

ennyi volt a differencialas eleg is volt

tipusok:

1. alapintegralok es ezekre vezetheto tipusok

lasd tablazat

pl:

- $\sin' x = \cos x (x \in \mathbb{R}) \implies \int \cos x \, dx = \sin x + C$
- $\arctan' x = \frac{1}{1+x^2} \, dx \implies \int \frac{1}{1+x^2} \, dx = \arctan x + C$

stb

a tablazatot fejbol kene tudni mert anelkul nem fog menni semmi

$$\int (2x^4 - 3x^2 + x - 71) \, dx = 2 \int x^4 \, dx - 3 \int x^2 \, dx + \int x \, dx - 71 \int 1 \, dx = 2 \frac{x^5}{5} - 3 \frac{x^3}{3} + \frac{x^2}{2} - 71x + C \quad (C \in \mathbb{R})$$

$$\int_{(x>0)} \sqrt{x} \sqrt{x} \sqrt{x} \, dx = \int x^{\frac{1}{2}} \cdot x^{\frac{1}{4}} \cdot x^{\frac{1}{8}} \, dx = \int x^{\frac{1}{2} + \frac{1}{4} + \frac{1}{8}} \, dx = \int x^{\frac{7}{8}} \, dx = \frac{x^{\frac{7}{8}+1}}{\frac{7}{8}+1} + C = \frac{8}{15} \cdot \sqrt[8]{x^{15}} + C \quad (C \in \mathbb{R}, x > 0)$$

$$\begin{aligned} \int_{(x>0)} \frac{(x+1)^2}{\sqrt{x}} \, dx &= \int \frac{x^2 + 2x + 1}{x^{\frac{1}{2}}} \, dx = \text{osszegre bontas} = \int x^{2-\frac{1}{2}} \, dx + 2 \int x^{1-\frac{1}{2}} \, dx + \int x^{-\frac{1}{2}} \, dx = \\ &= \int x^{\frac{3}{2}} \, dx + 2 \int x^{\frac{1}{2}} \, dx + \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C = \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + 2 \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + 2\sqrt{x} + C = \frac{2}{5}\sqrt{x^5} + \frac{4}{3}\sqrt{x^3} + 2\sqrt{x} + C \quad (C \in \mathbb{R}) \end{aligned}$$

$$\int_{x \in \mathbb{R}} \frac{x^2}{1+x^2} \, dx = \int \frac{x^2 + 1 - 1}{x^2 + 1} \, dx = \int \left(1 - \frac{1}{x^2 + 1}\right) \, dx = \int 1 \, dx - \int \frac{1}{1+x^2} \, dx = x - \arctan x + C \quad (x \in \mathbb{R}, C \in \mathbb{R})$$

$$\begin{aligned} \int_{(-\frac{\pi}{2}, \frac{\pi}{2})} \frac{\cos^2 x - 2}{1 + \cos 2x} \, dx &= \int \frac{\cos^2 x - 2}{(\cos^2 x + \sin^2 x) + (\cos^2 x - \sin^2 x)} \, dx = \int \frac{\cos^2 x - 2}{2 \cos^2 x} \, dx = \\ &= \frac{1}{2} \int 1 - \frac{2}{\cos^2 x} \, dx = \frac{1}{2} \int 1 \, dx - \int \frac{1}{\cos^2 x} \, dx = \frac{1}{2} \cdot x - \tan x + C \quad (C \in \mathbb{R}) \end{aligned}$$

2. linearis helyettesites

bemelegites:

$$\begin{aligned} \int \sin x \, dx &= -\cos x + C \quad (x, C \in \mathbb{R}) \\ \int \sin(3x) \, dx &= -\frac{\cos(3x)}{3} + C \quad (x, C \in \mathbb{R}) \\ \int \sin(7x - 8) \, dx &= -\frac{\cos(7x-8)}{7} + C \quad (x, C \in \mathbb{R}) \\ \int e^{3-5x} \, dx &= \frac{e^{3-5x}}{-5} + C \quad (x, C \in \mathbb{R}) \end{aligned}$$

tehat

$$\int_{x \in \mathbb{R}} \sin^2 x \, dx = \int \frac{1 - \cos 2x}{2} \, dx = \frac{1}{2} \int 1 \, dx - \frac{1}{2} \int \cos 2x \, dx = \frac{1}{2}x - \frac{1}{2} \cdot \frac{\sin 2x}{2} + C$$

$$3. \int f'(x) \cdot f^\alpha(x) dx$$

ket dolgot hasznalhatunk ilyeknkor

$$\begin{cases} \frac{f^{\alpha+1}(x)}{\alpha+1} + c, & \text{ha } \alpha \neq 1 \\ \int \frac{f'(x)}{f(x)} dx = \ln f(x) + C \end{cases}$$

pl:

$$\int_{x \in (0; +\infty)} \frac{1}{x} dx = \ln x + C$$

$$\int_{x \in (-\infty; 0)} \frac{1}{x} dx = \ln(-x) + C$$

tehát

$$\int \frac{1}{x} dx = \ln|x| + C \quad (x > 0 \vee x < 0, C \in \mathbb{R})$$

$$\int_{x \in \mathbb{R}} \frac{x}{x^2 + 3} dx = \int \frac{1}{2} dx \int \frac{2x}{x^2 + 3} dx = \frac{1}{2} \int \frac{(x^2 + 3)'}{x^2 + 3} dx = \frac{1}{2} \ln|x^2 + 3| + C = \frac{1}{2} \ln(x^2 + 3) + C$$

$$\int_{x \in (0,1) \vee x \in (1,\infty)} \frac{dx}{x \ln x} = \int \frac{1}{x \ln x} dx = \int \frac{(\ln x)'}{\ln x} dx = \ln|\ln x| + C = \begin{cases} \ln(\ln x) + C, & \text{ha } \ln x > 0 \Leftrightarrow x \in (1, \infty) \\ \ln(-\ln x) + C, & \text{ha } \ln x < 0 \Leftrightarrow x \in (0, 1) \end{cases}$$

$$\int_{x \in (-\frac{\pi}{2}, \frac{\pi}{2})} \tan x dx = \int \frac{\sin x}{\cos x} dx = - \int \frac{(\cos x)'}{\cos x} dx = -\ln|\cos x| + C = -\ln(\cos x) + C$$