$G = (V, E)$ is a finite 2-connected graph. Let x be any vertex of G. There exists a cycle C in G that contains x, and a vertex $y \neq x$ such that every neighbour of y is on C.

Definition: a C-trail (where C is a cycle) is either a path that has only its end vertices (but no edges) in common with C, or a cycle with exactly one vertex in common with C.

E' is a set of new edges that we needed to add, each edge in E' is parallel to an existing edge in G. No pair of vertices has more than two edges between them, the purpose of these is to increase the degree of certain vertices.

Let D be a component of $G - C$. For each such D, there exists a set of C-trails (that are edge disjoint but not vertex disjoint) in $G^2 + E'$, such that every vertex of D is in exactly one C-trail, and every edge that is on some C-trail and also incident with a vertex of C is in either E or E'. Denoting E_D as the edge set of the component D, so E_D is the union of these trails.

Let $G' = (V, E(C) \cup E_D^*)$, where E_D^* is the union of all C-trails over all components of $G - C$. We know every vertex in $G' - C$ has degree two. next we will add edges to C in G' to construct an eulerian multigraph, which we will call G_α

We can write $C = xg_1z \dots d_1yd_2 \dots g_2$ (if an edge e has a parallel edge, we write it as e')

depending on whether g_is and d_is have parallel edges, there are three ways to split up C into paths P_1, P_2, P_3, while deleting certain pairs of parallel edges., this transforms G_α into G_β, which is also a eulerian multigraph.

there will either be a single path P_3, or two options of having only P_1, P_2*

write $P_i = x_0^i e_1^i x_1 e_2^i x_2 \dots e_{l_i}^i x_{l_i}^i$

next we construct J', an eulerian tour of G_β, so it is a walk that contains every edge exactly once. this implies some cyclic ordering on all the edges of G_β, so it makes sense to talk about consecutive edges of J'.

J' is constructed to have a few specific properties that are only used to prove correctness, it is chosen mostly arbitrarily, except that: within some path P_i, if e_{j+1} has a parallel edge e'_{j+1} the edges e_j, e'_{j+1} are consecutive in J'

define: a pass through v is a path of length two with v as an internal vertex of the path, such that both edges are consecutive in J'.

If a pass gets 'marked' this means it will eventually be replaced by an edge in G^2.

For the fixed vertex x, mark every pass through x except for one chosen arbitrarily.

pick any $v \neq x$. if v is in $G - C$, then there is only one pass through v(by the above construction with C-trails), so leave this pass unmarked

if v is in C, then there is some pair i, j such that $v = x_j^i$, $j \neq 0$. mark all passes through v except the unique pass containing e_j^i (in other words, the unique edge 'preceding' v on the cycle).

now we know the following property:

if $e = uv$ is any edge in J', there are exactly two passes of J' that contain e, and at most one of these is marked. If $u = x$, then the pass of J' through v that contains e is unmarked

finally, construct H' by replacing every marked pass of J' by an edge of G^2 (which will be a virtual edge, since J' contained every edge of the original graph already). H' is a hamiltonian cycle of $G^2 + E'$. finally, replace every edge in E' with the corresponding edge in G, to obtain hamiltonian cycle H of G^2.