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Research paper

Chimera regimes in a ring of oscillators with local nonlinear interaction

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ABSTRACT

One of important problems concerning chimera states is the conditions of their existence and stability. Until now, it was assumed that chimeras could arise only in ensembles with nonlocal character of interactions. However, this assumption is not exactly right. In some special cases chimeras can be realized for local type of coupling [1–3]. We propose a simple model of ensemble with local coupling when chimeras are realized. This model is a ring of linear oscillators with the local nonlinear unidirectional interaction. Chimera structures in the ring are found using computer simulations for wide area of values of parameters. Diagram of the regimes on plane of control parameters is plotted and scenario of chimera destruction are studied when the parameters are changed.

1. Introduction

Complex spatial structures, so-called chimeras, cause a great interest among researchers in the recent time [4–6]. Chimeras emerge in oscillatory ensembles with different dynamics of units, both regular [1,7–12] and chaotic [13–16]. They consist of alternating clusters with coherent and incoherent behavior of neighboring oscillators. One of the most important condition of chimera emergence is assumed to be the nonlocal character of unit interaction, i.e. each oscillator is linked directly with a group of neighbors. The models of ensembles with nonlocal coupling were researched in a number of studies (see, for example, the review [6]). In [17–19] the results, confirming the existence of chimera in an ensemble with global interaction were obtained. The question about possibility of chimera existence in ensembles with local coupling is yet open. Generally, chimeras disappear with interaction radius decrease. However, it is possible that chimera-like structures can exist for some cases of ensembles with local interaction. So, chimera examples were obtained in the ensemble with local inertial coupling in [2]. The coupling is introduced with help of special variable described by a linear differential equation. Besides, the metastable chimeras were observed in ensemble of harmonic oscillators in a case of local interaction [1] and in ensemble of oscillators close to a homoclinic bifurcation [3].

The special type of chimera exists in the system with delayed feedback [20,21]. This is so-called virtual chimera. It is a special regime of time intermittency, when several intervals with regular and irregular behavior alternate in a period of delay. Moreover, these intervals are practically repeated for every delay period. So, the delay period can be considered in such system as a virtual space [20,21]. Systems with delayed feedback constitute a special class of distributed dynamical systems. Instantaneous state of such system in a time moment t is defined by some function of time at the interval

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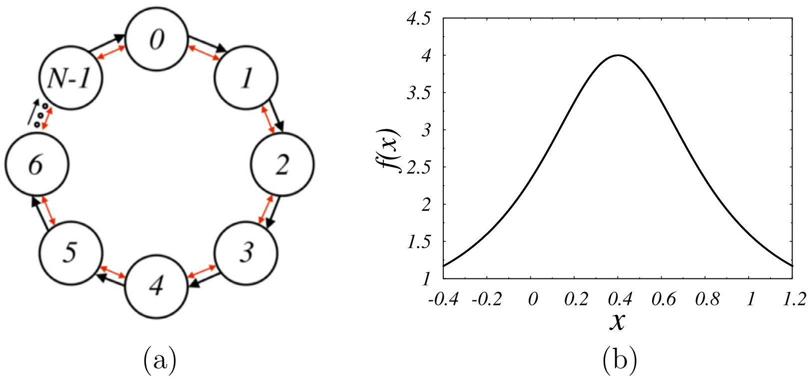

Fig. 1. (a) Schematic image of the system under study. Double arrows and one-side arrows represent diffusive coupling and unidirectional coupling, respectively; (b) graph of the nonlinear function describing the unidirectional coupling at $A = 4.7, \beta = 4, \phi = 0.4$.

$[t,\ t + T_d]$, where T_d is a delay period. Models with delayed feedback are widely used in mechanics [22], control problems [23–26], ecology [27–29], neurodynamics [30–32] and many other areas.

The behavior of oscillator with delayed feedback has similarity with one-dimensional spatially extended system. It has been shown that regime features such as the fractal dimension are extensive quantities, proportional to the time of delay, which appears to play a role very similar to the numbers of elements in a spatial system [33]. One of the main conditions of similarity between oscillator with delayed feedback and spatially distributed system is an asymmetric character of coupling between units [34]. The limit case of asymmetry is unidirectional coupling. So, it can be assumed that spatially distributed system, such as a ring of oscillators with a local unidirectional coupling under special conditions should demonstrate behavior, which is similar to the behavior of an oscillator with delayed feedback [35,36]. The existence of a virtual chimera in the systems with delayed feedback and the analogy mentioned above suggests that chimeras can emerge in a ring of oscillators with local coupling at least in a case of unidirectional character of coupling. If this hypothesis is true then the specific class of chimera structures can be obtained in the ensembles with local interaction of units. The results of numerical simulations, which prove this assumption, are presented in this article. The ensemble of linear oscillators in form of a ring with local nonlinear coupling are studied. Such system demonstrates chimeras, indeed. They are similar to the virtual chimeras in [20,21]. The evolution of chimeras is studied for variation of the system parameters. The influence of the characteristics of local coupling on the existence of the chimeric structures is considered.

2. Model of the ring

The model, studied in this work, is the ensemble consisting of identical linear dissipative oscillators. Interaction between oscillators is local and nonlinear. Boundary conditions are periodic. The schematic image of the model is shown in the Fig. 1a. The system equations are following (1):

$$\begin{cases} \dot{x}_j = -\alpha x_j - \omega_0^2 y_j + \sigma f(x_{j-1}) + \gamma (x_{j-1} + x_{j+1} - 2x_j), \\ \dot{y}_j = x_j, \end{cases} \\ x_{j+N} = x_j, \\ y_{j+N} = y_j. \tag{1}$$

where $j = 1, \ldots, N$ is an oscillator number (a discrete spatial coordinate), N is a number of units in the ring, α and ω_0 are parameters of oscillators (a dissipation coefficient and natural frequency, respectively). The two type of the local interactions are taken into account in the model(1). First type is a nonlinear unidirectional coupling with a force σ. Second type is a dissipative coupling with a coefficient γ. Unidirectional coupling provides the asymmetry of unit interactions and pumping of the energy into the dissipative oscillators, while the dissipative coupling leads to additional energy loss. In order to support the stationary oscillations in the ring, the unidirectional coupling force σ must be in direct ratio to dissipation α. So we can putting that $\sigma = k\alpha$, where k is a coefficient of unidirectional coupling, controlling its intensity. The nonlinearity is described by the following function (2):

$$f(x) = \frac{\beta}{1 + A\sin^2(x + \Phi)}. \tag{2}$$

This expression is known as Airy formula [37]. It describes intensity of light passing through the Fabry–Pérot interferometer. The similar function was used for setting of nonlinear delayed feedback in [21]. The parameters of nonlinearity are fixed as constant $A = 4.7, \beta = 4, \Phi = 0.4$. The graph of function $f(x)$ is shown in Fig. 1b.

The number of oscillators in the ensemble was chosen as $N = 300$. The frequency coefficient was fixed as $\omega_0 = 1$. The equations in (1) were integrated numerically using the Heun method [38]. The integration step was equal to $h = 0.0004$.

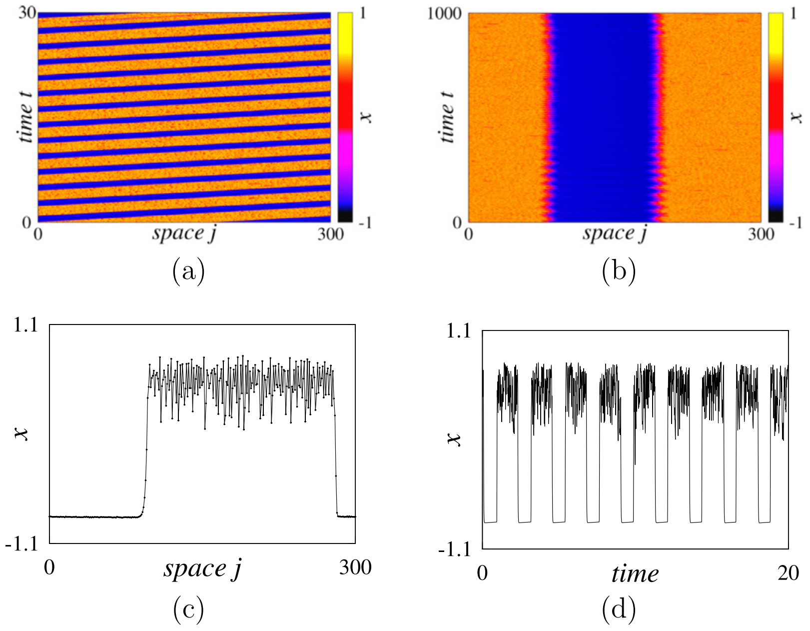

Fig. 2. Regime of a traveling chimera in the system (1), $\alpha = 100, \gamma = 0$ and $k = 0.508$: (a) space–time plot in a fixed coordinate system; (b) space–time plot in a co-moving frame; (c) instantaneous spatial profile x_j (snapshot); (d) form of oscillations in time for fixed spatial point $j = 150$. Transient time is $t_{set} = 25,000$. Initial conditions are random. (For interpretation of the references to color in the text, the reader is referred to the web version of this article.)

Space–time plots, snapshots, temporal realizations in fixed spatial points were applied for diagnostic of the regimes. Initial conditions were chosen as random or periodic with an additional random part.

3. The case of only unidirectional coupling

At first, let us consider the case of the most asymmetric coupling, when only nonlinear unidirectional links connect the oscillators in the ring, without diffusive interaction ($\gamma = 0$). The model (1) with $\gamma = 0$ is most relevant to the oscillator with nonlinear delayed feedback, studied in [20,21] in the case of small relaxation time (large dissipation). It can be physically realized, for example, as a ring of semiconducting lasers. The laser light beam passes through the Fabry–Pérot interferometer, and the signal is transformed nonlinearly as in (2). The fiber optic lines that are relatively short connect lasers in the ring. Therefore, the time of a signal passing through the lines is very small and delay time tends to zero.

Different spatio-temporal structures are observed in the ring (1) with variation of parameters α and k. The regime of stable traveling waves exists in the system for a large area of parameter values. This regime is illustrated in Fig. 2. It has the following feature: part of oscillators in a ring is located nearly the same metastable steady state, corresponding to dark (blue online) area in space–time plot. That is a coherent cluster. Another part is in state of spatiotemporal chaos (red–yellow area online). It forms an incoherent cluster. This structure rotates around the ring. The borders of coherent and incoherent clusters run around the ring with some constant velocity. If to represent a space–time plot in a co-moving frame, then one can obtain the fixed borders of chimera clusters. This regime is similar to the virtual chimera, which is observed in the oscillator with delayed feedback in [20,21]. In spatially distributed system, it can be named as a “traveling chimera”. The space–time plots of traveling chimera are shown in the Fig. 2a, b in fixed and rotating frames. A finiteness of number of the ring units forbids to specify exactly the velocity of chimera and eliminate a drift of the cluster borders. Therefore the borders of areas are not strictly vertical in the Fig. 2b. However, the described procedure allows to observe the chimera structure in a space–time plot on comparatively long time interval. The spatial structure is almost unchanged in time as seen in Fig. 2b. This allows us to speak about a stable in time chimera state. The form of corresponding oscillations in space and time are shown in Fig. 2c, d.

Fig. 2 illustrates the chimera regime for $\gamma = 0$ and a large dissipation when the ring of oscillators is similar to one oscillator with delayed feedback. For smaller dissipation, this similarity is violated, because the relaxation time becomes comparable with rotation of disturbance around the ring. Nevertheless, chimeras are observed for smaller value α until $\alpha \approx$ 2.5–3.0. They are similar to the chimeras in case of large dissipation. An example is shown in Fig. 3.

The independence from initial conditions is an important feature of the considered chimeras. The same regime of traveling chimera is observed in the system for various random and regular initial conditions. At the same time, the independence of traveling chimera from initial conditions is not total. In many cases, the same chimera is observed for changing of initial conditions only within certain limits. For example, chimeras with alternative numbers of “heads” (incoherent clusters) can

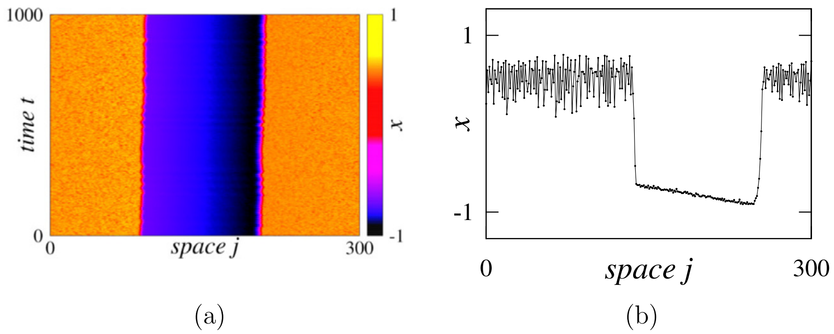

Fig. 3. Regime of a traveling chimera in the system (1) , $\alpha = 18, \gamma = 0$ and $k = 0.495$: (a) space–time plot in a comoving frame; (b) instantaneous spatial profile x_j (snapshot). Transient time is $t_{set} = 25,000$. Initial conditions are random.

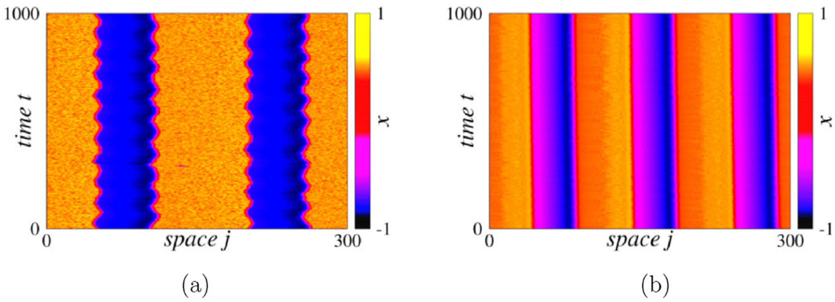

Fig. 4. Two additional examples of chimeras at $\alpha = 18,\ k = 0.4995$ observed for different random initial conditions. Transient time is $t_{set} = 25,000$.

be obtained for the different initial conditions at the same parameter values. That is the phenomenon of multistability. The examples of coexistence of two chimera structures for the same parameter values are shown in the Fig. 4.

Existence of areas of different spatiotemporal structures was studied for changing of parameters of the system (1). The diagram of regimes was plotted on the plane of control parameters α and k (Fig. 5). Different regimes are marked with different shading (colors and tones on line) in the diagram. Boxes with space–time plots, added to diagram, illustrate character of behavior for one or another area. The borders of areas are defined rather approximately, because their detail plotting is problematic due to duration of transient processes, multistability and complexity of the regimes, nevertheless, the diagram demonstrates adequate representation of the regime evolution.

When dissipation is small, the regular traveling waves are observed in the system (1). This regime is similar to the traveling chimera, but without chaotic component of oscillations. The area 1 of the diagram in Fig. 5 (yellow color online) corresponds to such regime. Intermittence in time between chaotic and space-homogeneous behavior is observed in the area 3 (white color). This behavior is specific for the system under study. It is observed when a coupling coefficient k is small and a dissipation is large. The various chimeras exist in the region marked by number 2 (tones of blue on line). The areas 2.1–2.4 correspond to the case of the maximal stability of chimera. Only the simplest chimera with one incoherent cluster (one-headed) exists in the area 2.1 (dark blue online). The area 2.2 (middle blue online) corresponds to coexistence of two- and one-headed chimeras. The one-, two- and three-headed chimeras coexist already in the area 2.3 (blue–gray online). Chimers with forth and more number of incoherent clusters can be observed in the area 2.4 (light blue online). In the area 4 (blue–green online) chimeras begin to destroy. The borders of incoherent clusters are gradually losing their stability with increase of parameter k.

4. Influence of coupling characteristics on the chimeras regimes

The nonlinear unidirectional interaction is a factor of generating chimera structures in a ring (1) composed of the simplest linear dissipative oscillators. At the same time, the specific type of nonlinear function $f(x)$ does not play a principal importance in the system behavior. This fact testifies the robustness of the system. The function should have at least one maximum. Numerical simulations have shown that the presence of other extrema (as in case (2)) is not required, but it is important that the maximum point was shifted from zero value.

The behavior of the system (1) for unidirectional coupling ($\gamma = 0$) has been studied for the following types of nonlinear functions, different from (2). In the first case, the function $f(x)$ is described by a following shifted Gaussian curve (3):

$$f(x) = A + Bexp\left\{-\frac{(x-\mu)^2}{2\sigma}\right\}, \tag{3}$$

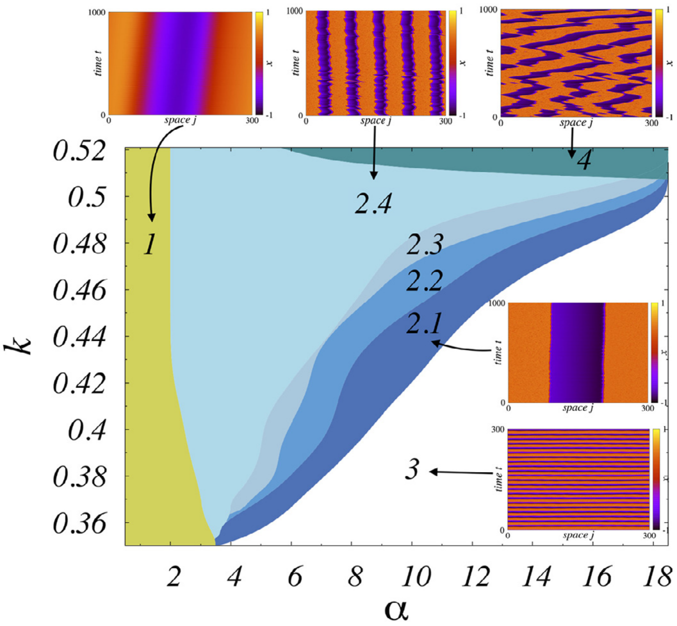

Fig. 5. Diagram of regimes in the system (1) with $\gamma = 0$ and unidirectional coupling on the plane of control parameters k and α. The space–time plots in boxes illustrate the behavior in different areas of diagram allocated to different tones (different coolers on line). (For interpretation of the references to color in the text, the reader is referred to the web version of this article.)

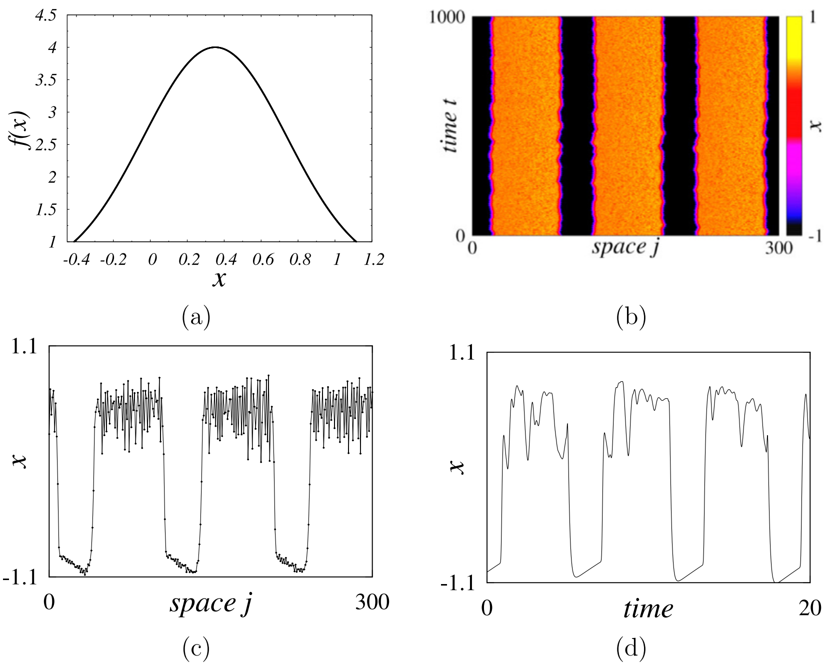

Fig. 6. Regime of a traveling chimera in the system (1) with nonlinear coupling function in form of (3) for $\alpha = 10$, $\gamma = 0$ and $k = 0.506$: (a) graph of the function (3); (b) space–time plot in a co-moving frame; (c) instantaneous spatial profile x_j; (d) form of oscillations in time for fixed spatial point $j = 150$. Initial conditions are random.

The curve parameters were chosen as follows: $A = 0.5, B = 3.5, \mu = 0.35, \sigma = 0.15$. The chimera example in the system (1) with such nonlinear coupling is given in Fig. 6.

In the second case, the nonlinear function has been taken in accordance with the shifted Lorentzian curve. This function is described by following Eq. (4):

$$f(x) = \frac{1}{\beta^2 + (x - A)^2}, \tag{4}$$

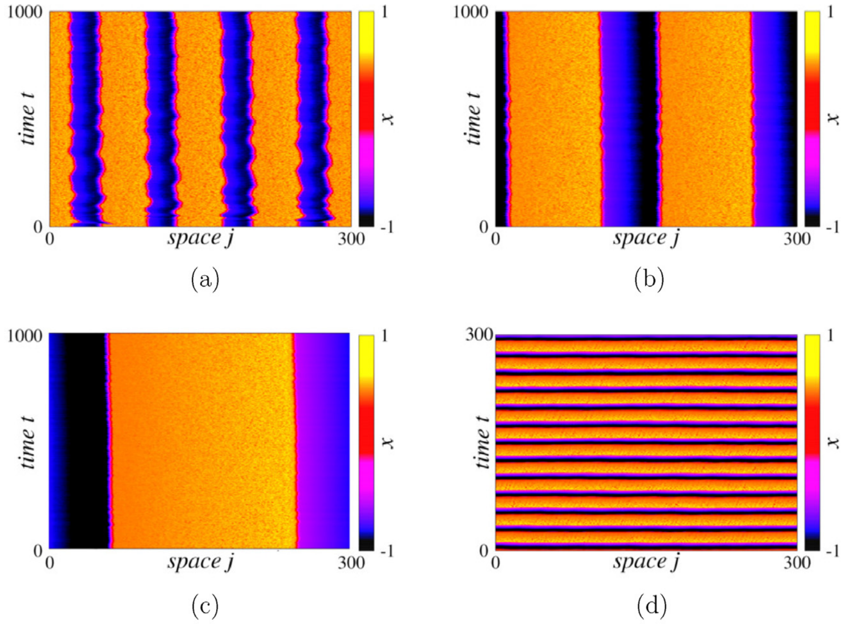

Fig. 7. Space–time plots corresponding to increase of diffusive coupling strength. (a) Four-headed chimera at $\gamma = 0.17$; (b) two-headed chimera at $\gamma = 0.297$; (c) one-headed chimera at $\gamma = 0.457$; (d) space-homogeneous regime at $\gamma = 0.52$. The initial conditions are random, and transient time is 20,000 for all cases.

where $\beta = 0.5$ and $A = 0.4$ are parameters of nonlinear function. The system (1) with this form of coupling nonlinearity demonstrate the same chimera states as in previous case. In both cases, the dynamics of the system (1) is similar to case of nonlinearity of type (2). Thus we determined the type of a nonlinear function of unidirectional coupling, when stable chimera is observed in the system (1) and all its features is similar to chimeras in all systems with nonlinearity of this type.

5. Influence of an additional diffusive coupling on the dynamical regimes in a ring (1)

The case of only unidirectional coupling was considered above. One of the important questions is the following: whether the system's behavior will change with the addition of a diffusive coupling. In presence of diffusive component of coupling the analogy between the ensemble (1) and one oscillator with delayed feedback is broken. Existence of chimera states is not obvious. However, our studies have shown that the regimes of travelling chimera continue to exist in (1) when $\gamma \neq 0$. Several specific effects of influence of addition diffusive coupling are observed. Let us consider the first of them. For fixed parameters $\alpha = 10$, k=0.51 and random initial conditions the behavior of the system (1) at $\gamma = 0$ corresponds to the regime of stable chimera with several incoherent clusters (their number is more, then three). Let us consider behavior of the system (1) at $\gamma > 0$ increasing value of γ. If γ is small a many-headed chimera is observed (see Fig. 7a). The number of incoherent clusters of chimera begins to decrease with increasing strength of diffusive coupling (Fig. 7b,c). Finally, only space-homogeneous regime is observed for a large value of γ (Fig. 7d). We must note, that for special initial conditions (corresponding to many-headed chimera) the number of chimera incoherent clusters remains constant even for very large value of γ.

The time form of oscillations for chimera regime changes with increasing strength of diffusive coupling. The number of chaotic oscillations in incoherent clusters decreases with increasing γ. Finally, chaotic oscillations disappear for the large diffusive strength. Thus, the regime of chaotic chimera evaluates to the periodic traveling wave. This phenomenon is illustrated with instantaneous spatial profile x_j given in Fig. 8a,b for relatively small and large values of γ.

6. Conclusions

The numerical studies have shown an example of a chimera regime, appearing in ensemble of units with local coupling. It is evidence that the local character of interaction does not exclude the possibility of formation of such complex structures as chimeras. Moreover, the elements of the studied ensemble are the simplest linear oscillators with large dissipation. The nonlinear unidirectional coupling plays in this case the main role in the chimera formation. The system demonstrates the property of roughness as the form of nonlinearity is changed or the diffusive coupling is added. Chimeras exist in a large area of values of unidirectional coupling coefficient and dissipation parameter of the oscillators, and they become impossible as the dissipation decrease too small or the diffusive component of coupling increases too large. The question arises: can

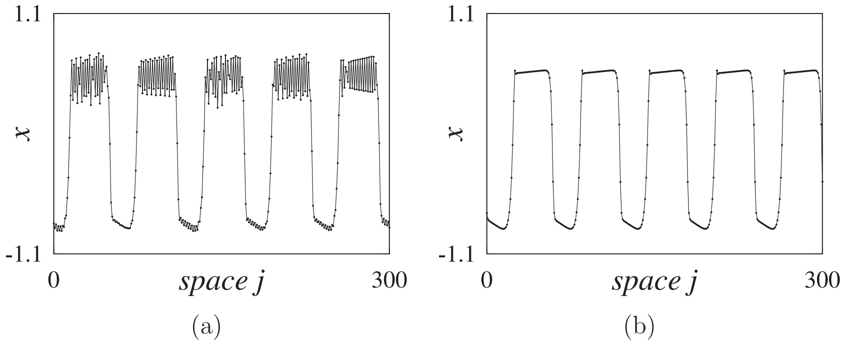

Fig. 8. Instantaneous spatial profile x_j of realizations from special initial conditions at small and large diffusive coupling strength: (a) $\gamma = 1.5$, (b) $\gamma = 3.67$.

such chimeras be obtained in a ring of units with linear local coupling when nonlinearity is characteristic of the units himself? To give the answer on this question the special new studies must be accomplished.

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