gy04/6

$$\begin{pmatrix} l_1 & 0 & 0 & 0 \\ l_2 & l_3 & 0 & 0 \\ l_4 & l_5 & l_6 & 0 \\ l_7 & l_8 & l_9 & l_{10} \end{pmatrix} \cdot \begin{pmatrix} l_1 & l_2 & l_3 & l_4 \\ l_5 & l_6 & l_7 & 0 \\ l_8 & l_9 & 0 & 0 \\ l_{10} & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 4 & -4 & 12 & 4 \\ -4 & 5 & -13 & -1 \\ 12 & -13 & 38 & 8 \\ 4 & -1 & 8 & 18 \end{pmatrix}$$

sorfolyt:

$$4 = l_1^2 \to l_1 = 2$$
$$-4 = l_2 \cdot l_1 = 2l_2 \to l_1 = -2$$
$$5 = l_2^2 + l_3^2 = 4 + l_3^2 \to l_3^2 = 1 \to l_3 = 1$$
$$12 = l_4 \cdot l_1 = 2l_4 \to l_4 = 6$$
$$-13 = l_4 \cdot l_2 + l_5 \cdot l_3 = -12 \to l_5 = -1$$
$$38 = l_4^2 + l_5^2 + l_6^2 = 36 + 1 + l_6^2 \to l_6^2 = 1 \to l_6 = 1$$
$$4 = l_7 \cdot l_1 = 2l_7 \to l_7 = 2$$
$$-1 = l_4 \cdot l_2 + l_8 \cdot l_3 = 2 \cdot (-2) + l_8 \to l_8 = 3$$
$$8 = l_7 \cdot l_4 + l_8 \cdot l_5 + l_9 \cdot l_6 = 2 \cdot 6 + 3 \cdot (-1) + l_9 \to l_9 = -1$$
$$18 = l_7^2 + l_8^2 + l_9^2 + l_{10}^2 = 4 + 9 + 1 + l_{10}^2 \to l_{10}^2 = 4 \to l_{10} = 2$$

1/a

$$A = \begin{pmatrix} 4 & -2 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} q_1 & q_2 \end{pmatrix} \cdot \begin{pmatrix} r_{11} & r_{12} \\ 0 & r_{21} \end{pmatrix}$$
$$r_{11} = \|a_1\|_2 = \sqrt{4^2 + 3^2} = 5$$
$$q_1 = \frac{1}{5} \cdot \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$
$$r_{12} =< a_2; q_1 >=< \begin{pmatrix} -2 \\ 1 \end{pmatrix}; \frac{1}{5}\begin{pmatrix} 4 \\ 3 \end{pmatrix} >= \frac{1}{5} \cdot ((-2) \cdot 4 + 1 \cdot 3) = -\frac{5}{5} = -1$$
$$s_2 = a_2 - r_{12}q_1 = \begin{pmatrix} -2 \\ 1 \end{pmatrix} - (-1) \cdot \frac{1}{5} \cdot \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \frac{1}{5}\left(\begin{pmatrix} -10 \\ 5 \end{pmatrix} + \begin{pmatrix} 4 \\ 3 \end{pmatrix}\right) = \frac{1}{5} \cdot \begin{pmatrix} -6 \\ 8 \end{pmatrix} = \frac{2}{5}\begin{pmatrix} -3 \\ 4 \end{pmatrix}$$
$$r_{22} = \|s_2\|_2 = \left\| \frac{2}{5} \cdot \begin{pmatrix} -3 \\ 4 \end{pmatrix} \right\|_2 = \frac{2}{5} \left\| \begin{pmatrix} -3 \\ 4 \end{pmatrix} \right\|_2 = \frac{2}{5} \cdot 5 = 2$$
$$q_2 = \frac{1}{r_{22}} \cdot s_2 = \frac{1}{5}\begin{pmatrix} -3 \\ 4 \end{pmatrix}$$

1/b

$$\begin{pmatrix} 2 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix} \cdot \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ 0 & r_{22} & r_{23} \\ 0 & 0 & r_{33} \end{pmatrix}$$

$$r_{11} = \|a_1\|_2 = \sqrt{4+1} = \sqrt{5}$$

$$q_1 = \frac{1}{r_{11}} a_1 = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$$

$$r_{12} = < q_1, a_2 > = \frac{1}{\sqrt{5}}(2-1) = \frac{1}{\sqrt{5}}$$

$$s_2 = a_2 - q_1 r_{12} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} - \frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} \cdot \frac{1}{\sqrt{5}} = \frac{1}{5} \begin{pmatrix} 3 \\ 6 \\ 0 \end{pmatrix} = \frac{3}{5} \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$

$$r_{22} = \|s_2\|_2 = \frac{3}{5}\sqrt{1^2+2^2} = \frac{3}{5}\sqrt{5} = \frac{3}{\sqrt{5}}$$

$$q_2 = \frac{1}{r_{22}} s_2 = \frac{\sqrt{5}}{3} \cdot \frac{3}{5} \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = \frac{\sqrt{5}}{5} \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$

$$r_{13} = < q_1, a_3 > = < \frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} > = 0$$

$$r_{23} = < q_2, a_3 > = < \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} > = 0$$

$$s_3 = a_3 - q_1 r_{13} - q_2 r_{23} = a_3 = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$$

$$r_{33} = \|s_3\|_2 = 2$$

$$q_3 = \frac{1}{r_{33}} s_3 = \frac{1}{2} \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} & 0 \\ -\frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \sqrt{5} & \frac{1}{\sqrt{5}} & 0 \\ 0 & \frac{3}{\sqrt{5}} & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

custom

$$A = \begin{pmatrix} 2 & 6 & 5 \\ -1 & -4 & 1 \\ -1 & -2 & -3 \end{pmatrix} = \begin{pmatrix} \tilde{q_1} & \tilde{q_2} & \tilde{q_3} \end{pmatrix} \cdot \begin{pmatrix} 1 & \widetilde{r_{12}} & \widetilde{r_{13}} \\ 0 & 1 & \widetilde{r_{23}} \\ 0 & 0 & 1 \end{pmatrix}$$

$$\tilde{q_1} = \tilde{a_1}$$

$$\widetilde{r_{12}} = \frac{< a_2; \tilde{q_1} >}{< \tilde{q_1}; \tilde{q_2} >} = \frac{< \begin{pmatrix} 6 \\ -4 \\ -2 \end{pmatrix}; \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} >}{6} = \frac{18}{6} = 3$$

$$\tilde{q_2} = a_2 - \widetilde{r_{12}} \tilde{q_1} = \begin{pmatrix} 6 \\ -4 \\ -2 \end{pmatrix} - 3 \cdot \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

$$\widetilde{r_{13}} = \frac{< a_3, \tilde{q_1} >}{< \tilde{q_1}, \tilde{q_1} >} = \frac{2 \cdot 5 + (-1) \cdot 1 + (-1) \cdot (-3)}{6} = \frac{12}{6} = 2$$

$$\widetilde{r_{23}} = \frac{< a_3, \tilde{q_2} >}{< \tilde{q_2}, \tilde{q_2} >} = \frac{< \begin{pmatrix} 5 \\ 1 \\ -3 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} >}{2} = \frac{-4}{2} = -2$$

$$\tilde{q_3} = a_3 - \widetilde{r_{13}} \tilde{q_1} - \widetilde{r_{23}} \tilde{q_2} = \begin{pmatrix} 5 \\ 1 \\ -3 \end{pmatrix} - 2 \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} - (-2) \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 2 & 6 & 5 \\ -1 & -4 & 1 \\ -1 & -2 & -3 \end{pmatrix} = \begin{pmatrix} \tilde{q_1} & \tilde{q_2} & \tilde{q_3} \end{pmatrix} \cdot \begin{pmatrix} 1 & \widetilde{r_{12}} & \widetilde{r_{13}} \\ 0 & 1 & \widetilde{r_{23}} \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 1 \\ -1 & -1 & 1 \\ -1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 & 2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

// mindegyik oszlopot leosztom a hosszaval.

minden osztast az egyik oldalon szorzassal kompenzalom a masik oldalon

$$A = QR = (QD^{-1})(DR)$$

$$Q = \begin{pmatrix} \frac{2}{\sqrt{6}} & 0 & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \end{pmatrix}$$

$$R = \begin{pmatrix} \sqrt{6} & 3\sqrt{6} & 2\sqrt{6} \\ 0 & \sqrt{2} & -2\sqrt{2} \\ 0 & 0 & \sqrt{3} \end{pmatrix}$$