Esercizi_Dstribusioni_discrete

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12/11/2022

Exercise 3.20

A quality control engineer tests the quality of produced computers. Suppose that 5% of computers have defects, and defects occur independently of each other. a) Find the probability of exactly 3 defective computers in a shipment of twenty. b) Find the probability that the engineer has to test at least 5 computers in order to find 2 defective ones.

$$a) \quad X = \text{number of defective components in a shipment} \quad P[X=1] = 0.05$$

$$P[X=3] = \text{dbinom}(3, 20, 0.05) = 0.05958215$$

$$b) \quad Y = \text{number of failures occuredd before two success (defected components)}$$

$$P[Y \geq 3] = 1 - P[Y < 3] = 1 - P[Y \leq 2] = 1 - (\text{pnbinom}(3-1, 2, 0.05)) = 0.9859813$$

Exercise 3.21

A lab network consisting of 20 computers was attacked by a computer virus. This virus enters each computer with probability 0.4, independently of other computers. Find the probability that it entered at least 10 computers.

$$X = \text{Number of computer attecked by a virus} \quad P[X=1] = 0.4$$

$$P[X \geq 10] = 1 - P[X < 10] = 1 - P[X \leq 9] = 1 - \text{pbinom}(9, 20, 0.4) = 0.2446628$$

Exercise 3.22

Five percent of computer parts produced by a certain supplier are defective. What is the probability that a sample of 16 parts contains more than 3 defective ones?

$$X = \text{Number of computer parts produced by a certain supplier are defective} \quad P[X=1] = 0.05$$

$$P[X > 10] = 1 - P[X \leq 10] = 1 - \text{pbinom}(3, 16, 0.05) = 0.007003908$$

Exercise 3.23

Every day, a lecture may be canceled due to inclement weather with probability 0.05. Class cancelations on different days are independent. a) There are 15 classes left this semester. Compute the probability that at least 4 of them get canceled. b) Compute the probability that the tenth class this semester is the third class that gets canceled

$$a) \quad X = \text{Number of classes clancelled} \quad P[X=1] = 0.05$$

$$P[X \geq 4] = 1 - P[X < 4] = 1 - P[X \leq 3] = 1 - \text{pbinom}(3, 15, 0.05) = 0.005467259$$

$$b)\text{\quadY} = \text{number of class cancelled before get 3 cancelled classes}$$

$$P[Y=7] = \text{dnbinom}(7, 3, 0.05) = 0.003142518$$

Exercise 3.24

An internet search engine looks for a certain keyword in a sequence of independent web sites. It is believed that 20% of the sites contain this keyword. a) Compute the probability that at least 5 of the first 10 sites contain the given keyword. b) Compute the probability that the search engine had to visit at least 5 sites in order to find the first occurrence of a keyword.

$$a) \quad X = \text{Number of sites that contain a specific keyword} P[X=1] = 0.2$$

$$P[X \geq 5] = 1 - P[X < 5] = 1 - P[X \leq 4] = 1 - \text{pbinom}(4, 10, 0.2) = 0.0327935$$

$$b) \quad Y = \text{numer of site that a serach engine had to visit before finding the first occurrence of a keyword}$$

$$P[Y \geq 5 - 1] = 1 - P[Y < 4] = 1 - P[Y \leq 3] = 1 - \text{pgeom}(3, 0.2) = 0.4096$$

Exercise 3.25

About ten percent of users do not close Windows properly. Suppose that Windows is installed in a public library that is used by random people in a random order. a) On the average, how many users of this computer do not close Windows properly before someone does close it properly? b) What is the probability that exactly 8 of the next 10 users will close Windows properly?

$$a) \quad np$$

$$b) \quad \text{dbinom}(8, 10, 0.9) = 0.1937102$$

Exercise 3.26

After a computer virus entered the system, a computer manager checks the condition of all important files. She knows that each file has probability 0.2 to be damaged by the virus, independently of other files. a) Compute the probability that at least 5 of the first 20 files are damaged. b) Compute the probability that the manager has to check at least 6 files in order to find 3 undamaged files.

$$a) \quad X = \text{number of demaged files} P[X=1] = 0.2$$

$$P[X \geq 5] = 1 - P[X < 4] = 1 - \text{pbinom}(4, 20, 0.2) = 0.3703517$$

$$b) \quad Y = \text{number of demages file that he has to check until see the third undamaged file}$$

$$P[Y \geq 3] = 1 - P[Y < 2] = 1 - \text{pnbinom}(2, 3, 0.8) = 0.05792$$

Exercise 3.27

Messages arrive at an electronic message center at random times, with an average of 9 messages per hour. a) What is the probability of receiving at least five messages during the next hour? b) What is the probability of receiving exactly five messages during the next hour?

$$a) \quad X = \text{number of messages received in the next hour}$$

$$P[X \geq 5] = 1 - P[X < 4] = 1 - \text{ppois}(4, 9) = 0.9450364$$

$$P[X = 5] = \text{dpois}(5, 9) = 0.06072688$$

Exercise 3.29

An insurance company divides its customers into 2 groups. Twenty percent of customers are in the high-risk group, and eighty percent are in the low-risk group. The high-risk customers make an average of 1 accident per year while the low-risk customers make an average of 0.1 accidents per year. Eric had no accidents last year. What is the probability that he is a high-risk driver?

H = high-risk group L = low-risk group A = costumer make an accident

$$P[H] = 0.2 \quad P[L] = 0.8 \quad P[A|H] \sim Pois(0,1) \quad P[A|L] \sim Pois(0, 0.1$$

$$P[H|A] = \frac{P[A|H]P[H]}{P[A]}$$

$$P[A] = P[H]P[H|A] + P[L]P[L|A] = 0.2 \times Pois(0,1) + 0.8 \times Pois(0, 0.1) = 0.7974458$$

$$P[H|A] = \frac{0.2 \times Pois(0,1)}{0.7974458} = 0.09226444$$

Exercise 3.30

Eric from Exercise 3.29 continues driving. After three years, he still has no traffic accidents. Now, what is the conditional probability that he is a high-risk driver?

H = high-risk group L = low-risk group A = costumer make an accident

$$P[H] = 0.2 \quad P[L] = 0.8 \quad P[A|H] \sim Pois(0,3) \quad P[A|L] \sim Pois(0,0.3$$

$$P[H|A] = \frac{P[A|H]P[H]}{P[A]}$$

$$P[A] = P[H]P[H|A] + P[L]P[L|A] = 0.2 \times Pois(0,3) + 0.8 \times Pois(0,0.3) = 0.602612$$

$$P[H|A] = \frac{0.2 \times Pois(0,3)}{0.602612} = 0.01652376$$

Exercise 3.31

Before the computer is assembled, its vital component (motherboard) goes through a special inspection. Only 80% of components pass this inspection. a) What is the probability that at least 18 of the next 20 components pass inspection? b) On the average, how many components should be inspected until a component that passes inspection is found?

$$a) \quad X = \text{number of component that pass the inspection}$$

$$P[X \geq 18] = 1 - P[X < 17] = 1 - \text{pbinom}(17, 20, 0.8) = 0.2060847$$

$$b) \quad Y = \text{number of components that should be inspected until a component that passes inspection is found}$$

$$\mathbf{E}[Y] = \frac{1}{p} = 1.25 \approx 1$$

Exercise 3.32

On the average, 1 computer in 800 crashes during a severe thunderstorm. A certain company had 4,000 working computers when the X area was hit by a severe thunderstorm. a) Compute the probability that less than 10 computers crashed. b) Compute the probability that exactly 10 computers crashed. You may want to use a suitable approximation.

$$a) \quad X = \text{number of crashed computers by asevere thunderstorm}$$

$$X \sim Binomial(n = 4000, p = 1/800) \approx Pois(\lambda = np = 5$$

$$P[X < 10] = P[X \leq 9] = \text{ppois}(9, 5) = 0.9681719$$

$$b) \quad P[X = 10] = \text{dpois}(10, 5) = 0.01813279$$

Exercise 3.33

The number of computer shutdowns during any month has a Poisson distribution, averaging 0.25 shutdowns per month. a) What is the probability of at least 3 computer shutdowns during the next year? b) During the next year, what is the probability of at least 3 months (out of 12) with exactly 1 computer shutdown in each?

$$a) \quad X = \text{number of computer that shutdowns during the next year}$$

$$X \sim Pois(0.25) \quad P[X \geq 3] = 1 - P[X \leq 2] = 1 - \text{ppois}(2, 0.25 \times 12) = 0.5768099$$

$$b) \quad Y = \text{number months (out of 12) with exactly 1 computer shutdown in each}$$

$$K = \text{dpois}(1, 0.25)$$

$$P[Y \geq 3] = 1 - P[Y < 2] = 1 - \text{pbinom}(2, 12, \text{K}) = 0.4228186$$

Exercise 3.34

A dangerous computer virus attacks a folder consisting of 250 files. Files are affected by the virus independently of one another. Each file is affected with the probability 0.032. What is the probability that more than 7 files are affected by this virus?

$$X = \text{number of files affected by a virus}$$

$$X \sim Binomial(250, 0.032) \approx Poisson(\lambda = np = 8)$$

$$P[X > 7] = 1 - P[X \leq 6] = 1 - \text{ppois}(6, 8) = 0.6866257$$

Exercise 3.35

In some city, the probability of a thunderstorm on any day is 0.6. During a thunderstorm, the number of traffic accidents has Poisson distribution with parameter 10. Otherwise, the number of traffic accidents has Poisson distribution with parameter 4. If there were 7 accidents yesterday, what is the probability that there was a thunderstorm?

T = thunderstorm NT = non thunderstorm A = traffic accidents

$$P[T] = 0.6 \quad P[NT] = 0.4 \quad P[A|T] \sim Pois(10) \quad P[A|NT] \sim Pois(7)$$

$$P[T|A] = \frac{P[A|T]P[T]}{P[A]}$$

$$P[A] = P[T]P[T|A] + P[NT]P[NT|A] = 0.6 \times dpois(7, 10) + 0.4 \times dpois(7, 4) = 0.07786368$$

$$P[H|A] = \frac{0.6 \times dpois(7, 10)}{0.07786368} = 0.6941302$$

Exercise 3.36

An interactive system consists of ten terminals that are connected to the central computer. At any time, each terminal is ready to transmit a message with probability 0.7, independently of other terminals. Find the probability that exactly 6 terminals are ready to transmit at 8 o'clock.

$$dpois(6, 10, 0.7) = 0.06305546????????????$$

Exercise 3.37

Network breakdowns are unexpected rare events that occur every 3 weeks, on the average. Compute the probability of more than 4 breakdowns during a 21-week period.

$$X = \text{Network breakdowns during a week} \quad X \sim Pois(3)$$

$$X = \text{Network breakdownsduring a 21-week period} \quad X \sim Pois(21/3)$$

$$P[X > 4] = 1 - P[X \leq 4] = 1 - \text{ppois}(4, 7) = 0.8270084$$