emlekezteto

valamely $f \in \mathbb{R} \to \mathbb{R}$ fuggveny, es $a \in \operatorname{int} D_f$ eseten

$$f \in D\{a\} \Longleftrightarrow \exists f'(a) := \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \in \mathbb{R}$$

feladatok

1.

egynel nagyon valos szamok halmazan ertelmes a kifejezes

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to a} \frac{\sqrt{x^2 - 1} - \sqrt{a^2 - 1}}{x - a} =$$

$\frac{0}{0}$ lenne ezert gyoktelenitunk

$$\lim_{x \to a} \frac{\sqrt{x^2 - 1} - \sqrt{a^2 - 1}}{x - a} \cdot \frac{\sqrt{x^2 - 1} + \sqrt{a^2 - 1}}{\sqrt{x^2 - 1} + \sqrt{a^2 - 1}}$$

felsimerjuk az azonossagokat

$$\lim_{x \to a} \frac{(x^2 - 1) - (a^2 - 1)}{(x - a)\left(\sqrt{x^2 - 1} + \sqrt{a^2 - 1}\right)} = \lim_{x \to a} \frac{x^2 - a^2}{(x - a)\left(\sqrt{x^2 - 1} + \sqrt{a^2 - 1}\right)} =$$

$$\text{mivel } (a^2 - b^2) = (a - b)(a + b)$$

$$\lim_{x \to a} \frac{(x - a)(x + a)}{(x - a)\left(\sqrt{x^2 - 1} + \sqrt{a^2 - 1}\right)} = \lim_{x \to a} \frac{x + a}{\sqrt{x^2 - 1} + \sqrt{a^2 - 1}} = \frac{a + a}{\sqrt{a^2 - 1} + \sqrt{a^2 - 1}} = \frac{2a}{2\sqrt{a^2 - 1}} = \frac{a}{\sqrt{a^2 - 1}}$$

2/a

$$f(x) = 4x^3 - 2x^2 + 5x - 3 \quad (x \in \mathbb{R})$$

$$f'(x) = 12x^2 - 4x + 5$$

2/b

$$f(x) = \sqrt{x\sqrt{x\sqrt{x}}} \quad (x > 0)$$

$$f'(x) = x^{\frac{7}{8}} = \frac{7}{8} \cdot x^{-\frac{1}{8}} = \frac{7}{8\sqrt[8]{x}}$$

2/c

$$f(x) = x^3 + \frac{1}{x^2} - \frac{1}{5x^5} \quad (x \in \mathbb{R} \setminus \{0\})$$

$$f'(x) = 2x^2 - \frac{2}{x^3} + \frac{1}{x^6}$$

2/d

$$f(x) = x^a + a^x + ax + \frac{x}{a} + \frac{a}{x} \qquad (x > 0), a > 0 \text{ parameter}$$

$$f'(x) = a \cdot x^{a-1} + a^x \ln a + a + \frac{1}{a} - \frac{a}{x^2}$$

2/e

szorzas azonossag

$$f(x) = x^2 \cdot \sin x \qquad (x \in \mathbb{R})$$

$$f'(x) = 2x \cdot \sin x + x^2 \cdot \cos x$$

2/f

osztas azonossag

$$f(x) = \frac{x^3 + 2}{x^2 + x + 5} \qquad (x \in \mathbb{R})$$

$$f'(x) = \frac{2x^2 \cdot (x^2 + x + 5) - (x^3 + 2) \cdot (2x + 1)}{(x^2 + x + 5)^2}$$

2/g

kompozicio azonossag

$$f(x) = (5x^2 + 3x)^{2023} \qquad (x \in \mathbb{R})$$

$$f'(x) = 2023(5x^2 + 3x)^{2022} \cdot (10x + 3)$$

2/h

kompozicio azonossag

$$f(x) = \sqrt{x + \sqrt{x}} \qquad (x > 0)$$

$$f'(x) = \frac{1}{2\sqrt{x + \sqrt{x}}} \cdot \left(1 + \frac{1}{2\sqrt{x}}\right)$$

2/i

megint kompozicio

$$f(x) = \sin\left(\frac{x^2 + 1}{x + 3}\right) \qquad (x > -3)$$

$$f'(x) = \cos\left(\frac{x^2 + 1}{x + 3}\right) \cdot \frac{2x \cdot (x + 3) - (x^2 + 1)}{(x + 3)^2} = \cos\left(\frac{x^2 + 1}{x + 3}\right) \cdot \frac{x^2 + 6x - 1}{x^2 + 6x + 9}$$

2/j

hat en ezt meg nem csinalom

$$f'(x) = -\frac{\sin\left(2\left(\ln\sqrt{1+\cos^2 x}+1\right)\right)}{\sqrt{2(1+\cos^2 x)}} \cdot \sin 2x$$

megjegyzes

lehetne a fentit ugy is megoldani hogy ne oregedjek meg kozen

van olyan dolog is hogy logaritmikus derivalas ami roviden annyi hogy vesszuk mind a ket oldal logarimusat es utana derivaljuk mind ket oldalt

ez akkor hasznos ha nagyon bonyolult a fuggveny es vannak hatvanyok es ha hanyados szerint derivalnank akkor eltelne az egesz zh mire vegeznenk vele