# 数学分析作业

第3章第1节

1.(6)

$$orall arepsilon \in (0,1), \exists X(arepsilon) = \ln rac{1}{arepsilon}, orall x > X: |e^{-x} - 0| < arepsilon \Rightarrow \lim_{x o \infty} e^{-x} = 0$$

1.(8)

$$egin{aligned} orall G > 0, \exists X(G) = \min\{-G, -2\}, orall x < 1 - G: rac{x^2}{x+1} < rac{x^2-1}{x+1} = x-1 < -G \ \Rightarrow \lim_{x o -\infty} rac{x^2}{x+1} = -\infty \end{aligned}$$

2.(10)

$$\lim_{x \to 0} \frac{\tan x - \sin x}{x^3} = \lim_{x \to 0} \frac{\sin x}{x} \cdot \frac{1 - \cos x}{x^2 \cos x}$$

$$\lim_{x \to 0} \frac{x^2 \cos x}{1 - \cos x} = \lim_{x \to 0} (\frac{x^2}{2 \sin^2 \frac{x}{2}} - x^2) = 2$$

$$\lim_{x \to 0} \frac{\tan x - \sin x}{x^3} = \frac{1}{2}$$

3.(2)

$$egin{aligned} \sqrt{x} &> \ln x \ x^{rac{1}{x}} &\geq 1 \ x^{rac{1}{x}} &= e^{rac{\ln x}{x}} &\leq e^{rac{1}{\sqrt{x}}} \ \lim_{x o \infty} e^{rac{1}{\sqrt{x}}} &= 1 \ \lim_{x o \infty} x^{rac{1}{x}} &= 1 \end{aligned}$$

4.(2)

当
$$x>e^{(k+1)^2}$$
时, $k+1<\sqrt{\ln x}=rac{\ln x}{\sqrt{\ln x}}<rac{\ln x}{\ln \ln x}\Rightarrow (k+1)\ln \ln x<\ln x\Rightarrow \ln^{k+1}x< x$  因此当 $x>e^{(k+1)^2}$ 时  $rac{\ln^k x}{x}<rac{1}{\ln x}\rightarrow 0 (x\rightarrow \infty)$  又因为当 $x>2$ 时  $rac{\ln^k x}{x}>0$ ,因此  $\lim_{x\rightarrow \infty}rac{\ln^k x}{x}=0$ 

5.(2)

$$orall arepsilon > 0, \exists \sigma = rac{1}{\log_2(rac{2+arepsilon}{arepsilon})}, orall 0 < x < \sigma: |rac{2^{rac{1}{x}}+1}{2^{rac{1}{x}}-1}-1| = |rac{2}{2^{rac{1}{x}}-1}| < arepsilon \ \lim_{x o 0^+} rac{2^{rac{1}{x}}+1}{2^{rac{1}{x}}-1} = 1$$

$$orall arepsilon > 0, \exists \sigma = rac{1}{\log_2(rac{2+arepsilon}{arepsilon})}, orall - \sigma < x < 0: |rac{2^{rac{1}{x}}+1}{2^{rac{1}{x}}-1} - (-1)| = |rac{2(2^{rac{1}{x}})}{2^{rac{1}{x}}-1}| < arepsilon \ \lim_{x o 0^-} rac{2^{rac{1}{x}}+1}{2^{rac{1}{x}}-1} = -1$$

6.(1)

$$orall arepsilon > 0, \exists X(arepsilon) = rac{1}{arepsilon}, orall |x| > X(arepsilon) : |rac{\sin x}{x} - 0| < |rac{1}{x}| < arepsilon \ \lim_{x o \infty} rac{\sin x}{x} = 0$$

6.(4)

$$orall G>0, \exists X(G)=\max\{1+\ln G,10\}, orall x>X(G): (1+rac{1}{x})^{x^2}=((1+rac{1}{x})^{x+1})^{rac{x^2}{x+1}}>e^{rac{x^2}{x+1}}>e^{x-1}>G \ \lim_{x o\infty}(1+rac{1}{x})^{x^2}=+\infty$$

6.(6)

说 
$$a_n=rac{1}{n+rac{1}{2}}
ightarrow 0^+(n
ightarrow\infty), b_n=rac{1}{n+rac{1}{3}}
ightarrow 0^+(n
ightarrow\infty).$$
  $A=\lim_{n
ightarrow\infty}(rac{1}{a_n}-\left[rac{1}{a_n}
ight])=\lim_{n
ightarrow\infty}(n+rac{1}{2}-n)=rac{1}{2}$   $B=\lim_{n
ightarrow\infty}(rac{1}{b_n}-\left[rac{1}{b_n}
ight])=\lim_{n
ightarrow\infty}(n+rac{1}{3}-n)=rac{1}{3}$   $A
eq B \Rightarrow$  极限不存在.

10.(2)

$$\exists \varepsilon > 0, \forall N \in \mathbb{N}^+, \exists n > N: |x_n| > \varepsilon$$

10.(4)

$$\exists G > 0, \forall N \in \mathbb{N}^+, \exists n > N : x_n < G$$

10.(6)

$$\exists G > 0, \forall X \in \mathbb{R}, \exists x > X : f(x) > -G$$

13.

必要性:

$$egin{aligned} orall arepsilon > 0, \exists X(arepsilon), orall x > X(arepsilon): |f(x) - A| < arepsilon \ orall G > 0, \exists N(G), orall n > N(G): x_n > G \ \Rightarrow orall arepsilon > 0, \exists N_1(arepsilon) = N(X(arepsilon)), orall n > N_1(arepsilon): x_n > X(arepsilon), |f(x_n) - A| < arepsilon \end{aligned}$$

充分性:

用反证法证明.

假设
$$\exists \varepsilon > 0, \forall X, \exists x > X: |f(x) - A| > \varepsilon$$
 取 $x_1 = 1$ ,可以找到一个 $x_n > x_{n-1} + 1$ ,使得 $|f(x_n) - A| > \varepsilon_0$ ,此时 $\{f(x_n)\}$ 不收敛. 矛盾,假设不成立,因此  $\lim_{x \to +\infty} f(x) = A$ 存在.

14.(2)

 $\lim_{x o x_0^+}f(x)$ 存在  $\Leftrightarrow orall arepsilon > 0, \exists \sigma_2(arepsilon), orall x_0 < x_1, x_2 < x_0 + \sigma_2(arepsilon): |f(x_1) - f(x_2)| \leq arepsilon$ 

必要性:

$$egin{aligned} \exists A \in \mathbb{R}, orall arepsilon > 0, \exists \sigma_1(arepsilon), orall x_0 < x < x_0 + \sigma_1(arepsilon) : |f(x) - A| < arepsilon \ \Rightarrow |f(x_1) - f(x_2)| = |f(x_1) - A + A - f(x_2)| < |f(x_1) - A| + |f(x_2) - A| < 2arepsilon \ \Rightarrow orall arepsilon > 0, \exists \sigma_2(arepsilon) = \sigma_1(rac{arepsilon}{2}), orall x_0 < x_1, x_2 < x_0 + \sigma_2(arepsilon) : |f(x_1) - f(x_2)| \leq arepsilon \end{aligned}$$

充分性:

对于任意数列
$$\{x_n\}, x_n \in D, x_n o x_0^+(n o \infty)$$
  $orall arepsilon > 0, \exists N(arepsilon), orall n_1, n_2 > N: |f(x_{n_1}) - f(x_{n_2})| < arepsilon$   $\Rightarrow \{f(x_n)\}$ 是基本数列,即 $\{f(x_n)\}$ 收敛  $\Rightarrow \lim_{x o x_0^+} f(x)$ 存在

15.

反证法.

假设
$$f(x_0)=B \neq A,$$
则 $f(2^nx_0)=B.$   $\Rightarrow \exists \varepsilon=|A-B|, \forall X\in (0,+\infty), \exists x=2^{\lceil\log_2\frac{X}{x_0}\rceil}x_0: f(x)=B, |f(x)-A|\geq \varepsilon$  与  $\lim_{x\to +\infty}f(x)=A$ 矛盾,假设不成立,因此 $f(x)\equiv A, x\in (0,+\infty)$ 

第3章第2节

1.(2)

 $在(0,+\infty)$ 

$$orall x_0 \in (0,+\infty), orall arepsilon > 0, \exists \sigma = arepsilon \sqrt{x_0}, orall x(x>0,0 < |x-x_0| < \sigma): |\sqrt{x}-\sqrt{x_0}| = rac{|x-x_0|}{\sqrt{x}+\sqrt{x_0}} < rac{|x-x_0|}{\sqrt{x_0}} < arepsilon \ \lim_{x o x_0} \sqrt{x} = \sqrt{x} \ \lim_{x o x_0} \sqrt{x} = 0, orall x = arepsilon^2, orall x(0 < x < \sigma): |\sqrt{x}-0| < \sqrt{\sigma} = arepsilon \ \lim_{x o 0^+} \sqrt{x} = \sqrt{0} \$$

综上, $y = \sqrt{x}$ 在定义域内连续.

1.(3)

2.(2)

$$(2k\pi-\frac{\pi}{2},2k\pi+\frac{\pi}{2}),k\in\mathbb{Z}$$

2.(4)

$$(-1,1)\cup [n,n+1), n\in \mathbb{N}^+$$

2.(6)

$$(2k\pi,2k\pi+\pi),k\in\mathbb{Z}$$

4.

(1) 不能. 反例:
$$x_0=0, f(x)=x, g(x)=egin{cases} 0, x=0 \\ 1, x \neq 0 \end{cases}$$
 (2) 不能. 反例: $x_0=0, f(x)=egin{cases} x, x \neq 0 \\ 1, x=0 \end{cases}, g(x)=egin{cases} \frac{1}{x}, x \neq 0 \\ 1, x=0 \end{cases}$ 

6.(1)

$$egin{aligned} orall x_0 &\in (a,b), \exists \delta = \min\{rac{x_0-a}{2}, rac{b-x_0}{2}\} : x_0 \in (a+\delta,b-\delta) \subset [a+\delta,b-\delta] \ &egin{aligned} eta f \Xi[a+\delta,b-\delta] η identify, 所以 \lim\limits_{x o x_0} f(x) &= f(x_0). \ &egin{aligned} eta \forall x_0 &\in (a,b) : \lim\limits_{x o x_0} f(x) &= f(x_0), egin{aligned} eta f(x) \Delta(a,b) &= f(x_0). \end{aligned}$$

6.(2)

不能.反例:
$$a=1,b=2,f(x)=egin{cases} x,x\in(1,2) \\ 0,x=1$$
或 $x=2$ 

7.

$$\lim_{x o x_0} f(x)^{g(x)} = \lim_{x o x_0} e^{g(x) \ln f(x)} = e^{\lim_{x o x_0} g(x) \ln \lim_{x o x_0} f(x)} = e^{eta \ln lpha} = lpha^eta$$

7.(3)

$$\lim_{x \to a} (\frac{\sin x}{\sin a})^{\frac{1}{x-a}} = \lim_{x \to a} (\frac{\sin x}{\sin a})^{\frac{1}{\sin x - \sin a}} \frac{\sin x - \sin a}{x - a} = (\lim_{x \to a} (\frac{\sin x}{\sin a})^{\frac{1}{\sin x - \sin a}})^{\frac{\sin x - \sin a}{x - a}}$$

$$\lim_{x \to a} \frac{\sin x - \sin a}{x - a} = \lim_{x \to a} \frac{2 \sin \frac{x - a}{2} \cos \frac{x + a}{2}}{x - a} = \lim_{x \to a} \cos \frac{x + a}{2} = \cos a$$

$$\lim_{x \to a} \frac{\sin x - \sin a}{x - a} = \lim_{x \to a} (\frac{\sin x}{\sin a})^{\frac{1}{\sin x - \sin a}} = \lim_{x \to \sin a} (\frac{x}{\sin a})^{\frac{1}{x - \sin a}} = \lim_{y \to 0} (y + 1)^{\frac{1}{y \sin a}} = e^{\frac{1}{\sin a}}$$

$$\lim_{x \to a} (\frac{\sin x}{\sin a})^{\frac{1}{x - a}} = e^{\frac{\cos a}{\sin a}}$$

7.(5)

$$\lim_{n\to\infty}\tan^n(\frac{\pi}{4}+\frac{1}{n})=\frac{\lim_{n\to\infty}(1+\tan\frac{1}{n})^n}{\lim_{n\to\infty}(1-\tan\frac{1}{n})^n}$$
 
$$\lim_{n\to\infty}(1+\tan\frac{1}{n})^n=\lim_{n\to\infty}(1+\tan\frac{1}{n})^{\frac{1}{\tan\frac{1}{n}}\cdot n\tan\frac{1}{n}}=e^{\lim_{n\to\infty}\frac{\sin\frac{1}{n}}{n\cos\frac{1}{n}}}=e^{\lim_{n\to\infty}\frac{\sin\frac{1}{n}}{n\cos\frac{1}{n}}}=e$$
 同理 
$$\lim_{n\to\infty}(1-\tan\frac{1}{n})^n=\frac{1}{e}$$
 
$$\lim_{n\to\infty}\tan^n(\frac{\pi}{4}+\frac{1}{n})=e^2$$

8.(2)

x=0 是第二类间断点,  $x\in\mathbb{Z}\setminus\{0\}$  是跳跃间断点.

8.(4)

 $x=rac{n}{2},n\in\mathbb{Z}$  是跳跃间断点.

8.(6)

x=0 是可去间断点.

8.(10)

$$x \in \mathbb{Q} \setminus \{x | x = \frac{1}{n}, n \in \mathbb{Z} \setminus \{0\}\}$$

9.

反证法.

假设设 
$$f(x_0)=a, f(y_0)=b, x_0\neq y_0, a\neq b, x_n=\sqrt{x_{n-1}}, y_n=\sqrt{y_{n-1}}$$
.显然 $f(x_n)=a, f(y_n)=b, x_n\to 1 (n\to\infty), y_n\to 1 (n\to\infty)$ 

由Heine定理,  $\lim_{x \to 1} f(x) = \lim_{n \to \infty} f(x_n) = a = \lim_{n \to \infty} f(y_n) = b$  与 $a \neq b$ 矛盾.假设不成立,因此f(x)在 $(0, +\infty)$ 上为常数函数.

### 第3章第3节

1.(4)

$$x \rightarrow 0+, a=1, \alpha=\frac{1}{8} \ x \rightarrow +\infty, a=1, \alpha=\frac{1}{2}$$

1.(7)

$$x \to 0+, a = 1, \alpha = \frac{1}{2}$$

1.(9)

$$x 
ightarrow 0, a = -\frac{3}{2}, lpha = 2$$

1.(10)

$$x \rightarrow 0, a = 1, \alpha = 1$$

2.(1)

$$\ln^k x, x^{\alpha}, a^x, [x]!, x^x$$

$$rac{\ln^k x}{x^lpha} = (rac{\ln x}{x^{rac{lpha}{lpha}}})^k = (rac{rac{2k}{lpha}\ln x^{rac{lpha}{2k}}}{x^{rac{lpha}{k}}})^k \leq (rac{rac{2k}{lpha}(x^{rac{lpha}{2k}}-1)}{x^{rac{lpha}{k}}})^k 
ightarrow 0(x
ightarrow +\infty)$$

$$rac{x^{lpha}}{lpha^x} = e^{a \ln x - x \ln a} < e^{2a(\sqrt{x}-1) - x \ln a} 
ightarrow 0 (x 
ightarrow + \infty)$$

设
$$n \leq x < n+1, rac{a^x}{n!} = C rac{a^{[n]-[a]}}{n!/[a]!} (C$$
是一个常数 $) \leq C rac{a^{[n]-[a]}}{([a]+1)^{[n]-[a]}} o 0 (x o + \infty)$ 

设
$$n \leq x < n+1, rac{n!}{x^x} \leq rac{n!}{n^n} \leq rac{1}{n} o 0 (x o +\infty)$$

2.(2)

$$(rac{1}{x})^{-rac{1}{x}}, rac{1}{[rac{1}{x}]!}, a^{-rac{1}{x}}, x^{lpha}, \ln^{-k}rac{1}{x}$$

理由同(1)

3.(1)

$$\lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt[3]{1+2x^2}}{\ln(1+3x)} = \lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt[3]{1+2x^2}}{3x} \cdot \frac{3x}{\ln(1+3x)} = \lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt[3]{1+2x^2}}{3x} = \lim_{x \to 0} \frac{(\frac{x}{2} + 1 + o(x)) - (1 + o(x))}{3x} = \frac{1}{6}$$

3.(2)

$$\lim_{x \to 0^+} \frac{1 - \sqrt{\cos x}}{1 - \cos \sqrt{x}} = \lim_{x \to 0^+} \frac{1 - \cos x}{(1 - \cos \sqrt{x})(1 + \sqrt{\cos x})} = \lim_{x \to 0^+} \frac{\frac{x^2}{2}}{(\frac{x}{2})(1 + \sqrt{\cos x})} = 0$$

3.(4)

$$\lim_{x\to\infty}(\sqrt{1+x+x^2}-\sqrt{1-x+x^2})=\lim_{x\to\infty}\frac{2}{\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}}\text{=1}$$

3.(6)

$$\lim_{x\to a} \tfrac{x^\alpha-a^\alpha}{x-a} = a^\alpha \lim_{x\to a} \tfrac{\left(\tfrac{x}{a}\right)^\alpha-1}{x-a} = a^\alpha \lim_{x\to a} \tfrac{e^{\alpha(\ln x - \ln a)}-1}{x-a} = a^\alpha \lim_{x\to a} \tfrac{\alpha(\ln x - \ln a)}{x-a} = a^{\alpha-1} \lim_{x\to a} \tfrac{\alpha\ln\frac{x}{a}}{\frac{x}{a}-1} = \alpha a^{\alpha-1}$$

3.(8)

$$\lim_{x\to a} \tfrac{\ln x - \ln a}{x-a} = \tfrac{1}{a} \lim_{x\to a} \tfrac{\ln \frac{x}{a}}{\frac{x}{a}-1} = \tfrac{1}{a}$$

3.(11)

$$\lim_{n o\infty}n(\sqrt[n]{x}-1)=\lim_{n o\infty}n(e^{rac{\ln x}{n}}-1)=\lim_{n o\infty}n(rac{\ln x}{n})=\ln x$$

设对每个自然数n,数集 $A_n\subset [0,1]$ 是有限集,而且 $A_i\cap A_j=\emptyset,\ \forall i,j\in\mathbb{N}_+,\ i\neq j$ . 定义函数 $f(x)=\begin{cases} 1/n, & \text{if }x\in A_n, \\ 0, & \text{if }x\in [0,1]\setminus (\cup_{i=1}^\infty A_i). \end{cases}$ .对每个 $a\in [0,1],\ ext{求}\lim_{x\to a}f(x)$ . (提示: 仿照Riemann函数的讨论)

orall arepsilon>0,取 $n=\lfloorrac{1}{arepsilon}
floor$ , $\exists \delta=\min\{s|x\in\cup_{i\in[n]}A_i,s=|x-a|\}, orall x\in[0,1](|x-a|<\delta):|f(x)|\leqrac{1}{n+1}\leqarepsilon$  。因此  $\lim_{x o a}f(x)=0$ 

## 第3章第4节

1.

由于  $\lim_{x\to +\infty} f(x)=A, \exists X, \forall x\geq X, |f(x)-A|\leq 1$  由于f(x)在 $[a,\infty)$ 连续,  $\forall x\in [a,X], m\leq f(x)\leq M.$  因此  $\min\{A-1,m\}\leq f(x)\leq \max\{A+1,M\}, \mathbb{p}f(x)$ 在 $[a,+\infty]$ 有界.

6.

10.

设  $f(x) = x - a \sin x - b$ .  $f(0) = -b < 0, f(a+b+1) = 1 + a(1 - \sin(a+b)) > 0, f(a+b+1) \cdot f(0) < 0$  因为 f(x) 连续,由零点存在定理,f(x)在(0,a+b+1)上至少有一个零点,即 $x = a \sin x + b$ 至少有一个正根.

设函数 $f:\mathbb{R} \to \mathbb{R}$ 且存在一个常数q(0 < q < 1)使得 $\forall x,y \in \mathbb{R}: |f(x) - f(y)| < q|x-y|$ ,则称f是 $\mathbb{R}$ 上一个压缩映射. 证明:  $\exists ! \xi \in \mathbb{R}: f(\xi) = \xi$ .

取 $x=x_0,\lim_{y\to x_0}|f(y)-f(x_0)|=0,$ 因此f(x)是连续函数. 设g(x)=x-f(x) 设x< y,则g(x)-g(y)=x-y-(f(x)-f(y))>x-y-|f(x)-f(y)|>(1-q)(x-y)>0,因此g(x)单调递增. 任取 $x_0\in\mathbb{R},$ 若 $g(x_0)=0,$ 则 $\xi=x_0$  若 $g(x_0)<0,$ 由于 $|g(x)-g(y)|>|x-y|-|f(x)-f(y)|>(1-q)|x-y|,那么<math>g(x_0+\frac{-g(x_0)}{1-q})>g(x_0)+(-g(x_0))=0,$ 由零点存在定理,存在 $\xi\in(x_0,x_0+\frac{-g(x_0)}{1-q})s.t.g(\xi)=0.$ 若 $g(x_0)>0,$ 同理可知存在 $\xi\in\mathbb{R}s.t.g(\xi)=0$ 

### 第3章第4节

8.(2)

取 $\varepsilon=2, orall \delta>0: x_1=\sqrt{2k\pi-\frac{\pi}{2}}, x_1=\sqrt{2k\pi}, x_2-x_1=\frac{\frac{\pi}{2}}{\sqrt{2k\pi-\frac{\pi}{2}}+\sqrt{2k\pi}}<\sqrt{\frac{\pi}{8k}},$ 当 $k=\lceil\frac{\pi}{8\varepsilon^2}\rceil$ 时, $|x_2-x_1|<\delta$ ,但 $|f(x_1)-f(x_2)|=1$ ,因此  $\sin x^2$ 在(0,1)上不一致连续.

 $\forall \varepsilon > 0, \exists \delta = \frac{\varepsilon}{1+A}, \forall x_1, x_2 \in [1,A](|x_1-x_2| < \delta) : |\sin x_1^2 - \sin x_2^2| = |2\sin \frac{x_1^2 - x_2^2}{2}\cos \frac{x_1^2 + x_2^2}{2}| \leq |x_1^2 - x_2^2| < (1+A)|x_2 - x_1| \leq \varepsilon.$  因此在[1,A]上一致连续.

8.(3)

 $orall arepsilon>0,\exists \delta=arepsilon^2, orall x_1, x_2\in [0,+\infty)(|x_1-x_2|<\delta,x_1>x_2):|\sqrt{x_1}-\sqrt{x_2}|\leq |\sqrt{x_1-x_2}|<arepsilon,$  因此在 $[0,+\infty)$ 一致连续

8.(4)

 $orall arepsilon > 0, \exists \delta = arepsilon, orall x_1, x_2 \in [1, +\infty)(|x_1 - x_2| < \delta, x_1 > x_2): |\ln x_1 - \ln x_2| = \ln rac{x_1}{x_2} < \ln rac{x_2 + \delta}{x_2} < rac{x_2 + \delta}{x_2} - 1 \leq arepsilon,$  因此在 $[0, +\infty)$ 一致连续.

9.

设 $l_1(\theta), l_2(\theta)$ 表示弦与x轴的角度为 $\theta$ 时P将弦分成的两段长度.

设 $l(\theta) = l_1(\theta) - l_2(\theta), l(0) + l(\pi) = 0$ 

 $\overline{ } eta l(0) = 0,$ 则l存在零点. $\overline{ } eta l(0) 
eq 0,$ 则 $l(0)l(\pi) = -l^2(0) < 0,$ 由零点存在定理,l在 $(0,\pi)$ 存在零点.

因此 $\exists \theta_0 \in [0,\pi], l_1(\theta_0) = l_2(\theta_0)$ 

11.

 $orall arepsilon > 0, \exists \delta > 0, orall x_1, x_2 \in (a,b)(|x_1-x_2| \leq \delta): |f(x_1)-f(x_2)| \leq arepsilon$ 

取 $\varepsilon = \varepsilon_0$ ,此时 $\delta = \delta_0$ .设 $|f(\frac{a+b}{2})| = y_0$ 

関則  $\eta \in \mathbb{N}(\frac{a+b}{2} + n\delta_0 < b): |f(\frac{a+b}{2} + n\delta_0)| = |(f(\frac{a+b}{2} + n\delta_0) - f(\frac{a+b}{2} + (n-1)\delta_0)) + (f(\frac{a+b}{2} + (n-1)\delta_0)) - f(\frac{a+b}{2} + (n-2)\delta_0)) + \dots + (f(\frac{a+b}{2} + \delta_0) - f(\frac{a+b}{2})) + f(\frac{a+b}{2})| \le |(f(\frac{a+b}{2} + n\delta_0) - f(\frac{a+b}{2} + (n-1)\delta_0))| + |(f(\frac{a+b}{2} + (n-1)\delta_0) - f(\frac{a+b}{2} + (n-2)\delta_0))| + \dots + |(f(\frac{a+b}{2} + \delta_0) - f(\frac{a+b}{2}))| + |f(\frac{a+b}{2})| \le n\varepsilon_0 + y_0$  同理,  $\forall n \in \mathbb{Z}^-(\frac{a+b}{2} + n\delta_0 > a): |f(\frac{a+b}{2} + n\delta_0)| \le |n|\varepsilon_0 + y_0$ 

那么,  $\forall x \in (a,b)$ ,  $\exists n \in Z: |x-(\frac{a+b}{2}+n\delta_0)| \leq \delta_0$ , 则 $|f(x)| \leq (|n|+1)\varepsilon + y_0$ , 即f(x)在(a,b)上有界

12.(1)

证:

 $\forall \varepsilon>0, \exists \delta_1(\varepsilon)>0, \forall x_1,x_2\in D, (|x_1-x_2|\leq \delta_1(\varepsilon)): |f(x_1)-f(x_2)|<\varepsilon\\ \forall \varepsilon>0, \exists \delta_2(\varepsilon)>0, \forall x_1,x_2\in D, (|x_1-x_2|\leq \delta_2(\varepsilon)): |g(x_1)-g(x_2)|<\varepsilon\\ \textcircled{則}\forall \varepsilon>0, \exists \delta(\varepsilon)=\min\{\delta_1(\frac{\varepsilon}{2}),\delta_2(\frac{\varepsilon}{2})\}, \forall x_1,x_2\in D, (|x_1-x_2|<\delta(\varepsilon)): |f(x_1)+g(x_1)-f(x_2)-g(x_2)|\leq |f(x_1)-f(x_2)|+|g(x_1)-g(x_2)|<\varepsilon\\ \textcircled{因此}f+g在D上一致连续$ 

12.(2)

反例: f(x) = g(x) = x

14.

设 $g(x)=f(x)-rac{f(x_1)+f(x_2)+\cdots+f(x_n)}{n}=rac{(f(x)-f(x_1))+(f(x)-f(x_2))+\cdots+(f(x)-f(x_n))}{n}$ 若 $f(x_1)=f(x_2)=\cdots=f(x_n)$ ,那么 $\xi=x_1$ 否则,设 $x_u=\max\{x_1,x_2,...,x_n\},x_v=\min\{x_1,x_2,...,x_n\},$ 则 $f(x_u)>0,f(x_v)<0$ ,由零点存在定理, $\exists \xi\in(x_u,x_v)(\vec{\mathbf{u}}(x_v,x_u)):f(\xi)=0$ 

# 得证.

15.

 $orall arepsilon > 0, \exists X(arepsilon), orall x_1, x_2 > X(arepsilon): |f(x_1) - f(x_2)| < arepsilon$  那么 $orall arepsilon > 0, \exists \delta, orall x_1, x_2 \in [a, +\infty)(|x_1 - x_2| < \delta): 在[a, X(arepsilon) + \delta] oxedsymbol{L} f(x)$ 一致连续因此对于 $x_1, x_2 \in [a, X(arepsilon) + \delta](|x_1 - x_2| < \delta)$ 满足 $|f(x_1) - f(x_2)| < arepsilon, oxedsymbol{\Pi}[X(arepsilon), +\infty)$ 上显然也满足.因此f(x)在 $[a, +\infty)$ 上一致连续.