custom hazi

1. lokalis szelsoertek: $f(x) = x^2 \cdot e^{-x} \qquad (x \in \mathbb{R})$

$$f'(x) = 2x \cdot e^{-x} + x^2 \cdot (-e^{-x}) = 2xe^{-x} - x^2e^{-x} = xe^{-x}(2-x)$$

$$f'(x) = 0 \Longleftrightarrow 2xe^{-x} = -x^2e^{-x} \Longrightarrow x_1 = 0, x_2 = 2$$

x	$x < 0$	0	$0 < x < 2$	2	$x > 2$
$f'(x)$	$+$	0	$+$	0	$-$
$f(x)$	$\downarrow$	0	$\uparrow$	$\frac{4}{e^2}$	$\downarrow$

lokalis minimum: $f(0) = 0$
lokalis maximum: $f(2) = \frac{4}{e^2}$

2. globalis szelsoertek: $f(x) = \sin^4 x + \cos^4 x \qquad x \in \left[-\frac{2\pi}{3}, \frac{\pi}{2}\right]$

$$\mathcal{D}_f = \left[-\frac{2\pi}{3}, \frac{\pi}{2}\right] \text{ korlatos es zart (kompakt), es } \; f \in C\left[-\frac{2\pi}{3}, \frac{\pi}{2}\right] \Longrightarrow \exists \min \mathcal{R}_f, \exists \max \mathcal{R}_f$$

lehetnek:
- belso pontok: $x \in \left(-\frac{2\pi}{3}, \frac{\pi}{2}\right)$ ahol $f'(x) = 0$
- vegpontok: $x = -\frac{2\pi}{3}, x = \frac{\pi}{2}$

ha 1)

$$x \in \left(-\frac{2\pi}{3}, \frac{\pi}{2}\right) \Longrightarrow f \in D\{a\} \;\text{ es }\; f'(x) = -\sin 4x = 0 \Longleftrightarrow x_1 = 0 + \frac{k\pi}{4} \qquad (k \in \mathbb{N})$$

ha 2)
kuka

regular hazi

1

$$f(x) := x^5 - 5x^4 + 5x^3 + 1 \qquad (x \in \mathbb{R})$$

$$f'(x) = 5x^4 - 20x^3 + 15x^2 = 5x^2(x^2 - 4x + 3) = 5x^2(x-3)(x-1) \Longleftrightarrow x_1 = 0 \vee x_2 = 3 \vee x_3 = 1$$

x	$x < 0$	0	$0 < x < 1$	1	$1 < x < 3$	3	$3 < x$
$f'(x)$	$-$	0	$+$	0	$-$	0	$+$
$f(x)$	$\downarrow$	1	$\uparrow$	2	$\downarrow$	-26	$\uparrow$

$$f \uparrow\text{: } (0; 1), (3; +\infty)$$

$$f \downarrow\text{: } (-\infty; 0), (1; 3)$$

$$\text{lokalis max} : 2$$

$$\text{lokalis min} : -26$$

2

$$f(x) := \frac{e^x}{x} \qquad (x \in \mathbb{R} \setminus \{0\})$$

$$f'(x) = \frac{e^x x - e^x}{x^2} = \frac{e^x(x-1)}{x^2} = 0 \iff x_1 = 0 \ (x \in \mathbb{R} \setminus \{0\}!) \lor x_2 = 1 \implies x = 1$$

x	$x < 1$	1	$x < 1$
$f'(x)$	$-$	0	$+$
$f(x)$	$\downarrow$	e	$\uparrow$

$$f \uparrow : (1; +\infty)$$
$$f \downarrow : (-\infty; 1)$$
$$\text{lokalis max} : \nexists$$
$$\text{lokalis min} : e$$

3

$$f(x) := \frac{x}{x^2 + x + 1} \qquad (x \in [-2, 0])$$

$$f'(x) = \frac{(x^2 + x + 1) - x(2x + 1)}{(x^2 + x + 1)^2} = \frac{x^2 + x + 1 - 2x^2 - x}{(x^2 + x + 1)^2} = \frac{-x^2 + 1}{(x^2 + x + 1)^2} = \frac{1 - x^2}{(x^2 + x + 1)^2}$$

x	-2	$-2 < x < 0$	0
$f'(x)$	-1	$+$	1
$f(x)$	$-\frac{2}{3}$	$\uparrow$	0

$$f \uparrow : [-2; 0]$$
$$\text{lokalis max} : -1$$
$$\text{lokalis min} : -\frac{2}{3}$$

4

$$6x + y = 9 \text{ legkozelebbi pont } (-3, 1)$$

$$y = -6x + 9$$
$$d = \sqrt{(x+3)^2 + (y-1)^2} = \sqrt{(x+3)^2 + (-6x+8)^2}$$
$$f(x) = (x+3)^2 + (-6x+8)^2$$
$$f'(x) = 2(x+3) + 2(-6x+8) \cdot (-6) = 2x + 6 + 72x - 96 = 74x - 90$$
$$\begin{cases} x = \frac{90}{74} \\ y = -6\left(\frac{90}{74}\right) + 9 = \frac{63}{37} \end{cases} \implies \text{a legkozelebbi pont } \left(\frac{90}{74}, \frac{63}{37}\right)$$

gyakorlo

1/b

$$f(x) := (x^2 - x + 1)e^{-x} \qquad (x \in [-2, 3])$$
$$\mathcal{D}_f = [-2, 3] \implies f \in C[-2, 3] \implies \exists \mathcal{R}_{\max}, \mathcal{R}_{\min}$$

$$x \in (-2,3) \Longrightarrow f \in D(-2,3)$$

$$f'(x) = (2x-1)e^{-x} - (x^2-x+1)e^{-x} = e^{-x}((2x-1)-(x^2-x+1)) = e^{-x}(2x-1-x^2+x-1) =$$

$$= e^{-x}(-x^2+3x-2) = e^{-x}(-x+1)(x-2) = 0 \Longleftrightarrow x_1 = 0 \vee x_2 = 1 \vee x_3 = 2$$

$$f(-2) = 7e^2,\ f(0) = 1,\ f(1) = \frac{1}{e},\ f(2) = \frac{3}{e^2},\ f(3) = \frac{7}{e^3}$$

$$\text{min helye} : 3,\ \text{erteke} : \frac{7}{e^3}$$

$$\text{max helye} : -2,\ \text{erteke} : 7e^2$$

1/c

$$f(x) := x^2 e^{-x} \qquad (x \in \mathbb{R})$$

$$f'(x) = 2xe^{-x} - x^2e^{-x} = xe^{-x}(2-x) = 0 \Longleftrightarrow x_1 = 0 \vee x_2 = 2$$

x	$x<0$	0	$0<x<2$	2	$x>2$
$f'(x)$	$+$	0	$+$	0	$-$
$f(x)$	$\downarrow$	0	$\uparrow$	$\frac{4}{e^2}$	$\downarrow$

lokalis minimum: $f(0) = 0$
lokalis maximum: $f(2) = \frac{4}{e^2}$

3

$$y^2 - x^2 = 4 \text{ melyik a legkozelebb pontja } (2,0)\text{-hoz}$$

$$d = \sqrt{(x-2)^2 + (y-0)^2} = \sqrt{(x-2)^2 + y^2}$$

$$d = \sqrt{(x-2)^2 + 4 + x^2}$$

$$f(x) = 2x^2 - 4x + 8 = 2(x^2 - 2 + 4) = 2[(x-1)^2 + 3]$$

$$\text{min} :\ \text{ha x} = 1 : \sqrt{5}$$