Restye János Barnabás - F8U9I2 — Analízis II. - 50 derivált

1. (845.)

$$f(x) := \frac{2x}{1-x^2}$$

$$u(x) = 2x \quad u'(x) = 2$$

$$v(x) = 1 - x^2 \quad v'(x) = -2x$$

$$f'(x) = \frac{u'(x)v(x) - u(x)v'(x)}{v^2(x)} = \frac{2(1-x^2) - 2x \cdot (-2x)}{\left(1-x^2\right)^2} = \frac{2+2x^2}{\left(1-x^2\right)^2}$$

2. (846.)

$$f(x) := \frac{1+x-x^2}{1-x+x^2}$$

$$u(x) = 1 + x - x^2 \quad u'(x) = -2x + 1$$

$$v(x) = 1 - x + x^2 \quad v'(x) = 2x - 1$$

$$f'(x) = \frac{(-2x+1)(1-x+x^2) - (1+x-x^2)(2x-1)}{\left(1-x+x^2\right)^2} =$$

$$= \frac{-2x + 2x^2 - 2x^3 + 1 - x + x^2 - 2x + 1 - 2x^2 + x + 2x^3 - x^2}{\left(1-x+x^2\right)^2} =$$

$$= \frac{-4x+2}{\left(1-x+x^2\right)^2}$$

3. (847.)

$$f(x) := \frac{x}{(1-x)^2(1+x)^3}$$

$$u(x) = x \quad u'(x) = 1$$

$$v(x) = (1-x)^2(1+x)^3$$

$$v'(x) = 2(1-x)(-1)(1+x)^3 + 3(1+x)^2(1)(1-x)^2 = (1-x)(1+x)^2[(-2)(1+x) + 3(1-x)] =$$

$$= -2(1-x)(1+x)^3 + 3(1-x)^2(1+x)^2$$

$$f'(x) = \frac{(1-x)^2(1+x)^3 - x\left[-2(1-x)(1+x)^3 + 3(1-x)^2(1+x)^2\right]}{\left[(1-x)^2(1+x)^3\right]^2} =$$

$$= \frac{(1-x)^2(1+x)^3 2x(1-x)(1+x)^3 - 3x(1-x)^2(1+x)^2}{(1-x)^4(1+x)^6} =$$

$$= \frac{(1-x)(1+x)^2[(1-x)(1+x) + 2x(1+x) - 3x(1-x)]}{(1-x)^4(1+x)^6} =$$

$$= \frac{(1-x)(1+x)^2(1-x+4x^2)}{(1-x)^4(1+x)^6} = \frac{1-x+4x^2}{(1-x)^3(1+x)^4} =$$

4. (849.)

$$f(x) := \frac{(1-x)^p}{(1+x)^q}$$

$$u(x) = (1-x)^p \qquad u'(x) = p(1-x)^{p-1}$$

$$v(x) = (1+x)^q \qquad v'(x) = q(1+x)^{q-1}$$

$$f'(x) = \frac{p(1-x)^{p-1}(-1)(1+x)^q - (1-x)^p q(1+x)^{q-1}}{(1+x)^{2q}}$$

5. (853.)

$$f(x) := \sqrt[3]{x^2} - \frac{2}{\sqrt{x}}$$

$$\left(\sqrt[3]{x^2}\right)' = \frac{2}{3\sqrt[3]{x}}$$

$$\left(-\frac{2}{\sqrt{x}}\right)' = \frac{2\frac{1}{2\sqrt{x}}}{x} = \frac{\frac{1}{\sqrt{x}}}{x} = \frac{1}{x\sqrt{x}}$$

$$f'(x) = \frac{2}{3\sqrt[3]{x}} + \frac{1}{x\sqrt{x}}$$

6. (855.)

$$f(x) := (1+x)\sqrt{2+x^2}\,\sqrt[3]{3+x^3}$$

$$u(x) = 1+x \qquad u'(x) = 1$$

$$v(x) = \sqrt{2+x^2} \qquad v'(x) = \frac{1}{2\sqrt{2+x^2}} \cdot 2x = \frac{x}{\sqrt{2+x^2}}$$

$$w(x) = \sqrt[3]{3+x^3} \qquad w'(x) = \frac{1}{3\sqrt[3]{(3+x^3)^2}} \cdot 3x^2 = \frac{x^2}{\sqrt[3]{(3+x^3)^2}}$$

$$f'(x) = u'(x)v(x)w(x) + u(x)v'(x)w(x) + u(x)v(x)w'(x) =$$

$$= \sqrt{2+x^2}\,\sqrt[3]{3+x^3} + (1+x)\frac{x}{\sqrt{2+x^2}}\sqrt[3]{3+x^3} + (1+x)\sqrt{2+x^2}\frac{x^2}{\sqrt[3]{(3+x^3)^2}}$$

7. (856.)

$$f(x) := \sqrt[m+n]{(1-x)^m(1+x)^n} = [(1-x)^m(1+x)^n)]^{\frac{1}{m+n}}$$

$$f'(x) = \frac{1}{m+n}[(1-x)^m(1+x)^n)]^{\frac{1}{m+n}-1} \cdot [(1-x)^m(1+x)^n]'$$

$$[(1-x)^m(1+x)^n]' = -m(1-x)^{m-1}(1+x)^n + (1-x)^m n(1+x)^{n-1}$$

$$f'(x) = \frac{1}{m+n}[(1-x)^m(1+x)^n)]^{\frac{1}{m+n}-1} \cdot \left[-m(1-x)^{m-1}(1+x)^n + (1-x)^m n(1+x)^{n-1}\right]$$

8. (857.)

$$f(x) := \frac{x}{\sqrt{a^2 - x^2}}$$

$$u(x) = x \qquad u'(x) = 1$$

$$v(x) = \sqrt{a^2 - x^2} \qquad v'(x) = -\frac{2x}{2\sqrt{a^2 - x^2}} = -\frac{x}{\sqrt{a^2 - x^2}}$$

$$f'(x) = \frac{\sqrt{a^2 - x^2} - x\left(-\frac{x}{\sqrt{a^2-x^2}}\right)}{a^2 - x^2}$$

9. (858.)

$$f(x) := \sqrt[3]{\frac{1 + x^3}{1 - x^3}}$$

$$\left(\frac{1 + x^3}{1 - x^3}\right)' = \frac{3x^2(1 - x^3) - (1 + x^3)(-3x^2)}{\left(1 - x^3\right)^2}$$

$$f'(x) = \frac{\frac{1+x^3}{1-x^3}}{3\sqrt[3]{\frac{3x^2(1-x^3)-(1+x^3)(-3x^2)}{\left(1-x^3\right)^2}}^2}$$

10. (859.)

$$f(x) := \frac{1}{\sqrt{1 + x^2}\left(x + \sqrt{1 + x^2}\right)}$$

$$\left[\sqrt{1 + x^2}\left(x + \sqrt{1 + x^2}\right)\right]' = \frac{x}{\sqrt{1 + x^2}}\left(x + \sqrt{1 + x^2}\right) + \sqrt{1 + x^2}\left(1 + \frac{x}{\sqrt{1 + x^2}}\right)$$

$$f'(x) = -\frac{\frac{x}{\sqrt{1+x^2}}\left(x + \sqrt{1 + x^2}\right) + \sqrt{1 + x^2}\left(1 + \frac{x}{\sqrt{1+x^2}}\right)}{\left[\left(\frac{x}{\sqrt{1+x^2}}\right)\left(x + \sqrt{1 + x^2}\right) + \sqrt{1 + x^2}\left(1 + \frac{x}{\sqrt{1+x^2}}\right)\right]^2}$$

11. (860.)

$$f(x) := \sqrt{x + \sqrt{x + \sqrt{x}}} = \sqrt{x + \sqrt{x \cdot x^{\frac{1}{2}}}} = \sqrt{x \cdot \left(x^{\frac{3}{2}}\right)^{\frac{1}{2}}} = \left(x \cdot x^{\frac{3}{2}}\right)^{\frac{1}{2}} = \left(x^{\frac{7}{4}}\right)^{\frac{1}{2}} = x^{\frac{7}{8}} = \sqrt[8]{x^7}$$

$$f'(x) = \frac{7}{8\sqrt[8]{x}}$$

12. (861.)

$$f(x) := \sqrt[3]{1 + \sqrt[3]{1 + \sqrt[3]{x}}}$$

$$\sqrt[3]{1 + \sqrt[3]{1 + \sqrt[3]{x}}} = \sqrt[3]{1 + \sqrt[3]{1 + x^{\frac{1}{3}}}} = \sqrt[3]{1 + \left(1 + x^{\frac{1}{3}}\right)^{\frac{1}{3}}} = \left(1 + \left(1 + x^{\frac{1}{3}}\right)^{\frac{1}{3}}\right)^{\frac{1}{3}}$$

$$\left[\left(1 + x^{\frac{1}{3}}\right)^{\frac{1}{3}}\right]' = \frac{1}{9\sqrt[3]{\left(x + \sqrt[3]{x}\right)^2}}$$

$$f'(x) = \frac{1}{27\sqrt[3]{\left(x + x\sqrt[3]{x} + x\sqrt[3]{1 + \sqrt[3]{x}} + x\sqrt[3]{x + \sqrt[3]{x}x}\right)^2}}$$

13. (862.)

$$f(x) := \cos 2x - 2\sin x$$
$$f'(x) = -\sin 2x - 2\cos x$$

14. (863.)

$$f(x) := (2 - x^2)\cos x + 2x\sin x$$
$$\left[(2 - x^2)\cos x\right]' = -2x\cos x + (2 - x^2)(-\sin x)$$
$$[2x\sin x]' = 2\sin x + 2x\cos x$$
$$f'(x) = -2x\cos x + (2 - x^2)(-\sin x) + 2\sin x + 2x\cos x = (2 - x^2)(-\sin x) + 2\sin x$$

15. (2.gy/gy1/a)

$$f(x) := \sin\sqrt{1 + x^3}$$
$$\left[\sqrt{1 + x^3}\right]' = \frac{3x^2}{2\sqrt{1 + x^3}}$$
$$f'(x) = \cos\sqrt{1 + x^3} \cdot \frac{3x^2}{2\sqrt{1 + x^3}}$$

16. (2.gy/gy1/b)

$$f(x) := \frac{(x + 1)^3}{x^{\frac{3}{2}}}$$
$$\left[(x + 1)^3\right]' = 3(x + 1)^2 = 3x^2 + 6x + 3$$
$$\left[\left(x^{\frac{3}{2}}\right)\right]' = \frac{3}{2x^{\frac{1}{2}}} = \frac{3\sqrt{x}}{2}$$
$$f'(x) = \frac{(3x^2 + 6x + 3)x^{\frac{3}{2}} - (x + 1)^3 \frac{3\sqrt{x}}{2}}{x^3}$$

17. (2.gy/gy1/c)

$$f(x) := \ln(e^{-x}\sin x)$$
$$[e^{-x}\sin x]' = -e^{-x}\sin x + e^{-x}\cos x$$
$$f'(x) = -\frac{1}{e^{-x}\sin x} \cdot e^{-x}\sin x + e^{-x}\cos x = -1 + \frac{\cos x}{\sin x}$$

18. (2.gy/gy1/d)

$$f(x) := \sqrt{1 + \sin^2 x} \cdot \cos x$$
$$\left[\sin^2 x\right]' = 2\sin x\cos x$$
$$\left[\sqrt{1 + \sin^2 x}\right]' = \frac{1}{2\sqrt{1 + \sin^2 x}} \cdot 2\sin x\cos x = \frac{\sin x\cos x}{\sqrt{1 + \sin^2 x}}$$
$$f'(x) = \frac{\sin x\cos x}{\sqrt{1 + \sin^2 x}} \cdot \cos x + \sqrt{1 + \sin^2 x} \cdot (-\sin x)$$

19. (2.gy/gy1/e)

$$f(x) := e^x\sin x$$
$$f'(x) = e^x\cos x + e^x\sin x$$

20. (2.gy/gy1/f)

$$f(x) := x^2 \sqrt[3]{x}$$

$$f'(x) = 2x\sqrt[3]{x} + x^2 \frac{1}{3\sqrt[3]{x^2}}$$

21. (2.gy/gy1/g)

$$f(x) := (x+2)^8 (x+3)^6$$

$$\left[(x+2)^8\right]' = 8(x+2)^7 \qquad \left[(x+3)^6\right]' = 6(x+3)^5$$

$$f'(x) = 8(x+2)^7(x+3)^6 + (x+2)^8 6(x+3)^5$$

22. (2.gy/gy1/h)

$$f(x) := \left(\sin^3 x\right)\cos x$$

$$\left[\sin^3 x\right]' = 3\sin^2 x \cdot \cos x$$

$$f'(x) = 3\sin^2 x \cos x \cos x + \sin^3 x \cdot (-\sin x) = 3\sin^2 \cos^2 x - \sin^4 x$$

23. (2.gy/gy1/i)

$$f(x) := \frac{1}{\sqrt[3]{x+\sqrt{x}}}$$

$$\left[\sqrt[3]{x+\sqrt{x}}\right]' = \left[\left(x+\sqrt{x}\right)^{\frac{1}{3}}\right]' = \frac{1+2\sqrt{x}}{6\sqrt[6]{x^3\left(x+\sqrt{x}\right)^4}}$$

$$f'(x) = -\frac{\frac{1+2\sqrt{x}}{6\sqrt[6]{x^3(x+\sqrt{x})^4}}}{\left(\sqrt[3]{x+\sqrt{x}}\right)^2}$$

24. (2.gy/gy1/j)

$$f(x) := \frac{\sin(2x^2)}{3-\cos(2x)}$$

$$\left[\sin(2x^2)\right]' = \cos(2x^2) \cdot 4x$$

$$[\cos(2x)]' = -\sin(2x) \cdot 2$$

$$f'(x) = \frac{\cos(2x^2)(4x)(3-\cos(2x)) - \sin(2x^2) \cdot (-\sin(2x) \cdot 2)}{(3-\cos(2x))^2}$$

25. (2.gy/gy1/k)

$$f(x) := \ln\left(x^2 e^x\right)$$

$$\left[x^2 e^x\right]' = 2xe^x + x^2 e^x$$

$$f'(x) = \frac{2xe^x + x^2 e^x}{x^2 e^x} = \frac{2}{x} + 1$$

26. (2.gy/gy1/l)

$$f(x) := e^{\cos x} + \cos(e^x)$$
$$\left[e^{\cos x}\right]' = e^{\cos x} \cdot (-\sin x)$$
$$\left[\cos(e^x)\right]' = -\sin(e^x) \cdot e^x$$
$$f'(x) = -e^{\cos x} \cdot \sin(x) - \sin(e^x) \cdot e^x$$

27. (2.gy/gy1/m)

$$f(x) := \left(x + \frac{1}{x^2}\right)^{\sqrt{7}}$$
$$f'(x) = \sqrt{7}\left(x + \frac{1}{x^2}\right)^{\sqrt{7}-1} \cdot \left(1 - \frac{2}{x^3}\right)$$

28. (2.gy/gy1/n)

$$f(x) := \ln(\cos x)$$
$$f'(x) = -\frac{\sin x}{\cos x} = -\operatorname{tg} x$$

30. (2.gy/gy1/o)

$$f(x) := x^x$$
$$f'(x) = x^x + x^x \ln(x)$$

31. (3.gy/gy1/a)

$$f(x) := \frac{x+1}{x-1} \qquad (x \in \mathbb{R} \setminus \{1\}) \qquad a = 3$$
$$f'(x) = \frac{(x-1)-(x+1)}{(x-1)^2} = -\frac{2}{(x-1)^2}$$
$$f'(3) = -\frac{2}{4} = -\frac{1}{2}$$
$$y = -\frac{1}{2}(x-3) + \frac{4}{2} = -\frac{1}{2}x + \frac{3}{2} + \frac{4}{2} =$$
$$y = -\frac{1}{2}x + \frac{7}{2}$$

32. (3.gy/gy1/b)

$$f(x) := \sqrt{1+x^2} \qquad (x \in \mathbb{R}) \qquad a = \frac{1}{2}$$
$$f'(x) = \frac{x}{\sqrt{1+x^2}}$$
$$f'\left(\frac{1}{2}\right) = \frac{\frac{1}{2}}{\sqrt{1+\left(\frac{1}{2}\right)^2}} = \frac{\frac{1}{2}}{\sqrt{1+\frac{1}{4}}} = \frac{\frac{1}{2}}{\frac{\sqrt{5}}{2}} = \frac{\sqrt{5}}{5}$$
$$y = \frac{\sqrt{5}}{5}\left(x - \frac{1}{2}\right) + \frac{\sqrt{5}}{2} = \frac{\sqrt{5}}{5}x - \frac{3\sqrt{5}}{10} + \frac{5\sqrt{5}}{10} =$$
$$y = \frac{\sqrt{5}}{5} + \frac{2\sqrt{5}}{5}$$

33.

$$f(x) := (4x^2 - x)(x^3 - 8x^2 + 12)$$
$$f'(x) = (8x - 1)(x^3 - 8x^2 + 12) + (4x^2 - x)(3x^2 - 16x)$$

34.

$$f(x) := \left(1 + \sqrt{x^3}\right)\left(x^{-3} - 2\sqrt{x}\right)$$
$$f'(x) = \left(\frac{3x^2}{2\sqrt{x^3}}\right)\left(x^{-3} - 2\sqrt{x}\right) + \left(1 + \sqrt{x^3}\right)\left(-3x^{-4} - \frac{1}{\sqrt{x}}\right)$$

35.

$$f(x) := (1 + 2x + 3x^2)(5x + 8x^2 - x^3)$$
$$f'(x) = (2 + 6x)(5x + 8x^2 - x^3) + (1 + 2x + 3x^2)(5 + 16x - 3x^2)$$

36.

$$f(x) := \frac{6x^2}{2 - x}$$
$$f'(x) = \frac{12x(2 - x) + 6x^2}{(2 - x)}$$

37.

$$f(x) := \frac{3x + x^4}{2x^2 + 1}$$
$$f'(x) = \frac{(3 + 4x^3)(2x^2 + 1) - (3x + x^4)(4x)}{(2x^2 + 1)^2}$$

38.

$$f(x) := \frac{\sqrt{x} + 2x}{7x - 4x^2}$$
$$f'(x) = \frac{\left(\frac{1}{\sqrt{x}}\right)(7x - 4x^2) - (\sqrt{x} + 2x)(7 - 8x)}{(7x - 4x^2)^2}$$

39.

$$f(x) := 2e^x - 8^x$$
$$f'(x) = 2e^x - \ln(8) \cdot 8^x$$

40.

$$f(x) := 4\log_3(x) - \ln(x)$$
$$f'(x) = \frac{4}{x \ln 3} - \frac{1}{x}$$

41.

$$f(x) := 3^x \log(x)$$
$$f'(x) = 3^x \ln(3) \log(x) + \frac{3^x}{x \ln 10}$$

42.

$$f(x) := x^5 - e^x \ln(x)$$
$$f'(x) = 5x^4 - \left(\frac{e^x}{x} + e^x \ln(x)\right)$$

43.

$$f(x) := \frac{x}{1 - e^x}$$
$$f'(x) = \frac{-e^x - x(-e^x)}{(1 - e^x)^2}$$

44.

$$f(x) := \frac{1 + 5x}{\ln(x)}$$
$$f'(x) = \frac{5\ln(x) - \frac{1+5x}{x}}{\ln^2 x}$$

45.

$$f(x) := \frac{1 + 4\ln(x)}{5x^3}$$
$$f'(x) = \frac{\frac{4}{x} \cdot 5x^3 - (1 + 4\ln(x))(15x^2)}{25x^6}$$

46.

$$f(x) := \frac{x^2 + \log_7(x)}{7^x}$$
$$f'(x) = \frac{\left(\frac{2x}{x} \ln 7\right)(7^x) - \left(x^2 + \log_7(x)\right)(\ln 7 \cdot 7^x)}{7^{2x}}$$

47.

$$f(x) := \frac{x^4 e^x}{\ln(x)}$$
$$f'(x) = \frac{(4x^3 e^x)(\ln(x)) - \frac{x^4 e^x}{x}}{\ln^2 x}$$

48.

$$f(x) := (1 - 8x)e^x \qquad a = -1$$
$$f'(x) = -8e^x + (1 - 8x)e^x$$
$$f'(-1) = -8e^{-1} + (1 + 8)e^{-1} = -\frac{8}{e} + \frac{9}{e} = \frac{1}{e}$$
$$y = \frac{1}{e}(x + 1) + \frac{9}{e} = \frac{1}{e}x + \frac{1}{e} + \frac{9}{e} = \frac{1}{e}x + \frac{10}{e}$$

49.

$$f(x) := 3x^2 \ln(x) \qquad a = 1$$
$$f'(x) = 6x\ln(x) + \frac{3x^2}{x} = 6x\ln(x) + 3x$$
$$f'(1) = 0 + 3$$
$$y = 3(x-1) + 0 \iff y = 3x - 3$$

50.

$$f(x) := 3e^x + 8\ln(x) \qquad a = 2$$
$$f'(x) = 3e^x + \frac{8}{x}$$
$$f'(2) = 3e^2 + 4$$
$$y = (3e^2 + 4)(x-2) + (3e^2 + 8\ln(2))$$
$$y = 3e^x - 3e^2 + 4x - 8 + 8\ln(2)$$