hazi

a

$$\int \frac{e^x}{1+e^{2x}}\,\mathrm{d}x, \quad u = e^x \Longrightarrow x = \ln u \Longrightarrow x' = \frac{1}{u},$$

$$\int \frac{u}{1+u^2}\frac{1}{u}\,\mathrm{d}u = \int \frac{1}{1+u^2}\,\mathrm{d}u = \arctan(u) + C \Longrightarrow \underline{\arctan(e^x) + C}$$

b

$$\int \frac{1}{x - \sqrt[3]{x+1} + 1}\,\mathrm{d}x, \quad u = \sqrt[3]{x+1} \Longrightarrow x = u^3 - 1 \Longrightarrow x' = 3u^2,$$

$$\int \frac{1}{u^3 - 1 - u + 1} \cdot 3u^2\,\mathrm{d}u = 3\int \frac{u^2}{u^3 - u}\,\mathrm{d}u = 3\int \frac{u}{u^2 - 1}\,\mathrm{d}u = \frac{3}{2}\int \frac{2u}{u^2-1}\,\mathrm{d}u = \frac{3}{2}\ln\left|u^2 - 1\right| + C \Longrightarrow$$

$$\Longrightarrow \underline{\frac{3}{2}\ln\left(\left(\sqrt[3]{x+1}\right)^2 - 1\right) + C}$$

c

$$\int \frac{1}{x}\sqrt{\frac{2x-3}{x}}\,\mathrm{d}x, \quad u = \sqrt{\frac{2x-3}{x}} \Longrightarrow u^2 = 2 - \frac{3}{x} \Longrightarrow x = -\frac{3}{u^2 - 2} \Longrightarrow x' = \frac{6u}{(u^2-2)^2},$$

$$\int \frac{1}{-\frac{3}{u^2-2}} \cdot u \cdot \frac{6u}{(u^2-2)^2}\,\mathrm{d}u = \int u \cdot \left(-\frac{2u}{u^2-2}\right) = -2\int \frac{u^2}{u^2-2}\,\mathrm{d}u = -2\int \frac{u^2 - 2 + 2}{u^2 - 2}\,\mathrm{d}u = -2\int 1 + \frac{2}{u^2-2}\,\mathrm{d}u =$$

$$= -2\left[\int 1\,\mathrm{d}u + 2\int \frac{1}{u^2-2}\,\mathrm{d}u\right],$$

$$\frac{A}{u-\sqrt{2}} + \frac{B}{u+\sqrt{2}} = 1 \Longrightarrow A\left(u+\sqrt{2}\right) + B\left(u-\sqrt{2}\right) = 1 \Longrightarrow (A+B)u + (A-B)\sqrt{2} = 1 \Longrightarrow A = \frac{1}{2\sqrt{2}},\ B = -\frac{1}{2\sqrt{2}},$$

$$-2u + -4\int \frac{\frac{1}{2\sqrt{2}}}{u-\sqrt{2}} - \frac{\frac{1}{2\sqrt{2}}}{u+\sqrt{2}}\,\mathrm{d}u = -2u - 4\ln\left|u-\sqrt{2}\right| - 4\ln\left|u+\sqrt{2}\right| \quad \text{waa nemjo}$$

gyakorlo

2

$$\int \frac{e^x + 4}{e^{2x} + 4e^x + 3}\,\mathrm{d}x, \quad u = e^x \Longrightarrow x = \ln u \Longrightarrow x' = \frac{1}{u},$$

$$\int \frac{u+4}{u^2 + 4u + 3}\frac{1}{u}\,\mathrm{d}u = \int \frac{u+4}{u(u+1)(u+3)}\,\mathrm{d}u,$$

$$\frac{A}{u} + \frac{B}{u+1} + \frac{C}{u+3} = u+4 \Longrightarrow A(u+1)(u+3) + B(u)(u+3) + C(u)(u+1) = u+4 \Longrightarrow$$

ha $u = 0$: $A = \frac{4}{3}$
ha $u = -1$: $B = -\frac{3}{2}$
ha $u = -3$: $C = \frac{1}{6}$

$$\int \frac{\frac{4}{3}}{u}\,\mathrm{d}u - \int \frac{\frac{3}{2}}{u+1}\,\mathrm{d}u + \int \frac{\frac{1}{6}}{u+3}\,\mathrm{d}u = \frac{4}{3}\int \frac{1}{u}\,\mathrm{d}u - \frac{3}{2}\int \frac{1}{u+1}\,\mathrm{d}u + \frac{1}{6}\int \frac{1}{u+3}\,\mathrm{d}u = \frac{4}{3}\ln|u| - \frac{3}{2}\ln|u+1| + \frac{1}{6}\ln|u+3| + C \Longrightarrow$$

$$\Longrightarrow \underline{\frac{4}{3}\ln(e^x) - \frac{3}{2}\ln(e^x + 1) + \frac{1}{6}\ln(e^x + 3) + C}$$

3/b

$$\int \frac{1}{(x+1)^2} \cdot \sqrt{\frac{x+1}{x}} \, dx, \quad u = \sqrt{\frac{x+1}{x}} \Longrightarrow u^2 = 1 + \frac{1}{x} \Longrightarrow x = \frac{1}{u^2-1} \Longrightarrow x' = -\frac{2u}{(u^2-1)^2},$$

$$\int \frac{1}{\left(\frac{1}{u^2-1}+1\right)^2} \cdot u \cdot \left(-\frac{2u}{(u^2-1)^2}\right) du = \int \frac{1}{\left(\frac{1+u^2-1}{u^2-1}\right)^2} \cdot u \cdot \left(-\frac{2u}{(u^2-1)^2}\right) du = \int \frac{1}{\left(\frac{u^2}{u^2-1}\right)^2} \cdot u \cdot \left(-\frac{2u}{(u^2-1)^2}\right) du =$$

$$= \int u \cdot \left(-\frac{2u}{(u^2-1)^2}\right)\left(\frac{(u^2-1)^2}{u^4}\right) du = -\int u \cdot \frac{2}{u^3} \, du = -\int \frac{2}{u^2} \, du = -2\int \frac{1}{u^2} \, du = -2\frac{u^{-1}}{-1} = \frac{2}{u} + C \Longrightarrow$$

$$\Longrightarrow \frac{2}{\sqrt{\frac{x+1}{x}}} + C$$