

Szótárak:  $\text{init}(t)$  (irescre  $\text{in} \cdot \text{it}$ )

Bin.

K fák

|  
|

Voltak

— →

$Q_0 Q_0 Q_0$

$\text{ins}(t, k, \dots)$

$\text{hossz}(t, k) := \dots$

$\text{del}(t, k)$

$\text{rem}(t, k) := \dots$

$\text{min}(t) := \dots$

$\text{max}(t) := \dots$

$\text{remMin}(t) := \dots$

$\text{remMax}(t) := \dots$

HASH TABLE

AVL fákban.  
 $O(\log t)$

időben

max.  
 $O(\log t)$   
időben

rossz  
csakben

D AVL fa : kiegészítő hozott BST  
(balanced)

(magasság szerint)

Szept. 12.

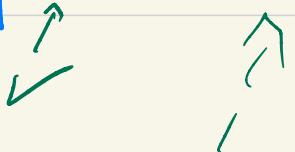
D<sub>1</sub> ( $\neq p$ ) csin legyen művek:  $p \rightarrow b \stackrel{\text{def}}{=} \begin{cases} h(p \rightarrow \text{right}) \\ -h(p \rightarrow \text{left}) \end{cases}$

$\neq p$  kiegészítő hozott  $\stackrel{\text{def}}{\Rightarrow} p \rightarrow b \in \{-1, 0, 1\}$

t bin. fa hozzájárható ( $\stackrel{\text{def}}{\Rightarrow}$  minden bármely

T<sub>1</sub> (belsőszintűkkel bin. fa magassága)  
t hossz - bin. fa;  $w := w(t)$ ;  $h = h(t)$

$$\Rightarrow \lfloor \log_2 w \rfloor \leq h \leq 1.55 \log_2 w$$



Biz. alapgrundolásra:

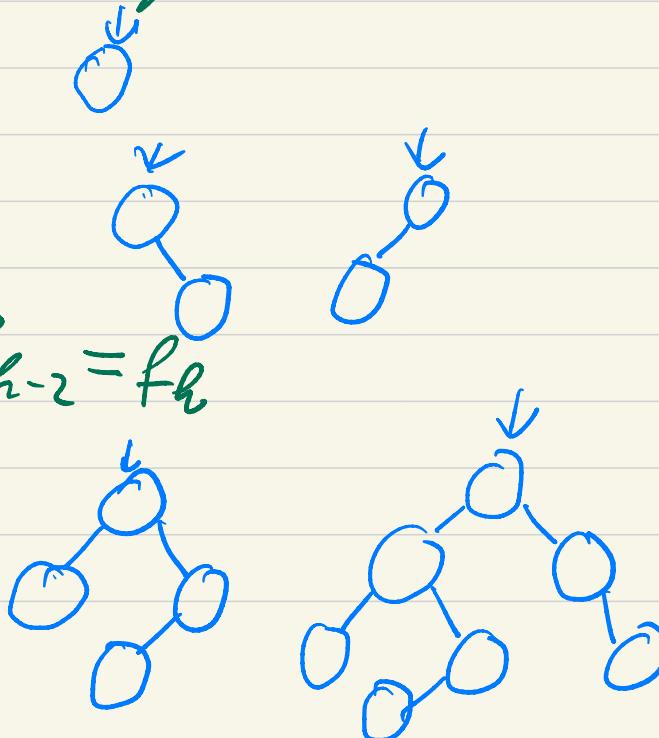
F  
I  
B F  
D A  
W K  
A C  
C C  
I f<sub>0</sub> = 1  
f<sub>1</sub> = 2

$f_h := h$  magassághoz tartozó legkisebb  
belegyelhető bin. fa méret.

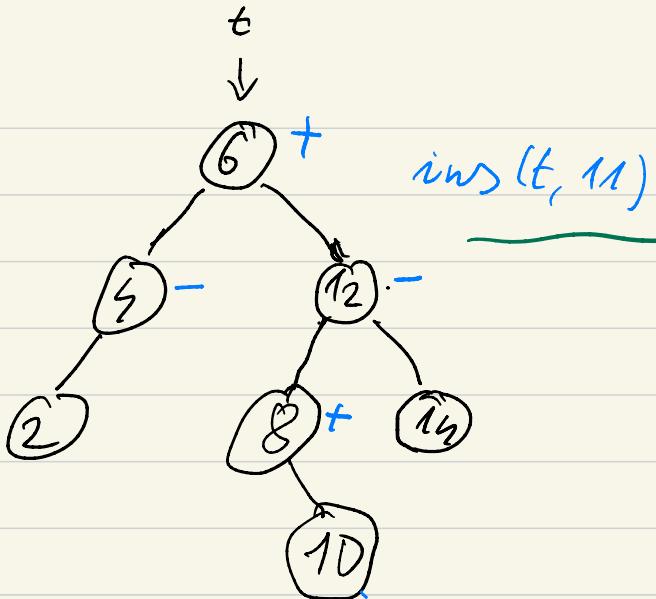
$$h \geq 2 \Rightarrow 1 + f_{h-1} + f_{h-2} = f_h$$

Tetsz-h mag. báligy.

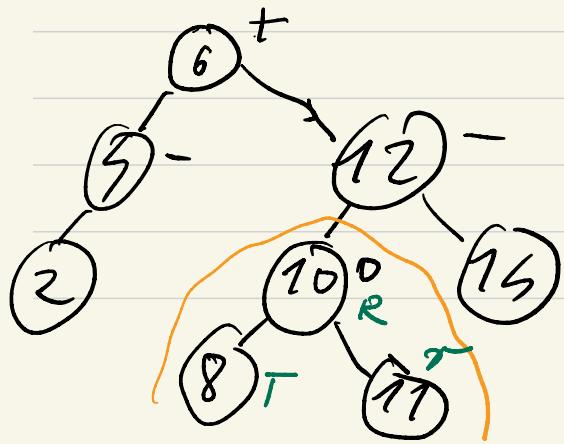
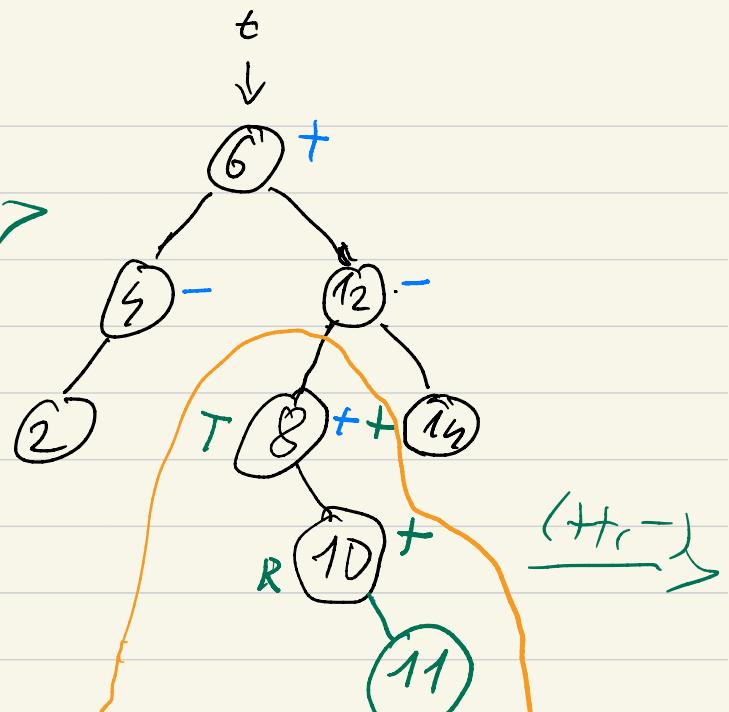
Bin. fára:  $n \geq f_h$



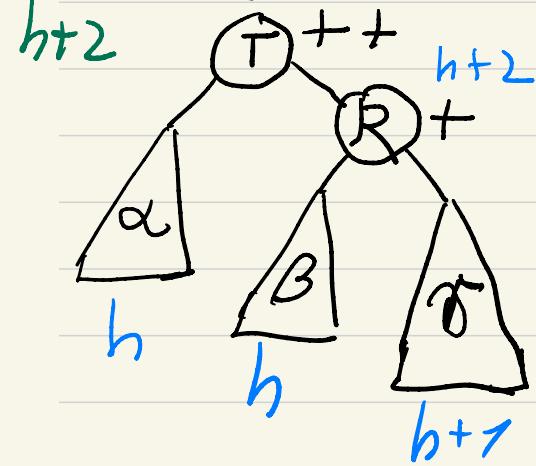
Szótár  
(AVL fa)  
műveletek:  
t := Ø  
- ins(t, k)  
- search(t, k): B  
min(t): NodeA  
max(t) -/-  
- remMin(t) -/-  
- remMax() -/-  
- del(t, k)



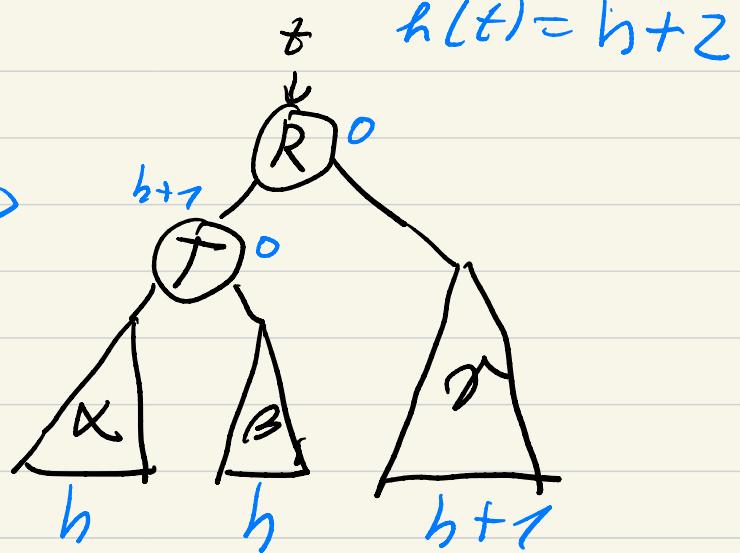
$\text{ins}(t, 11)$



$$\text{Münz. } \leftarrow h \\ h(t) = h+3$$

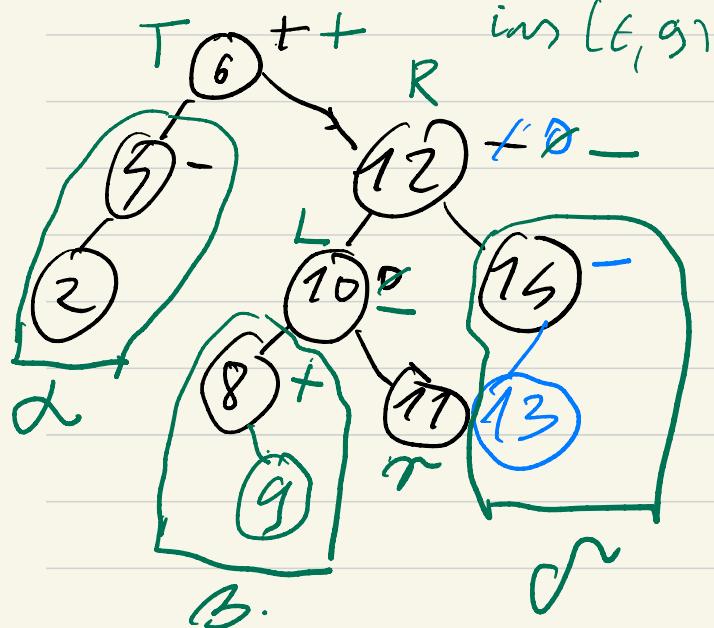


$(++_1+)$



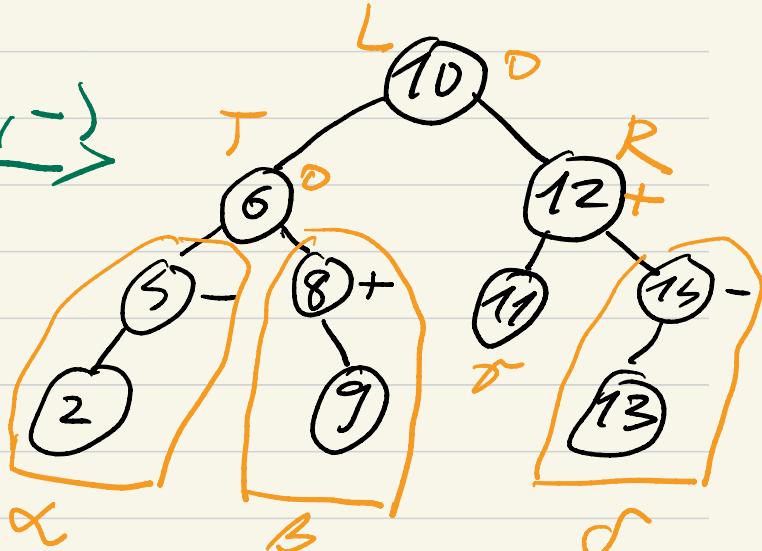
$$\alpha < T < \beta < R < \gamma$$

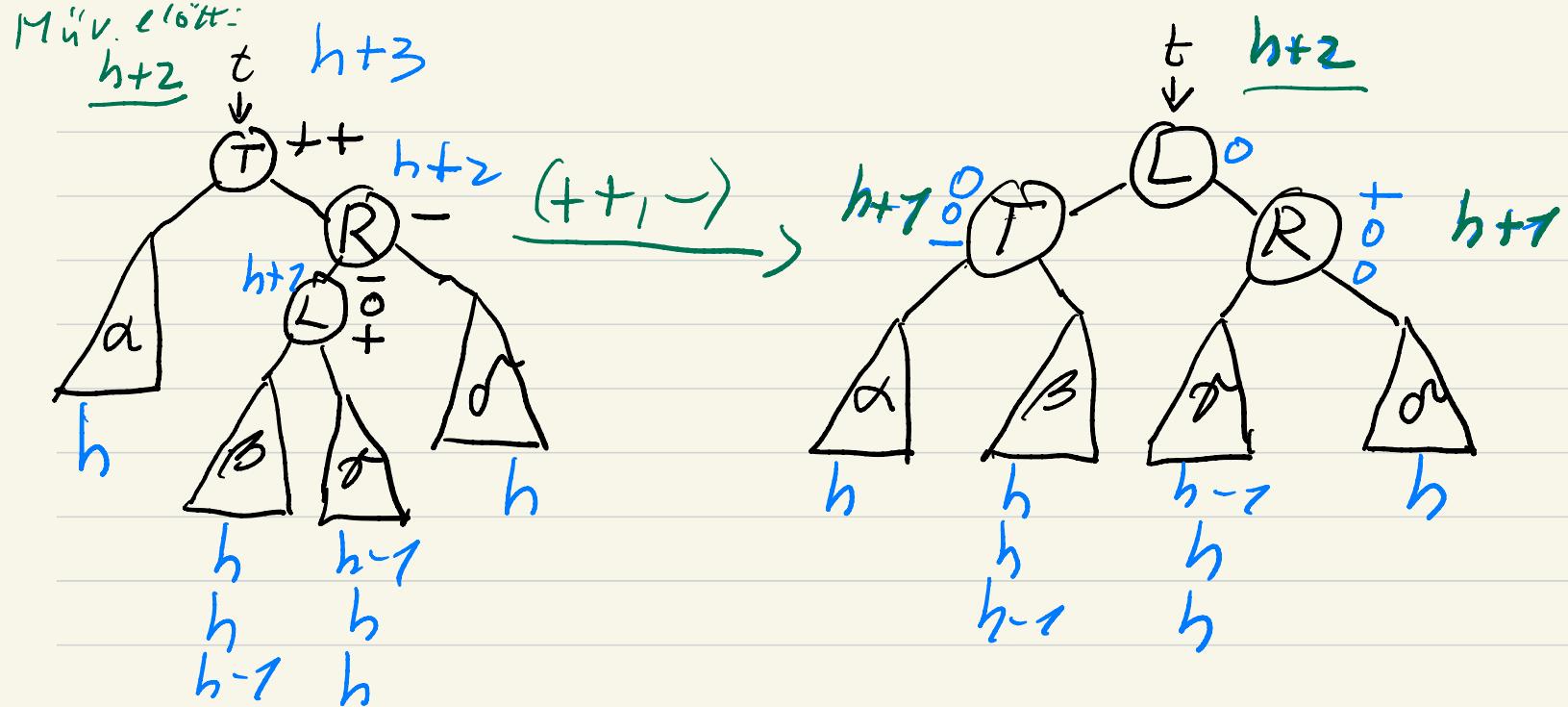
$\text{inv}(E, 13)$



$\text{inv}(E, 9)$

$L++I-,-\rightarrow$





## Node

+  $key : \mathcal{T}$  //  $\mathcal{T}$  is some known type  
 +  $b : -1..1$  // the balance of the node  
 +  $left, right : Node^*$   
 +  $Node() \{ left := right := \emptyset ; b := 0 \}$  // create a tree of a single node  
 +  $Node(x:\mathcal{T}) \{ left := right := \emptyset ; b := 0 ; key := x \}$

**AVLInsert(  $\&t:Node^*$  ;  $k:\mathcal{T}$  ;  $\&d:\mathbb{B}$  )**

	$t = \emptyset$		
	$k < t \rightarrow key$	$k > t \rightarrow key$	ELSE
$t := \mathbf{new}$ $Node(k)$ $d := true$	$AVLInsert(t \rightarrow left, k, d)$ $d$ $leftSubTreeGrown(t, d)$	$AVLInsert(t \rightarrow right, k, d)$ $d$ $rightSubTreeGrown(t, d)$	$d := hamis$

(leftSubTreeGrown( &t:Node\* ; &d: $\mathbb{B}$  ))

$t \rightarrow b = -1$

$l := t \rightarrow left$

$l \rightarrow b = -1$

balanceMMm( $t, l$ )

balanceMMp( $t, l$ )

$d := false$

$t \rightarrow b := t \rightarrow b - 1$

$d := (t \rightarrow b < 0)$

(rightSubTreeGrown( &t:Node\* ; &d: $\mathbb{B}$  ))

$t \rightarrow b = 1$

$r := t \rightarrow right$  //  $t \rightarrow b := 2$

$r \rightarrow b = 1$

balancePPp( $t, r$ )

balancePPm( $t, r$ )

$d := false$

$t \rightarrow b := t \rightarrow b + 1$

$d := (t \rightarrow b > 0)$

balancePPP( &t, r : Node\* )

$t \rightarrow right := r \rightarrow left$

$r \rightarrow left := t$

$r \rightarrow b := t \rightarrow b := 0$

$t := r$

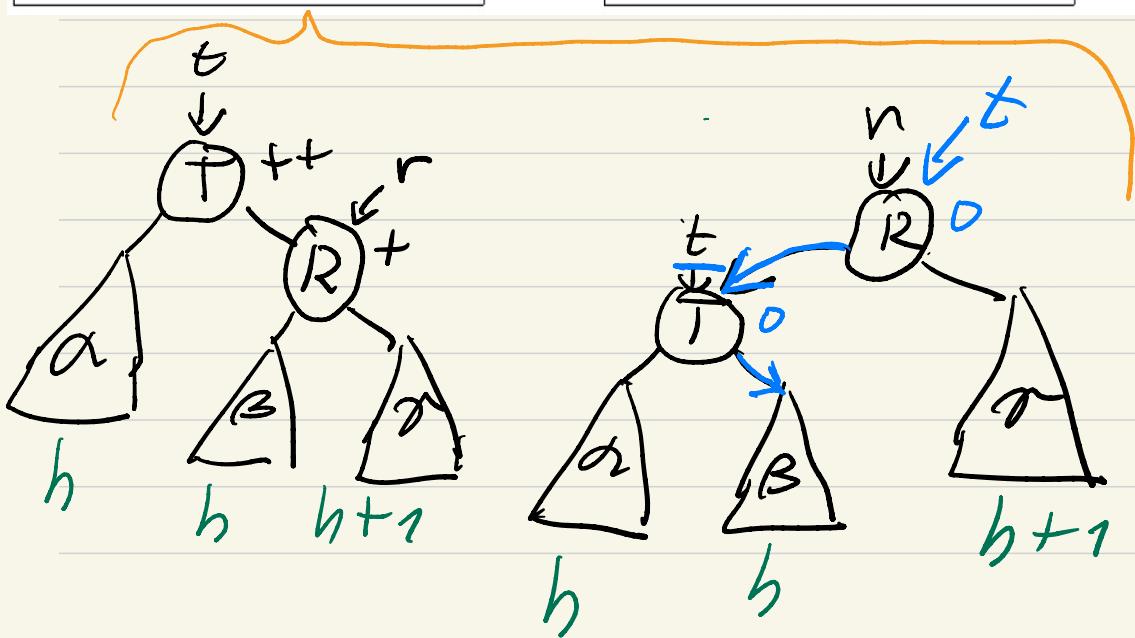
balanceMMm( &t, l : Node\* )

$t \rightarrow left := l \rightarrow right$

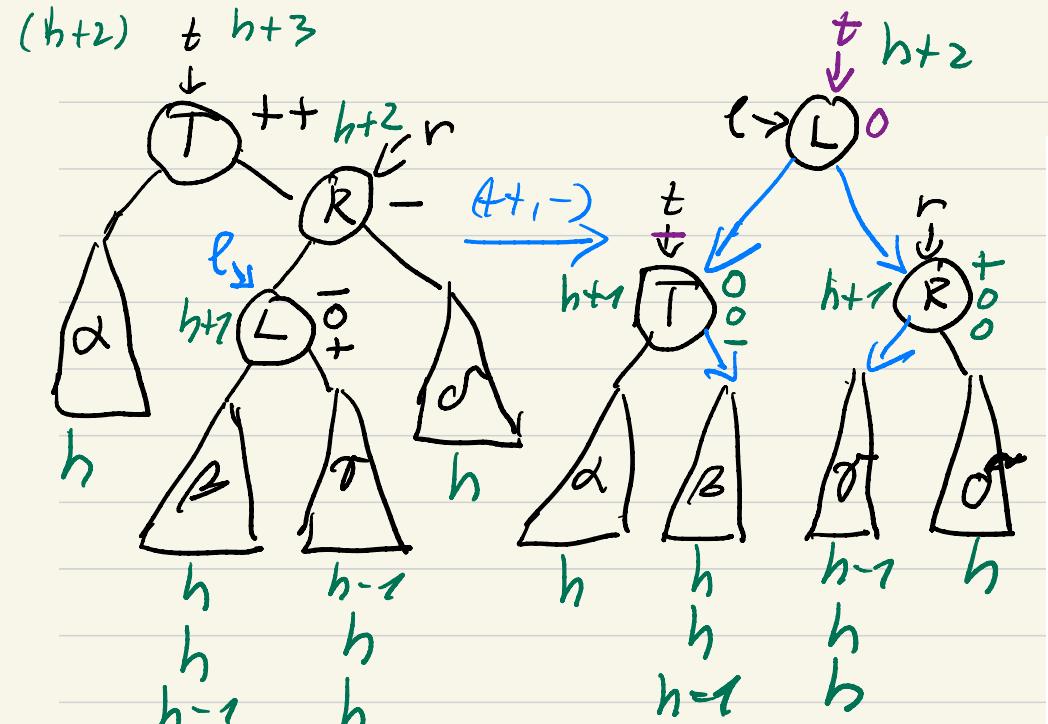
$l \rightarrow right := t$

$l \rightarrow b := t \rightarrow b := 0$

$t := l$



Szept 93.



$\alpha < T < \beta < L < T < R < \sigma$

balancePPm( &t, r : Node\* )

$l := r \rightarrow \text{left}$
$t \rightarrow \text{right} := l \rightarrow \text{left}$
$r \rightarrow \text{left} := l \rightarrow \text{right}$
$l \rightarrow \text{left} := t$
$l \rightarrow \text{right} := r$
$t \rightarrow b := -\lfloor (l \rightarrow b + 1)/2 \rfloor$
$r \rightarrow b := \lfloor (1 - l \rightarrow b)/2 \rfloor$
$l \rightarrow b := 0$
$t := l$

remMin( &t, &minp : Node\* )

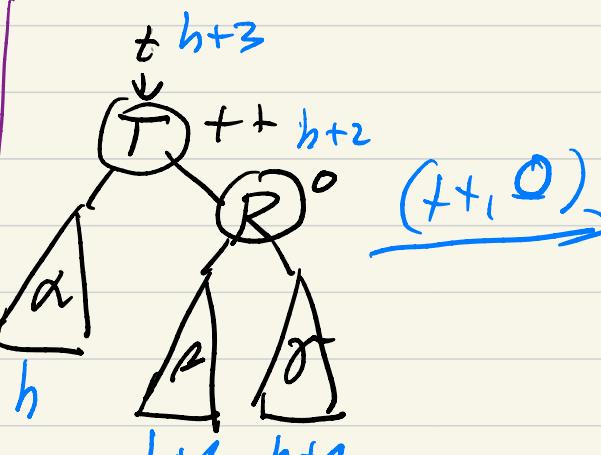
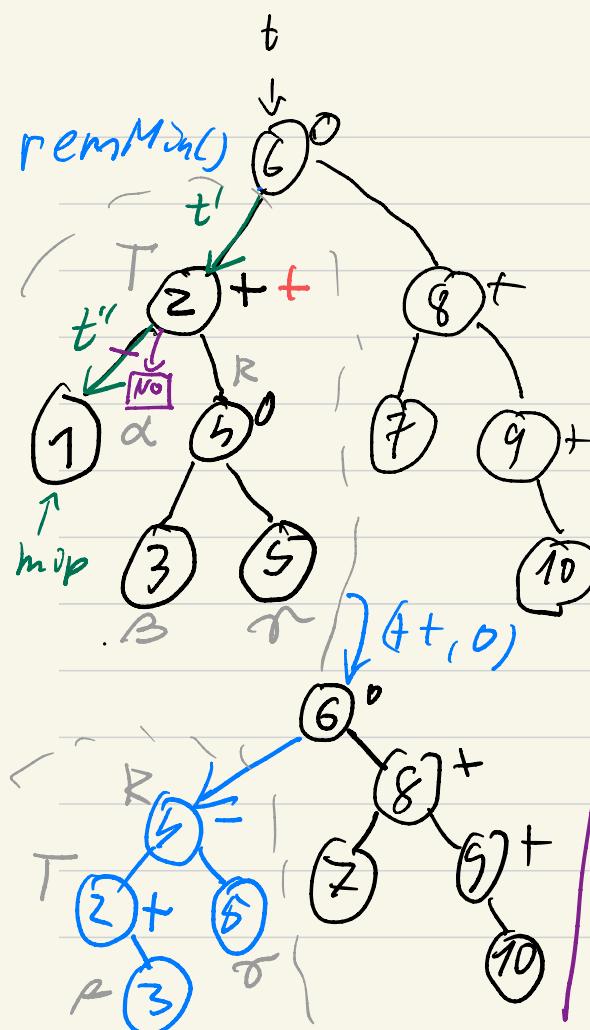
$t \rightarrow left = \emptyset$

$minp := t$

$t := minp \rightarrow right$

$minp \rightarrow right := \emptyset$

remMin( $t \rightarrow left, minp$ )



$\alpha < T < \beta < R < \tau$

AVLremMin(  $\&t, \&minp:\text{Node}^*$  ;  $\&d:\mathbb{B}$  )

$t \rightarrow left = \otimes$		
$minp := t$		AVLremMin( $t \rightarrow left, minp, d$ )
$t := minp \rightarrow right$		
$minp \rightarrow right := \otimes$	$d$	
$d := true$	leftSubTreeShrunk( $t, d$ )	SKIP

leftSubTreeShrunk(  $\&t:\text{Node}^*$  ;  $\&d:\mathbb{B}$  )

$t \rightarrow b = 1$		
$\text{balance\_PP}(t, d)$ <i>// <math>t \rightarrow b := 2</math></i>	$t \rightarrow b := t \rightarrow b + 1$	
		$d := (t \rightarrow b = 0)$

balance\_PP(  $\&t:\text{Node}^*$  ;  $\&d:\mathbb{B}$  )

$r := t \rightarrow \text{right}$

$r \rightarrow b = -1$

balancePPM( $t, r$ )

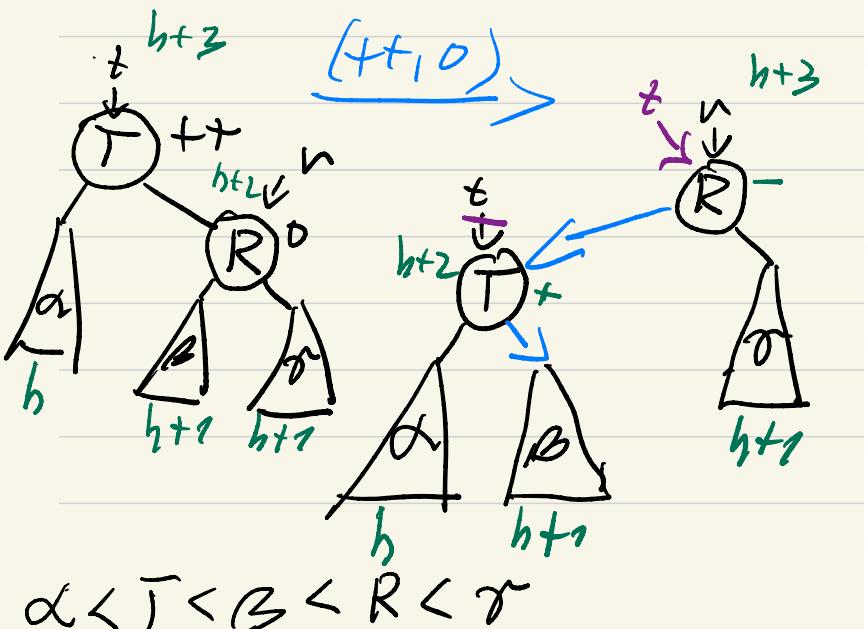
$r \rightarrow b = 0$

balancePP0( $t, r$ )

$r \rightarrow b = 1$

$d := \text{false}$

balancePPP( $t, r$ )



balancePP0(  $\&t, r : \text{Node}^*$  )

$t \rightarrow \text{right} := r \rightarrow \text{left}$

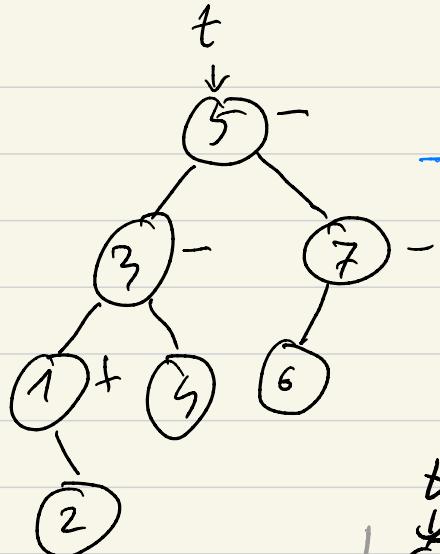
$r \rightarrow \text{left} := t$

$t \rightarrow b := 1$

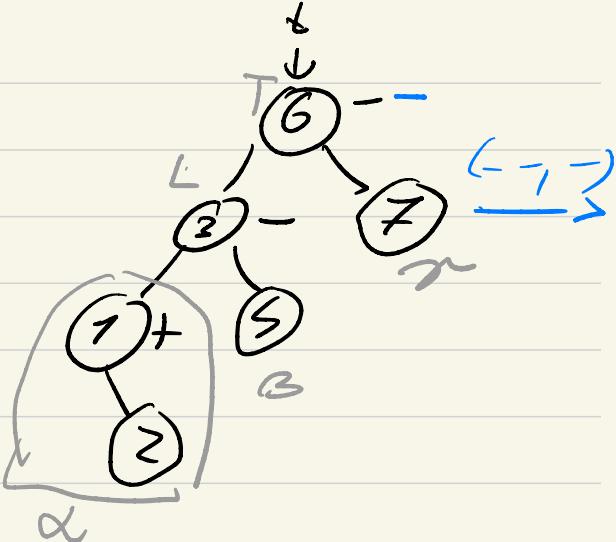
$r \rightarrow b := -1$

$t := r$

del(L, 5)



AVLbaum  
(L=links, R=rechts)



$\alpha < L < \beta < T < \gamma$

