# Restye János Barnabás - F8U9I2 - 50 integrál

## 1

$$\int \frac{\cos^2 x - 5}{1+\cos 2x}\,\mathrm{d}x = \int \frac{\cos^2 x - 5}{2\cos^2 x}\,\mathrm{d}x = \int \frac{1}{2}\frac{\cos^2 x}{\cos^2 x} - \frac{5}{\cos^2 x}\,\mathrm{d}x = \int \frac{1}{2} - \frac{5}{2}\frac{1}{\cos^2 x}\,\mathrm{d}x = \int \frac{1}{2}\,\mathrm{d}x - \frac{5}{2}\int \frac{1}{\cos^2 x}\,\mathrm{d}x =$$

$$= \underline{\frac{1}{2}x - \frac{5}{2}\operatorname{tg} x + C}$$

## 2

$$\int \frac{8x+14}{\sqrt[4]{(2x^2+7x+8)^5}}\,\mathrm{d}x = \int \frac{8x+14}{(2x^2+7x+8)^{\frac{5}{4}}}\,\mathrm{d}x,$$

$$(2x^2+7x+8)' = 4x+7,$$

$$2\int \frac{4x+7}{(2x^2+7x+8)^{\frac{5}{4}}}\,\mathrm{d}x = 2\int (4x+7)(2x^2+7x+8)^{-\frac{5}{4}}\,\mathrm{d}x = 2\frac{(2x^2+7x+8)^{-\frac{1}{4}}}{-\frac{1}{4}} + C = \underline{-\frac{8}{\sqrt[4]{2x^2+7x+8}} + C}$$

## 3

$$\int x \cdot \ln^2 x\,\mathrm{d}x = \int \left(\frac{x^2}{2}\right)' \cdot \ln^2 x\,\mathrm{d}x = \frac{x^2}{2}\cdot \ln^2 x - \int \frac{x^2}{2}\cdot\frac{2\ln x}{x}\,\mathrm{d}x = \frac{x^2}{2}\cdot\ln^2 x - \int x\cdot \ln x\,\mathrm{d}x,$$

$$\int x\cdot \ln x\,\mathrm{d}x = \int \left(\frac{x^2}{2}\right)' \cdot \ln x\,\mathrm{d}x = \frac{x^2}{2}\cdot \ln x - \int \frac{x^2}{2}\cdot\frac{1}{x}\,\mathrm{d}x = \frac{x^2}{2}\cdot \ln x - \int \frac{x}{2}\,\mathrm{d}x = \frac{x^2}{2}\cdot\ln x - \frac{1}{2}\frac{x^2}{2} + C,$$

$$\frac{x^2}{2}\cdot \ln^2 x - \int x\cdot \ln x\,\mathrm{d}x = \frac{x^2}{2}\cdot \ln^2 x - \frac{x^2}{2}\cdot \ln x + \frac{x^2}{4} + C = \underline{\frac{x^2}{2}\left(\ln^2 x - \ln x + \frac{1}{2}\right) + C}$$

## 4

$$\int \frac{\cos 2x}{\sin x + \cos x}\,\mathrm{d}x = \int \frac{\cos^2 x - \sin^2 x}{\cos x + \sin x}\,\mathrm{d}x = \int \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x + \sin x}\,\mathrm{d}x = \int \cos x - \sin x\,\mathrm{d}x =$$

$$= \int \cos x\,\mathrm{d}x - \int \sin x\,\mathrm{d}x = \underline{\cos x + \sin x + C}$$

## 5

$$\int \frac{x^2+1}{x^4-x^2+1}\,\mathrm{d}x = \int \frac{1+\frac{1}{x^2}}{x^2-1+\frac{1}{x^2}}\,\mathrm{d}x,$$

$$x^2 - 1 + \frac{1}{x^2} = x^2 - 2 + \frac{1}{x^2} + 1 = \left(x - \frac{1}{x}\right)^2 + 1,$$

$$\int \frac{1+\frac{1}{x^2}}{x^2-1+\frac{1}{x^2}}\,\mathrm{d}x = \underline{\arctan\left(x - \frac{1}{x}\right) + C}$$

## 6

$$\int (x+e^x)^2\,\mathrm{d}x = \int x^2 + 2xe^x + e^{2x}\,\mathrm{d}x = \frac{x^3}{3} + 2xe^x - 2e^x + \frac{e^{2x}}{2} + C$$

**7**

$$\int \csc^3 x \sec x \,\mathrm{d}x = \int \frac{1}{\sin^3 x \cos x}\,\mathrm{d}x = \int \frac{\sin^2 x + \cos^2 x}{\sin^3 x \cos x}\,\mathrm{d}x = \int \frac{\sin^2 x}{\sin^3 x \cos x} + \frac{\cos^2 x}{\sin^3 x \cos x}\,\mathrm{d}x =$$

$$= \int \frac{\sin^2 x}{\sin x \cos x} + \frac{\cos^2 x}{\sin x \cos x} + \frac{\cos x}{\sin^3 x}\,\mathrm{d}x = \int \frac{\sin x}{\cos x}\,\mathrm{d}x + \int \frac{\cos x}{\sin x}\,\mathrm{d}x + \int \frac{\cos x}{\sin^3 x}\,\mathrm{d}x = \ln|\sec x| + \ln|\sin x| - \frac{1}{2\sin^2 x} + C =$$

$$= \underline{\ln|\tan x| - \frac{1}{2}\csc^2 x + C}$$

**8**

$$\int \frac{\cos x}{\sin^2 x - 5\sin x - 6}\,\mathrm{d}x \underset{\substack{y=\sin x \\ \mathrm{d}y=\cos x\,\mathrm{d}x}}{=} \int \frac{1}{y^2 - 5y - 6}\,\mathrm{d}y,$$

$$D = 25 + 24 > 0, \quad y^2 - 5y - 6 = (y-6)(y+1),$$

$$\int \frac{\frac{1}{7}}{y-6} + \frac{-\frac{1}{7}}{y+1}\,\mathrm{d}y = \frac{1}{7}\int (y-6)^{-1}\,\mathrm{d}y - \frac{1}{7}\int (y+1)^{-1}\,\mathrm{d}y = \frac{1}{7}\ln|y-6| - \frac{1}{7}\ln|y+1| = \underline{\frac{1}{7}\left(\ln\left|\frac{\sin x - 6}{\sin x + 1}\right|\right) + C}$$

**9**

$$\int \frac{1}{\sqrt{e^x}}\,\mathrm{d}x = \int e^{-\frac{x}{2}}\,\mathrm{d}x = -2e^{-\frac{x}{2}} = \underline{\frac{-2}{\sqrt{e^x}} + C}$$

**10**

$$\int \frac{1}{x + \sqrt{x}}\,\mathrm{d}x = \int \frac{1}{\sqrt{x}(\sqrt{x}+1)}\,\mathrm{d}x,$$

$$\frac{A}{\sqrt{x}} + \frac{B}{\sqrt{x}+1} = 1 \Longrightarrow A\big(\sqrt{x}+1\big) + B\big(\sqrt{x}\big) = 1 \Longrightarrow (A+B)\sqrt{x} + A = 1 \Longrightarrow A = 1,\ B = -1,$$

$$\int \frac{1}{\sqrt{x}} - \frac{1}{\sqrt{x}+1}\,\mathrm{d}x = \int \frac{1}{\sqrt{x}}\,\mathrm{d}x - \int \frac{1}{\sqrt{x}+1}\,\mathrm{d}x = \underline{2\ln\big|\sqrt{x}+1\big| + C}$$

**11**

$$\int \frac{2\sin x}{\sin 2x}\,\mathrm{d}x = \int \frac{2\sin x}{2\sin x\cos x}\,\mathrm{d}x = \int \frac{1}{\cos x}\,\mathrm{d}x = \int \sec x\,\mathrm{d}x = \underline{\ln|\sec x + \tan x| + C}$$

**12**

$$\int \cos^2 2x\,\mathrm{d}x = \int \frac{1+\cos 4x}{2}\,\mathrm{d}x = \frac{1}{2}\int 1 + \cos 4x\,\mathrm{d}x = \frac{1}{2}\left(x + \frac{1}{4}\sin(4x)\right) = \underline{\frac{x}{2} + \frac{\sin 4x}{8} + C}$$

**13**

$$\int \frac{1}{x^3+1}\,\mathrm{d}x = \int \frac{1}{(x+1)(x^2-x+1)}\,\mathrm{d}x,$$

$$\frac{A}{x+1} + \frac{Bx+C}{x^2-x+1} = 1 \Longrightarrow A(x^2-x+1) + (Bx+C)(x+1) \Longrightarrow Ax^2 - Ax + A + Bx^2 + Bx + Cx + C =$$

$$= (A+B)x^2 + (B+C-A)x + (A+C) = 1 \Longrightarrow A = \frac{1}{3},\ C = \frac{2}{3},\ B = -\frac{1}{3},$$

$$\int \frac{\frac{1}{3}}{x+1} + \frac{-\frac{1}{3}x + \frac{2}{3}}{x^2-x+1}\,\mathrm{d}x = \frac{1}{3}\int \frac{1}{x+1}\,\mathrm{d}x - \frac{1}{3}\int \frac{x-2}{x^2-x+1}\,\mathrm{d}x,$$

$$\left(x^2-x+1\right)' = 2x-1,$$

$$\frac{1}{3}\int \frac{1}{x+1}\,\mathrm{d}x - \frac{1}{3}\frac{1}{2}\int \frac{2(x-2)-3}{x^2-x+1}\,\mathrm{d}x = \frac{1}{3}\int \frac{1}{x+1}\,\mathrm{d}x - \frac{1}{6}\left[\int \frac{2x-1}{x^2-x+1}\,\mathrm{d}x - \int \frac{3}{(x-\frac{1}{2})^2 + \left(\frac{\sqrt{3}}{2}\right)^2}\,\mathrm{d}x\right] =$$

$$= \frac{1}{3}\ln|x+1| - \frac{1}{6}\ln(x^2-x+1) + \frac{1}{2}\cdot\frac{2}{\sqrt{3}}\arctan\left(\frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) = \underline{\frac{1}{3}\ln|x+1| - \frac{1}{6}\ln(x^2-x+1) + \frac{1}{\sqrt{3}}\arctan\left(\frac{2}{\sqrt{3}}x - \frac{1}{\sqrt{3}}\right) + C}$$

**14**

$$\int x\sin^2 x\,\mathrm{d}x = \int x + \frac{1-\cos(2x)}{2}\,\mathrm{d}x = \frac{1}{2}\int x(1-\cos 2x)\,\mathrm{d}x = \frac{1}{2}\left[\int x\,\mathrm{d}x + \int -x\cos 2x\,\mathrm{d}x\right] =$$

$$= \frac{1}{2}\left[\frac{1}{2}x^2 - \frac{1}{2}x\sin 2x - \frac{1}{4}\cos 2x\right] = \underline{\frac{1}{4}x^2 - \frac{1}{4}x\sin 2x - \frac{1}{8}\cos 2x + C}$$

**15**

$$\int \left(x + \frac{1}{x}\right)^2\mathrm{d}x = \int x^2 + 2 + \frac{1}{x^2}\,\mathrm{d}x = \underline{\frac{x^3}{3} + 2x - \frac{1}{x} + C}$$

**16**

$$\int \frac{3}{x^2+4x+29}\,\mathrm{d}x = \int \frac{3}{x^2+4x+4+25}\,\mathrm{d}x = \int \frac{3}{(x+2)^2+5^2}\,\mathrm{d}x = 3\int \frac{1}{(x+2)^2+5^2}\,\mathrm{d}x = \underline{\frac{3}{5}\arctan\left(\frac{x+2}{5}\right) + C}$$

**17**

$$\int \sin^3 x\cos^2 x\,\mathrm{d}x = \int \sin x\sin^2 x\cos^2 x\,\mathrm{d}x = \int \underbrace{\sin x}_{-(\cos x)'}\left(1-\cos^2 x\right)\cos^2 x\,\mathrm{d}x =$$

$$= -\int \cos^2 x(\cos x)'\,\mathrm{d}x + \int \cos^4 x(\cos x)'\,\mathrm{d}x = \underline{\frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} + C}$$

**18**

$$\int \sin x\sec x\tan x\,\mathrm{d}x = \int \sin x\frac{1}{\cos x}\tan x\,\mathrm{d}x = \int \tan^2 x\,\mathrm{d}x = \int \left(\sec^2 - 1\right)\mathrm{d}x = \underline{\tan x - x + C}$$

**19**

$$\int e^{2x}\cos x\,\mathrm{d}x = e^{2x}$$

| | D | I |
|---|---|---|
| + | $e^{2x}$ | $\cos x$ |
| - | $2e^{2x}$ | $\sin x$ |
| + | $4e^{2x}$ | $-\cos x$ |

$$\int e^{2x}\cos x\,\mathrm{d}x = e^{2x}\sin x + 2e^{2x}\cos x - 4\int e^{2x}\cos x\,\mathrm{d}x,$$

$$\int e^{2x}\cos x\,\mathrm{d}x = \underline{\frac{1}{5}e^{2x}\sin x + \frac{2}{5}e^{2x}\cos x + C}$$

**20**

$$\int \frac{\ln x}{\sqrt{x}}\,\mathrm{d}x,$$

| | D | I |
|---|---|---|
| + | $\ln x$ | $\frac{1}{\sqrt{x}}$ |
| - | $\frac{1}{x}$ | $2\sqrt{x}$ |

$$\int \frac{\ln x}{\sqrt{x}}\,\mathrm{d}x = 2\sqrt{x}\ln x - 2\underbrace{\int \frac{1}{\sqrt{x}}}_{2\sqrt{x}}\,\mathrm{d}x = \underline{2\sqrt{x}\ln x - 4\sqrt{x} + C}$$

**21**

$$\int \frac{1}{e^{x}+e^{-x}}\,\mathrm{d}x = \int \frac{e^{x}}{e^{2x}+1}\,\mathrm{d}x = \underline{\arctan(e^{x}) + C}$$

**22**

$$\int \log_2 x\,\mathrm{d}x = \int \frac{\ln x}{\ln 2}\,\mathrm{d}x = \frac{1}{\ln 2}\int \ln x\,\mathrm{d}x,$$

| | D | I |
|---|---|---|
| + | $\ln x$ | 1 |
| - | $\frac{1}{x}$ | $x$ |

$$= \frac{1}{\ln x}(x\ln x - x) = \frac{x\ln x}{\ln 2} - \frac{x}{\ln 2} = \underline{x\log_2(x) - \frac{x}{\ln x} + C}$$

**23**

$$\int x^{3}\sin 2x\,\mathrm{d}x,$$

| | D | I |
|---|---|---|
| + | $x^3$ | $\sin 2x$ |
| - | $3x^2$ | $-\frac{1}{2}\cos 2x$ |
| + | $6x$ | $-\frac{1}{4}\sin 2x$ |
| - | 6 | $\frac{1}{8}\cos 2x$ |
| + | 0 | $\frac{1}{16}\sin 2x$ |

$$= \underline{-\frac{1}{2}\int x^{3}\cos 2x\,\mathrm{d}x + \frac{3}{4}x^{2}\sin 2x + \frac{3}{4}x\cos 2x - \frac{3}{8}\sin 2x + C}$$

**24**

$$\int \sinh x \, \mathrm{d}x = \underline{\cosh x + C}$$

**25**

$$\int \sinh^2 x \, \mathrm{d}x,$$

$$\sinh^2 x = \left(\frac{e^x - e^{-x}}{2}\right)^2 = \frac{1}{4}(e^x - 2 - e^{-2x}) = \frac{1}{4}(-2) + \frac{1}{2 \cdot 2}(e^{2x} + e^{-2x}) = -\frac{1}{2} + \cosh(2x),$$

$$\frac{1}{2}\int -1 + \cosh 2x \, \mathrm{d}x = \frac{1}{2}\left(-x + \frac{1}{2}\sinh 2x\right) = \underline{-\frac{1}{2}x + \frac{1}{4}\sinh 2x + C}$$

**26**

$$\int \sinh^3 x \, \mathrm{d}x = \int \sinh^2 x \sinh x \, \mathrm{d}x = \int (\cosh^2 x - 1)\sinh x \, \mathrm{d}x = \int \cosh^2 x \sinh x \, \mathrm{d}x - \int \sinh x \, \mathrm{d}x =$$

$$= \underline{\frac{\cosh^3 x}{3} - \cosh x + C}$$

**27**

$$\int \frac{1}{\sqrt{x^2+1}} \, \mathrm{d}x = \int \frac{1}{\sqrt{1+x^2}} \, \mathrm{d}x = \underline{\sinh^{-1} + C}$$

**28**

$$\int \ln\left(x + \sqrt{1+x^2}\right) \mathrm{d}x = \int \sinh^{-1} \mathrm{d}x,$$

| | D | I |
|---|---|---|
| + | $\sinh^{-1}$ | 1 |
| - | $\frac{1}{(\sqrt{1+x^2}}$ | $x$ |

$$= \underline{x \sinh^{-1} x - \sqrt{1+x^2} + C}$$

**29**

$$\tanh x \, \mathrm{d}x = \int \frac{\sinh x}{\cosh x} \, \mathrm{d}x = \underline{\ln|\cosh x| + C}$$

**30**

$$\int \ln(1+x^2) \, \mathrm{d}x,$$

| | D | I |
|---|---|---|
| + | $\ln(1+x^2)$ | 1 |
| - | $\frac{2x}{1+x^2}$ | $x$ |

$$= x\ln(1+x^2) - 2\int \frac{1+x^2-1}{1+x^2} \, \mathrm{d}x = \underline{x\ln(1+x^2) - 2x + 2\arctan(x) + C}$$

**31**

$$\int \frac{1}{x^4+x} \, \mathrm{d}x = \int \frac{1}{x^4(1+x^{-3})} \, \mathrm{d}x = \int \frac{x^{-4}}{1+x^{-3}} \, \mathrm{d}x = \underline{-\frac{1}{3}\ln\left|1+x^{-3}\right| + C}$$

## 32

$$\int \frac{1-\tan x}{1+\tan x}\,\mathrm{d}x = \int \frac{1-\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}}\,\mathrm{d}x = \int \frac{\cos x - \sin x}{\cos x + \sin x}\,\mathrm{d}x = \underline{\ln|\cos x + \sin x| + C}$$

## 33

$$\int x \sec x \tan x \,\mathrm{d}x,$$

| | D | I |
|---|---|---|
| + | $x$ | $\sec x \tan x$ |
| - | 1 | $\sec x$ |
| + | 0 | $\ln\lvert\sec x \tan x\rvert$ |

$$= \underline{x \sec x - \ln|\sec x \tan x| + C}$$

## 34

$$\int x^3 e^{x^2}\,\mathrm{d}x,$$

| | D | I |
|---|---|---|
| + | $x^2$ | $xe^{x^2}$ |
| - | $2x$ | $\frac{1}{2}e^{x^2}$ |

$$= x^2\frac{1}{2}e^{x^2} - \int 2x\frac{1}{2}e^{x^2}\,\mathrm{d}x = \frac{1}{2}x^2e^{x^2} - \int xe^{x^2}\,\mathrm{d}x = \underline{\frac{1}{2}x^2e^{x^2} - \frac{1}{2}e^{x^2} + C}$$

## 35

$$\int 2^{\ln x}\,\mathrm{d}x = \int e^{(\ln 2)^{\ln x}}\,\mathrm{d}x = \int e^{(\ln x)^{\ln 2}}\,\mathrm{d}x = \int x^{\ln 2}\,\mathrm{d}x = \int \frac{1}{1+\ln 2}x^{(\ln 2)+1}\,\mathrm{d}x = \int \frac{1}{1+\ln 2}x' x^{\ln 2}\,\mathrm{d}x = \underline{\frac{x2^{\ln x}}{1+\ln 2} + C}$$

## 36

$$\int \sqrt{1+\cos 2x}\,\mathrm{d}x = \int \sqrt{2\cos^2 x}\,\mathrm{d}x = \sqrt{2}\int \cos \mathrm{d}x = \underline{\sqrt{2}\sin x + C}$$

## 37

$$\int \frac{1}{1+\tan x}\,\mathrm{d}x = \frac{1}{2}\int \frac{1-\tan x + 1 + \tan x}{1+\tan x}\,\mathrm{d}x = \frac{1}{2}\left(\int \frac{1-\tan x}{1+\tan x}\,\mathrm{d}x + \frac{1+\tan x}{1+\tan x}\,\mathrm{d}x\right) = \underline{\frac{1}{2}\ln|\cos x + \sin x| + \frac{1}{2}x + C}$$

## 38

$$\int (\sin x + \cos x)^2\,\mathrm{d}x = \int (\sin^2 x + 2\sin x\cos x + \cos^2 x)\,\mathrm{d}x = \int (1+\sin 2x)\,\mathrm{d}x = \underline{x - \frac{1}{2}\cos 2x + C}$$

## 39

$$\int \sqrt{\frac{1-x}{1+x}}\,\mathrm{d}x = \int \sqrt{\frac{(1-x)(1-x)}{(1+x)(1-x)}}\,\mathrm{d}x = \int \frac{1-x}{\sqrt{1+x^2}}\,\mathrm{d}x = \int \frac{1}{\sqrt{1+x^2}}\,\mathrm{d}x - \int \frac{x}{\sqrt{1+x^2}}\,\mathrm{d}x =$$

$$= \underline{\arcsin x + \sqrt{1-x^2} + C}$$

**40**

$$\int x^{\frac{x}{\ln x}}\,\mathrm{d}x,$$

$$\int e^{(\ln x)\frac{x}{\ln x}}\,\mathrm{d}x = \int e^{x}\,\mathrm{d}x = \underline{e^{x} + C},$$

$$\int x^{\frac{x}{\ln x}}\,\mathrm{d}x = \underline{x^{\frac{x}{\ln x}} + C}$$

**41**

$$\int \arctan x\,\mathrm{d}x,$$

| | D | I |
|---|---|---|
| + | $\arctan x$ | 1 |
| - | $\frac{1}{1+x^2}$ | $x$ |

$$= \underline{x\arctan x - \frac{1}{2}\ln\left(1+x^2\right) + C}$$

**42**

$$\int \frac{\sin\frac{1}{x}}{x^3}\,\mathrm{d}x,$$

| | D | I |
|---|---|---|
| + | $\frac{1}{x}$ | $\frac{\sin\frac{1}{x}}{x^2}$ |
| - | $-\frac{1}{x^2}$ | $\cos\frac{1}{x}$ |

$$= \frac{\cos\frac{1}{x}}{x} - \int -\frac{1}{x^2}\cos\frac{1}{x} = \underline{\frac{\cos\frac{1}{x}}{x} - \sin\frac{1}{x} + C}$$

**43**

$$\int \frac{x-1}{x^4-1}\,\mathrm{d}x = \int \frac{x-1}{(x-1)(x+1)(x^2+1)}\,\mathrm{d}x = \int \frac{1}{(x+1)(x^2+1)}\,\mathrm{d}x,$$

$$\frac{A}{x+1} + \frac{Bx+C}{x^2+1} = 1 \Longrightarrow A(x^2+1) + (Bx+C)(x+1) = 1 \Longrightarrow Ax^2 + A + Bx^2 + Bx + Cx + C = 1 \Longrightarrow$$

$$\Longrightarrow (A+B)x^2 + (B+C)x + (A+C) = 1 \Longrightarrow A = \frac{1}{2},\ B = -\frac{1}{2},\ C = \frac{1}{2},$$

$$\int \frac{\frac{1}{2}}{x+1}\,\mathrm{d}x - \frac{1}{2}\int \frac{x}{x^2+1}\,\mathrm{d}x + \frac{1}{2}\int \frac{1}{x^2+1}\,\mathrm{d}x = \underline{\frac{1}{2}\ln|x+1| - \frac{1}{4}\ln(x^2+1) + \frac{1}{2}\arctan x + C}$$

**44**

$$\int \sqrt{1 + \left(x - \frac{1}{4x}\right)^2}\,\mathrm{d}x = \int \sqrt{1 + x^2 - \frac{1}{2} + \frac{1}{16x^2}}\,\mathrm{d}x = \int \sqrt{x^2 + \frac{1}{2} + \frac{1}{16x^2}}\,\mathrm{d}x = \int \sqrt{\left(x + \frac{1}{4x}\right)^2}\,\mathrm{d}x = \int x + \frac{1}{4x}\,\mathrm{d}x =$$

$$= \underline{\frac{1}{2}x^2 + \frac{1}{4}\ln|x| + C}$$

**45**

$$\int \frac{e^{\tan x}}{1-\sin^2 x}\,\mathrm{d}x = \int \frac{e^{\tan x}}{\cos^2 x}\,\mathrm{d}x = \int e^{\tan x}\sec^2 x\,\mathrm{d}x = \underline{e^{\tan x} + C}$$

**46**

$$\int \frac{\arctan x}{x^2}\,\mathrm{d}x,$$

| | D | I |
|---|---|---|
| + | $\arctan x$ | $\frac{1}{x^2}$ |
| - | $\frac{1}{1+x^2}$ | $-\frac{1}{x}$ |

$$= \underline{-\frac{\arctan x}{x} - \frac{1}{2}\ln\left(x^{-2}+1\right) + C}$$

**47**

$$\int \frac{\arctan x}{1+x^2}\,\mathrm{d}x = \underline{\frac{1}{2}\arctan^2 x + C}$$

**48**

$$\int \ln^2 x\,\mathrm{d}x,$$

| | D | I |
|---|---|---|
| + | $\ln^2 x$ | 1 |
| - | $\frac{2\ln x}{x}$ | $x$ |

$$= x\ln^2 x - \int 2\ln x\,\mathrm{d}x,$$

| | D | I |
|---|---|---|
| + | $2\ln x$ | 1 |
| - | $\frac{2}{x}$ | $x$ |

$$= x2\ln x - \int 2\,\mathrm{d}x = x2\ln x - 2x,$$

$$\int \ln^2 x\,\mathrm{d}x = \underline{x\ln^2 x - x2\ln x + 2x + C}$$

**49**

$$\int \frac{x}{1+x^4}\,\mathrm{d}x = \int \frac{x}{1+\left(x^2\right)^2}\,\mathrm{d}x = \frac{1}{2}\int \frac{2x}{1+\left(x^2\right)^2}\,\mathrm{d}x = \underline{\frac{1}{2}\arctan x^2 + C}$$

**50**

$$\int \sqrt{1+\sin 2x}\,\mathrm{d}x = \int \sqrt{\sin^2 x + \cos^2 x + 2\sin x\cos x}\,\mathrm{d}x = \int \sqrt{(\sin x + \cos x)^2}\,\mathrm{d}x = \int \sin x\,\mathrm{d}x + \int \cos x\,\mathrm{d}x =$$

$$= \underline{\sin x - \cos x + C}$$