$$(a, b) = c \qquad c = ax + by \qquad x \in \mathbb{Z}$$

1. $a = 13, b = 14$

14	x	1	0
13	x	0	1
1	1	1	-1
0	13	-13	14

1. $14 = 1 \cdot 14 + 0 \cdot 13$
2. $13 = 0 \cdot 14 + 1 \cdot 13$
3. $1 = 1 \cdot 14 + -1 \cdot 13$
4. $0 = (-13) \cdot 14 + 14 \cdot 13$

$$1 = (14, 13) = (1 + (-13)k) \cdot 14 \cdot + ((-1) + 14k) \cdot 13 \qquad (k \in \mathbb{N})$$

1/b $a = 16, b = 37$

17	x	1	0
16	x	0	1
5	2	1	-2
1	3	-3	7
0	5	-16	-37

1. $1 = (-3) \cdot 37 + 7 \cdot 16$
2. $0 = 16 \cdot 37 + (-37) \cdot 16$

$$1 = ((-3) + 16k) \cdot 37 + (7 + (-37k)) \cdot 16$$

1/c $a = 90, b = (-111)$

-111	x	1	0
90	x	0	1
69	-2	1	2
21	1	-1	-1
6	3	4	5
3	3	-13	-16
0	2	30	37

vagy

90	x	1	0
-111	x	0	1
90	x	0	1
69	-2	1	2
21	1	-1	-1
6	3	4	5
3	3	-13	-16
0	2	30	37

c_0	x	x_0	y_0
c_1	x	x_1	y_1
c_2	q_1	x_2=y_0 - x_1 dot q_2	y_1 = y_0 - q_1 dot q_2
c_3			

szazlabuak

232 lab 14,20 labuak vannak cases 323 = 14x + 20y

20	x	1	0
14	x	0	1
6	1	1	-1
2	2	-2	3
0	3	7	-10

$$2 = (-2) \cdot 20 + 3 \cdot 14 \quad / \cdot \frac{232}{2}$$
$$232 = (-232) \cdot 20 + (348) \cdot 14 \quad / + k \cdot 0$$
$$232 = (-232 + 7k) \cdot 20 + (348 - 10k) \cdot 14$$
$$232 = (-232 + 7 \cdot 34) \cdot 20 + (348 - 1034) \cdot 14$$
$$= (6 \cdot 20 + 8 \cdot 14) \quad (4 = 36)$$
$$6 + 8 = 14$$

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$a, b \in \mathbb{Z}$ a: $8^a \cdot 16^b = 32$

$$\left(2^3\right)^0 \cdot \left(2^4\right)^b = 2^5$$
$$2^{3a} \cdot 2^{4b} = 2^5$$
$$2^{3a+46} - 2^5 \implies 3a + 46 = 5$$

4	x	1	0
3	x	0	1
1	1	1	-1
0	3	-3	4

$$3(-1) + 4(1) \quad / \cdot 5$$
$$3(-5) + 4(5) = 5$$
$$3(-5 + 4) + 4(5 - 3k) = 5$$
$$\implies \left(2^3\right)^{-5+4k} \cdot \left(2^4\right)^{5-3k} = 2^5$$
$$\text{(valami mas is)}$$

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a:

$$3^{3n+1} \cdot 5^{2n+1} + 2^{5n+1} \cdot 11^n \equiv 0 \bmod 17$$
$$3 \cdot 3^{3n} \cdot 5 \cdot 5^{2n} + 2 \cdot 2^{5n} \cdot 11^n \equiv 0 \bmod 17$$
$$3 \cdot 5 \cdot 27^n \cdot 25^n + 2 \cdot 32^n \cdot 11^n \equiv 0 \bmod 17$$
$$15 \cdot 10^n \cdot 8^n + 2 \cdot 15^n \cdot 11^n \equiv 0 \bmod 17$$
$$15 \cdot 80^n + 2 \cdot 165^n \equiv 0 \bmod 17$$
$$15 \cdot 12^n + 2 \cdot 12^n \equiv 0 \bmod 17$$
$$17 \cdot 12^n \equiv 0 \bmod 17$$
$$0 \equiv 0 \bmod 17 \implies n \in \mathbb{Z}$$

2 $2x \equiv 1 \bmod 3$

felirjuk a 2 es a 3-ra bovitett euklideszit

3	x	1	0
2	x	0	1
1	1	1	-1
0	2	-2	3

$$1 = (1 - 2k) \cdot 3 + (-1 + 3k) \cdot 2$$

$$2x \equiv \underbrace{(1 - 2k)}_{0} \cdot 3 + (-1 + 3k) \cdot 2 \bmod 3$$

$$2x \equiv= (-1 + 3k) \cdot 2 \bmod 3$$

$$x \equiv= (-1 + 3k) \bmod 3$$

$$x \equiv -1 \equiv 2 \bmod 3$$