriately reward the contributions ISPs make to the interconnection coalition, which will result in a situation where large ISPs are reluctant to peer with other ISPs.

6.3. Different internet network structures

To check the effectiveness of the profit allocations under different network structures, we carry out numerical experiments in the case of three ISPs, i.e., $N = \{ISP_1, ISP_2, ISP_3\}$, where the three ISPs have the same intrinsic demand potential. We are interested in the magnitude of percentage increase in profit for each ISP under three allocations respectively. The results of percentage increase in profit for each ISP in various network structures are presented in Table 1.

Table 1 reveals that no matter in the market where three ISPs are equal in network size, or in the market where one ISP is particularly large and the others are small, or in any other network structure, the profit for all three ISPs will increase considerably under all three allocations. Non-settlement allocation benefits ISPs with smaller network size the most, while the other two allocations favor ISPs with larger network size. Furthermore, CPA allocates more profit to ISPs with relatively large network than Shapely allocation.

These results indicate that CPA appreciates the value of network the most, and gives the most reward to ISPs with larger network among three allocations analyzed in this study. Thus, if the proposed settlement rule **s** that leads to CPA instead of non-settlement



$$* \ N = \{ISP_1, ISP_2\}, D_1 = D_2 = 100, e_2 = 10, \alpha = 0.8, \beta = 0.1, t = 0.1, c_0 = \frac{1}{3}t, c_t = \frac{2}{3}t.$$

Fig. 5. (a) Fairness of different allocations under quadratic network externality (b). Fairness of different allocations under linear network externality.

Table 1
Percentage increase in profit under three allocations in different network structures.

Network structure			Non-settlement allocation			CPA			Shapley allocation		
e_1	e_2	e_3	ISP_1	ISP_2	ISP_3	ISP_1	ISP_2	ISP_3	ISP_1	ISP_2	ISP_3
1	2	7	19.61	19.08	9.91	5.73	11.01	28.98	11.85	15.01	19.67
1	4	5	19.61	16.65	14.78	5.73	20.00	23.62	11.40	18.63	19.54
2.5	3.5	4	18.38	17.17	16.39	13.47	17.97	20.00	15.56	17.59	18.32
3.33	3.33	3.33	17.22	17.22	17.22	17.22	17.22	17.22	17.22	17.22	17.22
5	2.5	2.5	14.11	17.95	17.95	23.62	13.47	13.47	19.35	15.68	15.68
7.5	1.5	1	7.45	18.27	18.47	29.92	8.43	5.73	19.38	13.73	12.05

$^*N = \{ISP_1, ISP_2, ISP_3\}$, $D_1 = D_2 = D_3 = 100$, $\alpha = 0.8$, $\beta = 0.1$, $t = 0.1, c_0 = \frac{1}{3}t, c_t = \frac{2}{3}t$.

Table 2
Rules used to generate random parameters.

Price coefficient: α_1	$\alpha_1 \sim Uni[0.1, 1]$
Price coefficient: α_2	$\alpha_2 = \rho\alpha_1,\ \rho \sim Uni[0.1, 0.5]$
Network size coefficient: β	$\beta \sim Uni[0.1, 1]$
Unit data transmission cost: t	$t \sim Uni[0,\ 0.1]$
Intrinsic demand potential: **D**	$D_i \sim Uni[100, 2000]$
Network size: **e**	$e_i \sim Uni[5, 50]$

is implemented at NAP, ISPs will be motivated to expand their network to pursue higher profit.

6.4. Competitive market

In this subsection, we first carry out a numerical test to show that the total profit of two competing and interconnecting ISPs under proposed settlement rule **s** is close to the jointly optimal total profit. 200 problem instances are generated randomly. For each problem instance, the price coefficients, network size coefficient, unit data transmission cost, the intrinsic demand potential vector **D** and the network size vector **e** are randomly assigned. For each instance, we have $\alpha_2 \leq \frac{1}{2}\alpha_1$ to limit the degree of competition. Table 2 summarizes the rules to generate random parameters in the numerical test.

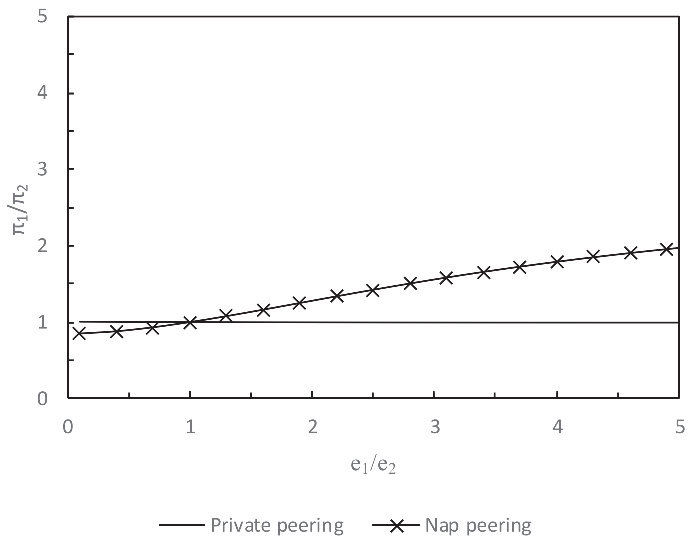
Table 3 gives the results of the numerical test. The "Average gap" measures the gap between the profit under settlement rule **s** and the optimal total profit on average over the 200 randomly generated instances. The "Median gap" and "80 percent percentile gap" list the median and the 80 percent percentile result among these instances respectively. For example, if the "80 percent Percentile" is 7 percent, it indicates that 80 percent of the instances have a gap no larger than 7 percent. The "Max gap" measures the maximum gap among these instances. The results in Table 3 demonstrate that the proposed settlement rule **s** performs well and can lead to a total profit close to optimum.

Table 3
The Gap between the profit under proposed settlement rule and the optimal profit.

Average gap	Median gap	80 percent percentile gap	Max gap
2.94 percent	1.95 percent	5.56 percent	10.55 percent

Then we illustrate the fairness performance of the proposed settlement rule **s** in competitive market. Profit ratio curves similar to that in Section 6.2 are used to reflect the changes in profit distribution among two competing ISPs as network size ratio increases. The results in Fig. 6 show that as network size ratio e_1/e_2 increases, the profit ratio π_1/π_2 decreases slightly when ISPs interconnect via private peering and no payments is paid between them. On the contrary, the profit ratio increases with network size ratio when ISPs interconnect via NAP and make decisions according to proposed settlement rule **s**. This suggests that our proposed settlement rule can transfer part of the benefits of interconnection from ISP with smaller network size to ISP with larger network size, so it can serve the competitive market as well by fairly apportioning the total profit to ISPs.

To sum up, the numerical experiments in this section verify that the proposed CPA can most effectively increase ISPs' profit and fairly distribute the total profit among three allocations analyzed in this study, and the corresponding proposed settlement rule **s** can



* $N = \{ISP_1, ISP_2\}, D_1 = D_2 = 100, e_2 = 10, \alpha = 0.8, \alpha_1 = 0.6, \alpha_2 = 0.2, \beta = 0.1, c_0 = \frac{1}{30}, c_t = \frac{1}{15}$

Fig. 6. Percentage increase in profit under private peering and NAP peering.

encourage ISPs to exchange traffic through NAP in a wide range of settings, including markets with different market structures and competitive market.

7. Conclusion

ISPs in a specific Internet market exchanging data through NAP can give rise to the welfare of the whole society. For the Internet users, they can visit more resources with shorter delay. For ISPs, as long as there is a rational settlement rule, exchanging data with other ISPs through NAP will also increase their profit. If there is no such settlement rule, ISPs' profit could be hindered, and would rather choose to operate alone or form small coalitions with some of the other ISPs. In our study, we have designed such a settlement rule **s**. For the whole society, if all the ISPs access to the NAP, the size of the total network will increase remarkably, and the value of the Internet will be enhanced.

This study analyzes the profit allocation among ISPs in a cooperative game framework, and proposes a Characterized Profit Allocation that has a much more brief and comprehensible formulation than Shapley-value based profit allocation, and can increase all ISPs' profit in all circumstances. Implementing the settlement rule **s** that induces ISPs to make jointly optimal pricing decisions and leads to Characterized Profit Allocation, the aim of encouraging ISPs to interconnect with each other through NAP can be achieved, and the network response speed and data exchange quality of the Internet can be greatly improved. We also show that with a little adjustment, the settlement rule $\mathbf{s}^{\mathbf{Q}}$ can lead to jointly optimal pricing and interconnection quality decisions when taking into account the asymmetric incentives of ISPs to provide interconnection quality. Our settlement rule **s** works well for a competitive market with two ISPs as well.

The theoretical contribution of this study is to introduce quadratic network externality and show that linear network externality is inappropriate in Internet interconnection context, and the CPA and the corresponding settlement rule **s** we propose is instructive for NAPs in operation across the world. For future research, as Internet users are becoming important resources themselves as we are entering an era of "We media", we suggest to include installed customer base in the demand function. Second, studies that extend our two-player competition to a general case should be fruitful. Practically, measurement items for network size also need to be specified to facilitate the enforcement of the settlement rule **s**.

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Appendix

A.1. Proof of Theorem 3

Theorem 3. *The non-settlement allocation is in the core of the game* $(N, \Pi^*(S))$.

Proof. As $\sum_{ISP_i \in N} r_{Ai} = \sum_{ISP_i \in N} \{ \frac{[D_i + \beta E_N^2 - \alpha(c_o + c_t)E_N][D_i + \beta E_N^2 + \alpha(c_o - c_t)E_N]}{4\alpha} - \frac{c_o e_i}{2} \sum_{ISP_j \in N} (D_j + \beta E_N^2 - \alpha t E_N)\} = \frac{\sum_{ISP_i \in N} (D_i + \beta E_N^2 - \alpha t E_N)^2}{4\alpha} = \Pi^*(N)$, r_A satisfies the efficiency principle. □

Then we rewrite the total allocation a set of ISPs can get from the grand coalition as $\sum_{ISP_i \in S} r_{Ai} = \frac{\sum_{ISP_i \in S} \{(D_i + \beta E_N^2 - \alpha t E_N)^2 + 2\alpha c_o [E_N (D_i + \beta E_N^2 - \alpha t E_N) - e_i \sum_{ISP_j \in N} (D_j + \beta E_N^2 - \alpha t E_N)]\}}{4\alpha}$

By observing $\Pi^*(S) = \frac{\sum_{ISP_i \in S} (D_i + \beta E_S^2 - \alpha t E_S)^2}{4\alpha}$ and $\sum_{ISP_i \in S} r_{Ai}$, we can conclude the sign of $\sum_{ISP_i \in S} r_{Ai} - \Pi^*(S)$ is not definite, and it depends on the value of e_i and D_i of ISPs in set S. Indeed, when ISPs have large e_i and small D_i, the profit allocation they can get from the grand coalition is less than what they can earn by deviating from the grand coalition and forming associated network by themselves. For example, when there are two ISPs in the market, and the parameters are as follows: $N = \{ISP_1, ISP_2\}$, $(e_1, e_2) = (10, 1)$, $(D_1, D_2) = (100, 1000)$, $\alpha = 0.8$, $\beta = 0.1$, $t = 0.1$, $c_0 = \frac{1}{3}t$, $c_t = \frac{2}{3}t$. In this setting, ISP_1's profit is

3726 when operating independently and 3699 when interconnecting with ISP_2 through NAP, which is 27 less than the independent profit. □

A.2. Proof of Theorem 4

Theorem 4. *The Characterized Profit Allocation is in the core of the game* $(N, \Pi^*(S))$ *and is a fair allocation rule.*

Proof. Characterized Profit Allcoation: $r_{Ci} = \frac{1}{4\alpha}[D_i^2 + e_i \sum_{ISP_j \in N} (\beta E_N - \alpha t)(2D_j + \beta E_N^2 - \alpha t E_N)]$.

As $\Pi^*(N) = \frac{\sum_{ISP_i \in N}(D_i + \beta E_N^2 - \alpha t E_N)^2}{4\alpha}$, which can be rewritten as $\frac{1}{4\alpha}[\sum_{ISP_i \in N} D_i^2 + \sum_{ISP_i \in N} E_N(\beta E_N - \alpha t)(2D_i + \beta E_N^2 - \alpha t E_N)]$, it's straight forward to see that $\sum_{ISP_i \in N} r_{Ci} = \Pi^*(N)$, so efficiency principle is satisfied.*NN*

Denote the smallest network size as e_{min}. According to our assumption that $2\beta e_i - \alpha t > 0$ for any $ISP_i \in N$, as long as $n \geq 2$, we have:

$$\beta E_N - \alpha t \geq n\beta e_{min} - \alpha t \geq 2\beta e_{min} - \alpha t > 0$$

so for any coalition $S \subseteq N$

$$\sum_{ISP_i \in S} r_{Ci} = \frac{1}{4\alpha}\left[\sum_{ISP_i \in S} D_i^2 + E_S \sum_{ISP_j \in N} (\beta E_N - \alpha t)\left(2D_j + \beta E_N^2 - \alpha t E_N\right)\right]$$

$$> \frac{1}{4\alpha}\left[\sum_{ISP_i \in S} D_i^2 + E_S \sum_{ISP_j \in S} (\beta E_S - \alpha t)\left(2D_j + \beta E_S^2 - \alpha t E_S\right)\right] = \Pi^*(S)$$

Thus, coalition rationality principle that $\sum_{ISP_i \in S} r_{Ci} \geq \Pi^*(S)$ is satisfied for any coalition $S \subseteq N$. Together with efficiency principle, we can conclude that $\mathrm{r_C}$ is in the core of the game.

As r_{Ci} is an increasing function of e_i, the ISP with larger network size can share a larger portion of profit under this allocation rule. Therefore, the CPA also preserves the fairness property. □

A.3. Proof of Theorem 5

Theorem 5. When ISPs make decisions independently, the equilibrium uplink quality, downlink quality and price are $\gamma_i^{U^*} = \underline{\gamma}$, $\gamma_i^{D^*} = 1$, $p_i^*(S) = \frac{D_i + \beta(\sum_{j \neq i} \underline{\gamma} e_j + e_i)^2 + \alpha c_o e_i + \alpha c_t E_S}{2\alpha}$ respectively.

Proof. As $\frac{\partial \pi_i(S)}{\partial \gamma_i^{\mathrm{U}}} = -c_o e_i^2 \sum_{ISP_j \in S,\ j \neq i} 2\beta \hat{y}_j < 0$, ISPs will set the equilibrium uplink quality $\gamma_i^{\mathrm{U}^*}$ at lower bound $\underline{\gamma}$.

Assume $p_i(S) \geq c_o e_i + c_t E_S$ for each ISP_i.

As $\frac{\partial \pi_i(S)}{\partial \gamma_i^{\mathrm{D}}} = (2\beta \underline{\gamma} \hat{y}_i \sum_{ISP_j \in S,\ j \neq i} e_j)[p_i(S) - c_o e_i + c_t E_S] \geq 0$, the equilibrium downlink quality $\gamma_i^{D^*}$ is the upper bound 1.

And we have $\frac{\partial \pi_i(S)}{\partial p_i(S)} = D_i - 2\alpha p_i(S) + \beta \hat{y}_i^2 + \alpha c_{oe_i} + \alpha c_t E_S$, $\frac{\partial^2 \pi_i(S)}{\partial p_i^2(S)} = -2\alpha$, so we use first order condition to obtain the equilibrium price $p_i^*(S)$. We can calculate that $p_i^*(S)$ is $\frac{D_i + \beta(\sum_{j \neq i} \underline{\gamma} e_j + e_i)^2 + \alpha c_o e_i + \alpha c_t E_S}{2\alpha}$, and the corresponding equilibrium demand $d_i^*(S)$ is $\frac{D_i + \beta(\sum_{j \neq i} \underline{\gamma} e_j + e_i)^2 - (\alpha c_o e_i + \alpha c_t E_S)}{2}$. Because demand should be no less than zero, $D_i + \beta(\sum_{j \neq i} \underline{\gamma} e_j + e_i)^2 - (\alpha c_o e_i + \alpha c_t E_S) > 0$. So $p_i^*(S) = \frac{D_i + \beta(\sum_{j \neq i} \underline{\gamma} e_j + e_i)^2 + \alpha c_o e_i + \alpha c_t E_S}{2\alpha} > c_o e_i + c_t E_S$, which is in accordance with our assumption $p_i(S) \geq c_o e_i + c_t E_S$. □

A.4. Proof of Theorem 6

Theorem 6. *The jointly optimal uplink and downlink quality decisions and pricing decision are* $\gamma_i^U = \gamma_i^D = 1$ *and* $p_i^*(S) = \frac{D_i + \beta E_S^2 + \alpha t E_S}{2\alpha}$ *respectively*.

Proof. Assume $p_i(S) \geq tE_S$ for each ISP_i.

We have $p_i(S) \in [tE_S, \frac{D_i + \beta E_S^2}{\alpha}]$ and $\gamma_i^{\mathrm{U}}, \gamma_i^{\mathrm{D}} \in [\underline{\gamma}, 1]$, so the feasible set $(\boldsymbol{\gamma_i^U}, \boldsymbol{\gamma_i^D}, \boldsymbol{p}) \in \mathcal{B} \subset \mathrm{R}^S \times R^S \times R^S$ is compact. Furthermore, as $\Pi(S) = \sum_{ISP_i \in S}[p_i(S)d_i(S) - td_i(S)E_S] = \sum_{ISP_i \in S}\{(p_i(S) - tE_S)[D_i - \alpha p_i(S) + \beta(\gamma_i^D \sum_{j \neq i} \gamma_j^U e_j + e_i)^2]\}$ is continuous on $\mathcal{B}$, we can conclude that a maximum exists on $\mathcal{B}$ with an appeal to the Weierstrass theorem. As $\frac{\partial \Pi(S)}{\partial \gamma_i^U} = 2\beta e_i \sum_{j \neq i} \gamma_j^D (\gamma_j^D \sum_{k \neq j} \gamma_k^U e_k + e_j)$ $[p_j(S) - tE_S] \geq 0$ and $\frac{\partial \Pi(S)}{\partial \gamma_i^U} = 2\beta(\gamma_i^D \sum_{j \neq i} \gamma_j^U e_j + e_i) \sum_{j \neq i} \gamma_j^U e_j$ $[p_j(S) - tE_S] \geq 0$ by assumption, $\Pi(S)$ is increasing in γ_i^U and γ_i^D, so the maximum is achieved at the boundary where $\gamma_i^U = \gamma_i^D = 1$ for $ISP_i \epsilon S$.

Substituting $\boldsymbol{\gamma^U} = \boldsymbol{\gamma^D} = 1$ to $\Pi(S)$, the maximization problem of $\Pi(S)$ is exactly the same as in section 4.1, so the optimal price is $\frac{D_i + \beta E_S^2 + \alpha t E_S}{2\alpha}$. As there are at least two ISPs in a coalition, $p_i^*(S) = \frac{D_i + \beta E_S^2 + \alpha t E_S}{2\alpha} \geq \frac{D_i + (2\beta e_{min} + \alpha t)E_S}{2\alpha} > \frac{D_i + 2\alpha t E_S}{2\alpha} > tE_S$, which is in accordance with our assumption $p_i(S) \geq tE_S$. □

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