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Modelling the Earth's magnetic field with magnetometer data from a Raspberry Pi on board the ISS

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Abstract

We used data from a magnetometer on board the ISS to simulate the Earth's magnetic field. For that purpose, we generated two models from the data and compared them: an off-centered dipole model and a centered multipolar model. We found out that measuring the magnetic field along only two orbits is not enough to provide a good estimate of the Earth's magnetic field, as they chart a too small portion of the Earth's surface. This was our original work within the "Astro Pi Challenge", a project ran by ESA and the Raspberry Pi Foundation, and the magnetometer used was the one incorporated in the Raspberry Pi Sense Hat. Here, we improved by performing the same analysis using data from ten orbits and learned that the multipolar model already provides a good approximation to the Earth's magnetic field with this number of orbits. We also noticed a deviation of the data collected from what would be expected from the IGRF model, in both sets of data.

1 Introduction

This work started in the context of the "European Astro Pi Challenge Mission Space Lab 2020-21", a contest promoted by ESA and the Raspberry Pi Foundation, where secondary school students are invited to write a code to be run on a Raspberry Pi device on board the International Space Station (ISS). The team of students (the last three authors of this article) wrote a code to register the magnetic field at regular intervals, with the purpose of evaluating the form of the Earth's magnetic field from the data obtained along the two orbits of the ISS during which the code was run. The resulting report [1] was then improved with the help of the first author and using data for ten orbits available at the
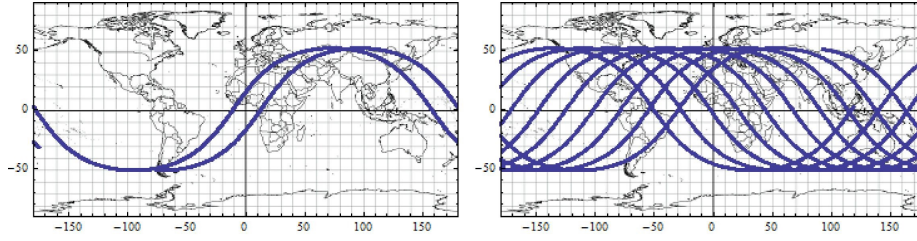
Figure 1: The two orbits of the ISS mapped by our team (left) and the ten orbits available at the "Raspberry Pi Learning Resources" webpage (right).

Raspberry site [2] in order to produce the current article. In Fig. 1 we plot both sets of orbits.

The Earth's magnetic field plays important roles. It serves as a shield against dangerous radiation from deep space, and as a navigation tool through the use of a compass, helping us find where the North is. Because of this, it is natural that humans have tried to describe and predict its behaviour. The International Geomagnetic Reference Field (IGRF) is one of these models and it is widely used in scientific studies. Our goal in this work was to develop a model of a stellar body's magnetic field (if it has one) and locate its magnetic poles. In this case, our study object was Earth, but procedures and calculations were made so that they could be applied in other cases too. This way, we aimed to have an equation that describes the Earth's magnetic field up to a certain degree of detail. This could be useful for many purposes, including the study of the geology of planets or other celestial bodies, and even interstellar exploration (for example, discovering if a planet is suitable for life).

To achieve this, we need to know the magnetic field intensity and direction at different points in space, so that, by interpolation, we can build our model. We used the magnetometer included in the Raspberry Pi SenseHat. After running our code, we received the data in the form of a spreadsheet, containing the coordinates for the position of the ISS, the magnetometer raw data, and the time at which the measurements were taken, the latter used only for checking the integrity of the data.

The data gathered was processed to best fit two different models:

- A dipolar model (the simplest possible), by which the data is fitted to a magnetic field described by one off-centered magnetic dipole.

- A multipolar model, based on the expansion of the magnetic field in spherical harmonics (in our case, up to order 3), which includes more components and, therefore, is more accurate.

We used Wolfram Mathematica to perform all the calculations, including the best fit which was, in both models, obtained by the method of the minimum of the square of differences. The detailed calculations are exposed in the next section. One major obstacle was our lack of knowledge about the orientation of the Raspberry Pi within the ISS, which was, however, overcome.

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2 The theoretical models

In the dipole approximation the magnetic field is given by

$$\vec{B}\left(\vec{r}\right) = \frac{\mu_0}{4\pi}\left[\frac{3\left(\vec{m}\cdot\vec{s}\right)\vec{s}}{s^5} - \frac{\vec{m}}{s^3}\right] \quad \text{with} \quad \vec{s} = \vec{r} - \vec{R} \tag{1}$$

where μ_0 is the vacuum magnetic permeability, $\vec{r}$ is the distance between the ISS and the center of the Earth, $\vec{m}$ is the Earth's magnetic dipole and $\vec{R}$ is the distance between the center of the Earth and the placement of the magnetic dipole that best fits the data. The two vectors $\vec{m}$ and $\vec{R}$ form a set of 6 parameters to be determined.

In the multipolar approximation the magnetic field is given as the gradient of a potential V

$$\vec{B} = -\vec{\nabla}V = -\left(\frac{\partial V}{\partial r}\hat{e}_r + \frac{1}{r}\frac{\partial V}{\partial \phi}\hat{e}_\phi + \frac{1}{r\cos\phi}\frac{\partial V}{\partial \theta}\hat{e}_\theta\right) \tag{2}$$

where we use spherical coordinates since the potential is to be expanded in spherical harmonics to order N

$$V = a\sum_{n=1}^{N}\left(\frac{a}{r}\right)^{n+1}\sum_{m=0}^{n}\left[g_n^m\cos\left(m\theta\right) + h_n^m\sin\left(m\theta\right)\right]P_n^m\left(\sin\phi\right) \tag{3}$$

with r being the distance to the Earth's center, ϕ is the latitude and θ is the longitude. Here we follow the IGRF [3], with the mean Earth's radius

$$a = 6,3712 \times 10^6 \text{ m} \tag{4}$$

The g_n^m and h_n^m are the multipolar moments. They form a set of $N\left(N+2\right)$ parameters to be determined to fit the data.

The P_n^m are the associated Legendre polynomials with the normalisation used in the IGRF ([3] and [4], Ap.B), e.g., up to order 3 the potential is:

$$
\begin{aligned}
V \quad = \quad & \frac{a^3}{r^2}\left\{g_1^0\sin\phi + \left[g_1^1\cos\left(\theta\right) + h_1^1\sin\left(\theta\right)\right]\cos\phi\right\} + \\
& + \frac{a^4}{r^3}\left\{g_2^0\left(1 - \frac{3}{2}\cos^2\phi\right) + \sqrt{3}\left[g_2^1\cos\left(\theta\right) + h_2^1\sin\left(\theta\right)\right]\sin\phi\cos\phi + \right. \\
& \left. + \frac{\sqrt{3}}{2}\left[g_2^2\cos\left(2\theta\right) + h_2^2\sin\left(2\theta\right)\right]\cos^2\phi\right\} + \\
& + \frac{a^5}{r^4}\left\{g_3^0\sin\phi\left(1 - \frac{5}{2}\cos^2\phi\right) + \sqrt{\frac{3}{2}}\left[g_3^1\cos\left(\theta\right) + h_3^1\sin\left(\theta\right)\right] \times \right. \\
& \times\cos\phi\left(2 - \frac{5}{2}\cos^2\phi\right) + \sqrt{\frac{15}{2}}\left[g_3^2\cos\left(2\theta\right) + h_3^2\sin\left(2\theta\right)\right]\sin\phi\cos^2\phi + \\
& \left. + \sqrt{\frac{15}{8}}\left[g_3^3\cos\left(3\theta\right) + h_3^3\sin\left(3\theta\right)\right]\cos^3\phi\right\}
\end{aligned}
\tag{5}
$$

For $N = 1$ the multipolar model reduces to a dipole located at the origin (i. e., with $\vec{R} = 0$). Taking for z axis the polar axis and for x axis the axis perpendicular to the polar axis that passes through the Greenwich meridian, the magnetic dipolar moment is

$$\vec{m} = \frac{4\pi a^3}{\mu_0} \left(g_1^1, h_1^1, g_1^0 \right) \tag{6}$$

3   Converting to the Raspberry Pi frame

In order to proceed with the calculations, we first convert the magnetic field components in (2) to a cartesian earth-centered, earth-fixed frame (ECEF) by

$$\begin{aligned}
\hat{e}_r &= \cos\phi\cos\theta\hat{e}_x + \cos\phi\sin\theta\hat{e}_y + \sin\phi\hat{e}_z \tag{7} \\
\hat{e}_\phi &= -\sin\phi\cos\theta\hat{e}_x - \sin\phi\sin\theta\hat{e}_y + \cos\phi\hat{e}_z \tag{8} \\
\hat{e}_\theta &= -\sin\theta\hat{e}_x + \cos\theta\hat{e}_y \tag{9}
\end{aligned}$$

We have access to the ISS position coordinates in the form of altitude h, latitude ϕ and longitude θ, which can be converted to the ECEF frame by

$$\begin{aligned}
x &= [h + s(\phi)]\cos\phi\cos\theta \tag{10} \\
y &= [h + s(\phi)]\cos\phi\sin\theta \tag{11} \\
z &= [h + s(\phi)]\sin\phi \tag{12}
\end{aligned}$$

where

$$s(\phi) = \frac{A^2}{\sqrt{A^2\cos^2\phi + B^2\sin^2\phi}} \tag{13}$$

is the equation for the reference ellipsoid with

$$A = 6378137 \text{ m} \qquad \text{and} \qquad B = 6356752.314245 \text{ m} \tag{14}$$

Next we have to convert the expected magnetic field, which is expressed in the ECEF reference frame to the LVLH (local vertical, local horizontal) reference frame of the ISS. Here we should note that the ISS is made to orbit the Earth with the same face permanently facing the Earth, and with the same face facing the direction of motion too (Fig. 2). In the LVLH the z axis points towards the Earth's center and the y axis is normal to the orbit, pointing in the opposite direction to the angular velocity; the frame is completed with the x axis being orthogonal to the other two. Since the orbit of the ISS is approximately circular, the x axis points in the direction of movement of the ISS. This frame is fixed with respect to the ISS. Denoting by capital letters the latter reference frame,
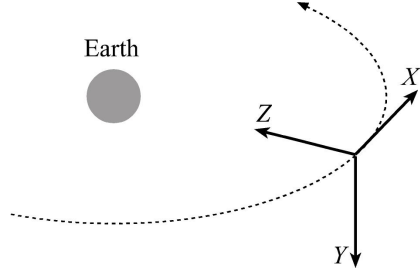
Figure 2: The LVLH reference frame of the ISS.

we can, in a fairly good approximation, write

$$\hat{e}_X = \frac{\vec{r}_i \times (\vec{r}_i \times \vec{r}_{i-1})}{r_i \|\vec{r}_i \times \vec{r}_{i-1}\|} = \frac{(\vec{r}_i \cdot \vec{r}_{i-1})\,\vec{r}_i - r_i^2 \vec{r}_{i-1}}{r_i \|\vec{r}_i \times \vec{r}_{i-1}\|} \tag{15}$$

$$\hat{e}_Y = \frac{\vec{r}_i \times \vec{r}_{i-1}}{\|\vec{r}_i \times \vec{r}_{i-1}\|} \tag{16}$$

$$\hat{e}_Z = -\frac{\vec{r}_i}{r_i} \tag{17}$$

where $\vec{r}_i$ is the position of the ISS at a certain instant t_i and $\vec{r}_{i-1}$ is its position at the instant t_{i-1}, immediately before t_i.

Hence the components of the magnetic field in the LVLH reference frame are

$$B_X = \frac{(\vec{r}_i \cdot \vec{r}_{i-1})\left(\vec{B}_i \cdot \vec{r}_i\right) - r_i^2 \left(\vec{B}_i \cdot \vec{r}_{i-1}\right)}{r_i \|\vec{r}_i \times \vec{r}_{i-1}\|} \tag{18}$$

$$B_Y = \frac{\vec{B}_i \cdot (\vec{r}_i \times \vec{r}_{i-1})}{\|\vec{r}_i \times \vec{r}_{i-1}\|} \tag{19}$$

$$B_Z = -\frac{\vec{B}_i \cdot \vec{r}_i}{r_i} \tag{20}$$

Finally, in order to compare the predicted values of the magnetic field with the measured ones, we should still convert from the LVLH frame to the Raspberry Pi frame. Unfortunately, it was not possible to know the orientation of the Raspberry Pi inside the ISS. But since the Raspberry Pi is fixed with respect to the ISS, this transformation is simply given by a rotation matrix:

$$\begin{bmatrix} B_a \\ B_b \\ B_c \end{bmatrix} = M \begin{bmatrix} B_X \\ B_Y \\ B_Z \end{bmatrix} \tag{21}$$

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with

$$
M = \begin{bmatrix} \cos\alpha\cos\beta & \begin{pmatrix} \cos\alpha\sin\beta\sin\gamma- \\ -\sin\alpha\cos\gamma \end{pmatrix} & \begin{pmatrix} \cos\alpha\sin\beta\cos\gamma+ \\ +\sin\alpha\sin\gamma \end{pmatrix} \\ \sin\alpha\cos\beta & \begin{pmatrix} \sin\alpha\sin\beta\sin\gamma+ \\ +\cos\alpha\cos\gamma \end{pmatrix} & \begin{pmatrix} \sin\alpha\sin\beta\cos\gamma- \\ -\cos\alpha\sin\gamma \end{pmatrix} \\ -\sin\beta & \cos\beta\sin\gamma & \cos\beta\cos\gamma \end{bmatrix} \tag{22}
$$

For each instant i our model predicts a magnetic field (B_a, B_b, B_c) which depends on the parameters of the model ($\vec{m}$ and $\vec{R}$ in the displaced dipole model and the g_n^m and h_n^m in the centered multipolar model), plus α, β and γ, which we have no way to know since we do not know the exact placement of the Raspberry Pi inside the ISS. These predicted values are to be compared with the measured values of the magnetic field for each point along the orbit, and the values of the parameters are to be chosen that make the best fit between the measured and predicted values.

4   Determination of the parameters

The determination of the parameters was done, for each model, by the method of least squares, by minimizing the sum on the P measurements of the magnetic field along the orbit

$$
S = \sum_{i=1}^{P} \left[(B_a^{\mathrm{meas}} - B_a)^2 + (B_b^{\mathrm{meas}} - B_b)^2 + (B_c^{\mathrm{meas}} - B_c)^2 \right] \tag{23}
$$

where B_a^{meas}, B_b^{meas} and B_c^{meas} are the measured values of the magnetic field and B_a, B_b and B_c are the predicted ones from (21).

We should note that this was done in the program *Mathematica* using the built-in function *FindMinimum*. However, the presence of trigonometric functions in (22) produces several local minima. Therefore, the absolute minimum had to be determined by running the program for different initial values of the angles α, β and γ until all the local minima were found. The Mathematica programs used to perform all calculations described in this article are available at the repository [5].

5   Comparison of the data with the IGRF prediction

Since the orientation of the Raspberry Pi inside the ISS is irrelevant for the purpose of computing the absolute value of the magnetig field, we could at once compare the measured magnitude of the magnetic field along the orbit of the ISS with the prediction from the IGRF model and with the prediction from a
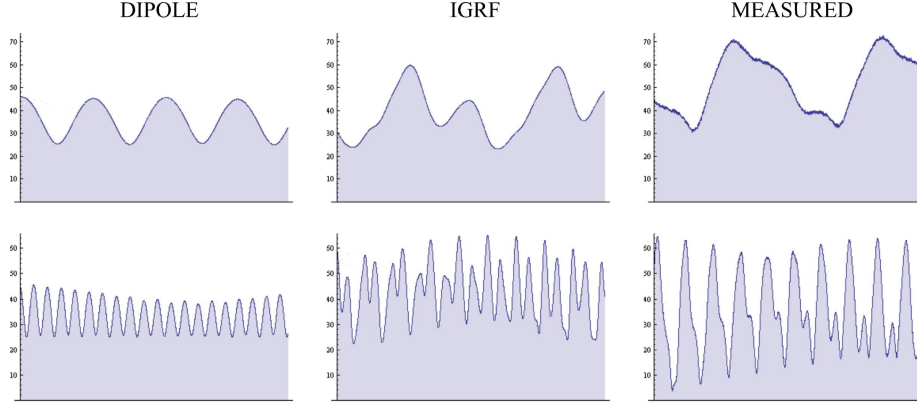
Figure 3: The measured magnitude of the magnetic field (in μT), for the data from 2 orbits (up) and from 10 orbits (down), compared with the expected magnitude from a dipolar model and from the IGRF multipolar model as a function of time along the trajectory of the ISS.

dipolar model with magnetic moment and poles at [6]:

$$m \quad = \quad 7.71 \times 10^{22} \text{ J/T} \tag{24}$$

$$\text{NP} \quad \rightarrow \quad (86.5\text{N}, 162.9\text{E}) \tag{25}$$

$$\text{SP} \quad \rightarrow \quad (64.1\text{S}, 135.9\text{E}) \tag{26}$$

This is depicted in Fig. 3. The measured values fit much better the IGRF model than the dipole model, as should be expected. However, they still differ significatively from the IGRF model.

6 Results using the dipolar model

For the dipolar model, the best fit gave the following values (we omit the values for α, β and γ that came out of the minimization process too)

$$\vec{m} \quad = \quad (-0.03, 6.95, -14.48) \times 10^{22} \text{ J/T} \tag{27}$$

$$\vec{R} \quad = \quad (-0.62, -1.31, 2.41) \times 10^{6} \text{ m} \tag{28}$$

for the set of two orbits, and

$$\vec{m} \quad = \quad (-0.40, 0.39, -7.04) \times 10^{22} \text{ J/T} \tag{29}$$

$$\vec{R} \quad = \quad (-0.11, 0.28, 0.40) \times 10^{6} \text{ m} \tag{30}$$

for the set of ten orbits.

To determine the position of the magnetic poles, we used an algorithm to minimize the angle between the field and a vector normal to the Earth's surface.

The location of the poles and the magnitude of the dipole (to be compared with (39)-(40)) determined were

$$m \quad = \quad 16.06 \times 10^{22} \text{ J/T} \tag{31}$$
$$\text{NP} \quad \rightarrow \quad (61.1\text{N}, 108.7\text{W}) \tag{32}$$
$$\text{SP} \quad \rightarrow \quad (37.2\text{S}, 172.6\text{E}) \tag{33}$$

for the set of two orbits, and

$$m \quad = \quad 7.06 \times 10^{22} \text{ J/T} \tag{34}$$
$$\text{NP} \quad \rightarrow \quad (85.9\text{N}, 88.1\text{E}) \tag{35}$$
$$\text{SP} \quad \rightarrow \quad (75.7\text{S}, 120.5\text{E}) \tag{36}$$

for the set of ten orbits.

The results obtained using our two orbits were poor, but using the set of ten orbits they improved substantially.

7 Results using the multipolar model

For the multipolar model - that we used to order $N = 3$ - we got an array of parameters (the multipolar moments, as opposed to just one vector), which are given in the Table 1, along with the IGRF's values for comparison. The poles' locations were calculated as previously and we got the values

$$\text{NP} \quad \rightarrow \quad (38.6\text{N}, 96.2\text{W}) \tag{37}$$
$$\text{SP} \quad \rightarrow \quad (26.7\text{S}, 92.1\text{E}) \tag{38}$$

for the set of two orbits, and

$$\text{NP} \quad \rightarrow \quad (82.7\text{N}, 77.2\text{E}) \tag{39}$$
$$\text{SP} \quad \rightarrow \quad (75.4\text{S}, 132.0\text{E}) \tag{40}$$

moment	IGRF	2 orb.	10 orb.	moment	IGRF	2 orb.	10 orb.
g_1^0	-29.40	-56.06	-27.14				
g_1^1	-1.45	0.14	-1.66	h_1^1	4.65	61.43	1.11
g_2^0	-2.50	-13.40	-3.19				
g_2^1	2.98	6.08	0.17	h_2^1	-2.99	12.83	-1.91
g_2^2	1.68	4.32	1.83	h_2^2	-0.73	-6.20	0.19
g_3^0	1.36	11.05	0.13				
g_3^1	-2.38	1.46	-0.15	h_3^1	-0.08	2.26	-0.73
g_3^2	1.24	18.19	0.81	h_3^2	0.24	-3.89	1.03
g_3^3	0.53	-0.73	0.72	h_3^3	-0.54	-15.32	0.29

Table 1: Comparison of the multipolar moments obtained from our analysis and the IGRF reference values (in μT).
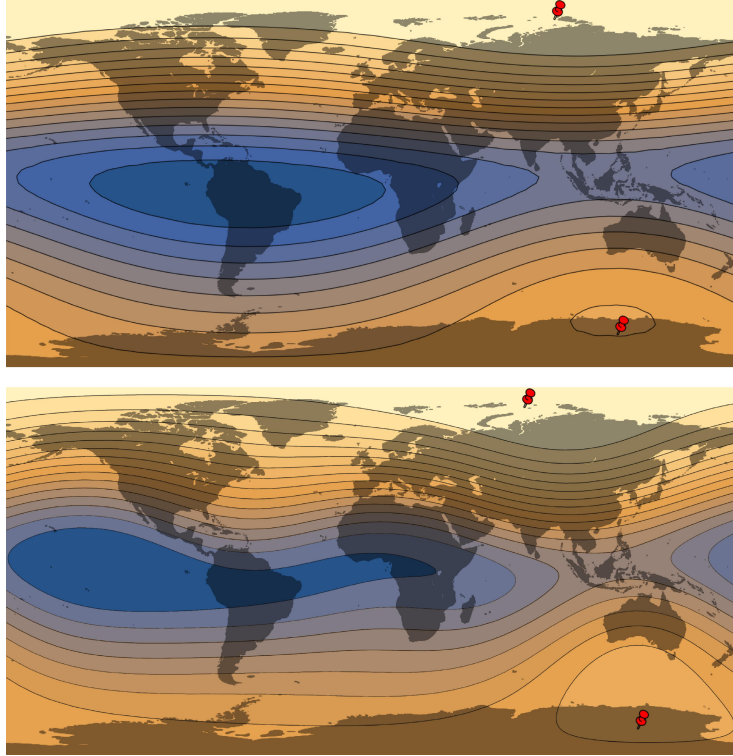
Figure 4: Contour plots for the magnitude of the magnetic field, using the data from 10 orbits for the dipolar model (up) and for the multipolar model (down). The field ranges from 25 to 65 μT.

for the set of ten orbits.

Once again, there is a notorious difference between the results obtained using ten and just two orbits, the former being quite poor.

In Fig. 4 we make contour plots of the expected field intensity for both the dipolar and the multipolar models, obtained using the set of ten orbits (the corresponding plots for the set of two orbits are quite unrealistic, given the low precision obtained with just two orbits). The South Atlantic anomaly is visible and the poles are not far from their actual location (to be compared with the corresponding map in [3]).

8   Conclusions

The models constructed with our original data, from only two orbits, are not reliable, either using a dipole approximation or the multipolar model to order 3. This should not be surprising since the two orbits only provide data from a thin ring over the surface of the Earth.

The models constructed with data from the 10 orbits are considerably better; the 10 orbits already provide data from most of the Earth between 52ºN and 52ºS. In the multipolar model the main features of the field are already reproduced: the dipole is correct within 10% error, both in magnitude and direction, and the multipolar moments calculated agree with the ones from the IGRF in order of magnitude and almost all of them in the sign. This came out as a surprise since we used no data from higher latitudes and it suggests that this method already provides a reasonable approximation to the magnetic field to be studied.

As for the choice between the dipolar model and the multipolar model, the latter proved superior, as expected.

It is not clear to us why the measured field along the orbits was not closer to the predicted one by the IGRF model, particularly since the seasonal variations of the field are not expected to be that large [7, 8]. The variations of the magnetic field on time scales greater than one year are of origin internal to the Earth, and they are accounted for by the IGRF model. The shorter time variations, which are not accounted for by the IGRF model, may originate in the lithospheric field, from ionospheric currents, or be magnetospheric. They should occur on the scale of tens of nT, while we got deviations reaching 10 μT, almost a thousand times greater! Moreover, this discrepancy appears both in the data collected by us and in the data that we downloaded from the Raspberry Pi site. We are therefore inclined to believe that the discrepancy originated in the operation of the magnetometer of the Raspberry Pi Sense Hat itself.

Acknowledgments

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