$$\deg(f \cdot g) = \deg(f) + \deg(g)$$
$$\deg(f + g) \leq \max\{\deg(f), \deg(g)\}$$
f, g polinomok

1.

$\deg(f \cdot g) =?$

a:

$$f = 3x^7 + 5x^6 + 4x^5 - 7x + 1 \in \mathbb{Q}[x], \quad g = -x^8 + 5x^7 - 11x^6 + 7x^3 + 5x - 9 \in \mathbb{Q}[x]$$

$$\deg(f) = 7, \quad \deg(g) = 8$$

$c_0 = 1 \cdot (-9) = (-9)$
$c_2 = (-7) \cdot 5 = (-35)$
$c_{14} = (-1) \cdot 5 + 5 \cdot 3 = 10$
$c_{15} = 3 \cdot (-1) = (-3)$

b:

$$f = 3x^7 + 3x^6 + 4x^5 + 3x + 1 \in \mathbb{Z}_5[x], \quad g = 4x^8 + 3x^7 + 4x^6 + 7x^3 + 4x + 1 \in \mathbb{Z}_5[x]$$

megjegyzes

$$p = c_0 \cdot x^0 + c_1 \cdot x^1 + ...$$

$$\deg(f \cdot g) = \deg(f) + \deg(g) = 7 + 8 + 15$$

$c_0 = 1 \cdot 1 = 1$
$c_2 = 3 \cdot 4 = 12 \equiv 2$
$c_{14} = 3 \cdot 3 + 3 \cdot 4 \equiv 9 + 12 \equiv 21 \equiv 1$
$c_{15} = 3 \cdot 4 = 12 \equiv 2$

2.

a:

$$f = 3x^7 + 5x^6 + 4x^5 - 7x + 1 \in \mathbb{Q}[x], \quad c = 2 \quad f(c) =?$$

	3	5	4	0	0	0	-7	1
2		3	11	26	52	104	208	409

$\Longrightarrow 819 = f(2)$

b:

$$f = 3x^7 + 3x^6 + 4x^5 + 3x + 1 \in \mathbb{Z}_5[x], \quad c = 1 \quad f(1) =?$$

	3	3	4	0	0	0	3	1
1		3	6	$10 \equiv 0$	$10 \equiv 0$	$10 \equiv 0$	$10 \equiv 0$	$13 \equiv 3$

$\Longrightarrow 14 \equiv 4 = f(1)$

3

f-et oszd el maradekosan g-vel

a:

$$f = 6x^6 - 4x^5 + 11x^4 + 15x^3 - 20x^2 + 18x - 14 \in \mathbb{Q}[x], \quad g = 2x^4 + 3x^2 + 7x - 3 \in \mathbb{Q}[x]$$

$\begin{pmatrix} 6 & -4 & 11 & 15 & -20 & 18 & -14 \end{pmatrix} : \begin{pmatrix} 2 & 0 & 3 & 7 & -3 \end{pmatrix} = \begin{pmatrix} 3 & -2 & 1 \end{pmatrix}$

$\begin{pmatrix} 6 & 0 & 9 & 21 & -9 \end{pmatrix}$

$\begin{pmatrix} 0 & 2 & -6 & -11 & 18 & -14 \end{pmatrix}$

$\begin{pmatrix} -4 & 0 & -6 & -14 & 6 \end{pmatrix}$

$\begin{pmatrix} 2 & 0 & 3 & 12 & -14 \end{pmatrix}$

$\begin{pmatrix} 2 & 0 & 3 & 7 & -3 \end{pmatrix}$

$\begin{pmatrix} 0 & 0 & 5 & -11 \end{pmatrix} = 5x = 11$

c:

$$f = x^8 + x^7 + x^6 + 2x^5 + 2x^4 + x^3 + x \in \mathbb{Z}_3[x], \quad g = x^5 + x^4 + 2x^2 + 2 \in \mathbb{Z}_3[x]$$

$\begin{pmatrix} 1 & 1 & 1 & 2 & 2 & 1 & 0 & 1 & 0 \end{pmatrix} : \begin{pmatrix} 1 & 1 & 0 & 2 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 2 \end{pmatrix} = x^3 + x + 2$

$\begin{pmatrix} 1 & 1 & 0 & 2 & 0 & 2 \end{pmatrix}$

$\begin{pmatrix} 0 & 1 & 0 & 2 & 2 & 0 & 1 & 0 \end{pmatrix}$

$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

$\begin{pmatrix} 1 & 0 & 2 & 2 & 0 & 1 & 0 \end{pmatrix}$

$\begin{pmatrix} 1 & 1 & 0 & 2 & 0 & 2 \end{pmatrix}$

$\begin{pmatrix} 2 & 2 & 0 & 0 & 2 & 0 \end{pmatrix}$

$\begin{pmatrix} 2 & 2 & 0 & 1 & 0 & 1 \end{pmatrix}$

$\begin{pmatrix} 0 & 0 & 2 & 2 & 2 \end{pmatrix} = 2x^2 + 2x + 2$

4

$$f = (1\ \ 0\ \ 0\ \ -3\ \ 1\ \ -2\ \ 2\ \ -1\ \ 2), \quad g = (1, 0, -1, -3, 0, 1, 2)$$

$x^8 - 3x^5 + x^4 - 2x^3 + 2x^2 - x + 2$		1	0
$x^6 - x^4 - 3x^3 + x + 2$		0	1
$2x^4 - 2x$	$x^2 + 1$	1	$-x^2 - 1$
$-2x^3 + 2$	$\frac{1}{2}x^2 - \frac{1}{2}$	$-\frac{1}{2}x^2 + \frac{1}{2}$	$\frac{1}{2} + \frac{1}{2}x^4$
0	$-x$	$-\frac{1}{2}x^3 + \frac{1}{2}x + 1$	$\frac{1}{2}x^5 - x^2 + \frac{1}{2}x - 1$

$\begin{pmatrix}1 & 0 & 0 & -3 & 1 & -2 & 2 & -1 & 2\end{pmatrix} : \begin{pmatrix}1 & 0 & -1 & -3 & 0 & 1 & 2\end{pmatrix} = \begin{pmatrix}1 & 0 & 1\end{pmatrix} = x^2 + 1$

$\begin{pmatrix}1 & 0 & -1 & -3 & 0 & 1 & 2\end{pmatrix}$

$\begin{pmatrix}0 & 1 & 0 & 1 & -3 & 0 & -1 & 2\end{pmatrix}$

$\begin{pmatrix}0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{pmatrix}$

$\begin{pmatrix}1 & 0 & 1 & -3 & 0 & -1 & 2\end{pmatrix}$

$\begin{pmatrix}1 & 0 & -1 & -3 & 0 & 1 & 2\end{pmatrix}$

$\begin{pmatrix}0 & 2 & 0 & 0 & -2 & 0\end{pmatrix} = 2x^4 - 2x$

$\begin{pmatrix}1 & 0 & -1 & -3 & 0 & 1 & 2\end{pmatrix} : \begin{pmatrix}2 & 0 & 0 & -2 & 0\end{pmatrix} = \begin{pmatrix}\frac{1}{2} & 0 & -\frac{1}{2}\end{pmatrix} = \frac{1}{2}x^2 - \frac{1}{2}$

$\begin{pmatrix}1 & 0 & 0 & -1 & 0\end{pmatrix}$

$\begin{pmatrix}0 & -1 & -2 & 0 & 1 & 2\end{pmatrix}$

$\begin{pmatrix}0 & 0 & 0 & 0 & 0 & 0\end{pmatrix}$

$\begin{pmatrix}-1 & -2 & 0 & 1 & 2\end{pmatrix}$

$\begin{pmatrix}-1 & 0 & 0 & 1 & 0\end{pmatrix}$

$\begin{pmatrix}-2 & 0 & 0 & 2\end{pmatrix} = -2x^3 + 2$

$\begin{pmatrix}2 & 0 & 0 & -2 & 0\end{pmatrix} : \begin{pmatrix}-2 & 0 & 0 & 2\end{pmatrix} = \begin{pmatrix}-1 & 0\end{pmatrix} = -x$

$\begin{pmatrix}2 & 0 & 0 & -2\end{pmatrix}$

$\begin{pmatrix}0 & 0 & 0 & 0\end{pmatrix}$

$\begin{pmatrix}0 & 0 & 0 & 0\end{pmatrix}$

$\begin{pmatrix}0\end{pmatrix} = 0$

5

$$x^3 + px + q \quad \mathbb{C} \text{ felett oszthato legyen } x^2 + mx - 1$$

$\begin{pmatrix}1 & 0 & p & q\end{pmatrix} : \begin{pmatrix}1 & m & -1\end{pmatrix} = \begin{pmatrix}1 & -m\end{pmatrix} =$

$\begin{pmatrix}1 & m & -1\end{pmatrix}$

$\begin{pmatrix}-m & p{+}1 & q\end{pmatrix}$

$\begin{pmatrix}-m & -m^2 & m\end{pmatrix}$

$\begin{pmatrix}p{+}1{+}m^2 & q{-}m\end{pmatrix} = 0$

$$\left.\begin{array}{l} p + 1 + m^2 = 0 \\ q - m = 0 \end{array}\right\} \Longrightarrow q = m, \quad p = -1 - m^2$$