



The predictive power of local properties of financial networks



Petre Caraiani

Institute for Economic Forecasting, Romanian Academy, Romania

HIGHLIGHTS

- We construct correlation based financial networks in time.
- We derive local properties in such networks.
- Some local properties Granger-cause the overall dynamics of the stock market.

ARTICLE INFO

Article history:

Received 3 February 2016

Received in revised form 23 July 2016

Available online 26 August 2016

Keywords:

Networks

Prediction

Forecasts

ABSTRACT

The literature on analyzing the dynamics of financial networks has focused so far on the predictive power of global measures of networks like entropy or index cohesive force. In this paper, I show that the local network properties have similar predictive power. I focus on key network measures like average path length, average degree or cluster coefficient, and also consider the diameter and the s -metric. Using Granger causality tests, I show that some of these measures have statistically significant prediction power with respect to the dynamics of aggregate stock market. Average path length is most robust relative to the frequency of data used or specification (index or growth rate). Most measures are found to have predictive power only for monthly frequency. Further evidences that support this view are provided through a simple regression model.

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1. Introduction

Financial networks are widely used today both in academia and in the practical financial analysis. The initial contributions on the use of correlation based financial networks can be traced back to Mantegna [1] who proposed the use of minimum spanning tree to filter the information in financial networks. Significant refinements of this approach have been done by Tumminello, Aste, Di Matteo, and Mantegna [2] or Tumminello, Di Matteo, Aste, and Mantegna [3].

While the initial research has been rather focused on the analysis of static financial networks, there is an increasing number of studies addressing the dynamics of financial networks.

For example, Kenett, Shapira, Madi, Bransburg-Zabary, Gur-Gershgoren, and Ben-Jacob [4] constructed an index cohesive force index using SP500 data between 1999 and 2010. They showed that this index can characterize the state of the stock market as a whole and can also indicate the probability of a market crash.

Later studies focused on the dynamics of the relationships among different stock markets, see Ref. [5] or Ref. [6]. Using daily data for various stock markets, Song et al. [5] uncovered two types of dynamics: slow dynamics related to globalization, and fast dynamics related to critical events on the stock markets.

Kenett et al. [6] built on the previous study by Kenett et al. [4] and analyzed the index cohesive force and meta-correlations for world financial markets. The key findings were that there are different patterns for the Western developed stock markets relative to the emerging Asian financial markets.

E-mail address: caraiani@ipe.ro.

More recently, several studies focused on the predictability of the stock market dynamics using network based measures.

Caraiani [7], for the case of DOW Jones index, used the singular value decomposition based entropy to show that it Granger causes the aggregate stock market dynamics. Later studies confirmed this finding. For example, Ref. [8] also found that the singular value decomposition entropy has predictive ability for Shenzhen stock market. However, the predictive power is affected by the size of the windows and the structural breaks. In a later work, Gu and Shao [9] introduced the concept of multi-scale singular value decomposition to study its predictive power for Dow Jones Industrial Index.

Although not strictly related to the issue of predicting the dynamics of financial networks, but still related to the analysis of financial networks through various measures, we can also notice the research by Sandoval Junior and De Paula Franca [10], who used the eigenvalues of the matrix of correlations, or by Peron, da Fontoura Costa, and Rodrigues [11], who used the singular value decomposition entropy in analyzing the financial crises. Kumar and Deo [12] used random matrix theory to study the correlation and network properties of several financial indexes. They also found changing structures in the networks that are related to the financial crisis.

Related research has shown that the strength of the correlation among stocks during financial stress changes, signaling the fact that we can use this information to track potential systemic changes in the financial markets, see Ref. [13]. The analysis has been expanded in Ref. [14] by the use of meta-correlations. The latter research has shown that changes in the returns are related to changes in the mean correlation. The findings were also confirmed in Ref. [15] who studied several Dow Jones economic sector indexes and showed that changes in correlations (as uncovered through principal component analysis) can be connected to systemic risk and the occurrence of financial crisis.

While the research above did show that we can use the global property of financial networks to derive measures that can predict the dynamics of the stock market, it is not clear up to now whether the same holds for the local properties of such networks.

The aim of this paper is to study several key network measures related to local properties and test whether they hold any predictive power with respect to the dynamics of the aggregate stock markets.

The paper is organized as follows. Section 2 is dedicated to a discussion of the methods used in the paper. In Section 3, I derive the local properties of the financial networks using the available data. I test for Granger causality between these local measures and the aggregate dynamics of the stock market and present additional statistical analyses in Section 4. Finally, in Section 5, I conclude and suggest potential extensions of the results in this paper.

2. Methodology

I discuss in this section in a succinct manner the techniques and methodology used in this paper.

2.1. Correlation networks of stocks

I use the standard approach in constructing correlation networks of financial stocks, i.e. I derive the network based on the Pearson correlation coefficient among the different stocks, see Ref. [4] for a comparative review of the approaches used in the literature.

The procedure involves building first a correlation matrix R using the standard Pearson correlation (although there are other methods proposed in the literature too). We can construct the correlation matrix with the formula:

$$\rho_{i,j} = \frac{\text{cov}(r_i, r_j)}{\sigma_i \sigma_j} \quad (1)$$

here r_i is the return of the stock i , while σ_i is the standard deviation of the return of the same stock i . The return of a stock i is computed in a standard way as the logarithmic difference of the value of that stock, namely by:

$$r_i(t) = \log[P_i(t)] - \log[P_i(t-1)] \quad (2)$$

where $r_i(t)$ is the return of the stock i at time t , while $P_i(t)$ is the value of the same stock i at moment t .

This way, I get a matrix R of dimensions $N \times N$, with N as the number of stocks included. I eliminate weak correlations by setting a threshold of 0.3 (a standard value in the literature), namely the correlations lower than 0.3 are set to zero. I further compute the distances as 1 less the correlation. This way, I can obtain an adjacency matrix representation of a weighted undirected network, with each node representing the distance between the return of a stock i and the return of a stock j .

2.2. Local properties of correlation networks

As shown in the introduction, the previous literature has almost exclusively focused on systemic-type properties, mostly on entropy type measures, see Ref. [4] or Ref. [7]. This paper proposes the use of local properties of correlation networks in predicting the dynamics of the stock market.

The most robust measures typically employed in characterizing a network from a statistical point of view are the clustering coefficient, the degree distribution and the average path length. Since the paper is focused on deriving a dynamic measure for each local characteristic, I do not use the degree distribution (given the fact that this gives a distribution and the

focus is rather on scalar values that can be derived each time for a certain local property of a given network). I also consider a few other measures, like average degree, the diameter of the network and the s -metric. The list is not exhaustive, but rather covers the most significant and relevant measures for the type of network used in this paper.

We can denote by $l(i, j)$ the length of the shortest path between node i and node j (or distance between the two nodes). The average path length is taken as the average distance between two nodes from the network. Formally:

$$\text{average path length} = \frac{\sum_{i \geq j} l(i, j)}{\frac{n(n-1)}{2}}. \quad (3)$$

The diameter represents the largest distance between any two nodes:

$$\text{diameter} = \max_{i, j} l(i, j). \quad (4)$$

Clustering can be defined in many ways—globally or at individual node level. Our focus is on local properties. This is why I compute an average cluster coefficient. I start from clustering at an individual node i , computed as:

$$Cl_i(g) = \frac{\text{number of triangles connected to vertex } i}{\text{number of triples centered at } i}. \quad (5)$$

I can simply determine the average clustering degree by using:

$$Cl_i^{avg}(g) = 1/n \sum_i Cl_i(g). \quad (6)$$

A fourth measure used is the average degree. This is computed as:

$$\text{average degree} = 2 \frac{E}{N} \quad (7)$$

where E is the total number of edges in the network, while N is the total number of nodes.

Besides the above mentioned metrics, I use one further metric through which local properties are characterized: the s -metric. The s -metric, due to Li, Alderson, Doyle, and Willinger [16], is meant to measure the interconnectedness between the hub nodes within a network. It is computed as the sum of the products of degrees of all edges. Formally, we can write as:

$$s = \sum_{(i, j) \in E(Z)} \omega_i \omega_j \quad (8)$$

here the degree sequence in a network Z can be defined as $\omega = \omega_1, \omega_2, \dots, \omega_N$. The degree of a node i in this network is ω_i .

2.3. Granger causality tests

Originating from economics, the test for causality in the Granger sense has become widely adopted by other fields too. The Granger causality tests, see Ref. [17] or Ref. [18], is meant to uncover whether a given time series can help predicting the future dynamics of a second time series.

I formally describe the test in the following equations.

$$y_t = \beta_0 + \sum_{k=1}^N \beta_k y_{t-k} + \sum_{l=1}^N \alpha_l x_{t-l} + u_t \quad (9)$$

here y_t and x_t are the time series of interest, u_t stands for the uncorrelated disturbances. The parameters k and l indicate the number of lags.

The null hypothesis of the Granger causality test is that the series y does not cause the series x . In other words, there is no α_l significantly different from zero. The null and alternative hypotheses are specified below:

H0: $\alpha_l = 0$ for any l ;

H1: $\alpha_l \neq 0$ for at least some l .

In order to apply the Granger causality test, we run the regression using the specification in (7) and obtain the residual sum of squares (RSS, hereafter), given by:

$$RSS_u = \sum_{t=1}^N u_t^2. \quad (10)$$

As well as a restricted version without y_{t-k} (for this particular case, the residuals are denoted by v_t):

$$RSS_v = \sum_{t=1}^N v_t^2. \quad (11)$$

In order to test the null hypothesis, we use an F test, in other words, an F distribution under the null hypothesis, given by:

$$F = \frac{(RSS_v - RSS_u)/l}{RSS_u/(N - 2l - 1)} \quad (12)$$

characterized by an F distribution with l and $N - l - 1$ degrees of freedom.

I employ the Bayesian Information Criterion to choose the optimal lag length when running the Granger causality test, due to Schwarz [19]. This criterion is of the most employed ones when discriminating among different models. Conceptually, it is similar to Bayes Factor, as it is derived within a Bayesian framework, see Ref. [20]. Formally, we can denote by $L_n(k)$ the maximum likelihood of a estimated model. Here, k is the number of parameters of the model and n is the sample size. The BIC criterion can be formally written as:

$$c_n(k) = -\frac{2 \ln(L_n(k))}{n} + k \frac{\ln(n)}{n}. \quad (13)$$

Basically, through the BIC criterion, we select the most probable model conditional on the data.

3. Data selection

Following Caraiani [7] I select data on the main US stock market index, the Dow Jones Industrial Average. There are several reasons for doing this. First of all, it has been widely used in the literature. Second, and most important, we can contrast the results in this paper with the earlier results in Ref. [7]. I use both daily and monthly data, having in mind a check on the effects of the data frequency. The data on Kraft Foods Inc. have been excluded since there is limited availability (only after 2001). For similar reasons, for daily data, the data series on Travelers Companies Inc. has been excluded due to missing observations.

For daily data, the following sample is used: from July, 1, 1991 to August, 8, 2012, while for the monthly data the sample spans between July, 1991 and July, 2012.

The data sample for the daily frequency is between July, 1, 1991 and August, 8, 2012, while for monthly data it is between July, 1991 and July, 2012. Appendix A presents the stocks included in the sample as well as their acronyms. The data series are shown in Figs. B.11 and B.12. in Appendix B.

4. Results

4.1. Correlation based financial networks

Paralleling the analysis in Ref. [7], I perform an analysis of the dynamic properties of networks, by constructing networks and deriving their properties over a sliding window. The sliding window corresponds to one year, for daily data, respectively two years for monthly ones, which is a reasonable time length for deriving financial networks.

For monthly data, the sliding window consists of 24 observations, corresponding exactly to two years. For daily data, the window consists of 250 observations, corresponding roughly to one year. For each iteration, the window is moved one period to the right (day, or month).

4.2. Local properties of financial networks

For each iteration, I construct financial networks using the data included in the sliding window. For each such financial network, I derive the local properties, resulting a vector of values for each such local property: average path length, diameter, cluster coefficient, and so on. The results are presented below in Figs. 1–10.

The literature has discussed in detail the meaning of system-wide measures of networks. For example, as pointed out by Kenett et al. [4], a market index derived from the overall entropy can measure the information at a system level. Thus, significant changes in this index could signal changes in the state of the market, while an increased value might signal higher informational disorder in the market.

The literature is not clear however about the local properties and especially their significance. Comparing with the derived measure of singular value decomposition entropy, see Ref. [7], we see that the dynamics are strikingly similar. Thus, the local measures are shown, at an intuitive level, to contain significant information with respect to the dynamics of the market, as global measures of networks do (like entropy).

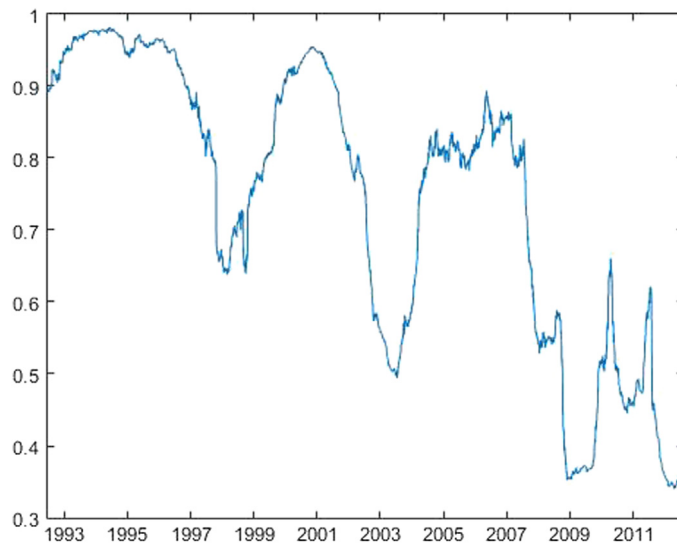


Fig. 1. Average path length—daily data.

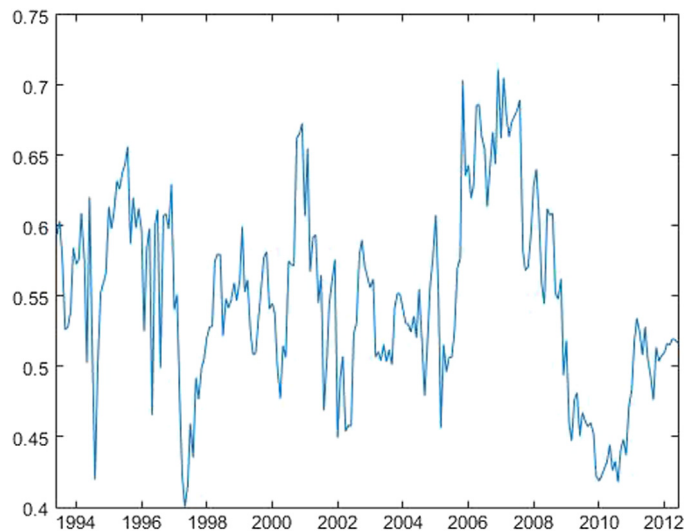


Fig. 2. Average path length—monthly data.

What could determine this behavior? This behavior might simply originate from the way the local measures are constructed—they indicate changes in the topology of the networks. For example, a lower diameter might indicate that the overall correlation among stocks is becoming weaker during certain periods, which might indicate a changing state in the financial market. Similarly, changes in the average path length, or the average distance between two nodes, might indicate changes in the strength of the connection among the nodes. Again, in times of turbulence, the connections might become weaker. The average degree tells us how many edges one can find on average in this network for each node. Since an edge between two nodes will be absent only if the correlation is weaker than 0.3, this is just another way to measure how strongly move together the returns. The graphics depict a similar behavior as for the average path length. The *s*-metric is a less intuitive measures as it focuses on the connections among hubs and not nodes *per se*. However, it is well known that in networks of financial stocks, stocks tend to group together according to the sectors they are part of. In the end, although in an indirect way, the *s*-metric depicts again the strength of the connection between stocks (nodes). Finally, the cluster coefficient indicates the degree in which nodes (stocks) tend to group together. The figures show that indeed the clustering coefficient behaves differently than the other measures. We see however that the value of this coefficient fell during the last two market crashes (2001 and 2007). This could come from the fact that during market turbulences, shares tend to move more erratically, and thus they tend to group less. In the end the clustering coefficient will fall during such periods.

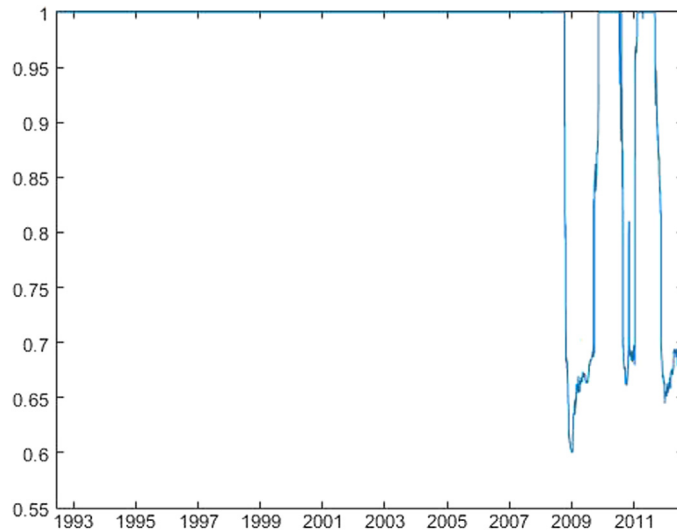


Fig. 3. Diameter—daily data.

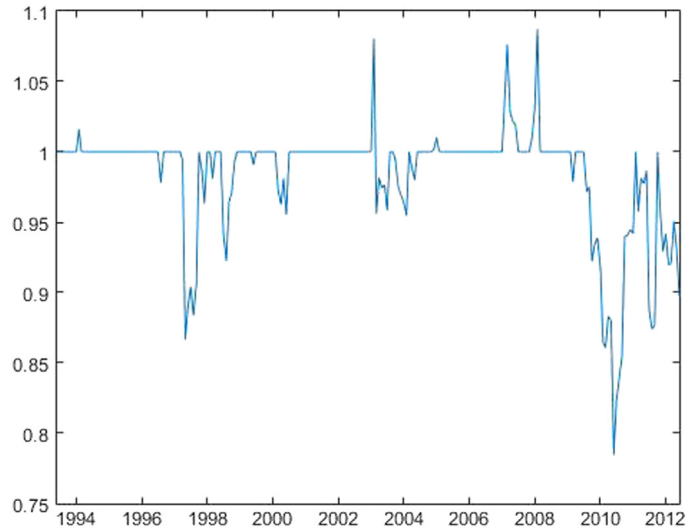


Fig. 4. Diameter—monthly data.

Comparing the different measures of local properties, we can see that while most are robust across the different frequencies (daily or monthly) some are not at all (there is no valuable information contained in the cluster coefficient for daily series), or present significantly different behavior along the different frequencies (the diameter and the average path length). This is interesting and important since it is usually thought that the most significant and robust local measures of networks are the average path length, the cluster coefficient and the degree distribution, see also Ref. [21].

It is interesting that for two cases, the diameter and the cluster coefficient, there are sharp differences between the daily data and the monthly data (see Figs. 3 and 4 as well as 9 and 10, respectively). For the case of daily data, there is some variability for the diameter starting with 2008 (corresponding roughly to the financial crisis). Thus, the changes in these variables appear only at lower frequencies, while at higher frequencies (daily data) they are not significant. A potential explanation is that the lack of variation in the cluster coefficient for daily data, for example, might suggest that changes in the way stocks tend to be grouped together emerge rather in the longer run.

4.3. Granger causality results

As suggested in Figs. 1–10, the indexes I derived from the local properties of the networks might indicate the state of the stock market. In order to back up this proposition with statistical evidences, in Section 1 I test whether the constructed

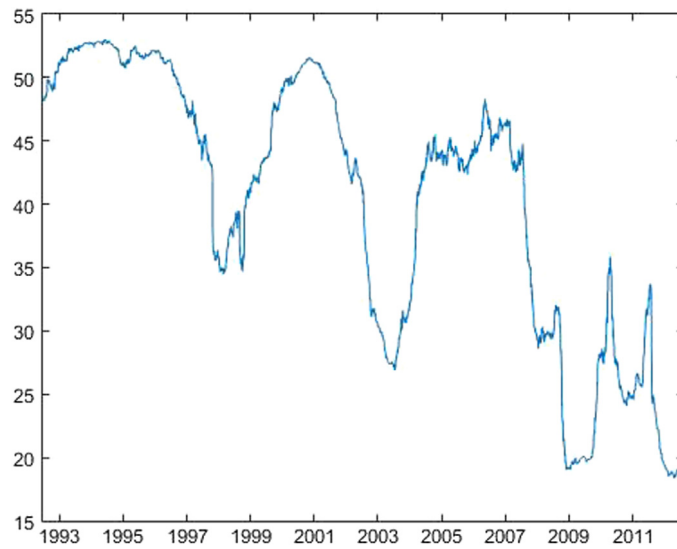


Fig. 5. Average degree—daily data.

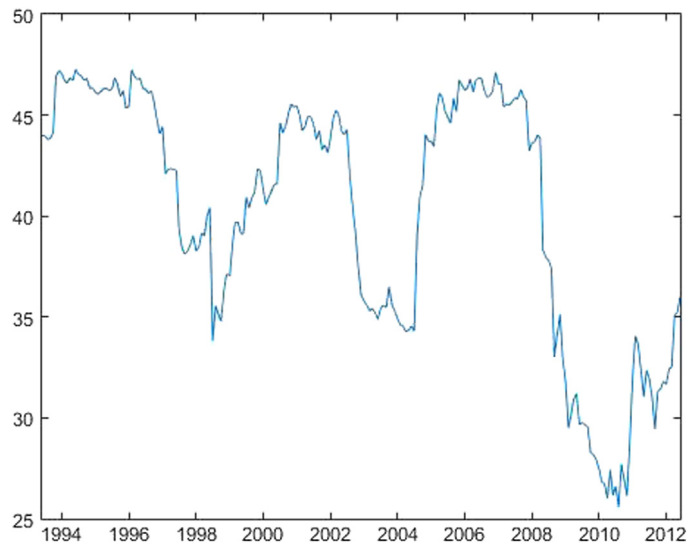


Fig. 6. Average degree—monthly data.

Table 1
Granger causality for daily data.

Variable	DOW Jones index	DOW Jones returns
Average degree	0.50	1.11
Average path length	8.54 [*]	15.15 [*]
s-metric	0.42	1.08
Diameter	0.02	1.11
Cluster coefficient	0.00	0.39

Note:

^{*} Denotes statistical significance of the F -test at 0.1 level.

dynamic measures have any predictive ability with respect to the DOW Jones Industrial Index Average, i.e. relative to the overall dynamics of the market.

In Tables 1 and 2, I present the results of Granger causality tests between the dynamic local measures presented in the last section and the Dow Jones Industrial Index Average at both daily and monthly frequencies. For each test, the optimal lag order was chosen using the BIC criterion (allowing for a selection of lags up to the order of 12 lags).

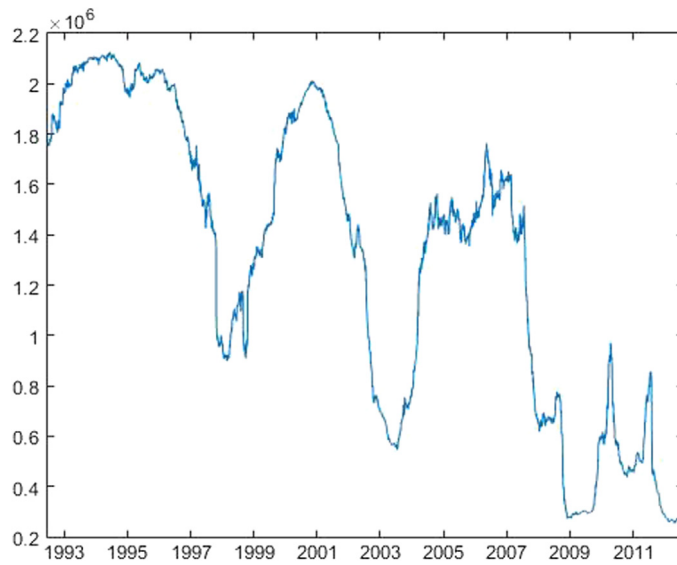


Fig. 7. s-metric—daily data.

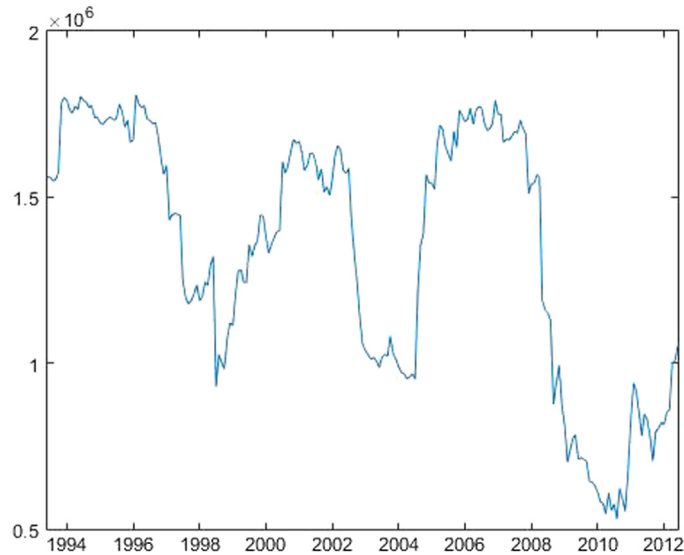


Fig. 8. s-metric—monthly data.

Table 2
Granger causality for monthly data.

Variable	DOW Jones index	DOW Jones returns
Average degree	8.22 [*]	17.02 [*]
Average path length	3.87 [*]	1.30
s-metric	8.51 [*]	17.15 [*]
Diameter	0.31	1.64
Cluster coefficient	3.52 [*]	6.58 [*]

Note:

^{*} Denotes statistical significance of the *F*-test at 0.1 level.

The tables above, [Table 1](#) for daily data and [Table 2](#) for monthly data, present the results of the *F*-test for Granger causality tests between the local measures and Dow Jones Industrial Index Average. The findings differ sharply along the different frequencies: most measures are not statistically significant for daily data, except for the average path length which is statistically significant taken both as index and growth rate. For monthly data, most series, except the diameter, are

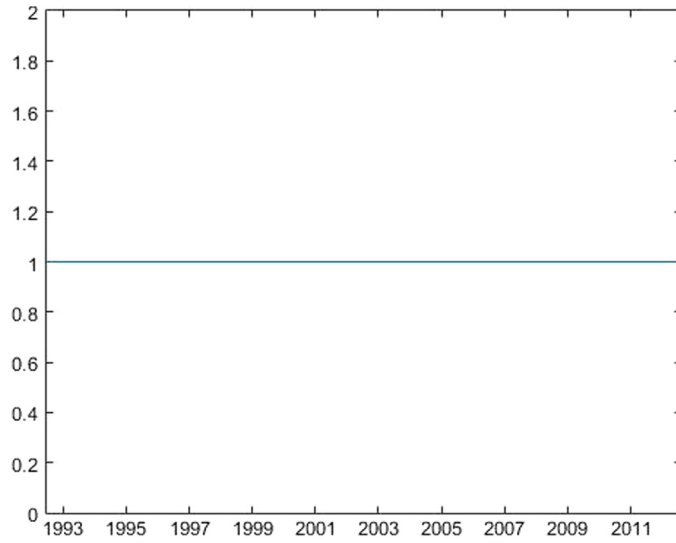


Fig. 9. Cluster coefficient—daily data.

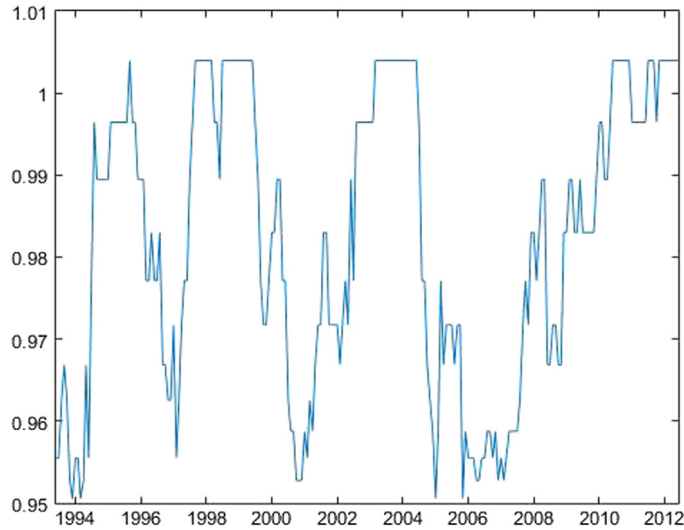


Fig. 10. Cluster coefficient—monthly data.

statistically significant both as an index and growth rate. The average path length is statistically significant taken as an index but not as a growth rate for monthly data.

While there are changes in most measures at both daily and monthly frequencies, in terms of Granger causality, we can see that except average path length, the other local measures do not Granger-cause Dow Jones Index. A potential explanation might come from the fact that the changes from one day to another in the data do not necessarily lead to changes in the properties of the network, while the more significant changes from one month to another do. In other words, small changes in the stock returns are not reflected in changes in the derived networks and their associated properties, while larger changes are reflected (from month to month).

4.4. Further statistical evidences

In this section, I further extend the previous analysis by considering a regression model. I focus on the monthly data, to simplify the analysis. The dependent variable is the Dow Jones Industrial Index, while the explanatory variables are the various local measures derived above. Table 3 presents the results of the regression analysis. I tested for the presence of unit roots in the series before performing the regression analysis, see Table 4 in Appendix B.

I estimate a regression using the following specification:

$$DJIA_t = \alpha + \beta * localmeasure_t + \gamma * DUMMY2001 + \delta * DUMMY2008 + \theta * DUMMY2011 + \epsilon_t \quad (14)$$

Table 3
Regression analysis.

Variable	Model 1 (average degree)	Model 2 (average path length)	Model 3 (cluster coefficient)	Model 4 (s-metric)	Model 5 (diameter)
α	0.007*** (0.0025)	0.0075*** (0.0026)	0.0074*** (0.0026)	0.0076*** (0.0025)	0.0065*** (0.0028)
<i>Local measure</i>	0.003* (0.0021)	0.0408 (0.0505)	0.0055 (0.3322)	0.00006** (0.0025)	−0.1496* (0.0082)
<i>DUMMY2001</i>	−0.12*** (0.0020)	−0.1234*** (0.0322)	−0.1212*** (0.0321)	−0.1212*** (0.0319)	−0.1496*** (0.0826)
<i>DUMMY2008</i>	−0.17*** (0.0068)	−0.1761*** (0.0323)	0.1748*** (0.0323)	−0.1796*** (0.0322)	−0.1113*** (0.0331)
<i>DUMMY2011</i>	−0.11*** (0.0046)	−0.1103*** (0.0322)	0.1092*** (0.0322)	0.1140*** (0.0321)	
R^2	0.25	0.24	0.23	0.25	0.22

Note:

* Denotes statistical significance at 0.1 level.

** Denotes statistical significance at 0.05 level.

*** Denotes statistical significance at 0.01 level.

where $DJIA_t$ is the return of DOW Jones Industrial Index Average at time t , α is a constant, *localmeasure* is one of dynamic local measures previously derived, *DUMMY2001*, *DUMMY2008* and *DUMMY2011* stand for the dummy variables that capture the large movements due to financial crises or other events, while ϵ_t is the residual.

Table 3 shows the results of the estimations of models including each of the local measures previously derived. I also included a few dummy variables that capture unexpected changes, usually tied to the financial crises in the past (for 2001, the dotcom crash, for 2008, the financial crisis). Again, we find a positive relationship between a local measure (the average degree as well as s-metric with the latter having a smaller coefficient due to its scale) and the dynamics of the aggregate stock market index, a finding that parallels the earlier findings for entropy in Ref. [7], for example, or Ref. [4]. There is also a significant coefficient, though negative, attached to the diameter. Since the shorter diameter means more powerful connections among the stocks, the diameter moves counter-cyclically relative to the stock market index.

5. Conclusion

Earlier research has emphasized the predictive role of system-wide measures of financial networks like entropy relative to the aggregate dynamics of stock markets. In contrast to previous research, this paper has asked whether *local properties* of networks can also help predict the movements of stock markets.

Analyzing several such local measures like average degree, cluster coefficient, average path length, the diameter or the s-metric, the paper found solid statistical evidences that some local measures do have predictive power relative to the aggregate stock market. The evidences comprised Granger causality tests for both daily and monthly data, though this approach has known limitations, as well as simple regression models, with the dependent variable the return of the DJIA index and the explanatory variable the various local measures. The coefficient attached to average degree and s-metric was found positive and statistically significant (while the one corresponding to the diameter was found statistically significant but negative), paralleling earlier results on the role of entropy in explaining the aggregate dynamics of the stock market.

Future research might as well consider other factors that can influence the Granger causality between the properties of networks and the dynamics of the aggregate market, like long memory, a property specific to financial time series. Additional work might be also carried using a larger number of structural measures which were not considered here.

Acknowledgment

This paper contains partial results from the 2016 research project Predicting Network Dynamics part of Advanced theories and models of economic analysis and forecasting research program at the Institute for Economic Forecasting, Romanian Academy.

Appendix A. DOW Jones industrial average components

DOW Jones industrial average components	
Countries	Abbreviation
Alcoa Inc. Common Stock	AA
American Express Company	AXP
Boeing Company	BA

(continued on next page)

DOW Jones industrial average components	
Countries	Abbreviation
Bank of America Corporation	BAC
Caterpillar, Inc.	CAT
Cisco Systems, Inc.	CSCO
Chevron Corporation	CVX
E.I. du Pont de Nemours and Com	DD
Walt Disney Company (The)	DIS
General Electric Company	GE
Home Depot, Inc. (The)	HD
Hewlett–Packard Company	HPQ
International Business Machines	IBM
Intel Corporation	INTC
Johnson & Johnson	JNJ
JP Morgan Chase & Co.	JPM
Kraft Foods Inc.	KFT
Coca-Cola Company (The)	KO
McDonald's Corporation	MCD
3M Company	MMM
Merck & Company, Inc.	MRK
Microsoft Corporation	MSFT
Pfizer, Inc.	PFE
Procter & Gamble Company (The)	PG
AT&T Inc.	T
The Travelers Companies, Inc.	TRV
United Technologies Corporation	UTX
Verizon Communications Inc.	VZ
Wal-Mart Stores, Inc.	WMT
Exxon Mobil Corporation	XOM

Appendix B. Data

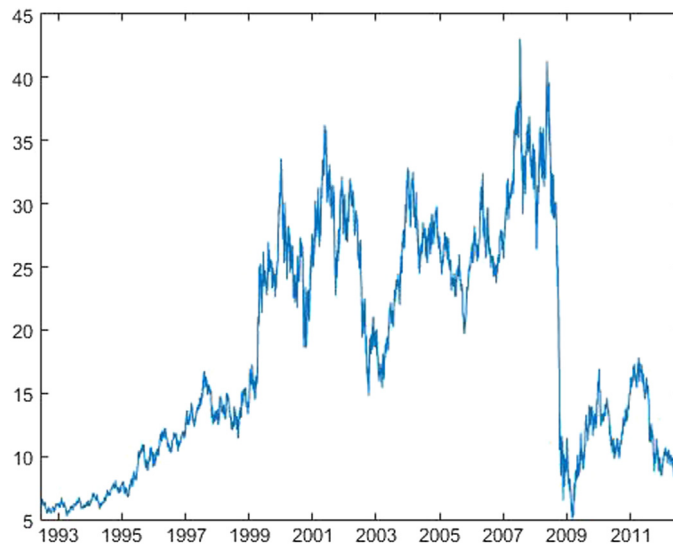


Fig. B.11. DJIA: Daily data.

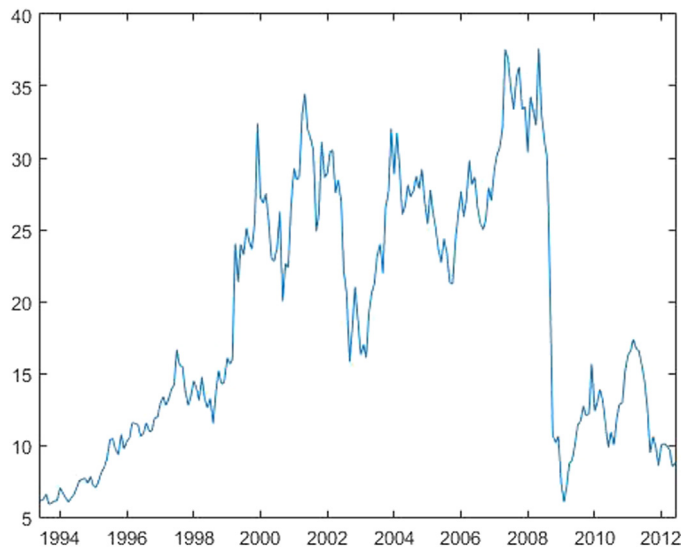


Fig. B.12. DJIA: Monthly data.

Appendix C. Unit root tests

Table 4
Unit root tests.

Variable	ADF test	Phillips–Perron test
DJIA returns	−12.14***	−14.14***
Average degree	−14.34***	−14.49***
Average path length	−19.68***	−22.61***
s-metric	−14.44***	−14.28***
Diameter	−13.10***	−19.31***
Cluster coefficient	−14.25***	−16.46***

Note:

* Denotes statistical significance at 0.1 level.

** Denotes statistical significance at 0.05 level.

*** Denotes statistical significance at 0.01 level.

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