Diszkret matek hazi VI

a) $(1-i)^2(1-\sqrt{3}i)$;

$r_1 = (\sqrt{2})^2 = 2 \qquad z_1 = 2\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

$r_2 = 2 \qquad z_2 = 2\left(\cos\frac{5\pi}{3} + i\cdot\sin\frac{5\pi}{3}\right)$

$$4\left(\cos\frac{7\pi}{6} + i\cdot\sin\frac{7\pi}{6}\right)$$

b) $\frac{(\sqrt{3}+i)^5}{-1-i}$;

$r_1 = 2^5 = 32 \qquad z_1 = 32\left(\cos\frac{5\pi}{6} + i\cdot\sin\frac{5\pi}{6}\right)$

$r_2 = \sqrt{2} \qquad z_2 = \sqrt{2}\left(\cos\frac{5\pi}{4} + i\cdot\sin\frac{5\pi}{4}\right)$

$$\frac{32}{\sqrt{2}}\left(\cos\frac{19\pi}{12} + i\sin\frac{19\pi}{12}\right)$$

$$\frac{5\pi}{6} - \frac{5\pi}{4} = \frac{10\pi - 15\pi}{12} = \frac{-5\pi}{12} + \frac{24\pi}{12} = \frac{19\pi}{12}$$

c) $(1-i)^{100}$.

$r = (\sqrt{2})^{100} = 2^{50} \qquad z = 2^{50}(\cos\pi + i\cdot\sin\pi)$

$$\frac{3\pi}{2} + \frac{\pi}{4} = \frac{7\pi}{4}$$

$$\frac{7\pi}{4}\cdot 100 = 175\pi = \pi$$

Számítsa ki a $(\sqrt{3}+i)^5/(1-i)^7$ komplex szám harmadik gyökeit! (1, 2)

$r_1 = 2^5 \qquad z_1 = 32\left(\cos\frac{5\pi}{6} + i\cdot\sin\frac{5\pi}{6}\right), \qquad r_2 = (\sqrt{2})^7 \qquad z_2 = (\sqrt{2})^7\left(\cos\frac{49\pi}{4} + i\cdot\sin\frac{49\pi}{4}\right)$

$z_2 = (\sqrt{2})^7\left(\cos\frac{\pi}{4} + i\cdot\sin\frac{\pi}{4}\right)$

$/:\ 2\sqrt{2}\left(\cos\frac{7\pi}{12} + i\cdot\sin\frac{7\pi}{12}\right)$

$\frac{32}{(\sqrt{2})^7} = 2\sqrt{2}, \qquad \frac{5\pi}{6} - \frac{\pi}{4} = \frac{10\pi - 3\pi}{12} = \frac{7\pi}{12}$

$$\sqrt{2}\left(\cos\left(\frac{7\pi}{12} + \frac{2k\pi}{3}\right) + i\cdot\sin\left(\frac{7\pi}{12} + \frac{2k\pi}{3}\right)\right) \quad k = 0,1,2$$

$\left(r = \sqrt[3]{2\sqrt{2}} = 2^{\frac{1}{3}}\cdot 2^{\frac{1}{6}} = 2^{\frac{1}{2}} = \sqrt{2}\right)$