hatodik gyakorlat anyaga switchup ropzhra is a hatodikbol kell keszulni

taylor formula a lagrange-fele maradekkal

Legyen $n \in \mathbb{N}$
TFH $f \in D^{n+1}(K(a))$
Ekkor

$$\forall x \in \dot{K}(a) \quad \text{ponthoz} \quad \exists \xi \ a \quad \text{es} \quad x \quad \text{kozott} \ :$$
$$f(x) - T_{a,n}(f,x) = f(x) - \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-a)^{n+1}.$$

1/a

$$f(x) := \frac{1}{\sqrt[3]{x+1}} \qquad x \in (-1, +\infty) =: I; \quad a = 0; \quad T_3 f(x) = ?; \quad x \in \left(0, \frac{1}{10}\right]; \quad \text{hiba?}$$

$$T_3 f(x) = f(0) + f'(0)x + \left(\frac{f''(0)}{2!}\right)x^2 + \left(\frac{f'''(0)}{3!}\right)x^3$$

$$f(x) = (x+1)^{-\frac{1}{3}} \Longrightarrow f(0) = 1$$

$$f'(x) = -\frac{1}{3}(x+1)^{-\frac{4}{3}} \cdot 1 \Longrightarrow f'(0) = -\frac{1}{3}$$

$$f''(x) = \frac{4}{9}(x+1)^{-\frac{7}{3}} \cdot 1 \Longrightarrow f''(0) = \frac{4}{9}$$

$$f'''(x) = -\frac{28}{27}(x+1)^{-\frac{10}{3}} \cdot 1 \Longrightarrow f'''(0) = -\frac{28}{27}$$

$$\text{eleg lenne de a hiba miatt kell n+1 edik derivalt is}$$

$$f^{(4)}(x) = \frac{280}{81}(x+1)^{-\frac{13}{3}} \cdot 1$$

$$T_3 f(x) = T_3(x) = 1 - \frac{1}{3}x + \frac{\frac{4}{9}}{2}x^2 - \frac{\frac{28}{27}}{6}x^3 = 1 - \frac{1}{3}x + \frac{2}{9}x^2 - \frac{14}{81}x^3 \ (x \in \mathbb{R})$$

$$f(x) - T = \left| \frac{f^{(4)}(\xi)}{4!} x^4 \right|$$

$$0 < \xi < x \leq \frac{1}{10}$$

$$|f(x) - T_{0,3}(f,x)| = \frac{1}{24} \cdot \frac{280}{81} \cdot \left| \frac{1}{\sqrt[3]{x+1}} \right| \cdot |x^4| = \frac{70}{681}$$

$$\text{tehat a vegeredmeny}$$

$$\frac{35}{243} \cdot \frac{1}{10^4} = \frac{7}{486 \cdot 10^3}$$

1/b

$$\text{adjunk egy kozelitoerteket: } A := \frac{1}{\sqrt[3]{1.03}} \approx ? \text{ es hiba}$$

$$\frac{1}{\sqrt[3]{1.03}} = \frac{1}{\sqrt[3]{x+1}} = f(x) \Longleftrightarrow x+1 = 1+\frac{3}{100} \Longleftrightarrow x = \frac{3}{100} \in \left(0, \frac{1}{10}\right]$$

$$A = f\left(\frac{3}{100}\right) \approx T_3 f\left(\frac{3}{100}\right) = 1 - \frac{1}{3}\cdot\frac{3}{100} + \frac{2}{9}\cdot\frac{9}{10000} - \frac{14}{81}\cdot\frac{27}{10^6} = \frac{1485293}{1500000} = 0,99153$$

$$\text{Hiba:}$$

$$\left|\frac{1}{\sqrt[3]{1.03}} - T_3 f\left(\frac{3}{100}\right)\right| \le \frac{1}{4!}\cdot\left|f^{(4)}\left(\frac{2}{3}?\right)\right|\cdot\left(\frac{3}{100}\right)^4 \le \frac{7}{6}\cdot 10^{-7}$$

2 hazi

L'Hospital szabaly

$$\lim \frac{\operatorname{tg} x - x}{x - \sin x} = \frac{0}{0}$$

$$\text{akkor mukodik a hospital ha } \frac{0}{0} \text{ vagy } \frac{\infty}{\infty}\text{. altalaban jobb oldali hatarertekre mondjak ki}$$

$$\lim_{0} \frac{f}{g} = \lim_{0} \frac{f'}{g'}$$

$$\lim_{x\to 0} \frac{\operatorname{tg} x - x}{x - \sin x} = \frac{0}{0} = L'H = \lim_{x\to 0} \frac{\frac{1}{\cos^2 x} - 1}{1 - \cos x} =$$

$$= \lim_{x\to 0} \frac{1-\cos^2 x}{\cos^2 x(1-\cos x)} = \lim_{x\to 0} \frac{(1-\cos x)(1+\cos x)}{\cos^2 x(1-\cos x)} = \frac{2}{1} = 2$$

2

$$\lim_{x\to 0}\left(\frac{1}{x} - \frac{1}{e^x - 1}\right) = \lim_{x\to 0} \frac{e^x - 1 - x}{x(e^x-1)} = \frac{0}{0} = L'H = \lim_{x\to 0} \frac{e^x - 1}{1(e^x-1) + xe^x} =$$

$$= e^x - 1 + xe^x = \frac{0}{0} = L'H = \lim \frac{e^x}{e^x + e^x + xe^x} = \frac{1}{2}$$

3

$$\lim_{x\to 1-0} \ln x \cdot \ln(1-x) = 0\cdot(-\infty)$$

$$f\cdot g = \frac{f}{\frac{1}{g}} = \frac{g}{\frac{1}{f}}$$

$$\lim_{x\to 1-0} \ln x\cdot\ln(1-x) = \frac{\ln(1-x)}{\frac{1}{\ln x}} = \frac{-\infty}{-\infty} = L'H = \lim_{x\to 1-0} \frac{\frac{1}{1-x}\cdot(-1)}{(-1)\cdot(\ln x)^2\cdot\frac{1}{x}} =$$

$$= \lim_{x\to 1-0} x \cdot \lim_{x\to 1-0} \frac{(\ln x)^2}{1-x} = \frac{0}{0} = L'H = \lim_{x\to 1-0} \frac{2\ln x\cdot\frac{1}{x}}{-1} = \frac{2\ln 1\cdot 1}{-1} = 0$$

e

1^{∞}-rol az euler szamoknak kellett volna eszunkbe jutnia

$$\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{\frac{1}{x}} \qquad (a,b,c>0) =$$

$$= 1^{\infty} = e^{\ln\left(f(x)^{g(x)}\right)} = e^{g(x)\cdot\ln f(x)} \lim_{x\to 0} e^{\frac{1}{x}\cdot\ln\left(\frac{a^x+b^x+c^x}{3}\right)}$$

$$\text{kitevo}: \lim_{x\to 0}\frac{\ln\left(\frac{a^x+b^x+c^x}{3}\right)}{x} = \frac{0}{0} = L'H = \left(\ln\left(\frac{a^x+b^x+c^x}{3}\right)\right)' = \frac{1}{\frac{a^x+b^x+c^x}{3}}\cdot\left(\frac{a^x\ln a+b^x\ln b+c^x\ln c}{3}\right) =$$

$$= \frac{a^x\ln a+b^x\ln b+c^x\ln c}{a^x+b^x+c^x},$$

$$\text{kitevo hatarerteke}: \lim_{x\to 0}\frac{\ln\left(\frac{a^x+b^x+c^x}{3}\right)}{x} = \frac{0}{0} = L'H = \lim_{x\to 0}\frac{a^x\ln a+b^x\ln b+c^x\ln c}{3} = \frac{\ln a+\ln b+\ln c}{3} = \ln\sqrt[3]{abc}$$

$$\text{Mivel exp fuggveny folytonos } \mathbb{R}\text{-en, ezert}:$$

$$\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{\frac{1}{x}} = \exp\left(\ln\sqrt[3]{abc}\right) = \sqrt[3]{abc}$$