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Understanding the multifractality in portfolio excess returns

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HIGHLIGHTS

- The multifractality in portfolio excess returns has not been considered in the literature.
- The significant multifractality is revealed via MF-DFA.
- The multifractality is mainly attributed to long-range dependence.
- The cross-correlations between portfolio and market returns are multifractal.

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ABSTRACT

The multifractality in stock returns have been investigated extensively. However, whether the autocorrelations in portfolio returns are multifractal have not been considered in the literature. In this paper, we detect multifractal behavior of returns of portfolios constructed based on two popular trading rules, size and book-to-market (BM) ratio. Using the multifractal detrended fluctuation analysis, we find that the portfolio returns are significantly multifractal and the multifractality is mainly attributed to long-range dependence. We also investigate the multifractal cross-correlation between portfolio return and market average return using the detrended cross-correlation analysis. Our results show that the cross-correlations of small fluctuations are persistent, while those of large fluctuations are anti-persistent.

1. Introduction

The efficient market hypothesis of Fama [1] advocates that the current financial asset prices can respond quickly to the new information and therefore market investors have rare chance to get excess returns. However, more and more researchers find that capital markets do not work efficiently as EMH depicts but like a complex system. Many phenomena such as long-term reversal effect [2], momentum effect [3] and some other stylized facts that are pervasive in both developed and emerging market also reflect that investors can get excess return if they could find the law hidden in the market. These studies all bring big challenges to the EMH.

In the area of econophysics, a wide range of methods have been developed to test for EMH by examining the independence of financial asset returns. Peng et al. [4] propose the detrended fluctuation analysis (DFA) when studying the auto-correlations of molecular chains in deoxyribonucleic acid (DNA). Then Kantelhardt [5] extend DFA to the multifractal form and introduce a multifractal detrended fluctuation analysis (MF-DFA). MF-DFA is widely employed to reveal the existence of multifractality in the capital markets which again highlights the fact of market inefficiency. For example, Onali and Goddard [6] investigate the performance of Italian stock market, and the evidence shows that the market is multifractal.

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Narouzzadeh and Rahmani [7] find the multifractal temporal dependence in Iranian rial–US dollar exchange rate using MF-DFA, also indicating that the exchange market is inefficient.

The interdependence or cross-correlation which suggests that a variable can predict the other variable also implies the market inefficiency. Podobnik and Stanley [8] extend the DFA to bivariate case and design a new algorithm for cross-correlation, named detrended cross-correlation analysis (DCCA). This method can be used to effectively detect cross-correlation between two non-stationary time series. Until now, DCCA has been the most popular method for cross-correlation. For example, Dutta et al. [9] employ DCCA and find the strong cross-correlations between exchange rates and stock markets in India. Sequeira Jr. et al. [10] also use the DCCA and find that the cross-correlations between the Brazilian stock and commodity markets are stronger than what would be expected from simple combinations of auto-correlations of individual series. Wang et al. [11] quantitatively investigate the cross-correlations between Chinese A-share and B-share markets using the method of DCCA, and point out that the cross-correlations are strongly multifractal in the short-term and weakly multifractal in the long-term. Dutta et al. [12] consider that the cross-correlation coefficients can be linked with the stability of the market, and the market is more stable when the two series are more heavily correlated. In addition, the DCCA method has been also used in some other related papers [13–18].

Moreover, the sources for multifractality are investigated from several aspects, such as the long-range correlation and the fat-tailed distribution. Barunik [19] compute the generalized Hurst exponents of different financial time series including the stock market indices, interests and exchange rates and finds that the major reason for formatting their multifractal characteristics lies in the fat-tailed distribution. Wang [20] shows that long-term dependence is the primary cause of the multifractal phenomenon of the NASDAQ stock market.

Although the multifractality in financial markets has been studied extensively, in this paper we revisit this topic by contributing to the literature in two aspects. First, we focus on the excess returns, rather than the nominal returns in most studies since investors are usually more concerned about the return in excess of risk-free rate. For most investors, one of the important reasons for choosing stocks instead of riskless Treasury bill is that they have chance to get higher return as the compensation of undertaking the higher risk. If the return that investors get from the stock market is even lower than the risk-free rate, then they would only buy the Treasury bill and undertake no risk. Furthermore, we investigate the multifractality in portfolio returns, instead of returns of individual stocks or market index. The motivation is from the market segmentation theory which suggests that investors always pay attention to only a fraction of stocks and portfolio diversification theory which argues that investing in a basket of assets performs better than investing in an individual asset. We detect the multifractality in returns of portfolios formed by rules of size and book-to-market ratio via a multifractal detrended fluctuation analysis (MF-DFA). We also find the source of multifractality to further understand the origins of portfolio uncertainty. Second, we investigate the long-range cross-correlations between portfolio returns and market returns using the well-known detrended cross-correlation analysis (DCCA). This has important implications for asset pricing. For example, if returns of portfolio and market index are long-range cross-correlated, the classical Capital Asset Pricing Model (CAPM) which implies short-term contemporaneous correlated behavior is not suitable to capture their relationships.

The remainder of this paper is organized as follows: Section 2 provides the methodology. Data description is provided in Section 3. Section 4 shows the empirical results. The last section concludes the paper.

2. Methodology

In this section, we will give a brief description about the multifractal detrended fluctuation analysis (MF-DFA) and its bivariate extension, i.e., multifractal detrended cross-correlation analysis (MF-DCCA). These two methods have been considered powerful tools in analyzing multifractality in autocorrelation and cross-correlation, respectively.

2.1. Multifractal detrended fluctuation analysis

Traditional detrended fluctuation analysis (DFA) can only be used to detect the mono-fractal characteristics of the time series. Kantelhardt [7] extend the DFA method to the multifractal form and propose the new method known as the multifractal DFA (MF-DFA). The MF-DFA algorithm can be described as follows:

Step 1. Consider one time series, $\{x_t,\ t = 1, \dots, N\}$, where N is the length of the series. Then we describe the "profile" and get a new series,

$$y_k = \sum_{t=1}^{k}(x_t - \bar{x}), \quad k = 1, 2, \dots, N \tag{1}$$

where $\bar{x}$ denotes the average over the whole time series.

Step 2. Divide the profile $\{y_k\}_{k=1,\dots,N}$ into $N_s \equiv int(\frac{N}{s})$ nonoverlapped segments of equal length s. Since the length N of the series is often not a multiple of the considered time scale s, a short part at the end of the profile may remain. In order not to disregard this part of the series, the same procedure is repeated starting from the opposite end of the profile. In this way, $2N_s$ segments are obtained altogether. Following the suggestion of Peng et al. [4], we set $10 < s < N/5$.

Step 3. Calculate the local trend for each of the $2N_s$ segments by a least-square fit of the series. Then determine the variance

$$F^2(s,\lambda) \equiv \frac{1}{s}\sum_{j=1}^{s}\left[y_{(\lambda-1)s+j} - P_\lambda(j)\right]^2 \tag{2}$$

for $\lambda = 1, 2, \ldots, N_s$ and

$$F^2(s,\lambda) \equiv \frac{1}{s}\sum_{j=1}^{s}\left[y_{N-(\lambda-N_s)s+j} - P_j(j)\right]^2 \tag{3}$$

for $\lambda = N_s + 1, N_s + 2, \ldots, 2N_s$. Here, $P_j(j)$ is the fitting polynomial with order m in segment λ (conventionally, called mth order MF-DFA and wrote MF-DFAm).

Step 4. Average over all segments to obtain the qth order fluctuation function

$$F_q(s) = \left\{\frac{1}{2N_s}\sum_{\lambda=1}^{2N_s}\left[F^2(s,\lambda)\right]^{\frac{q}{2}}\right\}^{\frac{1}{q}} \tag{4}$$

for any real value $q \neq 0$ and

$$F_0(s) = \exp\left\{\frac{1}{4N_s}\sum_{\lambda=1}^{2N_s}\ln\left[F^2(s,\lambda)\right]\right\}. \tag{5}$$

We repeat steps 2 to 4 for several time scales s. It is apparent that $F_q(s)$ will increase with larger values of s. Of course, $F_q(s)$ depends on the DFA order m. By construction, $F_q(s)$ is only defined for $s \geq m + 2$.

Step 5. Determine the scaling behavior of the fluctuation functions by analyzing log–log plots of $F_q(s)$ versus s for each value of q. If the series x_t are long-range power-law correlated, $F_q(s)$ increases, for larger values of s, as a power-law,

$$F_q(s) \sim s^{h(q)}. \tag{6}$$

In general, the exponent $h(q)$ in Eq. (6) may depend on q. When $h(q) < 0.5$, the kinds of fluctuations related to q have persistence, while $h(q) > 0.5$, the kinds of fluctuations related to q have anti-persistence. If $h(q) = 0.5$, the kinds of fluctuations related to q display random walk behavior. Obviously, the larger bias between $h(q)$ and 0.5 implies the corresponding fluctuations are more inefficient.

2.2. Detrended cross-correlation analysis

In order to investigate the cross-correlations between two time series, Podobnik and Stanley [8] introduce the detrended cross-correlation analysis (DCCA). This is a robust method of detecting the cross-correlation even between two nonstationary financial time series. DCCA, to some extent, can be taken as the bivariate extension of DFA. The multifractal form of DCCA (MF-DCCA) can be described as follows:

Step 1. Consider two time series, $\{x_t, t = 1, \ldots, N\}$ and $\{y_t, t = 1, \ldots, N\}$, where N is the equal length of these two series. Then, we describe the "profile" of each series and get two new series, $xx_k = \sum_{t=1}^{k}(x_t - \bar{x})$ and $yy_k = \sum_{t=1}^{k}(y_t - \bar{y})$, $k = 1, \ldots, N$.

Step 2. Divide the both profiles $\{xx_k\}$ and $\{yy_k\}$ into $N_s \equiv int(\frac{N}{s})$ segments as the MF-DFA does.

Step 3. Calculate the local trends

$$F^2(s,\lambda) \equiv \frac{1}{s}\sum_{j=1}^{s}\left[xx_{(\lambda-1)s+j} - \widetilde{xx}_{(\lambda-1)s+j}\right]\left[yy_{(\lambda-1)s+j} - \widetilde{yy}_{(\lambda-1)s+j}\right] \tag{7}$$

for $\lambda = 1, 2, \ldots, N_s$ and

$$F^2(s,\lambda) \equiv \frac{1}{s}\sum_{j=1}^{s}\left[xx_{N-(\lambda-1)s+j} - \widetilde{xx}_{N-(\lambda-1)s+j}\right]\left[yy_{N-(\lambda-1)s+j} - \widetilde{yy}_{N-(\lambda-1)s+j}\right] \tag{8}$$

for $\lambda = N_S + 1,\ N_S + 2, \ldots,\ 2N_S$. The trends $\widetilde{xx}_{(\lambda-1)s+j}$ and $\widetilde{yy}_{(\lambda-1)s+j}$ can be computed from linear, quadratic or high order polynomial fit of each profile for segment λ.

Step 4 and Step 5 of DCCA are similar to those of MF-DFA. And if two series are long-range cross-correlated, as a power-law

$$F_q(s) = s^{\alpha(q)}. \tag{9}$$

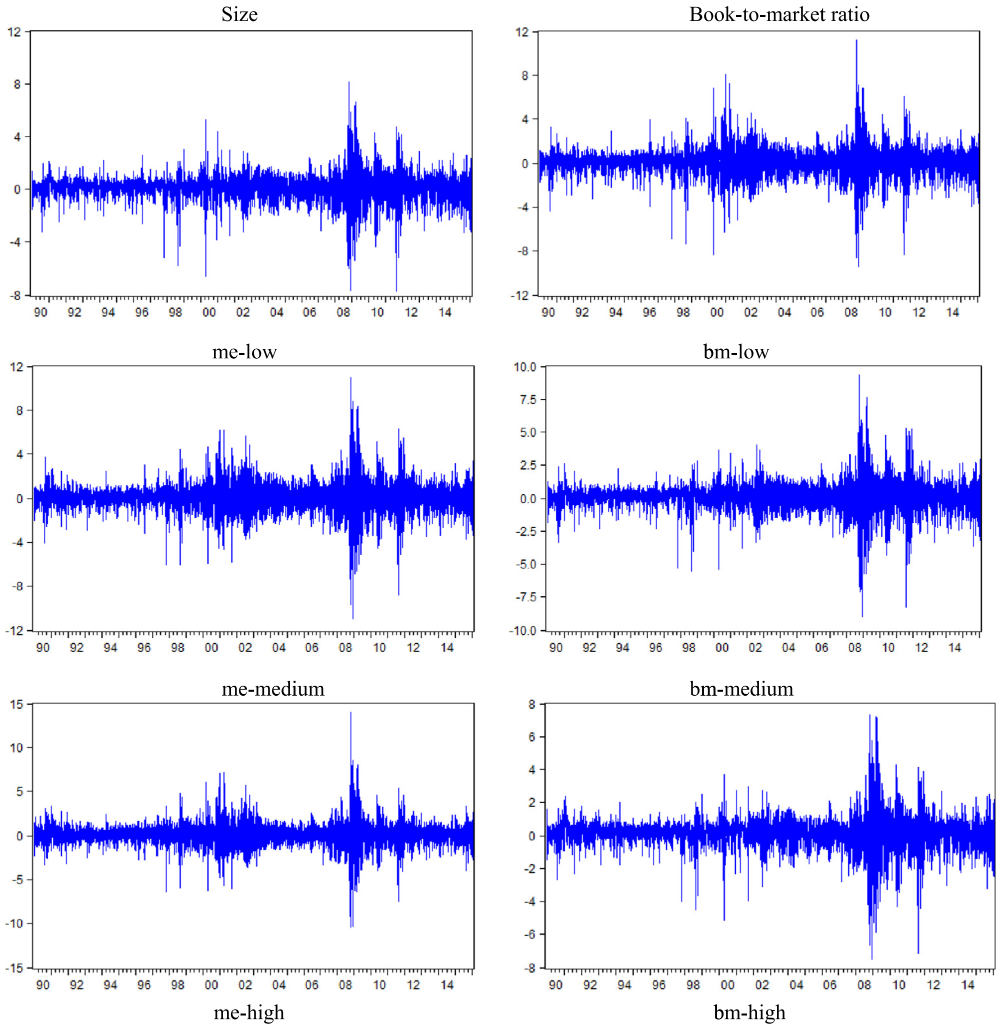

Fig. 1. Excess returns of six portfolios.

Here, if the scaling exponent $\alpha(q) > 0.5$, the cross-correlation between the kinds of fluctuations related to q are persistent. An increase of one price is likely to be followed by an increase of the other price. If scaling exponent $\alpha(q) < 0.5$, the cross-correlations between the kinds of fluctuations related to q are anti-persistent. An increase of one price is likely to be followed by a decrease of the other price. If $\alpha(q) = 0.5$, one series is not cross-correlated with the other, and the change of one price cannot affect the behavior of the other price.

3. Data

In this paper, we construct portfolios of stocks of all the companies that listed in NYSE, AMEX and NASDAQ. We consider two popular rules to construct portfolios. These two rules are based on the companies' sizes and book-to-market (BM) ratios, respectively. In detail, we form the portfolios belonging to the bottom 30%, the middle 40% and the top 30% of the stocks of all the companies classified by size or BM ratio. In this way, we can obtain a total of six portfolios.

We get daily excess return data of these six portfolios from the homepage of Prof. Kenneth French.[1] Our data covers the period from January 2nd, 1990 to January 29th, 2016. For convenience, three excess return series formed by size rule are denoted as me_low, me_medium, me_high, respectively. Similarly, the return series formed by BM ratio are denoted as bm_low, bm_medium and bm_high, respectively. The graphical representations of these six portfolio return series are illustrated in Fig. 1.

1 http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

Table 1
Descriptive statistics of excess return series.

	me_low	me_medium	me_high	bm_low	bm_medium	bm_high
Mean	0.108	0.05	0.047	0.068	0.085	0.121
Median	0.18	0.11	0.09	0.15	0.15	0.17
Maximum	8.12	11	13.99	11.24	9.39	7.28
Minimum	−7.74	−11	−10.39	−9.45	−9	−7.47
Std. Dev.	0.975	1.296	1.229	1.232	1.034	0.902
Skewness	−0.487	−0.15	0.008	−0.263	−0.361	−0.463
Kurtosis	10.564	10.015	13.684	9.734	11.653	12.938
Jarque–Bera	15925.98***	13498.63***	31260.15***	12494.27***	20645.82***	27277.34***
ADF	−39.225***	−78.012***	−60.190***	−74.301***	−43.030***	−39.381***

Note: The Jarque–Beta statistic tests for the null hypothesis of normality in sample returns distribution. ADF are statistics of the Augmented Dickey–Fuller unit root test based on the AIC criterion.
*** Indicates significance at the 1% significance level.

4. Empirical results

From Fig. 1, we can see that returns of different portfolios experienced an overall similar trend from 1990 to 2016, and the big swings also happened during the same time intervals. For instance, because of the big shock of the 1998 Asian financial crisis, the volatility of each portfolio return significantly widened during 1998–2002 period. From 2008 to 2012, each portfolio return experienced continuously large fluctuations which can be attributed to several occasional events, including the global financial crisis in mid-2008, and the European debt crisis in March 2010.

Further comparing these two types y portfolios, we find that returns of size-sorted portfolios are more volatile than returns of BM-sorted ones, particularly when financial crisis occurred. Portfolios of companies with larger sizes display lower return volatility than those with smaller sizes. Also, high-BM portfolio returns are usually less volatile than small-BM portfolio during crisis periods.

Table 1 shows the descriptive statistics for the six return series. The means values are all positive, indicating that investors can on average get positive returns. The portfolios of smaller-size and higher-BM companies present higher returns, consistent with the well-known "small firm effect" and "BM effect". Meanwhile, we can observe that portfolios with higher returns also display higher volatility, implying the existence of risk–return trade-off. On the distribution property, the Jarque–Bera statistics significantly reject the null hypothesis of Gaussian distribution at 1% level, suggesting the existence of stylized fact of fat-tailed distribution. This finding is also confirmed by the non-zero skewness and the values of kurtosis greater than 3. We use the augmented Dickey and Fuller test to examine whether the excess returns are stationary. The statistics for all series reject the null hypothesis of unit root in favor of the stationarity.

To further investigate how the portfolio returns deviate from Gaussian distribution, we show the Quantile–Quantile plots in Fig. 2. We can see that the Quantile–Quantile of both size-formed and BM-formed portfolio returns share the similar pattern with a downward slope at the top and an upward slope at the bottom, indicating that these return series have a sharper peak and a fatter tail than the normal distributed series. The return densities shown in Fig. 3 also confirm our finding on fat-tailed distributions.

4.1. Detrended fluctuation analysis

We use MF-DFA to investigate the multifractality in size- and BM-formed portfolio returns. The graphical illustrations of generalized Hurst exponents $h(q)$ of size-formed portfolios are shown in Fig. 4. We set the fluctuation order to change from -10 to 10 with a fixed step of 1. We can clearly see that the values of $h(q)$ decrease with the increase of q, implying the existence of multifractality in each of three return series.

Specifically, the values of $h(q)$ of small-size portfolio returns are always greater than 0.5, indicating that the fluctuations are persistent. In other words, a positive return of the small-size portfolio is likely to be followed by another positive return. The $h(q)$'s of returns of medium-size and big-size company portfolios are larger than 0.5 when q is negative and smaller than 0.5 when q is positive. This indicates that small returns of medium- and big-size portfolios are persistent whereas large returns are anti-persistent. In other words, a small positive (negative) return is always followed by another small positive (negative) return but a large positive (negative) return is more likely to be followed by a large negative (positive) return (i.e., mean-reverting behavior).

Fig. 5 shows $h(q)$ values of BM-formed portfolio returns. The monotonically decreasing pattern of $h(q)$ also implies the existence of multifractality. In detail, the generalized Hurst exponents of three BM-formed portfolios are rather close, suggesting the similar multifractal behavior. When $q < 0$, the $h(q)$'s are greater than 0.6, indicating the strongly persistent behavior of small fluctuations. When $q > 0$, the $h(q)$'s are close to 0.5, indicating that the auto-correlations of large fluctuations are relatively weak. Notably, when q is positive, the $h(q)$'s of high-BM portfolios still change around 0.5 and larger than those of both low-BM and medium-BM portfolios, which indicates that even in face of big market volatility can high-BM companies react better than low-BM and medium-BM ones.

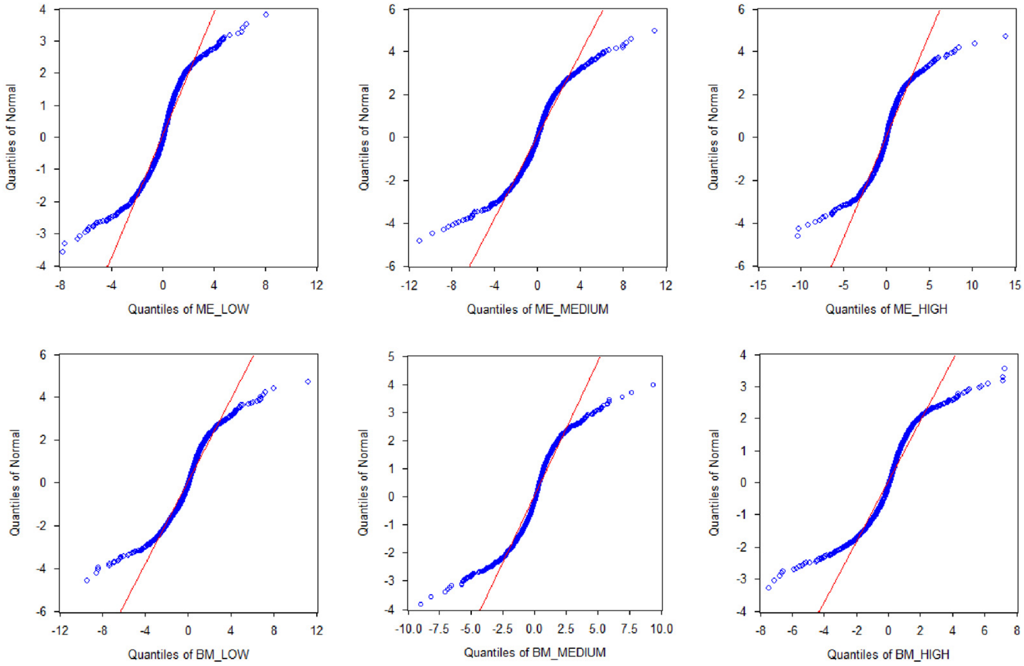

Fig. 2. Quantile–Quantile of excess returns.

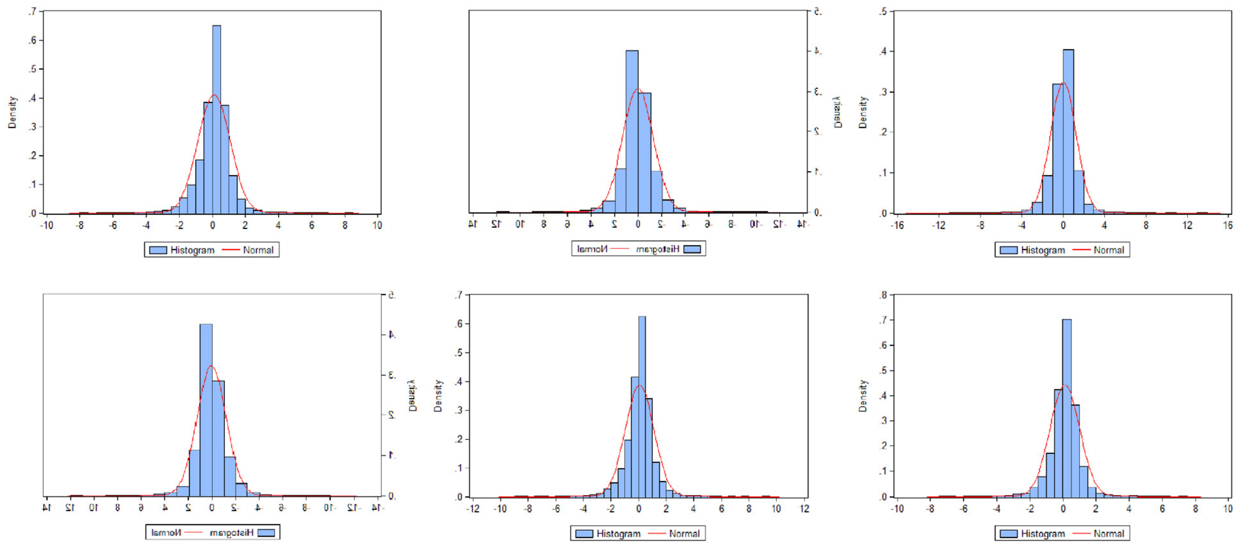

Fig. 3. Density of excess returns.

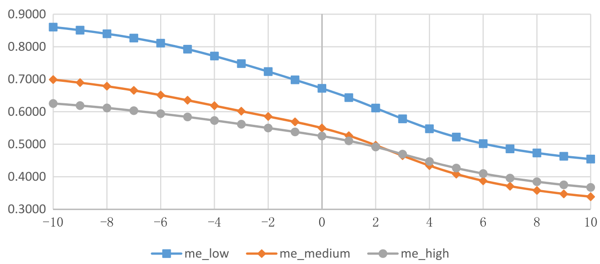

Fig. 4. Generalized Hurst exponents of size-formed portfolios.

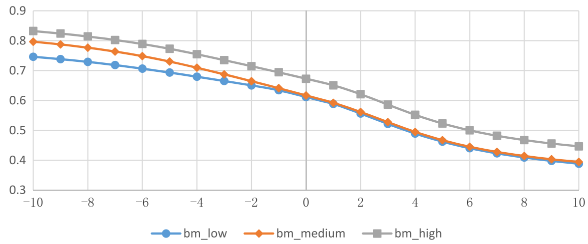

Fig. 5. Generalized Hurst exponents of BM-formed portfolios.

Table 2
The degree of multifractality during two subperiods.

Δh	me_low	me_medium	me_high	bm_low	bm_medium	bm_high
Whole sample	0.4061	0.3604	0.2581	0.3581	0.4024	0.3859
Period I	0.4980	0.2661	0.1910	0.4281	0.2853	0.3068
Period II	0.2202	0.1933	0.1826	0.2163	0.2000	0.2044

Note: $\Delta h = h(-10) - h(10)$. Period I is from January 2nd, 1990 to December 31st, 2002, and Period II covers the remaining period.

To investigate the change of multifractality over time, we divide the whole sample into two subsamples with the equal length. The first and second sample periods are denoted as Period I and Period II, respectively. Table 2 shows how multifractality of each portfolio returns change over the time. We define the multifractality degree as the range of generalized Hurst exponents. It is clear that each portfolio's Δh during period I is obviously larger than that during period II, which tells us that multifractality of portfolio returns gradually becomes weaker over the time. This is a piece of 'good news' to EMH's supporters to some extent, since the weaker multifractal behavior also implies the higher degree of market efficiency.

There are many factors which can result in the multifractality such as long memory and fat-tailed distribution. By comparing the multifractality of the original series and randomly shuffled series, we can obtain the contribution of the long-range correlations. The procedure of getting the shuffled series can be described as follows:

(1) Generate a random natural logarithm (j, k) with j, k not bigger than N, and N is the length of time series to be shuffled.
(2) Exchange the jth and kth data in original sequence.
(3) Repeat the above steps for $20N$ times to ensure the original series to be fully shuffled.

We can compare the multifractality between the original series and surrogated series to obtain the contribution of fat-tailed distribution. The method to acquire surrogated series can be described as follows:

(1) For a given series, we can generate a series of random numbers with Gaussian distributions $\left\{r_t',\ t = 1, \ldots, N\right\}$.
(2) Rearrange the series generated in (1) to get the rearranged series $\{r_t,\ t = 1, \ldots, N\}$ which has the same rank ordering as the original series.

Figs. 6a and 6b show the generalized Hurst exponents of original, shuffled and surrogated series. We can see that both shuffled and surrogated procedures can decrease the dependence of the generalized Hurst exponents $h(q)$ to the fluctuation order q. This evidence manifests that long-term correlation and fat-tailed distribution are both the sources for the multifractality. Furthermore, the multifractality degrees of shuffled series are the weakest such that the values of $h(q)$ maintain between 0.45 and 0.6. This evidence indicates that long-term correlation is the main factor of multifractality characterization while fat-tailed distribution just makes limited contribution.

4.2. *Detrended cross-correlation analysis*

The classical Capital Asset Pricing Model (CAPM) claims that the market return is an important factor explaining stock returns. Market average return reflects investors' expectation and assessment of the whole capital market, and the returns of the size-formed company portfolios and BM-formed company portfolios certainly both have some connection to it. In this subsection, we use the multifractal detrended cross-correlation analysis to detect the relationship between the portfolio return and the market average return.

We get the scaling behavior of the fluctuation functions by analyzing log–log plots $F_q(s)$ versus s for each value of q. From Table 3 we can see that scaling exponents for different values q are different, indicating that the cross-correlation between the returns of different company portfolios and the market average return are nonlinear, specifically multifractal.

In order to learn the underlying reasons, we investigate the overall correlation by breaking up it into different amplitudes of fluctuations. For $q = 10$, scaling exponents $\alpha(10)$ of different portfolio returns are around 0.4, which indicates the cross-correlated behaviors of large fluctuations are weakly anti-persistent. That is to say, when large fluctuations happen to market average return, the company portfolio returns tend to act in a direction that weakly contrary to market average return.

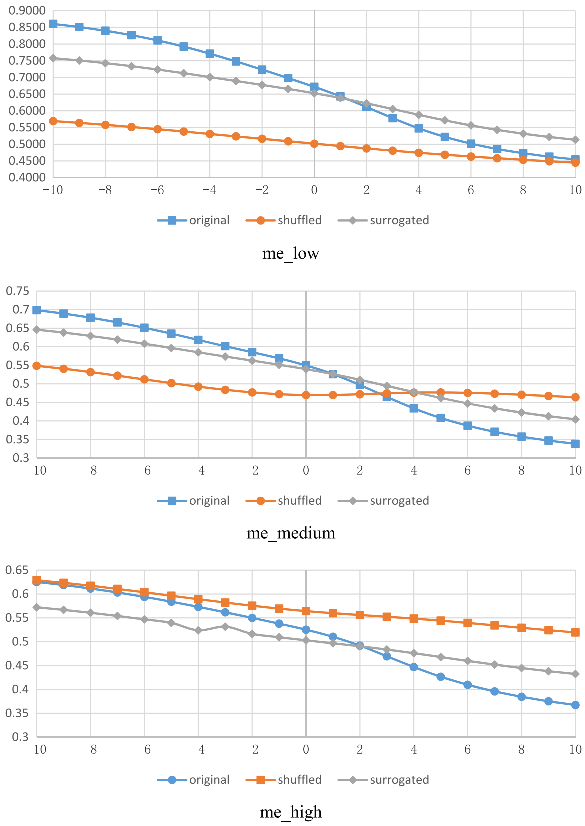

Fig. 6a. Generalized Hurst exponents of original, shuffled and surrogated series for size-formed portfolio returns.

Table 3
Scaling exponents for cross-correlations between portfolio return and market return.

q	me_low	me_medium	me_high	bm_low	bm_medium	bm_high
−10	0.7435	0.6736	0.6279	0.6900	0.7143	0.7310
−5	0.6825	0.6120	0.5803	0.6366	0.6518	0.6743
−2	0.6315	0.5629	0.5411	0.5952	0.5977	0.6256
−1	0.6142	0.5473	0.5282	0.5805	0.5801	0.6099
1	0.5767	0.5133	0.5026	0.5441	0.5441	0.5783
2	0.5536	0.4921	0.4875	0.5202	0.5222	0.5579
5	0.4803	0.4233	0.4319	0.4471	0.4495	0.4824
10	0.4162	0.3615	0.3749	0.3841	0.3858	0.4153

Such fluctuations of portfolios are generally considered to be temporary and will bounce back in future. For $q = -10$, scaling exponents $\alpha(-10)$ are all larger than 0.5 and change around 0.7, indicating that there exists strong persistent cross-correlation between the behaviors of small fluctuations. This is understandable. In general, small changes in the market are caused by factors within the market. For example, an overall declining expectation of some certain industry could lead to an outflow of the capital market, which not only reduces the market average return but also the return of each company portfolio.

Notably, scaling exponents $\alpha(-10)$ for returns of small-size and high-BM company portfolios are both larger than 0.5, while the scaling exponents for the other portfolios are all less than 0.5, all indicating that returns of small-size and high-BM companies usually response in the same direction as the market when large volatility happens. Given the above descriptive result that the market average return is positive and portfolios' same-direction response to the market fluctuations, returns of these two portfolios usually change positively. This is why investing in small-size and high-BM portfolios can get higher excess returns than investing in the other portfolios.

5. Conclusions

In this paper, we investigate the multifractal behaviors of excess returns of portfolios formed by size and book-to-market rules. Using the multifractal detrended fluctuation analysis, we find the strong multifractal characterization of these

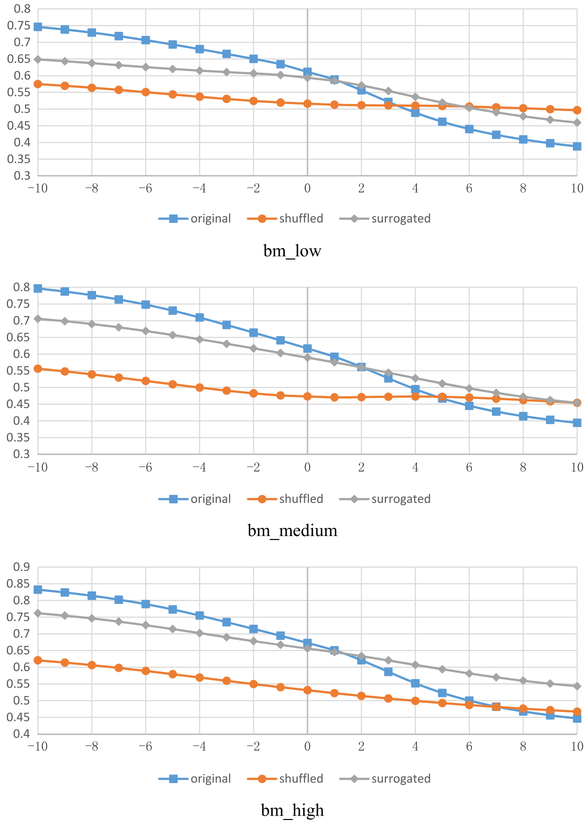

Fig. 6b. Generalized Hurst exponents of original, shuffled and surrogated series for BM-formed portfolio returns.

portfolio returns, which challenges the traditional EMH. We further investigate the sources of return multifractality, and find that long-term correlation and fat-tailed distribution are both contributing factors and the former one plays the critical role. Furthermore, we explore the cross-correlation between the portfolio return and market average return using the method of multifractal detrended cross-correlation analysis. The significant multifractality is also observed. Moreover, the cross-correlation of small fluctuations are persistent, while those of large fluctuations are usually anti-persistent.

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