1

a)

$$\arccos\left(\frac{\sqrt{3}}{2}\right)$$

$$\arccos x \quad (x \in [-1, 1]) = y \quad (y \in [0, \pi]) \iff \cos y = x$$

$$\arccos\left(\frac{\sqrt{3}}{2}\right) = y \in [0, \pi] \iff \cos y = \frac{\sqrt{3}}{2} \iff (y = 30°) \quad y = \frac{\pi}{6}$$

b)

$$\log_{\frac{1}{4}}\left(\frac{1}{1024}\right) = \frac{\ln\left(\frac{1}{1024}\right)}{\ln\left(\frac{1}{4}\right)} = \frac{-\ln 1024}{-\ln 4} = \frac{\ln 1024}{\ln 4} = \frac{10\ln 2}{2\ln 2} = 5$$

c)

$$e^{-2\ln 3} = e^{\ln(3^{-2})} = 3^{-2} = \frac{1}{9}$$

d)

$$8^{\log_4 9} = \left(2^3\right)^{\log_4 9} = \left(2^3\right)^{\log_2 3} = 2^{3\log_2 3} = \left(2^{\log_2 3}\right)^3 = 3^3 = 27$$

2

$$f(x) := \left(\frac{x-3}{x+1}\right)^2 \quad (x \in \mathbb{R} \setminus \{-1\})$$

1 ÉT:

$D_f = (-\infty, -1) \cup (-1, +\infty)$

$x = 0 : \left(-\frac{3}{1}\right)^2 = 9$

$y = 0 : x = 3$ A fv atmegy a $(0, 9)$ es $(3, 0)$ pontokon

2 f' elojele:

$$f'(x) = 2\left(\frac{x-3}{x+1}\right) \cdot \frac{x+1-(x-3)}{(x+1)^2} = 2\left(\frac{x-3}{x+1}\right) \cdot \frac{4}{(x+1)^2} = \frac{8(x-3)}{(x+1)^3}$$

$$f' = 0 \iff x = 3$$

$$\operatorname{sign} f'(x) = \operatorname{sign}(8(x-3)(x+1)^3)$$

- $(-\infty, -1) : +$
- $(-1, 3) : -$
- $(3, +\infty) : +$

3 f'' elojele:

nevezo': $8x - 24$

$$f''(x) = \frac{8(x+1)^3 - (8x-24)3(x+1)^2}{(x+1)^6} = \frac{8(x+1) - (8x-24)3}{(x+1)^4} = \frac{8x+8-24x+72}{(x+1)^4} = \frac{-16x+80}{(x+1)^4} = \frac{-16(x+5)}{(x+1)^4}$$

$$f'' = 0 \iff x = -5$$

$$\text{sign } f''(x) = \text{sign}\left(-16(x+5)(x+1)^4\right) = (\forall x \in \mathbb{R} \setminus \{-1\}) \text{ sign}(-16(x+5))$$

- $(-\infty, 5) \setminus \{-1\} : +$
- $(5, +\infty) : -$

4 aszimptota:

$$\lim_{x \to \pm\infty} \left(\frac{x-3}{x+1}\right)^2 = \frac{+\infty}{+\infty} = \lim_{x \to \pm\infty} \left(\frac{1}{1}\right)^2 = 1 \implies y = 1$$

$$\lim_{x \to -1} \left(\frac{x-3}{x+1}\right)^2 = \left(\frac{-4}{0}\right)^2 = +\infty$$

valami nem jo de nem fogom kideriteni

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