Hazi

1.

$$f(x) := x^3 + x, \quad (x \in \mathbb{R})$$

megoldas

$$f'(x) = 3x^2 + 1 > 0 \quad (\forall x \in \mathbb{R})$$

tehat invertalhato, es

$$\left(f^{-1}\right)'(2) = \frac{1}{f'(1)} = \frac{1}{3 \cdot 1^2 + 1} = \frac{1}{4}$$

2.

$$f(x) := \sin\left(\frac{x-1}{x^2+1}\right) \quad (x \in \mathbb{R}), \quad a = \frac{1}{2}$$

megoldas

Mivel

$$f(a) = \sin\left(\frac{-\frac{1}{2}}{\frac{5}{4}}\right) = \sin\left(-\frac{4}{10}\right) = -\sin\left(\frac{2}{5}\right)$$

$$f'(x) = \cos\left(\frac{x-1}{x^2+1}\right) \cdot \frac{x^2+1-(x-1)(2x)}{\left(x^2+1\right)^2} = \cos\left(\frac{x-1}{x^2+1}\right) \cdot \frac{x^2+1-2x^2+2x}{\left(x^2+1\right)^2} = \cos\left(\frac{x-1}{x^2+1}\right) \cdot \frac{-x^2+2x+1}{\left(x^2+1\right)^2}$$

Ekkor

$$f'\left(\frac{1}{2}\right) = -\cos\left(\frac{2}{5}\right) \cdot \frac{-\frac{4}{25}+2}{\left(\frac{4}{25}+1\right)^2} = -\cos\left(\frac{2}{5}\right) \cdot \frac{\frac{46}{25}}{\left(\frac{29}{25}\right)^2}$$

hagyjuk inkabb oszinten

3.

$$f(x) := \begin{cases} 1-x, & x < 0 \\ e^{-x}, & x \geq 0 \end{cases}$$

megoldas

ha $x < 0$

$$f'(x) = -1$$

ha $x \geq 0$

$$f'(x) = e^{-x}$$

ha $x = 0$

$$\frac{f(x)-f(0)}{x-0} = \frac{f(x)-1}{x} = \begin{cases} \frac{1-x-1}{x} = -1 \\ \frac{e^{-x}-1}{x} \end{cases}$$

$$\lim_{x \to 0} \frac{e^{-x}-1}{x} = \lim_{x \to 0} \frac{e^0 - e^0}{x} = ? = -1$$

Gyakorlo

1/d.

$$f(x) := \frac{1}{\ln^2\left(x - \frac{1}{x}\right)} \qquad (x > 1), \quad a = 2$$

$$u(x) = x - \frac{1}{x} \qquad u(a) = \frac{3}{2}$$

$$v(x) = \ln(u(x)) \qquad v(a) = \ln\left(\frac{3}{2}\right)$$

$$f(a) = \frac{1}{[v(a)]^2} = \frac{1}{\left[\ln\left(\frac{3}{2}\right)\right]^2}$$

$$f(x) = [v(x)]^{-2} \Longrightarrow f'(x) = -2[v(x)]^{-3} \cdot v'(x)$$

$$v'(x) = \frac{1}{x - \frac{1}{x}} \cdot \left(1 + \frac{1}{x^2}\right) = \frac{1}{\frac{x^2-1}{x}} \cdot \frac{x^2+1}{x^2} = \frac{x}{x^2-1} \cdot \frac{x^2+1}{x^2} = \frac{x^2+1}{x(x^2-1)}$$

$$f'(x) = -2 \cdot \left[\ln\left(x - \frac{1}{x}\right)\right]^{-3} \cdot \frac{x^2+1}{x(x^2-1)}$$

$$f'(2) = -2 \cdot \left[\ln\left(\frac{3}{2}\right)\right]^{-3} \cdot \frac{5}{6} = -\frac{5}{3} \cdot \frac{1}{\ln\left(\frac{3}{2}\right)^3}$$

$$y = f'(2)(x-2) + f(2) = -\frac{5}{3} \cdot \frac{1}{\ln\left(\frac{3}{2}\right)^3} \cdot (x-2) + \frac{1}{\ln^2\left(\frac{3}{2}\right)}$$

1/e.

$$f(x) := x^{\ln x} \qquad (x > 0) \qquad a = e^2$$

$$u(x) = x \qquad u'(x) = 1 \qquad u(a) = e^2$$

$$v(x) = \ln x \qquad v'(x) = \frac{1}{x} \qquad v(a) = \ln e^2$$

$$f(a) = e^{2 \cdot \ln e^2} = e^4$$

$$f'(x) = e^{(\ln x)^2} \cdot \left[(\ln x)^2\right]' = f(x) \cdot 2\ln x \cdot \frac{1}{x} = x^{\ln x} \frac{2\ln x}{x}$$

$$f(a) = e^4$$

$$f'(a) = e^4 \frac{2 \cdot 2}{e^2} = e^4 \frac{4}{e^2} = 4e^2$$

$$y = f'(a)(x-a) + f(a) = 4e^2\left(x - e^2\right) + e^4 = 4e^2x - 3e^4$$

3/c.

$$\sqrt{x}+\sqrt{y}=3, \quad (4,1)$$

$$\sqrt{x}+\sqrt{y}=3 \Longrightarrow \sqrt{y}=3-\sqrt{x} \Longrightarrow y=\left(3-\sqrt{x}\right)^2 \Longrightarrow y=x-6\sqrt{x}+9$$

$$m=\lim_{x\to 4}\frac{y(x)-y(4)}{x-4}=\lim_{x\to 4}\frac{x-6\sqrt{x}+9-1}{x-4}=\lim_{x\to 4}\frac{x-6\sqrt{x}+8}{x-4}=\lim_{x\to 4}\frac{(\sqrt{x}-4)(\sqrt{x}-2)}{x-4}=\lim_{x\to 4}\frac{(\sqrt{x}-4)(\sqrt{x}-2)}{(\sqrt{x}-2)(\sqrt{x}+2)}=$$

$$=\lim_{x\to 4}\frac{\sqrt{x}-4}{\sqrt{x}+2}=-\frac{2}{4}=-\frac{1}{2}$$

$$y=-\frac{1}{2}x+3$$

6/d.

$$f(x):=\begin{cases}\cos x, & x\le 0\\ a\sin x+x+b, & x>0\end{cases}$$

1. x < 0:

$$f'(x)=-\sin x$$

1. x > 0:

$$f'(x)=a\cos x+1$$

1. x = 0

1. folytonossag

$$\left.\begin{array}{l}\lim\limits_{x\to 0-0}\cos x=\cos 0=1\\ \lim\limits_{x\to 0+0}a\sin x+x+b=a\sin 0+0+b=b\end{array}\right\}\Longrightarrow \text{akkor folytonos ha } b=1$$

2. jobb-/bal oldali derivalt egyezes

$$\left.\begin{array}{l}f'_-(0)=-\sin 0=0\\ f'_+(0)=a\cos 0+1=a+1\end{array}\right\}\Longrightarrow \text{akkor egyezik ha } a+1=0 \Longleftrightarrow a=-1$$

eredmeny:

$$f'(x)=\begin{cases}-\sin x, & x<0\\ 0, & x=0\\ -\cos +1, & x>0\end{cases}$$