householder matrixot altalaban sosem szamoljuk ki es sosem szorzunk vele ezert nem fogjuk sosem felirni

hasonlit a ge-hez, mert itt is csak sormuveleteket fogunk vegezni

ez es csak ez nagyon kelleni fog:

$$x \in \mathbb{R}^n H(v)x = (I - 2vv^T)x = x - 2v \underbrace{(v^T x)}_{\in \mathbb{R}}$$

vizualisan az x es v a ket szelen es kozepen a skalaris szorzatuk ketszereset vonjuk ki.

tulajdonsagok:

$$H^T = H(\text{szimmetrikus})$$

$$H^2 = I, \text{azaz } H^{-1} = H(\text{ortogonalis})$$

$$H(v) \cdot v = -v$$

$$\forall y \perp v : H(v) \cdot y = y$$

tetel

$$\text{legyen } a, b \in \mathbb{R}^n, a \neq b, \|a\|_2 = \|b\|_2! + 0 :$$

$$v = \pm \frac{a - b}{\|a - b\|_2}$$

3

a =

$$\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$$

b =

$$\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$

$$v \text{ melyre } H(v)a = b$$

$$\|a\|_2 = \|b\|_2 = \sqrt{5}$$

$$a - b = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \to \|a - b\|_2 = \sqrt{6}$$

$$v = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

ellenorzo lepes (nem kell ha jol csinaltuk ig)

$$H(v)\cdot a = a - 2(v^T a)\cdot v = \begin{pmatrix}2\\0\\1\end{pmatrix} - 2\frac{1}{\sqrt{6}}(1\ \ -2\ \ 1)\cdot\begin{pmatrix}2\\0\\1\end{pmatrix}\cdot\frac{1}{\sqrt{6}}\begin{pmatrix}1\\-2\\1\end{pmatrix} = \begin{pmatrix}2\\0\\1\end{pmatrix}\cdot\frac{2}{6}\cdot 3\cdot\begin{pmatrix}1\\-2\\1\end{pmatrix} = \begin{pmatrix}1\\2\\0\end{pmatrix}$$

4

a =

$$\begin{pmatrix}-1\\1\\-1\\1\end{pmatrix}$$

$$k\cdot e_1 = \begin{pmatrix}k\\0\\0\\0\end{pmatrix}$$

$$2 = \|a\|_2 = \|k\cdot e_1\|_2 = \|k\|_2 \to k = +2$$

pozitiv mert az elso elem a-ban negativ

$$a - k\cdot e_1 = \begin{pmatrix}-1-2\\1\\-1\\1\end{pmatrix} = \begin{pmatrix}-3\\1\\-1\\1\end{pmatrix}$$

$$\|a - k\cdot e_1\|_2 = \sqrt{12} \Longrightarrow v = \frac{1}{\sqrt{12}}\cdot\begin{pmatrix}-3\\1\\-1\\1\end{pmatrix}$$

HF: ell: $H(v)\cdot a = k\cdot e_1 = \begin{pmatrix}2\\0\\0\\0\end{pmatrix}$

$$H(v)\cdot b = b - 2(v^T b)\cdot v = \begin{pmatrix}2\\1\\1\\0\end{pmatrix} - 2\cdot\frac{1}{\sqrt{12}}\cdot(-3\ \ 1\ \ -1\ \ 1)\cdot\begin{pmatrix}2\\1\\1\\0\end{pmatrix}\cdot\frac{1}{\sqrt{12}}\cdot\begin{pmatrix}-3\\1\\-1\\1\end{pmatrix} = \begin{pmatrix}2\\1\\1\\0\end{pmatrix} - \frac{2}{12}\cdot(-6)\begin{pmatrix}-3\\1\\-1\\1\end{pmatrix} = \begin{pmatrix}-1\\2\\0\\1\end{pmatrix}$$

5

$$\begin{pmatrix} 4 & -2 & | & 0 \\ 3 & 1 & | & 5 \end{pmatrix} \to \text{householder} \to \begin{pmatrix} -5 & 1 & | & -3 \\ 0 & 2 & | & 4 \end{pmatrix} \Longrightarrow$$

$$\Longrightarrow 2x_2 = 4 \to x_2 = 2$$

$$x_1 = 1$$

mert

$$\|a_1\|_2 = 5 \to \sigma_1 = -5$$

$$a_1 - \sigma_1 e_1 = \begin{pmatrix} 5 & -(-5) \\ 3 \end{pmatrix} - \begin{pmatrix} 9 \\ 3 \end{pmatrix} = 3 \cdot \begin{pmatrix} 3 \\ 1 \end{pmatrix} \Longrightarrow \|...\|_2 = 3 \cdot \frac{1}{\sqrt{10}}$$

$$v_1 = \frac{1}{\sqrt{10}} \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

Tudjuk hogy $H(v_1) \cdot a_1 = -5 \cdot a_1$

$$H(v_1)\text{-et alkalmazzuk } a_2\text{-re :}$$

$$H(v_1) \cdot a_{22} = a_2 - 2(v_1^T \cdot a_2)v_1 = \begin{pmatrix} -2 \\ 1 \end{pmatrix} - 2 \cdot \frac{1}{\sqrt{10}}(3 \;\; 1) \cdot \begin{pmatrix} -2 \\ 1 \end{pmatrix} \cdot \frac{1}{\sqrt{10}} \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} - \frac{2}{10} \cdot (-5) \cdot \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -2+3 \\ 1+1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$H(v_1)\text{-et alkalmazzuk } b\text{-re :}$$

$$\begin{pmatrix} 0 \\ 5 \end{pmatrix} - 2 \cdot \frac{1}{\sqrt{10}}(3 \;\; 1) \cdot \begin{pmatrix} 0 \\ 5 \end{pmatrix} \cdot \frac{1}{\sqrt{10}} \cdot \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} - \frac{2}{10} \cdot 5 \cdot \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$$

6

$$\begin{pmatrix} 1 & 0 & 1 & | & 2 \\ 0 & 2 & 0 & | & 2 \\ 1 & 0 & 3 & | & 4 \end{pmatrix} \to \text{householder} \to \begin{pmatrix} -\sqrt{2} & 0 & -2\sqrt{2} & | & -3\sqrt{2} \\ 0 & 2 & 0 & | & 2 \\ 0 & 0 & \sqrt{2} & | & \sqrt{2} \end{pmatrix}$$