Physica A 462 (2016) 67–83

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

A study on Improvisation in a Musical performance using Multifractal Detrended Cross Correlation Analysis

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HIGHLIGHTS

- MFDFA technique was applied to assess the musical improvisation made by a performer in different music signals of Hindustani classical music.
- MFDXA technique was applied to assess the degree of cross-correlation in similar parts of different music signals.
- The variation of multifractal widths within parts of different sample suggest how much the performer improvised in each rendition.
- Improvisational cues are also obtained from those parts which have lower degree of cross correlation as compared to other parts.
- A naïve listener, who can recognize the changes in different performance perceptually, will now be able to conclusively identify the improvisational cues in each performance.

ARTICLE INFO

Article history:
Received 18 September 2015
Received in revised form 6 April 2016
Available online 15 June 2016

Keywords:
Hindustani Raga music
Hurst exponent
MFDFA
MFDXA
Musical improvisation

ABSTRACT

MFDFA (the most rigorous technique to assess multifractality) was performed on four Hindustani music samples played on same '*raga*' sung by the same performer. Each music sample was divided into six parts and 'multifractal spectral width' was determined for each part corresponding to the four samples. The results obtained reveal that different parts of all the four sound signals possess spectral width of widely varying values. This gives a cue of the so called 'musical improvisation' in all music samples, keeping in mind they belong to the *bandish* part of the same *raga*. Formal compositions in Hindustani raga are juxtaposed with the improvised portions, where an artist manoeuvers his/her own creativity to bring out a mood that is specific for that particular performance, which is known as 'improvisation'. Further, this observation hints at the association of different emotions even in the same *bandish* of the same raga performed by the same artist, this interesting observation cannot be revealed unless rigorous non-linear technique explores the nature of musical structure. In the second part, we applied MFDXA technique to explore more in-depth about 'improvisation' and association with emotion. This technique is applied to find the degree of cross-correlation (γ_x) between the different parts of the samples. Pronounced correlation has been observed in the middle parts of the all the four samples evident from higher values of γ_x whereas the other parts show weak correlation. This gets further support from the values of spectral width from different parts of the sample — width of those parts is significantly different from other parts. This observation is extremely new both in respect of musical structure of so called improvisation and associated emotion. The importance of this study in application area of cognitive music therapy is immense.

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http://dx.doi.org/10.1016/j.physa.2016.06.013

1. Introduction

From a physical point of view, musical signals are approximately periodic in micro and macro forms. In this approach, musical signals seem to have a deterministic behavior but this is not really the case, as music would then be a deterministic issue of rational human thought [1]. On the other hand, there is a widespread opinion (in linguistic, aesthetic and cognitive philosophy) that music is a complex, and multidimensional nonlinear system [2]. A number of earlier studies are based on rhythmic and harmonic structure of the musical notes, while frequency analysis may fail to decipher the real dynamics in case of polyphonic recordings. A few studies have been done to correlate complex actions coordinated by people with complex rhythmic musical sequence [3,4]. One such study says [3] that as people listen to rhythmic structure of music; a stable multi-periodicity pattern arises psychologically, which is a manifestation of the temporal structure of the rhythm. In this study, we want to specify some parameters with which we can quantify the improvisational cues in four different renditions of a single "*raga*" performance of a Hindustani music performer.

The *raga* is a sequence of musical notes and the play of sound which delights the hearts of people. The word Raga is derived from the Sanskrit word "Ranj" which literally means to delight or please and gratify [6]. Although there are a number of definitions attributed to a Raga, it is basically a tonal multifarious module. The listener has to listen to several pieces of the Raga in order to recognize the Raga. The goal of a performer of Hindustani music is to convey the musical structure and expression so that the audience gets pleasantness. The presentation of a Raga is started with *Alap*. The *Alap* is the opening section of a typical Hindustani Music (HM) performance [7]. In the *alap* part, the *raga* is introduced and the paths of its development are revealed using all the notes used in that particular *raga* and allowed transitions between them with proper distribution over time. *Alap* is usually accompanied by the tanpura drone only and sung at a slow tempo or sometimes without tempo. Then comes the vilambit *bandish* part where the lyrics and *tala* are introduced. *Bandish* is a fixed, melodic composition in Hindustani vocal or instrumental music, set in a specific *raga*, performed with rhythmic accompaniment by a tabla or pakhawaj, a steady drone, and melodic accompaniment by a sarangi, harmonium etc. [8]. *Vilambit* is a type of *bandish* which is sung at a very slow tempo, or *laya*, of 10–40 beats per minute. In HM the existing phrases are stretched or compressed, and the same may happen to motives from the phrases; further motives may be prefixed, infixed and suffixed. Phrases may be broken up or telescoped with others, and motives or phrases may be sequenced through different registers [9]. Thus, during a performance, a singer steadily loosens the strangle hold of the rules of music in a subtle way. He does not flout them, he merely interprets them in a new way, which is the beauty of Hindustani classical music and there comes the wisdom that *Raga* and its grammar are only means and not ends in themselves. The way in which a performer interprets a *raga* during each specific performance is unique and is the very essence of improvisation in Hindustani music (HM). Unlike symphony or a concerto, *Raga* is unpredictable; it is eternally blooming, blossoming out into new and vivid forms during each and every performance which is the essence of "improvisation" [10].

Improvisation is a common form of musical practice across cultures, and yet remains scarcely studied or understood from a scientific musical analysis point of view. It is said that—in Hindustani music (HM), other than *Aarohan* (ascending), *Aborohan* (descending), *Chalan* (main phrase) and *Bandish* (composition), everything depends on the artist's own imagination, creativity, *Talim* (learning) and *Riyaz* (intense practice) [11]. There is no notation in HM system like western music and the musician is himself the composer. A musician while performing, expresses the raga according to his mood and environment surrounding him. Thus, there are differences from one rendition to another. Even if an artist sings or play same *Raga* and same *Bandish* twice, then there is supposed to be some dissimilarity in between the two performances. These differences in the rendition of a raga several times on different days are generally called improvisation. A number of studies in ethnomusicology reports musical tradition among performers and the interactions that play an important role shaping the social hierarchy of North Indian Classical music [12–14]. In Western musicology, improvisation is considered as an opposite of composition, hence traditionally been regarded as an inferior to art music, where the importance of pre-composition is paramount [15]. The situation is a stark contrast in Hindustani classical music, where "improvisation" is the central and defining term in any performance. Improvisation is crucial and indispensible feature of Hindustani Music (HM) which depends upon the imagination, originality and ingenuity of a particular artist [8] and can be best identified by analyzing the variation imposed by the artist in different renditions of the same musical piece. There have been a number of approaches to study improvisations, especially in jazz and folk music [16–18], while in music therapy, the analysis of improvisations is gaining more ground in recent years, informing directly the therapeutic process [19–22]. Another recent study [23] using cross wavelet spectral analysis sheds new light on the spontaneous improvisation made by the coordination of the musician with his/her co-performers to produce novel musical expressions. Performative gestures are considered important to listening amongst all genres of music [24]. For e.g., in an analysis of B. B. King's music, it was found that some gestures have the effect of drawing the listeners' attention to local aspects of music, specifically to the nuanced treatment of individual notes, and away from larger scale musical structure [25]. The importance of gesture has been realized until recently [26,27] as something outside language; Indian music, with its emphasis on note combinations has often regarded gestures as something outside music. In Hindustani classical music, the gestures that accompany improvisation are closely coordinated with the vocal action; they are never taught explicitly and seem to come as an expression for melody. The importance of gestural dispositions in Hindustani *raga* performances has been extensively studied in Ref. [28]. A study [29] on search for emotion in Hindustani vocal music based on human response data showed that segments from the same raga elicit different emotions which can be assigned into prescribed categories. Also cross-cultural similarity of the elicited response is significant. Another recent study on Indian classical instrumental music also based on human response data categorizes the *alap* and *gat* portion of *raga* as elicitor of specific distinct emotions [30]. In the present study, for the first time, we attempt to quantify the improvisational cues in a Hindustani music performance with the help of different non-linear parameters.

At first sight music shows a complex behavior: at every instant components (in micro and macro scale: pitch, timbre, accent, duration, phrase, melody etc.) are close linked to each other [5]. All these properties (above stated in a heuristic characterization) are peculiar of systems with chaotic, self organized, and generally, nonlinear behavior. Therefore, the analysis of music using linear and deterministic frameworks seems not to be useful.

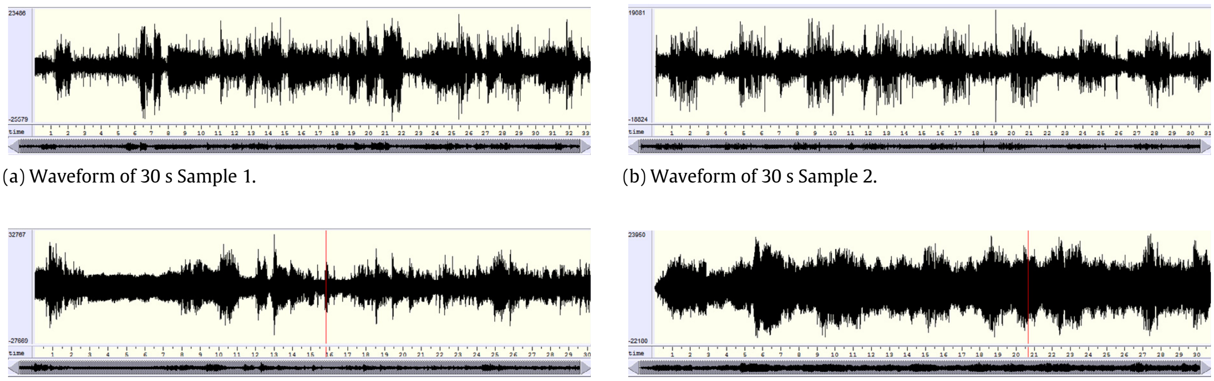
Non-linear dynamical modeling for source clearly indicates the relevance of non-deterministic/chaotic approaches in understanding the speech/music signals [31–37]. In this context fractal analysis of the signal which reveals the geometry embedded in signal assumes significance. Fractal analysis of audio signals was first performed by Voss and Clarke [38], who analyzed amplitude spectra of audio signals to find out a characteristic frequency f_c, which separates white noise (which is a measure of flatness of the power spectrum) at frequencies much lower than f_c from very correlated behavior ($\sim 1/f^2$) at frequencies much higher than f_c. Some other studies [39,40] applied fractal tools to the pitch variations and revealed irregularities in scaling behavior and long range characteristics. Music data is a quantitative record of variations of a particular quality over a period of time. One way of analyzing it is to look for the geometric features to help towards categorizing the data in terms of concept [41]. However, it is well-established experience that naturally evolving geometries and phenomena are rarely characterized by a single scaling ratio; different parts of a system may be scaling differently. That is, the clustering pattern is not uniform over the whole system. Such a system is better characterized as 'multifractal' [42]. A multifractal can be loosely thought of as an interwoven set constructed from sub-sets with different local fractal dimensions. Real world systems are mostly multifractal in nature. Music too, has non-uniform property in its movement [43,44]. In another recent study [45], multifractal technique has been applied to separate different genres of music based on their multifractal spectral width.

Therefore, the melodic fragments of a raga sung by a musician of HM on different days will produce certain changes in nature of the phrases due to improvisation. It has been shown earlier that fractal dimension calculated from a song is a good measure of the complexity of its notation [46,43]. Also earlier studies showed that same song when sung by different performers, the fractal dimension changes [47]. The fractal character is only one of many aspects that define a composition. The human mind may use one or more models of perception in order to determine whether a given melody or musical structure is ugly or beautiful [48]. Fractal dimensions of time series data might reveal the presence of non-linearity in the art of production mechanism and therefore the complexity of the time sequences of the phrases where the performer have improvised, might vary. These may be reflected through the change in their fractal dimensions. Another major objective of the present study is to see whether fractal dimensions are related to the improvisations made by the artist. Voss and Clarke [38] showed that the frequency characteristics of musical signal behave similar to the $1/f$ noise. Interestingly, this type of noise, called pink noise occurs very commonly in nature [49] and it is this noise which sounds most pleasant to the human ear. Therefore fractal analysis would be a good technique to obtain the power exponent that defines the scale invariant structure of the whole signals.

The fractal signal is of two types-multifractal and monofractal. Fractal analysis is used to find the Hurst-exponent, singularity exponent and singularity dimension and therefore to find the multifractal spectrum. A number of studies have used Detrended fluctuation analysis (DFA) of EEG signals elicited by a variety of stimuli because of its robustness to non-stationarity [50]. Detrending involves the isolation of the low frequency variation (i.e. trend) and to decompose the residual signal into a seasonal (or cyclic) and a random walk type variation (i.e. white noise or high frequency noise). There are also a few reports that have used this technique to study the scaling behavior of the fluctuations in the music signal, as well as in the detection of arousal based effects in music induced EEG signals [46,43,47,48,51].

The multifractal signals are complex self-similar structures with spatial and temporal variation in the scale invariant structures. Therefore, a single power law exponent cannot reveal the self-similarity and a power law spectrum with different exponents are used for their analysis. In this way, this technique is able to decipher much accurately the amount of self similarity present in a signal. The spectrum in multifractal detrended fluctuation analysis (MFDFA) is obtained by summarizing the local RMS variation into an overall RMS. This overall RMS is then expanded into a q-order RMS to distinguish between segments of different size.

The MFDFA technique is much more accurate than the conventional DFA technique in case of music signals because of the fact that there are segments with extremely large variation as well as segments with very small variation (i.e. they are multifractals) therefore, the normal distribution considering second order RMS variation cannot be applied and all the q-order moments need to be considered. This method is very useful in the analysis of various non-stationary time series and it also gives information regarding the multifractal scaling behavior of non-stationary signals. MFDFA technique has been widely applied in various fields ranging from stock market to biomedical fields for prognosis of diseases [52–56]. Also dynamical systems theory provides a systematic approach to the study of complex systems along with the mathematical tools needed to identify regularities in their behavior and track their evolution over time [57]. In the domain of music analysis, using multifractal detrended fluctuation analysis (MF-DFA) method, frequency series of Bach pitches have been analyzed and multifractality due to long range correlation and broad probability distribution function have been identified [58]. In Ref. [43], the authors show that both melody and rhythm can be considered as multifractal objects by separating both of them as series of geometric points, while in Ref. [59] the authors use the DFA technique to relate body movements of performers to the expression embedded in it. Live performances encompass a variety of such musical features



(a) Waveform of 30 s Sample 1.

(b) Waveform of 30 s Sample 2.

(c) Waveform of 30 s Sample 3.

(d) Waveform of 30 s Sample 4.

Fig. 1. 30 second waveforms of the 4 music signals.

including tempo fluctuations [60], notation and timbre variation to name a few. Several other researchers have used the fractal analysis technique to examine musical movements and musical structure [61–64]. Thus, the multifractal nature of music signals is well established and could prove to be an important tool when analyzing improvisational cues in a specific performance of Hindustani *raga*. In this context, taking the entire signal as a time series for analysis can be interesting as we are considering all the properties as a whole to ratify the multifractal nature of music and to investigate cues which distinctly separates on performance from another.

As a generalization of the DFA method, the detrended cross-correlation analysis (DCCA) is proposed to investigate the long-term cross-correlations between two non stationary time series [65–71], and Multifractal Detrended Cross-Correlation Analysis (MF-DXA) can unveil the multifractal features of two cross-correlated signals [72–74,52]. The noisy signals in many real-world systems display long-range autocorrelations and these cross-correlations can be accurately quantified with the help of DCCA [75] technique. In this paper we try to see the presence of multifractality as well as multifractal cross correlations in the music signal of Hindustani classical music rendered by an eminent maestro. Four such renderings of the same maestro of raga *Sur Malhar* have been taken for analysis. Our objective is to see the difference in complexity in the four signals though he sung the same raga. The results reveal interesting new information regarding the improvisation cue in Hindustani music which are discussed in detail.

2. Experimental details

Four different renderings of *raga Sur Malhar* by an eminent maestro of Hindustani Vocal Music was taken. The *raga* falls under the broad class *Kafi that*. Three minutes from the portion of *vilambit bandish* part in mid tempo was cut out from each rendering. The signals are digitized at the rate of 22,050 samples/second in 16 bit format. A *Bandish* provides the literature ingredient of the *raga* in each individual rendition for traditional structured singing. The *bandish* part was taken so that the notes used in all the renderings are same and hence the changes in musical structure will be mainly due to the improvisations made by the artist. Also, the artist makes his own improvisations in the *raga* predominantly in the *bandish* part. Each three minutes signal is divided into six equal segments of 30 s each. This was done to see the change of complexity in each time window for each song. Part 1 of all the four signals which were analyzed for multifractality have been plotted in the representative following Fig. 1.

3. Methodology

3.1. Method of multifractal analysis of music signals

The analysis of the music signals is done using MATLAB in this paper and for each step an equivalent mathematical representation is given which is taken from the prescription of Kantelhardt et al. [76].

The complete procedure is divided into the following steps:

Step 1: converting the noise like structure of the signal into a random walk like signal. It can be represented as:

$$Y(i) = \sum (x_k - \bar{x}) \tag{1}$$

where $\bar{x}$ is the mean value of the signal

The integration reduced the level of noise present in experimental records and finite data.

Step 2: the whole length of the signal is divided into N_s no of segments consisting of certain no. of samples. For s as sample size and N the total length of the signal the segments are

$$N_s = \mathrm{int}\left(\frac{N}{s}\right). \tag{2}$$

Step 3: The local RMS variation for any sample size s is the function $F(s, v)$. This function can be written as follows:

$$F^2(s, v) = \frac{1}{s}\sum_{i=1}^{s}\{Y[(v-1)s+i] - y_v(i)\}^2.$$

For $\nu = N_s + 1 \ldots, 2N_s$, where $y_v(i)$ is the least square fitted value in the binν. In this work, a least square linear fit using first order polynomial (MF-DFA-1) is performed. The study can also be extended to higher orders by fitting quadratic, cubic, or higher order polynomials.

Step 4: The q-order overall RMS variation for various scale sizes can be obtained by the use of following equation

$$F_q(s) = \left\{\frac{1}{N_s}\sum_{v=1}^{N_s}\left[F^2(s, v)\right]^{\frac{q}{2}}\right\}^{\left(\frac{1}{q}\right)} \tag{3}$$

where q is an index that can take all possible values except zero, because in that case the factor $1/q$ is infinite.

Step 5: The scaling behavior of the fluctuation function is obtained by drawing the log–log plot of $F_q(s)$ vs. s for each value of q.

$$F_q(s) \sim s^{h(q)}. \tag{4}$$

The $h(q)$ is called the generalized Hurst exponent. The Hurst exponent is a measure of self-similarity and correlation properties of time series produced by fractal. The presence or absence of long range correlation can be determined using Hurst exponent. A monofractal time series is characterized by unique $h(q)$ for all values of q.

The generalized Hurst exponent $h(q)$ of MF-DFA is related to the classical scaling exponent $\tau(q)$ by the relation

$$\tau(q) = qh(q) - 1. \tag{5}$$

A monofractal series with long range correlation is characterized by linearly dependent q order exponent $\tau(q)$ with a single Hurst exponent H. Multifractal signal on the other hand, possess multiple Hurst exponent and in this case, $\tau(q)$ depends non-linearly on q [77].

The singularity spectrum $f(\alpha)$ is related to $h(q)$ by

$$\begin{aligned}\alpha &= h(q) + qh'(q)\\ f(\alpha) &= q[\alpha - h(q)] + 1\end{aligned} \tag{6}$$

where α is the singularity strength and $f(\alpha)$ specifies the dimension of subset series that is characterized by α. The multifractal spectrum is capable of providing information about relative importance of various fractal exponents in the series e.g., the width of the spectrum denotes range of exponents. A quantitative characterization of the spectra may be obtained by least square fitting it to a quadratic function [78] around the position of maximum α_0,

$$f(\alpha) = A(\alpha - \alpha_0)^2 + B(\alpha - \alpha_0) + C$$

where C is an additive constant $C = f(\alpha 0) = 1$. B indicates the asymmetry of the spectrum. It is zero for a symmetric spectrum. The width of the spectrum can be obtained by extrapolating the fitted curve to zero.

Width W is defined as,

$$\begin{aligned}&W = \alpha_1 - \alpha_2,\\ &\text{with } f(\alpha_1) = f(\alpha_2) = 0.\end{aligned} \tag{7}$$

The width of the spectrum gives a measure of the multifractality of the spectrum. Greater is the value of the width W greater will be the multifractality of the spectrum. For a monofractal time series, the width will be zero as $h(q)$ is independent of q. The origin of multifractality in music signal time series can be verified by randomly shuffling the original time series data [79, 56]. All long range correlations that existed in the original data are destroyed by this random shuffling and what remains is a totally uncorrelated sequence. Hence, if the multifractality of the original data was due to long range correlation, the shuffled data will show non-fractal scaling.

On the other hand, if the initial $h(q)$ dependence does not change; i.e. if $h(q) = h_{\text{shuffled}}(q)$, then the multifractality is not due to long range correlation but is a result of broad probability density function of the time series. If any series has multifractality both due to long range correlation as well as probability density function, then the shuffled series will have smaller width W than the original series.

3.2. Multifractal Detrended Cross Correlation Analysis (MF-DXA)

We have performed a cross-correlation analysis of correlation between different samples of the same raga following the prescription of Zhou [80].

$$x_{\text{avg}} = 1/N \sum_{i=1}^{N} x(i) \quad \text{and} \quad y_{\text{avg}} = 1/N \sum_{i=1}^{N} y(i). \tag{8}$$

Then we compute the profiles of the underlying data series $x(i)$ and $y(i)$ as

$$X(i) \equiv \left[\sum_{k=1}^{i} x(k) - x_{\text{avg}} \right] \quad \text{for } i = 1 \dots N \tag{9}$$

$$Y(i) \equiv \left[\sum_{k=1}^{i} x(k) - x_{\text{avg}} \right] \quad \text{for } i = 1 \dots N. \tag{10}$$

The integration also reduces the level of measurement noise present in experimental records and finite data. Each of the integrated time series was divided to N_s non-overlapping bins where $N_s = \text{int}(N/s)$ where s is the length of the bin. Now, since N is not a multiple of s, a short part of the series is left at the end. So in order to include this part of the series the entire process was repeated starting from the opposite end thus leaving a short part at the beginning thus obtaining $2N_s$ bins. For each bin, least square linear fit was performed and the fluctuation function is given by:

$$F(s, \nu) = 1/s \sum_{i=1}^{s} \{Y[(\nu - 1)s + i] - y_\nu(i)\} \times \{X[(\nu - 1)s + i] - x_\nu(i)\}$$

for each bin ν, $\nu = 1, \dots, N_s$ and

$$F(s, \nu) = 1/s \sum_{i=1}^{s} \{Y[(\nu - N_s)s + i] - y_\nu(i)\} \times \{X[N - (\nu - N_s)s + i] - x_\nu(i)\}$$

for $\nu = N_s + 1, \dots, 2N_s$ where $x_\nu(i)$ and $y_\nu(i)$ is the least square fitted value in the bin ν.
The qth order detrended covariance $F_q(s)$ is obtained after averaging over $2N_s$ bins.

$$F_q(s) = \left\{ 1/2N_s \sum_{\upsilon=1}^{2N_s} [F(s, \upsilon)]^{q/2} \right\}^{1/q} \tag{11}$$

where q is an index which can take all possible values except zero because in that case the factor $1/q$ blows up. The procedure can be repeated by varying the value of s. $F_q(s)$ increases with increase in value of s. If the series is long range power correlated, then $F_q(s)$ will show power law behavior

$$F_q(s) \sim s^{\lambda(q)}.$$

If such a scaling exists $\ln F_q$ will depend linearly on $\ln s$, with $\lambda(q)$ as the slope. Scaling exponent $\lambda(q)$ represents the degree of the cross-correlation between the two time series. In general the exponent $\lambda(q)$ depends on q. We cannot obtain the value of $\lambda(0)$ directly because F_q blows up at $q = 0$. F_q cannot be obtained by the normal averaging procedure; instead a logarithmic averaging procedure is applied

$$F_0(s) = \left\{ 1/4N_s \sum_{\upsilon=1}^{2N_s} [F(s, \upsilon)] \right\} \sim s^{\lambda(0)}. \tag{12}$$

For $q = 2$ the method reduces to standard DCCA. If scaling exponent $\lambda(q)$ is independent of q, the cross-correlations between two time series are monofractal. If scaling exponent $\lambda(q)$ is dependent on q, the cross-correlations between two time series are multifractal. Furthermore, for positive q, $\lambda(q)$ describes the scaling behavior of the segments with large fluctuations and for negative q, $\lambda(q)$ describes the scaling behavior of the segments with small fluctuations. Scaling exponent $\lambda(q)$ represents the degree of the cross-correlation between the two time series $x(i)$ and $y(i)$. The value $\lambda(q) = 0.5$ denotes the absence of cross-correlation. $\lambda(q) > 0.5$ indicates persistent long range cross-correlations where a large value in one variable is likely to be followed by a large value in another variable, while the value $\lambda(q) < 0.5$ indicates anti-persistent cross-correlations where a large value in one variable is likely to be followed by a small value in another variable, and vice versa [81].
Zhou found that for two time series constructed by binomial measure from p-model, there exists the following relationship [80]:

$$\lambda(q = 2) \approx [h_x(q = 2) + h_y(q = 2)]/2. \tag{13}$$

Podobnik and Stanley have studied this relation when $q = 2$ for monofractal Autoregressive Fractional Moving Average (ARFIMA) signals and EEG time series [67].

Table 1
Variation of multifractal spectral width in the four samples.

Part No.	Sample 1		Sample 2		Sample 3		Sample 4	
	Spectral width (W_{original})	Shuffled (W_{shuffled})	Spectral width (W_{original})	Shuffled (W_{shuffled})	Spectral width (W_{original})	Shuffled (W_{shuffled})	Spectral width (W_{original})	Shuffled (W_{shuffled})
1	0.733 ± 0.08	0.187	0.577 ± 0.07	0.107	0.370 ± 0.03	0.084	1.158 ± 0.08	0.076
2	0.645 ± 0.04	0.111	0.579 ± 0.05	0.080	0.482 ± 0.08	0.072	0.531 ± 0.05	0.048
3	0.587 ± 0.06	0.084	0.467 ± 0.06	0.110	0.521 ± 0.07	0.104	0.378 ± 0.04	0.0710
4	0.555 ± 0.07	0.077	0.443 ± 0.02	0.083	0.565 ± 0.06	0.098	0.495 ± 0.06	0.071
5	0.811 ± 0.13	0.242	0.558 ± 0.05	0.112	0.738 ± 0.05	0.080	0.270 ± 0.06	0.076
6	0.758 ± 0.08	0.072	0.639 ± 0.11	0.065	0.440 ± 0.07	0.115	0.380 ± 0.03	0.081

In case of two time series generated by using two uncoupled ARFIMA processes, each of both is autocorrelated, but there is no power-law cross correlation with a specific exponent [81]. According to auto-correlation function given by:

$$C(\tau) = \langle [x(i+\tau) - \langle x\rangle][x(i) - \langle x\rangle]\rangle \sim \tau^{-\gamma}. \tag{14}$$

The cross-correlation function can be written as

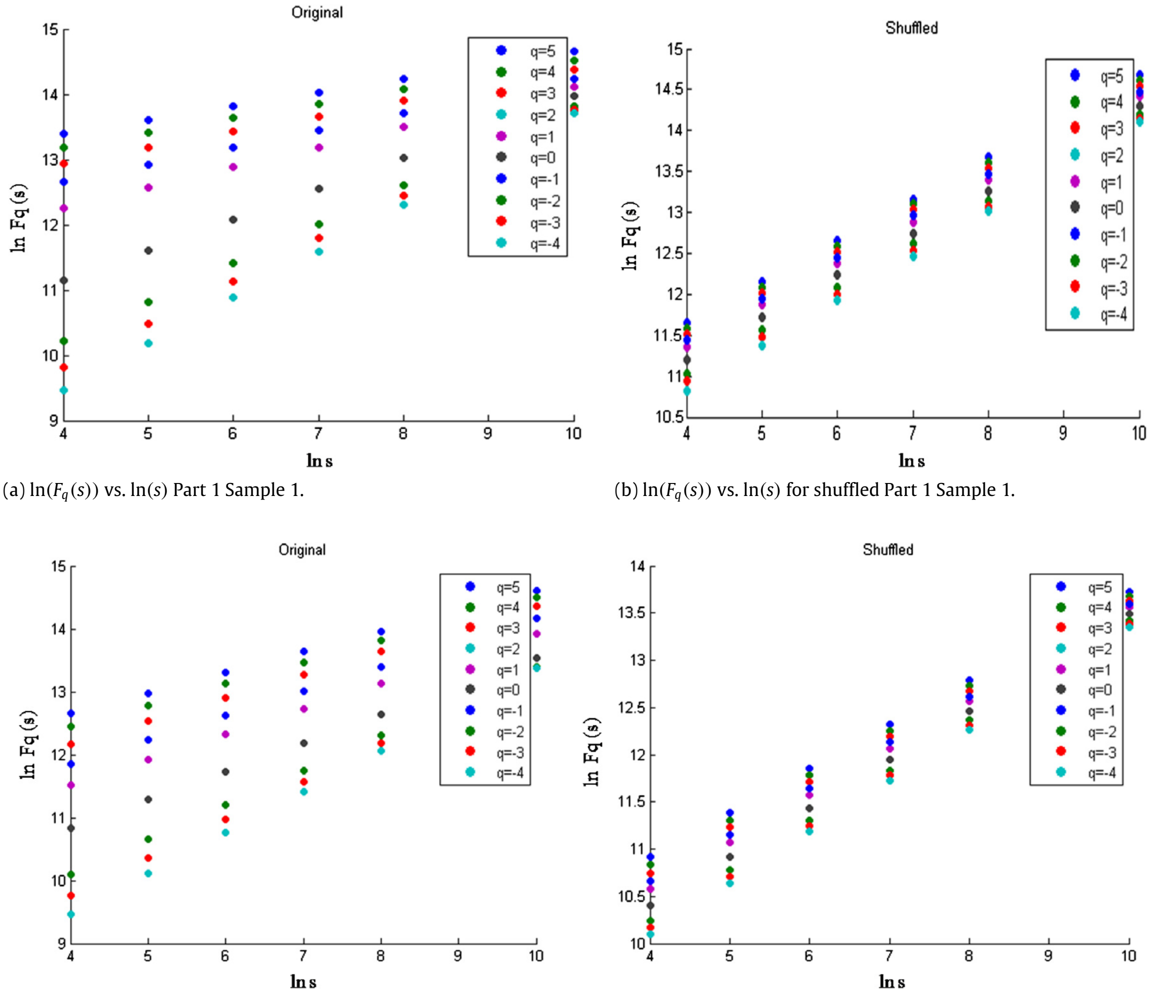
$$C_x(\tau) = \langle [x(i+\tau) - \langle x\rangle][y(i) - \langle y\rangle]\rangle \sim \tau_x^{-\gamma} \tag{15}$$

where γ and γ_x are the auto-correlation and cross-correlation exponents, respectively. Due to the non-stationarities and trends superimposed on the collected data, direct calculation of these exponents are usually not recommended; rather the reliable method to calculate auto-correlation exponent is the DFA method, namely $\gamma = 2 - 2h(q = 2)$ [81]. Recently, Podobnik et al., have demonstrated the relation between cross-correlation exponent, γ_x and scaling exponent $\lambda(q)$ derived by Eq. (4) according to $\gamma_x = 2 - 2\lambda(q = 2)$ [65]. For uncorrelated data, γ_x has a value 1 and lower the value of γ and γ_x more correlated is the data.
In general, $\lambda(q)$ depends on q, indicating the presence of multifractality. In other words, we want to point out how two sound signals are cross-correlated in various time scales. For this, we consider the $\lambda(q = 2)$ case to evaluate the cross correlation exponent, γ_x.

4. Results and Discussion

Musical structures can be explored on the basis of multifractal analysis and nonlinear correlations in the data. Traditional signal processing techniques are not capable of identifying such relationships, nor do they provide quantitative measurement of the complexity or information content in the signal. Music signals can therefore generate fluctuations that are not best described by linear decomposition [82]. On the other hand, the classical nonlinear dynamics method such as correlation dimension and Lyapunov exponents are very sensitive to noise and require the stationary condition.
The multifractal music signals analyzed here are complex self-similar structures with spatial and temporal variation in the scale invariant structures. Therefore, a single power law exponent cannot reveal the self-similarity and a power law spectrum with different exponents are used for their analysis. In this way, this technique is able to decipher much accurately the amount of self similarity present in a signal. The spectrum in multifractal detrended fluctuation analysis (MFDFA) is obtained by summarizing the local RMS variation into an overall RMS. This overall RMS is then expanded into a q-order RMS to distinguish between segments of different size. The q-order weights the influence of segments with large and small fluctuations, RMS{1}. $F_q(n_q)$ for negative q's (i.e., $n_q = 1-5$) are influenced by segments v with small RMS{1}(v) fluctuations, while $F_q(n_q)$ for positive q's (i.e., $n_q = 5 - 10$) are influenced by segments v with large RMS{1}(v) fluctuations. The local fluctuations RMS{1} in the musical signal with large and small magnitudes are graded by the magnitude of the negative or positive q-order, respectively.

The MFDFA technique is much more accurate than the conventional DFA technique in case of music signals because of the fact that there are segments with extremely large variation as well as segments with very small variation (i.e. they are multifractals) therefore, the normal distribution considering second order RMS variation cannot be applied and all the q-order moments need to be considered. This method is very useful in the analysis of various non-stationary time series and it also gives information regarding the multifractal scaling behavior of non-stationary signals.
Every musical composition/element can be considered as a nonlinear complex time series—the multifractal width (w) being a quantitative measure of its complexity. In other words, more w—more local fluctuations in temporal scale and thus this parameter is very sensitive to characterize and quantify a particular music signal to a level which is not possible with any other method. In a similar manner, small w implies less local fluctuations in temporal scale. Thus, similar w means that the two musical signals have similar complexity (or same local fluctuations) in the temporal scale. Hence, multifractal spectral width can be considered as the best parameter for the characterization of a music sample. In this paper we verify the presence of multifractality in the same musical signals sung in four different days by an eminent vocalist. For this, we have taken 3 min from the *bandish* part of the rendering by the vocalist. The vocalist has rendered the same raga (Sur Malhar) in four different days. Since the raga is same, the notes and the phrases are same, though the vocalist and the raga rendered are same but there should be some difference in the phrasal structure. Hence, we have divided the 3 min song signal into six equal segments and studied multifractality using MFDFA technique in all the segments for all the signals. Each of the 30 s segment was divided into 5 windows of 6 s each and the average multifractal width has been given in Table 1 along with the variance.



(a) $\ln(F_q(s))$ vs. $\ln(s)$ Part 1 Sample 1.

(b) $\ln(F_q(s))$ vs. $\ln(s)$ for shuffled Part 1 Sample 1.

(c) $\ln(F_q(s))$ vs. $\ln(s)$ Part 1 Sample 2.

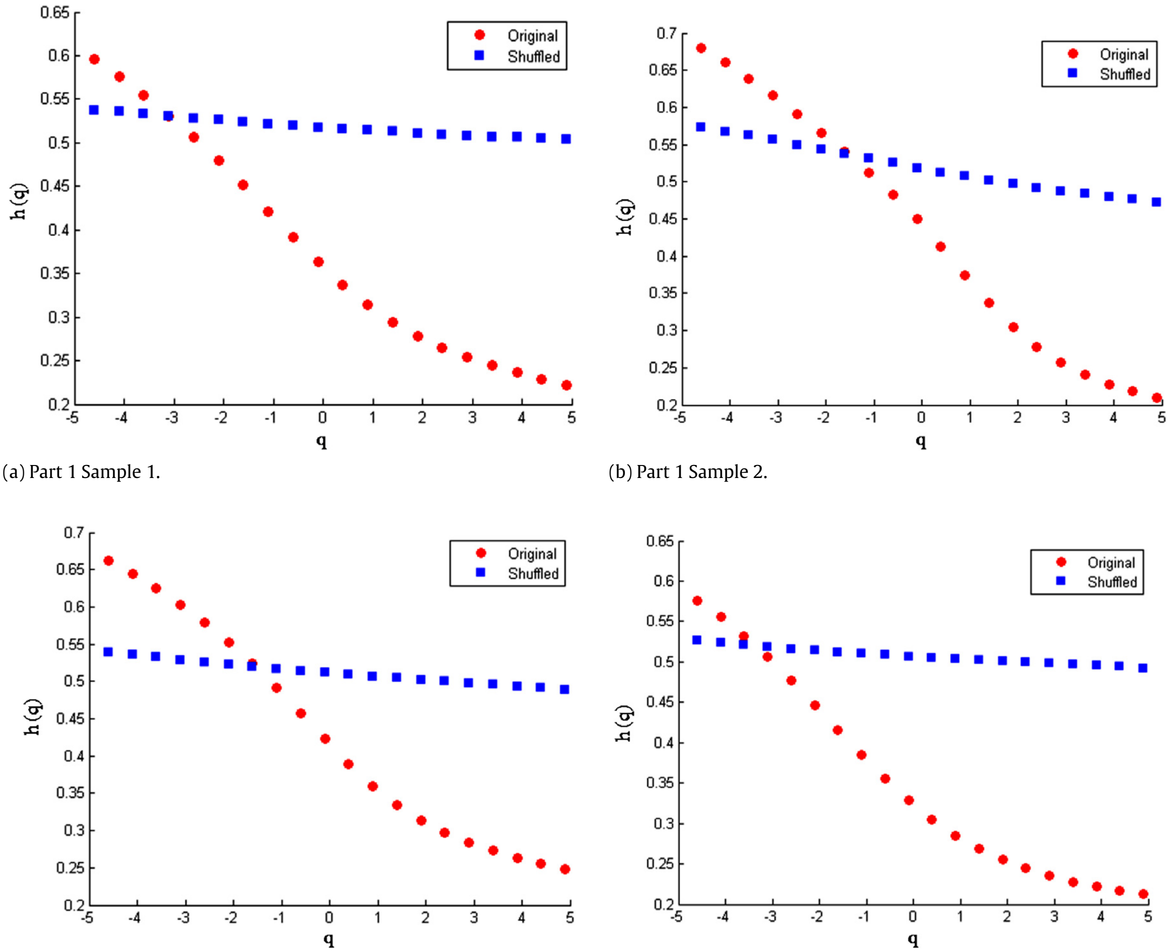
(d) $\ln(F_q(s))$ vs. $\ln(s)$ for shuffled Part 1 Sample 2.

Fig. 2. Sample regression plots of $\ln F_q(s)$ vs. $\ln s$.

The qth order fluctuation function $F_q(s)$ for 10 points of q in between -5 and $+5$ was obtained. Fig. 2 depicts the plot of $\ln F_q$ vs. $\ln s$ for Part 1 of music signals 1 and 2. Linear dependence of $\ln F_q$ on $\ln s$ is observed suggesting scaling behavior. The shuffled plots of $\ln F_q$ vs. $\ln s$ reveal that the shuffled values do not change with the values of q, and thus has a fixed slope $h(q) = H$, which is the conventional Hurst exponent for monofractal time series. The values of $h(q)$ are thus obtained from the slope of linear fit of $\ln F_q(s)$ vs. $\ln s$ plots shown in Fig. 2(a)–(d).
For a monofractal time series we get unique value of $h(q)$ for all q. If the small and large fluctuations scale vary differently, then $h(q)$ will depend on q or in other words the time series is multifractal. A representative figure for variation of $h(q)$ with q for a particular part of each signal for four different music signals have been shown in Fig. 3(a)–(d). The shuffled values of $h(q)$ has also been shown in the same figure. The following representative figures show the variation of Hurst exponent $h(q)$ with q for the Part 1 of the four samples:

The variation of $h(q)$ with q clearly indicates multifractal behavior, the shuffled values showing remarkable difference from that of the original values. It is also evident from Fig. 3 that in all cases the values of $h(q)$ decreases with increasing q. Also, the shuffled values of $h(q)$ remains constant with the change of q, which is a characteristic of a monofractal scaling.

The amount of multifractality can be determined quantitatively in each of the windows of each signal from the width of the multifractal spectrum ($f(\alpha)$ vs. α). The multifractal nature of the scaling properties can be depicted by the multifractal spectrum $f(\alpha)$ vs. α as shown in Fig. 4(a)–(d). The multifractal spectrums were then fitted to Eq. (7) and the multifractal widths were obtained for all parts of the four samples. To ascertain the origin of multifractality the corresponding randomly shuffled series was also analyzed. The randomly shuffled series exhibits weaker multifractality indicating that the origin of multifractality is due to both long range correlations and broad probability distribution. In an ideal case, for a sufficiently long series the shuffled series would have monofractal properties when the randomly shuffled series has smaller width as compared to the original series. Fig. 4(a)–(d) is a representative figure which shows that in Part 1 of all the four signals,



(a) Part 1 Sample 1.

(b) Part 1 Sample 2.

(c) Part 1 Sample 3.

(d) Part 1 Sample 4.

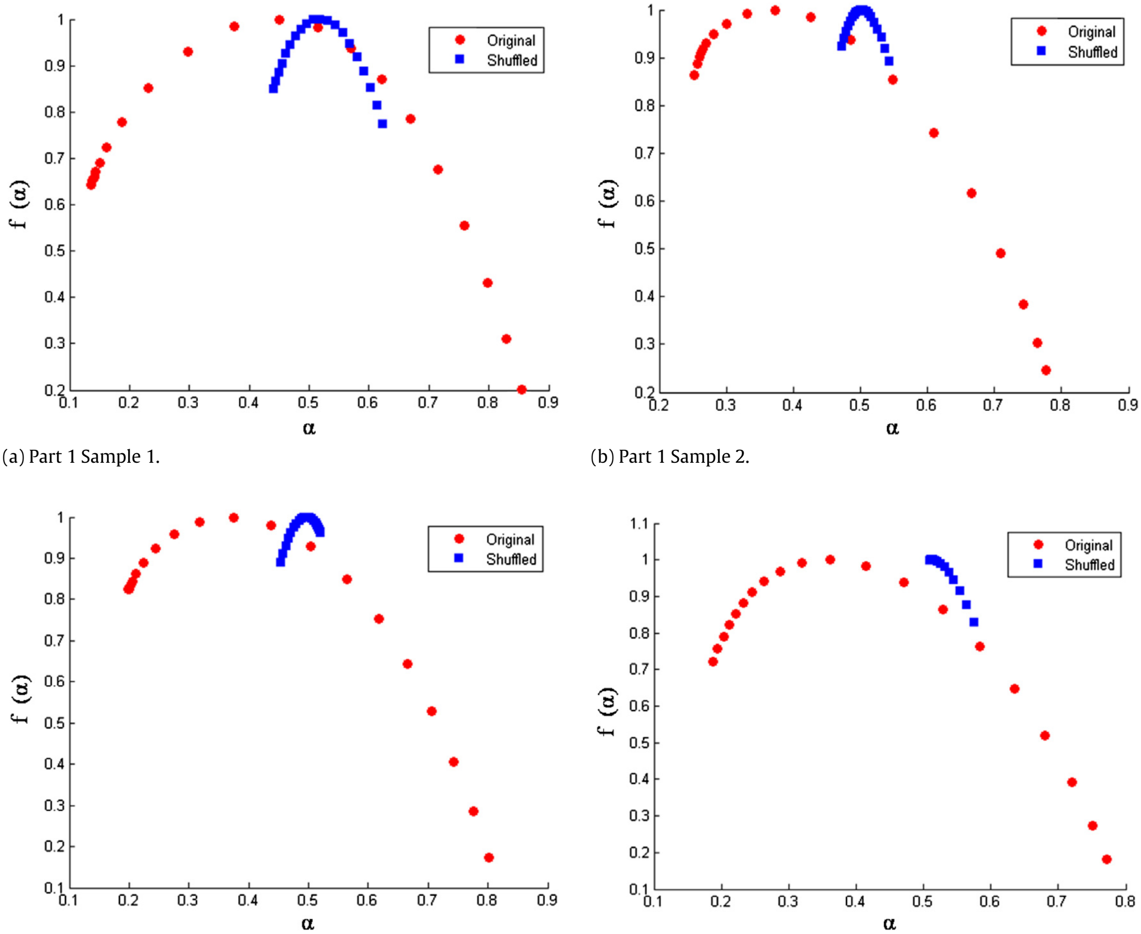
Fig. 3. The variation of Hurst exponent for Part 1 of the 4 samples.

the shuffled series $f(\alpha)$ vs. α has a weaker multifractality as compared to the original signal. The same trend is observed in all the other parts of the four samples. The destruction of all long range correlations in the data makes the shuffled series monofractal in nature. Ideally $f(\alpha)$ should be independent of α. Since the data size is quite large in this case, the inference drawn from the results are reasonably significant and the difference in means is also relevant statistically as affirmed later by ANOVA and subsequent post hoc tests. Table 1 gives the values of mean multifractal spectral width for the six different parts of the four samples along with their Standard Deviation (SD) values computed analytically from the different parts of the same sample. The shuffled widths are also given in the adjacent column.

The variation of multifractal width for the 1st parts of the four music signals (both original and shuffled) is shown in a representative Fig. 4. The blue graphs give the randomly shuffled width. As is evident from the figure, the shuffled width (W_{shuffled}) in all cases is much smaller than the original width (W_{original}). This confirms that the multifractality in the music signal is due to both broad probability distribution as well as long range correlation

The following figures (Fig. 5) depict the change in multifractal width for each of six parts of the four different samples. We see that in each part complexity is significantly different for the four renditions indicating specific improvisations during each performance. For the purpose of comparison, they have been drawn in the same scale.

Analysis of the values show that though the artist have sung the same raga having the same phrasal structure, still all the six parts for all the four signals show remarkably different values of multifractal spectral width. This difference can be attributed to the change in the duration of notes, note to note transitions and the variation in the use of pauses within the phrasal structure. Thus, we can say that there is a change in amount of multifractality when the rendering of same raga music was on different days by the same vocalist. This may be caused due to the musical improvisation done by the singer on each day while singing the same raga. The audience is pleased on all the four days when the performer creates different mood or ambience while singing the same *raga*, in spite of the different modulations made by the artist keeping in mind the demand of the audience.



(a) Part 1 Sample 1.

(b) Part 1 Sample 2.

(c) Part 1 Sample 3.

(d) Part 1 Sample 4.

Fig. 4. $f(\alpha)$ vs. α plot showing the original data as well as randomly shuffled data. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

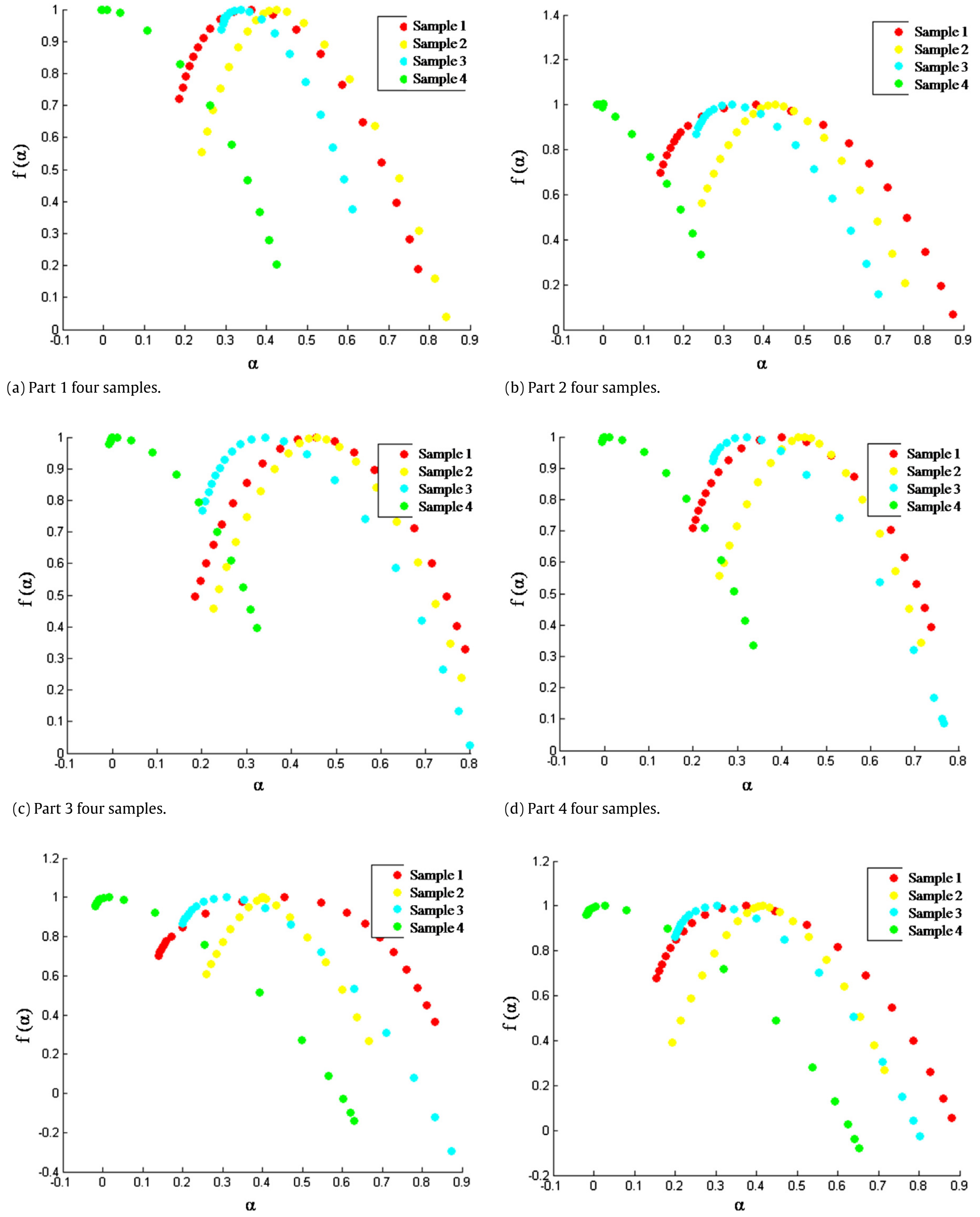
Apart from studying the variation of multifractality in different renditions, it will also be interesting to study the change of multifractal values in different parts within each sample. In that respect, Fig. 6 gives the variation of spectral widths in the six parts of each of the four samples.

As is evident from the figure, there is considerable variation of multifractal width within different parts of the same sample. Thus there is significant variation of complexity in musical structure even in the same musical signal, for e.g. in Sample 4 the spectral width varies from as low as 0.27 to as high as 1.15. Such a large variation in complexity within different parts of the same sample can be ascribed to varying emotions conveyed to the audience by the artist during the performance. It can also be ascribed to the varying style of rendition of the same *raga* in different days which has led to the significant difference in their complexity.

One Way ANOVA [83] was performed to test the significance of the results obtained in Table 1. The six parts of four samples were analyzed with the help of ANOVA technique. The significance level was set to $p < 0.05$. Post hoc analysis in the form of Tukey–Kramer multiple comparison tests were performed only if ANOVA results were significant between the groups. The ANOVA results are elaborated in Table 2. All the tests were performed with 95% confidence intervals between the parts analyzed.

Tukey–Kramer multiple comparison tests were performed for Parts 2–6 which yielded significant results in the one way ANOVA tests. The results of post-hoc analysis are reported in Table 3. Sample 1 vs. 2 and Sample 2 vs. 4 were reported to be highly significant in all the parts, while Sample 1 vs. 3 and Sample 3 vs. 4 reported to be mostly significant. Except with a few spurious aberrations, we can say that the means of the reported data varied significantly within themselves as well as within groups.

Next, MFDXA was performed for each part between all the samples using the methodology given above. All the data sets were first transformed according to Eqs. (9) and (10) to reduce noise in the data. The integrated time series were then divided to N_s bins where $N_s = \mathrm{int}(N/s)$, N is the length of the series. The qth order detrended covariance $F_q(s)$ was obtained from Eq. (11) for values of q from -5 to $+5$ in steps of 1 just like the MFDFA part. Power law scaling of $F_q(s)$ with s is observed for all



(a) Part 1 four samples.
(b) Part 2 four samples.
(c) Part 3 four samples.
(d) Part 4 four samples.
(e) Part 5 four samples.
(f) Part 6 four samples.

Fig. 5. $f(\alpha)$ vs. α plot showing the variation of multifractal width in the four samples.

values of q as is seen from Fig. 2(a)–(d) same as those found for MFDFA. The values of cross correlation coefficient γ_x ($q = 2$), are provided in Table 4. We have also shown variation of $h(q)$ with q for Part 1 of the four clips by means of MF-DFA in Fig. 3. The plot depicts multifractal behavior of cross-correlations because for different q, there are different exponents; that is, for different q, there are different power-law cross-correlations. Further from the same figure we can see that the value

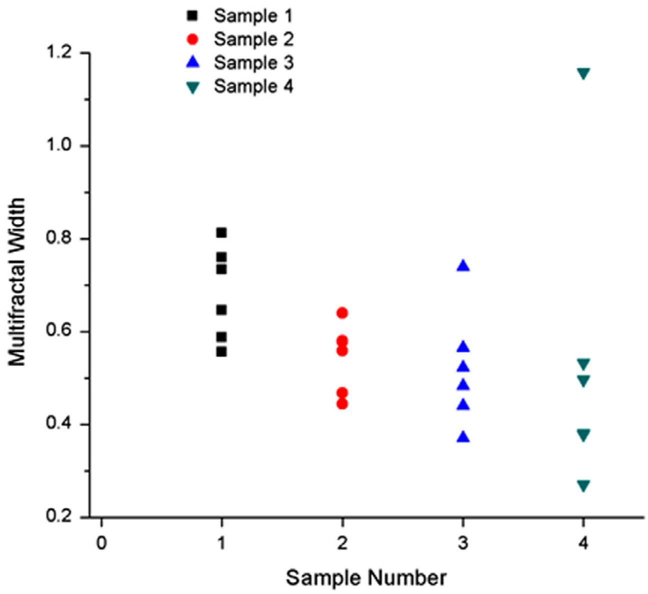


Fig. 6. Variation of multifractal width within each sample.

Table 2
ANOVA values for the different parts of music samples.

Parts of music sample		Part 1		Part 2		Part 3		Part 4		Part 5		Part 6	
Source	*df*	*F*	*p*	*F*	*p*	*F*	*p*	*F*	*p*	*F*	*p*	*F*	*p*
Treatment (between experimental conditions)	3	0.43	0.736	11.5	0.001	25.54	<0.001	4.58	0.032	25.99	<0.001	10.04	0.003
Residual (within experimental conditions)	9												
Total	12												

df: degrees of freedom.

of $H(q)$ depends on q for all the four samples that we have taken in this study. We know that $H(q) = 0.5$ indicates that the series is an independent random process, and for $H(q) < 0.5$ it is characterized by long-range anti-correlations while for $0.5 < H(q) < 1$, it is featured by long-term correlations. In this case the signal is stationary. The exponent $H(q = 2)$ is equivalent with the well-known Hurst index. A representative figure (Fig. 7) reports the variation of cross correlation exponent $\lambda\ (q)$ with q for two particular samples (Part 1 for Sample 1 and Sample 2), also the variation of $h(q)$ with q for those two samples obtained from MFDFA technique are also shown in the same figure for comparison.

The variation of $\lambda(q)$ with q for the two cross correlated signals (Part 1 for Sample 1 and Sample 2) show that they are multifractal in nature. To illustrate further the presence of multifractality in the cross-correlated music signals, i.e. to have information about the distribution of degree of cross-correlation in various time scales, a representative multifractal spectrum was plotted for the two signals in Fig. 8 The way to characterize multifractality of cross correlation between two samples is to relate via a $\lambda(q)$ Legendre Transform as in the case of single series [84]. The growth of the width of $f(\alpha)$ or equivalently $\Delta\alpha$ shows the increase in degree of multifractality of the coupled signals. Again, it becomes evident from the spectrum that the cross correlated signals are multifractal in various time scales. It is worth mentioning here that, in this study, we are not considering the entire cross-correlated multifractal spectrum, but have restricted ourselves to the $\lambda(q = 2)$ case, to evaluate the cross correlation exponent, γ_x. This is because, we want to see the degree of cross-correlation between the two music signals, or in other words, how the two signals are correlated. The $q = 2$ case is a direct fallout of the standard DCCA technique. The analysis of the entire cross correlated multifractal spectrum (along with the cross correlated multifractal spectral width) may yield more interesting results in the domain of music signal analysis.

Jones and Kaul [85] were the first to reveal a stable negative cross-correlation between oil prices and stock prices. The negative cross-correlations were also found by Refs. [86–88]. A negative value of cross correlation is an indication of strong cross-correlation between the two samples for which the cross correlation is being carried out. The cross correlation exponent γ_x is reported in Table 4 along with the corresponding SD values. MFDXA was carried out amongst six parts of all the four signals taken for analysis. Fig. 9 depicts the variation of cross correlation coefficient among the various parts of the 4 music signals, S1, S2… denote Sample Numbers in the Figure.

As is evident from the figure, strong cross-correlation is observed in Parts 2, 3 and 4 for almost all the samples i.e. for these parts the value of γ_x is negative for all the cases. Part 1 as well as the last two parts (Parts 5 and 6), in general do not have strong cross-correlation between them as is evident from the positive values of γ_x. The four renditions of the same raga

Table 3
Tukey–Kramer multiple comparison test results.

Comparison	Mean difference	q	p-value
Part 2			
Sample 1 vs. 2	14.45	12.546	<0.001
Sample 1 vs. 3	2.53	4.587	<0.0448
Sample 1 vs. 4	1.96	2.1688	0.0979
Sample 2 vs. 3	4.78	7.956	<0.0031
Sample 2 vs. 4	12.35	10.378	<0.001
Sample 3 vs. 4	1.34	2.419	0.228
Part 3			
Sample 1 vs. 2	18.8	12.51	<0.0001
Sample 1 vs. 3	15.76	11.33	<0.0001
Sample 1 vs. 4	6.42	6.2125	<0.002
Sample 2 vs. 3	4.93	9.104	0.003
Sample 2 vs. 4	19.24	9.901	<0.0001
Sample 3 vs. 4	8.42	7.337	<0.0065
Part 4			
Sample 1 vs. 2	6.92	9.4027	0.0004
Sample 1 vs. 3	7.3	6.4988	0.002
Sample 1 vs. 4	1.58	2.218	0.16
Sample 2 vs. 3	4.85	6.904	0.0029
Sample 2 vs. 4	12.85	7.1849	<0.0001
Sample 3 vs. 4	7.19	9.712	0.0003
Part 5			
Sample 1 vs. 2	7.19	8.8636	0.0004
Sample 1 vs. 3	4.04	6.663	0.0397
Sample 1 vs. 4	2	2.3495	0.092
Sample 2 vs. 3	5.23	5.9675	0.0119
Sample 2 vs. 4	7.45	6.5138	<0.0003
Sample 3 vs. 4	4.82	4.636	0.063
Part 6			
Sample 1 vs. 2	10.44	11.23	<0.0001
Sample 1 vs. 3	4.24	6.46	<0.037
Sample 1 vs. 4	3.24	3.81	0.048
Sample 2 vs. 3	4.75	7.37	0.003
Sample 2 vs. 4	11.18	9.326	<0.0001
Sample 3 vs. 4	2.32	3.87	0.059

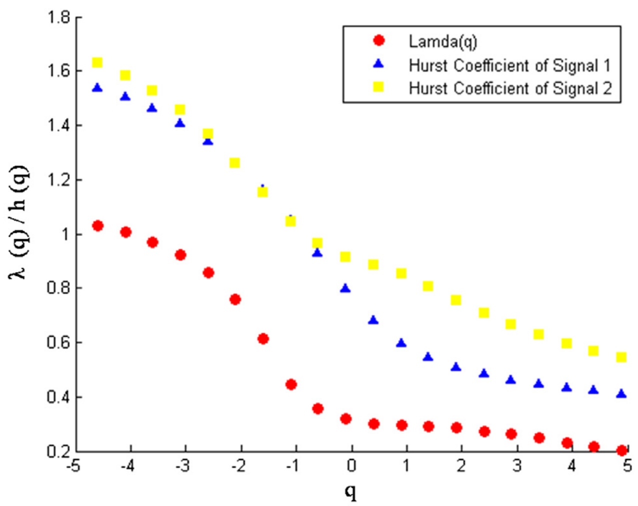


Fig. 7. Variation of $\lambda\ (q)$ and $h(q)$ for two sound signals.

are different from one another due to the variation of note-to-note sequences, interval between notes and other modulating factors which corresponds to the "improvisational modulation" made by the artist during each and every rendition. Thus we

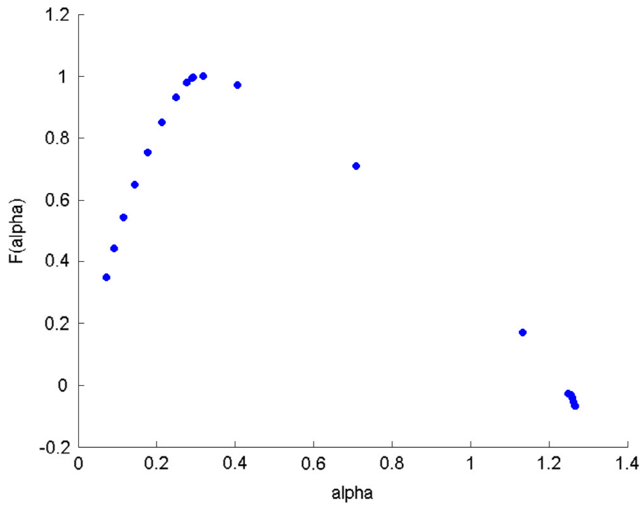


Fig. 8. Multifractal cross-correlated Spectrum of Sample 1 and 2 (Part 1).

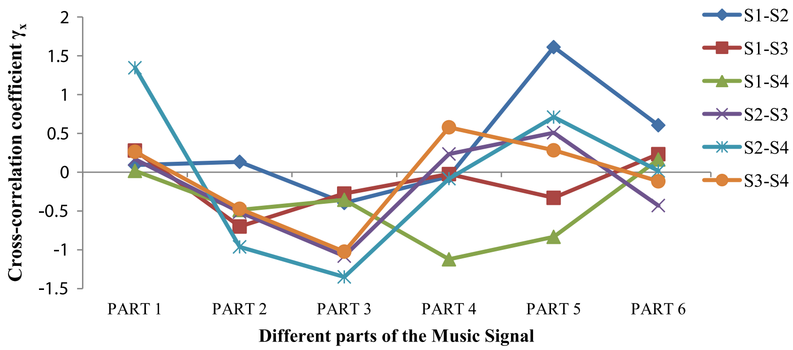


Fig. 9. Variation of cross-correlation coefficient among different samples.

Table 4
Cross correlation coefficient (γ_x) for various combinations.

Cross correlation coefficient (γ_x)						
	Sample 1–Sample 2	Sample 1–Sample 3	Sample 1–Sample 4	Sample 2–Sample 3	Sample 2–Sample 4	Sample 3–Sample 4
Part 1	0.092 ± 0.09	0.279 ± 0.02	0.017 ± 0.04	0.167 ± 0.03	1.345 ± 0.09	0.263 ± 0.06
Part 2	0.133 ± 0.05	-0.700 ± 0.17	-0.487 ± 0.05	-0.515 ± 0.05	-0.963 ± 0.13	-0.472 ± 0.03
Part 3	-0.396 ± 0.04	-0.276 ± 0.04	-0.356 ± 0.07	-1.081 ± 0.14	-1.349 ± 0.08	-1.021 ± 0.11
Part 4	-0.055 ± 0.07	-0.025 ± 0.06	-1.122 ± 0.16	0.235 ± 0.07	-0.083 ± 0.04	0.579 ± 0.07
Part 5	1.612 ± 0.03	-0.330 ± 0.05	-0.833 ± 0.09	0.509 ± 0.06	0.709 ± 0.05	0.282 ± 0.04
Part 6	0.605 ± 0.08	0.232 ± 0.09	0.163 ± 0.04	-0.430 ± 0.03	0.016 ± 0.07	-0.118 ± 0.06

may hypothesize that the artist has made variation in the rendition of the raga in those parts for which we are not getting any strong cross correlation, in this case especially in Part 1 while to some extent in Parts 5 and 6. Every rendition of the raga is different and unique as it embodies elements of the musician's vision, as well as his interpretation and this uniqueness might be manifested in those parts which do not have strong cross correlation coefficient among one another.
Further, the multifractal width (obtained from MFDFA technique) of the different parts of the four samples has been plotted in Fig. 10.

It is clearly observed from the figure that part 1, part 5 and part 6 of all the signals have varied multifractal width showing that the signals are quite different in complexity in those time windows. For other parts of the four signals, i.e. Parts 2–4, the multifractal widths form a cluster i.e. their spectral widths are almost same, depicting similar complexity. We can therefore say that the observed fluctuations of the multifractal width along the music sequences confirm the non-uniformity feature in the structures of melodic and rhythmic motions of music. In Parts 2–4, the multifractal widths are almost of the similar order for all the four signals studied, implies that the local fluctuations are comparable for these. This in turn substantiates our notion that the pattern of rendition in these parts mostly remains similar and the performer has stuck to the protocol during these parts. While in Parts 1, 5 and 6, the multifractal widths (and hence complexity of the signal) have shown

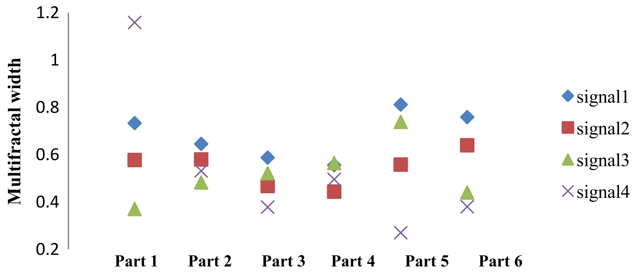


Fig. 10. Variation of multifractal spectral widths in different parts of the four samples.

considerable variation implying that the vocalist has made a number of subtle improvisations in the *raga* which has lead to the huge variety in the complexity patterns of these parts. This curve of multifractal width characterizes the melodic as well as the rhythmic phrasal patterns of music in different time windows. Thus, it is evident that multifractal width can be used as parameter with which it may be possible to characterize and quantify improvisational cues in Hindustani music performances.

With the help of different rigorous non-linear techniques we have thus studied the variation of complexity of musical structure in different portions of *bandish* of a raga and the results show that complexity varies appreciably within the same performance as well as within different performances. A high negative value of cross correlation coefficient γ_x signifies those portions of the *raga* which are strongly correlated i.e. they are bound by a tight framework of notes, note sequences etc., while the less correlated portions are those where the artist improvises and shows his uniqueness. We have therefore provided an innovative mean to disclose the intrinsic property of music called improvisation.

5. Conclusion

"Improvisation" refers to those elements of a musical performance which are generated spontaneously by the performer. Even during the performance of a musical composition, there will be some elements that are not pre-conceived, which will become the amazement factor for the audience. Examples of such improvisation can be variations in different parts of the *raga* added by the performer during the course of performance. Hindustani classical vocal music stands apart as one of the more difficult vocal forms, wherein the artist acts both as composer as well as singer; improvising at every instant during the performance. Improvisation is crucial and indispensible feature of Indian Classical Music which depends upon the imagination, originality and ingenuity of a particular vocal artist. Keeping within a fixed framework of the *raga*, a musician makes variation in the scansion of the lyrics over the period of rhythm as well as in the intricate details of the melodic structure. This is the essence of improvisation in Hindustani classical music, whose cues can only be identified with rigorous nonlinear techniques; such as MFDFA or MFDXA. The work leads to the following interesting conclusions:

1. MFDFA performed on the four vocal musical performances (in a 30 s window) by the same artist based on the same *raga* show multifractal nature. This multifractal nature of music signals may be coming from the multidimensional nature of music. The origin of multifractality in the music signals can be ascribed to the presence of long range correlation and broad probability density function as confirmed by randomly shuffling the original data.
2. The multifractal widths (obtained from MFDFA technique) show significant variation for different samples. But, we have found a clustering area which includes Parts 2, 3 and 4 for each sample, where the values of multifractal widths are more or less close to each other indicating close proximity of temporal fluctuations and complexity features of all the signals in these parts. We thus hypothesize, that these are the parts where the four different renditions are quite similar to one another and the performer is following the rigid framework of notes to establish the *raga*.
3. In Parts 1, 5 and 6 the multifractal widths (obtained from MFDFA) for each of the sample are considerably different from one another; i.e. the complexity features of the signal are greatly different. We hypothesize these portions as the "improvisation" part which makes each rendition unique from the other even though they have the same characteristic of that particular *raga* (here Sur Malhar). Each unique rendition is able to create a different mood or ambience in which the audience is captivated because of the improvisation part. The performers use his/ her own artistic experience and tradition to improvise and reinvent each individual performance in a new light.
4. Multifractal Detrended Cross Correlation Analysis (MFDXA) was done between the different samples for specific parts to ascertain the degree of cross correlation (in the form of cross correlation exponent, γ_x) among the signals. The results corroborated our previous findings and strong correlation was noted for Parts 2, 3 and 4 in case of all the possible combinations of correlation between samples. Other parts showed varying degree of cross correlation. Thus, it can be concluded that those parts which are strongly cross correlated are the parts which are alike one another and those parts which are not strongly correlated (but also not anti-correlated) are the parts in which improvisation takes place. Here, we have evaluated a particular property of the MFDXA technique, i.e. the cross correlation exponent, γ_x (for $q = 2$) and not the complete range of the cross-correlated multifractal spectra.

5. ANOVA test was performed on the four samples for all the parts to test the significance of the results. It was found that except for Part 1, all parts gave significant results with 95% confidence level. Post hoc analysis also revealed significant output for most of the Samples used.

Thus, with the help of rigorous latest nonlinear analysis techniques (MFDFA and MFDXA), we have proposed an automated algorithm with which one can identify the improvisational cues in a performance. MFDFA technique is by far the most suitable method—where in the measurement of 'w' (the multifractal spectral width), ideas of complexity and determinism are reintroduced and embedded. This method exhibits its novelty to use 'w' as the cue for improvisation at the deepest level of understanding and measurement. One should note that the multifractal width of the cross correlated signals (obtained from the MFDXA technique) can show signs of improvisation, but the cross correlation exponent (used in this study) is a more rigorous parameter for the assessment of improvisation. A naïve listener, who can recognize the changes in different performance perceptually, will now be able to conclusively identify the improvisational cues in each performance. From the performer's point of view, this study will help the performer to have a quantitative assessment of his improvisation. This in turn will help in the popularization of Hindustani classical music. We conclude emphasizing that the importance of this study in application area for cognitive music therapy is also immense. Analysis with a wide variety of *raga* and also the importance of gestural movements in the rendition of a *raga* is an interesting area for future research. We intend to do the analysis of entire multifractal cross-correlated spectrum of the two music signals, in future, for the study of improvisational cues in Hindustani music which might reveal more interesting results.

Acknowledgments

One of the authors, AB acknowledges the Department of Science and Technology (DST), Govt. of India for providing (A.20020/11/97-IFD) the DST Inspire Fellowship to pursue this research work. The first author, SS acknowledges the West Bengal State Council of Science and Technology (WBSCST), Govt. of West Bengal for providing the S.N. Bose Research Fellowship Award to pursue this research (193/WBSCST/F/0520/14).

The authors gratefully acknowledge the reviewers for their insightful comments which have helped in enriching the contents of this paper in numerous ways.

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