$$\int f'(x) f^{\alpha}(x)\,\mathrm{d}x \quad \text{tipus mar meg van beszelve}$$

a

$$\int \frac{x}{\sqrt{x^2-1}}\,\mathrm{d}x = \int x\left(x^2-1\right)^{-\frac{1}{2}}\,\mathrm{d}x = \frac{1}{2}\int 2x\cdot\left(x^2-1\right)^{-\frac{1}{2}}\,\mathrm{d}x =$$

$$= \frac{1}{2}\int \left(x^2-1\right)'\left(x^2-1\right)^{-\frac{1}{2}}\,\mathrm{d}x = \frac{1}{2}\frac{\left(x^2-1\right)^{\frac{1}{2}}}{\frac{1}{2}} + C = \sqrt{x^2-1}+C$$

f

$$\int \frac{1}{\cos^2 x\sqrt{\tan^3 x}}\,\mathrm{d}x = \int (\tan x)'(\tan x)^{-\frac{3}{2}}\,\mathrm{d}x = \frac{(\tan x)^{-\frac{1}{2}}}{-\frac{1}{2}} + C$$

masik tipus

$$\int \sin^n x\cdot\cos^m x\,\mathrm{d}x \qquad (n,m\in\mathbb{N};\mathbb{Z};\mathbb{Q})$$

pl

a paratlan kitevobol kell levalasztani egy tagot

$$\int \sin^3 x\cdot\cos^4 x\,\mathrm{d}x = \int \underbrace{\sin^2 x}_{1-\cos^2 x}\cdot\cos^4 x\cdot\underbrace{\sin x}_{-(\cos x)'}\,\mathrm{d}x = -\int \cos^4\cdot(\cos x)'\,\mathrm{d}x + \int \cos^6\cdot(\cos x)'\,\mathrm{d}x = -\frac{\cos^5 x}{5}+\frac{\cos^7 x}{7}+C$$

ha mind a ketto paratlan, a kisebbol kell levalasztani

$$\int \sin^6\cdot\cos^3 x\,\mathrm{d}x = \int \sin^6 x\cdot\cos^2 x\cdot\cos x\,\mathrm{d}x = \int \sin^6 x(1-\sin^2 x)(\sin x)'\,\mathrm{d}x =$$

$$= \int \sin^6 x(\sin x)'\,\mathrm{d}x - \int \sin^8 x(\sin x)'\,\mathrm{d}x = \frac{\sin^7 x}{7}-\frac{\sin^9 x}{9}+C$$

ha mindegyik paros az egesz kuka
a fo ilyenkor otlet a linearizalas

$$\int \sin^2 x\cdot\cos^4 x\,\mathrm{d}x = \int \sin^2 x\cos^2 x\cos^2 x\,\mathrm{d}x = \int \left(\frac{\sin 2x}{2}\right)^2\cdot\frac{1+\cos 2x}{2}\,\mathrm{d}x = \frac{1}{8}\int \sin^2 x\,\mathrm{d}x + \frac{1}{8}\int \sin^2(2x)\cos 2x\,\mathrm{d}x =$$

$$= \frac{1}{8}\int \frac{1-\cos(4x)}{2}\,\mathrm{d}x + \frac{1}{2}\frac{1}{8}\int \sin^3(2x)\cdot(\sin(2x))'\,\mathrm{d}x = \frac{1}{16}x - \frac{1}{16}\cdot\frac{\sin 4x}{4}\cdot\frac{1}{48}(\sin^3 2x)+c$$

hazi

$$\int \sin^3 x\,\mathrm{d}x$$

$$\int \cos^4 x\,\mathrm{d}x$$

$$\int \frac{\cos^3 x}{\sqrt{\sin^5 x}}\,\mathrm{d}x$$

megegy ilyen tipusu pelda

$$\int \sqrt{\frac{\arcsin x}{1-x^2}}\,\mathrm{d}x = \int \underbrace{\frac{1}{\sqrt{1-x^2}}}_{(\arcsin x)'} \cdot (\arcsin x)^{\frac{1}{2}}\,\mathrm{d}x = \frac{2}{3} \cdot \sqrt{\arcsin^3 x} + C$$

hazi

$$\int \frac{1}{(x^2+1)\arctan^2 x}\,\mathrm{d}x$$

II. parcialis integralas

$$(f \cdot g)' = f' \cdot g + f \cdot g' \Longrightarrow \int (f \cdot g)' = \int f' \cdot g + \int f \cdot g' \Longrightarrow f \cdot g = \int f' \cdot g + \int f \cdot g' \Longrightarrow$$

$$\Longrightarrow \int f' \cdot g = f \cdot g - \int f \cdot g' \quad \text{vagy} \quad \int f'(x) \cdot (x)\,\mathrm{d}x = f(x) \cdot g(x) - \int f(x) \cdot g'(x)\,\mathrm{d}x$$

1. tipus

a

a polinom · exp tipusnal mindig az exp legyen a derivalt

$$\int (x^2+2x-1)e^{-2x}\,\mathrm{d}x = \int (x^2+2x-1) \cdot \left(\frac{e^{-2x}}{-2}\right)'\mathrm{d}x = (x^2+2x-1) \cdot \frac{e^{-2}}{-2} - \int (x^2+2x-1)' \cdot \frac{e^{-2}}{-2}\,\mathrm{d}x =$$

$$= -\frac{1}{2}(x^2+2x-1)e^{-2x} + \int (x+1)e^{-2x}\,\mathrm{d}x = -\frac{1}{2}(x^2+2x-1)e^{-2x} + (x+1) \cdot \frac{e^{-2x}}{2} - \int (x+1)' \cdot \frac{e^{-2x}}{-2}\,\mathrm{d}x =$$

$$= \ldots = -\frac{1}{4}(2x^2+6x+1) \cdot e^{-2x} + C$$

hazi

$$\int (x^2-3x)e^{3x}\,\mathrm{d}x$$

b

$$\int x^2 \cdot \sin(2x)\,\mathrm{d}x = \int x^2 \left(-\frac{\cos 2x}{2}\right)'\mathrm{d}x = -\frac{1}{2}\int x^2 (\cos 2x)'\,\mathrm{d}x = -\frac{1}{2} \cdot \left(x^2 \cos 2x - \int 2x \cdot \cos 2x\right)\mathrm{d}x =$$

$$= \int x \cdot \cos 2x\,\mathrm{d}x - \frac{1}{2}x^2 \cos 2x = x \cdot \frac{\sin 2x}{2} - \int 1 \frac{\sin 2x}{2}\,\mathrm{d}x - \frac{1}{2}x^2 \cos 2x = \frac{1}{2}x \cdot \sin 2x + \frac{1}{2}\frac{\cos 2x}{2} - \frac{1}{2}x^2 \cos 2x + C$$

hazi

$$\int (x^2+1) \cdot \cos 2x\,\mathrm{d}x$$

2. tipus

inverzfuggvenyeket tartalmazo integralok

a

$$\int \ln x \,\mathrm{d}x = \int 1 \cdot \ln x \,\mathrm{d}x = \int (x)' \ln x \,\mathrm{d}x = x \cdot \ln x - \int x \cdot \frac{1}{x} \,\mathrm{d}x = x \ln x - \int 1 \,\mathrm{d}x = x \ln x - x + C$$

b

$$\int (x^2 - 3x + 1) \cdot \ln x \,\mathrm{d}x = \int \left(\frac{x^3}{3} - 3\frac{x^2}{2} + x \right)' \cdot \ln x \,\mathrm{d}x = ... = \left(\frac{x^3}{3} - \frac{3}{2}x^2 + x \right) \cdot \ln x - \frac{1}{9}x^3 + \frac{3}{4}x^2 - x + C$$

c

$$\int \arctan(3x) \,\mathrm{d}x = \int 1 \cdot \arctan(3x) \,\mathrm{d}x = \int x' \cdot \arctan 3x \,\mathrm{d}x =$$

$$= x \cdot \arctan(3x) - \int x \cdot (\arctan(3x))' \,\mathrm{d}x = x \arctan 3x - \frac{1}{6} \ln(9x^2 + 1) + C$$

hazi

$$\int \arcsin(2x) \,\mathrm{d}x$$

$$\int (\arcsin x)^2 \,\mathrm{d}x$$

3. tipus

egyenlettel megoldhato esetek

3/b

$$\int e^{2x} \cdot \sin x \,\mathrm{d}x = \int e^{2x} \cdot (-\cos x)' \,\mathrm{d}x = -\int e^{2x} (\cos x)' \,\mathrm{d}x = -\left(e^{2x} \cos x - 2 \int e^{2x} \cos x \,\mathrm{d}x \right) =$$

$$= -e^{2x} \cos x + 2 \int e^{2x} \cos x \,\mathrm{d}x = -e^{2x} \cos x + 2\left(e^{2x} \cdot \sin x - \int 2e^{2x} \sin x \,\mathrm{d}x \right) = 2e^{2x} \sin x - e^{2x} \cos x - 4 \int e^{2x} \sin x \,\mathrm{d}x$$

tehat

$$I(x) = e^{2x} \cdot (2 \sin x - \cos x) - 4I(x) \Longrightarrow I(x) = \frac{1}{5} e^{2x} (\sin x - \cos x) + C$$

hazi ("jo kis zh jelolt")

$$\int e^{-x} \cdot \cos 5x \,\mathrm{d}x$$

b

$$\int \sqrt{1-x^2}\,\mathrm{d}x = \int (x)' \cdot (1-x^2)\,\mathrm{d}x = x \cdot \sqrt{1-x^2} - \int x \cdot \frac{-2x}{2\sqrt{1-x^2}}\,\mathrm{d}x =$$

$$= x\sqrt{1-x^2} - \int \frac{-x^2}{\sqrt{1-x^2}}\,\mathrm{d}x = x\sqrt{1-x^2} - \int \frac{1-x^2-1}{\sqrt{1-x^2}}\,\mathrm{d}x = x\sqrt{1-x^2} - \int \sqrt{1-x^2}\,\mathrm{d}x + \int \frac{1}{\sqrt{1-x^2}}\,\mathrm{d}x \Longrightarrow$$

$$\Longrightarrow 2 \cdot \int \sqrt{1-x^2}\,\mathrm{d}x = x\sqrt{1-x^2} + \arcsin x \Longrightarrow \int \sqrt{1-x^2}\,\mathrm{d}x = \frac{1}{2} \cdot x \cdot \sqrt{1-x^2} + \frac{1}{2}\arcsin x + C$$