1

$$\int \sin^3 x \, \mathrm{d}x = \int \sin x \cdot \sin^2 x \, \mathrm{d}x = \int \sin x \cdot (1 - \cos^2 x) \, \mathrm{d}x = \int \sin x - \sin x \cos^2 x \, \mathrm{d}x = \int \sin x \, \mathrm{d}x - \int \sin x \cos^2 x \, \mathrm{d}x =$$

$$= -\cos x + C + \int (\cos x)' \cos^2 x \, \mathrm{d}x = -\cos x + \frac{\cos^3 x}{3} + C$$

2

$$\int \cos^4 x \, \mathrm{d}x = \int \left(\cos^2 x\right)^2 \mathrm{d}x = \int \left(\frac{1 - \cos(2x)}{2}\right)^2 \mathrm{d}x = \frac{1}{4} \int (1 - 2\cos(2x) + \cos^2(2x)) \, \mathrm{d}x =$$

$$= \frac{1}{4}\left[\int 1 \, \mathrm{d}x - \int 2\cos(2x) \, \mathrm{d}x + \int \frac{1 - \cos(4x)}{2} \, \mathrm{d}x\right] =$$

$$= \frac{1}{4}x + C + \frac{1}{4}\left[2 \cdot \frac{\sin(2x)}{2} + C + \frac{1}{2}\int 1 \, \mathrm{d}x - \frac{1}{2}\int \cos(4x) \, \mathrm{d}x\right] =$$

$$= \frac{x}{4} + \frac{\sin(2x)}{4} + \frac{x}{8} + \frac{1}{4}\frac{\sin(4x)}{8} + C = \frac{3}{8}x + \frac{\sin(2x)}{4} + \frac{\sin(4x)}{32} + C$$

3

$$\int \frac{\cos^3 x}{\sqrt{\sin^5 x}} \, \mathrm{d}x = \int \cos^3 x \cdot \left(\sin^5 x\right)^{-\frac{1}{2}} \mathrm{d}x = \int \cos^3 x \cdot \sin^{-\frac{5}{2}} x \, \mathrm{d}x = \int \cos^2 x \cdot \sin^{-\frac{5}{2}} x \cdot \cos x \, \mathrm{d}x =$$

$$= \int (1 - \sin^2 x) \cdot \sin^{-\frac{5}{2}} x \cdot \cos x \, \mathrm{d}x = \int \left(\sin^{-\frac{5}{2}} x - \sin^{-\frac{1}{2}} x\right) \cos x \, \mathrm{d}x = \int \sin^{-\frac{5}{2}} x \cos x \, \mathrm{d}x - \int \sin^{-\frac{1}{2}} x \cos x \, \mathrm{d}x =$$

$$= \int \sin^{-\frac{5}{2}} x (\sin x)' \, \mathrm{d}x - \int \sin^{-\frac{1}{2}} x (\sin x)' \, \mathrm{d}x = \frac{\sin^{-\frac{3}{2}} x}{-\frac{3}{2}} - \frac{\sin^{\frac{1}{2}} x}{\frac{1}{2}} + C = -\frac{2}{3\sqrt{\sin^3 x}} - 2\sqrt{\sin x} + C$$

4

$$\int \frac{1}{(x^2+1)\arctan^2 x} \, \mathrm{d}x = \int (\arctan x)' \cdot \arctan^{-2} x \, \mathrm{d}x = \frac{\arctan^{-1}}{-1} + C = -\frac{1}{\arctan x} + C$$

5

$$\int (x^2 - 3x) e^{3x} \, \mathrm{d}x = \int (x^2 - 3x) \cdot \left(\frac{e^{3x}}{3}\right)' \mathrm{d}x = (x^2 - 3x) \cdot \frac{e^{3x}}{3} - \int (x^2 - 3x) \cdot \frac{e^{3x}}{3} \, \mathrm{d}x =$$

$$= \frac{1}{3}(x^2 - 3x)e^{3x} - \int (2x - 3) \cdot \frac{e^{3x}}{3} \, \mathrm{d}x,$$

$$\int (2x - 3) \cdot \frac{e^{3x}}{3} \, \mathrm{d}x = \int (2x - 3) \cdot \left(\frac{e^{3x}}{9}\right)' \mathrm{d}x =$$

$$= (2x - 3) \cdot \frac{e^{3x}}{9} - \int \frac{2e^{3x}}{9} \, \mathrm{d}x = (2x - 3) \cdot \frac{e^{3x}}{9} - \frac{2}{9} \cdot \frac{e^{3x}}{3} = \frac{(6x - 11)e^{3x}}{9} + C,$$

$$\frac{1}{3}(x^2 - 3x)e^{3x} - \frac{(6x - 11)e^{3x}}{9} + C = \frac{3(x^2 - 3x)e^{3x}}{9} - \frac{(6x - 11)e^{3x}}{9} + C = \text{inkabb nem}$$

6

$$\int (x^2+1)\cdot\cos(2x)\,\mathrm{d}x = \int (x^2+1)\cdot\left(\frac{\sin(2x)}{2}\right)'\mathrm{d}x = (x^2+1)\cdot\cos(2x) - \int \frac{2x\sin(2x)}{2}\,\mathrm{d}x =$$

$$= (x^2+1)\cdot\cos(2x) - \left[\frac{x\cos(2x)}{2} - \int -\frac{\cos(2x)}{2}\,\mathrm{d}x\right] =$$

$$= (x^2+1)\cdot\cos(2x) - \frac{x\cos(2x)}{2} + \frac{1}{2}$$

7

$$\int \arcsin(2x)\,\mathrm{d}x$$

8

$$\int (\arcsin x)^2\,\mathrm{d}x \underset{u=\arcsin(x)}{=}$$

9

$$\int e^{-x}\cdot\cos 5x\,\mathrm{d}x = \int (-e^{-x})'\cdot\cos 5x\,\mathrm{d}x =$$

$$= -e^{-x}\cdot\cos 5x - \int e^{-x}\cdot 5\sin(5x)\,\mathrm{d}x =$$

$$= -e^{-x}\cdot\cos 5x - \left(-e^{-x}\cdot 5\sin 5x - \int -e^{-x}\cdot 25\cos 5x\,\mathrm{d}x\right) =$$

$$= -e^{-x}\cdot\cos 5x - \left(-e^{-x}\cdot 5\sin 5x + 25\int e^{-x}\cos 5x\,\mathrm{d}x\right) \Longrightarrow$$

$$\Longrightarrow \frac{5e^{-x}\sin(5x) - e^{-x}\cdot\cos(5x)}{26} + C = \frac{e^{-x}(5\sin(5x) - \cos(5x))}{26} + C$$