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Research paper

# Modeling and predicting opinion formation with trust propagation in online social networks

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ABSTRACT

While many opinion models often concentrate on the stable state and phase transition of the dynamics, the temporal evolution pattern of public opinion rarely is investigated. If the variation of opinions with time can be formulated, it is probable to effectively characterize the intrinsic process or even predict the future trend of the global state. In this paper, we study a trust-aware voter model in which individual trust co-evolves with their opinions and the trust on a target agent propagates from common neighbors. When agents often have similar opinions with their neighbors, they may develop trust on the neighbors, and vice versa. Individual trust changes opinion interactions, and agents tend to adopt opinions of the neighbors they trust. Mean-field analysis and simulations are conducted to explore the transient opinion profiles of the model. Results prove that public opinion changes as an exponential mixture form in both homogeneous and heterogeneous networks. Most agents quickly stick to the majority opinion, and the conservation of magnetization is broken. Indirect trust promotes opinion interactions and drives the system towards consensus. In addition, we conduct empirical experiments on topic discussions of a real-world network, and the results show that the model well fits and predicts opinion formation over online social networks.



## 1. Introduction

The formation of collective human behaviors in social systems has attracted wide attention in the last few decades. As a typical representative, opinion dynamics has been studied through the methods of statistical physics, applied probability and graph theory, to investigate how local interactions change the macroscopic state [1–3]. Opinion models have been used to simulate the evolution process and describe social phenomena, such as the voter model [4–6], the majority model [7,8] and the Sznajd model [9–11]. The system may reach different regimes, i.e., consensus, polarization or fragmentation [12–14]. The underlying topology plays a significant role in opinion dynamics [15]. In recent years, heterogeneous individual characteristics have been introduced to opinion dynamics, greatly changing local behaviors and the stable global state [16, 17]. Individual inertia makes agents reluctant to change their opinions. The inertia slows the microscopic dynamics but accelerates the macroscopic dynamics [18]. The phase transition from a disordered state to an ordered state is affected by individual influence [19] and personal conviction [20]. When agents with different confidence levels are considered, the crit-

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ical confidence above which a single opinion cluster is formed correlates with the differentiation of confidence levels [21]. In actual systems, when meeting a neighbor, agents consider whether the neighbor is trustworthy before they start the conversation. Many studies have addressed the problem of individual trust in opinion models [22,23]. Further, empirical studies prove that individual trust is closely related to agents' past interactions [24]. Opinion models often describe the direct trust on the target agent, while the evaluation of common neighbors is neglected. In this paper, we explore trust propagation during interactions. Both self-trust and interpersonal trust are included in the dynamics, and the indirect evaluation of common neighbors has a important role in opinion updates.

With the extension of opinion models, real data in social life or online social media are used to verify opinion models. New properties and novel conclusions are determined by empirical analysis from actual networks [25]. The study of online political discussions showed that online opinion exchanges do not lead to consensus, implying that a substantially different process may be involved in online discussions [26]. The intensity of personal conflict among users advances the growth of discussions, in contrast to link growth in scientific collaboration networks. In Ref. [27], an opinion model was presented to describe discussions in online forums. In the model, negative emotions boost users' activity and make them take actions energetically. The model reproduces several essential characteristics of real networks. In addition, individual opinions evolve very quickly in online social networks, and a few users maintain the minority opinion during the interaction [28]. Users' participation activity and inter-event time were found to be heterogeneous [29]. These studies indicate that actual interactions may be different from theoretical models and that distinct individual behaviors exist in social systems. Therefore, large-scale real data should be used to validate the models, providing a realistic description and explanation of social phenomena.

Most recent studies have concentrated on the stable state and phase transition of opinion dynamics, and they have investigated the dependence on parameters in detail to give a clear illustration of macroscopic dynamics. However, the evolution pattern of the collective opinion generally has been ignored. This suggests further exploration concerning how individual opinions change with time and how rapidly the system levels off in real social networks. In Ref. [30], an opinion model characterizing the election procedure was presented, and the results proved that transient opinion configurations generated by the model perform in statistical agreement with the data from Finland's election in 2003. The empirical confirmation of opinion dynamic configurations is relatively rare. To justify the credibility of the temporary results of models, big data in real social systems are required to determine the concrete form of opinion evolution. If we grasp the pattern by which opinions evolve, we can describe the dynamic system more precisely, and we even may be able to use early data to predict the future trend and the stable state of public opinion. In this paper, we investigate a trust-aware voter model in which opinion interactions correlate with individual trust. Individual trust depends on past opinion updates, but conversely, it affects the dynamics. As individual opinions about a certain subject change, so do their opinions about the person who influences them. In reverse, it is also true that the same information can differently be interpreted depending on its source [31]. If an agent often has a similar opinion with its neighbors, it may develop trust on the neighbors and like to interact with them. On the contrary, if neighbors of the agent are often against it, it will decrease interactions with them. For instance, hyperlinks between blogs in online blogging websites can be regarded as indications of trust among bloggers. Hyperlinks are created on the basis of bloggers' past acquaintance with neighbors [32]. Once hyperlinks exist, they influence interactions among bloggers. Then, we conduct mean-field analysis and numerical simulations in regular and heterogeneous networks, showing that the evolution of the average opinion follows an exponential mixture pattern. Empirical experiments are implemented to validate the theoretical model using real-world data from online social networks.

The rest of the paper is structured as follows. Section 2 presents the voter model with individual trust propagation. Mean-field analysis is presented in Section 3. Section 4 provides simulation and empirical results of the model. Our conclusions are presented in Section 5.

## 2. The model

### 2.1. Opinion model with individual trust

Assume that agents can hold either of two opinions, i.e., $\sigma = 0$ or $\sigma = 1$. Therefore, the average opinion $O(t)$ equals the overall density $f(t)$ of opinion 1. Here, $t$ means time. In the voter model, an agent during an update selects one of its neighbors at random, and adopts the neighbor's opinion. Each neighbor has the same influence on agents, and they contribute equally to individual opinions. The average magnetization is conserved, and the final average opinion $O(\infty)$ at $t\to\infty$ equals the initial average opinion $O(0)$ at $t = 0$. However, in actual situations, neighboring influence is often heterogeneous, and agents prefer to talk with neighbors that they trust. Therefore, individual trust plays a significant part in opinion interactions, and agents update their opinions following different neighbors with different probability. We will introduce our opinion model with individual trust in the following.

We consider the opportunity that an agent changes its opinion following a neighbor correlates positively with the trust on the neighbor. In an update, an agent $i$ and one of its neighbors $j$ are selected at random, and then agent $i$ adopts $j$'s opinion according to the individual trust. For agent $i$, its trust on $j$ at time $t$ is defined as $tr_{i\to j}(t)$, and the trust on itself at time $t$ is given by $tr_{i\to i}(t)$. The probability of changing its opinion for agent $i$ at time $t$ is

$$p = \frac{tr_{i\to j}(t)}{tr_{i\to j}(t) + tr_{i\to i}(t)} \tag{1}$$

Therefore, if agent $i$ is deeply convinced by agent $j$, it is more likely to accept $j$'s opinion; when agent $i$ develops strong confidence for its own opinion, it tends to keep its current opinion. Ref. [33] presented a measure of individual trust among users in online social networks, considering information published by users. Individual trust can be calculated by analyzing the sentiments of users' posts, and it increases monotonously with the proportion of positive comments. If an agent publishes a negative opinion frequently toward a neighbor, it generates strong distrust on the neighbor, and vice versa. Here, we consider that the evaluator $i$'s interpersonal trust on the target agent $j$ at time $t$ is

$$tr_{i\to j}(t) = \frac{pos_{i,j}(t)}{pos_{i,j}(t) + neg_{i,j}(t)} \tag{2}$$

where $pos_{i,j}(t)$ is the number of times evaluator $i$ adopts $j$'s opinion until time $t$, and $neg_{i,j}(t)$ is the number of times evaluator $i$ keeps its opinion during interactions with $j$ until time $t$. Note that the denominator of Eq. (2) is 0 before agent $i$ interacts with agent $j$. We give the default value of interpersonal trust by $\varepsilon$ ($0 < \varepsilon \le 1$), so that, in the beginning, all interpersonal trust is set to $\varepsilon$. The parameter represents the social credit in a system. It cannot be set to 0, since in that situation, agents initially have zero trust on any neighbor. Then, agents always maintain their original opinions, and the dynamics is frozen. From Eq. (2), interpersonal trust does not remain constant, but it undergoes an evolution process.

The trust of evaluator $i$ on itself can be measured according to the conviction towards it own opinion. Self-trust $tr_{i\to i}(t)$ is reinforced when agent $i$'s opinion is approved by its neighbors. We define $tr_{i\to i}(t)$ as:

$$tr_{i\to i}(t) = \frac{pos_i(t)}{pos_i(t) + neg_i(t)} \tag{3}$$

where $pos_i(t)$or $neg_i(t)$ is the number of times agent $i$ holds the same or a different opinion from its neighbors during interactions until time $t$. In self-trust, all of the interactions with agent $i$ are taken into consideration. Similarly to $\varepsilon$, we define the default value of self-trust as $\delta$ ($0 \le \delta \le 1$). The parameter $\delta$ can take the value of 0, meaning that, in the beginning, all agents follow their neighbors' opinions absolutely.

From the above model, it is realized that agents with weak self-trust are likely to adopt the opinions of neighbors that they trust. Once an agent forms strong self-trust, the probability of updating its opinion does not exceed 0.5.

In our model, individual trust is calculated according to accumulative opinion updates during the evolution process. Since agents have the memory of past interactions, their actions in the evolution can be recorded and will influence their future decisions. Ref. [33, 34] measured social trust by analyzing users' comments on neighbors' information. Individual trust was defined in terms of the difference between the number of positive and negative comments (i.e., like and dislike). Considering the memory effect, individual trust at time $t$ was accumulated over a period of time before $t$. Large-scale empirical data were used to validate the concrete form of individual trust through machine learning methods, and it was found that individual trust reflects users' relationships and affects users' behaviors. Experiment results proved that the definition of social trust can predict users' relationships effectively. Inspired by their work, we define individual trust as the accumulative proportion of past actions, and make the trust no less than 0. Here, we assume each agent can remember all of its opinion updates.

Our model is similar to the Ising model, since agents change their opinions with the probability that depends on neighboring opinions. In the Ising model, an agent is influenced by all of its neighbors and the whole system, and its decision correlates with the current states of other agents. However, in our model, individual trust exists between any pair of agents, and an agent changes its opinion following a neighbor according to its trust only on the neighbor. Moreover, individual trust is determined by accumulative opinion updates.

### *2.2. Trust propagation in the opinion model*

In actual situations, when agents talk with others, they consider the direct trust on the target agent and take the evaluation of its neighbors into account. If agent $i$ trusts agent $u$, and agent $u$ trusts agent $j$, then agent $i$ is likely to trust agent $j$. Therefore, interpersonal trust propagates among agents, and indirect trust also affects opinion interactions. If agent $i$ interacts with agent $j$, the trust $tr_{i\to j}(t)$ contains the direct trust $dtr_{i\to j}(t)$ and the indirect trust $indtr_{i\to j}(t)$ at time $t$. The direct trust can be calculated by Eq. (2). To measure the indirect trust, the evaluation of common neighbors toward the target agent $j$ should be included. In Ref. [35], a real social network was used to explore trust propagation among users, and the results demonstrated the multiplicative transitivity of trust. The study proved the phenomenon that is often stated as 'The friend of my friend is my friend, and the enemy of my friend is my enemy'. Thus, we define the indirect trust of evaluator $i$ on $j$ at time $t$ as follows:

$$indtr_{i\to j}(t) = \sum_{u\in\Gamma(i,j)} dtr_{i\to u}(t)\cdot dtr_{u\to j}(t) \tag{4}$$

where $\Gamma(i, j)$ is the set of common neighbors for agent $i$ and $j$, and $dtr_{i\to u}(t)$ means the direct trust of evaluator $i$ on $u$ at time $t$. Note that the value of $indtr_{i\to j}(t)$ may be above 1, so we normalize it and obtain the total interpersonal trust of agent $i$ on $j$.

$$tr_{i\to j}(t) = \frac{dtr_{i\to j}(t) + indtr_{i\to j}(t)}{\sum_{u\in\Gamma(i,j)} dtr_{i\to u}(t) + 1} = \frac{dtr_{i\to j}(t) + \sum_{u\in\Gamma(i,j)} dtr_{i\to u}(t)\cdot dtr_{u\to j}(t)}{\sum_{u\in\Gamma(i,j)} dtr_{i\to u}(t) + 1} \tag{5}$$

The value 1 of the denominator of Eq. (5) accounts for the normalization weight of direct trust on $j$. Assuming $dtr_{i\to i}(t) = 1$, Eq. (5) can be simply denoted by the following:

$$tr_{i\to j}(t) = \frac{\sum_{u\in\Gamma(i,j)\cup i} dtr_{i\to u}(t)\cdot dtr_{u\to j}(t)}{\sum_{u\in\Gamma(i,j)\cup i} dtr_{i\to u}(t)} \tag{6}$$

It should be noticed that we introduce the notation $dtr_{i\to i}(t)$ just for the simplification of Eq. (6), and $dtr_{i\to i}(t) = 1$ only holds true here.

## 3. Theoretical analysis

### 3.1. Without indirect trust

We make an analytical approach of the opinion model. We assume that the underlying network is a regular network in which each agent has the same degree, $k$. First, we do not consider self-trust, and investigate the average magnetization. Under this condition, agents adopt a neighbor's opinion with the probability that equals the direct trust. In the mean-field assumption, an agent meets a neighbor holding opinion 1 at time $t$ with the probability $f(t)$ that is the overall density of opinion 1. Also, an agent has concordant interactions with all of the neighbors, so the trust of the agent on each neighbor is identical. However, different agents may update their opinions at different times. To describe interpersonal trust, we divide the population into different groups according to the opinion update frequency until time $t$, i.e., $pos(t)$. Noting that the frequency satisfies $pos(t) \le t$, we label the groups by $pos(t) = 0, 1, 2, \cdots, t$, so agents in group $l$ have the update frequency $pos(t) = l$ where $l$ is a nonnegative integer. Therefore, the trust $tr_{i\to j}(t)$ of evaluators in group $l$ is simply written as $tr_l(t)$. The density of agents in group $l$ at time $t$ is given by $\rho_l(t)$. When an agent in group $l$ meets a neighbor at time $t$, the probability that the agent will adopt the neighbor's opinion is

$$tr_l(t) = \frac{pos(t)}{pos(t)+neg(t)} = \frac{l}{t-1} \tag{7}$$

where $t-1$ is the average number of interactions for each agent until time $t$.

Analogously, the density $f(t)$ should be calculated for different groups. We define the overall density of opinion 1 in group $l$ at time $t$ as $f_l(t)$. In the mean-field approach, the density satisfies $f_l(t) = \rho_l(t)\cdot f(t)$.

We focus on the time variation of $f(t)$. $f(t)$ increases when agents with opinion 0 update to opinion 1 following neighbors, but deceases when agents holding opinion 1 change their opinion. The probability of changing opinions depends on the update frequency, and therefore, agents in different groups should be considered separately when calculating $f(t)$. In the aspect of increase for $f(t)$, agents holding opinion 0 in group $l$ change their opinion to opinion 1 with the probability of $f(t)\cdot l/(t-1)$ at time $t$, and their density is $\rho_l(t) - f_l(t)$. Similarly, in the aspect of decrease, an agent holding opinion 1 in group $l$ changes to opinion 0 with the probability of $(1-f(t))\cdot l/(t-1)$. Therefore, the variation of $f(t)$ can be identified.

$$df(t)/dt = \sum_{l=0}^{t-1}\left((\rho_l(t) - f_l(t))f(t)\frac{l}{t-1} - f_l(t)(1-f(t))\frac{l}{t-1}\right) \equiv 0 \tag{8}$$

Without self-trust, the average opinion remains constant, indicating the conservation of magnetization.

Then, the effect of self-trust is included in the dynamics, and agents change their opinions in terms of self-trust and direct interpersonal trust. The self-trust of an agent is related to the number of times its opinion is approved by its neighbors. If some difference exists in the densities of the two opinions, the probability of being approved is different for these two opinions. Therefore, we define $n_1(t)$ and $n_0(t)$ as the number of times opinion 1 or opinion 0 is approved by neighboring agents until time $t$, on average. For agents holding opinion 1, their self-trust in the mean-field limit is $n_1(t)/(t-1)$. At time $t$, the probability of being approved for opinion 1 is $f(t)$, so $n_1(t)$ and $n_0(t)$ can be calculated as follows:

$$\begin{aligned} n_1(t) &= \int_0^{t-1} f(\tau)d\tau \\ n_0(t) &= \int_0^{t-1} (1-f(\tau))d\tau \end{aligned} \tag{9}$$

Opinion updates continue according to Eq. (1). An agent holding opinion 1 in group $l$ follows a neighbor's opinion with the probability $l/(n_1(t)+l)$, and the probability for opinion 0 is $l/(n_0(t)+l)$. Analogously to Eq. (8), it is easy to obtain the variation of $f(t)$ under the impact of self-trust. After some algebraic manipulations, we have the following equation:

$$df(t)/dt = f(t)(1-f(t))\sum_{l=0}^{t-1}\rho_l(t)\left(\frac{l}{n_0(t)+l} - \frac{l}{n_1(t)+l}\right) \tag{10}$$

If the initial density of opinion 1 is greater than 0.5, the number of approval times yields $n_1(t) > n_0(t)$, and thus, we have $df(t)/dt > 0$. For $f(0) < 0.5$, we have $\partial f(t)/\partial t < 0$. Therefore, the conservation of magnetization is broken. From Eq. (10), we find that the stationary points are $f(t) = 0$ and $f(t) = 1$. Now we analyze the stability of these stationary points. If we let $f'(t)$=$df(t)/dt$, the stable stationary points should satisfy $df'(t)/df(t) < 0$ at $f(t) = 0$ and $f(t) = 1$. Thus, at these two stationary points, we have the constraint.

$$(0.5 - f(t)) \sum_{l=0}^{t-1} \rho_l(t) \left( \frac{l}{n_0(t) + l} - \frac{l}{n_1(t) + l} \right) < 0 \tag{11}$$

For $f(0) < 0.5$, the stable solution is $f(t) = 0$, and, for $f(0) > 0.5$, the stable solution is $f(t) = 1$. However, note that when agents do not update their opinions at the first time, their direct trust on others becomes 0 and it does not change during the evolution process. The opinions of these agents do not evolve. Therefore, the population can be divided into two parts, i.e., the evolving and non-evolving categories. In the evolving category, all agents change to the same opinion, and consensus is achieved. In the non-evolving category, the dynamics is frozen, so the minority opinion exists.

Now, we calculate the variation of $\rho_l(t)$. $\rho_l(t)$ increases when agents in group $l$-1 change their opinions following their neighbors and move to group $l$. Agents holding different opinions should be considered separately since they have different self-trust, so the update probability for agents in group $l$-1 is $f(t) \cdot (l-1)/(n_1(t) + l - 1)$+$(1 - f(t)) \cdot (l-1)/(n_0(t) + l - 1)$. Meanwhile, $\rho_l(t)$ decreases when agents in group $l$ update their opinions and move to group $l+1$. Moreover, in an update event, an agent with degree $k$ interacts with only one of its neighbors and changes the trust on the neighbor, so the update rate of interpersonal trust is $1/k$. Then, the variation equation of $p_l(t)$ is obtained as follows:

$$\begin{aligned} dp_l(t)/dt &= \rho_{l-1}(t)\frac{f(t)}{k}\frac{l-1}{n_1(t)+l-1} + \rho_{l-1}(t)\frac{(1-f(t))}{k}\frac{l-1}{n_0(t)+l-1} \\ &\quad -\rho_l(t)\frac{f(t)}{k}\frac{l}{n_1(t)+l} - \rho_l(t)\frac{(1-f(t))}{k}\frac{l}{n_0(t)+l}, l = 2, \cdots, t-1 \end{aligned} \tag{12}$$

From Eq. (12), the variation of $p_l(t)$ contains four parts. The first term $\rho_{l-1}(t)f(t)(l-1)/(k(n_1(t)+l-1))$ means the density of agents with opinion 1 in group $l-1$ that move to group $l$ at time $t$, where $\rho_{l-1}(t)f(t)$ denotes the density of agents holding opinion 1 in group $l-1$, $1/k$ denotes the update rate of interpersonal trust, and $(l-1)/(n_1(t)+l-1)$ denotes the probability of these agents updating their opinion. The second term $\rho_{l-1}(t)(1-f(t))(l-1)/(k(n_0(t)+l-1))$ means the density of agents with opinion 0 in group $l-1$ that move to group $l$. The third term $\rho_l(t)f(t)l/(k(n_1(t)+l))$ indicates that agents holding opinion 1 in group $l$ move to group $l+1$, where $l/(n_1(t)+l)$ means the probability that agents with opinion 1 in group $l$ *update* their opinion. The last term accounts for that agents with opinion 0 in group $l$ move to group $l+1$.

Noticing the density of agents in group $l = t$ only increases at time $t$, the variation is

$$d\rho_l(t)/dt = \rho_{l-1}(t)\frac{f(t)}{k}\frac{l-1}{n_1(t)+l-1} + \rho_{l-1}(t)\frac{(1-f(t))}{k}\frac{l-1}{n_0(t)+l-1}, l = t \tag{13}$$

For group 0, some agents update their opinions and enter group 1, while the others keep their opinions unchanged and update their interpersonal trust to 0. Once the interpersonal trust is 0, the agents become non-evolving agents. As mentioned above, when agents have not participated in interactions with others yet, the initial value of interpersonal trust is $\varepsilon$, so we have the following special case:

$$\begin{aligned} dp_l(t)/dt &= \rho_{l-1}(t)\frac{f(t)}{k}\frac{\varepsilon}{n_1(t)/(t-1)+\varepsilon} + \rho_{l-1}(t)\frac{(1-f(t))}{k}\frac{\varepsilon}{n_0(t)/(t-1)+\varepsilon} \\ &\quad -\rho_l(t)\frac{f(t)}{k}\frac{l}{n_1(t)+l} - \rho_l(t)\frac{(1-f(t))}{k}\frac{l}{n_0(t)+l}, l = 1 \\ d\rho_l(t)/dt &= -\rho_l(t)/k, l = 0 \end{aligned} \tag{14}$$

From the last equation of Eq. (14), we obtain $\rho_0(t) = e^{-t/k}$. The density of agents in group 0 decreases exponentially, and some of them enter the non-evolving category. Based on the equations above, we conclude that the dynamics does not rely on the default value of self-trust,$\delta$, since all agents generate their self-trust after the first interaction. Thus, we have two time scales for the evolution of trust, and the dynamics of self-trust is much faster than that of interpersonal trust. The densities of non-evolving agents holding opinion 0 or opinion 1 at time $t$ are given by $h_{non0}(t)$and $h_{non1}(t)$, respectively. The densities evolve as follows:

$$\begin{aligned} dh_{non0}(t)/dt &= e^{-t/k}\frac{(1-f(t))}{k}\left(1 - \frac{\varepsilon}{n_0(t)/(t-1)+\varepsilon}\right) \\ dh_{non1}(t)/dt &= e^{-t/k}\frac{f(t)}{k}\left(1 - \frac{\varepsilon}{n_1(t)/(t-1)+\varepsilon}\right) \end{aligned} \tag{15}$$

When $f(t) > 0.5$, $h_{non0}(t)$ represents the minority opinion that prevents the total consensus and cannot be invaded, and vice versa. When the degree $k$ is small, the stable solution of Eq. (15) is $e^{-t/k} \approx 0$, but, when $k$ is large enough, the stable

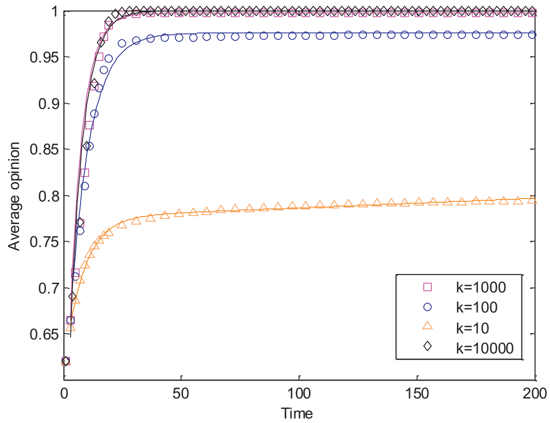



**Fig. 1.** Time evolution of $f(t)$ given by Eqs. (12, 15, 16). The initial density of opinion 1 is 0.6, and $\varepsilon = 0.2$. The discrete symbols denote numerical solutions of mean-field equations, while the real lines represent fittings by exponential mixture functions.

solution is $f(t) \approx 0$ or $f(t) \approx 1$. Non-evolving agents do not update their opinions, but they propagate their opinions to neighbors. In terms of non-evolving agents, the evolution of $f(t)$ is modified as:

$$df(t)/dt = (1 - f(t) - h_{non0}(t))f(t)\sum_{l=0}^{t-1}\rho_l(t)\frac{l}{n_0(t)+l} - (f(t) - h_{non1}(t))(1 - f(t))\sum_{l=0}^{t-1}\rho_l(t)\frac{l}{n_1(t)+l} \quad (16)$$

Considering the special case $k \to \infty$, the underlying network is treated as a complete graph. Then, we have $\rho_0(t) \approx 1$, $h_{non0}(t) \approx 0$ and $h_{non1}(t) \approx 0$. Therefore, for $f(t) > 0.5$ Eq. (16) is approximated by:

$$df(t)/dt = f(t)(1 - f(t))e^{-t/k}/(1+\varepsilon) \quad (17)$$

By solving the equation, it is evident that $f(t)$ in Eq. (17) changes as an exponential mixture function. Furthermore, we calculate the numerical solutions of Eqs. (12, 15, 16) in other cases of $k$, as shown in Fig. 1. Here, we assign the parameter $\varepsilon$ and $k$ in the beginning, and set the initial average opinion at $f(0) = 0.6$. We have the initial condition $p_l(0) = 1$ for $l = 0$, and $p_l(0) = 0$ for any other value of $l$. Substituting the condition into Eqs. (12, 15, 16), we calculate the evolution of these variables with time, and obtain the numerical solutions. Obviously, the time evolution of average opinion can be fitted by

$$f(t) = \alpha_1 \cdot e^{\lambda_1 t} + \alpha_2 \cdot e^{\lambda_2 t} \quad (18)$$

where $\alpha_1$, $\alpha_2$, $\lambda_1$ and $\lambda_2$ are parameters for an evolutionary process. The parameters of exponential fittings for $k = 10000$ are $\alpha_1 = 0.9997, \alpha_2 = -0.6039$, and $\lambda_1 = 0, \lambda_2 = -0.1783$. Those for $k = 1000$ are $\alpha_1 = 0.9971, \alpha_2 = -0.5842$, and $\lambda_1 = 0, \lambda_2 = -0.1882$. The parameters for $k = 100$ are $\alpha_1 = 0.9757, \alpha_2 = -0.444$, and $\lambda_1 = 0, \lambda_2 = -0.1214$, while for $k = 10$ they are $\alpha_1 = 0.7759, \alpha_2 = -0.1721$, and $\lambda_1 = 0.0001313, \lambda_2 = -0.1242$. The root mean square errors of fittings for $k = 10000$, $k = 1000$, $k = 100$, and $k = 10$ are 0.001878, 0.002298, 0.006206, and 0.001774 respectively. The system reaches an ordered state in which one opinion predominates, and the advantage of the initial majority opinion is enhanced in the dynamics. The dynamics with large $k$ is driven towards the consensus state, in accordance with Eq. (17). The average opinion for $k = 10000$ and that for $k = 1000$ are approximately the same, implying the system becomes independent of $k$ when $k$ is large enough. We also find that the system for $k = 10000$ achieves consensus a litter earlier. Decreasing $k$ leads to more non-evolving agents, and thus, the total consensus is prevented.

If the underlying network is a heterogeneous network, the densities of opinion 1 at different degrees are inhomogeneous. The proportion of opinion 1 belonging to degree $k$ is defined as $f(k, t)$. Considering uncorrelated networks, the probability $P(k'|k)$ that an agent with degree $k$ connects to another agent with degree $k'$ can be written as $P(k'|k) = k'P(k')/\bar{k}$. $P(k)$ means the degree distribution, and $\bar{k}$ denotes the average degree of the network. Agents with degree $k$ are affected by neighbors with different degrees, and the average density of neighbors holding opinion 1 at time $t$ is $\sum_{k'} k'P(k')f(k',t)/\bar{k}$. Similarly to Eq. (10), the evolution of $f(k, t)$ can be calculated as:

$$\partial f(k,t)/\partial t = \frac{(1 - f(k,t))}{\bar{k}}\sum_{k'} k'P(k')f(k',t)\sum_{l=0}^{t-1}\rho_l(t)\frac{l}{n_0(t)+l} - \frac{f(k,t)}{\bar{k}}\sum_{k'} k'P(k')(1 - f(k',t))\sum_{l=0}^{t-1}\rho_l(t)\frac{l}{n_1(t)+l} \quad (19)$$

From Eq. (19), all factors on the right side of the equation except $f(k, t)$ and $1-f(k, t)$ are independent of $k$. Therefore, it is easy to infer that the variation of $f(k, t)$ does not rely on the degree $k$ if the initial proportion of $f(k, t)$ is the same for all degrees. The evolution of $\rho_l(t)$ for group 0 in inhomogeneous networks is modified as :

$$d\rho_l(t)/dt = -\rho_l(t)\sum_k P(k)/k, l = 0 \tag{20}$$

Thus, $\rho_0(t)$ is obtained as $\rho_0(t) = e^{-t\sum_k P(k)/k}$. By integrating Eqs. (19, 20), it is evident that the evolution of the average opinion in heterogeneous networks also follows an exponential mixture pattern.

### *3.2. With indirect trust*

When indirect trust takes effect, the evaluation of common neighbors on the target agent is taken into account by evaluators. In this situation, even if the direct trust of an agent is 0, it will not immediately become non-evolving, since it also changes its opinion according to the indirect trust. Therefore, an agent enters the non-evolving category on the condition that both direct trust and indirect trust decrease to 0. From Eq. (4), when $indtr_{i\to j}(t) = 0$, at least one of the equations $dtr_{i\to u}(t) = 0$ and $dtr_{u\to j}(t) = 0$ is satisfied for any common neighbor, $u$. From Eq. (15), agents holding opinion 1 in group 0 update their direct trust to 0 with the probability of $1 - \varepsilon/(n_1(t)/(t-1)+\varepsilon)$. Assume that each pair of agents has $kc$ common neighbors, and $kc$ is the number of common neighbors. Under the effect of indirect trust, agents holding opinion 1 in group 0 become non-evolving with the probability

$$p = \left(1 - \frac{\varepsilon}{n_1(t)/(t-1)+\varepsilon}\right)\left(1 - \left(\frac{\varepsilon}{n_1(t)/(t-1)+\varepsilon}\right)\left(\frac{\varepsilon f(t)}{n_1(t)/(t-1)+\varepsilon} + \frac{\varepsilon(1-f(t))}{n_0(t)/(t-1)+\varepsilon}\right)\right)^{kc} \tag{21}$$

The first factor of Eq. (21) refers to direct trust, while the second factor accounts for indirect trust. When the average degree $\bar{k}$ of the network is small, consensus is still prevented by non-evolving agents, and the local dynamics may be frozen. The density of the minority opinion increases as $kc$ decreases. For large values of $kc$, all agents change to the majority opinion, and there is no doubt that the consensus state is promoted.

## 4. Simulation results

We conduct Monte–Carlo simulations to explore the dynamics of the voter model with individual trust, and we concentrate on the evolution pattern of public opinion. At the beginning of simulations, individual opinions are assigned uniformly with a given ratio between two opinions. Simulations are implemented asynchronously. In an update event, an agent and one of its neighbors are selected randomly, and the agent updates its opinion according to its self-trust and interpersonal trust on the neighbor. Then, the agent updates its trust in terms of the opinion interaction. Time is increased by 1 after $N$ update events when the system size is $N$. Simulation results are divided into two subsections. First, we use synthetic networks as interaction topology to investigate the dynamic trend of average opinion and essential characteristics of the model. Then, we analyze real data to verify the model and to determine whether it is possible to predict the evolution trend.

### *4.1. Synthetic networks*

Fig. 2 illustrates the evolution of the average opinion in regular networks. The average opinion varies following an exponential mixture form, in accordance with the analytical results. The root mean square errors of the fittings for $k = 2000$, $k = 1000$, $k = 100$ and $k = 10$ are 0.003706, 0.005219, 0.005632, and 0.003044, respectively. The density of the initial majority opinion increases rapidly towards consensus in the early stage, and it levels off after a while. Finally, one opinion takes the absolute majority. We notice that the relaxation process in simulations is slower than that in the mean-field approach, and the final average opinion in simulations is a little smaller, because random fluctuations exist in a finite-sized system. The difference can be diminished by increasing the size of the system. From Eq. (15), smaller values of $\varepsilon$ and $k$ cause more non-evolving agents, and therefore, the final density of the majority opinion becomes smaller. The total consensus cannot be reached for $k < 1000$, especially for $k = 10$. The system with $k$=2000 achieves consensus earlier than that with $k = 1000$. We also find large values of $\varepsilon$ make the relaxation process much slower. In the dynamics, agents holding opinion 1 in group 0 update their opinion with the probability of $\varepsilon/(n_1(t)/(t-1)+\varepsilon)$. When opinion 1 is the majority opinion, the self-trust $n_1(t)/(t-1)$ for opinion 1 approaches 1. In this situation, if $\varepsilon$ is small, agents holding the majority opinion in group 0 hardly change their opinion. However, for large values of $\varepsilon$, the probability of opinion updates is relatively large, and many agents in group 0 will also change to the minority opinion, leading to large fluctuations. Therefore, for large values of $\varepsilon$, the dynamics requires a long relaxation time.

Fig. 3 shows the effect of indirect trust on the final average opinion after 1000 time steps. A Barabasi–Albert scale-free and Watts-Strogatz small-world network with $\bar{k} = 10$ are used as interaction topology. From Fig. 3, it is clear that non-conservative magnetization exists in the dynamics with individual trust in both networks, especially when indirect trust is included in interactions. When $f(0) < 0.2$ or $f(0) > 0.8$, the consensus state almost is achieved. Individual trust changes target agents in local discussions, and makes agents choose the ones they trust for interactions. Therefore, initial minority opinion

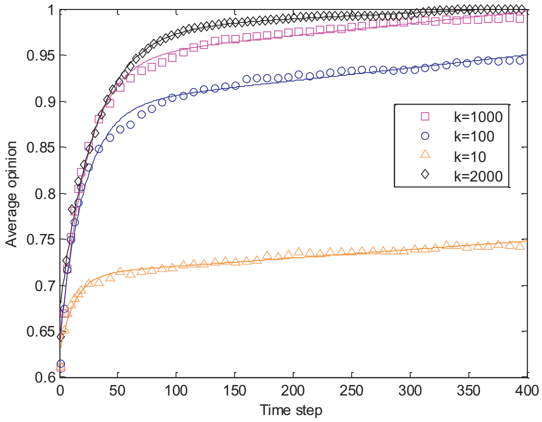



**Fig. 2.** Time evolution of $f(t)$ by simulations. The initial density of opinion 1 is 0.6, and $\varepsilon = 0.2$. The discrete symbols denote simulation results, while the real lines represent fittings by exponential mixture functions. The results are averaged over 50 realizations.

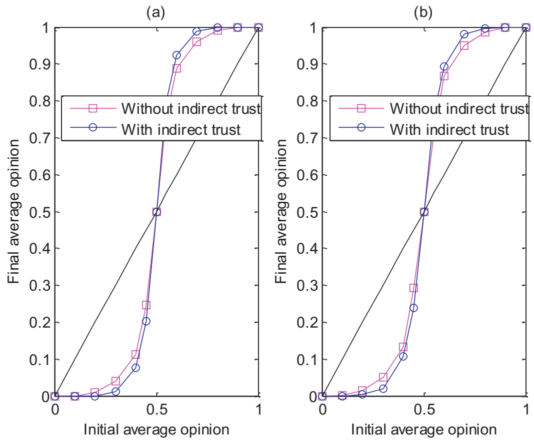
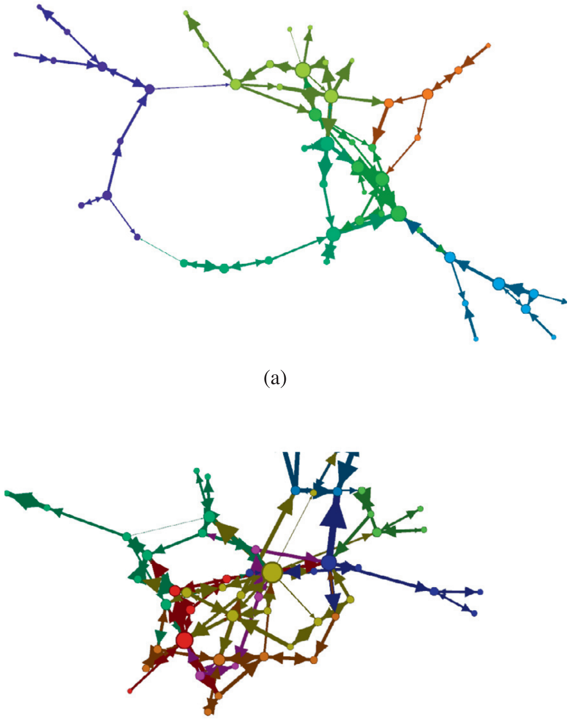



**Fig. 3.** Final average opinion versus initial average opinion, $N = 1000$, $\varepsilon = 0.5$ and $\delta = 0$. The straight lines in both plots account for the standard voter model. Each plot is an average of 50 simulations. (a) A scale-free network mediates the interactions; (b) The underlying network is a small-world network.

is less possible to diffuse, and its density decreases gradually with time, leading to the phase transition of average opinion around $f(0) = 0.5$. Considering the evaluation of common neighbors, agents may talk with the neighbors that they do not trust directly, and thus, indirect trust promotes opinion interactions. With indirect trust, the majority opinion occupies more agents, and the system converges nearly to consensus for $f(0) < 0.3$ or $f(0) > 0.7$. Comparing both plots in Fig. 3, the final density of majority opinion is greater in the scale-free network than in the small-world network, due to the role of the hub nodes which propagate their opinions to a larger extent in the scale-free network.

To determine how individual trust evolves with opinions, we plot the dimensional configuration of interpersonal trust after 500 time steps in a scale-free network. We choose 50 nodes and obtain the trust among them, as shown in Fig. 4. In the figure, nodes denote agents, and directed edges denote interpersonal trust. Nodes are identical for both subplots. Source nodes on one side of edges represent the evaluators. The sizes of nodes depend on their degrees, while the widths of edges and sizes of arrows illustrate the values of individual trust. We use the community detection algorithm [36] to classify agents, and agents in different communities are marked with different colors. The colors of arrows in the figure correlate with their source nodes. Weak trust always exists between nodes that belong to different communities, and strong trust is often generated inside a community. Nodes with a large degree tend to receive strong trust from others, and reciprocal trust is likely to occur between agents with large degrees in the same community. Comparing subplots (a) and (b), we find that agents form much stronger interpersonal trust towards others if they consider the evaluation of common neighbors. Therefore, indirect trust promotes opinion interactions.

We assign the initial density of opinion 1 at $f(0) = 0.4$ to explore the impact of parameters on the dynamics. Fig. 5 shows the stable state as a function of $\varepsilon$ with different default values of self-trust $\delta$. Indirect trust is considered in this figure. A small-world network and scale-free network are used for comparison. From Fig. 5, the final average opinion decreases as $\varepsilon$ increases. For $\varepsilon < 0.5$, the average opinion decreases markedly, but for $\varepsilon > 0.5$, the decrement becomes smaller. The relationship between the final average opinion and $\varepsilon$ also can be approached by an exponential mixture function. For



(a)

(b)

**Fig. 4.** Dimensional configuration plots of interpersonal trust, $\varepsilon = 0.5$ and $\delta = 0$. The initial average opinion is 0.4, and a scale-free network is used as interaction topology. (a) Without indirect trust; (b) With indirect trust. (For interpretation of the references to color in this figure texr, the reader is referred to the web version of this article.)

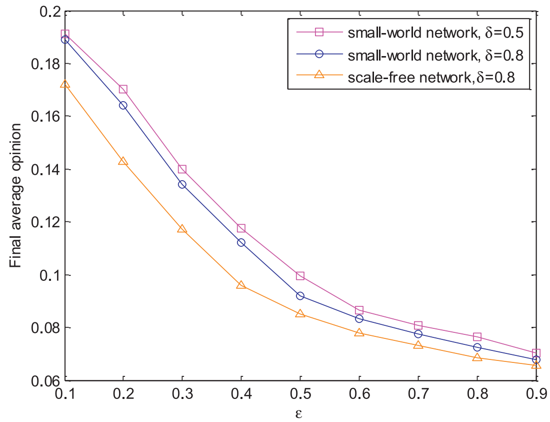



**Fig. 5.** Final average opinion as a function of $\varepsilon$. The initial average opinion $f(0)$ is 0.4, and $N = 1000$. Each plot is an average of 50 simulations.

instance, in a scale-free network, the average opinion is approximately expressed as $0.1895 \cdot e^{-2.773\varepsilon} + 0.02755 \cdot e^{0.6822\varepsilon}$, and the root mean square error is 0.002. The parameter $\delta$ does not have a distinct effect on opinion evolution, and the phenomenon is in agreement with the analytical results. However, a slight difference for $\delta = 0.5$ and for $\delta = 0.8$ is observed, because not all agents have their self-trust after the first interaction due to random fluctuations. Also, the system converges more markedly in the scale-free network than in the small-world network, similarly to Fig. 3. Furthermore, we calculate the opinion update frequency and final average self-trust to study local interacting behaviors. We find that the opinion update frequency increases linearly with $\varepsilon$, but the final self-trust decreases as $\varepsilon$ increases. In the small-world network, the update frequency is 0.1354 for $\varepsilon = 0.1$ and 0.1617 for $\varepsilon = 0.3$, while the final self-trust is 0.8679 for $\varepsilon = 0.1$ and 0.8444 for $\varepsilon = 0.3$.

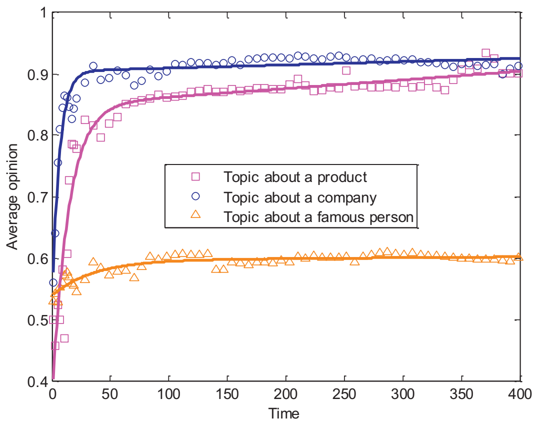



**Fig. 6.** Time evolution of proportion of positive sentiments for 3 real topics on Twitter. The discrete symbols denote Twitter data, while real lines represent fittings by exponential mixture functions.

Thus, a large default value of interpersonal trust, $\varepsilon$, restrains individual self-trust, and promotes opinion exchanges, leading to a more ordered state.

### 4.2. Real data evaluation

We collected large-scale data from a real social network (Twitter) and analyzed users' sentiments. The methods of data processing and mining are available in Ref. [37]. We obtained more than two million user profiles and six million related posts. All of the data were anonymized. We analyzed the sentiments of users' posts about different topics, and calculated the public opinion that was reflected by the posts in order to determine the evolution of individual opinions as a function of time. After sentiment analysis, users' posts were labeled either as positive posts or negative posts.

We choose three topics, i.e. a topic about a product, a topic about a company, and a topic about a famous person, and we investigate the real opinion dynamics for these topics. We calculate the proportion of accumulative positive posts at different time, and determine the evolution pattern of average opinion, as shown in Fig. 6. The proportion is counted by hours. The fitting functions for the topic about a product, a company, and a person are $f(t) = 0.8487 \cdot e^{0.0001584t} - 0.4803 \cdot e^{-0.07128t}$, $f(t) = 0.9041 - 0.3838 \cdot e^{-0.1574t}$, and $f(t) = 0.5962 - 0.05806 \cdot e^{-0.02862t}$, respectively. The root mean square errors for the fittings are 0.01913, 0.01295, and 0.007931, respectively. From the figure, our trust-aware opinion model fits the real-world data very well, implying that our model adequately characterizes the process of topic discussions in real social networks. In the model, agents interact with others in terms of individual trust that is determined by agents' past behaviors. Individual trust in social networks has been verified extensively by real data, and it can be used for personalized recommendation, information diffusion prediction, and opinion interaction modeling [38]. In addition, compared with simulation results in synthetic networks, the public opinion on Twitter evolves very rapidly in the early stage and stabilizes within a shorter time. The reason is that we cannot collect all of the real data due to the limited access to the data on the Twitter website. However, the evolution pattern of public opinion still is captured by our model.

We examine whether our model can predict the future trend in the evolution of public opinion for topic discussions in social networks. We use parts of the data in the early stage to estimate the parameters in our model and to obtain the curve fitting. Then, we use the model to predict the remaining opinion dynamics and compare the results with the real data, as shown in Fig. 7. We choose a topic about 'iphone' and calculate the proportion of accumulative positive sentiments at different hours. In subplots (a), (b), (c), and (d), different proportions of the real data are used for fitting, and the root mean square errors for the fittings are 0.02077, 0.0166, 0.01358, and 0.006592, respectively. When abundant data are used, the fitting apparently is more accurate. In subplot (a), when 10% of the data are used for parameter evaluation, our model predicts that the average opinion will reach a plateau quickly, coinciding with the Twitter topic. We check the transition point at which the average opinion becomes stable. The transition point predicted by the model is a little earlier than the transition in the real topic, and the density of the majority opinion in the prediction result is noticeably smaller than that in the real topic. In online social networks, users publish posts openly, and other users can read the posts conveniently. A few users may sometimes comment on the posts of some other users even though they are not neighbors. Therefore, they communicate with more users, and it is harder for them to enter the non-evolving category. When non-evolving users become less, the majority opinion is adopted by more users. We realize that 10% of the data may be not enough to estimate the evolution pattern and final system state. In subplot (b), 16% of the data are used for estimation, the transition point of average opinion around $t = 128$ is predicted effectively, and the final average opinion predicted by the model approaches that in the real-world evolution. Since the transition point and stable system state are important information for opinion dynamics, the data before $t = 80$ can be used to characterize individual opinion interactions in social networks. In subplots (c) and (d), with more data for estimation, the model performs better.

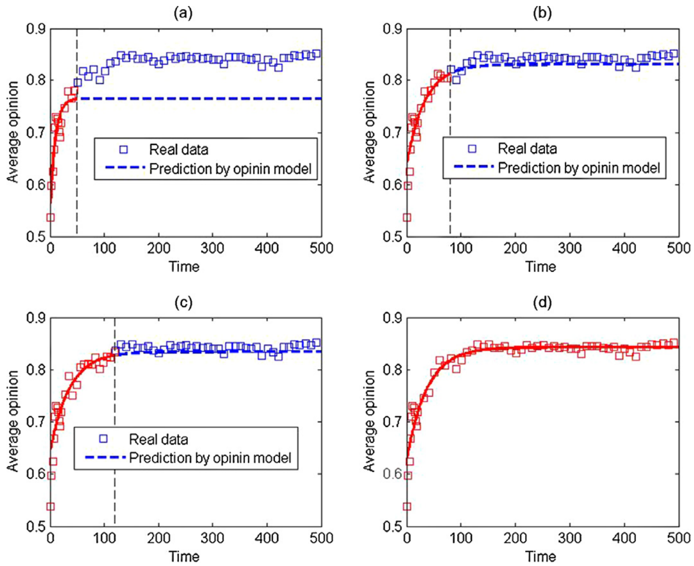



**Fig. 7.** Prediction simulation results for a Twitter topic. The red symbols denote Twitter data that are used for curve fitting, while the solid red lines are fittings by exponential mixture functions. The dashed blue lines represent prediction results by proposed opinion model. (a) Curve fitting using 10% of the data; (b) Curve fitting using 16% of the data; (c) Curve fitting using 24% of the data; (d) Curve fitting using all of the data. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

## 5. Conclusions

In this paper, we explored the temporary evolution pattern of public opinion in opinion dynamics. We proposed an opinion model with trust propagation. When an agent interacts with a neighbor, it takes into account the trust on itself and on the neighbor, i.e., self-trust and interpersonal trust; in return, individual trust is determined by past opinion updates. Trust propagates among agents, and the evaluation of common neighbors on the target agent affects individual decisions. We conducted mean-field analysis and numerical simulations of the trust-aware opinion model in regular networks and inhomogeneous networks.

Both the analytical and simulation results show that public opinion changes with time as an exponential mixture function. Individual trust changes local behaviors, and the conservation of magnetization is broken. The system will achieve an ordered state in which one opinion accounts for the absolute majority. Self-trust forms faster than interpersonal trust, so the initial value of interpersonal trust takes effect more obviously in opinion evolution. Indirect trust from common neighbors promotes opinion interactions, and drives the system towards consensus. Empirical results are consistent with our theoretical results, and the exponential mixture pattern is observed in real-world topic discussions. Experiment results indicate that our model is practical for modeling opinion dynamics in online social networks.

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