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Multifractal detrended Cross Correlation Analysis of Foreign Exchange and SENSEX fluctuation in Indian perspective

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HIGHLIGHTS

- Auto and cross correlations of SENSEX and FX rate for Indian currency.
- MFDFA and MFDXA methodologies were employed.
- Degree of correlation with time is studied.
- Results are found to be consistent with data and explained qualitatively.

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ABSTRACT

The manuscript studies autocorrelation and cross correlation of SENSEX fluctuations and Forex Exchange Rate in respect to Indian scenario. Multifractal detrended fluctuation analysis (MFDFA) and multifractal detrended cross correlation analysis (MFDXA) were employed to study the correlation between the two series. It was observed that the two series are strongly cross correlated. The change of degree of cross correlation with time was studied and the results are interpreted qualitatively.

1. Introduction

The last few decades have experienced that concepts and methods of Statistical Physics have been effective in elucidating the properties and complex dynamics exhibited in economic and financial time series. The study of the Foreign Exchange market and the stock market is very important. The two markets are considered to be the barometers measuring the economic growth of a country. In finance, exchange rate between the two currencies is the rate at which one currency is exchanged for another. The Foreign Exchange rate is the key variable, that affects the Foreign Exchange investors, bankers, policymakers, economic institutions, the imports and exports of a country, tourists visits in a country in terms of the value of their foreign currency. A value of a local currency appreciates whenever the demand for it is greater than the available supply while the reverse will depreciate the value. Thus the movements in the exchange rates have important impacts on the economy's business cycle, country's trade, inflow of foreign funds and are therefore extremely vital for understanding the financial developments, industry policies, future cash flows and stock prices of the firms. The stock markets give

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benchmark indices that are representative of the entire market. The stock markets are based on the demand and supply forces prevailing in the market and therefore are highly volatile. A growing stock market would attract capital flows from foreign investors, which may cause an increase in the demand for a country's currency. While in case of a falling stock market the investors would try to sell their stocks to avoid further losses and would convert their money into foreign currency to move out of the country. There would be demand for foreign currency in exchange of local currency and it would lead to the depreciation of local currency. A knowledge about the daily changes in the stock prices is very crucial for the investors. This shows that it is necessary to study the existence of a relation between the Foreign Exchange market and the stock market. Several researchers have tried to investigate the relationship between the two markets in the Indian scenario [1–3]. Gulati and Kakhani [1] attempted to examine whether or not a casual relationship existed between the two markets. Patel and Kagalwala [2] analysed the relationship between exchange rates and Indian stock exchanges. This paper attempts at a deeper insight into the subject by utilizing the multifractal properties of the time series to reveal the correlation between the two. Such an investigation was not attempted in past in the Indian scenario.

Stanley et al. [4] have observed that physical systems which consist of a large number of interacting particles obey universal laws that are independent of the microscopic details. Since economic systems also consist of a large number of interacting units, scaling theory can also be applied to economics. Lux and Marchesi [5] have shown that in financial Stock markets, the dynamical change of price over time illustrates a highly complex behaviour because it is generated by the non-linear interactions among heterogeneous agents. The economic systems are characterized with extreme complexities and have recently become an interesting area of study for the physicists as well as economists [6,7]. Many studies have found the financial time series to exhibit some non-linear properties such as long-memory in volatility [8–11], a multifractal nature [12–16] and fat tails [16–20]. Various empirical studies have also shown financial time series to exhibit scaling like characteristics. Muller et al. [21] and Guillaume et al. [22] have reported an empirical scaling law for mean absolute price changes over a time interval for Foreign Exchange Rate. Dacorogna et al. [23] have presented empirical scaling laws for US Dollar–Japanese Yen (USD–JPY) and British Pound–US Dollar (GBP–USD). Mantegna and Stanley [17] also found scaling behaviour in the Standard and Poor's index (S&P 500) by examining high frequency data. Gencay et al. [24] suggested that financial time series may not follow a single-scaling law across all horizons. They have used a wavelet multi-scaling approach to show that Foreign Exchange Rate volatilities follow different scaling laws at different horizons. It was also evident from the study that there was no unique global scaling in financial time series but rather scaling was time varying.

Various time series in the financial markets are reported to possess multifractal properties [18,25,26], such as the Foreign Exchange Rate [27–40], Gold price [12,41,42], commodity price [43], Stock price [43–54], to list a few. Extensive methods have been adopted to extract the empirical multifractal properties in financial data sets, for instance, Wavelet Transform Modulus Maxima (WTMM) [55–57], and the Multifractal Detrended Fluctuation Analysis (MFDFA) [58], Multifractal Detrended Cross Correlation analysis [59], Multifractal Detrending Moving-Average Cross Correlation Analysis [60], Multifractal Cross Correlation Analysis [61]. A time series of the price fluctuations possessing multifractal nature usually has either fat tails in the distribution or long-range temporal correlation or both [58]. However, possessing long memory is not sufficient for the presence of multifractality and one has to have a non-linear process with long-memory in order to have multifractality [62].

The present investigation studies the multifractal properties of Foreign Exchange Rate in terms of Indian Rupee and US Dollar (abbreviated as FX) and SENSEX, an index of Bombay Stock Exchange, India. The term Foreign Exchange refers to the exchange of one currency for another. The value of a country's currency is very much linked with its economic conditions, policies and political conditions. The origin of the Foreign Exchange market in India could be traced to the year 1978. The currency price is always stated in relation to another currency. So when one currency appreciates, the other depreciates. A currency will tend to become valuable whenever the demand for it is greater than the available supply causing the currency to appreciate. The exchange rate changes daily with the international supply and demand for currency depending on the country's economy such as imports, exports, inflation, employment, etc.

The SENSEX or Sensitive Index was first compiled in the year 1986. Over the years it has become a prime indicator of the Indian Stock Market. The index is calculated based on a free float capitalization method and accordingly the level of index at any point of time reflects the free float market value of 30 largest and most actively traded stocks relative to a base period. Stock prices change every day because of pressure from the markets. If the SENSEX goes up it means that the prices of the stocks of most of the major companies on the BSE have gone up and vice versa. SENSEX is highly volatile and the Foreign Exchange Institutional Investors (FIIs) have a major impact on the movement of the SENSEX and are responsible for the rise and fall of the SENSEX. The global financial crisis that appeared by September 2008 had a negative impact on the Indian economy. The recession in the US resulted in the outflow of the Foreign Institutional Investment (FII) and the value of the Indian Rupee underwent a steep depreciation and the SENSEX experienced a sharp fall. Figs. 1 and 2 represent the variation of FX and SENSEX over a period from January 1995 to December 2012. At the onset, it can be observed that both the series show fluctuations with respect to time which are huge in the recent years from 2006 to 2012. In the present study autocorrelation and cross correlation between FX and SENSEX are studied using Multifractal Detrended Fluctuation Analysis (MFDFA) and Multifractal Detrended Cross Correlation Analysis (MFDXA).

Fractal geometry is associated with systems that are basically irregular at all scales [63]. They have two important properties, self-similarity and non-integer dimensions. Fractals can be classified into two categories: monofractals and multifractals. The monofractal systems are those, whose scaling properties are the same in different regions of the systems and a single scaling exponent is sufficient to describe such systems. On the other hand, the multifractal systems which

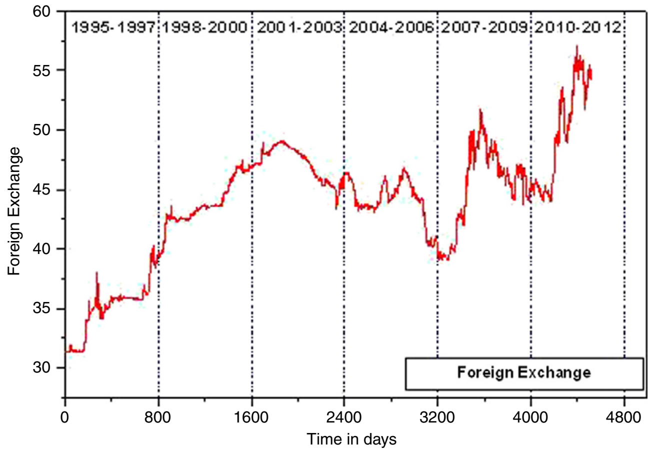

Fig. 1. Plot of the fluctuation of the Foreign Exchange Rate (FX) over a period from January 1995 to December 2012.

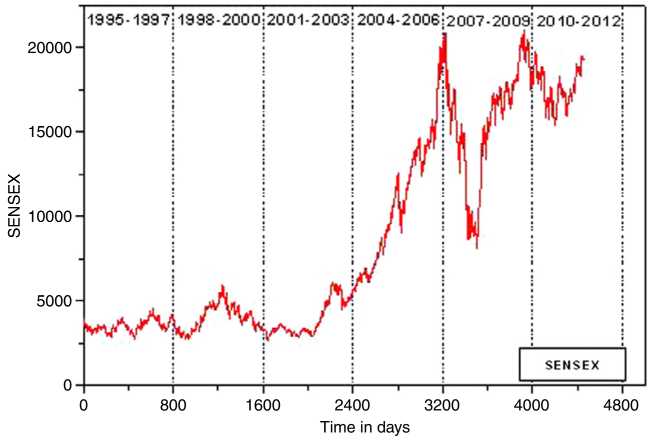

Fig. 2. Plot of the fluctuation of the SENSEX over a period from January 1995 to December 2012.

are much more complicated and in which a single scaling exponent is not enough to describe their dynamics, instead a continuous spectrum of exponents (the singularity spectrum) is needed.

Peng et al. [64] introduced the Detrended Fluctuation Analysis (DFA) methodology to investigate the fractal scaling properties and long-range correlations of noisy, non-stationary time series. It has been applied successfully over wide variety of fields extending from the study of the DNA sequences, heart rate dynamics, neuron spiking and human gait to studying geological data, economic time-series and also to weather related and earthquake signals [8,65–71]. However, many real world data, such as many geophysical signals, medical patterns and even economic and financial time-series do not exhibit monofractal scaling behaviour, which can be explained by a single scaling exponent [72,73], therefore different scaling exponents are required for analysing different parts of the series [74]. So, a method of multifractal analysis was conceived. The Multifractal Detrended Fluctuation Analysis (MFDFA) was first proposed by Kantelhardt et al. [58] as a generalization of the standard DFA found its application for studying the multifractal scaling behaviour of various non-stationary time series [75–82]. The analysis of many financial time series shows that they exhibit scaling of multifractal nature and a single scaling exponent is not sufficient to describe their dynamical properties and instead a spectrum of exponents is required [43,58,83–86].

Multifractal properties of a number of financial markets, particularly the Foreign Exchange markets [27–40] and stock markets [43–54] have been unveiled in the past. MFDFA has been applied by Liu et al. [53] to study Chinese stock index futures market, Dutta [54] to study the fluctuation pattern of SENSEX in the Indian stock market, Oh et al. [38] to investigate the multifractal properties of the Foreign exchange rates for Japan, Hong Kong, Korea and Thailand with respect to US Dollar, Stosic et al. [39] to study the dynamic properties of the BRL/USD exchange rate while Norouzzadeh [40] observed the multifractal properties of Iranian Rial–US Dollar exchange rate.

DFA and later MFDFA have been important tools for detecting the multifractal nature of the signal and exploring the long-range autocorrelations for single non-stationary time series. But there may be situations where more than one variable is simultaneously recorded which exhibit long-range dependence or multifractal nature, such as temperature, rainfall, river flow fluctuations [87], cross correlation between air temperature and air relative humidity [88], seismic data [89], asset prices, indexes and trading volumes in financial markets [90], Foreign Exchange rates [91]. The Detrended Cross Correlation Analysis (DCCA) was conceived by Podobnik and Stanley to investigate the long-range cross correlations between two non-stationary time series as a generalization of DFA [92,93]. E.L.S Junior et al. [94] studied the autocorrelations and cross correlations of the volatility time series in the Brazilian stock and commodity market using Detrended Cross Correlation Analysis. The multifractal generalization of DCCA, the Multifractal Detrended Cross Correlation analysis (MFDXA) has also been successfully applied to investigate the cross correlations between signals [59,87,95], including different financial variables [96–108], in physiology [109] as well as in other domains [89,110].

Dutta et al. [95] have studied the auto and cross correlations between SENSEX and Gold Price Fluctuation using MFDXA. Cao et al. [96] have applied MFDXA to identify the cross correlations between the RMB exchange market and the stock market in China. He and Chen have found the existence of power-law cross correlation between China's and US agricultural future markets [97] and have also explained theoretically the relationship between the bivariate cross correlation exponent and the generalized Hurst exponents for time series of respective variables [98]. Ma et al. [99] have been successful in confirming the presence of the cross correlations between the stock market in China and markets in Japan, South Korea and Hong Kong by employing MFDXA method. The multifractality in the price–volume correlation in China metal futures market was studied by Guo et al. [100] and it was observed that long-range correlation and non Gaussian probability distribution are mainly responsible for the existence of multifractality. Ma et al. [101] investigated the cross correlations between the crude oil market and the six GCC stock markets. They also applied the method of the MFDFA and MFDXA and found that the autocorrelations of the crude oil market and the six GCC stock markets and cross correlations between them are all the multifractal. Zhang and Wang [102] analysed the multifractal characteristics of the cross correlations and autocorrelations between the Spot prices of Brent Crude Oil and US Dollar indexes and observed that both the long-range correlations and the fat-tail distributions are both responsible for multifractality. Yuan et al. [103] showed that both Shanghai stock market and Shenzhen stock market show pronounced long-range cross correlations between stock price and trading volume. Zhuang et al. [104] employed MFDXA and detected the cross correlations between carbon and crude oil markets and studied their dynamic behaviour as well.

In this respect it will be very interesting to study the cross correlation of FX and SENSEX which will throw light on the nonlinear dynamics of the Indian market.

2. Method of analysis

In this paper we have performed MFDXA to investigate the cross correlation between the USD–INR Foreign Exchange (FX) Rate and SENSEX for a period from January 1995 to December 2012 following the prescription of Zhou [59] and compared the results with that obtained from the MFDFA.

The important steps involved in MFDXA are mentioned here:

Step 1: Computing the average,

Let us suppose $x(i)$ for $i = 1, 2, \ldots, N$ and $y(i)$, for $i = 1, 2, \ldots, N$, be the two non-stationary time series of length N. The mean of the above series is given by,

$$x_{avg} = \frac{1}{N}\sum_{i=1}^{N} x(i), \qquad y_{avg} = \frac{1}{N}\sum_{i=1}^{N} y(i). \tag{1}$$

Step 2: Computing the integrated time series of the two underlying data series, x_k and y_k as,

$$X(i) = \sum_{k=1}^{i}[x(k) - x_{avg}] \tag{2}$$

for $i = 1, 2, \ldots, N$

$$Y(i) = \sum_{k=1}^{i}[y(k) - y_{avg}] \tag{3}$$

for $i = 1, 2, \ldots, N$.

The integration also reduces the level of noise present in experimental records and finite data.

Step 3: Dividing both the integrated time series into N_s non-overlapping bins (where $N_s = \text{int}\,(N/s)$, s being the length of the bin) and computing the fluctuation function for both the time series. Since N is not a multiple of s, a short part of the series is left at the end and so in order to include this part of the series the entire process is repeated starting from the opposite end thus leaving a short part at the beginning. Thus $2N_s$ bins are obtained and for each bin we perform least square linear fit of the series and then determine the variance,

For each bin, we determine the variance,

$$F(s,\nu) = \frac{1}{s}\sum_{i=1}^{s}[X[(\nu-1)s+i] - x_\nu(i)][Y[(\nu-1)s+i] - y_\nu(i)]$$

for each bin ν, $\nu = 1, 2, 3, \ldots, N_s$ and

$$F(s,\nu) = \frac{1}{s}\sum_{i=1}^{s}[X[N-(\nu-N_s)s+i]-x_\nu(i)][Y[N-(\nu-N_s)s+i]-y_\nu(i)]$$

for $\nu = N_s+1, N_s+2, N_s+3\ldots 2N_s$. Where $x_\nu(i)$ and $y_\nu(i)$ are the least square fitted value in the bin ν.

Step 4: Computing fluctuation function.

The qth order fluctuation function $F_q(s)$ is obtained after averaging over $2N_s$ bins.

$$F_q(s) = \left[\frac{1}{2N_s}\sum_{\nu=1}^{2N_s}\{F(s,\nu)\}^{\frac{q}{2}}\right]^{\frac{1}{q}} \tag{4}$$

where q is an index which can take all possible values except zero because in that case the factor $1/q$ blows up. F_q for $q = 0$ cannot be obtained by the normal averaging procedure; instead a logarithmic averaging procedure is applied for both the time series.

$$F_0(s) \equiv \exp\left[\frac{1}{4N_s}\sum_{\nu=1}^{2N_s}\ln\{F(s,\nu)\}\right] \sim s^{\lambda(0)}. \tag{5}$$

Step 5: The procedure is repeated by varying the value of s. $F_q(s)$ is observed to increase with increasing value of s. If the series is long-range power correlated, then $F_q(s)$ will show power law behaviour,

$$F_q(s) \propto s^{\lambda(q)}.$$

When $x = y$, the above method reduces to the standard MFDFA. $F(s,\nu)$ may obtain negative values in general. To eliminate the problem in evaluation of fluctuation functions which may be complex valued for different values of q we have taken the modulus of to eliminate the negative values. However there is a very recent work by Oswiecimka et al. [61] in which the authors have suggested an alternative more rigorous method Multifractal Cross Correlation Analysis (MFCCA) to take care of the negative values in cross covariances. The authors suggest that the proposed method is a more natural generalization of DCCA compared to MFDXA. It prohibits losing information that is stored in the negative cross covariance. The method is yet to be tested in various systems.

The final step is calculating the slope from the $\log F_q$ vs. $\log s$ plots, which determines the scaling exponent $\lambda(q)$.

If such a scaling exists, $\log F_q$ will depend linearly on $\log s$, with $\lambda(q)$ as the slope. The variation of $\lambda(q)$ with q indicates a multifractal scaling otherwise if the scaling exponent $\lambda(q)$ is independent of q, the cross correlation between the two series is monofractal. For positive q, $\lambda(q)$ describes scaling behaviour of the segments with large fluctuations and vice versa for negative q. The values of $\lambda(q) = 0.5$ indicate the absence of cross correlation. $\lambda(q) > 0.5$ indicates persistent long-range cross correlations and $\lambda(q) < 0.5$ indicates anti-persistent cross correlations [111,112].

An empirical approximation for $q = 2$ has been obtained,

$$\lambda(q) = (h_{FX}(q=2) + h_{SENSEX}(q=2))/2.$$

The degree of multifractality of the two cross correlated time series can be estimated from the singularity spectrum. The singularity spectrum $f(\alpha)$ is related to $\lambda(q)$ by,

$$\alpha = \lambda(q) + q\lambda'(q) \tag{6}$$

$$f(\alpha) = q[\alpha - \lambda(q)] + 1 \tag{7}$$

where α is the singularity strength and $f(\alpha)$ specifies the dimension of subset series that is characterized by α.

The singularity spectrum in general quantifies the long-range correlation property of a time-series [113]. The multifractal spectrum is capable of providing information about relative importance of various fractal exponents in the series e.g. the width of the spectrum denotes range of exponents. A quantitative characterization of the spectra may be obtained by least square fitting it to a quadratic function [114] around the position of maximum α_0,

$$f(\alpha) = A(\alpha-\alpha_0)^2 + B(\alpha-\alpha_0) + C \tag{8}$$

where C is an additive constant $C = f(\alpha_0) = 1$. B indicates the asymmetry of the spectrum. It is zero for a symmetric spectrum. The width of the spectrum can be obtained by extrapolating the fitted curve to zero. The width W is defined as $W = \alpha_1 - \alpha_2$ with $f(\alpha_1) = f(\alpha_2) = 0$. It has been proposed by some groups [115] that the width of the multifractal spectra is a measure of degree of multifractality. For a monofractal series, $\lambda(q)$ is independent of q and the width of the spectrum will be zero. Hence from relation (6) and (7) it follows that the width of the spectrum will be zero for a monofractal series. The more the width, the more multifractal is the spectrum.

According to the autocorrelation function given by,

$$C(\tau) = \langle[x(i+\tau) - x_{avg}][x(i) - x_{avg}]\rangle = \tau^{-\gamma}.$$

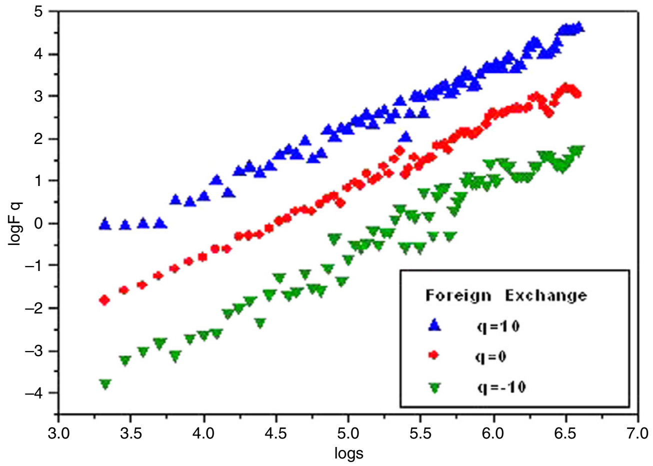

Fig. 3. Plot of the fluctuation function $\log F_q$ vs. $\log s$ for Foreign Exchange data.

Similarly the long-range cross correlation function is given by,

$$C(\tau) = \langle [x(i+\tau) - x_{avg}][x(i) - x_{avg}] \rangle = \tau^{-\gamma_X}$$

where γ and γ_X are the autocorrelation and cross correlation exponents respectively. Due to the superimposition of the non-stationarities and trends on the collected data, the direct calculations of these exponents are avoided, one of the reliable methods applied to calculate the autocorrelation exponent is the DFA method [72,111], where

$$\gamma = 2 - 2h(q = 2). \tag{9}$$

Podobnik and Stanley [92] have recently demonstrated the relation between cross correlation exponent, γ_X and scaling exponent, $\lambda(q)$ determined by Eq. (6) according to

$$\gamma_X = 2 - 2\lambda(q = 2). \tag{10}$$

For uncorrelated data, γ_X has a value 1 and the lower the value, the more correlated is the data.

The origin of multifractality in a time-series can be determined. Basically two different types of multifractality may be present in a time-series: (i) Multifractality due to broad probability density function for the values of the time-series. (ii) Multifractality due to different long-range correlations of the small and large fluctuations. The origin of the multifractality can be ascertained by analysing the corresponding randomly shuffled series. In the shuffling procedure, the values are put into random order and hence all correlations are destroyed. Hence, if the multifractality is due to long-range correlations, then the shuffled series exhibits a non-fractal scaling. On the other hand, if the original $h(q)$ dependence does not change, i.e. $\lambda(q) = \lambda_{shuffled}(q)$, then the multifractality is due to the broad probability density, which is not affected in the shuffling procedure. If both kinds of multifractality are present in a given series, the shuffled series will show weaker multifractality than the original series.

3. Results and discussion

MFDFA and MFDXA were applied to investigate the autocorrelation and cross correlation between USD–INR Foreign Exchange (FX) Rate and SENSEX from January 1995 to December 2012. The data sets over the entire period in both the cases were divided into the following six subsets—(i) January 1995–December 1997, (ii) January 1998–December 2000, (iii) January 2001–December 2003, (iv) January 2004–December 2006, (v) January 2007–December 2009 and (vi) January 2010–December 2012.

The data sets were first transformed to obtain the integrated signals for both the cases. This process reduces the noise in the collected data sets. The integrated time-series was then divided into N_s bins (where $N_s = \text{int}(N/s)$), N is the length of the series and s is the length of the bin. The value of s was suitably chosen. The qth order fluctuation function $F_q(s)$ is obtained for all values of q from -10 to $+10$ in steps of 1 including zero. Figs. 3–5 represent the plots of $\log F_q$ vs. $\log s$ for a particular set of FX, SENSEX and cross correlated data. It was observed that $\log F_q$ depends linearly on $\log s$ for different values of q, which indicates a scaling behaviour while studying the cross correlations between the two non-stationary time series. The slope of the linear curves gives an estimation of the values of the scaling exponents $\lambda(q)$. The values of $\lambda(q)$ were found to decrease with increasing values of q. For the sake of comparison, the values of $h(q)$ obtained from MFDFA were also examined for all the cases. The variation of the scaling exponents $\lambda(q)$ with q depicted in Fig. 6 indicates presence of multifractal behaviour for the individual as well as cross correlated series. The figure also suggests different scaling for small and large fluctuations. The dependence is not the same for all the subsets, reflecting different degrees of multifractality for different subsets.

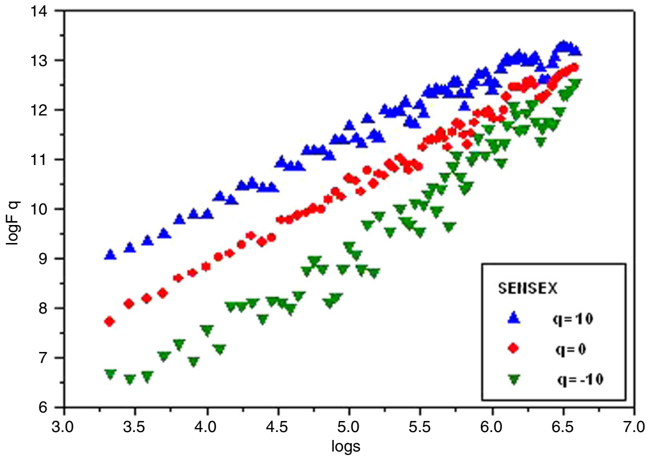

Fig. 4. Plot of the fluctuation function log F_q vs. log s for SENSEX data.

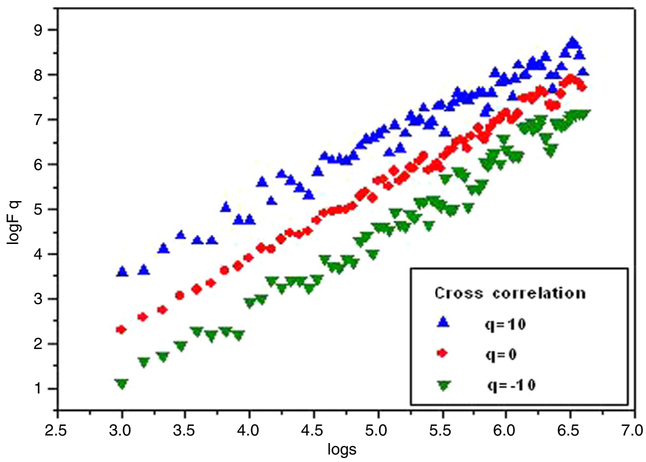

Fig. 5. Plot of the fluctuation function log F_q vs. log s for Cross correlations.

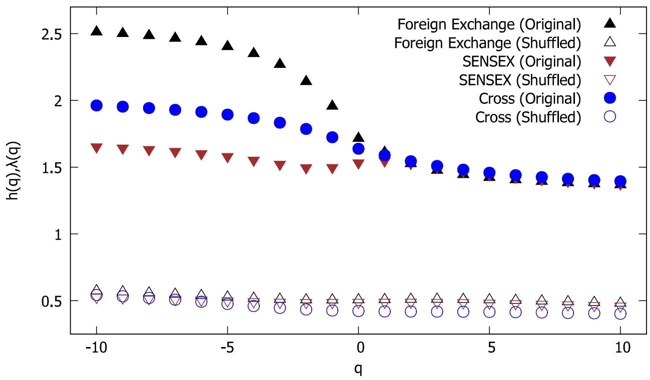

Fig. 6. Plot of $h(q)$ and $\lambda(q)$ vs. q for the Foreign Exchange Rate, SENSEX and Cross correlations of the Original and the Shuffled series for set (1995–2012).

The values of α and $f(\alpha)$ were estimated using Eqs. (6) and (7). The width of the singularity spectrum $f(\alpha)$ vs. α gives a quantitative estimation of the degree of multifractality. The values of the width obtained from the singularity spectrum for FX, SENSEX and cross correlated series for the different subsets as well as the complete set are listed in Tables 1–3. Tables 1 and 2 display the values of the autocorrelation coefficients and Table 3 shows the values of the cross correlation coefficients obtained from Eqs. (9) and (10). The non-zero values of W_X are a clear evidence in support of the multifractal nature of the

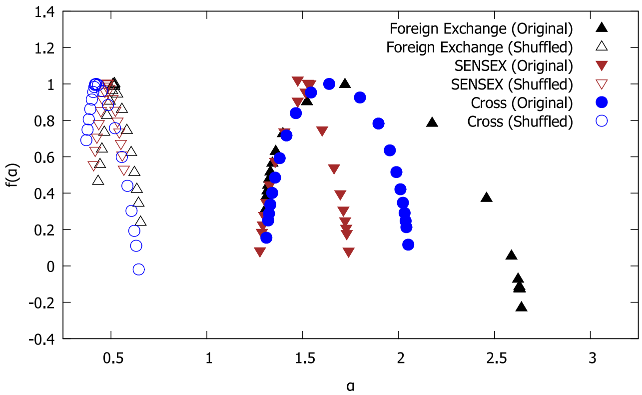

Fig. 7. Plot of $f(\alpha)$ vs. α for the Foreign Exchange Rate, SENSEX and Cross correlations of the Original and the Shuffled series for the set (1995–2012).

Table 1
Values of W_{FX}, $W_{FXshuffled}$, γ_{FX}, $\gamma_{FXshuffled}$ for FX data.

Set	W_{FX}	$W_{FXshuffled}$	γ_{FX}	$\gamma_{FXshuffled}$
1995–2012 (complete set)	1.45 ± 0.06	0.28 ± 0.01	-1.063 ± 0.03	0.968 ± 0.04
1995–1997	1.13 ± 0.04	0.67 ± 0.06	-0.919 ± 0.04	0.939 ± 0.03
1998–2000	0.87 ± 0.02	0.59 ± 0.01	-1.125 ± 0.05	1.153 ± 0.03
2001–2003	1.33 ± 0.04	0.33 ± 0.005	-1.217 ± 0.04	1.006 ± 0.04
2004–2006	0.51 ± 0.03	0.36 ± 0.004	-1.137 ± 0.04	1.011 ± 0.03
2007–2009	0.75 ± 0.02	0.39 ± 0.004	-1.106 ± 0.03	1.132 ± 0.02
2010–2012	0.51 ± 0.03	0.33 ± 0.01	-1.054 ± 0.04	0.973 ± 0.03

Table 2
Values of W_{SENSEX}, $W_{SENSEXshuffled}$, γ_{SENSEX}, $\gamma_{SENSEXshuffled}$ for SENSEX data.

Set	W_{SENSEX}	$W_{SENSEXshuffled}$	γ_{SENSEX}	$\gamma_{SENSEXshuffled}$
1995–2012 (complete set)	0.49 ± 0.02	0.23 ± 0.01	-1.046 ± 0.01	1.033 ± 0.03
1995–1997	0.81 ± 0.02	0.45 ± 0.01	-0.814 ± 0.02	1.075 ± 0.03
1998–2000	0.82 ± 0.04	0.44 ± 0.03	-1.019 ± 0.02	0.959 ± 0.03
2001–2003	0.84 ± 0.03	0.52 ± 0.02	-1.195 ± 0.03	0.956 ± 0.03
2004–2006	0.74 ± 0.02	0.37 ± 0.003	-1.142 ± 0.02	1.134 ± 0.03
2007–2009	0.58 ± 0.02	0.38 ± 0.02	-1.089 ± 0.01	1.161 ± 0.03
2010–2012	0.83 ± 0.03	0.42 ± 0.01	-0.793 ± 0.02	0.949 ± 0.03

Table 3
Values of W_X, $W_{Xshuffled}$, γ_X, $\gamma_{Xshuffled}$ for Cross correlation.

Set	W_X	$W_{Xshuffled}$	γ_X	$\gamma_{Xshuffled}$
1995–2012 (complete set)	0.829 ± 0.03	0.27 ± 0.01	-1.089 ± 0.06	1.162 ± 0.01
1995–1997	0.704 ± 0.02	0.28 ± 0.02	-0.931 ± 0.05	0.917 ± 0.03
1998–2000	0.639 ± 0.01	0.41 ± 0.01	-1.148 ± 0.01	0.979 ± 0.02
2001–2003	0.879 ± 0.02	0.48 ± 0.01	-1.267 ± 0.04	0.954 ± 0.03
2004–2006	0.481 ± 0.04	0.31 ± 0.01	-1.229 ± 0.05	0.977 ± 0.03
2007–2009	0.722 ± 0.04	0.22 ± 0.02	-1.102 ± 0.01	1.025 ± 0.02
2010–2012	0.522 ± 0.03	0.37 ± 0.02	-0.972 ± 0.05	0.931 ± 0.01

cross correlations. Singularity spectra $f(\alpha)$ vs. α for the complete set 1995–2012 for FX, SENSEX and cross correlated series are depicted in Fig. 7.

To detect the origin of multifractality, the corresponding randomly shuffled series were also analysed for all the cases. The variations of $\lambda(q)$ and $h(q)$ with q and $f(\alpha)$ with α for the original series and the corresponding randomly shuffled series are displayed in Figs. 6 and 7 respectively for the complete set 1995–2012. The figures clearly show that the shuffled series exhibit weaker multifractality compared to the original series. The values of the width of the singularity spectrum and auto and cross correlation coefficients for original series and corresponding randomly shuffled series for all the sets are listed in Tables 1–3. The auto and cross correlation coefficients for the shuffled series are close to 1 indicating that the correlations present in the series are destroyed due to shuffling procedure. The values of multifractal width are considerably reduced for the shuffled series indicating that both probability distribution and long-range correlation are responsible for the presence of multifractality in the time series. Drozdz et al. [116] have shown that a relatively short time series would leave traces of multifractality for shuffled series. With increase in number of data points the results systematically and steadily approach

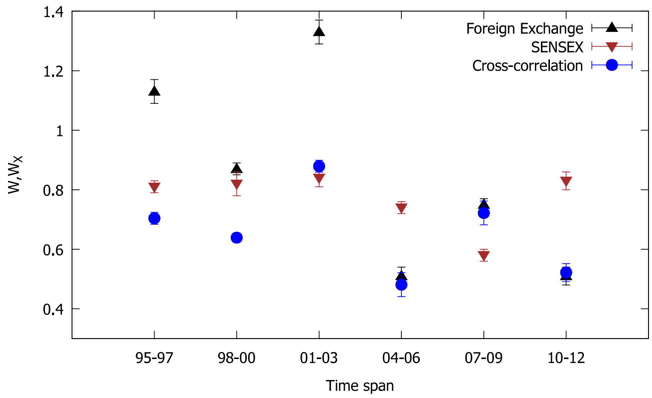

Fig. 8. Variation of Multifractal width W and W_X with time for Foreign Exchange Rate, SENSEX and Cross correlations for all the subsets.

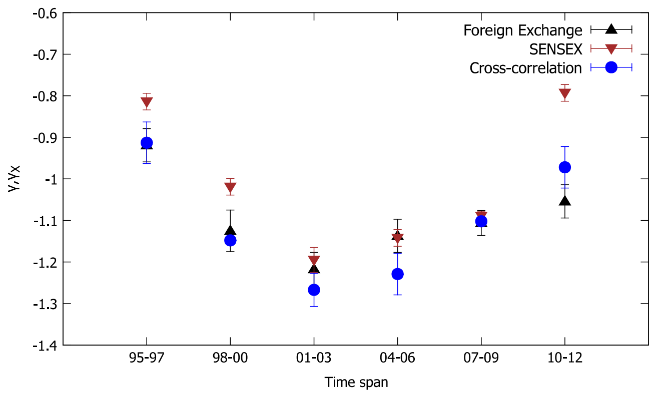

Fig. 9. Variation of γ and γ_X with time for Foreign Exchange Rate, SENSEX and Cross correlations for all the subsets.

monofractal behaviour. The variation of multifractal width with time is shown in Fig. 8. It is observed that the degree of multifractality is highest for the subset 2001–2003 for FX, SENSEX and the cross correlated series.

A close observation of the values of the autocorrelation γ and the cross correlation γ_X exponents reveals the fact that both the time series and their cross correlations have low negative values. We know that the lower the values of γ, the more the autocorrelation and the lower γ_X suggests better cross correlations. According to MFDXA, it may be considered that each of the two variables at any time may not only depend on its own past records but also on the past records of the other variable [105]. The fluctuation in the values of one variable may influence the other variable.

Large negative values of γ and γ_X indicate strong autocorrelation and cross correlation between FX and SENSEX fluctuations. Drozdz et al. [116] have also observed negative values of correlation. Jones and Kaul [117] were the first to reveal a stable negative cross correlation between oil prices and stock prices. The negative cross correlations were also found by Refs. [118–120]. The degree of long-range correlation for FX and SENSEX fluctuations as well as the cross correlated series is maximum for the period 2001–2003. Variation of auto and cross correlation coefficients with time span is shown in Fig. 9.

4. Conclusion

The study of cross correlation between FX and SENSEX is very significant and has yielded some interesting results. At the onset if we compare Figs. 1 and 2, we find an interesting behaviour of the fluctuation of both the time series especially in the period 2004–2009. A steep decrease in the value of FX rate during 2004–2006 is accompanied by a huge increase in values of SENSEX. On the contrary during 2007–2009 a disastrous fall in the value of SENSEX is reflected in a sharp increase in the value of FX rate. Now on approaching further, the multifractal analysis of the time series reveals that both the time series as well as the cross correlated series possess multifractal properties for the entire set as well as the subsets. Huge degree of multifractality and large negative values of the cross correlation coefficient indicate that the two series are hugely cross correlated. It is observed that the degree of correlation as well as the degree of multifractality is maximum for the subset 2001–2003 for SENSEX, FX as well as the cross correlated series. A steady fall in the value of FX is reflected in a relatively steady rise in the value of SENSEX. This is quite an interesting observation. Telesca et al. [76] have observed an increase in multifractal width prior to the occurrence of the largest earth quake. Dutta et al. [54,95] have observed similar increases in multifractal width prior to volatile behaviour of SENSEX and huge increases in price of Gold. The period 2001–2003 has

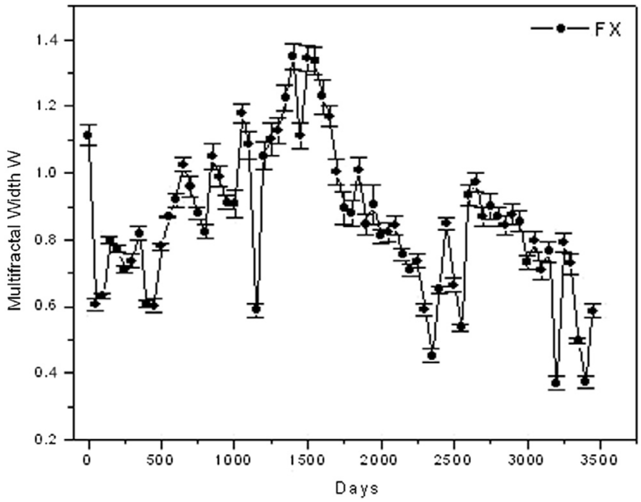

Fig. 10. Variation of multifractal width with starting day of each rolling window of length 1000 days for FX rate data.

experienced high autocorrelation and cross correlation coefficients as well as high degree of multifractality. As a result in the subsequent years 2004–2006 and 2007–2009 there is huge fluctuation in values of FX and SENSEX. If a series is correlated, if it is increasing presently, then it will have a tendency to increase in future with a probability more than 0.5. However in this case since the two series are cross correlated, their future values will not only depend on their present values but also on the values of the other series. Thus when SENSEX depreciates FX increases and vice versa. In the years 1995–1997 and the recent period 2010–2012 the series experience comparatively lower values of cross correlation coefficients. During this period the series are comparatively less cross correlated. Thus in spite of a huge increase in value of FX during 1995–1997 SENSEX is relatively stable during that period. During 2010–2012 the degree of multifractality as well as degree of cross correlation is low. It is observed that near the end of 2010–2012 both SENSEX and FX show an increasing trend. Thus the change in values of one hardly affects the other. The low values of degree of correlation and degree of multifractality in the period 2010–2012 suggest that possibly SENSEX/FX will not experience huge fluctuations in near future.

The study has however certain limitations. Statistical tests would [121] be helpful to check whether the correlations present in the series are genuine. It has been shown by Podobnik et al. [122,123] that if the autocorrelations are stronger, then the cross correlations will also be stronger. A χ^2 test would be indicative of presence of strong cross correlations in the series. The data has been divided into six subsets which has shortened the length of the data considerably. A considerably longer series would provide better results. A comparison with shuffled series has however suggested presence of multifractality due to long-range correlations. The data uses daily variations of the considered time series. More accurate results are expected if the data of variations of the parameters throughout the day are available. Though MFDXA is a widely tested methodology, a comparison of the results with the recently proposed methods MFCCA [61] and MFDMCA [124] would be interesting. It has been evidenced by the shuffling procedure that long-range correlations are predominant contributors to multifractality. However an investigation of the fat-tails would complete the analysis.

Thus the above analysis reveals interesting results and can be effective in Indian economic scenario in providing future predictions for the fluctuations of SENSEX and FX rate. The study can be further extended including variations in daily price which will be effective in predicting short term fluctuations.

5. Method of rolling windows

Since the limitation of this study in regard to the division of the series into six subsets which has shortened the length of the data considerably, the method of rolling windows [125–127] is used to investigate time-varying cross correlation exponents and scaling exponents to study the dynamics of the market. We have considered a window of size 1000 days and evaluated the Hurst exponents for the first 1000 days of the series. Then data for first 50 days were dropped and the window was shifted starting at 51 days and so on and the process was repeated. The variation of W and γ is shown in Figs. 10–15 for the autocorrelated and cross correlated series. The multifractal width variation for cross correlated series shows a sharp decrease for the window 2501–3500. This is very significant as it can be related to the meteoric rise and catastrophic fall in values of SENSEX during this period resulting in high instability of the market. For the FX data we observe the series is most correlated for the window 801–1800. We observe a steady increase in value of the FX rate during this period. On the contrary it is least correlated in the window 2501–3500. During this period the FX rate experiences huge fluctuations. The variation of γ for SENSEX on the other hand does not reveal such sharp maxima or minima. However there is increasing trend of values of γ for the days 1051–2800. For this period we observe an increasing trend in values of SENSEX. The steady

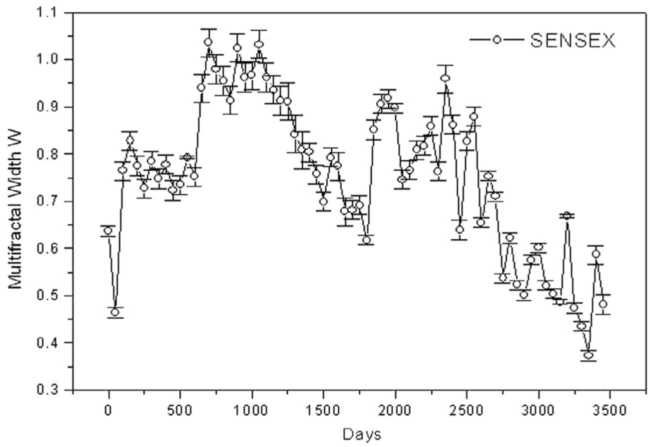

Fig. 11. Variation of multifractal width with starting day of each rolling window of length 1000 days for SENSEX data.

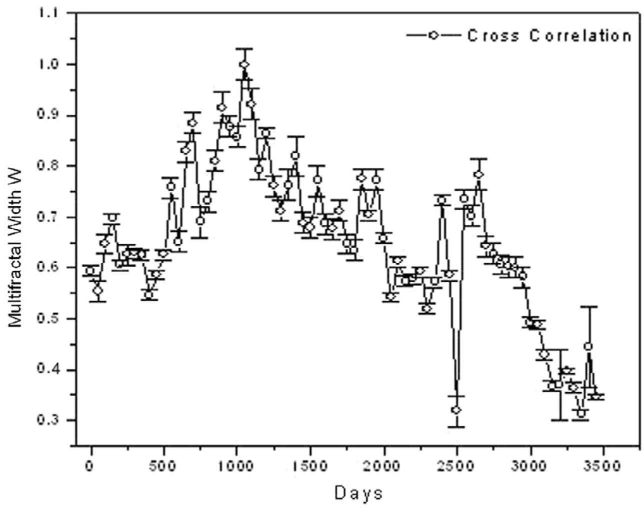

Fig. 12. Variation of multifractal width with starting day of each rolling window of length 1000 days for Cross correlations.

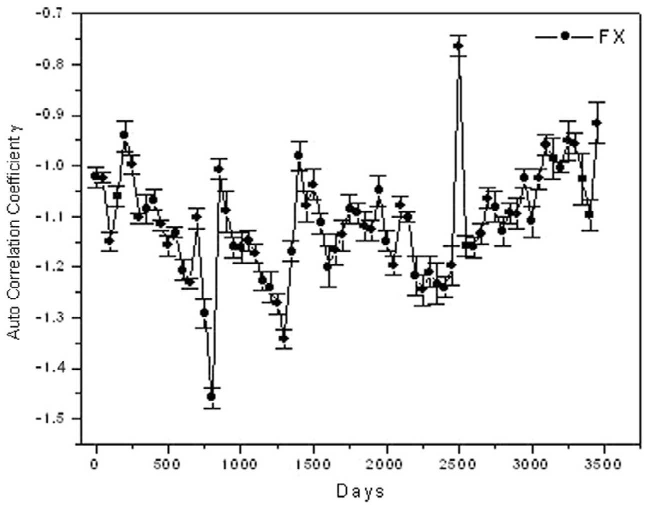

Fig. 13. Variation of Autocorrelation coefficient with starting day of each rolling window of length 1000 days for FX rate data.

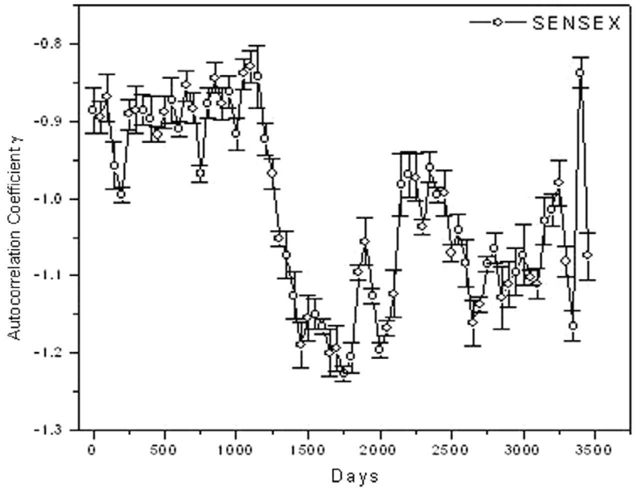

Fig. 14. Variation of Autocorrelation coefficient with starting day of each rolling window of length 1000 days for SENSEX data.

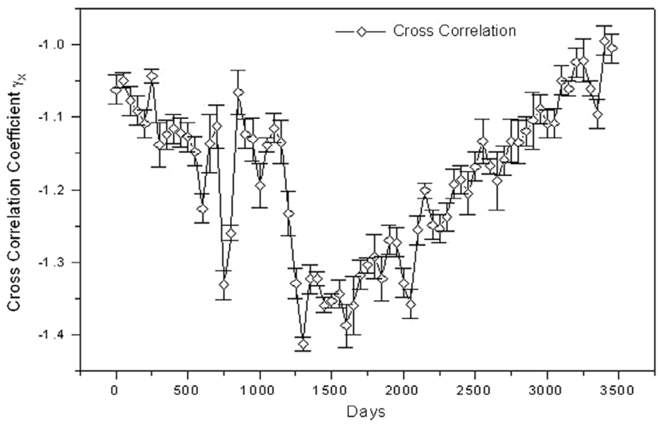

Fig. 15. Variation of Cross correlation coefficient with starting day of each rolling window of length 1000 days for Cross correlations.

increase in values of cross correlation coefficient in recent years predicts that the fluctuations in SENSEX and FX are less likely to have a huge impact on each other.

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