

# Probabilistic Machine Learning and AI

# Outline of the lecture

This lecture introduces you to the fascinating subject of classification and regression with convolutional neural networks.

# Types of Learning

- **Supervised (inductive) learning**
  - Training data includes desired outputs
- **Unsupervised learning**
  - Training data does not include desired outputs
- **Semi-supervised learning**
  - Training data includes a few desired outputs
- **Reinforcement learning**
  - Rewards from sequence of actions

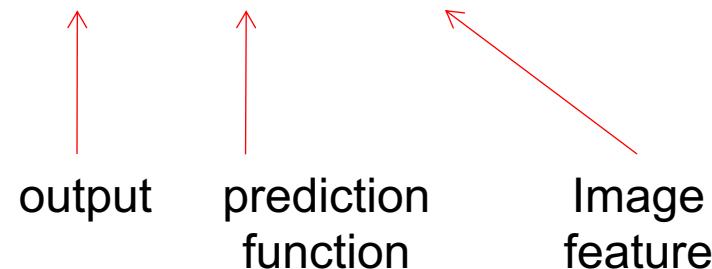
# The machine learning framework

- Apply a prediction function to a **feature representation** of the image to get the desired output:

$$h(\text{apple}) = \text{"apple"}$$
$$h(\text{cow}) = \text{"cow"}$$

# The machine learning framework

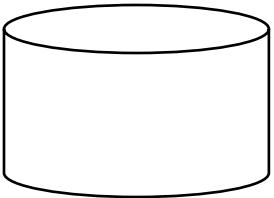
$$y = h(\mathbf{x})$$



- **Training:** given a *training set* of labeled examples  $\{(\mathbf{x}_1, y_1), \dots, (\mathbf{x}_N, y_N)\}$ , estimate the prediction function  $h$  by minimizing the prediction error on the training set
- **Testing:** apply  $h$  to a never before seen *test example*  $\mathbf{x}$  and output the predicted value  $y = h(\mathbf{x})$

# Generalization

## Data Training



Training  
Labels



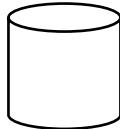
Training



$h$

Learned  
model

## Data Testing



$h$

Learned  
model



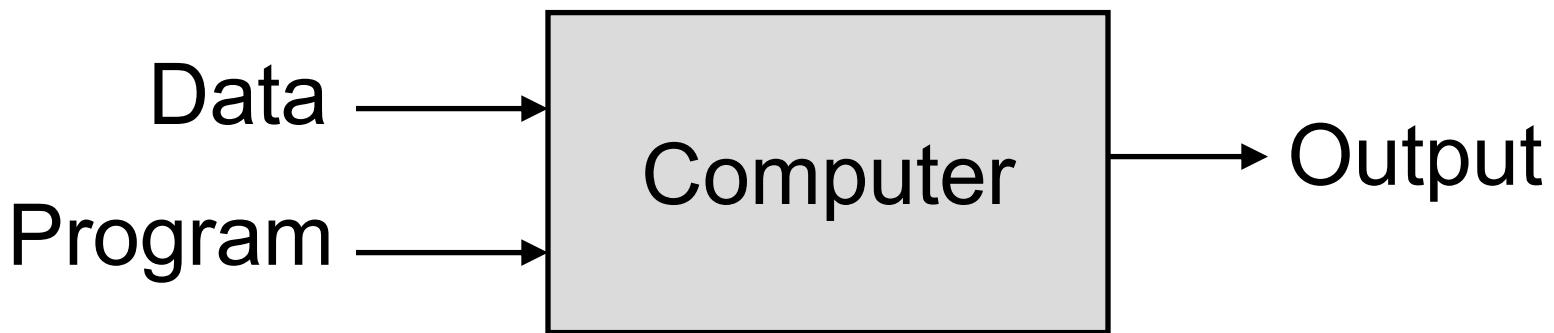
Prediction

How well does a learned model generalize from the data it was trained on to a new test set?

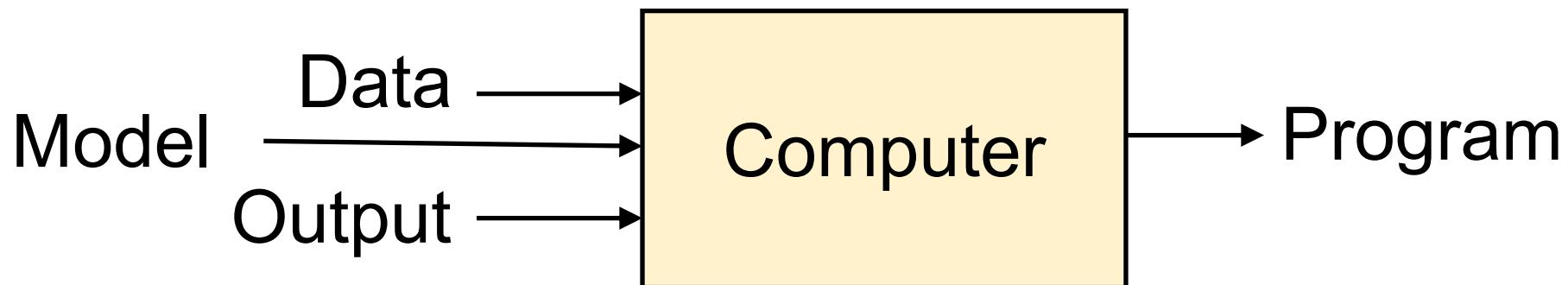
# Generalization

- Components of generalization error
  - **Bias:** how much the average model over all training sets differ from the true model?
    - Error due to inaccurate assumptions/simplifications made by the model
  - **Variance:** how much models estimated from different training sets differ from each other
- **Underfitting:** model is too “simple” to represent all the relevant class characteristics
  - High bias and low variance
  - High training error and high test error
- **Overfitting:** model is too “complex” and fits irrelevant characteristics (noise) in the data
  - Low bias and high variance
  - Low training error and high test error

## Traditional Programming

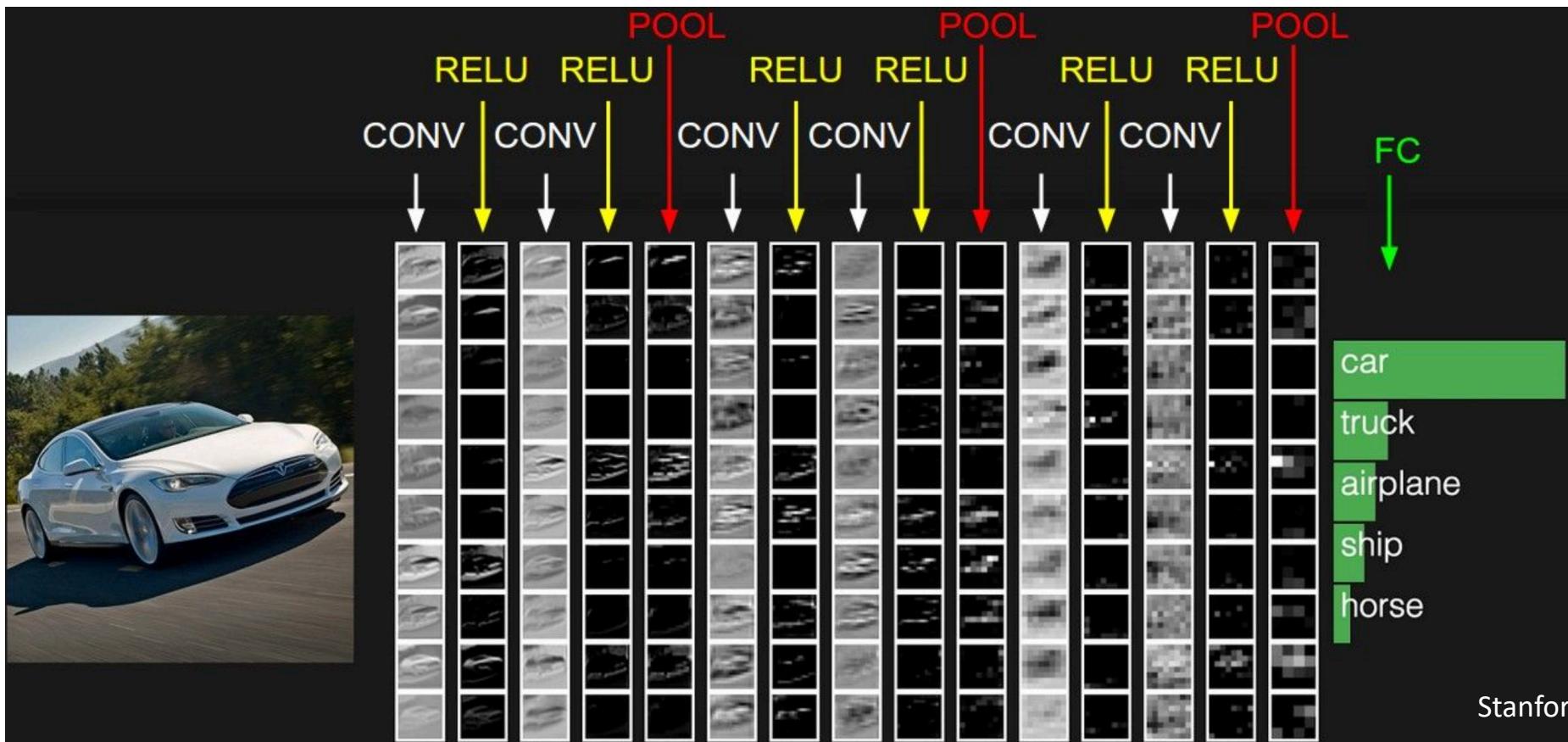


## Machine Learning



# Convolution Neural Network (CNN)

Neural Networks that share their parameters across space.



# What is convolution?

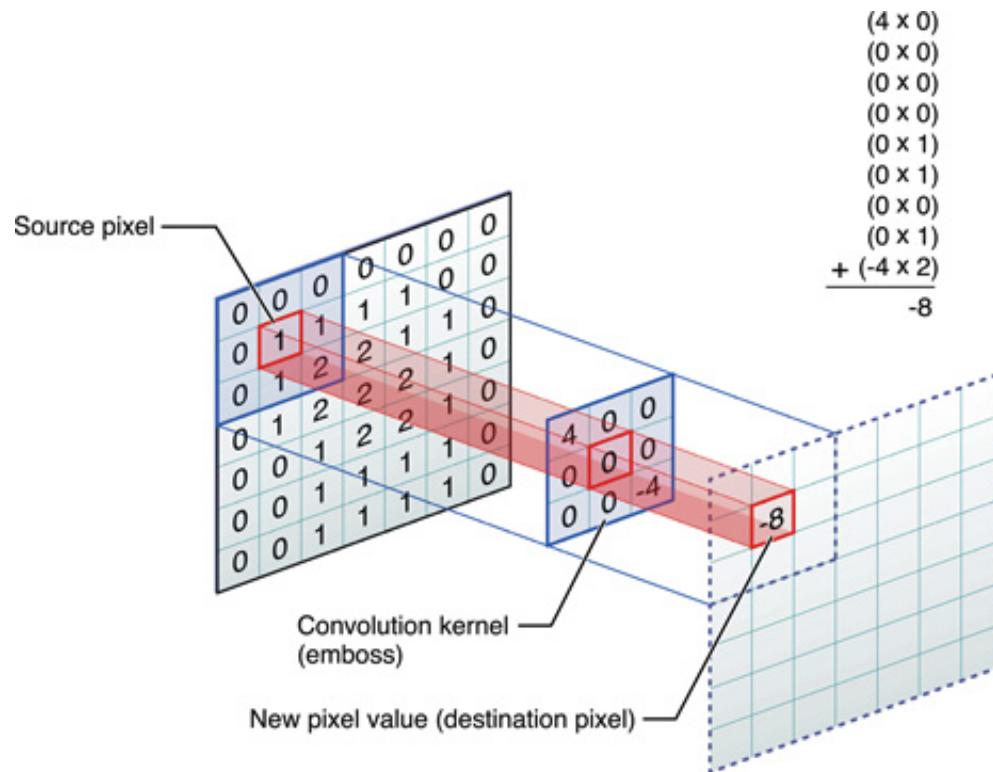
- Convolution is an important operation from signal processing
- A convolution is an integral that expresses the amount of overlap of one function as it is shifted over another function

$$f * g = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau = \int_{-\infty}^{\infty} g(\tau)f(t-\tau)d\tau$$

- 2-Dimensional Discrete Function (Image)

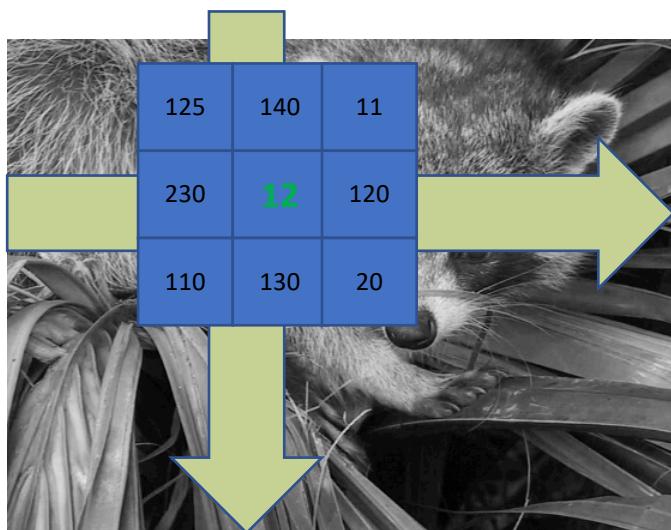
$$f[x,y] * g[x,y] = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} f[n_1, n_2] \cdot g[x - n_1, y - n_2]$$

# 2-Dimensional Convolution



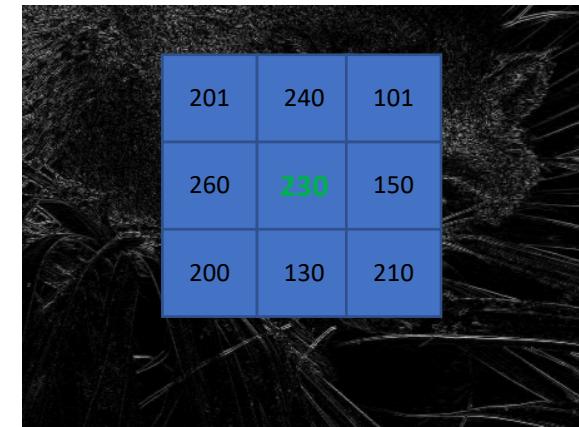
# Example: 2-Dimensional Convolution

A convolution is an integral (**discrete signals :Matrix Dot Product**) that expresses the amount of overlap of one function as it is shifted over another function

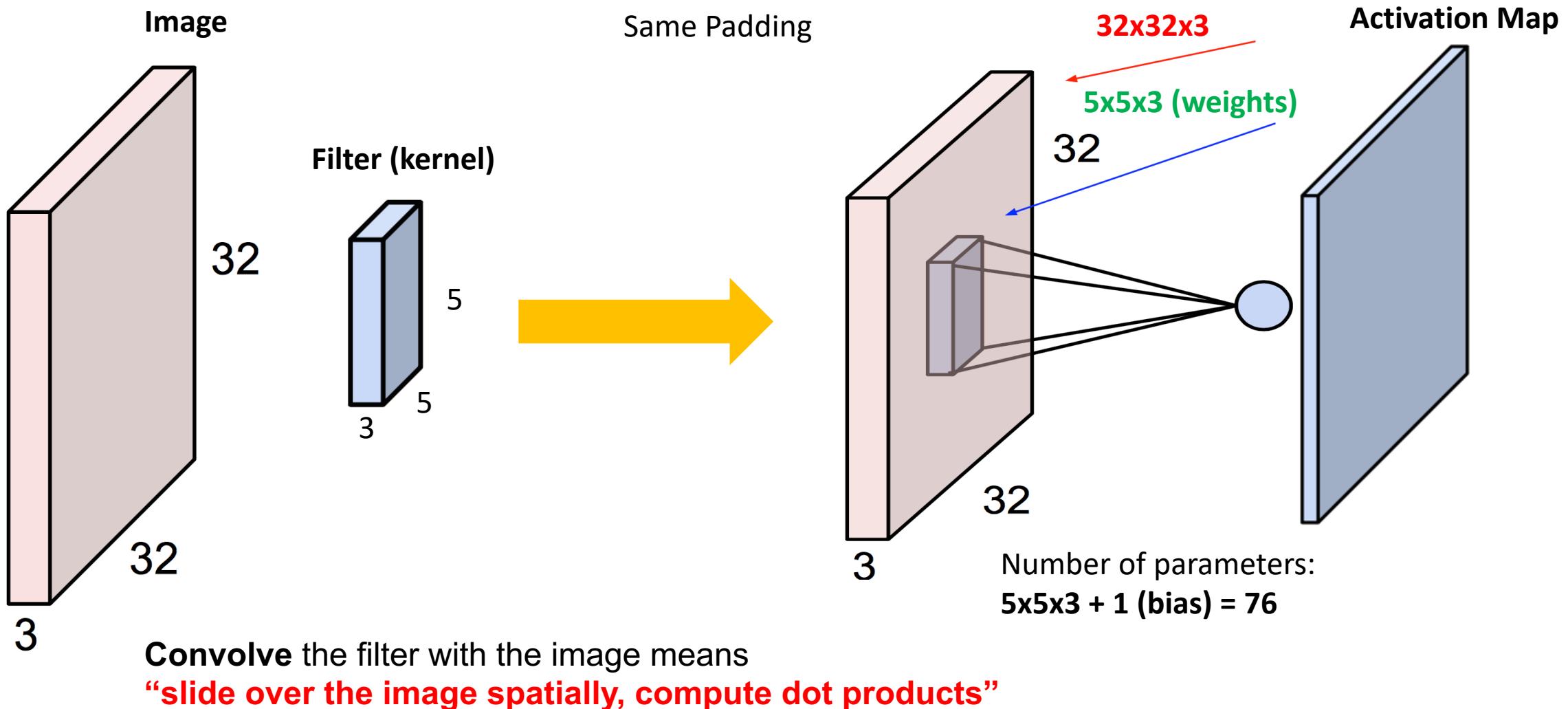


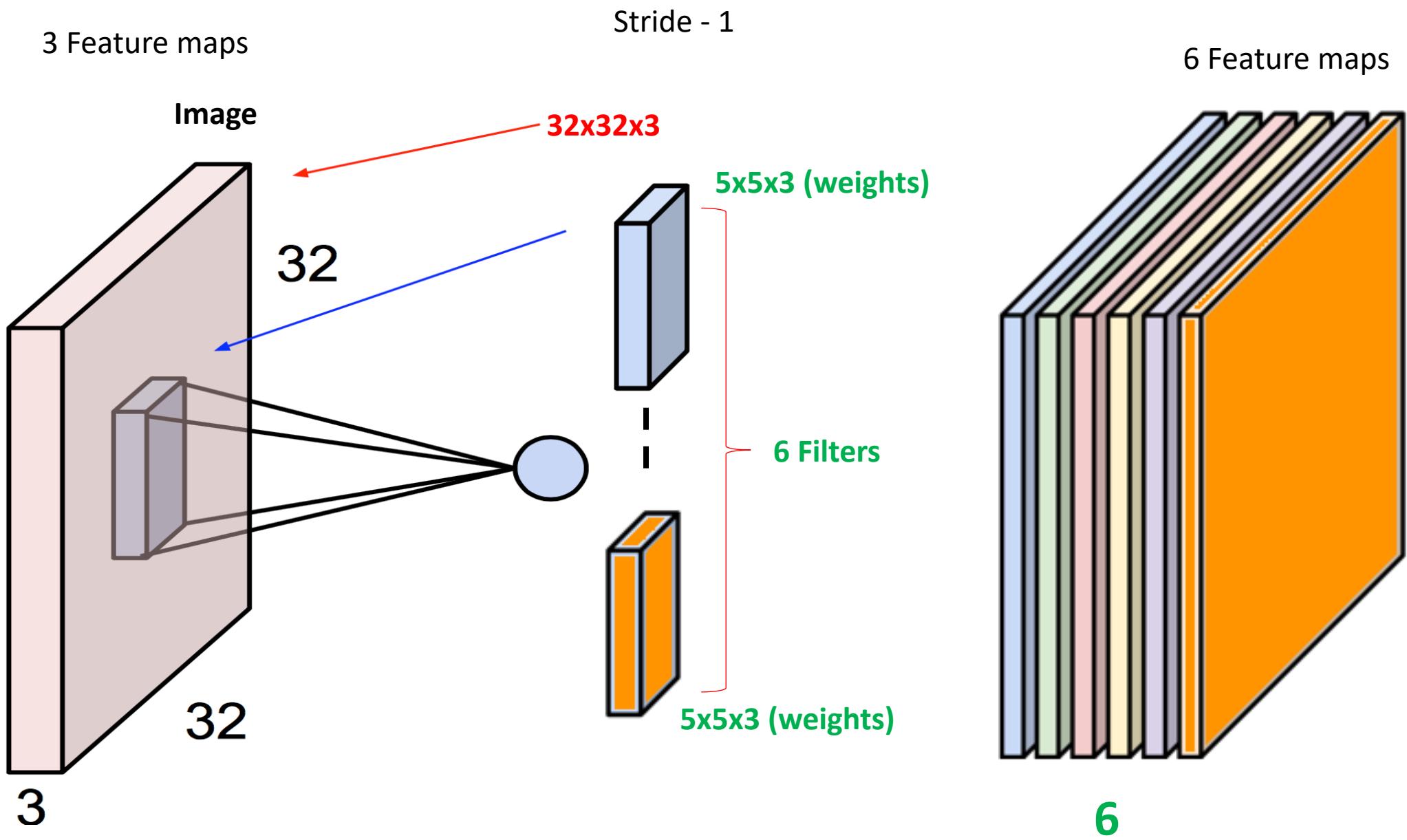
-1	-2	-1
0	0	0
1	2	1

-1	0	1
-2	0	2
-1	0	1



# Convolution Layer





Input Volume (+pad 1) (7x7x3)

$x[:, :, 0]$	Filter W0 (3x3x3)	Filter W1 (3x3x3)	Output Volume
0 0 0 0 0 0 0	w0[:, :, 0]	w1[:, :, 0]	$o[:, :, 0]$
0 2 0 1 1 0 0	1 1 1 1 1 1 1 1 1	1 1 0 0 0 -1 0 0 1	9
0 2 1 2 2 1 0	2 1 0 1 1 1 0 -1 0	w0[:, :, 1] w1[:, :, 1]	$o[:, :, 1]$
0 2 0 0 1 2 0	1 2 0 0 1 1 -1 1 -1	-1 0 -1 -1 1 -1 -1 0 1	2
0 1 1 2 2 1 0	2 1 0 0 1 1 -1 1 1	w0[:, :, 2] w1[:, :, 2]	$o[:, :, 2]$
0 0 1 0 2 2 0	2 2 0 1 2 0 -1 1 0	1 1 -1 -1 1 1 1 1 0	
0 0 0 0 0 0 0			
$x[:, :, 1]$	Bias b0 (1x1x1) $b0[:, :, 0]$	Bias b1 (1x1x1) $b1[:, :, 0]$	
0 0 0 0 0 0 0	0	0	
0 1 2 1 1 2 0	1 2 0 0 1 2 -1 1 0		
0 1 2 1 2 0 0	2 0 0 2 2 0 1 0 0		
0 2 0 1 2 2 0	2 2 0 1 0 0 0 0 0		
0 2 2 2 1 0 0	1 0 0 0 0 0 0 0 0		
0 0 1 0 2 2 0	2 2 0 1 0 0 0 0 0		
0 0 0 0 0 0 0			
$x[:, :, 2]$			
0 0 0 0 0 0 0			
0 0 0 2 0 0 0			
0 1 1 1 0 2 0	0 2 0 1 1 2 2 1 0		
0 2 1 1 2 1 0	2 1 0 1 2 1 1 0 0		
0 0 2 1 1 0 0	1 0 0 0 2 1 0 0 0		
0 0 0 2 1 2 0	1 2 0 0 0 2 0 0 0		
0 0 0 0 0 0 0			

Filter W0 (3x3x3)

$w0[:, :, 0]$
1 1 1
1 1 1
1 1 1
0 -1 0
0 1 1
-1 1 -1
-1 1 1
1 1 0

Bias b0 (1x1x1)  
 $b0[:, :, 0]$ 

1

Filter W1 (3x3x3)

$w1[:, :, 0]$
1 1 0
0 0 -1
0 0 1
-1 0 -1
-1 1 -1
-1 0 1
1 1 -1
1 1 1
1 1 0

Bias b1 (1x1x1)  
 $b1[:, :, 0]$ 

0

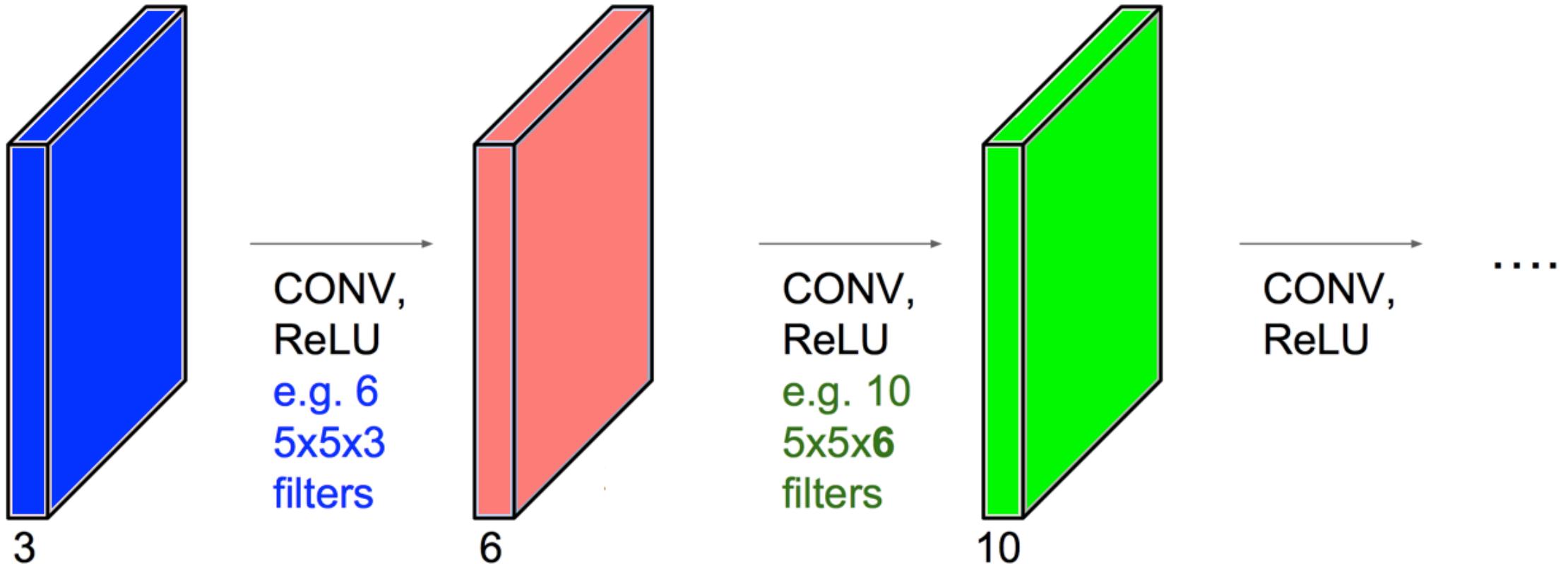
$$(2 \times 1) + (1 \times 1) + 0 + (1 \times 1) + (2 \times 1) + 0 + (2 \times 1) + (1 \times 1) + 0 = 9$$

$$0 + 0 + 0 + (2 \times 1) + 0 + (1 \times -1) + 0 + 0 = 1$$

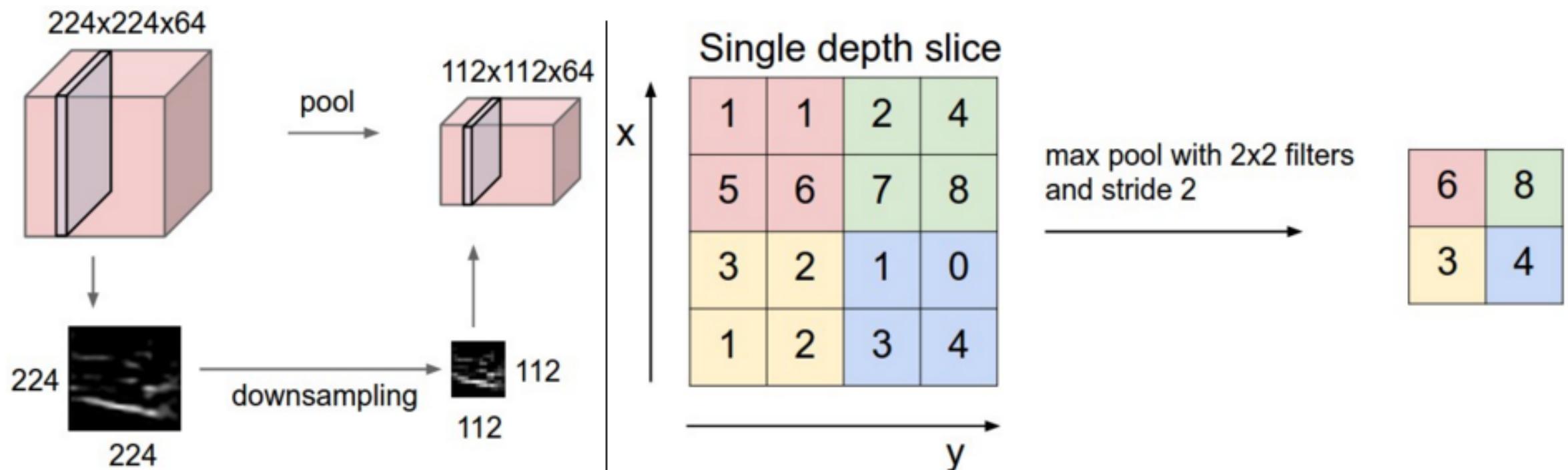
$$0 + 0 + 0 + (2 \times -1) + (1 \times 1) + 0 + (1 \times -1) + 0 + 0 = -2$$

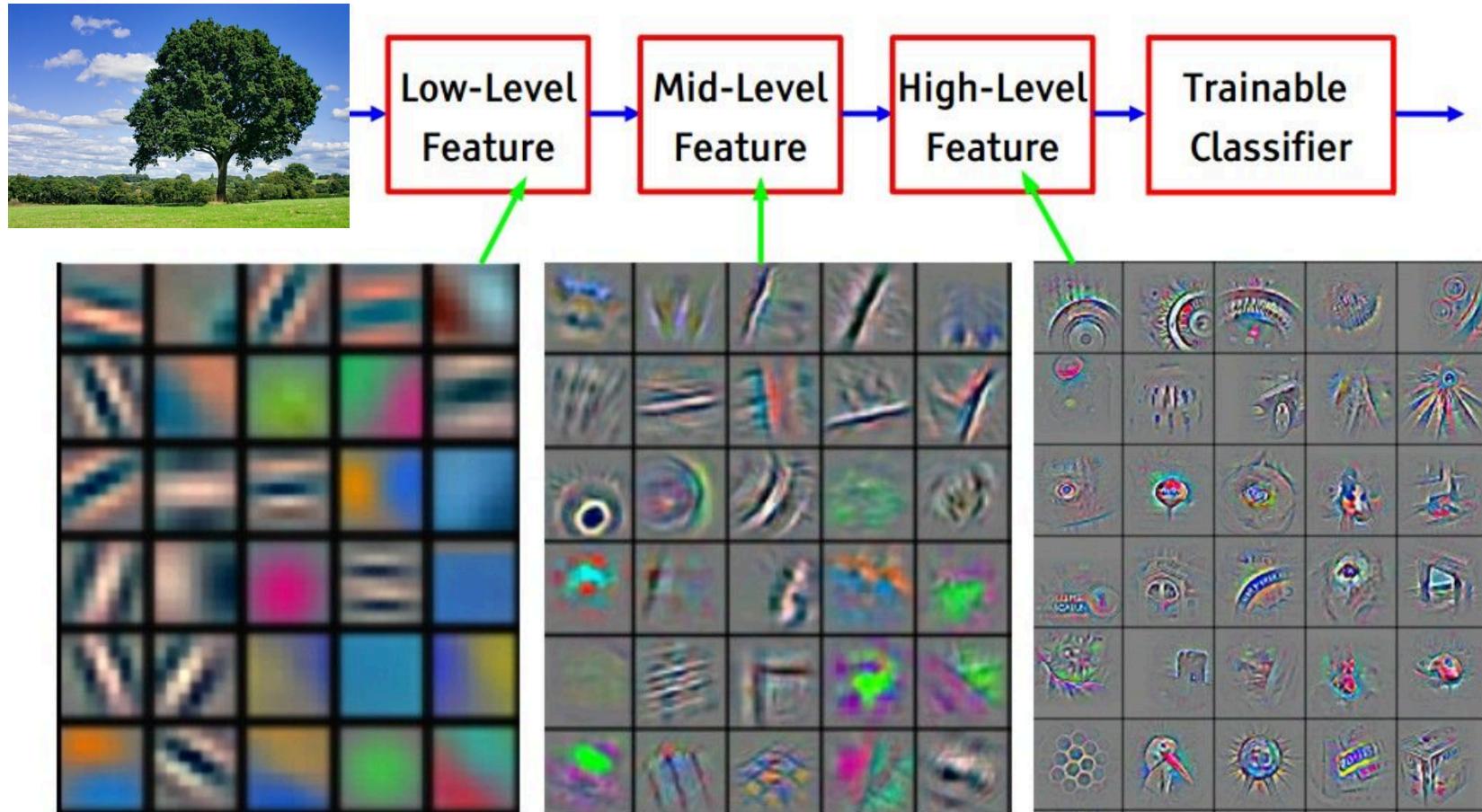
$$1 + 1 + (-2) + 9 = 9$$

# Convolution Network

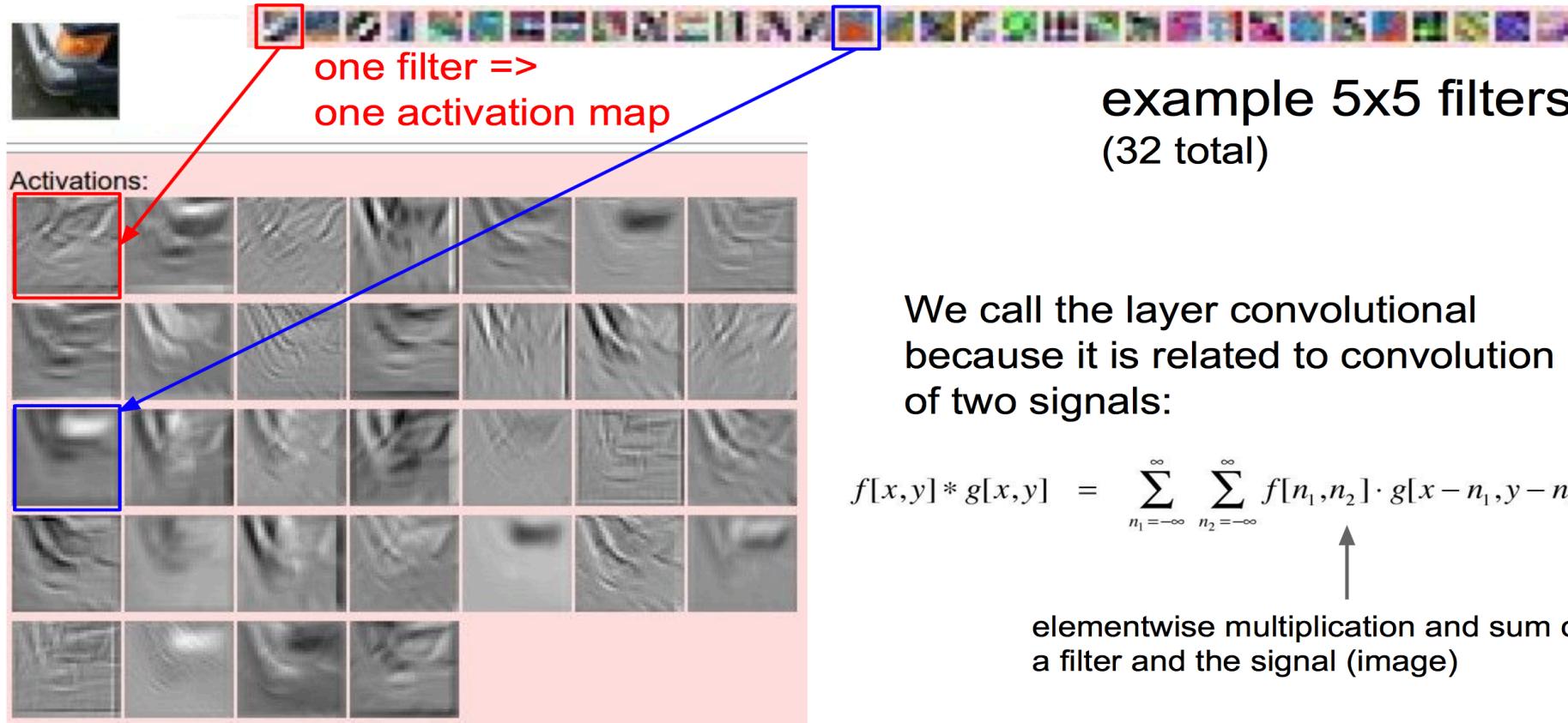
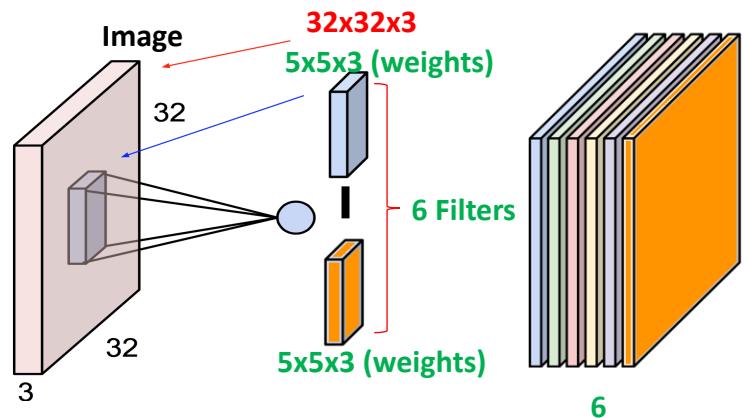


# Pooling





Feature visualization of convolution net trained on ImageNet from [Zeiler & Fergus 2013]



# Architecture of LeNet-5

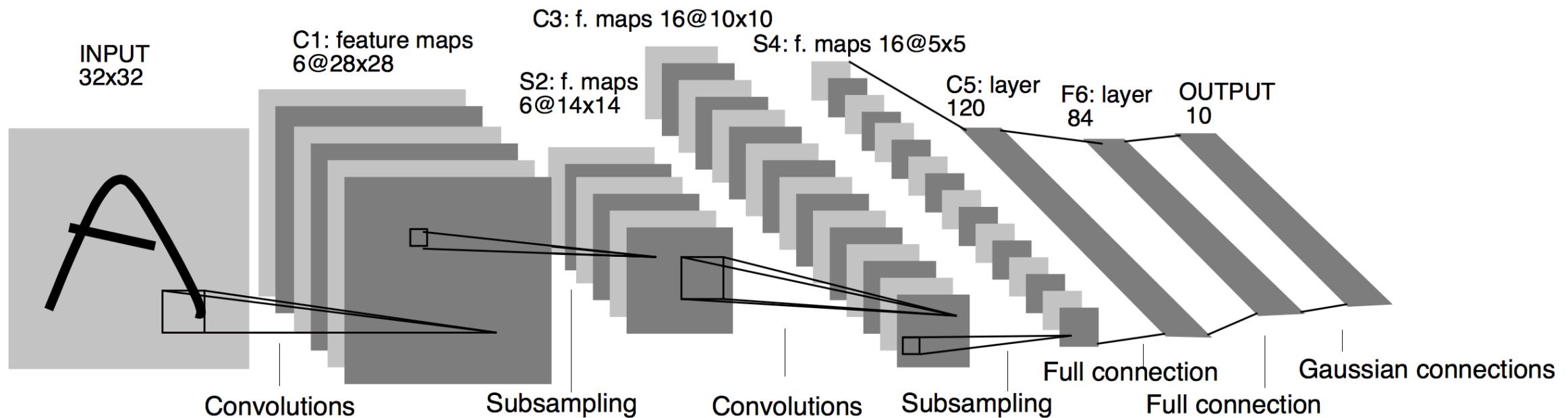


Fig. 2. Architecture of LeNet-5, a Convolutional Neural Network, here for digits recognition. Each plane is a feature map, i.e. a set of units whose weights are constrained to be identical.

Proc. Of the IEEE, November 1998, "Gradient-Based Learning Applied to Document Recognition"