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Dynamics analysis of a pest management prey–predator model by means of interval state monitoring and control

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ABSTRACT

In this work, a new pest management strategy by means of interval state monitoring is introduced into a prey–predator model, i.e. when the pest density exceeds the slightly harmful level but is below the damage level, the biological control is adopted in case of the predator density below a maintainable level, once the pest density exceeds the damage level, the chemical control is adopted. In order to determine the frequency of the chemical control and yield of releases of the predator, analysis on the existence of order-1 or order-2 periodic orbit is carried out by the construction of Poincaré map. The results could make the pest control strategy to be a periodic one without real-time monitoring the species. In addition, the stability and attractiveness of the periodic orbit are obtained by geometry approach, which ensures a certain robustness of control, i.e., even though the species densities are detected inaccurately or with a deviation, the system will be eventually stable at the periodic orbit under the control action. Furthermore, to obtain the optimum chemical control strength and yield releases of the predator, an optimization problem is constructed. The analytical results presented in the work are validated by numerical simulations for a specific model.

1. Introduction

Pest management is an interesting and significant issue in real life. The traditional and efficient method is to spray pesticide. However, unrestrained use of persistent pesticide not only increases the incidence of pesticide-resistant pest varieties, but also inflicts harmful effects on humans through the accumulation of hazardous chemicals in their food chain [1]. Moreover, pesticide pollution is a major threat to beneficial insects, which sometimes are more sensitive to pesticides than target pest. So if it is not necessary, the chemical control should not be easily considered.

Biological control, as an alternative control method, by releasing biological agents to increase their effectiveness plays an important role in suppressing pests growth [2,3]. Researches on augmentation as a biological control method have also shown that some practices are cost-effective [4] and others are not and sometimes can have disastrous consequences without being well planned [5]. Especially, when encountering the situation of some disaster, which is limited in a small range, the method is not very effective or ideal at this time.

Integrated pest management (IPM) is an effective method in controlling pests with minimal use of harmful pesticides and other undesirable measures, which has been proved to be more effective than the classic methods both experimentally [1,2]

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and theoretically [6,7]. Many researchers have introduced IPM strategy in modeling pest control, for example: the periodic release of predators and infected pests [8,9]; the periodic release of infected pests combined with periodic applications of pesticides [10]; the periodic release of predators, pests combined with periodic applications of pesticides [11–16] and state dependent release of predators combined with applications of pesticides [17–23].

By considering biological control and chemical control adopted at different pest levels, Nie et al. [24,25] studied two prey–predator models with twice impulsive controls. The idea is interesting, but the suppressing effect of natural predator is neglected at the biological control level. Based on this consideration, Tian et al. [26,27] introduced a predator density level into the models in [24,25], i.e. when the predator density in the system is below the level, the biological control strategy is adopted until the predator density is higher than that level. And followed, by choosing different predator control level, Zhao et al. [28] and Zhang et al. [29] proposed and analyzed the dynamics of other type of predator–prey models in detail.

The idea of involving biological and chemical controls at different prey densities is interesting and has certain practical significance. Before the chemical control has to be adopted, a biological control should be adopted in advance, which can extend the time for pest density to reach the pest damage level. However, there exists a key problem in this process that needs to be dealt with properly, i.e. the biological control is adopted when the pest density reaches the first control level and the predator density is lower than its maintainable level, but for a higher pest density, no control strategy is adopted. This is obviously unreasonable. Since the biological control and chemical control are taken at different pest levels, a more reasonable model should also consider the control action when the pest density lies between the two levels. Motivated by this control strategy, Tian [30] proposed a pest management Gompertz model with interval state feedback impulsive control. As a continuation, a prey–predator Logistic model by means of interval state monitoring and control is presented and control optimization is carried out based on the qualitative analysis on the proposed model.

This paper is organized as follows. In Section 2, a pest control prey–predator model by means of interval state monitoring and control is put forward, and some basic definitions are given. In Section 3, a detail dynamics analysis in case of the chemical control strength is carried out. In Section 4, numerical simulations are presented with a specific model to verify the theoretical results step by step. Finally, conclusions are presented in Section 5.

2. Model formulation and preliminaries

2.1. Model formulation

Let $x(t)$ and $y(t)$ denote the pest and its natural enemy densities at time t, which follow the logistic model

$$\begin{cases} \dfrac{\mathrm{d}x(t)}{\mathrm{d}t} = x(t)\left[r - \dfrac{rx(t)}{K} - by(t)\right] \\ \dfrac{\mathrm{d}y(t)}{\mathrm{d}t} = y(t)[\varrho bx(t) - d] \end{cases} \tag{2.1}$$

where $r > 0$ is the birth rate, $K > 0$ is the environmental carrying capacity for the prey in absence of predator, $b > 0$ is the contact rate, $0 < \varrho < 1$ is the conversion coefficient and $0 < d < 1$ is the death rate of predator. In this study, it is assumed that

$$(P1) : K > \overline{K} \triangleq \frac{d}{\varrho b}.$$

Let ET_1 denote the first pest control level, ET_2 ($ET_1 < ET_2 < K$) denote the pest damage level above which the chemical control is taken to suppress pests increasing. When the prey density locates in between ET_1 and ET_2 (i.e. $x(t) \in [ET_1, ET_2)$) and the predator density is below a level $\overline{y}_x^{\lambda}$, only the biological control is taken as a control method to suppress the prey. By the biological background, it is only necessary to consider the dynamic behavior of such a system in the region $\Omega = \{(x, y)|0 < x \le ET_2, 0 < y \le y_M - \varrho x\}$, where y_M is a sufficiently large constant satisfying $\mathrm{d}\chi_0/\mathrm{d}t|_{\chi_0=0} < 0$ where $\chi_0 : \varrho x + y - y_M = 0$. According to the above mentioned control strategy, the system can be modeled by the following impulsive differential equations:

$$\begin{cases} \left.\begin{aligned} &\dfrac{\mathrm{d}x(t)}{\mathrm{d}t} = x(t)\left[r - \dfrac{rx(t)}{K} - by(t)\right] \\ &\dfrac{\mathrm{d}y(t)}{\mathrm{d}t} = y(t)[\varrho bx(t) - d] \end{aligned}\right\} & \begin{aligned} &x < ET_1 \\ &\text{or } x \in [ET_1, ET_2), y > \overline{y}_x^{\lambda} \end{aligned} \\ \left.\begin{aligned} &\Delta x = 0 \\ &\Delta y = \alpha(x(t)) \end{aligned}\right\} & x \in [ET_1, ET_2), 0 < y \le \overline{y}_x^{\lambda} \\ \left.\begin{aligned} &\Delta x = -px(t) \\ &\Delta y = -qy(t) \end{aligned}\right\} & x = ET_2, 0 < y \le \overline{y}_x^{\lambda} \end{cases} \tag{2.2}$$

where $\overline{y}_x^{\lambda}$ is the predator maintainable level (or critical biological control level) at the pest level x, which is defined as follows

$$\overline{y}_x^{\lambda} = \overline{y}^{\lambda}(x) \triangleq (r(K - \overline{K})/bK)\lambda(x), \tag{2.3}$$

where $\lambda : [ET_1, ET_2] \longmapsto \mathbb{R}^+$ is called a *reference function*, $\bar{y}_x^{\bar{\lambda}}$ is the maximum biological control predator level at the pest level $x \in [ET_1, ET_2]$, where $\bar{\lambda}(x) \triangleq (K - x)/(K - \bar{K})$.

In this study, from practice point of consideration, the predator maintainable level $\bar{y}^{\lambda}(x)$ given in Eq. (2.3) is assumed to have the following properties:

- the consistency property (P2): the biological control taken at any pest level should also take effect for a higher pest level;
- is no more than half of the maximum predator maintainable level;
- is below the level $\bar{y}_{ET_2}^{\lambda}$ since above $\bar{y}_{ET_2}^{\lambda}$ the chemical control loses its practical significant, and so does the biological control.

The control parameters p, q represent the effect of the pesticide on the prey and predator, where q as a side effect is dependent on p. Since the pesticide is corresponding to the pest, the effect on predator is limited, then it is assumed that $0 < p < 1$ and $0 \le q \le q_{\max} \triangleq 1 - \bar{y}_{ET_1}^{\lambda}/\bar{y}_{ET_2}^{\bar{\lambda}}$, and the yield of releases of the predator $\alpha_x \triangleq \alpha(x)$ is assumed to satisfy

$$\alpha(x) = (1 - \theta_x)\alpha_{\min}(x) + \theta_x\alpha_{\max}(x), \quad x \in [ET_1, ET_2], \tag{2.4}$$

where $\alpha_{\min}(x) \triangleq \bar{y}^{\lambda}(x)$, $\alpha_{\max}(x) \triangleq \bar{y}^{\bar{\lambda}}(x) - \bar{y}^{\lambda}(x)$ and $\theta_x \in [0, 1]$.

Next, we will find an upper bound of $\bar{y}^{\lambda}$ with consistency property (P2). Denote the first integral of the continuous system of system (2.2) by $\Gamma(x, y) = C$. Then the trajectory $\mathbf{z} = (x, y)$ of the continuous system of system (2.2) which passing through the point $P_0(x_0, y_0)$ is

$$\Gamma(x, y) = \Gamma(x_0, y_0). \tag{2.5}$$

Let $y_{\Gamma}^{-}(x, (x_0, y_0))$ (or $y_{\Gamma}^{-}(x, P_0)$) denote the implicit function determined by the low branch of (2.5). Now the critical reference function λ^* is chosen as

$$\lambda^*(x) = y^{*-1} y_{\Gamma}^{-1}(x, (ET_1, \bar{y}_{ET_1}^{\lambda})), \quad x \in [ET_1, ET_2]. \tag{2.6}$$

Then the predator maintainable level $\bar{y}^{\lambda^*}$ is an upper bound of $\bar{y}^{\lambda}$ with property (P2), which is also called the critical predator maintainable level. Since the dynamics for the model (2.2) with the reference function λ is similar to the one with λ^*, thus without loss of generality, it is assumed that the reference function λ in the model (2.2) is the critical reference function λ^*.

2.2. Preliminaries

For a general model concerning IPM strategies

$$\begin{cases} \left.\begin{aligned} \frac{dx}{dt} &= P(x, y) \\ \frac{dy}{dt} &= Q(x, y) \end{aligned}\right\} & (x, y) \notin M_{\mathtt{imp}} \\ \left.\begin{aligned} \Delta x &= I_1(x, y) \\ \Delta y &= I_2(x, y) \end{aligned}\right\} & (x, y) \in M_{\mathtt{imp}} \end{cases} \tag{2.7}$$

where $M_{\mathtt{imp}} = \{(x, y) | \phi(x, y) = 0\}$ describes the states at which the control strategy is taken on, I_1 and I_2 describe the effects of the control strategy and $(x, y) \in \Omega_0 \in \mathbb{R}^2$. Denote $N_{\mathtt{pha}} = \{(x, y) | x = x' + I_1(x', y'), y = y' + I_2(x', y'), (x', y') \in M_{\mathtt{imp}}\}$.

Let $\mathbf{z}(t) = (x(t), y(t))$ be any solution of system (2.7) and the positive orbit through the point $\mathbf{z}_0 \in \Omega_0$ for $t \ge t_0$ is defined as

$$O^+(t, \mathbf{z}_0, t_0) = \{\mathbf{z}(t) \in \Omega_0, t \ge t_0, \mathbf{z}(t_0) = \mathbf{z}_0\}.$$

Denote $\mathbf{z}_k = \mathbf{z}(t_k^+) \in O^+(t, \mathbf{z}_0, t_0)$, where $t_k \in \prod \triangleq \{t_k | k = 1, 2, \ldots\}$ with $\mathbf{z}(t_k) \in M_{\mathtt{imp}}$.

Let $S \subseteq \mathbb{R}^2 = (-\infty, +\infty)^2$ be an arbitrary set and $P \in \mathbb{R}^2$ be an arbitrary point. Then the distance between the point P and the set S is denoted by

$$d(P, S) = \inf_{P_0 \in S} |P - P_0|.$$

For convenience, the control set at the pest level $x = ET_1$ is denoted by $\sum_{ET_1}^{-}$. i.e.

$$\sum_{ET_1}^{-} = \{(x, y) | x = ET_1, 0 \le y \le \bar{y}_{ET_1}^{\lambda}\},$$

the phase set at $x = ET_1$ is denoted by

$$\sum_{ET_1}^{+} := \{(x, y) | x = ET_1, \bar{y}_{ET_1}^{\lambda} < y \le \bar{y}_{ET_1}^{\lambda} + \alpha_{ET_1}\}$$

and the control set at the pest level $x = ET_2$ is denoted by

$$\sum_{ET_2}^{-} := \{(x, y)|x = ET_2, 0 \le y \le \bar{y}^{\lambda}_{ET_2}\}.$$

For $0 < p \le p_T \triangleq 1 - ET_1/ET_2$, the control set at $x = (1-p)ET_2$ is denoted by

$$\sum_{p}^{-} := \{(x, y)|x = (1-p)ET_2,\ y \le \bar{y}^{\lambda}_{(1-p)ET_2}\}$$

and the phase set at $x = (1-p)ET_2$ is denoted by

$$\sum_{p}^{+} := \{(x, y)|x = (1-p)ET_2, 0 < y - \bar{y}^{\lambda}_{(1-p)ET_2} \le \alpha_{(1-p)ET_2}\};$$

For $p_T < p < 1$, the phase set at $x = (1-p)ET_2$ is denoted by

$$\sum_{p}^{+} := \{(x, y)|x = (1-p)ET_2,\ y \le (1-q)\bar{y}^{\lambda}_{(1-p)ET_2}\}.$$

Definition 2.1 (*Poincaré Map*). Assume that the trajectory $O^+(L_n^+, t_n)$ starts from the point $L_n^+((1-p)ET_2, y_n)$ on $\sum_p^+$.

(i) If $0 < p \le p_T$, then it first reaches the point $L_{n+1}^-(ET_2, \tilde{y}_{n+1})$ on the section $\sum_{ET_2}^-$. Then it jumps from L_{n+1}^- to the point $L_{n+1}((1-p)ET_2, (1-q)\tilde{y}_{n+1})$ on $\sum_p^-$ due to the impulsive effects $\Delta x = (1-p)x$ and $\Delta y = (1-q)y$, and then jumps from L_{n+1} to $L_{n+1}^+((1-p)ET_2, \tilde{y}_{n+1} + \alpha_{(1-p)ET_2})$ on $\sum_p^+$ due to impulsive effects $\Delta x = 0$ and $\Delta y = \alpha_{(1-p)ET_2}$, as illustrated in Fig. 1(a));

(ii) If $p_T < p < 1$, the trajectory first reaches point $L_{n+1}(ET_1, \tilde{y}_{n+1})$ on the section $\sum_{ET_1}^-$. Then it jumps from L_{n+1} to the point $L'_{n+1}(ET_1, \tilde{y}_{n+1} + \alpha_{ET_1})$ on $\sum_{ET_1}^+$ due to the impulsive effects $\Delta x = 0$ and $\Delta y = \alpha_{ET_1}$. Then the trajectory starting from L'_{n+1} reaches the point $L_{n+1}^-(ET_2, \hat{y}_{n+1})$ on the section $\sum_{ET_2}^-$, then jumps from L_{n+1}^- to $L_{n+1}^+((1-p)ET_2, y_{n+1})$ on $\sum_p^+$ due to impulsive effects $\Delta x = -px$ and $\Delta y = -qy$, as illustrated in Fig. 1(b).

Thus, y_{n+1} can be determined by the parameters y_n, p, q, α_{ET_1} or $\alpha_{(1-p)ET_2}$. Therefore, the Poincaré map on $\sum_p^+$ is defined as follows

$$\begin{aligned} F = (1_x, F_y):\quad & \sum_{p}^{+} \to \sum_{p}^{+}, \\ & L_n^+((1-p)ET_2, y_n) \to L_{n+1}^+((1-p)ET_2, y_{n+1}) \\ & \text{i.e. } 1_x((1-p)ET_2) = (1-p)ET_2,\ \ F_y(y_n) \triangleq y_{n+1}. \end{aligned} \tag{2.8}$$

Definition 2.2 (*Successor Function*). For any $L^+((1-p)ET_2, y) \in \sum_p^+$, the successor function $f_{\mathtt{sor}}$ at L^+ is defined as

$$f_{\mathtt{sor}}(L^+) \triangleq F_y(y) - y, \tag{2.9}$$

which is continuous on $\sum_p^+$, where F_y is the Poincaré map determined by Eq. (2.8).

Definition 2.3 (*Periodic Orbit [31]*). An orbit $O^+(t, \mathbf{z}_0, t_0)$ of system (2.7) is said to be periodic if there exists positive integer $m \ge 1$ such that $\mathbf{z}_m = \mathbf{z}_0$. Denote $m_0 \triangleq \min\{m \in \mathbb{N}, \mathbf{z}_m = \mathbf{z}_0\}$. Then the orbit $O^+(t, \mathbf{z}_0, t_0)$ is said to be an order-k periodic orbit if $O^+(t_0 \le t \le t_{m_0}, \mathbf{z}_0, t_0)$ includes k ($k \le m_0$) different trajectories of system (2.7).

Definition 2.4 (*Orbital Stability [31]*). $\mathbf{z}^*(t)$ is said to be orbitally stable if, given $\varepsilon > 0$, there exists $\delta = \delta(\varepsilon) > 0$ such that, for any other solution $\mathbf{z}(t)$ of system (2.7) satisfying $|\mathbf{z}^*(t) - \mathbf{z}(t)| < \delta$, then $d(\mathbf{z}(t), O^+(\mathbf{z}_0, t_0)) < \varepsilon$ for $t > t_0$.

Definition 2.5 (*Asymptotic Orbital Stability [31]*). $\mathbf{z}^*(t)$ is said to be asymptotically orbitally stable if it is orbitally stable, and furthermore for any other solution $\mathbf{z}(t)$ of system (2.7), there exists a constant $\eta > 0$ such that if $|\mathbf{z}^*(t_0) - \mathbf{z}(t_0)| < \eta$, then $\lim_{t\to\infty} d(\mathbf{z}(t), O^+(\mathbf{z}_0, t_0)) = 0$.

Lemma 2.1 (*Poincaré–Bendixson Theorem [31]*). *Assume that there is a bounded closed region $A\widehat{BC}D\widehat{A}$, as shown in Fig. 2. The boundaries $\widehat{AD}$ and $\widehat{BC}$ are the no cut-arcs of system (2.7), the direction of the orientation field and on $\widehat{AD}$ and $\widehat{BC}$ determined by system (2.7) is pointing to the interior of ABCDA. In addition, there is no singularity in the interior of $A\widehat{BC}D\widehat{A}$ and the boundaries. The boundary AB is the impulse set of the system (2.7), the corresponding phase set satisfies $I(AB) \subset CD$, CD is also the no cut-arcs of system (2.7). The direction of the orientation field and on CD determined by system (2.7) is pointing to the interior of ABCDA. Then there exists an order-1 periodic of system (2.7) in region $A\widehat{BC}D\widehat{A}$.*

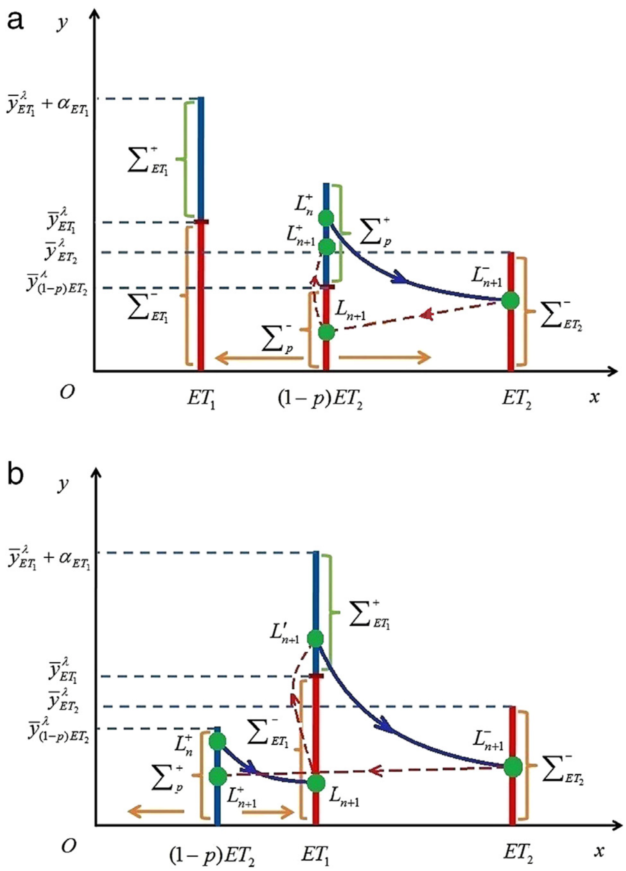

Fig. 1. Illustration of Poincaré map.

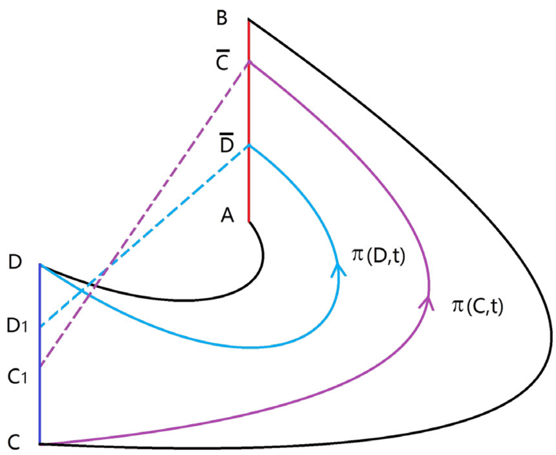

Fig. 2. Illustration of Bendixson field.

3. Dynamic analysis and control optimization

The continuous system of system (2.2) without control is called a free system, i.e.,

$$\begin{cases} \dfrac{\mathrm{d}x(t)}{\mathrm{d}t} = rx(t)\left[1 - \dfrac{x(t)}{K}\right] - bx(t)y(t) \\ \dfrac{\mathrm{d}y(t)}{\mathrm{d}t} = y(t)[\varrho bx(t) - d] \end{cases} \tag{3.1}$$

which was used as a basic model to model different systems with certain function in the literature.

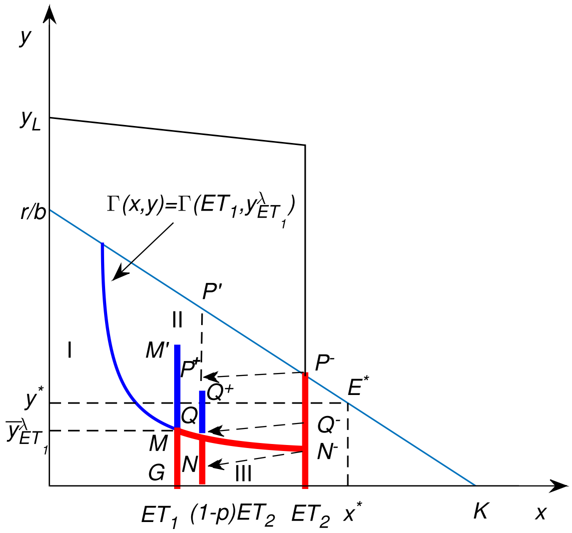

Fig. 3. The partition of the region Ω and illustration of impulse set and phase set for $0 < p \leq 1 - ET_1/ET_2$.

For the system (3.1), the following result holds true directly:

Theorem 3.1. *System (3.1) admits three equilibriums $O(0,0)$, $K(K,0)$ and $E^*(x^*,y^*)$, where $x^* = \overline{K}$ and $y^* \triangleq r(K-\overline{K})/bK$. Furthermore, $O(0,0)$ and $K(K,0)$ are saddle points and $E^*(x^*,y^*)$ is globally asymptotically stable.*

Next, we mainly discuss the dynamic behavior of the model (2.2) in the region Ω with $\lambda = \lambda^*$ defined by Eq. (2.6). Since the steady state $E^*(x^*,y^*)$ is globally asymptotically stable, i.e., without any control strategy, the solution of free system will infinitely approach the steady state, thus the prey chemical control level is assumed to be less than its steady state, i.e. $ET_2 \leq x^* = \overline{K}$. The chemical control strength level is described by p, that is to say, the larger the p, the higher the control strength.

In this study, two control strengths are considered. case 1: a lower strength, i.e. $0 < p \leq p_T$; case 2: a higher intensity, i.e. $p_T < p < 1$.

3.1. Dynamical analysis for a lower chemical control strength

The region Ω for this case can be divided into three subregions I–III, as shown in Fig. 3, where P^- represents the intersection point between the lines $x = ET_2$ and $\dot{x}/x = 0$, P^+ and N represents the phase points of $P^-(ET_2, \bar{y}^{\lambda}_{ET_2})$ and $N^-(ET_2, \bar{y}^{\lambda}_{ET_2})$ under the impulse mapping $x^+ = x - px$ and $y^+ = y - qy$ respectively; M' and Q^+ represent respectively the phase points of $M(ET_1, \bar{y}^{\lambda}_{ET_1})$ and $Q((1-p)ET_2, \bar{y}^{\lambda}_{(1-p)ET_2})$ under the impulse mapping $y^+ = y + \alpha(x)$; Q^- is the impulse point of Q, and G and K are respectively the intersection points between the lines $y = 0$ and $x = ET_1$, $\dot{x}/x = 0$.

Notice that any point P_0 in region *III* will jump into region *II* under the impulse mapping $y_0^+ = y_0 + \alpha(x)$, and the solution of the system (3.1) starting from any point P_0 in region *I* will arrive at the segment $\overline{MG}$ and then jumps to the segment $\overline{MM'}$, which belongs to the region *II*. In addition, the solution of the system (3.1) starting from any point P_0 in region *II* will arrive at the segment $\overline{P^-N^-}$ and then jumps to the segment $\overline{P^+N}$, and the point in $\overline{QN} \subset III$ will jump to $\overline{Q^+Q}$. Since the relation between P^+ and Q^+ does not affect the dynamics of the system (2.2), as illustrated in Fig. 3, it is assumed that P^+ is above Q^+. Thus, it is only necessary to analyze the tendency of the system (2.2)'s solution starting from the segment $\overline{Q^+Q}$. Let L_0^+ denote the point in $\overline{Q^+Q}$ with $y_{L_0^+} \triangleq \alpha_{(1-p)ET_2} + (1-q)\bar{y}^{\lambda}_{ET_2}$.

Theorem 3.2. *For a lower chemical control strength, i.e.* $0 < p \leq p_T$, *system (2.2) admits at least one periodic orbit.*

Proof. Let us consider the region $P^-\widehat{N^-Q}P^+$ with boundaries $\overline{P^-N^-}$, $\widehat{N^-Q}$, $\overline{QP^+}$ and $\overline{P^+P}$. The impulse set is $\overline{P^-N^-}$, and the phase set $I(\overline{P^-N^-}) \subset \overline{QP^+}$. Denote

$$\chi_1 : x - (1-p)ET_2 = 0$$

and

$$\chi_2 : y - \bar{y}^{\lambda}_{ET_2} - q\bar{y}^{\lambda}_{ET_2}(x - ET_2)/pET_2 = 0.$$

With a simple calculation, it yields that $\mathrm{d}\chi_1/\mathrm{d}t|_{\chi_1=0} > 0$ and $\mathrm{d}\chi_2/\mathrm{d}t|_{\chi_2=0} < 0$, which means that the trajectory of system (3.1) will pass through $\chi_1 = 0$ from left to right, pass through $\chi_2 = 0$ from up to down. In addition, $\widehat{N^-Q}$ is an orbit of the system (3.1), then by Lemma 2.1, system (2.2) admits at least one periodic orbit. $\square$

Theorem 3.3. *For a lower chemical control strength, i.e.* $0 < p \le p_T$, *system* (2.2) *admits an order-1 periodic orbit exists if and only if* $\alpha_{\min}((1-p)ET_2) \le \alpha_{(1-p)ET_2} \le \overline{\alpha}_1$, *where*

$$\overline{\alpha}_1 \triangleq \max_{\alpha \le \alpha_{\max}((1-p)ET_2)} \left\{ \alpha \mid \frac{(1-q)y_\Gamma^-(ET_2, Q_\alpha)}{\overline{y}^\lambda_{(1-p)ET_2}} \le 1 \right\}, \tag{3.2}$$

and $Q_\alpha \triangleq Q_\alpha((1-p)ET_2, \overline{y}^\lambda_{(1-p)ET_2} + \alpha)$.

Proof. "$\Leftarrow$" Assume that $\alpha_{\min}((1-p)ET_2) \le \alpha_{(1-p)ET_2} \le \overline{\alpha}_1$. Then

$$y_\Gamma^-(ET_2, Q^+) \le y_{Q^-} = \overline{y}^\lambda_{(1-p)ET_2}/(1-q).$$

If the equality holds, the trajectory $\mathbf{z} = (x, y_\Gamma^-(x, Q^+))$ will first intersect the impulse set P^-N^- at the point Q^-, and then jumps to the point Q under the impulse mapping $x^+ = x - px$ and $y^+ = y - qy$. Next, the point Q jumps to the point Q^+ under the impulse mapping $x^+ = x + \alpha(x)$ and $y^+ = y$, which forms a cycle $\widehat{Q^+Q^-}QQ^+$. Then by Definition 2.3, $\widehat{Q^+Q^-}QQ^+$ is an order-1 periodic orbit. If the inequality holds, the trajectory $\mathbf{z} = (x, y_\Gamma^-(x, Q^+))$ first intersects the impulse set P^-N^- at a point below Q^-, then jumps to a point below Q and next jumps to a point below Q^+, by Definition 2.2, there is $f_{\mathrm{sor}}(Q^+) < 0$. On the other hand, the trajectory $\mathbf{z} = (x, y_\Gamma^-(x, L_0^+))$ will intersect the impulse set P^-N^- at a point below Q^-, then jumps to a point below Q and next jumps to a point between L_0^+ and Q^+, by Definition 2.2, there is $f_{\mathrm{sor}}(L_0^+) > 0$. From the continuity of f_{sor} it can be concluded that there exists a point L^+ between L_0^+ and Q^+ such that $f_{\mathrm{sor}}(L^+) = 0$, i.e. system (2.2) exists an order-1 periodic orbit.

"$\Rightarrow$"

Since $\alpha_{\min}((1-p)ET_2) \le \alpha_{(1-p)ET_2} \le \overline{\alpha}_1$ also implies that $\alpha_{\min}((1-p)ET_2) \le \overline{\alpha}_1$. By the definition of $\overline{\alpha}_1$, there is $\overline{\alpha}_1 \le \alpha_{\max}((1-p)ET_2)$. If the equality holds, the necessity holds since $\alpha_{(1-p)ET_2}$ satisfies (P_3). If the inequality holds, then it only needs to show that system (2.2) does not admit an order-1 periodic orbit in case of $\overline{\alpha}_1 < \alpha_{(1-p)ET_2} \le \alpha_{\max}((1-p)ET_2)$.

Denote A^+ as the point below Q^+ such that

$$y_\Gamma^-(ET_2, A^+) = y_{Q^-} = (1-q)^{-1}\overline{y}^\lambda_{(1-p)ET_2}.$$

Obviously, the trajectory starting from $\overline{A^+Q^+}$ cannot form an order-1 periodic orbit. For any point B^+ in $\overline{L_0^+Q}$, there is $f_{\mathrm{sor}}(B^+) > 0$, which means that the trajectory starting from B^+ cannot also form an order-1 periodic orbit. If $y_{A^+} \le y_{L_0^+}$, then system (2.2) does not admit an order-1 periodic orbit for $\overline{\alpha}_1 < \alpha_{(1-p)ET_2} \le \alpha_{\max}((1-p)ET_2)$. For the case of $y_{A^+} > y_{L_0^+}$, let L_1^- be the intersection point between $y = y_\Gamma^-(x, L_0^+)$ and $x = ET_2$, L_1 be the phase point of L_1^- under the impulse $x^+ = x - px$ and $y^+ = y - qy$, L_1^+ be the phase point of L_1 under the impulse $x^+ = x + \alpha_{(1-p)ET_2}$.

Similarly, let L_k^- be the intersection point between $y = y_\Gamma^-(x, L_{k-1}^-)$ and $x = ET_2$, L_k be the phase point of L_k^- and L_k^+ be the phase point of L_k as long as $y_{L_k^-} < y_{Q^-}$. Denote $l_0 \triangleq 0$ and $l_k \triangleq d(L_k^-, N^-)$. Obviously, $l_k > l_{k-1}$. Then $y_{L_k^+} = \alpha_{(1-p)ET_2} + (1-q)\overline{y}^\lambda_{ET_2} + (1-q)l_k$ and $f_{\mathrm{sor}}(L_{k-1}^+) = y_{L_k^+} - y_{L_{k-1}^-} = (1-q)(l_k - l_{k-1})$. Next, it is shown that $\exists K_0 > 0$ such that $y_{L_k^-} \ge y_{Q^-}$, which also means that the trajectory starting from the point in $\overline{L_0^+A^+}$ cannot form an order-1 periodic orbit. Otherwise, a monotone increasing sequence $\{l_k\}_{k\in\mathbb{N}}$ is obtained, which is bounded by $l_{\max} \triangleq (1-q)^{-1}\overline{y}^\lambda_{(1-p)ET_2} - \overline{y}^\lambda_{ET_2}$, there exists a limit for the sequence denoted by l, i.e. $l_k \to l < l_{\max}$ when $k \to \infty$. In this case, $y_{L_k^+} \to y_L \triangleq \alpha_{(1-p)ET_2} + (1-q)\overline{y}^\lambda_{ET_2} + (1-q)l$, $L_k^- \to L^-$ and $L_k^+ \to L^+$ when $k \to \infty$. Therefore, by the continuity of the successor function f_{sor}, there is $f_{\mathrm{sor}}(L^+) = f_{\mathrm{sor}}(\lim_{k\to\infty} L_k^+) = \lim_{k\to\infty} f_{\mathrm{sor}}(L_k^+) = \lim_{k\to\infty}(1-q)(l_k - l_{k-1}) = 0$, i.e. system (2.2) admits an order-1 periodic orbit.

On the other hand, denote $y_{A^+}(x) = y_\Gamma^-(x, A^+)$ and $y_{L^+}(x) = y_\Gamma^-(x, L^+)$, where $ET_1 \le x \le ET_2$. Denote

$$\delta_{AL}(x) \triangleq y_{A^+}(x) - y_{L^+}(x).$$

Then

$$\begin{aligned}
\delta'_{AL}(x) &= y'_{A^+}(x) - y'_{L^+}(x) \\
&= \frac{y_{A^+}(\rho bx - d)}{x(r - rx/K - by_{A^+})} - \frac{y_{L^+}(\rho bx - d)}{x(r - rx/K - by_{L^+})} \\
&= \frac{\varrho bx - d}{x}[g(y_{A^+}) - g(y_{L^+})] \\
&= \frac{\varrho bx - d}{x} g'(ET_1 + \vartheta(ET_2 - ET_1))(y_{A^+} - y_{L^+}),
\end{aligned} \tag{3.3}$$

where

$$g(y) \triangleq \frac{y}{r - rx/K - by} \tag{3.4}$$

with $g'(y) = r(1 - x/K)/(r - rx/K - by)^2 > 0$, which means that $\delta'_{AL}(x) < 0$, i.e. $\delta_{AL}(x)$ is a monotone decreasing function on $[ET_1, ET_2]$. Then it yields that

$$\delta_{AL}(ET_1) > \delta_{AL}(ET_2) > (1-q)\delta_{AL}(ET_2),$$

i.e. $d(A^+, L^+) > d(A^-, L^-) > d(A, L)$. Combined with the result that $\widehat{L^+L^-}LL^+$ is an order-1 periodic orbit, there is

$$\begin{aligned}
\alpha_{(1-p)ET_2} &= d(L, Q) + d(Q, L^+) \\
&< d(Q, L^+) + d(L^+, A^+) \\
&< d(Q, L^+) + d(L^+, Q^+) \\
&= \alpha_{(1-p)ET_2},
\end{aligned}$$

which leads to a contradiction. □

Corollary 3.1. *For a lower chemical control strength, i.e.* $0 < p \le p_T$*, system (2.2) does not admit an order-1 periodic orbit if* $\overline{\alpha}_1 < \alpha_{\min}((1-p)ET_2)$.

For $\alpha_{(1-p)ET_2} > \overline{\alpha}_1$ in case of $\overline{\alpha}_1 < \alpha_{\max}((1-p)ET_2)$, let us define

$$\overline{\alpha}_2 \triangleq \overline{\alpha}_1 + \overline{y}^{\lambda}_{(1-p)ET_2} - (1-q)\overline{y}^{\lambda}_{ET_2} \tag{3.5}$$

and

$$\overline{\alpha}_3 \triangleq \max_{\alpha \in [\overline{\alpha}_1, \alpha_{\max}((1-p)ET_2)]} \left\{ \alpha \,|\, \frac{(1-q)y^-_{\Gamma}(ET_2, Q_\alpha)}{\overline{y}^{\lambda}_{(1-p)ET_2} + \overline{\alpha}_1} \le 1 \right\}, \tag{3.6}$$

where $Q_\alpha \triangleq Q_\alpha((1-p)ET_2, \overline{y}^{\lambda}_{(1-p)ET_2} + \alpha)$.

Theorem 3.4. *For a lower chemical control strength, i.e.* $0 < p \le p_T$*, system (2.2) admits an order-2 periodic orbit if and only if* $\overline{\alpha}_2 < \alpha_{(1-p)ET_2} \le \overline{\alpha}_3$ *in case of* $\overline{\alpha}_2 \ge \alpha_{\min}((1-p)ET_2)$ *or* $\alpha_{(1-p)ET_2} \le \overline{\alpha}_3$ *in case of* $\overline{\alpha}_2 < \alpha_{\min}((1-p)ET_2)$*, where* $\overline{\alpha}_3$ *is defined by Eq. (3.6).*

Proof. It is clear that $\overline{\alpha}_2 < \alpha_{\max}((1-p)ET_2)$ implies $\overline{\alpha}_2 < \overline{\alpha}_3$.

"$\Leftarrow$"

To show the sufficiency, it only needs to find a point $L^+ \in \overline{N^+Q^+}$, where N^+ is the phase point of N under the impulse effect $x^+ = x$ and $y^+ = y + \alpha_{(1-p)ET_2}$. Clearly, if $\overline{\alpha}_2 < \alpha_{(1-p)ET_2} \le \overline{\alpha}_3$, then the trajectory starting from N^+ first intersects the impulsive set $\overline{P^-N^-}$ at the point $\overline{N}^-$, and then jumps to the point $\overline{N} \in \overline{A^+Q}$ due to impulse effect $x^+ = x - px$ and $y^+ = y - qy$. Next, the trajectory starting from the point $\overline{N}$ intersects the impulsive set $\overline{P^-N^-}$ at the point $\widehat{N}^-$, and then jumps to the point $\overline{N} \in \overline{NQ}$ due to impulse effects $x^+ = x - px$ and $y^+ = y - qy$. Since $y_{\overline{N}} < y_Q = \overline{y}^{\lambda}_{(1-p)ET_2}$, then $\widehat{N}$ jumps to the point $\widehat{N}^+ \in \overline{N^+Q^+}$ due to impulse effects $x^+ = x$ and $y^+ = y - qy$. Thus, there is $f_{\mathtt{sor}}(N^+) = y_{\widehat{N}^+} - y_{N^+} > 0$. Similarly, there is $f_{\mathtt{sor}}(Q^+) < 0$. Then by the continuity of $f_{\mathtt{sor}}$, there exists $L^+ \in \overline{N^+Q^+}$ such that $f_{\mathtt{sor}}(L^+) = 0$, i.e. the orbit $\widehat{L^+\overline{L}^-}\widehat{\overline{L}\widehat{L}^-}\widehat{L}L^+$ trajectory from L^+ forms an order-2 periodic orbit.

"$\Rightarrow$"

The necessary is equivalent to show that for $\overline{\alpha}_3 < \alpha_{(1-p)ET_2} \le \alpha_{\max}((1-p)ET_2)$ in case of $\overline{\alpha}_3 < \alpha_{\max}((1-p)ET_2)$ or $\overline{\alpha}_1 < \alpha_{(1-p)ET_2} \le \overline{\alpha}_2$, there does not exist an order-2 periodic orbit. Firstly, for the case of $\overline{\alpha} < \alpha_{(1-p)ET_2} \le \alpha_{\max}((1-p)ET_2)$, there is $y_{Q^+} > y_{\overline{A}^+}$, where $\overline{A}^+$ denotes the point $\overline{A}^+((1-p)ET_2, \overline{y}^{\lambda}_{(1-p)ET_2} + \overline{\alpha}_3)$. Obviously, the orbit starting from any point in $\overline{\overline{A}^+Q^+} \cup \overline{QL_0^+}$ cannot form an order-2 periodic orbit. Thus assume that there exists an order-2 periodic orbit $\widehat{L^+\overline{L}^-}\widehat{\overline{L}\widehat{L}^-}\widehat{L}L^+$ starting from $L^+ \in \overline{L_0^+\overline{A}^+}$. Then by the definition of order-2 periodic orbit, i.e. Definition 2.3, there is

$$\begin{aligned}
\alpha_{(1-p)ET_2} &= d(\widehat{L}, Q) + d(Q, L^+) \\
&< d(L^+, \overline{A}^+) + d(Q, L^+) \\
&< d(Q, L^+) + d(L^+, Q^+) \\
&= \alpha_{(1-p)ET_2},
\end{aligned}$$

which leads to a contradiction.

Next, for the case of $\overline{\alpha}_1 < \alpha_{(1-p)ET_2} \le \overline{\alpha}_2$, there is $y_{Q^+} > y_{A^+}$. For any point $L^+ \in \overline{N^+A^+}$, there is

$$f_{\mathrm{sor}}(L^+) > d(A^+, Q^+) + q \times d(L^+, A^+) > 0,$$

which means that the trajectory starting from L^+ will arrive at $\overline{A^+Q^+}$ after at most $[d(N^+, A^+)/d(A^+, Q^+)]$ times of impulse effects, where $[\cdot]$ represents the top integral function. Thus the order-2 periodic orbit if exists can only start from $\overline{A^+Q^+}$. Assume that $\widehat{L^+\bar{L}^-}\widehat{\bar{L}\bar{L}^-}\widehat{\bar{L}L^+}$ is the order-2 periodic orbit. Then by the definition of order-2 periodic orbit, i.e. Definition 2.3, there is

$$\begin{aligned}
\alpha_{(1-p)ET_2} &= d(N, N^+) \\
&< d(N, \widehat{L}) + d(\widehat{L}, A^+) \\
&< d(\widehat{L}, A^+) + d(A^+, L^+) \\
&= \alpha_{(1-p)ET_2},
\end{aligned}$$

which leads to a contradiction. Therefore, if an order-2 periodic orbit exists, there must be $\overline{\alpha}_2 < \alpha_{(1-p)ET_2} \le \overline{\alpha}_3$. □

Corollary 3.2. *For a lower chemical control strength, system (2.2) admits a period-k ($k \ge 3$) orbit in case of $\overline{\alpha}_1 < \alpha_{(1-p)ET_2} \le \overline{\alpha}_2$ or $\overline{\alpha}_3 < \alpha_{(1-p)ET_2} \le \alpha_{\max}((1-p)ET_2)$.*

Theorem 3.5. *For a lower chemical control strength, i.e., $0 < p \le p_T$, the order-1 periodic orbit determined in Theorem 3.3 is unique, orbitally asymptotically stable and global attractive.*

Proof. By Theorem 3.3, system (2.2) admits at least one order-1 periodic orbit when $\alpha_{(1-p)ET_2} \le \overline{\alpha}_1$. Next, it is shown that the order-1 periodic solution determined in Theorem 3.3 is unique. Otherwise, let L^+ and $\overline{L}^+$ denote the two points on $\overline{L_0^+Q^+}$ with $y_{L^+} > y_{\overline{L}^+}$ such that $f_{\mathrm{sor}}(L^+) = f_{\mathrm{sor}}(\overline{L}^+) = 0$, i.e. $\widehat{L^+L^-}LL^+$ and $\widehat{\overline{L}^+\overline{L}^-}\overline{LL}^+$ are order-1 periodic orbits. Let $y_{L^+}(x) = y_{\Gamma}^-(x, L^+)$ and $y_{\overline{L}^+}(x) = y_{\Gamma}^-(x, \overline{L}^+)$, where $x \in [ET_1, ET_2]$. Let $\delta_{L\overline{L}}(x) = y_{L^+}(x) - y_{\overline{L}^+}(x)$. Denote $l \triangleq d(L^+, \overline{L}^+) = \delta_{L\overline{L}}((1-p)ET_2)$ and $\bar{l} \triangleq d(L^-, \overline{L}^-) = \delta_{LL^+}(ET_2)$. Then by Eq. (3.3) there is

$$\delta'_{L\overline{L}}(x) = y'_{L^+}(x) - y'_{\overline{L}^+}(x) = \frac{\rho c x - d}{x} g'(ET_1 + \theta(ET_2 - ET_1))(y_{L^+} - y_{\overline{L}^+}),$$

where $g(y)$ is defined by Eq. (3.4), which means that $\delta'_{L\overline{L}}(x) < 0$, i.e. $\delta_{L\overline{L}}(x)$ is a monotone decreasing function on $[ET_1, ET_2]$. Then it yields that $l > \bar{l} > (1-q)\bar{l}$. Combined with the result that $\widehat{L^+L^-}LL^+$ and $\widehat{\overline{L}^+\overline{L}^-}\overline{LL}^+$ are two order-1 periodic orbits, there is

$$\alpha_{(1-p)ET_2} = d(\overline{L}, L) + d(L, \overline{L}^+) < d(L, \overline{L}^+) + d(\overline{L}^+, L^+) = \alpha_{(1-p)ET_2},$$

which leads to a contradiction. Thus the order-1 periodic solution determined in Theorem 3.3 is unique.

Next, it discusses the stability and attractiveness of the order-1 periodic orbit when it exists. By the definition of $\overline{\alpha}_1$, there is $y_{L^+} \le y_Q + \alpha_{(1-p)ET_2} \le y_Q + \overline{\alpha}_1 \le \overline{y}^{\lambda}_{(1-p)ET_2}$. If $y_{L^+} = y_Q + \overline{\alpha}_1$, then the trajectory of system (2.2) starting from any point in Ω will arrive at the segment $\overline{QL^+}$. Thus it is only necessary to consider the tendency of the trajectory starting from $\overline{QL^+}$. For the case of $y_{L^+} < y_Q + \overline{\alpha}_1$, it should consider the tendency of the trajectory starting from $\overline{QL^+}$ and $\overline{L^+Q^+}$, respectively.

For any $P_0^+ \in \overline{QL^+}$, a sequence $\{P_k^+\}|_{k=0,1,2,\ldots}$ on $\overline{QL^+}$ can be obtained such that $y_{P_{k+1}^+} = y_{P_k^+} + f_{\mathrm{sor}}(P_k^+)$; and for any $\overline{P}_0^+ \in \overline{L^+Q^+}$, a sequence $\{\overline{P}_k^+\}|_{k=0,1,2,\ldots}$ on $\overline{L^+Q^+}$ can be obtained such that $y_{\overline{P}_{k+1}^+} = y_{\overline{P}_k^+} + f_{\mathrm{sor}}(\overline{P}_k^+)$. From the proof of the uniqueness of the order-1 periodic orbit, it can be observed that the successor function f_{sor} satisfies the following properties:

- $f_{\mathrm{sor}}(P_0^+) = 0$ if and only if $P_0^+ = L^+$;
- $f_{\mathrm{sor}}(P_0^+) > 0$ for $P_0^+ \in \overline{QL^+}$, $P_0^+ \ne L^+$;
- $f_{\mathrm{sor}}(P_0^+) < 0$ for $P_0^+ \in \overline{L^+Q^+}$, $P_0^+ \ne L^+$.

This means that $\{y_{P_k^+}\}$ is a monotone increasing sequence with an upper bound y_{L^+}, and $\{y_{\overline{P}_k^+}\}$ is a monotone decreasing sequence with an upper bound y_{L^+}, then there exist $y_{L_1^+}$ and $y_{L_2^+}$ such that $y_{P_k^+} \to y_{L_1^+}$ and $y_{\overline{P}_k^+} \to y_{L_2^+}$ when $k \to \infty$. But $f_{\mathrm{sor}}(L_1^+) = f_{\mathrm{sor}}(L_2^+) = 0$ implies that $L_1^+ = L_2^+ = L^+$. Thus the unique order-1 periodic orbit $\widehat{L^+L^-}LL^+$ is orbitally asymptotically stable. Since the points P_0^+ and $\overline{P}_0^+$ are arbitrary, the orbit asymptotical stability implies the global attractiveness. □

Theorem 3.6. *For a lower chemical control strength, i.e., $0 < p \le p_T$, the order-2 periodic orbit determined in Theorem 3.4 is unique, orbitally asymptotically stable and global attractive.*

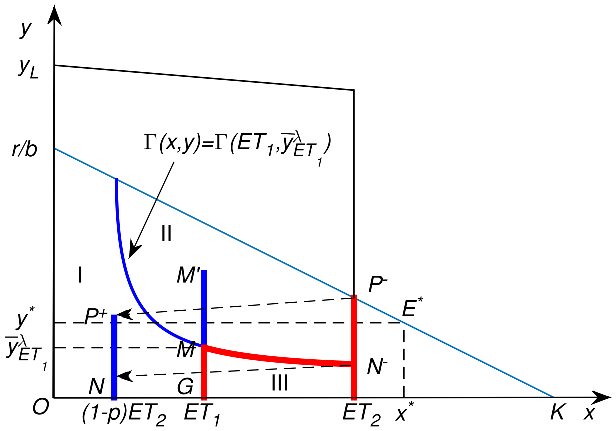

Fig. 4. The partition of the region Ω and illustration of impulse set and phase set for $1 - ET_1/ET_2 < p < 1$.

Proof. The proof is similar to that of Theorem 3.5 and omitted thereby.

Theorem 3.7. *For a lower chemical control strength, i.e., $0 < p \leq p_T$, system (2.2) admits a unique periodic orbit, which is orbitally asymptotically stable and global attractive.*

Proof. The proof is similar to that of Theorem 3.5 and omitted thereby.

Remark 3.1. Theorem 3.2 (i.e., Bendixson region method) guarantees the existence of the periodic orbit, but type of the periodic orbit cannot be determined. Theorems 3.3 and 3.4 (i.e., successor function method) give the condition to ensure the existence of a specific type of the periodic orbit, i.e. order-1 and order-2 periodic orbits, but the number of the periodic orbits cannot be determined. Theorems 3.5 and 3.6 (i.e., geometric method) give the number of the periodic orbits and the stability of the periodic orbit.

3.2. Dynamical analysis for a higher chemical control strength

In Section 3.1, it is shown that the natural enemy releasing quantity $\alpha_{(1-p)ET_2}$ plays a key effect in determining the existence of the order-1 periodic orbit for a lower chemical control strength. For a higher chemical control strength, i.e., $(1-p)ET_2 < ET_1$, system (2.2) does not admit an order-1 periodic orbit due to the impulse effect $x^+ = x - px$, $y^+ = y - qy$. Thus in this subsection, it mainly discusses the existence of period-2 orbit. Similar to the analysis in Section 3.1, it is only necessary to analyze the tendency of the system (2.2)'s solution starting from the segment $\overline{P^+N^+}$, as illustrated in Fig. 4.

Theorem 3.8. *For a higher chemical control strength, i.e. $p_T < p < 1$, system (2.2) admits at least one periodic orbit.*

Proof. Clearly, the region $P^+M^+P\widehat{NM}GN^+$ constructs a Bendixson region with boundaries $\overline{P^+M^+}$, $\overline{M^+P}$, $\overline{PN}$, $\widehat{NM}$, $\overline{MG}$ and $\overline{GN^+}$, then by Lemma 2.1, system (2.2) admits at least one periodic orbit in $P^+M^+P\widehat{NM}GN^+$. $\square$

Theorem 3.9. *For a higher chemical control strength, i.e. $p_T < p < 1$, system (2.2) admits an order-2 periodic orbit if α_{ET_1} satisfies that $\alpha_{\min}(ET_1) \leq \alpha_{ET_1} \leq \overline{\alpha}_4$; but for $\alpha_{ET_1} > \overline{\alpha}_4$, an order-2 periodic orbit exists if and only if $\overline{p} \leq p < 1$, where*

$$\overline{p} \triangleq \min_{p \geq p_T} \left\{ p | \frac{y^-_\Gamma(ET_1, ((1-p)ET_2, (1-q)\overline{y}^{\lambda}_{ET_2}))}{\overline{y}^{\lambda}_{ET_1}} \leq 1 \right\}, \tag{3.7}$$

and

$$\overline{\alpha}_4 \triangleq \max_{\alpha \leq \alpha_{\max}(ET_1)} \left\{ \alpha | \frac{y^-_\Gamma(ET_2, (ET_1, \overline{y}^{\lambda}_{ET_1} + \alpha))}{(1-q)^{-1} y^-_\Gamma((1-p)ET_2, \overline{M})} \leq 1 \right\}, \tag{3.8}$$

with $\overline{M} \triangleq \overline{M}((1-\overline{p})ET_2, (1-q)\overline{y}^{\lambda}_{ET_2})$.

Proof. To show the existence of order-2 periodic orbit in case of $\alpha_{ET_1} \leq \overline{\alpha}_4$, it is sufficient to find a point $L^+ \in \overline{P^+N^+}$ such that $f_{\mathtt{sor}}(L^+) = 0$. Since $y^-_\Gamma(ET_1, N^+) < \overline{y}^{\lambda}_{ET_1}$ for any $\overline{y}^{\lambda}_{ET_1} > 0$, then there is $f_{\mathtt{sor}}(N^+) > 0$. So the remaining problem is to find a point $Q^+ \in \overline{P^+N^+}$ such that $f_{\mathtt{sor}}(Q^+) \leq 0$.

Clearly, $\overline{p}$ defined by Eq. (3.7) satisfies that $p_T < \overline{p} < 1$.

If $\overline{p} \leq p < 1$, choose $Q^+ = P^+$, then for any α_{ET_1} satisfying (P_3), there is $f_{\mathrm{sor}}(Q^+) < 0$.

If $p_T < p < \overline{p}$, denote M^+ as the intersection point between the trajectory starting from $\overline{M}((1-\overline{p})ET_2, (1-q)\overline{y}^{\lambda}_{ET_2})$ and $x = (1-p)ET_2$, i.e. $M^+((1-p)ET_2, y_{M^+})$, where $y_{M^+} = y^-_{\Gamma}((1-p)ET_2, \overline{M})$.

If $\alpha_{ET_1} \leq \overline{\alpha}_4$, then $f_{\mathrm{sor}}(M^+) \leq 0$. If $f_{\mathrm{sor}}(M^+) = 0$, the orbit $\widehat{M^+M}\widehat{M'M^-}M^+$ forms an order-2 periodic orbit. Else, choose $Q^+ = M^+$, then there is $f_{\mathrm{sor}}(Q^+) < 0$.

Then by the continuity of f_{sor}, there exists a point $L^+ \in \overline{Q^+N^+}$ such that $f_{\mathrm{sor}}(L^+) = 0$, which means that the orbit $\widehat{L^+L}\widehat{L'L^-}L^+$ forms an order-2 periodic orbit.

For $\overline{\alpha}_4 < \alpha_{ET_1} < \alpha_{\max}(ET_1)$, the sufficiency is directly from the above proof since the existence of order-2 periodic orbit is not dependent on α_{ET_1} in case of $\overline{p} \leq p < 1$. To show the necessity, it only needs to prove that the order-2 periodic orbit does not exist for $p_T < p < \overline{p}$. Since $\alpha_{ET_1} > \overline{\alpha}_4$, then there is $f_{\mathrm{sor}}(M^+) = y_{\overline{M^+}} - y_{M^+} > 0$, where $\overline{M^+}$ is the successor point of M^+. Assume system (2.2) admits an order-2 periodic orbit denoted by $\widehat{L^+L}\widehat{L'L^-}L^+$, where $L^+ \in \overline{M^+N^+}$. Thus followed by the proof Theorem 3.3, there is

$$
\begin{aligned}
d(M^+, L^+) > d(M, L) &= d(M', L') \\
&> d(M^-, L^-) \\
&= d(\overline{M}^+, L^+)/(1-q) \\
&> d(M^+, L^+),
\end{aligned}
$$

which leads to a contradiction. Therefore, the order-2 periodic orbit does not exist. $\square$

Corollary 3.3. *For a higher chemical control strength, i.e. $p_T < p < 1$, in case of $\overline{\alpha}_4 < \alpha_{\min}(ET_1)$, system (2.2) admits an order-2 periodic orbit if and only if $\overline{p} \leq p < 1$.*

Even though Theorem 3.9 gives a condition to ensure the existence of order-2 periodic orbit in case of $\alpha_{ET_1} > \overline{\alpha}_4$, however in some cases the condition $\overline{p} \leq p < 1$ is a little extreme since the chemical control strength p in this case might be too high and also impossible in reality.

Theorem 3.10. *For a higher chemical control strength, i.e. $p_T < p < 1$, system (2.2) admits an order-2 periodic orbit if and only if $q \geq \overline{q}_1$ and $\alpha_{ET_1} \leq \overline{\alpha}_5$, where*

$$
\overline{q}_1 \triangleq 1 - \frac{\overline{\tau}_2}{y^-_{\Gamma}(ET_2, (ET_1, 2\overline{y}^{\lambda}_{ET_1}))} \tag{3.9}
$$

and

$$
\overline{\alpha}_5 \triangleq \max_{\alpha \geq \alpha_{\min}(ET_1)} \left\{ \frac{y^-_{\Gamma}(ET_2, (ET_1, \overline{y}^{\lambda}_{ET_1} + \alpha))}{(1-q)^{-1}\overline{\tau}_2} \leq 1 \right\} \tag{3.10}
$$

with

$$
\overline{\tau}_2 \triangleq \max_{\tau \geq \overline{y}^{\lambda}_{ET_1}} \left\{ \tau \middle| \frac{y^-_{\Gamma}(ET_1, ((1-p)ET_2, \tau))}{\overline{y}^{\lambda}_{ET_1}} \leq 1 \right\}.
$$

Proof. The proof is similar to that of Theorem 3.9 and omitted thereby.

Theorem 3.11. *For a higher chemical control strength, i.e., $p_T < p < 1$, the order-2 periodic orbit determined in Theorem 3.9 is unique, orbitally asymptotically stable and global attractive.*

Proof. By Theorem 3.9, system (2.2) admits at least one order-2 periodic orbit if $\alpha_{ET_1} \leq \overline{\alpha}_4$ or $\alpha_{ET_1} > \overline{\alpha}_4$ and $\overline{p} \leq p < 1$. Next, it is shown that the order-2 periodic orbit determined by Theorem 3.5 is unique. Otherwise, let L^+ and $\overline{L}^+$ denote two points on $\sum^+_p$ with $y_{L^+} > y_{\overline{L}^+}$ such that $f_{\mathrm{sor}}(L^+) = f_{\mathrm{sor}}(\overline{L}^+) = 0$, i.e. $\widehat{L^+L}\widehat{L'L^-}L^+$ and $\widehat{\overline{L}^+\overline{L}}\widehat{\overline{L}'\overline{L}^-}\overline{L}^+$ are two order-2 periodic orbits. Similar to the proof of Theorem 3.4, there is

$$
\begin{aligned}
d(L^+, \overline{L}^+) &= (1-q)d(L^-, \overline{L}^-) \\
&< (1-q)d(L', \overline{L}') \\
&= (1-q)d(L, \overline{L}) \\
&< (1-q)d(L^+, \overline{L}^+),
\end{aligned}
$$

which leads to a contradiction. Thus, the order-2 periodic solution is unique.

Next, it is shown that the order-2 periodic orbit is orbitally asymptotically stable and global attractive. Denote the order-2 periodic orbit by $\widehat{L^+LL'}\widehat{L'L^-}L^+$. In case of $\overline{p} \le p < 1$, the phase set is the segment $\overline{N^+P^+}$, thus $L^+ \in \overline{N^+P^+}$. While in case of $p_T < p < \overline{p}$, the real phase set is the segment $\overline{N^+M^+}$, thus $L^+ \in \overline{N^+M^+}$. If the point L^+ is the endpoint P^+ in case of $\overline{p} \le p < 1$ or M^+ in case of $p_T < p < \overline{p}$, it is only necessary to analyze one side (i.e., from below) stability, otherwise, it should consider two sides stability. Firstly, for the case of $L^+ \ne P^+$ (or $L^+ \ne M^+$), for any point $P_0^+ \in \overline{L^+P^+}$ (or $P_0^+ \in \overline{L^+M^+}$), a sequence $\{P_k^+\}|_{k=0,1,2,\ldots}$ on $\overline{QL^+}$ on $\overline{L^+P^+}$ (or $\overline{L^+M^+}$) can be obtained such that $y_{P_{k+1}^+} = y_{P_k^+} + f_{\mathtt{sor}}(P_k^+)$; and for any point $\overline{P}_0^+ \in \overline{L^+N^+}$, a sequence $\{\overline{P}_k^+\}|_{k=0,1,2,\ldots}$ on $\overline{L^+N^+}$ can be obtained such that $y_{\overline{P}_{k+1}^+} = y_{\overline{P}_k^+} + f_{\mathtt{sor}}(\overline{P}_k^+)$. From the proof of the uniqueness of the order-1 periodic orbit, it can be observed that the successor function $f_{\mathtt{sor}}$ satisfies the following properties:

- $f_{\mathtt{sor}}(P_0^+) = 0$ if and only if $P_0^+ = L^+$;
- $f_{\mathtt{sor}}(P_0^+) > 0$ for $P_0^+ \in \overline{N^+L^+}$, $P_0^+ \ne L^+$;
- $f_{\mathtt{sor}}(P_0^+) < 0$ for $P_0^+ \in \overline{L^+P^+}$ (or $P_0^+ \in \overline{L^+M^+}$), $P_0^+ \ne L^+$.

This means that $\{y_{P_k^+}\}$ is a monotone decreasing sequence with an upper bound y_{L^+}, and $\{y_{\overline{P}_k^+}\}$ is a monotone increasing sequence with an upper bound y_{L^+}, then there exist $y_{L_1^+}$ and $y_{L_2^+}$ such that $y_{P_k^+} \to y_{L_1^+}$ and $y_{\overline{P}_k^+} \to y_{L_2^+}$ when $k \to \infty$. But $f_{\mathtt{sor}}(L_1^+) = f_{\mathtt{sor}}(L_2^+) = 0$ implies that $L_1^+ = L_2^+ = L^+$. Thus the unique order-2 periodic orbit $\widehat{L^+LL'}\widehat{L'L^-}L^+$ is orbitally asymptotically stable. Since the points P_0^+ and $\overline{P}_0^+$ are arbitrary, the orbit asymptotical stability implies the global attractiveness. □

Corollary 3.4. *For a higher chemical control strength, i.e., $p_T < p < 1$, if $q \ge \overline{q}_2$, where*

$$\overline{q}_2 \triangleq 1 - \max_{\tau \ge \overline{y}^{\lambda}_{ET_1}} \left\{\tau \,|\, y^-_{\Gamma}(ET_1, ((1-p)ET_2, \tau)) \le \overline{y}^{\lambda}_{ET_1}\right\} / y^-_{\Gamma}(ET_2, (ET_1, \overline{y}^{\lambda_{ET_1}})),$$

then system (2.2) admits a unique order-2 periodic orbit for any $\alpha_{(1-p)ET_2}$ satisfying $(P3)$, which is orbitally asymptotically stable and global attractive.

3.3. Control optimization

To determine the optimum chemical control strength and yield of releases of the predator, the following optimization problem is considered.

Let c_1 denote the unit cost of releases of the predator, c_2 be the unit cost of the chemical control including the price of chemical agent and the damage to environment. The objective is to minimize the cost per unit time in this process. Let $F_{\mathtt{cost}}$ denote the total cost in one period $T = T(\alpha, p, q)$, which is a function of the chemical control strength p and yield of releases of predator α, i.e. $F_{\mathtt{cost}}(\alpha, p) = c_1 n_1 \alpha + c_2 n_2 p$, where n_1 is the number of biological control and n_2 is the number of chemical control. Since the strength q is linearly dependent on p, i.e. $q = \tau p$, where τ is a constant, then the period T is only dependent on p and α, i.e. $T = T(\alpha, p)$. Thus the optimization model can be formulated as

$$\begin{array}{ll} \min & P(\alpha, p) = F(\alpha, p)/T(\alpha, p) \\ \text{s.t.} & 0 < p < 1, \alpha_{\min} \le \alpha \le \alpha_{\max}. \end{array} \tag{3.11}$$

Solving the optimization problem (3.11) yields the optimum yield of releases of the predator α^* and chemical control strength p^*.

4. Numerical simulations

In this section, a specific example is presented to verify the theoretical results obtained in the previous section by considering the change of the control parameters p, q and α_x.

Let $r = 1.2$, $K = 100$, $b = 0.1$, $\varrho = 5\% = 0.05$, $d = 35\% = 0.35$. With a simple calculation, the free system's steady state $(x^*, y^*) = (70, 3.6)$ is obtained. The prey damage (or chemical control) level ET_2 is assumed to be about 60% of the environmental carrying capacity, i.e., $ET_2 = 60\%K = 60$, and the prey slightly harm (or biological control) level ET_1 is assumed to be about 30% of the environmental carrying capacity, i.e., $ET_1 = 30\%K = 30$. The predator maintainable level at ET_1 is assumed to be about 10% of steady predator density of the free system, i.e., $\overline{y}^{\lambda}_{ET_1} = 10\%y^* = 0.36$.

4.1. Verification for the lower chemical control strength

Let $p = 40\%$ and $q = 20\%$. The predator maintainable level at $(1-p)ET_2$ is about 95% of the one at ET_1, i.e., $\overline{y}^{\lambda}_{(1-p)ET_2} = 95\% \times 0.36 = 0.342$. The parameter θ_x in Eq. (2.4) is set to 0.01, then there is $\alpha_{ET_1} = \alpha_{\min}(ET_1) + \theta_x(\alpha_{\max}(ET_1) - \overline{y}^{\lambda}_{ET_1}) = 0.4368$ and $\alpha_{(1-p)ET_2} = 0.4145$.

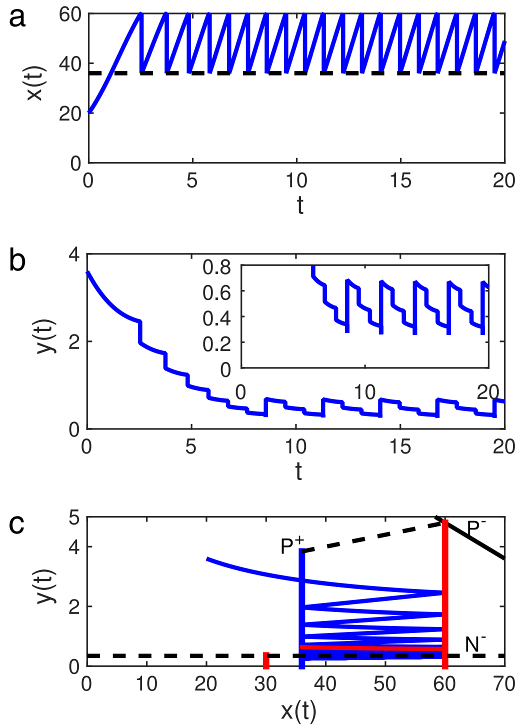

Fig. 5. The change in prey density (a), predator density (b) and phase diagrams (c) starting from $(x_0, y_0) = (10, 3.6)$. Control parameters: $p = 40\%$, $q = 20\%$, $ET_1 = 30$, $ET_2 = 60$, $\bar{y}^{\lambda}_{ET_1} = 0.36$, $\bar{y}^{\lambda}_{(1-p)ET_2} = 0.342$, $\alpha_{ET_1} = 0.4368$, $\alpha_{(1-p)ET_2} = 0.4145$.

The time series and phase diagrams of prey density and predator density are shown in Fig. 5, where the initial prey density is 10% of the environmental carrying capacity, i.e., $x_0 = 10\%K = 10$, and the initial predator density is the predator density at steady state of the free system, i.e., $y_0 = y^* = 3.6$. From Fig. 5(c) it can be observed that the orbit tends to be periodic.

For such control parameters, it is easily checked that $\overline{\alpha}_1$ defined in Theorem 3.3 is equal to 0.13, i.e., $\overline{\alpha}_1 = 0.13 < \alpha_{\min}((1-p)ET_2)$, by Corollary 3.1, system (2.2) does not admit order-1 periodic orbit. Since $\alpha_{(1-p)ET_2} > \overline{\alpha}_2 = 0.2214$ and $\overline{\alpha}_3 = 0.3111 < \alpha_{(1-p)ET_2}$, by Theorem 3.4, system (2.2) does not admit an order-2 periodic orbit. By Corollary 3.2, a period-k orbit exists in case of $\alpha_{(1-p)ET_2} > \overline{\alpha}_3$, where $k \geq 3$. From Fig. 5(b) it can be observed that the limit orbit forms a period-3 orbit, which is presented in Fig. 6.

When the natural enemy releasing quantity $\alpha_{(1-p)ET_2}$ increases to, for example, 0.694, i.e. θ_x in Eq. (2.4) is set to 0.05, the period-3 orbit disappears and a period-4 orbit occurs, as shown in Fig. 7.

Since $\alpha_{(1-p)ET_2} \geq \alpha_{\min}((1-p)ET_2)$ is guaranteed, thus to verify the existence of order-2 or order-1 periodic orbit, the threshold $\overline{\alpha}_1$ has to be enlarged. An easier way is to increase q, for example, from 20% to 35%, which leads to $\overline{\alpha}_1$ increased from 0.13 to 0.24, $\overline{\alpha}_3$ increased from 0.3111 to 0.658, i.e. $\overline{\alpha}_2 < \alpha_{(1-p)ET_2} < \overline{\alpha}_3$, then by Theorem 3.4, system (2.2) admits an order-2 periodic orbit, as shown in Fig. 8(b).

When q is increased to 40%, $\overline{\alpha}_1$ increases to 0.2924, then $\alpha_{(1-p)ET_2} < \overline{\alpha}_2$. By Corollary 3.2, a period-k orbit exists in case of $\alpha_{(1-p)ET_2} < \overline{\alpha}_2$, where $k \geq 3$. From Fig. 9(b) it can be observed that the limit orbit forms a period-8 orbit.

When q is increased to 50%, $\overline{\alpha}_1$ increases to 0.4184, then $\alpha_{(1-p)ET_2} < \overline{\alpha}_1$. By Theorem 3.3, system (2.2) admits an order-1 periodic orbit, as shown in Fig. 10(b).

4.2. Verification for the higher chemical control strength

Let $p = 60\%$ and $q = 20\%$, the other parameters are kept the same as above. The time series and phase diagrams of prey density and predator density with initial values $(x_0, y_0) = (10, 3.6)$ are shown in Fig. 11. From Fig. 11(c) it can be observed that the orbit tends to be periodic.

It is easily checked that $\overline{p}$ and $\overline{\alpha}_4$ defined in Theorem 3.9 are $\overline{p} = 99.965\%$ and $\overline{\alpha}_4 = 0.09$. Then by Corollary 3.3, system (2.2) does not admit an order-2 periodic orbit for $p < \overline{p}$ in case of $\overline{\alpha}_4 < \alpha_{\min}(ET_1)$. Fig. 11(b) indicates that the periodic orbit is a period-3 one. Since $\overline{p}$ and also $\overline{\alpha}_4$ are dependent on q, to verify the existence of order-2 periodic orbit, it is considered to increase the value of q. With a numerical calculation, there are $\overline{\tau}_2 = 0.382$ and $\overline{q}_1 = 38.6\%$. Then by increasing q, for example from 20% to 40% yields that $\overline{\alpha}_5 = 0.378$. The time series of prey density and predator density and phase diagrams

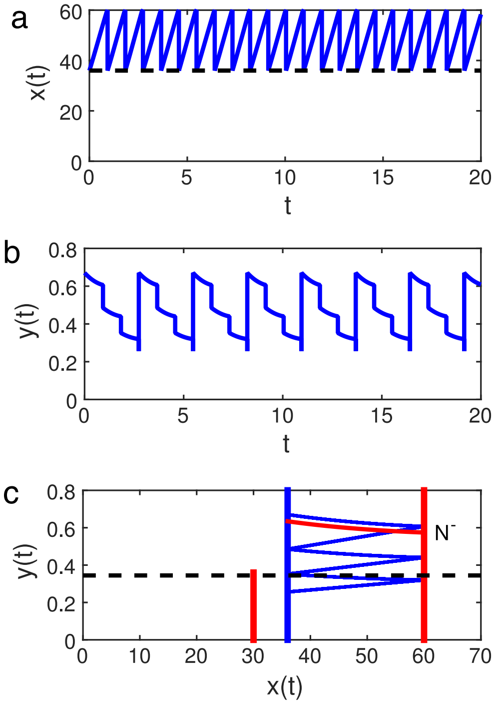

Fig. 6. The change in prey density (a), predator density (b) and phase diagrams (c) starting from $(x_0, y_0) = (36, 0.6706)$. Control parameters: $p = 40\%$, $q = 20\%$, $ET_1 = 30$, $ET_2 = 60$, $\overline{y}^{\lambda}_{ET_1} = 0.36$, $\overline{y}^{\lambda}_{(1-p)ET_2} = 0.342$, $\alpha_{ET_1} = 0.4368$, $\alpha_{(1-p)ET_2} = 0.4145$.

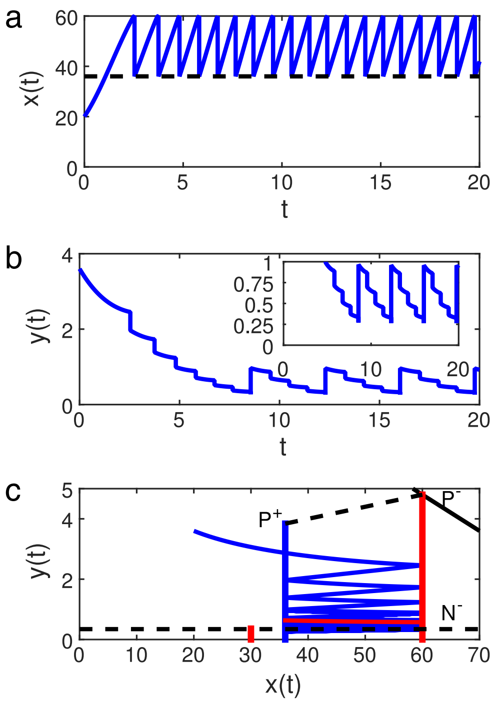

Fig. 7. The change in prey density (a), predator density (b) and phase diagrams (c) starting from $(x_0, y_0) = (10, 3.6)$. Control parameters: $p = 40\%$, $q = 20\%$, $ET_1 = 30$, $ET_2 = 60$, $\overline{y}^{\lambda}_{ET_1} = 0.36$, $\overline{y}^{\lambda}_{(1-p)ET_2} = 0.342$, $\alpha_{ET_1} = 0.4368$, $\alpha_{(1-p)ET_2} = 0.694$.

are shown in Fig. 12. In this case, by Theorem 3.10, system (2.2) does not admit an order-2 periodic orbit since $\alpha_{ET1} > \overline{\alpha}_5$. Fig. 12(b) indicates that a period-5 orbit exists.

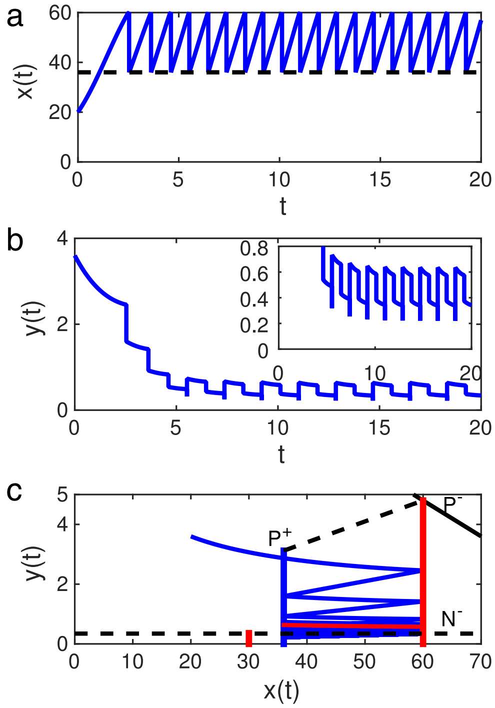

Fig. 8. The change in prey density (a), predator density (b) and phase diagrams (c) starting from $(x_0, y_0) = (20, 3.6)$. Control parameters: $p = 40\%$, $q = 35\%$, $ET_1 = 30$, $ET_2 = 60$, $\bar{y}^{\lambda}_{ET_1} = 0.36$, $\bar{y}^{\lambda}_{(1-p)ET_2} = 0.342$, $\alpha_{ET_1} = 0.4368$, $\alpha_{(1-p)ET_2} = 0.4145$.

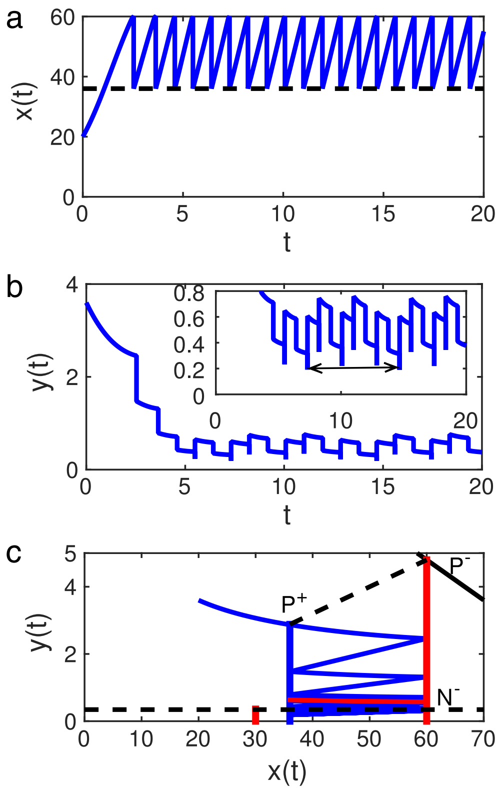

Fig. 9. The change in prey density (a), predator density (b) and phase diagrams (c) starting from $(x_0, y_0) = (10, 3.6)$. Control parameters: $p = 40\%$, $q = 40\%$, $ET_1 = 30$, $ET_2 = 60$, $\bar{y}^{\lambda}_{ET_1} = 0.36$, $\bar{y}^{\lambda}_{(1-p)ET_2} = 0.342$, $\alpha_{ET_1} = 0.4368$, $\alpha_{(1-p)ET_2} = 0.4145$.

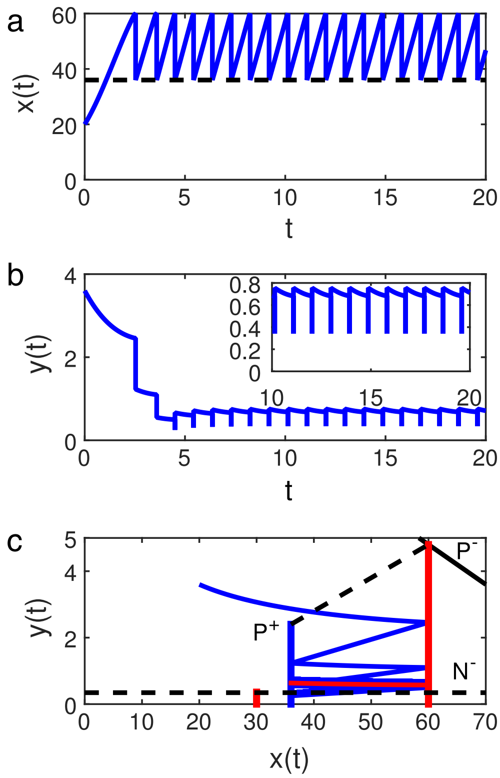

Fig. 10. The change in prey density (a), predator density (b) and phase diagrams (c) starting from $(x_0, y_0) = (10, 3.6)$. Control parameters: $p = 40\%$, $q = 50\%$, $ET_1 = 30$, $ET_2 = 60$, $\overline{y}^{\lambda}_{ET_1} = 0.36$, $\overline{y}^{\lambda}_{(1-p)ET_2} = 0.342$, $\alpha_{ET_1} = 0.4368$, $\alpha_{(1-p)ET_2} = 0.4145$.

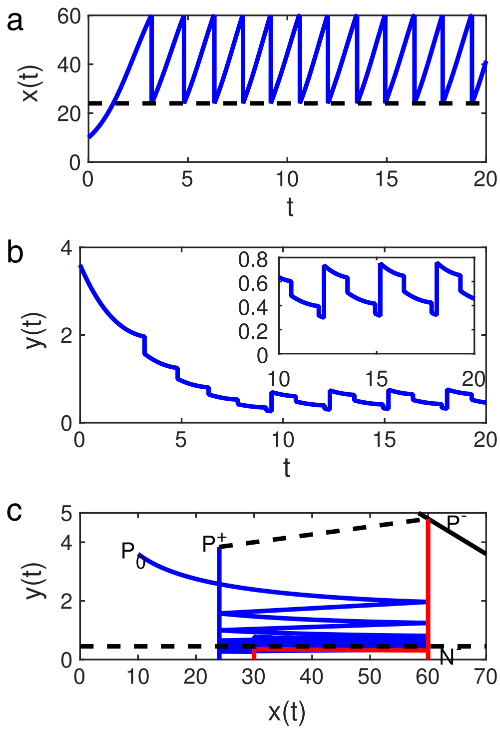

Fig. 11. The change in prey density (a), predator density (b) and phase diagrams (c) starting from $(x_0, y_0) = (10, 3.6)$. Control parameters: $p = 60\%$, $q = 20\%$, $ET_1 = 30$, $ET_2 = 60$, $\overline{y}^{\lambda}_{ET_1} = 0.36$, $\alpha_{ET_1} = 0.4368$.

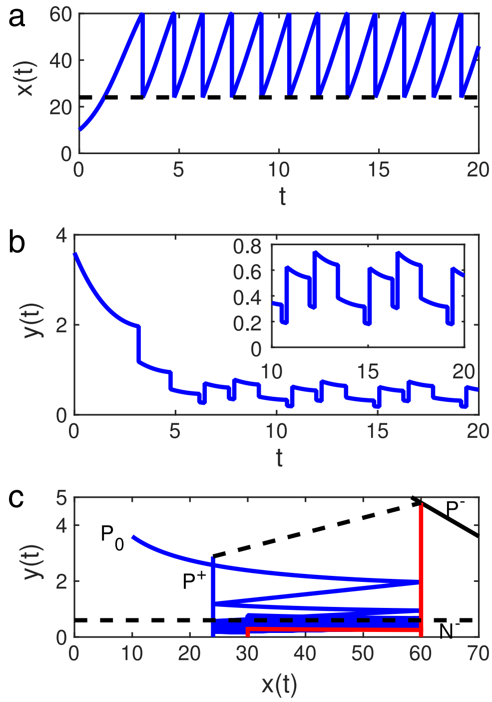

Fig. 12. The change in prey density (a), predator density (b) and phase diagrams (c) starting from $(x_0, y_0) = (10, 3.6)$. Control parameters: $p = 60\%$, $q = 40\%$, $ET_1 = 30$, $ET_2 = 60$, $\bar{y}^{\lambda}_{ET_1} = 0.36$, $\alpha_{ET_1} = 0.4368$.

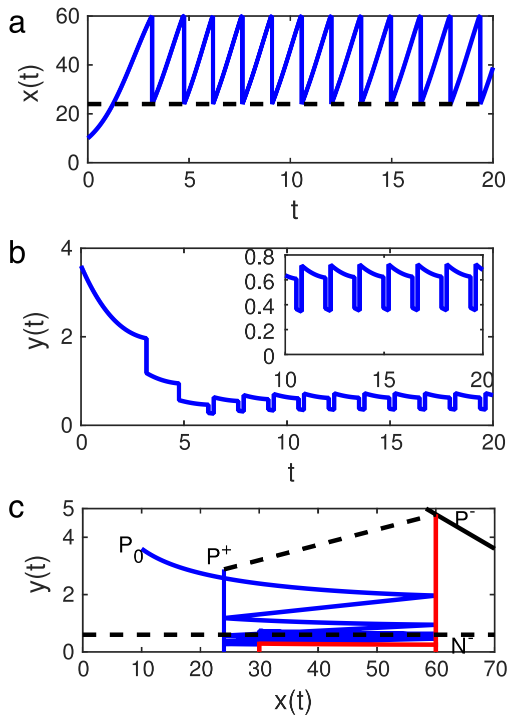

Fig. 13. The change in prey density (a), predator density (b) and phase diagrams (c) starting from $(x_0, y_0) = (10, 3.6)$. Control parameters: $p = 60\%$, $q = 40\%$, $ET_1 = 30$, $ET_2 = 60$, $\bar{y}^{\lambda}_{ET_1} = 0.36$, $\alpha_{ET_1} = 0.37$.

For $q = 40\%$, when the predator releasing quantity α_{ET_1} is less than $\overline{\alpha}_5$, for example $\alpha_{ET_1} = 0.37$, i.e. $\theta_x = 0.0014$, then by Theorem 3.10, system (2.2) admits an order-2 periodic orbit, as shown in Fig. 13(b).

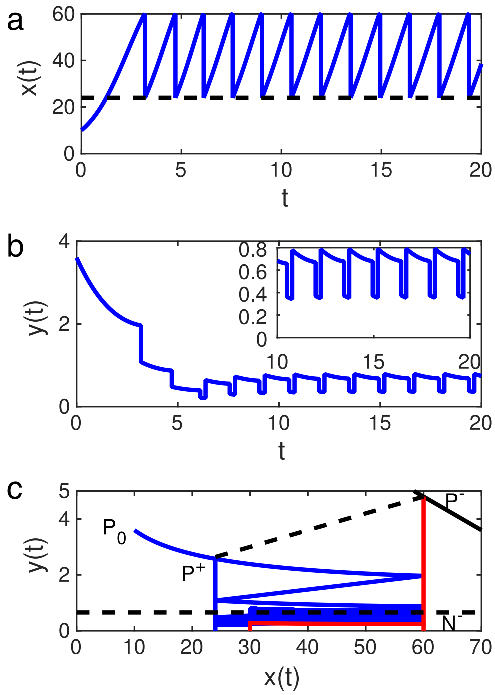

Fig. 14. The change in prey density (a), predator density (b) and phase diagrams (c) starting from $(x_0, y_0) = (10, 3.6)$. Control parameters: $p = 60\%$, $q = 45\%$, $ET_1 = 30$, $ET_2 = 60$, $\bar{y}^{\lambda}_{ET_1} = 0.36$, $\alpha_{ET_1} = 0.4368$.

Similarly, for $\alpha_{ET_1} = 0.4368$, when q increases greater than 44.39%, for example, $q = 45\%$, the time series of prey density and predator density and phase diagrams are shown in Fig. 14.

When q increases greater than $\bar{q}_2 = 90.57\%$, for example, $q = 91\% > q_2$, then for any α_{ET_1} satisfying $(P3)$, system (2.2) admits an order-2 periodic orbit, as shown in Fig. 15 for $\alpha_{ET_1} = \alpha_{\max}(ET_1)$.

The simulation results are consistent with the theoretical results, which is another aspect further validated the correctness of the conclusions.

4.3. Determination of the optimal control level

The optimal results depend on the choice of c_1, c_2 and τ, which are determined in practice. In general, for a given pest control problem, the chemical control strength is fixed, for example $p = 40\%$ and $q = 50\%p$, then the only variable to be determined is the yield release of the predator. The dependence of P_{cost} on $\alpha_{(1-p)ET_2}$ for $c_1 = 100$, $\tau = 1, 2, 5, 10$ is shown in Fig. 16. From Fig. 16 it can be seen that the optimum yield release of the predator increases with the increasing of τ. For $\tau = 1$, i.e. the price of chemical control is identical to biological control, then the yield release of the predator should be the minimum since the chemical control is more efficient than biological control. For $\tau \geq 5$, i.e., the price of chemical control is more higher than biological control, then the optimum yield release of the predator should be the maximum. While for the medial case ($\tau = 2$), the optimum yield release of the predator is $\alpha_{(1-p)ET_2} = 6.5 = \alpha_{\min} + 0.65(\alpha_{\max} - \alpha_{\min})$.

5. Conclusions and discussion

A pest management prey–predator model has been studied in which the biological control and chemical control are taken at different control thresholds. At the early stage of the pest damage outbreaks, the biological control is adopted to reduce the pests' growth in case of the predator density in the environment being lower than its maintainable level. Once the pest density reaches a critical level, above which pests may cause a serious damage to environment, the chemical control with a given strength is taken. Different from the existing models in the literature involving biological control and chemical control at different pest levels [24–29], the proposed model conforms better to the practice.

For the proposed model, it was shown that the periodic orbit always exists for any yield of releases of the predator and chemical control strength. However, to determine the frequency of the chemical control and yield of releases of the predator, only the existence of order-1 periodic orbit or order-2 periodic orbit (in case of order-1 periodic orbit does not exist) is valuable and beneficial. The theoretical analysis indicated that, the yield of releases of the predator plays a key role in determining the existence of order-1 periodic orbit. That is to say, for a lower chemical strength (i.e. $0 < p \leq p_T \triangleq 1-ET_1/ET_2$), in case of $\bar{\alpha}_1$ (defined in Eq. (3.2)) is not less than the minimum yield of releases of the predator $\alpha_{\min}((1-p)ET_2)$,

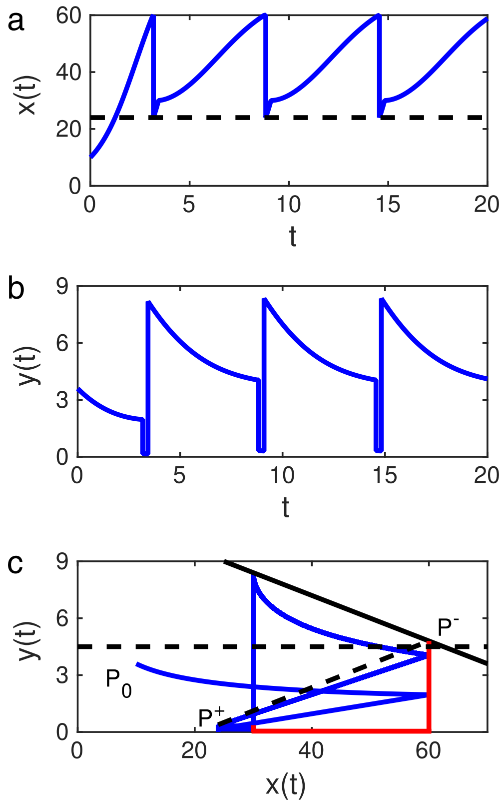

Fig. 15. The change in prey density (a), predator density (b) and phase diagrams (c) starting from $(x_0, y_0) = (10, 3.6)$. Control parameters: $p = 60\%$, $q = 92\%$, $ET_1 = 30$, $ET_2 = 60$, $\bar{y}^{\lambda}_{ET_1} = 0.36$, $\alpha_{ET_1} = \alpha_{\max}(ET_1)$.

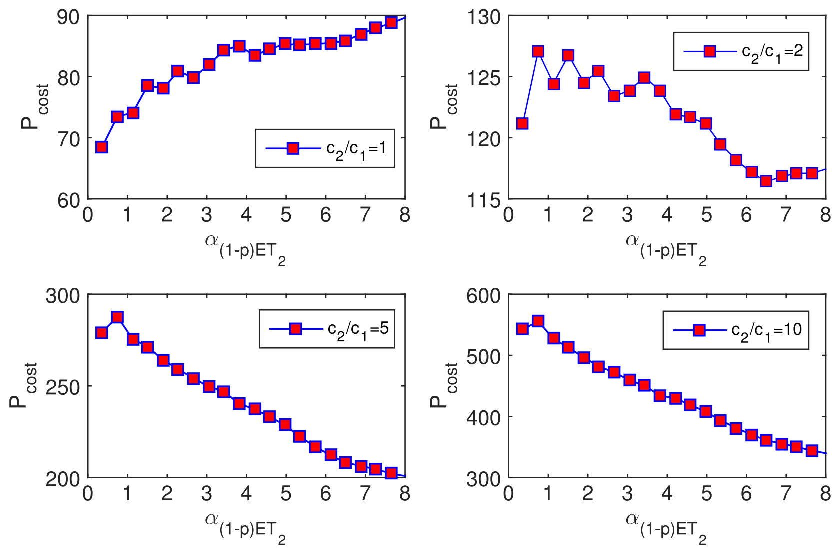

Fig. 16. The change in the cost per unit time $P_{\mathtt{cost}}$ on the yield of release of predator $\alpha_{(1-p)ET_2}$ for $c_2/c_1 = 1, 2.5, 5, 10$.

the system admits a unique, orbitally asymptotically stable and global attractive order-1 periodic orbit if the yield of releases of the predator $\alpha_{(1-p)ET_2}$ at the pest density $(1-p)ET_2$ is no more than the threshold $\bar{\alpha}_1$. When $\bar{\alpha}_1$ is less than $\alpha_{\min}((1-p)ET_2)$, the order-1 periodic orbit does not exist. But when $\alpha_{(1-p)ET_2}$ is higher than the threshold $\bar{\alpha}_2$ (defined in Eq. (3.5)) and no more than the threshold $\bar{\alpha}_3$ (defined in Eq. (3.6)), the system admits a unique, orbitally asymptotically stable and global attractive order-2 periodic orbit. When $\alpha_{(1-p)ET_2}$ lies between $\bar{\alpha}_1$ and $\bar{\alpha}_2$ or is greater than $\bar{\alpha}_2$, the system admits an order-k periodic orbit, where $k \geq 3$. While for a higher chemical strength (i.e. $p_T < p < 1$), if the yield of releases of predator α_{ET_1} at the pest density ET_1 is no more than the threshold $\overline{\alpha}_4$ (defined in Eq. (3.8)), the system admits a unique, orbitally asymptotically stable and global attractive order-2 periodic orbit. When α_{ET_1} is higher than the threshold $\overline{\alpha}_4$, the system admits an order-2 periodic orbit only if the predator chemical control strength q is higher than a threshold $\overline{q}_1$ (defined in Eq. (3.9)) and α_{ET_1} is no more than the threshold $\overline{\alpha}_5$ (defined in Eq. (3.10)). The practical significance to study the existence of order-1 or order-2 periodic orbit lies in that it could provide a possibility to determine the frequency of using chemical pesticide and yield of releases of the predator, which makes the control to be a periodic one without real-time monitoring the species while keeps the prey density below the damage level. The stability and attractiveness could ensure a certain robustness of control, i.e., even though the species density is detected inaccurately or with a deviation, the system will be eventually stable at the periodic orbit under the control action. As far as the optimum frequency of the chemical control and yield of releases of the predator of the optimization model (3.11) is concerned, it will be associated with an actual demand in practice.

Acknowledgments

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