ex_continuous

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Exercise 4.6

A program is divided into 3 blocks that are being compiled on 3 parallel computers. Each block takes an Exponential amount of time, 5 minutes on the average, independently of other blocks. The program is completed when all the blocks are compiled. Compute the expected time it takes the program to be compiled.

Exercise 4.7

The time it takes a printer to print a job is an Exponential random variable with the expectation of 12 seconds. You send a job to the printer at 10:00 am, and it appears to be third in line. What is the probability that your job will be ready before 10:01?

$$P[X \geq 3] = P[X \leq 2] = 1 - ppois(2, 5) = 0.875348$$

##Exercise 4.8

For some electronic component, the time until failure has Gamma distribution with parameters α = 2 and λ = 2 (years−1). Compute the probability that the component fails within the first 6 months.

$$X = \text{number of months of live component}$$

$$P[X \leq 6] = pgamma(0.5, 2, 2) = 0.2642411$$

##Exercise 4.9

On the average, a computer experiences breakdowns every 5 months. The time until the first breakdown and the times between any two consecutive breakdowns are independent Exponential random variables. After the third breakdown, a computer requires a special maintenance. (a) Compute the probability that a special maintenance is required within the next 9 months. (b) Given that a special maintenance was not required during the first 12 months, what is the probability that it will not be required within the next 4 months?

$$pgamma(9, 4, 5)$$

Exercise 4.10

Two computer specialists are completing work orders. The first specialist receives 60% of all orders. Each order takes her Exponential amount of time with parameter λ1 = 3 hrs−1. The second specialist receives the remaining 40% of orders. Each order takes him Exponential amount of time with parameter λ2 = 2 hrs−1. A certain order was submitted 30 minutes ago, and it is still not ready. What is the probability that the first specialist is working on it?

S1 = Specialist 1 S2 = Specialist 2 OT1 = order time of S1 OT2 = order time of S2 S = order submitted 30 minutes ago and it is still not ready

$$P[S1|S] = \frac{P[S|S1]P[S1]}{P[S]}$$

$$P[S] = P[S1] \times P[OT1|S1] + P[S2] \times P[OT2|S2] = 0.6 \times (1 - pexp(0.5, 3)) + 0.4 \times (1 - pexp(0.5, 2)) = 0.2810299$$

$$P[S1|S] = \frac{P[S1] \times P[OT1|S1]}{P[S]} = \frac{0.6 \times (1 - pexp(0.5, 3))}{0.2810299} = 0.4763838$$

Exercise 4.11

Consider a satellite whose work is based on a certain block A. This block has an independent backup B. The satellite performs its task until both A and B fail. The lifetimes of A and B are exponentially distributed with the mean lifetime of 10 years. (a) What is the probability that the satellite will work for more than 10 years? (b) Compute the expected lifetime of the satellite.

$$a) \quad X = \text{number of lifetime years of a satellite}$$

$$P[X > 10] = 1 - P[X \leq 10] = 1 - (P[A \leq 10] \times P[B \leq 10])$$

$$= 1 - (pexp(10, .1) * pexp(10, .1)) = 0.6004236$$

$$b)$$

Exercise 4.12

A computer processes tasks in the order they are received. Each task takes an Exponential amount of time with the average of 2 minutes. Compute the probability that a package of 5 tasks is processed in less than 8 minutes.

$$pgamma(8, 5, 1/2) = 0.3711631$$

Exercise 4.13

On the average, it takes 25 seconds to download a file from the internet. If it takes an Exponential amount of time to download one file, then what is the probability that it will take more than 70 seconds to download 3 independent files?

$$1 - pgamma(70, 3, 1/25) = 0.4694537$$

Exercise 4.14

The time X it takes to reboot a certain system has Gamma distribution with E(X) = 20 min and Std(X) = 10 min. (a) Compute parameters of this distribution. (b) What is the probability that it takes less than 15 minutes to reboot this system?

Exercise 4.15

A certain system is based on two independent modules, A and B. A failure of any module causes a failure of the whole system. The lifetime of each module has a Gamma distribution, with parameters α and λ given in the table,

|———|—|—————-| | Component | a | lambda (years^-1) | |———|—|—————| | A | 3 | 1 | |———|—|—————| | B | 2 | 2 | | ———|—|—————|

a. What is the probability that the system works at least 2 years without a failure?
b. Given that the system failed during the first 2 years, what is the probability that it failed due to the failure of component B (but not component A)?

$$a) \quad X = \text{lifetime years of the system}$$

$$P[X \geq 2] = \{P[A \geq 2] \cap P[B \geq 2]\} = \{(1 - P[A < 2]) \cup (1 - P[B < 2])\} = ((1 - pgamma(2, 2, 2)) * (1 - pgamma(2, 3, 1))) = 0.061$$

$$b) \quad Y = \text{the system failed due to the failure of component B (but not component A)}$$

$$P[Y|X < 2] = \frac{P[Y, X < 2]}{P[X < 2]} = \frac{(1 - P[A \leq 2]) \times P[B \leq 2]}{P[X < 2]} = \frac{((1 - pgamma(2, 3, 1)) * pgamma(2, 2, 2))}{0.9380312} = 0.6553168$$

Exercise 4.21

The average height of professional basketball players is around 6 feet 7 inches (79 inches), and the standard deviation is 3.89 inches. Assuming Normal distribution of heights within this group, a) What percent of professional basketball players are taller than 7 feet (84 inches)? b) If your favorite player is within the tallest 20% of all players, what can his height be?

$$a) \quad X = \text{height of professional basketball players} \sim N(79, 3.89)$$

$$P[X > 84] = 1 - P[X \leq 84] = 1 - \left(Z \leq \frac{84 - 79}{3.89}\right) = 1 - pnorm((84 - 79)/3.89) = 0.09933552$$

$$b) \quad P[X > x] = 1 - P[X \leq x] = 1 - \left(Z \leq \frac{x - 79}{3.89}\right) = 0.2$$

$$1 - \Phi\left(\frac{x - 79}{3.89}\right) = 0.2$$

$$\Phi\left(\frac{x - 79}{3.89}\right) = 0.8$$

$$\frac{x - 79}{3.89} = \Phi(0.8)$$

$$\frac{x - 79}{3.89} = 0.8416212 \quad x = (0.8416212 \times 3.89) + 79 = 82.27391$$

Exercise 4.22

Refer to the country in Example 4.11 on p. 91, where household incomes follow Normal distribution with μ = 900 coins and σ = 200 coins. a) A recent economic reform made households with the income below 640 coins qualify for a free bottle of milk at every breakfast. What portion of the population qualifies for a free bottle of milk? b) Moreover, households with an income within the lowest 5% of the population are entitled to a free sandwich. What income qualifies a household to receive free sandwiches?

$$a) \quad X = \text{income of households} \sim N(900, 200)$$

$$P[X < 640] = P\left\{\frac{X - \mu}{\sigma} < \frac{640 - \mu}{\sigma}\right\} = P\left\{Z < \frac{640 - 900}{200}\right\} = P[Z < -1.3] = \Phi(-1.3) = pnorm(-1.3) = 0.09680048$$

$$b) \quad P[X < x] = P[X \leq x] = \left(Z \leq \frac{x - 900}{200}\right) = 0.05$$

$$\Phi\left(\frac{x - 900}{200}\right) = 0.05$$

\frac{x - 900}{200}} = \Phi(0.05)

$$\frac{x - 900}{200} = -1.644854 \quad x = (-1.644854 \times 200) + 900 = 571.0292$$

Exercise 4.23

The lifetime of a certain electronic component is a random variable with the expectation of 5000 hours and a standard deviation of 100 hours. What is the probability that the average lifetime of 400 components is less than 5012 hours?

$$X = \text{The lifetime of a certain electronic component} \quad E[X] = 5000hours \quad SD[X] = 100hours \quad n = 400 \quad E[S_n] = 5012$$

$$S_n = \sum_{i=1}^{n} X_i = E[S_n] \times n = 2004800$$

$$P[S_n < 5012] = P\left\{ \frac{S_n - n\mu}{\sigma\sqrt{n}} < \frac{2004800 - 5000 * 400}{100\sqrt{400}} \right\} = \Phi(2.4) = pnomr(2.4) = 0.9918025$$

Exercise 4.24

Installation of some software package requires downloading 82 files. On the average, it takes 15 sec to download one file, with a variance of 16 sec^2. What is the probability that the software is installed in less than 20 minutes?

$$X = \text{Minutes take to install the software} \quad E[X] = 15s \quad SD[X] = 4s \quad n = 82$$

$$P[S_n \leq 20] = P\left\{ \frac{S_n - n\mu}{\sigma\sqrt{n}} < \frac{20 - 82 * (15/60)}{4/60\sqrt{82}} \right\} = \Phi\left(\frac{20 - 82 * (15/60)}{4/60\sqrt{82}} \right) = pnorm(-0.8282364) = 0.2037683$$

Exercise 4.25

Among all the computer chips produced by a certain factory, 6 percent are defective. A sample of 400 chips is selected for inspection. a) What is the probability that this sample contains between 20 and 25 defective chips (including 20 and 25)? b) Suppose that each of 40 inspectors collects a sample of 400 chips. What is the probability that at least 8 inspectors will find between 20 and 25 defective chips in their samples?

$$a) \quad p = 0.06 \quad n = 400 \quad X = \text{Number of defective computer chips in a sample of 400}$$

$$X \sim Binomial(n, p) \approx Normal(np, \sqrt{np(1-p)})$$

$$P[20 \leq X \leq 25] = P\left\{ \frac{(20 - 0.5) - \mu}{\sigma} \leq \frac{X - \mu}{\sigma} \leq \frac{(25 + 0.5) - \mu}{\sigma} \right\}$$

$$P\left\{ \frac{19.5 - 24}{4.749737} \leq Z \leq \frac{25.5 - 24}{4.749737} \right\}$$

$$\Phi\left\{ \frac{25.5 - 24}{4.749737} \right\} - \Phi\left\{ \frac{19.5 - 24}{4.749737} \right\}$$

$$\Phi(0.315807) - \Phi(-0.9474209)$$

$$pnorm(0.315807) - pnorm(-0.9474209) = 0.4522133$$

$$b) \quad Y = \text{inspectors will find between 20 and 25 defective chips in their samples}$$

$$P[Y \geq 8] = 1 - P[Y < 8] = 1 - P[Y \leq 7] = 1 - pbinom(7, 40, 0.4522133) = 0.9997762$$

Exercise 4.26

An average scanned image occupies 0.6 megabytes of memory with a standard deviation of 0.4 megabytes. If you plan to publish 80 images on your web site, what is the probability that their total size is between 47 megabytes and 50 megabytes?

Teorema limite centrale, non ho voglia di scriverlo bene

$$X = \text{size of my 80 images} \quad E[X] = 0.6MBs \quad SD[X] = 0.4MBs \quad n = 80$$

$$P[47 \leq X \leq 50]$$

$$pnorm((47 - (80 * 0.6))/(0.4 * sqrt(80))) - pnorm((50 - (80 * 0.6))/(0.4 * sqrt(80)))$$

$$0.7119249 - 0.3899273 = 0.3219976$$

Exercise 4.27

A certain computer virus can damage any file with probability 35%, independently of other files. Suppose this virus enters a folder containing 2400 files. Compute the probability that between 800 and 850 files get damaged (including 800 and 850).

$$p = 0.35 \quad n = 2400 \quad X = \text{Number of demaged files}$$

$$X \sim Binomial(n, p) \approx Normal(np, \sqrt{np(1-p)})$$

$$P[800 \leq X \leq 850] = P\left\{\frac{(800-0.5)-\mu}{\sigma} \leq \frac{X-\mu}{\sigma} \leq \frac{(850+0.5)-\mu}{\sigma}\right\}$$

$$P\left\{\frac{799.5-840}{23.36664} \leq Z \leq \frac{850.5-840}{23.36664}\right\}$$

$$\Phi\left\{\frac{799.5-840}{23.36664}\right\} - \Phi\left\{\frac{850.5-840}{23.36664}\right\}$$

$$\Phi(0.4493586) - \Phi(-1.73324)$$

$$pnorm(0.4493586) - pnorm(-1.73324) = 0.631887$$

Exercise 4.28

Seventy independent messages are sent from an electronic transmission center. Messages are processed sequentially, one after another. Transmission time of each message is Exponential with parameter λ = 5 min−1. Find the probability that all 70 messages are transmitted in less than 12 minutes. Use the Central Limit Theorem.

$$X = \text{Transmission time of each message in an electronic transmission center} \sim Exponential(5)$$

$$P[S_n \leq 12] = P\left\{\frac{S_n - E[S_n]}{\sigma \times \sqrt{n}} \leq \frac{12-(70\times 1/5)}{1/5\times\sqrt{70}}\right\} = \Phi\left\{\frac{12-(70\times 1/5)}{1/5\times\sqrt{70}}\right\}$$

$$\Phi(-1.195229) = pnorm(-1.195229) = 0.1159988$$

Exercise 4.29

A computer lab has two printers. Printer I handles 40% of all the jobs. Its printing time is Exponential with the mean of 2 minutes. Printer II handles the remaining 60% of jobs. Its printing time is Uniform between 0 minutes and 5 minutes. A job was printed in less than 1 minute. What is the probability that it was printed by Printer I?

P1 = Printer 1 P2 = Printer 2 T = printing time

$$P[P1] = 0.4 \quad T|P1 \sim Exp(1/2)$$

$$P[P2] = 0.6 \quad T|P2 \sim Uni(0,5)$$

$$P[P1|T<1] = \frac{P[T<1|P1]\times P[P1]}{P[T<1]}$$

$$P[T<1] = P[P1]\times P[T<1|P1] + P[P2]\times P[T<1|P2] = 0.4\times pexp(1,1/2) + 0.6\times punif(1,0,5) = 0.2773877$$

$$P[P1|T<1] = \frac{0.4 * pexp(1,1/2)}{0.2773877} = 0.5673926$$

Exercise 4.30

An internet service provider has two connection lines for its customers. Eighty percent of customers are connected through Line I, and twenty percent are connected through Line II. Line I has a Gamma connection time with parameters α = 3 and λ = 2 min−1 . Line II has a Uniform(a, b) connection time with parameters a = 20 sec and b = 50 sec. Compute the probability that it takes a randomly selected customer more than 30 seconds to connect to the internet.

L1 = Line 1 L2 = Line 2 C = connection time taken

$$P[L1] = 0.8 \quad C|L1 \sim Gam(3,2)$$

$$P[L2] = 0.2 \quad C|L2 \sim Uni(20/60, 50/60)$$

$$P[C > 30/60] = \{P[L1] * (1 - P[C \leq 1/2|L1]) + P[L2] * (1 - P[C \leq 1/2|L2])\} =$$

$$0.8 * (1 - pgamma(1/2,3,2)) + 0.2 * (1 - punif(1/2, 2/6, 5/6)) = 0.8690922$$

Exercise 4.31

Upgrading a certain software package requires installation of 68 new files. Files are installed consecutively. The installation time is random, but on the average, it takes 15 sec to install one file, with a variance of 11 sec^2. a) What is the probability that the whole package is upgraded in less than 12 minutes? b) A new version of the package is released. It requires only N new files to be installed, and it is promised that 95% of the time upgrading takes less than 10 minutes. Given this information, compute N.

$$a) \quad X = \text{Installation time} \quad E[X] = 15sec = 0.25m \quad SD[X] = 11sec = 0.1833333min$$

$$P[S_n < 12] = P\left\{\frac{S_n - E[S_n]}{\sigma\times\sqrt{n}} \leq \frac{12-(68\times 0.25)}{0.1833333\times\sqrt{68}}\right\} = \Phi\left\{\frac{12-(68\times 0.25)}{0.1833333\times\sqrt{68}}\right\}$$

$$\Phi(-3.307305) = pnorm(-3.307305) = 0.0004709913$$

$b)$