1.

$$z^3, z^{13}, z^{135} = ? \qquad \text{ha } z = i$$

$$z^3 = (z \cdot z) \cdot z = -1 \cdot i = -i$$

$$z^{13} = i^{13} = i^{3 \cdot 4 + 1} = \left(i^4\right)^3 \cdot i = 1 \cdot i = i$$

$$z^{135} = i^{4 \cdot 33 + 2 + 1} = \left(i^4\right)^{33} \cdot i^2 \cdot i = 1 \cdot (-1) \cdot i = -1$$

es ha $z = \frac{1+i}{\sqrt{2}}$?

$$z^3 = \left(\frac{1+i}{\sqrt{2}}\right)^3 = \left(\frac{1+i}{\sqrt{2}}\right)^2 \frac{1+i}{\sqrt{2}} = i \cdot \frac{1+i}{\sqrt{2}} = -\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$$

$$z^{13} = z^{2+4+1} = i^4 \cdot i^2 \cdot \left(\frac{1+i}{\sqrt{2}}\right) = 1 \cdot (-1) \cdot \left(\frac{1+i}{\sqrt{2}}\right) = -\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$$

$$z^{135} = z^{16 \cdot 8 + 7} = 1 \cdot \left(\frac{1+i}{\sqrt{2}}\right)^4 \cdot \left(\frac{1+i}{\sqrt{2}}\right)^3 = +\frac{1}{\sqrt{2}} + \ldots$$

es ha $z = 1 - i = \frac{1-i}{\sqrt{2}} \cdot \sqrt{2}$?

2.

$M^2, M^5, M^{123} = ? \qquad$ ha $M = \begin{pmatrix} 8 & -21 \\ 3 & -8 \end{pmatrix}$

$$M^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$M^5 = M^{2 \cdot 2 + 1} = I^2 \cdot M = M$$

$$M^{123} = M^{61 \cdot 2 + 1} = I^{61} \cdot M = M$$

$$M = \begin{pmatrix} -3 & 2 \\ -5 & 3 \end{pmatrix}$$

$$M^2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -I$$

$$M^5 = I \cdot M = M$$

$$M^{123} = M^{30 \cdot 4 + 2 + 1} = I^{30} \cdot (-I) \cdot M = -M$$

3.

a, b egeszek, ugy hogy $a, b \neq 0 \qquad \exists! q, r$ egeszek, hogy $a = b \cdot q = r$ es $0 \leq r < |b|$

jel: r = a mod b

q = szar jeloles, a,b > 0

a + b mod m?

$$a = m \cdot q_1 + r_1$$

$$b = m \cdot q_2 + r_2$$

- osszeadas

$$a + b \bmod m = (q_1 + q_2) \cdot m + (r_1 + r_2) \bmod m = r_1 + r_2 \bmod m$$

- szorzas

$$a \cdot b \bmod m = (m \cdot q_1 + r_1)(m \cdot q_2 + r_2) \bmod m = m^2 \cdot q_1 \cdot q_2 + m \cdot q_2 \cdot r_1 + m \cdot q_2 + r_1 + r_1 \cdot r_2 \bmod m = r_1 \cdot r_2 \bmod m$$

- hatvany

$$a^n \bmod m = (m \cdot q = r)(...)... \bmod m = r \cdot r \cdot ... \bmod m = r^n \bmod m$$

5.

a:

$$13 \cdot 15 + 31 \cdot 42 + 51^2 \bmod 2 = 1 \cdot 1 + 1 \cdot 0 + 1^2 \bmod 2 = 1 + 0 + 1 \bmod 2 = 2 \bmod 2 = 0$$

b:

$$73 \cdot 82 + 17 \cdot 71 \bmod 4 = 1 \cdot 2 + 1 \cdot 3 \bmod 4 = 1$$

c:

$$123 + 594 + 931 \bmod 10 = 3 + 4 + 1 \bmod 10 = 8$$

g:

$$3^{100} \bmod 7$$

gyorshatvanyozas kell mert ez cooked

$$100 = 54 + 32 + 4 \Rightarrow 3^{100} \bmod 7 = 3^{64} \cdot 3^{32} \cdot 3^4 \bmod 7$$

$$3^1 \bmod 7 = 3$$

$$3^2 \bmod 7 = 2$$

$$3^4 \bmod 7 = 2^2 \bmod 7 = 4$$

$$3^8 \bmod 7 = 4^2 \bmod 7 = 2$$

$$3^{16} \bmod 7 = 2^2 \bmod 7 = 4$$

$$3^{32} \bmod 7 = 4^2 \bmod 7 = 2$$

$$3^{64} \bmod 7 = 2^2 \bmod 7 = 4$$

igy

$$100 = 54 + 32 + 4 \Rightarrow 3^{100} \bmod 7 = 3^{64} \cdot 3^{32} \cdot 3^4 \bmod 7 = 4 \cdot 2 \cdot 4 \bmod 7 = 32 \bmod 7 = 4$$

i:

$$(583+57)\cdot 715+41^2 \bmod 7=(2+1)\cdot\left(1+6^2\right) \bmod 7=3+1 \bmod 7=4$$

2

d

$$M=\begin{pmatrix}0 & 0 & 1\\ 1 & 0 & 0\\ 0 & 1 & 0\end{pmatrix}$$

$$M^2=\begin{pmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 0 & 0\end{pmatrix}$$

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$$M=\begin{pmatrix}0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ 1 & 0 & 0 & 0 & 0 & 0\end{pmatrix}$$

$$M^2=\begin{pmatrix}0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ 1 & 0 & 0 & 0 & 0 & 0\end{pmatrix}\cdot\begin{pmatrix}0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ 1 & 0 & 0 & 0 & 0 & 0\end{pmatrix}=\begin{pmatrix}0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0\end{pmatrix}$$