1.

$$\begin{pmatrix} 4 & 2 & 6 & -2 \\ 2 & 10 & 9 & 5 \\ 6 & 9 & 14 & 2 \\ -2 & 5 & 2 & 7 \end{pmatrix} \to \begin{pmatrix} 4 & 2 & 6 & -2 \\ 0 & 9 & 6 & 6 \\ 0 & 6 & 5 & 5 \\ 0 & 6 & 5 & 6 \end{pmatrix} \to \begin{pmatrix} 4 & 2 & 6 & -2 \\ 0 & 9 & 6 & 6 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix} \to \begin{pmatrix} 4 & 2 & 6 & -2 \\ 0 & 9 & 6 & 6 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} = U$$

$$L_1^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ \frac{3}{2} & 0 & 1 & 0 \\ -\frac{1}{2} & 0 & 0 & 1 \end{pmatrix},\ L_2^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & \frac{2}{3} & 1 & 0 \\ 0 & \frac{2}{3} & 0 & 1 \end{pmatrix},\ L_3^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

$$L = L_1^{-1} L_2^{-1} L_3^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ \frac{3}{2} & \frac{2}{3} & 1 & 0 \\ -\frac{1}{2} & \frac{2}{3} & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 2 & 6 & -2 \\ 2 & 10 & 9 & 5 \\ 6 & 9 & 14 & 2 \\ -2 & 5 & 2 & 7 \end{pmatrix} \to \begin{pmatrix} 4 & 2 & 6 & -2 \\ \frac{1}{2} & 9 & 6 & 6 \\ \frac{3}{2} & 6 & 5 & 5 \\ -\frac{1}{2} & 6 & 5 & 6 \end{pmatrix} \to \begin{pmatrix} 4 & 2 & 6 & -2 \\ \frac{1}{2} & 9 & 6 & 6 \\ \frac{3}{2} & \frac{2}{3} & 1 & 1 \\ -\frac{1}{2} & \frac{2}{3} & 1 & 2 \end{pmatrix} \to \begin{pmatrix} 4 & 2 & 6 & -2 \\ \frac{1}{2} & 9 & 6 & 6 \\ \frac{3}{2} & \frac{2}{3} & 1 & 1 \\ -\frac{1}{2} & \frac{2}{3} & 1 & 1 \end{pmatrix} = U$$

$$\det(A) = 4 \cdot 9 \cdot 1 \cdot 1 = 36$$

1/c.

$$L \cdot U = A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ l_1 & 1 & 0 & 0 \\ l_2 & l_3 & 1 & 0 \\ l_4 & l_5 & l_5 & 1 \end{pmatrix} \cdot \begin{pmatrix} 4 & 2 & 6 & -2 \\ 0 & u_1 & u_2 & u_3 \\ 0 & 0 & u_4 & u_5 \\ 0 & 0 & 0 & u_6 \end{pmatrix} = \begin{pmatrix} 4 & 2 & 6 & -2 \\ 2 & 10 & 9 & 5 \\ 6 & 9 & 14 & 2 \\ -2 & 5 & 2 & 7 \end{pmatrix}$$

sorfolytonsoan haladok vegig A elemein

$$2 = l_1 \cdot 4 \to l_1 = \frac{1}{2}$$

$$10 = l_1 \cdot 2 + 1 \cdot u_1 = 1 + u_1 \to u_1 = 9$$

$$9 = l_1 \cdot 6 + 1 \cdot u_1 = \frac{1}{2} \cdot 6 + u_2 = 3 + u_2 \to u_2 = 6$$

$$5 = l_1 \cdot (-2) + 1 \cdot u_3 = \frac{1}{2}(-2) + u_3 = -1 + u_3 \to u_3 = 6$$

$$6 = l_2 \cdot 4 \to l_2 = \frac{3}{2}$$

$$9 = l_2 \cdot 2 + l_3 \cdot u_1 = 3 + l_3 \cdot 9 \to l_3 \cdot 9 = 6 \to l_3 = \frac{2}{3}$$

nem csinaljuk tovabb

2/a.

tarolos ge-val

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 3 & 0 \\ 2 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & -1 \\ 1 & 1 & 1 \\ 2 & -4 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & -1 \\ 1 & 1 & 1 \\ 2 & -4 & 6 \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & -4 & 1 \end{pmatrix} \quad U = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 6 \end{pmatrix}$$

2/b.

$$B = \begin{pmatrix} 2 & 4 & 6 \\ 4 & 8 & 2 \\ 8 & 12 & 14 \end{pmatrix} \to \begin{pmatrix} 2 & 4 & 6 \\ 2 & 0 & -10 \\ 4 & -4 & -10 \end{pmatrix} \to \text{sorcsere nelkul a ge nem folytathato. } \det(D_2) = 0$$

ideaig jutottunk volna:

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 4 & 6 \\ 0 & 0 & -10 \\ 0 & 0 & \end{pmatrix} = \begin{pmatrix} 2 & 4 & 6 \\ 4 & 8 & 2 \\ 8 & 12 & \end{pmatrix}$$

na de itt a baj mert

$$12 = 4 \cdot 4 + l \cdot 0 \text{ baj}$$

de

$$16 = 4 \cdot 4 + l \cdot 0 \quad \text{tehat} \quad l \in \mathbb{R} \text{ tetszoleges } \infty \text{ sok LU felbontas}$$

2/c.

$$A = LDU$$

man i ain doin allat