Diszkret matek hazi V

6

6a a) $\{z:\ \mathrm{Re}((1+i)z) \le 0\}$;

$$(1+i)z = (1+i)(x+yi) \le 0$$

$$x + yi + xi + yi^2$$

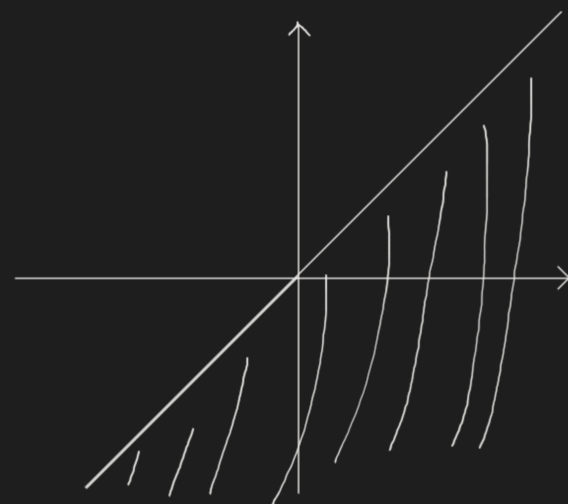
$$x + yi + xi - y$$

$$(x-y) + i(y+x)$$

$$\mathrm{Re}((1+i)z) = (x-y) \le 0$$

$$x - y \le 0$$

$$x \le y$$



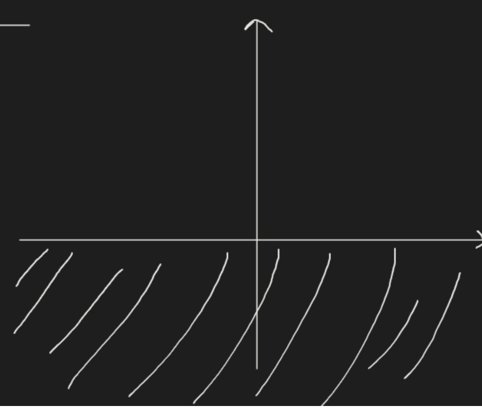
b) $\{z:\ \mathrm{Im}(1/z) \ge 0\}$;

$$\frac{1}{z} = \frac{1}{x+yi} = \frac{x-yi}{(x+yi)(x-yi)} = \frac{x-yi}{x^2+y^2}$$

$$\mathrm{Im}\left(\frac{1}{z}\right) = \frac{-y}{x^2+y^2} \ge 0$$

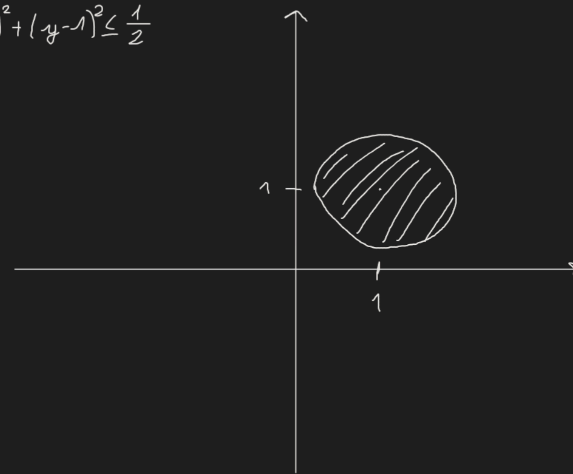
$$-y \ge 0$$

$$y \le 0$$



c) $\{z:\ |(1+i)(z-i-1)| \le 1\}$.

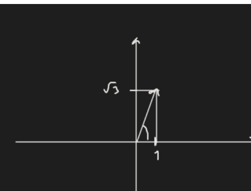
$$|(1+i)((x+yi)-i-1)| \le 1$$
$$|x+yi-i-1+xi+yi^2-i^2-1i| \le 1$$
$$|x+yi-i-\cancel{1}+xi-y+\cancel{1}-1i| \le 1$$
$$|x+yi-2i+xi-y| \le 1$$
$$|(x-y)+i(x+y-2)| \le 1$$
$$\sqrt{(x-y)^2+(x+y-2)^2} \le 1 \quad /()^2$$
$$(x-y)^2+(x+y-2)^2 \le 1$$
$$x^2-\cancel{2xy}+y^2+x^2+\cancel{2xy}+y^2-4x-4y+4 \le 1$$
$$2x^2+2y^2-4x-4y+4 \le 1$$
$$2(x^2-2x)+2(y^2-2y) \le -3$$
$$2(x-1)^2-2+2(y-1)^2-2 \le -3$$
$$2(x-1)^2+2(y-1)^2 \le 1$$
$$(x-1)^2+(y-1)^2 \le \frac{1}{2}$$



7

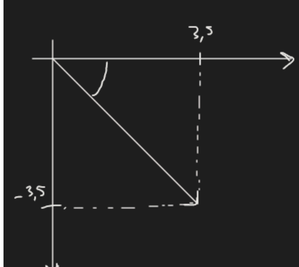
a) $1+\sqrt{3}i$;

$$r=\sqrt{1^2+(\sqrt{3})^2}=2 \qquad z=\left(\cos\frac{\pi}{6}+i\cdot\sin\frac{\pi}{6}\right)$$
$$\cos\frac{\sqrt{3}}{2}=\frac{\pi}{6}, \quad \sin\frac{1}{2}=\frac{\pi}{6}$$



b) $\frac{7}{1+i}$;

$$\frac{7}{1+i}\cdot\frac{1-i}{1-i}=\frac{7-7i}{1-i^2}=\frac{7-7i}{2}=\frac{7}{2}-\frac{7}{2}i, \qquad r=\sqrt{\left(\frac{7}{2}\right)^2+\left(-\frac{7}{2}\right)^2}=\sqrt{\frac{49}{4}+\frac{49}{4}}=\sqrt{\frac{49}{2}}=\frac{7}{\sqrt{2}}$$



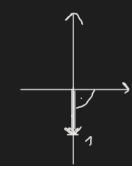
$$\cos\frac{-\frac{7}{2}}{\frac{7}{\sqrt{2}}}=-\frac{7}{2}\cdot\frac{\sqrt{2}}{7}=-\frac{7\sqrt{2}}{14}=-\frac{\sqrt{2}}{2}=-\frac{\pi}{4}$$

$$z=\frac{7}{\sqrt{2}}\left(\cos\left(-\frac{\pi}{4}\right)+i\cdot\sin\left(-\frac{\pi}{4}\right)\right)$$

c) $\frac{1-\sqrt{3}i}{\sqrt{3}+i}$.

$$\frac{1-\sqrt{3}i}{\sqrt{3}+i}\cdot\frac{\sqrt{3}-i}{\sqrt{3}-i}=\frac{\sqrt{3}-i-3i+\sqrt{3}i^2}{3-i^2}=\frac{\cancel{\sqrt{3}}-i-3i-\cancel{\sqrt{3}}}{3+1}=\frac{-4i}{4}=-i$$

$$r=\sqrt{0^2+(-1)^2}=1$$



$$\underline{z=\left(\cos-\frac{\pi}{2}+i\cdot\sin-\frac{\pi}{2}\right)}$$

8

$$R=\{(z,w)\in\mathbb{C}^2:|z|=|w|\},\quad S=\{(z,w)\in\mathbb{C}^2:\mathrm{Re}(z)=\mathrm{Re}(w)\}.$$

Mi lesz $(R\circ S)(\{1\})$, ill. $(S\circ R)(\{1\})$? (**1 pont**)

$S(\{1\})$:

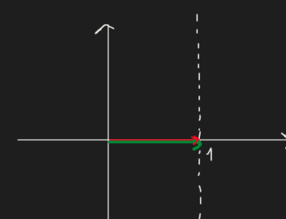
$\mathrm{Re}(1)=\mathrm{Re}(w) \Rightarrow \mathrm{Re}(w)=\mathrm{Re}(1)$

$S(\{1\})=\{(1,w) : w=1+iy,\ y\in\mathbb{R}\}$

$R\circ S(\{1\})$: $|1|=|w|$

$|w|=\sqrt{1^2+y^2}=1$ (ha $y=0$)

$R\circ S(\{1\})=\{(1,1)\}$



Mind az S és az R feltételében egyenlőség van, melynek nincs 'iránya'. Ebből következik hogy a kompozíció sorrendtől függetlenül ugyanazt az eredményt adja

$$(S\circ R)(\{1\})=(R\circ S)(\{1\})=(\{1,1\})$$