lokalis szelsoertek koran reggel

1/a.

$$f(x) := 3x^4 - 4x^3 - 12x^2 + 2$$

$$f'(x) = 12x^3 - 12x^2 - 24x = 12x(x^2 - x - 2) = 12x(x-2)(x+1) = 0 \Longleftrightarrow x_1 = 0 \vee x_2 = 2 \vee x_3 = -1;$$

x	$-\infty$	-1	$-1 < x < 0$	0	$0 < x < 2$	2	$+\infty$
$f'(x)$	$-$	0	$+$	0	$-$	0	$+$
$f(x)$	$\downarrow$	(-3)	$\uparrow$	(2)	$\downarrow (-30)$		$\uparrow$

lokalis minimum -3-nal, globalalis minimum a -30, lokalis max a 2

$$f(-1) = -3$$
$$f(0) = 2$$
$$f(1) = -30$$
$$f \downarrow: (-\infty; -1); f \downarrow (0; 2);$$
$$f \uparrow (-1; 0); f \uparrow (2; +\infty);$$

1/b.

$$f(x) := \frac{x}{x^2 - 10x + 16} \qquad (x \in \mathbb{R} \setminus \{2, 8\})$$

$$f(x) = \frac{x}{x^2 - 10x + 16} = \frac{x}{(x-2)(x-8)}$$

$$f'(x) = \frac{x(x^2 - 10x + 16) - x(2x - 10)}{(x-2)^2(x-8)^2} = \frac{16 - x^2}{(x-2)^2(x-8)^2} \qquad (\forall x \in \mathbb{R} \setminus \{2, 8\})$$

$$\text{sign } f'(x) = \text{sign}(16 - x^2)$$

x	$-\infty$	-4	$-4 < x < 2$	2	$2 < x < 4$	4	$4 < x < 8$	8	$+\infty$
$f'(x)$	$-$	0	$+$	$\vert$	$+$	0	$-$	$\vert$	$-$
$f(x)$	$\downarrow$		$\uparrow$	$\vert$	$\uparrow$		$\downarrow$	$\vert$	$\downarrow$

-4 lokalis min, 4 lokalis max, global min/max nem letezik (mert van benne $\pm\infty$)

$$f(-4) = \frac{-4}{-6 \cdot (-12)} = -\frac{1}{18}$$

$$f(4) = \frac{4}{2 \cdot (-4)} = -\frac{1}{2}$$

Tehat:

$$f \downarrow: (-\infty; 4); (4, 8); (8, +\infty);$$
$$f \uparrow: (-4; 2); (2, 4);$$

abszolut szelsoertek

2/a.

$$f(x) := x^4 - 4x^3 + 10 \ (x \in [-1, 4])$$

$$\mathcal{D}_f = [-1, 4] \ \text{ korlatos es zart (kompakt), es } \ f \in C[-1, 4] \Longrightarrow \exists \min \mathcal{R}_f, \exists \max \mathcal{R}_f$$

$$\text{hol lehetnek? } \begin{cases} \text{- } \ x \in (-1, 4) \text{ belso pontban ott ahol } f'(x) = 0 \\ \text{- vegpontokban: } \ x = -1 \ \text{ ill. } x = 4 \end{cases}$$

ha a)

$$x \in (-1, 4) \Longrightarrow f \in D\{a\} \quad \text{es} \quad f'(x) = 4x^3 - 12^2 = 4x^2(x - 3) = 0 \Longleftrightarrow$$

$$\Longleftrightarrow x_1 = 0 \in (-1, 4), x_2 = 3 \in (-1, 4)$$

ha b) osszevetes

$$f(0) = 10,$$

$$f(-1) = 15, \ \text{ abszolut max}$$

$$f(3) = -17, \ \text{ abszolut min}$$

$$f(4) = 10$$

2/b.

$$f(x) := \frac{x}{x^2 + 1} \qquad \left(x \in \left[-\frac{3}{2}, 2\right] = \mathcal{D}_f\right) \Longrightarrow D_f \text{ kompakt}, f \in C\left[-\frac{3}{2}, 2\right] \Longrightarrow \exists \min \mathcal{R}_f; \exists \max \mathcal{R}_f \Longrightarrow$$

$$\Longrightarrow f \in D \text{ es } f'(x) = \left(\frac{x}{x^2 + 1}\right)' = \frac{1 \cdot (x^2 + 1) - x \cdot 2x}{(x^2 + 1)^2} = \frac{1 - x^2}{(x^2 + 1)^2}$$

$$\left(x \in \left(-\frac{3}{2}, 2\right)\right) \ \text{ es } \ f'(x) = 0 \Longleftrightarrow 1 - x^2 = 0 \Longleftrightarrow x = \pm 1 \in \left(-\frac{3}{2}, 2\right)!$$

osszevetes

$$f(1) = \frac{1}{2},$$

$$f\left(-\frac{3}{2}\right) = \frac{-\frac{3}{2}}{\frac{9}{4} + 1} = -\frac{6}{13}, \text{absz min}$$

$$f(-1) = -\frac{1}{2},$$

$$f(2) = \frac{2}{5}, \text{absz max}$$

custom hazi

- 1. lokalis szelsoertek: $f(x) = x^2 \cdot e^{-x} (x \in \mathbb{R})$
- 2. globalis szelsoertek: $f(x) = \sin^4 x + \cos^4 x x \in \left[-\frac{2\pi}{3}, \frac{\pi}{2}\right]$

szoveges peldak

3.

tekintsunk egy a oldalu negyzetet. ($a > 0$).

a sarkokbol kivagunk x oldalu negzeteket es eltavolitjuk. a maradekbol csinaljunk egy dobozt.

hol vagjunk (mekkora legyen az x), ugy hogy V_{doboz} maximalis legyen?

V_{doboz} a celfuggveny amit minimalizalni/maximalizalni kell

alap hossza: $a - x$, magassaga x

ebbol terfogat:

$$(a-2x)^2 \cdot x = (2x-a)^2 \cdot x \qquad \left(x \in \left[0; \frac{a}{2}\right]\right)$$

ezt a fuggvenyt kell maximalizalni

most ne wierstrassal csinaljuk mert ugy tul konnyu lenne

$$\forall x \in \left(0, \frac{a}{2}\right) \Longrightarrow V \in D\{x\} \text{ es } V'(x) = 2(2x-a)^1 \cdot x + (2x-a)^2 \cdot 1 = (2x-a)(4x+2x-a) = (2x-a)(6x-a)$$

$$V'(x) = 0 \Longleftrightarrow x_1 = \frac{a}{2};\ x_2 = \frac{a}{6}$$

x	0	$0 < x < \frac{a}{6}$	$\frac{a}{6}$	$\frac{a}{6} < x < \frac{a}{2}$	$\frac{a}{2}$
$V'(x)$	0	$+$	0	$-$	0
$V(x)$	$\uparrow$	$\uparrow$		$\downarrow$	$\downarrow$

$$V\left(\frac{a}{6}\right) = \left(2 \cdot \frac{a}{6} - a\right)^2 \cdot \frac{a}{6} = \frac{4a^2}{9 \cdot 6} = \frac{2a^3}{27}$$

Tehat: $x = \frac{a}{6}$

4.

tekintsunk egy 1 literes hengeralaku zart konzervet, minimalizaljuk a koltseget ugy hogy az ar az egyenesen aranyos a felszinnel

$$V = 1 \text{ liter} = 1000\text{cm}^3$$

$R, h = ?$ hogy a felszin minimalis legyen

$$F(R,h) = 2\pi R^2 + 2\pi R h \text{ (celfuggveny)} \quad (R, h > 0)$$

nem tudunk ketvaltozos analizist ezert kene egy valtozo

$$V(R,h) = \pi R^2 h = 1000 \text{ (feltetel)} \Longrightarrow$$

$$\Longrightarrow h = \frac{1000}{\pi R^2} \Longrightarrow$$

$$\Longrightarrow f(R) = F\left(R, \frac{1000}{\pi R^2}\right) = 2\pi R^2 + 2\pi R \frac{1000}{\pi R^2} = 2\pi R^2 + \frac{2000}{R} = 2\left[\pi R^2 + \frac{1000}{R}\right] \qquad (R \in (0; +\infty))$$

nem kompakt tehat nincs weierstrass!

$$\Longrightarrow f \in D \text{ es } f'(R) = 2\left(2\pi R - \frac{1000}{R^2}\right) = 0 \Longleftrightarrow 2\pi R = \frac{1000}{R^2} \Longleftrightarrow R^3 = \frac{1000}{2\pi} \Longleftrightarrow R = \frac{10}{\sqrt[3]{2\pi}} > 0$$

$$f'(R) > 0 \iff 2R\pi > \frac{1000}{R^2}$$

$$R^2 \iff R^3 > \frac{1000}{2\pi} \implies R > \frac{10}{\sqrt[3]{2\pi}}$$

R	0	$0 < R < \frac{10}{\sqrt[3]{2\pi}}$	$\frac{10}{\sqrt[3]{2\pi}}$	$\frac{10}{\sqrt[3]{2\pi}} < R < +\infty$	$+\infty$
$f'(R)$	n.e	$-$	0	$+$	n.e.
$f(R)$	$n.e$	$\downarrow$		$\uparrow$	$n.e.$

$$\lim_{R\to 0+0} f(R) = +\infty$$
$$\lim_{R\to +\infty} f(R) = +\infty$$

absz min: $a \frac{10}{\sqrt[3]{2\pi}}$

$$f\left(\frac{10}{\sqrt[3]{2\pi}}\right) = 2\pi \cdot \frac{100}{\sqrt[3]{(2\pi)^2}} + \frac{2000}{\frac{10}{\sqrt[3]{2\pi}}} = 100\sqrt[3]{2\pi} + 200\sqrt[3]{2\pi} = 300\sqrt[3]{2\pi}$$

ropzh04:
- orai kozul barmi
- hf:
  - a ket custom
  - 1,2,3,4
- gyakorlo: 1/b,c es 3