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Multifractal detrended cross-correlations between crude oil market and Chinese ten sector stock markets

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HIGHLIGHTS

- Multifractality exists in the cross-correlations between crude oil and sector stock markets.
- The strength of multifractality is the highest for energy stock markets.
- VAR cannot capture the stock–oil relationships.

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ABSTRACT

Based on the daily price data of spot prices of West Texas Intermediate (WTI) crude oil and ten CSI300 sector indices in China, we apply multifractal detrended cross-correlation analysis (MF-DCCA) method to investigate the cross-correlations between crude oil and Chinese sector stock markets. We find that the strength of multifractality between WTI crude oil and energy sector stock market is the highest, followed by the strength of multifractality between WTI crude oil and financial sector market, which reflects a close connection between energy and financial market. Then we do vector autoregression (VAR) analysis to capture the interdependencies among the multiple time series. By comparing the strength of multifractality for original data and residual errors of VAR model, we get a conclusion that vector auto-regression (VAR) model could not be used to describe the dynamics of the cross-correlations between WTI crude oil and the ten sector stock markets.

1. Introduction

To reveal multifractal features of two cross correlated signals, Zhou extended the detrended cross-correlation (DCCA) which was proposed by Podobnik and Stanley [1], and put forward the multifractal detrended cross-correlation analysis (MF-DCCA) [2]. Recently, many empirical results investigated multifractal features of cross-correlations between financial time series and other time series by applying MF-DCCA. For example, Dutta et al. studied the cross-correlation between gold and Bombay Stock market using MF-DCCA [3]. Similarly, Sto et al. investigated the cross-correlation behaviors among thirteen global stock market indices using MF-DCCA [4]. Cao et al. proposed a method called the multifractal asymmetric detrended cross-correlation analysis (MF-ADCCA) which is an extension of MF-DCCA, to investigate the cross-correlations among the Chinese stock market, the RMB exchange market, and the US stock market [5]. Particularly, crude oil is one of the

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most important commodities in global financial markets [6]. It is considered as the life support of many economies and may serve as the underlying asset in the trading of various financial instruments. A large number of studies in finance literature have focused on the relationship between crude oil and financial market in different economies. For example, Jones and Kaul for the U.S., Canada, Japan, and the U.K. [7]; Faff and Brailsford for Australia [8]; Basher and Sadorsky for emerging economies for 15 countries in the Asia-Pacific regions [9]; Park and Ratti for the U.S. and 13 European countries [10]; and Driesprong, Jacobsen, and Maat for 18 developed countries and 30 emerging economies [11]. In recent years, some studies have analyzed the cross-correlations between crude oil market and stock market from multifractal perspective. For example, Wang and Xie investigated cross-correlations between the crude oil market and U.S. stock market [12], and they found that the cross-correlated behavior between WTI crude oil market and U.S. stock market was nonlinear and multifractal. Similarly, Ma et al. investigated the cross-correlations between the crude oil market and the six Gulf Cooperation Council (GCC) stock markets [13]. They found linear regression models cannot be used to describe the dynamics of cross-correlations between the crude oil and six GCC stock markets. In this paper, we will focus on the cross-correlations between crude oil market and Chinese sector stock markets. Many existing literatures research the multifractality of performance of stock market using the data of stock index [14–22]. For example, Wang et al. analyzed the efficiency for Shenzhen stock market using the daily closing price of Shenzhen Component Index [16]. Suárez-García and Gómez-Ullate used the data of price ticks of the index Ibex35 of the Madrid Stock Exchange to analyze multifractality of finance time series [17]. Wei et al. compared the direct and cross hedging effectiveness of the copula-MFV model using high-frequency intra day quotes of the spot Shanghai Stock Exchange Composite Index (SSEC), spot China Securities Index 300 (CSI 300), and CSI 300 index futures [18]. Therefore, to analyze the cross-correlations between crude oil and Chinese sector stock markets, we study the dynamics of ten sector stock indices daily returns in China A share market. Since Zhou proposed the MF-DCCA to reveal multifractal features of two cross correlated signals [2], MF-DCCA have been widely used in detection of cross-correlations between two time series in several papers [3,23,13,24–27]. Thus, in this paper, we try to employ MF-DCCA method to investigate cross-correlations between crude oil and Chinese sector stock markets. We consider that strengths of multifractality between crude oil market and sector stock markets are different, and the strength of multifractality can be regarded as an indicator to reflect how close the connection between crude oil market and sector stock market is. This is a new perspective to measure the relationship between sector stock market and crude oil market. Moreover, based on MF-DCCA method, we try to figure out whether vector autoregression (VAR) model is good enough to capture the interdependencies of non-stationary series of WTI crude oil markets and Chinese ten sector stock markets from the perspective of multifractality. This is a new idea, which contributes to intensively understanding the VAR model from the perspective of multifractality.

The remainder of this paper is organized as follows. Section 2 describes the data of WTI crude oil price and sector stock indices in Chinese stock market. Section 3 mainly focuses upon the description of MF-DCCA method. Section 4 provides the empirical results and Section 5 is the conclusions.

2. Data

We choose the daily closing prices of ten CSI300 sector indices from 4 January 2005 to 2 November 2015 to analyze different sector stock markets. The CSI300 sector indices are developed by China Securities Index co., Ltd to measure the performance of sectors of CSI 300 index and to provide analysis tools for investors. The ten sector stock indices' names are CSI 300 Energy Index(CSI300E), CSI 300 Materials Index(CSI300M), CSI 300 Industrials Index(CSI300I), CSI 300 Consumer Discretionary Index(CSI300CD), CSI 300 Consumer Staples Index(CSI300CS), CSI 300 Health Care Index(CSI300HC), CSI 300 Financials Index(CSI300F), CSI 300 Information Technology Index(CSI300IT), CSI 300 Telecommunication Services Index(CSI300TS), CSI 300 Utilities Index(CSI300U). We also choose the daily closing data of spot prices of West Texas Intermediate (WTI) crude oil from 4 January 2005 to 2 November 2015. Let P_t denote the closing price of stock index or crude oil price. The daily price return, r_t, is calculated as its logarithmic difference, $r_t = log(P_t) - log(P_{t-1})$. The descriptive statistics of the 11 returns are shown in Table 1.

3. Methodology

Various methods have been developed and applied to characterize the correlation behavior on time series, detrended fluctuation analysis (DFA) [28], detrended moving average method (DMA) [29], multifractal detrended fluctuation analysis (MF-DFA) [30], wavelet based fluctuation analysis (WBFA) [31], average wavelet coefficient method (AWC) [32], wavelet transform modulus maxima (WTMM) [33] etc. To reveal the multifractal features of two cross-correlated non-stationary signals, Zhou extended the detrended cross-correlation(DCCA), and proposed multifractal detrended cross-correlation analysis (MF-DCCA, or called MF-DXA) [2], Jiang and Zhou created MF-X-DMA which is an extension of DMA [34], Kristoufek proposed MF-HXA [35] based on the height–height correlation analysis of Barabási and Vicsek [36], and Hedayatifar et al. extended the MF-DCCA to the method of coupling detrended fluctuation analysis (CDFA) for the case when more than two series are correlated to each other [37]. Zhao et al. refer to three methods, to estimate the long-range cross-correlation of two variables. They include the linear cross-correlation function, the multifractal detrended cross-correlation analysis (MF-DCCA), and the multifractal height cross-correlation analysis (MF-HXA). These methods are applied to the artificial variables, the theoretical results of which can be prior inferred, to determine which one is more effective. And for two non-stationary variables, the MF-DCCA method seems to be the most effective one, both for monofractal and multifractal cases [38]. In

Table 1
The descriptive statistics of return series.

	Mean(%)	S. D(%)	Min(%)	Max(%)	Skew	Kurt	J-B	Q(20)
CSI300E	0.0243	2.217	−10.009	9.244	−0.189	5.373	632***	53.234***
CSI300M	0.0283	2.163	−9.527	8.639	−0.45	5.151	595***	63.675***
CSI300I	0.0432	2.049	−9.564	9.548	−0.467	5.914	1025***	69.247***
CSI300CD	0.0633	2.029	−10.284	9.204	−0.5	5.507	797***	58.047***
CSI300CS	0.0761	1.878	−9.856	8.772	−0.248	5.935	970***	62.232***
CSI300HC	0.0774	2.01	−10.061	9.541	−0.348	5.788	904***	77.114***
CSI300F	0.0641	2.142	−10.111	9.539	−0.168	5.582	742***	42.854***
CSI300IT	0.0298	2.325	−10.291	9.552	−0.518	4.891	509***	33.597**
CSI300TS	0.0436	2.25	−10.528	9.602	−0.151	6.037	1019***	42.636***
CSI300U	0.0316	1.85	−10.15	8.471	−0.517	6.678	1597***	67.277***
WTI	0.0018	2.459	−19.164	16.414	−0.078	9.066	4028***	71.074***

Note: Symbols "Max", "Min", "S. D", "Skew", "Kurt" denote Maximum, Minimum, Std. Dev, Skewness and Kurtosis respectively. Q(20) denotes the value of Ljung–Box–Pierce Q statistics with 20 lags.
** Indicates rejection at the 5% significance level.
*** Indicates rejection at the 1% significance level.

addition, as we mentioned in Section 1, MF-DCCA method has been widely used in detection of cross-correlations between two time series in several papers [3,23,13,24–27]. Therefore, we apply the MF-DCCA method to empirically analyze the cross-correlations between crude oil and Chinese sector stock markets.

There are two time series $x(i)$ and $y(i)$, $i = 1, 2, \ldots, N$, where N is the length of the series. The MF-DCCA method can be summarized as follows:

First, calculate the profile

$$X(i) = \sum_{k=1}^{i} (x(k) - \bar{x}), \qquad Y(i) = \sum_{k=1}^{i} (y(k) - \bar{y}), \quad i = 1, 2, \ldots, N, \tag{1}$$

where $\bar{x}$ and $\bar{y}$ denote the averaging over the two whole time series $x(i)$ and $y(i)$.

Second, divide the two profiles $X(i)$ and $Y(i)$ into $N_s = \mathrm{int}(N/s)$ non-overlapping segments of equal length s. Since the length N of the series is often not a multiple of the given time scale s, a short part at the end of each profile may remain. In order not to discard this part of the series, the same procedure is repeated starting from the opposite end of each profile. Thereby, $2N_s$ segments are obtained together. Then, estimate the trends for each of the $2N_s$ segments by means of the mth order polynomial fit. Then the detrended covariance is determined by

$$F^2(s, \lambda) = \frac{1}{s} \sum_{j}^{s} \left| X_{(\lambda-1)s+j}(j) - \tilde{X}_\lambda(j) \right| \left| Y_{(\lambda-1)s+j}(j) - \tilde{Y}_\lambda(j) \right| \tag{2}$$

for each segment λ, $\lambda = 1, 2, \ldots, N_s$ and

$$F^2(s, \lambda) = \frac{1}{s} \sum_{j}^{s} \left| X_{N-(\lambda-N_s)s+j}(j) - \tilde{X}_\lambda(j) \right| \left| Y_{N-(\lambda-N_s)s+j}(j) - \tilde{Y}_\lambda(j) \right| \tag{3}$$

for each segment λ, $\lambda = N_s + 1, N_s + 2, \ldots, 2N_s$. Here, $\tilde{X}_\lambda(j)$ and $\tilde{Y}_\lambda(j)$ denote the fitting polynomial with order m in segment λ.

Then average over all segments to obtain the qth-order fluctuation function

$$F_q(s) = \left\{ \frac{1}{2N_s} \sum_{\lambda=1}^{2N_s} \left[F^2(s, \lambda) \right]^{q/2} \right\}^{1/q} \tag{4}$$

for any $q \neq 0$, and

$$F_0(s) = \exp\left\{ \frac{1}{4N_s} \sum_{\lambda=1}^{2N_s} \ln\left[F^2(s, \lambda) \right] \right\} \tag{5}$$

for $q = 0$.

Finally, analyze the scaling behavior of the fluctuations by observing log–log plots of $F_q(s)$ versus s for each value of q. If the two series are long-range cross-correlated, $F_q(s)$ will increase for large values of s, as a power-law

$$F_q(s) \propto s^{H_{xy}(q)}. \tag{6}$$

The scaling exponent $H_{xy}(q)$, known as the generalized cross-correlation exponent, describes the power-law relationship of ordinary least squares (OLS). In particular, if $x(i)$ is identical to $y(i)$, then MF-DCCA is equivalent to MF-DFA.

Furthermore, the scaling exponent $H_{xy}(q)$ has similar properties and interpretation to the generalized Hurst exponent $H(q)$. For positive q, $H_{xy}(q)$ describes the scaling behavior of the segments with large fluctuations. On the contrary, for negative q, $H_{xy}(q)$ describes the scaling behavior of the segments with small fluctuations. When $H_{xy}(2) > 0.5$, the series are cross-persistent so that a positive (a negative) value $\Delta X_t \Delta Y_t$ is more statistically probable to be followed by another positive (negative) value of $\Delta X_{t+1} \Delta Y_{t+1}$. However, for $H_{xy}(2) < 0.5$, the series are cross-antipersistent so that a positive (a negative) value of $\Delta X_t \Delta Y_t$ is more statistically probable to be followed by a negative (a positive) value of $\Delta X_{t+1} \Delta Y_{t+1}$.

The multifractal spectrum $f_{xy}(\alpha)$ describes the singularity content of the time series one can obtain through the Legendre transform:

$$\alpha = H_{xy}(q) + qH'_{xy}(q) \quad \text{and} \quad f_{xy}(\alpha) = q\left(\alpha - H_{xy}(q)\right) + 1. \tag{7}$$

Here, α is the Holder exponent or the singularity strength, which is used to characterize the singularities of the time series. The multifractal spectrum $f_{xy}(\alpha)$ describes the singularity content of the time series. The strength of multifractality can be estimated by the width of singularity which is given by

$$\Delta\alpha = \alpha_{\max} - \alpha_{\min}. \tag{8}$$

The broader spectrum is evident for strong multifractality nature and narrow spectrum is evident for weak multifractality nature of the cross-correlated time series.

4. Empirical results

4.1. Cross-correlations test

In order to quantify the cross-correlation between WTI crude oil and Chinese ten sector stock markets, we apply a cross-correlation statistic proposed by Podobnik et al. [39]. The cross-correlation statistic between two series $x(i)$, $y(i)$, which have the same length N, functions as follows

$$C_i = \frac{\sum_{k=i+1}^{N} x_k y_{k-i}}{\sqrt{\sum_{k=1}^{N} x_k^2 \sum_{k=1}^{N} y_k^2}}. \tag{9}$$

Then the cross-correlation test statistic

$$Q_{cc}(m) = N^2 \sum_{i=1}^{m} \frac{C_i^2}{N-i}. \tag{10}$$

The cross-correlation statistic $Q_{cc}(m)$ is approximately $\chi^2(m)$ distributed with m degrees of freedom. If there are no cross-correlations between two time series, the cross-correlation test agrees well with the $\chi^2(m)$ distribution. If the cross-correlations test exceeds the critical value of the $\chi^2(m)$ distribution, then the cross-correlations are significant at a special significance level. As a comparison, we also describe the critical value of the $\chi^2(m)$ distribution at the 5% level of significance for the degrees of freedom varying from 1 to 1000 (see Fig. 1).

Based on Eqs. (9) and (10), we can obtain the cross-correlation statistic (logarithmic form) $Q_{cc}(m)$ for WTI crude oil and Chinese ten sector stock markets respectively (Fig. 1). In Fig. 1, the cross-correlation statistic $Q_{cc}(m)$ for the ten pairs are always larger than (or close to) the critical values for the $\chi^2(m)$ distribution at the 5% level of significance, which suggests there existed long-range cross-correlations.

Furthermore, to verify the results as above, we also apply another new method proposed by Podobnik et al. [40], the function as below:

$$\rho_{DCCA} = \frac{F^2_{DCCA}(n)}{F_{DFA1}(n)\, F_{DFA2}(n)}. \tag{11}$$

Then, the value of ρ_{DCCA} ranges between -1 and 1. If ρ_{DCCA} equal to zero, which the two series have no cross-correlation, and it splits the level of cross-correlation between positive and the negative case. We calculate the values of ρ_{DCCA} based on different values of window size $n (n = 16, 32, 64, 128, 256)$ (see Table 2) that we can compare with Table 1 (see Table 1 in Podobnik et al. [40]) and draw the conclusions that are consistent with the cross-correlation test as above.

4.2. Multifractal detrended cross-correlation analysis

Based on Eqs. (9) and (10), we can only test for the presence of cross-correlation qualitatively. To present the cross-correlations quantitatively, we use the MF-DCCA method to estimate a quantitative cross-correlation exponent.

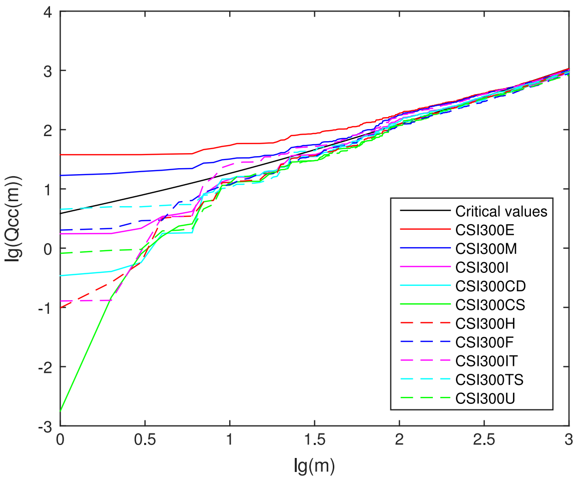


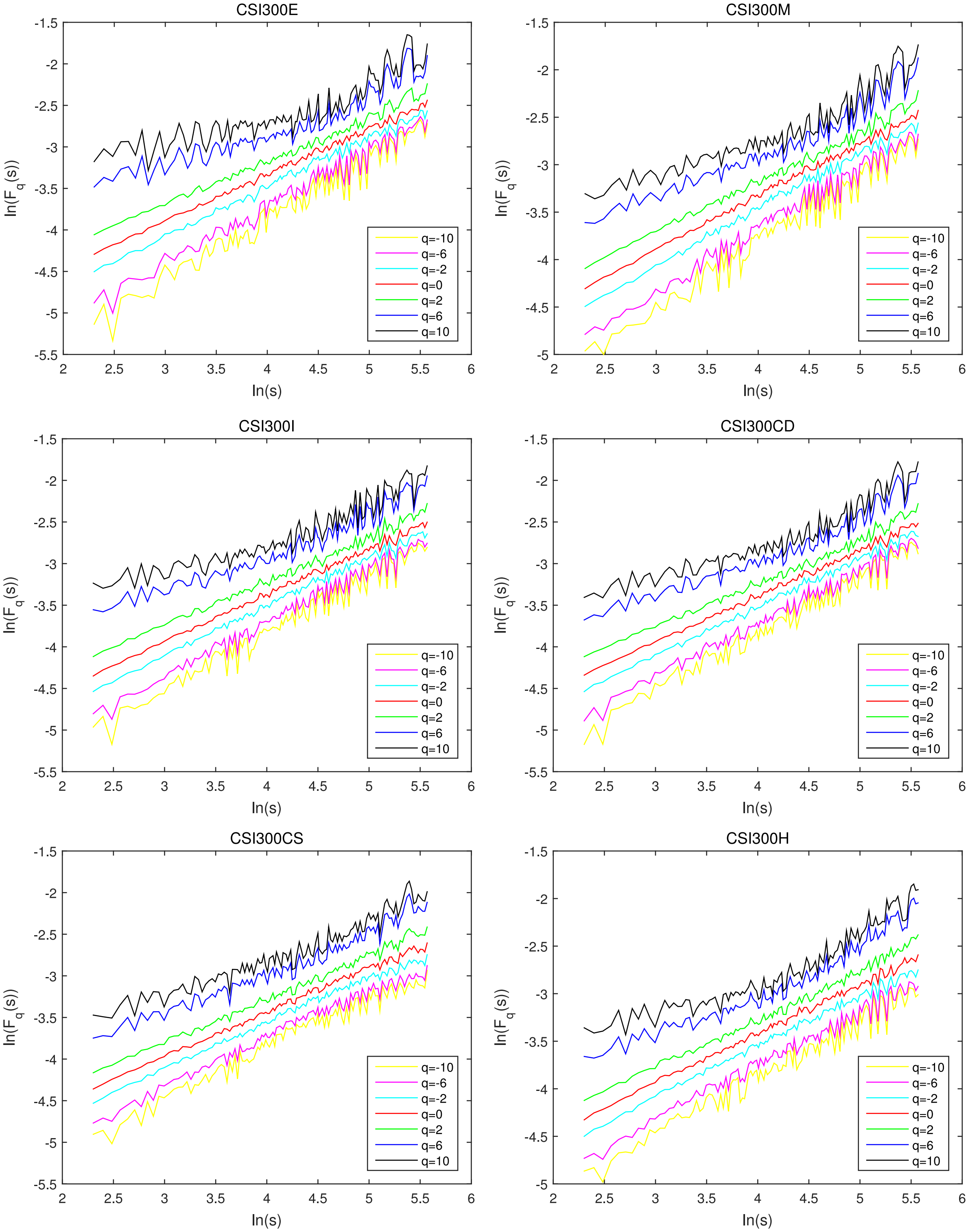
Fig. 1. The cross-correlation statistic between WTI crude oil and Chinese ten sector stock markets respectively. The black line stands for the critical values (at the 5% level of significance). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 2
The value of ρ_{DCCA} for a given window size n.

n	$n = 16$	$n = 32$	$n = 64$	$n = 128$	$n = 256$
CSI300E	0.569317	0.524596	0.612731	0.813602	0.48809
CSI300M	0.440799	0.39536	0.543992	0.725781	0.282536
CSI300I	0.416146	0.371993	0.533069	0.658024	0.271476
CSI300CD	0.363923	0.33745	0.499598	0.656163	0.309518
CSI300CS	0.31579	0.261551	0.411179	0.522801	0.454232
CSI300H	0.309794	0.323619	0.420873	0.568154	0.292132
CSI300F	0.381987	0.354897	0.491418	0.633523	0.279576
CSI300IT	0.364427	0.32409	0.52725	0.610425	0.284508
CSI300TS	0.465174	0.36567	0.486924	0.648022	0.309569
CSI300U	0.39637	0.411153	0.500335	0.564169	0.338515

We show the log–log plots of fluctuation function $F_q(s)$ versus time scale s for WTI crude oil and Chinese ten sector stock markets as $q = -10, -6, -2, 0, 2, 6, 10$ in Fig. 2 when polynomial order $m = 1$ (i.e., MF-DXA-1, when $m = 1, 2, 3, 4, 5$, the results are qualitatively similar). In this paper, we set the range of time scale s to be $10 < s < N/10$, where N is the length of each time series. As can be seen from Fig. 2, almost every line can fit the curve of fluctuation functions well, which can tell us that power-lower cross-correlations exist between WTI crude oil and Chinese ten sector stock markets. The $H_{xy}(q)$ spectra have been shown for WTI crude oil and the ten sector stock markets in Fig. 3, and the $H_{xy}(q)$ values with q varying from -10 to 10 have been shown in Table 3. If the cross-correlation exponent $H_{xy}(q)$ varies with different values of q, the two correlated series are multifractal; otherwise, it is monofractal. In Fig. 3 and Table 3, we find that all the cross-correlation exponents $H_{xy}(q)$ decrease with q varying from -10 to 10, implying strong multifractal features exist between the crude oil and the ten sector stock markets during the sample period. When q is equal to 2, the cross-correlation exponent $H_{xy}(2)$ between the crude oil and the ten sector stock markets are all larger than 0.5, which means that the two correlated series are persistent or long-range dependence.

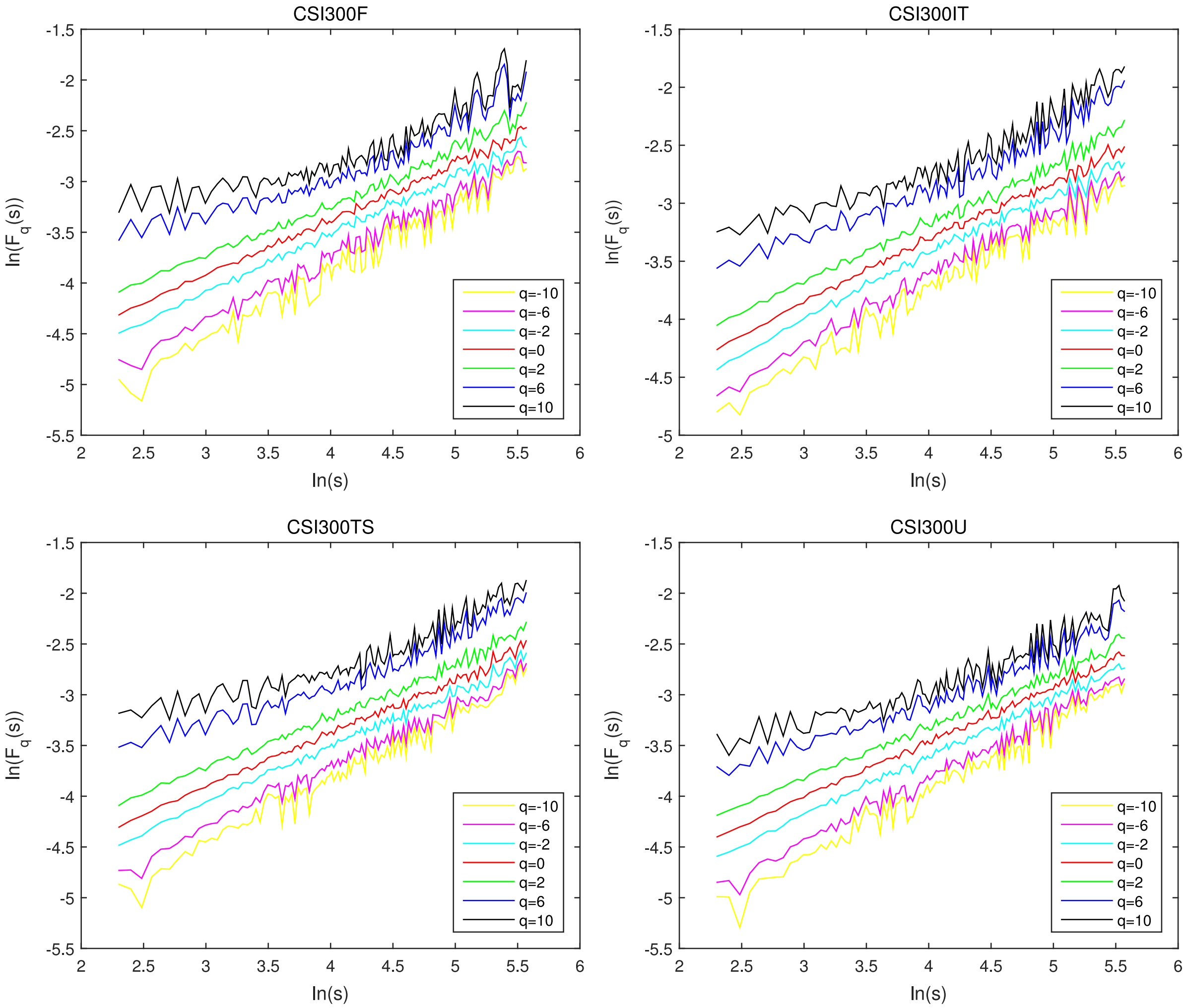
Fig. 4 shows that the multifractal spectra of WTI crude oil and Chinese ten sector stock markets are not points, so we can prove that multifractal features exhibit between the ten pairs. Moreover, the width of the multifractal spectrum can be used to estimate the strength of multifractality. The value of $\Delta\alpha$, width of the spectrum, is shown in Table 4. It is clearly that the strength of multifractality between WTI crude oil and energy sector stock market is the highest, followed by the strength of multifractality between WTI crude oil and financial sector market. One possible explanation is that the volatility of oil price could cause the volatility of value of oil field, which is an important asset for some energy firm. It will definitely result in the volatility of profits for these energy firms. So the energy sector stock market will be affected directly and the strength of multifractality between WTI crude oil and energy sector stock market is the highest. As we mentioned in Section 1, crude oil is one of the most important commodities in global financial markets [6]. It is considered as the life support of many economies and may serve as the underlying asset in the trading of various financial instruments. That is may be the reason why the strength of multifractality between WTI crude oil and financial sector stock market is the second-highest. As for the other sector stock markets, connections with crude oil market are not closer than energy or financial sector stock market in China, so the strengths of multifractality are relatively weaker. The results indicate that energy market and financial market is closely connected. Moreover, the cross-correlations between WTI crude oil and sector stock markets in China displayed



(a).

Fig. 2. Log–log plots of fluctuation functions $ln(F_q(s))$ versus time scale $\ln(s)$ for WTI crude oil and the ten sector stock markets.

complexity (multifractality) indicating that the cross-correlations were easily to be affected by large and small fluctuations. It is nearly impossible to predict the future prices based on the history of the other markets and the future prices in energy



(b).

Fig. 2. (*continued*)

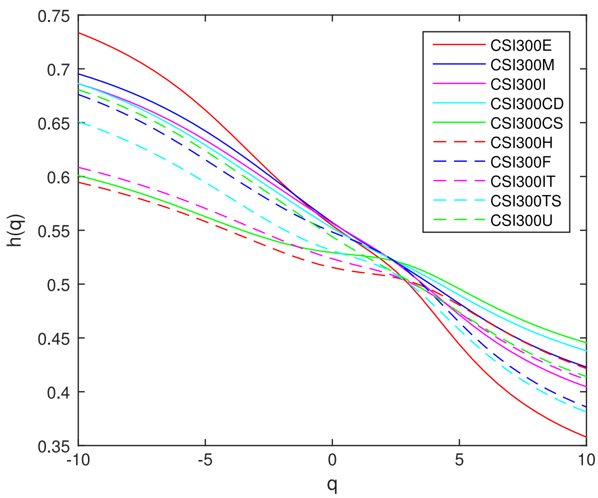


Fig. 3. Cross-correlation Hurst exponent, $H_{xy}(q)$ as a function of q, for WTI crude oil and the ten sector stock markets.

sector stock market is the most unpredictable based on the history of the WTI crude oil markets, followed by the future prices in financial sector stock.

Table 3
Cross-correlation Hurst exponents with q varying from −10 to 10 for WTI crude oil and Chinese ten sector stock markets.

q	CSI 300E	CSI 300M	CSI 300I	CSI 300CD	CSI 300CS	CSI 300H	CSI 300F	CSI 300IT	CSI 300TS	CSI 300U
−10	0.734	0.695	0.686	0.686	0.601	0.595	0.676	0.608	0.651	0.681
−9	0.724	0.688	0.679	0.678	0.595	0.589	0.667	0.602	0.642	0.672
−8	0.712	0.679	0.670	0.668	0.588	0.582	0.656	0.595	0.632	0.662
−7	0.698	0.668	0.659	0.656	0.580	0.575	0.644	0.588	0.621	0.651
−6	0.681	0.656	0.647	0.643	0.572	0.567	0.630	0.579	0.608	0.638
−5	0.662	0.642	0.634	0.629	0.563	0.558	0.616	0.570	0.595	0.624
−4	0.640	0.627	0.619	0.614	0.554	0.549	0.600	0.561	0.580	0.608
−3	0.617	0.610	0.603	0.598	0.546	0.539	0.585	0.551	0.565	0.592
−2	0.594	0.592	0.587	0.582	0.538	0.530	0.571	0.541	0.552	0.576
−1	0.573	0.574	0.572	0.567	0.533	0.522	0.559	0.531	0.540	0.560
0	0.554	0.557	0.557	0.552	0.529	0.515	0.548	0.524	0.531	0.544
1	0.538	0.542	0.543	0.539	0.527	0.511	0.539	0.517	0.524	0.530
2	0.522	0.527	0.528	0.527	0.524	0.508	0.528	0.511	0.515	0.517
3	0.500	0.513	0.511	0.516	0.518	0.503	0.511	0.502	0.501	0.503
4	0.473	0.498	0.491	0.503	0.508	0.493	0.489	0.489	0.480	0.489
5	0.444	0.482	0.471	0.490	0.496	0.480	0.464	0.473	0.457	0.474
6	0.418	0.467	0.453	0.477	0.483	0.466	0.442	0.457	0.436	0.459
7	0.398	0.453	0.438	0.465	0.472	0.453	0.423	0.443	0.418	0.445
8	0.382	0.441	0.425	0.455	0.462	0.441	0.408	0.431	0.403	0.433
9	0.368	0.431	0.414	0.446	0.453	0.431	0.396	0.42	0.391	0.423
10	0.358	0.423	0.405	0.438	0.445	0.422	0.386	0.411	0.381	0.414

Table 4
The α values for WTI crude oil and Chinese ten sector stock markets.

	α_{max}	α_{min}	$\Delta\alpha$
CSI300E	0.8241	0.2620	0.5621
CSI300M	0.7656	0.3452	0.4204
CSI300I	0.7570	0.3214	0.4356
CSI300CD	0.7653	0.3678	0.3975
CSI300CS	0.6575	0.3775	0.2801
CSI300H	0.6484	0.3405	0.3079
CSI300F	0.7606	0.2944	0.4663
CSI300IT	0.6652	0.3305	0.3348
CSI300TS	0.7302	0.2902	0.4400
CSI300U	0.7590	0.3331	0.4259

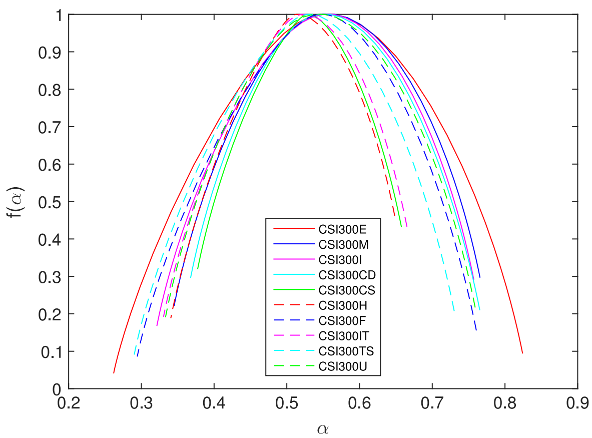


Fig. 4. Multifractal spectra, $f(\alpha)$ as a function of α, for WTI crude oil and the ten sector stock markets.

4.3. Vector autoregression analysis

The vector autoregression (VAR) model is an econometric model used to capture the interdependencies among multiple time series. In this paper, we do vector autoregression analysis to capture the interdependencies among WTI crude oil and Chinese ten sector stock markets by the equation below:

$$r_{(t)} = \varphi_1 r_{(t-1)} + \varphi_2 r_{(t-2)} + \cdots + \varphi_{21} r_{(t-21)} + \phi_0 r_{o,t} + \phi_1 r_{o,t-1} + \cdots + \phi_{21} r_{o,t-21} + \varepsilon_t \tag{12}$$

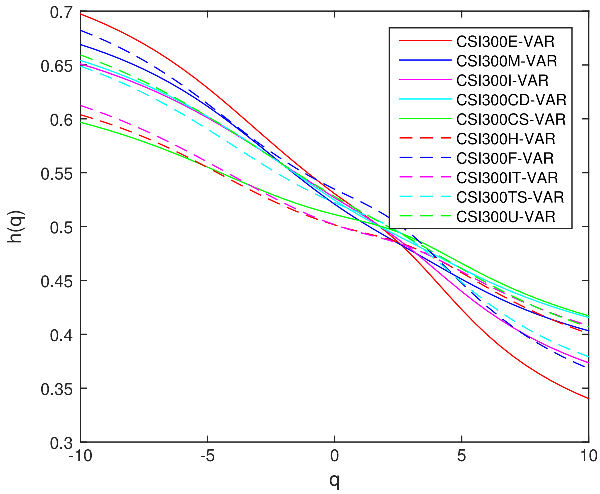


Fig. 5. Cross-correlation Hurst exponent, $H_{xy}(q)$ as a function of q, of VAR residual terms for WTI crude oil and the ten sector stock markets.

Table 5
Cross-correlation Hurst exponents for with q varying from −10 to 10 for WTI crude oil and Chinese ten sector stock markets.

q	CSI 300E	CSI 300M	CSI 300I	CSI 300CD	CSI 300CS	CSI 300H	CSI 300F	CSI 300IT	CSI 300TS	CSI 300U
−10	0.697	0.669	0.651	0.654	0.597	0.604	0.682	0.612	0.649	0.659
−9	0.687	0.661	0.643	0.646	0.590	0.596	0.672	0.604	0.640	0.650
−8	0.676	0.651	0.635	0.637	0.583	0.588	0.661	0.595	0.629	0.640
−7	0.662	0.640	0.625	0.627	0.574	0.578	0.647	0.584	0.618	0.629
−6	0.647	0.627	0.614	0.615	0.565	0.567	0.631	0.572	0.605	0.616
−5	0.629	0.612	0.601	0.602	0.555	0.555	0.614	0.560	0.590	0.602
−4	0.609	0.594	0.587	0.588	0.545	0.542	0.595	0.547	0.575	0.587
−3	0.588	0.576	0.572	0.573	0.535	0.530	0.578	0.534	0.560	0.572
−2	0.568	0.557	0.557	0.557	0.526	0.519	0.561	0.522	0.546	0.557
−1	0.549	0.538	0.542	0.540	0.518	0.509	0.547	0.511	0.533	0.542
0	0.530	0.521	0.527	0.524	0.511	0.502	0.535	0.502	0.522	0.528
1	0.514	0.505	0.512	0.510	0.505	0.495	0.523	0.494	0.513	0.514
2	0.496	0.491	0.497	0.497	0.499	0.489	0.511	0.488	0.502	0.501
3	0.474	0.478	0.479	0.485	0.490	0.481	0.493	0.481	0.488	0.488
4	0.449	0.464	0.459	0.473	0.478	0.470	0.471	0.470	0.469	0.475
5	0.423	0.451	0.440	0.461	0.466	0.457	0.448	0.458	0.449	0.461
6	0.400	0.439	0.422	0.450	0.454	0.444	0.426	0.446	0.430	0.448
7	0.380	0.428	0.406	0.439	0.442	0.431	0.407	0.435	0.413	0.435
8	0.364	0.418	0.394	0.430	0.433	0.419	0.392	0.424	0.400	0.425
9	0.351	0.410	0.383	0.422	0.424	0.409	0.379	0.416	0.389	0.415
10	0.340	0.403	0.374	0.416	0.417	0.401	0.368	0.408	0.379	0.407

$$r_{(o,t)} = \varphi_1 r_{(t-1)} + \varphi_2 r_{(t-2)} + \cdots + \varphi_{21} r_{(t-21)} + \phi_0 r_{o,t} + \phi_1 r_{o,t-1} + \cdots + \phi_{21} r_{o,t-21} + \varepsilon_{o,t}. \tag{13}$$

Here, $r_{(t)}$ is one of the ten sector stock indices daily return series, and $r_{(o,t)}$ is WTI crude oil daily return series. In order to test whether the VAR model above is good enough to capture the interdependencies between WTI crude oil and the ten sector stock markets, we do multifractal detrended cross-correlation analysis (MF-DCCA) for the residual term series, that is, ε_t and $\varepsilon_{t,o}$ series in Eqs. (12) and (13).

The $H_{xy}(q)$ spectra of the residual term series have been shown for WTI crude oil and the ten sector stock markets in Fig. 5 and the $h_{xy}(q)$ values with q varying from −10 to 10 have been shown in Table 5. We find that the cross-correlation exponent $H_{xy}(q)$ between WTI oil and the ten sector stock markets decreases with q varying from −10 to 10, implying that multifractal features still exist between the crude oil WTI and the ten sector stock markets residual term series during the sample period. When q is equal to 2, the cross-correlation exponent $H_{xy}(2)$ between WTI oil and the ten sector stock markets are all larger than 0.5, which means that the two correlated series are still persistent or long-range dependence.

Fig. 6 shows that the multifractal spectra of residual term series for the crude oil WTI and Chinese ten sector stock markets are not points, so we can prove that multifractal features exhibit between the ten pairs. The value of $\Delta\alpha$, $\Delta\alpha_{VAR}$ and $\Delta\alpha - \Delta\alpha_{VAR}$ are shown in Table 6. $\Delta\alpha - \Delta\alpha_{VAR}$ are small and some of them are positive and some of them are negative which implies that there is no distinct change of the width of the multifractal spectrum after we do vector autoregression analysis. So we can come to a conclusion that conventional linear models such as vector auto-regression (VAR) model could not be

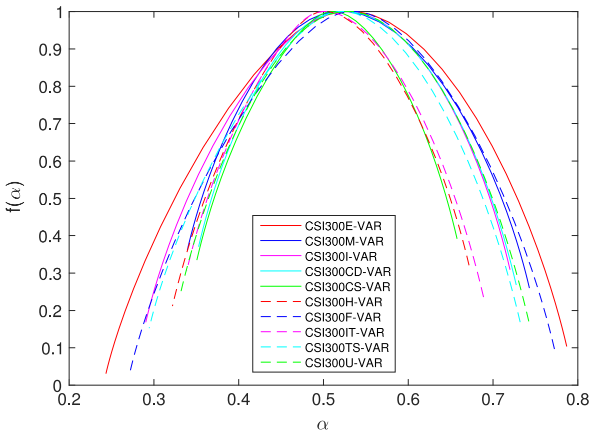


Fig. 6. Cross-correlation Hurst exponent, $H_{xy}(q)$ as a function of q, of VAR residual terms for WTI crude oil and the ten sector stock markets.

Table 6
The α values for WTI crude oil and Chinese ten sector stock markets.

	$\Delta\alpha$	$\Delta\alpha_{VAR}$	$\Delta\alpha - \Delta\alpha_{VAR}$
CSI300E	0.5621	0.5433	0.0188
CSI300M	0.4204	0.4033	0.0171
CSI300I	0.4356	0.4294	0.0062
CSI300CD	0.3975	0.3744	0.0231
CSI300CS	0.2801	0.3067	-0.0266
CSI300H	0.3079	0.3499	-0.0419
CSI300F	0.4663	0.5005	-0.0342
CSI300IT	0.3348	0.3502	-0.0154
CSI300TS	0.4400	0.4376	0.0024
CSI300U	0.4259	0.4110	0.0149

Note: $\Delta\alpha$ denotes the width of multifractal spectrum for the original series and $\Delta\alpha_{VAR}$ denotes the width of multifractal spectrum for the VAR residual term series.

used to describe the dynamics of the cross-correlations between WTI crude oil and the ten sector stock markets. Nonlinear methods are appealing.

5. Conclusions

In this paper, we investigated the cross-correlations between WTI crude oil and Chinese ten sector stock markets. The empirical results obtained through the MF-DCCA method imply that multifractality exists in the cross-relations between WTI crude oil and the ten sector stock markets. We find that the strength of multifractality between WTI crude oil and energy sector stock market is the highest, followed by the strength of multifractality between WTI crude oil and financial sector market, which reflects a close connection between energy and financial market. Then we do vector autoregression analysis to capture the interdependencies among multiple time series. By comparing the strength of multifractality for original data and residual errors of VAR model, we get a conclusion that the vector auto-regression (VAR) model could not be used to describe the dynamics of the cross-correlations between WTI crude oil and Chinese ten sector stock markets.

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