1. Számítsa ki az $f'(x)$-et, ha

a) $f(x) := \frac{2x^2-1}{x\sqrt{1+x^2}} \qquad (x > 0)$

$$u(x) = 2x^2 - 1 \qquad v(x) = x\sqrt{1+x^2}$$

$$u'(x) = 4x$$

$$v'(x) = 1(1+x^2)^{\frac{1}{2}} + x \cdot \frac{1}{2}(1+x^2)^{-\frac{1}{2}} \cdot 2x = (1+x^2)^{\frac{1}{2}} + \frac{x^2}{(1+x^2)^{\frac{1}{2}}} = \frac{1+x^2+x^2}{\sqrt{1+x^2}} = \frac{1+2x^2}{\sqrt{1+x^2}}$$

$$f'(x) = \frac{4x \cdot x\sqrt{1+x^2} - 2x^2 - 1 \cdot \frac{1+2x^2}{\sqrt{1+x^2}}}{\left(x\sqrt{1+x^2}\right)^2}$$

$$\text{szamlalo: } 4x \cdot x\sqrt{1+x^2} - 2x^2 - 1 \cdot \frac{1+2x^2}{\sqrt{1+x^2}} = 4x^2\sqrt{1+x^2} - 2x^2 \cdot \frac{(2x^2-1)(1+2x^2)}{\sqrt{1+x^2}} =$$

$$= \frac{4x^2(1+x^2) - (2x^2-1)(1+2x^2)}{\sqrt{1+x^2}}$$

$$\text{nevezo: } \left(x\sqrt{1+x^2}\right)^2 = x^2(1+x^2)$$

$$f'(x) = \frac{\frac{4x^2+4x^4-(2x^2+4x^4-1-2x^2)}{\sqrt{1+x^2}}}{x^2(1+x^2)} = \frac{\frac{4x^2+4x^4-2x^2-4x^4+1+2x^2}{\sqrt{1+x^2}}}{x^2(1+x^2)} = \frac{\frac{4x^2+1}{\sqrt{1+x^2}}}{x^2(1+x^2)} = \frac{4x^2+1}{x^2(1+x^2)\sqrt{1+x^2}}$$

b) $f(x) := \frac{e^x}{1+e^x} \qquad (x \in \mathbb{R})$

$$u(x) = e^x \qquad u'(x) = e^x$$

$$v(x) = 1 + e^x \qquad v'(x) = e^x$$

$$f'(x) = \frac{e^x(1+e^x) - e^x \cdot e^x}{(1+e^x)^2} = \frac{e^x + e^{2x} - e^{2x}}{(1+e^x)^2} = \frac{e^x}{(1+e^x)^2}$$

c) $f(x) := 3^{x^2} \qquad (x \in \mathbb{R})$

$$u(x) = 3^x \qquad u'(x) = \ln 3 \cdot 3^x$$

$$v(x) = x^2 \qquad v'(x) = 2x$$

$$f'(x) = \ln 3 \cdot 3^{x^2} \cdot 2x$$

d) $f(x) := \frac{1}{x} + \sqrt{1 + \frac{1}{x^2}} \qquad (x > 0)$

$$u(x) = \frac{1}{x} \qquad u'(x) = -\frac{1}{x^2}$$

$$v(x) = 1 + \frac{1}{x^2} \qquad \sqrt{v(x)} = (v(x))^{\frac{1}{2}} \qquad v'(x) = -\frac{2}{x^3}$$

$$\left(\sqrt{v(x)}\right)' = \frac{1}{2}v(x)^{-\frac{1}{2}} \cdot v'(x) = -\frac{1}{x^3\sqrt{\frac{2}{x^2}}}$$

$$f'(x) = -\frac{1}{x^2} + \frac{1}{2} - \frac{1}{x^3\sqrt{\frac{2}{x^2}}}$$

e) $f(x) := 2\operatorname{tg} x - 3\operatorname{ctg} x \qquad (x \in (0, \frac{\pi}{2}))$

$$\operatorname{tg}' x = \sec^2 x \qquad \operatorname{ctg}' x = -\csc^2 x$$

$$f'(x) = 2\sec^2 x + 3\csc^2 x$$

f) $f(x) := (2 + \sin x)^{\cos x} \qquad (x \in \mathbb{R})$

$$\ln f(x) = \cos x \ln(2 + \sin x)$$

$$f'(x) = -\sin x \ln(2 + \sin x) + \cos x \frac{\cos x}{2 + \sin x} = -\sin x \ln(2 + \sin x) + \frac{\cos^2 x}{2 + \sin x}$$

$$f'(x) = (2 + \sin x)^{\cos x} \left(-\sin x \ln(2 + \sin x) + \frac{\cos^2 x}{2 + \sin x} \right)$$