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Research paper

The transmissibility of nonlinear energy sink based on nonlinear output frequency-response functions

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ABSTRACT

For the first time, a new representation of transmissibility based on nonlinear output frequency-response functions (NOFRFs) is proposed in the present study. Furthermore, the transmissibility is applied to evaluate the vibration isolation performance of a nonlinear energy sink (NES) in frequency domain. A two-degree-of-freedom (2-DOF) structure with the NES attached system is adopted. Numerical simulations have been performed for the 2-DOF structure. Moreover, the effects of NES parameters on the transmissibility of the nonlinear system are evaluated. By increasing the viscous damping and mass, as well as decreasing the cubic nonlinear stiffness of the NES, the analytical results show that the transmissibility of the 2-DOF structure with NES is reduced over all resonance regions. Therefore, the present paper produces a novel method for NES design in frequency domain.

1. Introduction

Vibration isolation is very important in many engineering practices, such as vibration control of satellite systems and sensitive equipment [1–3]. Generally speaking, there are two kinds of vibration isolation systems: passive and active. Active control strategies were shown to be effective in suppressing oscillations. However, they often associate with problems like high cost and added weight. Also, active control usually require an independent energy supply. These limitations highly restrict its usage in many applications. Therefore, passive vibration control is still used more often these days.

Linear absorber is commonly used to attenuate unwanted vibration energy in engineering practices but it is only effective in the vicinity of its design frequency [4]. On the other hand, the excitation frequency in reality is not fixed in most cases. Therefore, the linear vibration absorber will not be able to work efficiently in these situations due to its narrowband effectiveness. Even worse, the linear absorber may also adversely affect the frequency response in the neighbourhood of resonant frequencies of the primary system [5].

To overcome the limitations of the linear absorber, nonlinear strategies in vibration suppression have been widely studied over these years. In Refs. [6–9], it has been shown that energy from transiently loaded linear subsystems may be passively absorbed by properly designed, essentially nonlinear local attachments known as nonlinear energy sink (NES). A valuable feature of a NES is its capability to realize targeted energy transfer (TET). Therefore, passive energy pumped from the pri-

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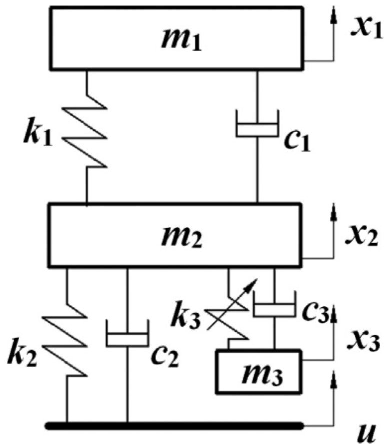

Fig. 1. A 2-DOF oscillator with a nonlinear energy sink attached.

mary system will be transferred to the NES irreversibly. Furthermore, these works also showed that a properly designed NES could dissipate vibrational energy within a wide frequency range as long as the input energy is above a certain threshold.

Over the last few years, much attentions have been paid to researches on NES. The effect of a NES on the steady-state dynamics of a weakly coupled system has been investigated theoretically and experimentally [10]. Many novel NES designs with different types of nonlinear stiffness, like cubic, non-smooth, non-polynomial, etc, have been proposed [11–15]. Capturing and storing the dissipated energy by the NES with a vibration energy harvester have been researched [16]. For better vibration isolation performance, some novel designs of NES, such as asymmetric NES and NES with negative linear and nonlinear coupling stiffness components, have also been proposed [17–18]. However, the above mentioned works regarding to NES analysis were conducted by evaluating displacement or energy dissipation in time-domain. Other works regarding to NES analysis in frequency domain were carried out by evaluating the frequency response through FFT [19–24]. For a linear system, transmissibility is usually represented as the ratio between the magnitude of the system output and input spectrum. Due to its simplicity, this representation is also widely used in industry for nonlinear frequency domain analysis and design. But, it should be noted that this representation for transmissibility is not correct theoretically for nonlinear systems [25].

Nonlinear output frequency response functions (NOFRFs) is a new concept proposed by Lang and Billings recently [25]. It has been used to detect the position of nonlinear components in periodic structures which is of great practical significance in system identification [26–27]. The concept of NOFRFs is regarded as an extension of the classical frequency response function for linear systems to the nonlinear case. By introducing the concept of NOFRFs, the analysis of nonlinear systems could be implemented in a way similar to the analysis of linear systems. Therefore, informative and physically meaningful interpretation for many nonlinear phenomena in the frequency domain are provided. In contrast to the representation of transmissibility based on FFT, the NOFRFs are well defined in the context of nonlinear systems. Therefore, the transmissibility results achieved based on NOFRFs should be more accurate in frequency domain analysis.

In the present study, a new representation of the transmissibility based on NOFRFs is proposed for the first time. Furthermore, the transmissibility is applied to evaluate the vibration isolation performance of a NES in frequency domain. A two-degree-of-freedom (2-DOF) structure with the NES attached system is adopted. Numerical simulations have been performed for the 2-DOF structure. Moreover, the effects of NES parameters on the transmissibility of the nonlinear system based on the concept of NOFRFs are evaluated. The analytical results show that the transmissibility is reduced over all resonance regions by increasing the viscous damping and mass, as well as decreasing the cubic nonlinear stiffness of the NES. These results are very useful for NES design in engineering practices.

2. Two degree of freedom system with NES

The systems considered in the present study are described by a typical 2-DOF oscillator as shown in Fig. 1 with the input $u(t)$ applied at the supporting base and a nonlinear energy sink attached to the 2th mass. The nonlinear oscillator is designated as the nonlinear energy sink (NES), and possesses a cubic nonlinear stiffness in parallel to a viscous damper.

The governing equations of motion of this system are given by

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} c_1 & -c_1 & 0 \\ -c_1 & c_1+c_2+c_3 & -c_3 \\ 0 & -c_3 & c_3 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix}$$
$$+\begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1+k_2 & -k_3 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ k_3(x_2-x_3)^3 \\ -k_3(x_2-x_3)^3 \end{bmatrix} = \begin{bmatrix} 0 \\ c_2\dot{u}+k_2u \\ 0 \end{bmatrix} \tag{1}$$

In Eq. (1), m_i ($i = 1, 2$) are the masses of the primary oscillator and m_3 is the mass of the NES. c_i ($i = 1, 2$) are damping coefficients of the primary oscillator and c_3 is the damping coefficient of the NES; k_i ($i = 1, 2$) are the linear stiffness of the primary oscillator and k_3 is the cubic nonlinear stiffness of the NES.

3. Transmissibility of NES based on the concept of NOFRFs

As complements of time domain analysis, and by providing a physically meaningful insight into the behaviour of systems under investigation, frequency domain methods play a very important role in the theory and practice of control and signal processing. The transmissibility concept based on FFT has been widely used in frequency domain analysis of linear systems for a very long time. As it is simple to use, it is also still widely applied in industry for nonlinear frequency domain analysis and design. However, it should be noted that this representation for transmissibility is not correct theoretically for nonlinear systems. Therefore, the NOFRFs based analysis can be used here to improve traditional methods for nonlinear system analysis and design in the frequency domain. In contrast to the transmissibility based on FFT, the NOFRFs are well defined in the context of nonlinear systems. Therefore, the transmissibility results achieved based on NOFRFs should be more accurate in frequency domain analysis of the system behaviour.

A wide class of nonlinear systems could be studied based on the Volterra series theory. For nonlinear systems which can be described by a Volterra series model, the concepts of Generalized Frequency Response Functions (GFRFs) [28–30] and NOFRFs could be used to conduct frequency domain analysis for the system from two different perspectives. However, unlike the GFRFs, one of the salient feature of the NOFRFs is that it is a one dimensional function of frequency, which allows the analysis of nonlinear systems in the frequency domain to be implemented in a manner similar to the analysis of linear system.

According to the Volterra series theory of nonlinear systems which are stable at zero equilibrium, the relationships between the output $x_i\ (t)$ and the input $u(t)$ can be expressed as [25]

$$x_i(t) = \sum_{n=1}^{N} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h_n(\tau_1, \ldots, \tau_n) \prod_{i=1}^{n} u(t-\tau_i) d\tau_i \tag{2}$$

in which $h_n(\tau_1,\ldots,\ \tau_n)$ is the nth order Volterra kernel, and N is the maximum order of the system nonlinearity in the Votlerra series. The output frequency response of this class of nonlinear systems to a general input is described in Ref. [28] as

$$\begin{cases} X(j\omega) = \sum_{n=1}^{N} X_n(j\omega) \\ X_n(j\omega) = \dfrac{1/\sqrt{n}}{(2\pi)^{n-1}} \displaystyle\int_{\omega_1+\cdots+\omega_n=\omega} H_n(j\omega_1, \ldots, j\omega_n) \times \prod_{i=1}^{n} U(j\omega_i) d\sigma_{n\omega} \end{cases} \tag{3}$$

where $X(j\varpi)$ is the spectrum of the system output and $X_n(j\varpi)$ is the nth order output frequency response of the system. Eq. (3) could be viewed as a natural extension of the well-known linear relationship $X(j\varpi) = H_1(j\varpi)U(j\varpi)$ to the nonlinear case [28].

$$H_n(j\omega_1, \ldots, j\omega_n) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h_n(\tau_1, \ldots, \tau_n) e^{-(\omega_1\tau_1+\cdots+\omega_n\tau_n)j} \times d\tau_1 \cdots d\tau_n \tag{4}$$

is known as the Generalized Frequency Response Functions (GFRFs) in which $\int_{\omega_1+\cdots+\omega_n=\omega} H_n(j\omega_1, \ldots, j\omega_n) \times \prod_{i=1}^{n} U(j\omega_i) d\sigma_{n\omega}$denotes the integration of $H_n(j\omega_1, \ldots, j\omega_n) \times \prod_{i=1}^{n} U(j\omega_i)$ over the n-dimensional hyper-plane $\omega_1 + \cdots + \omega_n = \omega$.

Based on the above results for output frequency responses of nonlinear systems, Lang and Billings [25] proposed a new concept known as NOFRFs recently. The concept was defined as

$$G_n(j\omega) = \frac{\int_{\omega_1+\ldots+\omega_n=\omega} H_n(j\omega_1, \ldots, j\omega_n) \prod_{i=1}^{n} U(j\omega_i) d\sigma_{n\omega}}{\int_{\omega_1+\ldots+\omega_n} \prod_{i=1}^{n} U(j\omega_i) d\sigma_{n\omega}} \tag{5}$$

under the condition that [25]

$$U_n(j\omega) = \int_{\omega_1+\ldots+\omega_n} \prod_{i=1}^{n} U(j\omega_i) d\sigma_{n\omega} \neq 0 \tag{6}$$

It should be noted that $G_n(j\varpi)$ is valid over the frequency range of $U_n(j\varpi)$, which can be determined using an explicit expression derived by Lang and Billings in [28].

By introducing the concept of NOFRFs $G_n(j\varpi)$, $n = 1, \ldots, N$, Eq. (3) can be written as [28]

$$X(j\omega) = \sum_{n=1}^{N} X_n(j\omega) = \sum_{n=1}^{N} G_n(j\omega) U_n(j\omega) \tag{7}$$

which is similar to the description of the output frequency response for linear systems. The NOFRFs, therefore, reflect a combined contribution of the input and the system to the frequency domain output behaviour according to Eq. (7).

When system (2) is subject to a harmonic input [31,32],

$$u(t) = A\cos(\omega_F t + \beta) \tag{8}$$

Lang and Billings showed that the output spectrum $X(j\varpi)$ of nonlinear systems can be described as [31,32]

$$X(jk\omega_F) = \sum_{n=1}^{[(N-k+1)/2]} G^H_{k+2(n-1)}(jk\omega_F)A_{k+2(n-1)}(jk\omega_F) \qquad (k = 0, 1, \cdots, n) \tag{9}$$

in which [·] means to take the integer part, and [31,32]

$$A_n(j(-n+2k)\omega_F) = \frac{1}{2^n}C_n^k|A|^n e^{j(-n+2k)\beta} \tag{10}$$

$$G^H_n(j(-n+2k)\omega_F) = H_n(\overbrace{j\omega_F, \cdots, j\omega_F}^{k}, \overbrace{-j\omega_F, \cdots, -j\omega_F}^{n-k}) \tag{11}$$

Traditionally, the transmissibility concept in linear system is normally represented as the magnitude of the ratio between the system output and input spectrum which is theoretically not correct for nonlinear system. In the present study, the Transmissibility based on the concept of NOFRFs is represented as

$$Tran(j\omega_F) = X(j\omega_F)/U(j\omega_F) = \sum_{n=1}^{N} G^H_n(j\omega_F)A_n(j\omega_F)/U(j\omega_F) \tag{12}$$

Where $U(j\varpi_F)$ is the spectrum of the harmonic input $u(t)$. In contrast to the transmissibility representation in linear system, instead of being a Fourier transform of output $u(t)$, the output spectrum $X(j\varpi_F)$ in the representation of transmissibility in the present paper is obtained through NOFRFs as described in Eq. (12). Therefore, the transmissibility representation proposed in this study is well defined in the context of nonlinear system and the results achieved based on this concept should be more accurate regarding the system frequency domain behaviour and be more informative for design.

4. The effects of NES parameters on the transmissibility

In order to analyse the vibration suppression performance of NES through transmissibility based on the concept of NOFRFs, numerical simulation studies were conducted on system (1) by applying the Runge-Kutta method to the 2-DOF structure [33,34]. The values of system parameters are taken as $m_1 = 5000$, $m_2 = 200$, $m_3 = 20$, $k_1 = 1.2 \times 10^6$, $k_2 = 2 \times 10^6$, $k_3 = 3000$, $c_1 = 3000$, $c_2 = 10$, $c_3 = 1000$.

According to reference [25–32], many researchers have verified that, in most cases, the NOFRFs up to fourth-order are sufficient to represent system output frequency responses. Besides, it can be observed in the simulation results that the magnitude of the $|X(j\omega)|$ is 10, while the magnitudes of $|X(j2\omega)|$, $|X(j3\omega)|$ and $|X(j4\omega)|$ are close to 10^{-4}. As the magnitude of additional terms in the expansion will be smaller, the more additional terms should be negligible compared to the retained terms. Therefore it can be assumed that the NOFRFs up to fourth-order are sufficient to represent the system output frequency response. According to Eqs. (9)–(11), and assume that the NOFRFs up to fourth-order are sufficient to represent the system output frequency response in this case, the frequency components of the system output can analytically be described as [31]

$$X(j\omega_F) = G^H_1(j\omega_F)A_1(j\omega_F) + G^H_3(j\omega_F)A_3(j\omega_F) \tag{13}$$

$$X(j2\omega_F) = G^H_2(j2\omega_F)A_2(j2\omega_F) + G^H_4(j2\omega_F)A_4(j2\omega_F) \tag{14}$$

$$X(j3\omega_F) = G^H_3(j3\omega_F)A_3(j3\omega_F) \tag{15}$$

$$X(j4\omega_F) = G^H_4(j4\omega_F)A_4(j4\omega_F) \tag{16}$$

Fig. 2. gives the NOFRFs of the nonlinear system (1) with a cubic stiffness.

For linear mechanical systems, it is well known that resonance occurs when the driving frequency of the force is close to the natural frequency of a vibrating system. Similarly, in the study of nonlinear systems under a harmonic input, resonance phenomenon may also occur when the driving frequency ϖ_F coincides with one of the resonant frequencies of a NOFRFs of the system. The magnitude of this NOFRFs will be significant at a high order harmonic of ϖ_F. Consequently, a significant input signal energy may be transferred through this NOFRFs from the driving frequency to the higher order harmonic component [31].

It can be seen from Fig. 2, for example, when system (1) is subjected to a harmonic excitation with driving frequency $\varpi_F = \varpi_{L1} = 2$ Hz which is the resonant frequency of $G^H_1(j\omega_F)$ and also one of the resonant frequencies of the primary linear system, a considerable input energy may be transferred through the first order NOFRFs to the first order harmonic component $\varpi_{L1} = 2$ Hz in the output according to Eq. (13). For $G^H_3(j\omega_F)$, resonant frequencies are $\varpi_{L1} = 2$ Hz and $\varpi_{L2} = 19$ Hz which

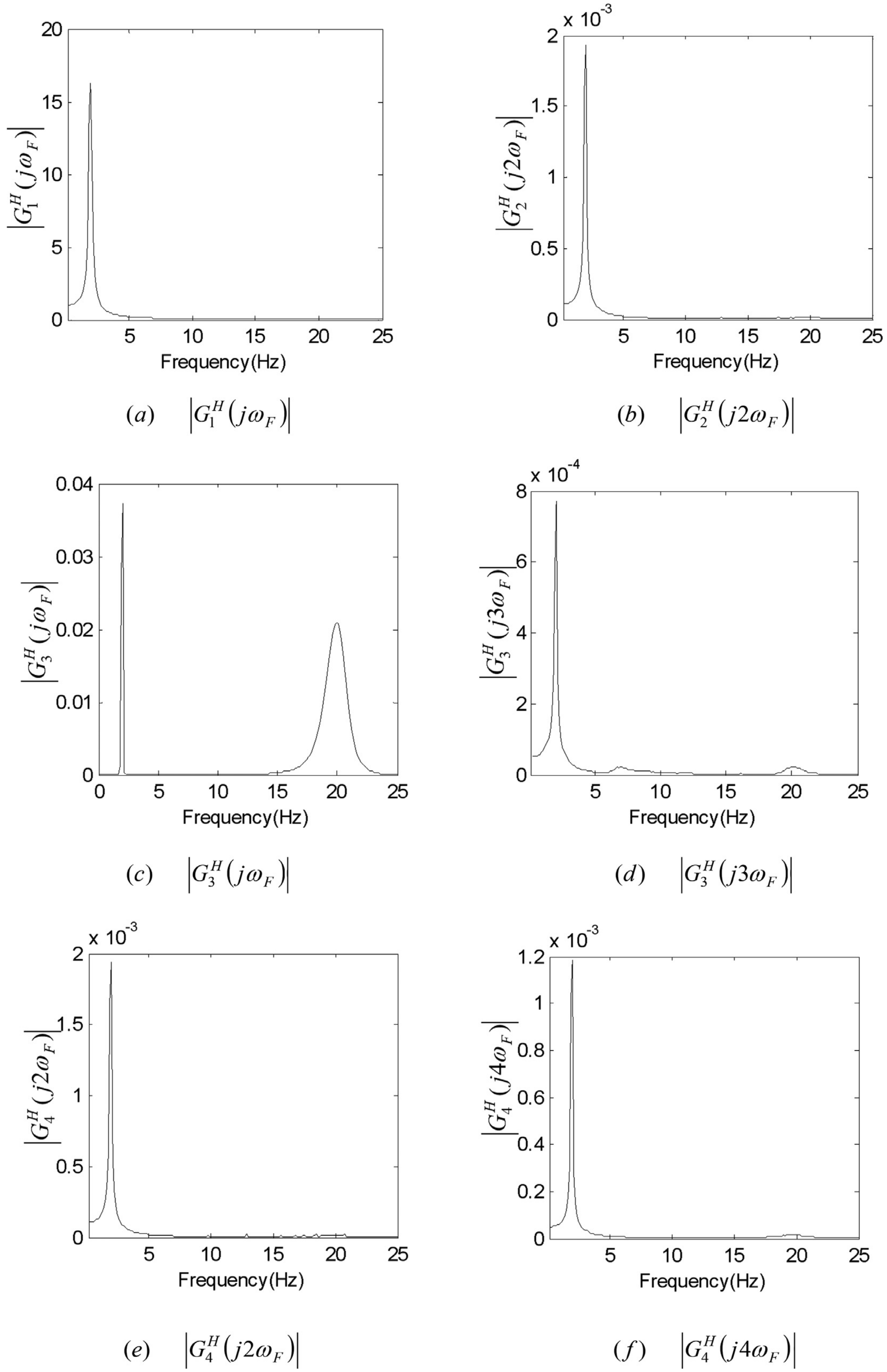

Fig. 2. The NOFRFs of the nonlinear system (1) under a harmonic loading: (a) $|G_1^H(j\omega_F)|$; (b) $|G_2^H(j2\omega_F)|$; (c) $|G_3^H(j\omega_F)|$; (d) $|G_3^H(j3\omega_F)|$; (e) $|G_4^H(j2\omega_F)|$; (f) $|G_4^H(j4\omega_F)|$.

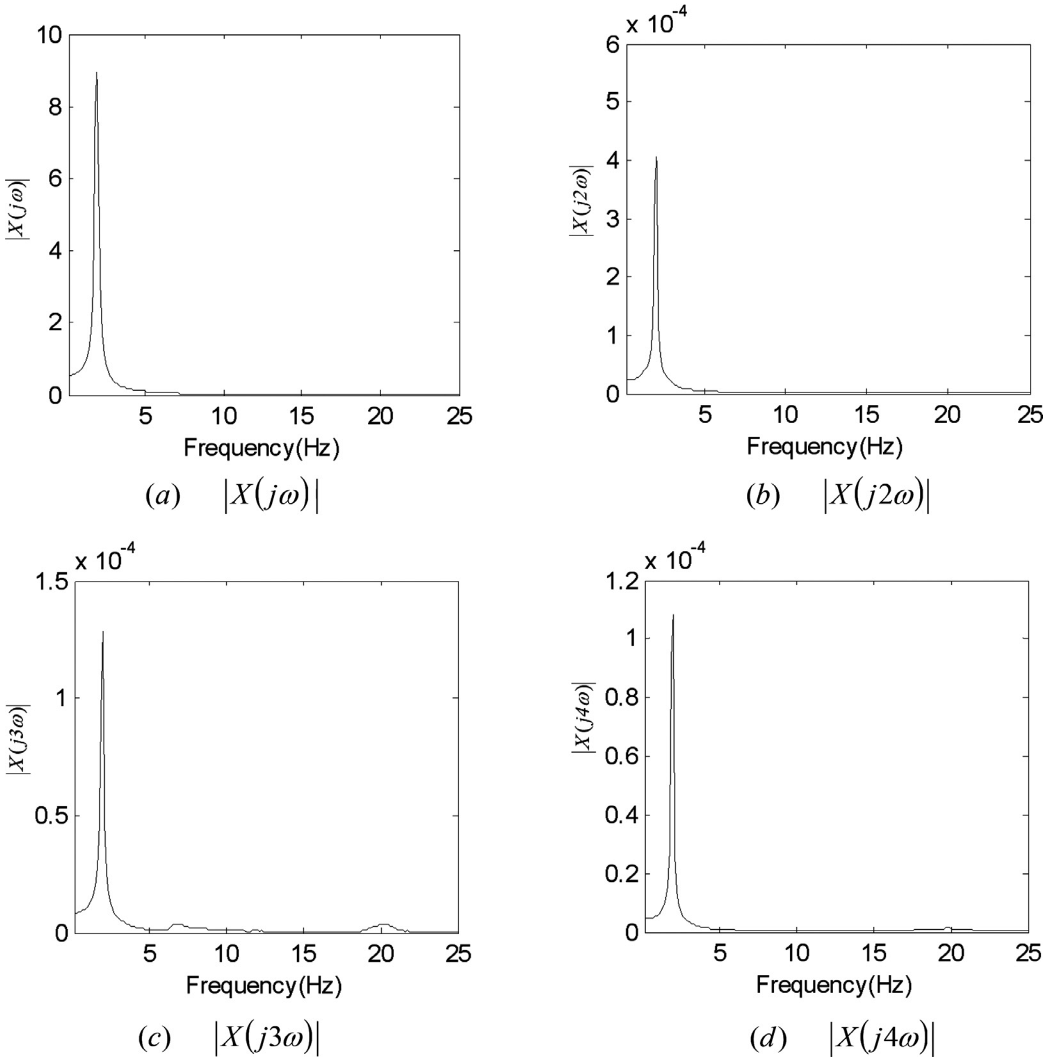

Fig. 3. The first four order output frequency responses of the nonlinear system under a harmonic loading: (a) $|X(j\omega)|$; (b) $|X(j2\omega)|$; (c) $|X(j3\omega)|$; (d) $|X(j4\omega)|$.

are also resonant frequencies of the primary linear system. This means that a proportion of energy may transfer through the 3rd order NOFRFs from the driving frequency $\omega_{L1} = 2$ Hz or $\omega_{L2} = 19$ Hz to the first order harmonic component according to Eq. (13).

It can also be observed in Fig. 2 that the other NOFRFs were all negligible compare to $G_1^H(j\omega_F)$ and $G_3^H(j\omega_F)$, this means very little energy will transfer through these NOFRFs to higher order harmonic component in the output according to Eqs. (13)–(16).

A further numerical study was conducted to obtain the first four orders output frequency responses of the nonlinear system under the same harmonic loading as shown in Fig. 3. Clearly, this result shows that higher order output frequency responses $|X(j2\omega)|$, $|X(j3\omega)|$ and $|X(j4\omega)|$ of the nonlinear system were all negligible compare to the first order output frequency response $|X(j\omega)|$. This means by introducing the NES into this linear system under harmonic loading, higher order output frequency responses will be negligible. In other words, resonant frequencies of the 2-DOF linear system can barely be shifted by introducing the NES into the linear system. This result also validated the conclusions obtained in Fig. 2.

In order to investigate the effect of viscous damping, mass and cubic nonlinear stiffness of the NES on the system transmissibility for NES design, numerical studies have been carried out and results have been obtained as shown in Fig. 4–6.

Fig. 4 shows the results of the transmissibility of system (1) with the viscous damping of the NES c_3 = 800, 1000, 1500 respectively and the transmissibility of the primary linear system. One thing should be noted that the values of the other parameters in Fig. 4 are kept the same as in Fig. 2 and Fig. 3. Clearly, this result shows that the transmissibility can be reduced over all the resonant frequency regions by increasing the viscous damping of the NES with little change of the system resonant frequencies. This analysis result suggest that increasing the viscous damping of the NES could be beneficial

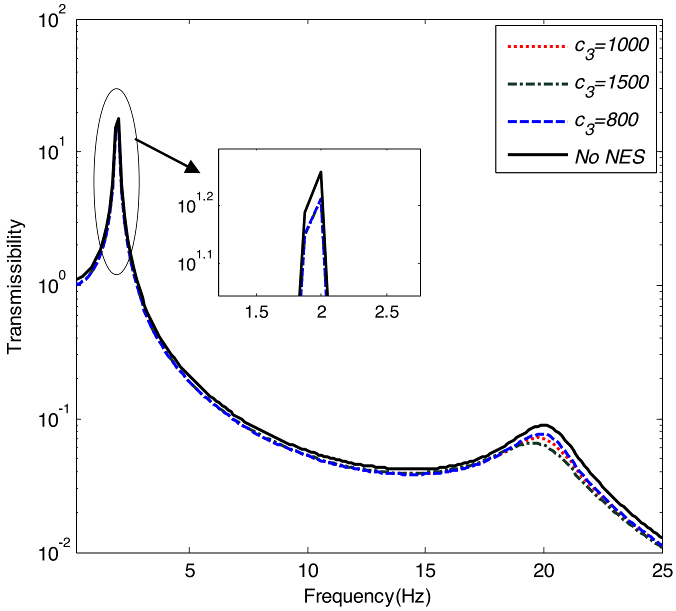

Fig. 4. The transmissibility of system (1) with $c_3 = 800$, 1000, 1500 and the transmissibility of the primary linear system.

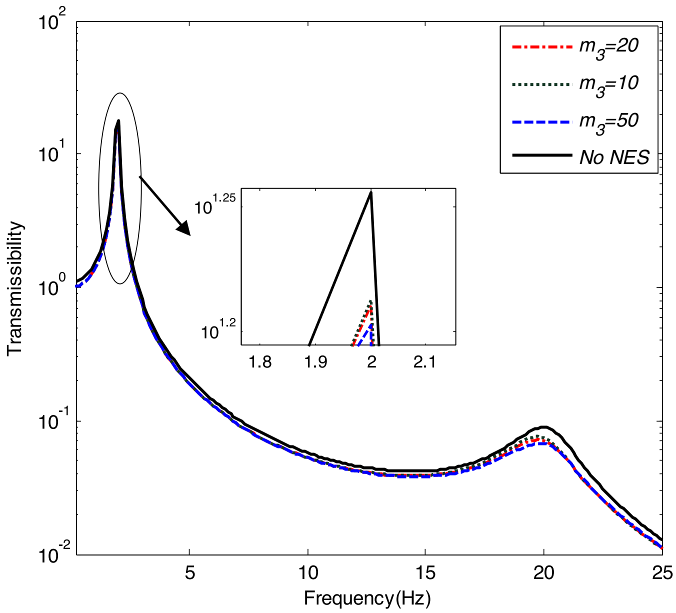

Fig. 5. The transmissibility of system (1) with $m_3 = 10$, 20, 50 and the transmissibility of the primary linear system.

in suppressing the vibration, but should not be unlimited because a too strong linear damping may cause a shift of the system frequencies as shown in Fig. 4.

Fig. 5 shows the results of the transmissibility of system (1) with the mass of the NES m_3=10, 20, 50 respectively and the transmissibility of the primary linear system. The values of the other parameters in Fig. 5 are kept the same as in Fig. 2 and Fig. 3. It can been observed from this result that the transmissibility can be reduced over all the resonant frequency regions by increasing the mass of the NES and the system resonant frequencies will be changed slightly. This means that increase of the mass of the NES can be useful in suppressing the vibration. However, a too large mass of the NES may not only cause a shift of the system frequencies as shown in Fig. 5, but is also unrealistic in engineering practices for reasons such as weight problems.

Fig. 6 shows the results of the transmissibility of system (1) with the cubic nonlinear stiffness k_3=1000, 3000, 8000 and the transmissibility of the primary linear system. The values of the other parameters in Fig. 6 are also kept the same as in

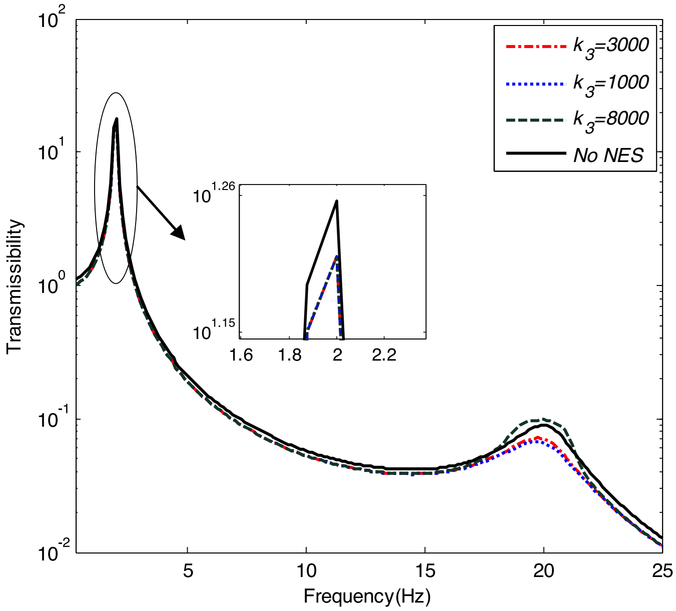

Fig. 6. The transmissibility of system (1) with $k_3 = 1000$, 3000, 8000 and the transmissibility of the primary linear system.

Fig. 2 and Fig. 3. It can be seen in this result that in order to reduce the transmissibility over all the resonant frequency regions, the cubic nonlinear stiffness of the NES should be decreased in this case.

5. Conclusions and remarks

In this study, a new representation of the transmissibility based on NOFRFs is proposed for the first time. Furthermore, the transmissibility is applied to evaluate the vibration isolation performance of a nonlinear energy sink (NES) in frequency domain. A two-degree-of-freedom (2-DOF) structure with a NES model has been built theoretically. The NOFRFs of this nonlinear system has been analysed based on the concept of resonant and resonance frequency. Furthermore, numerical simulations have been performed to evaluate the effects of the NES parameters on the transmissibility.

Numerical examples found that the first and the third order NOFRFs are much larger than the others in this case. Therefore, most of the energy will transfer through these two orders NOFRFs to higher order harmonic component in the output. Under the same harmonic loading, further studies of the first four order output frequency responses of the nonlinear system have also been obtained to validate this conclusion. These numerical results indicate that resonant frequencies of the linear system is barely shifted by introducing the NES into the linear system.

Numerical results also found that the transmissibility of the system over all resonant frequency regions is reduced by increasing the viscous damping and mass of the NES. Since a too large mass or viscous damping could shift the resonant frequencies of the linear system, this is an unlimited way. At the same time, by decreasing the cubic nonlinear stiffness of the NES, the transmissibility of the system is reduced to achieve a better vibration suppression performance over all resonant frequency regions.

Acknowledgments

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