

Fall 2022 MATH1205H Homework XXVI

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Exercise 1.

No. Because we can change the order of σ sequence.

Exercise 2.

$$\begin{aligned} Ax &= \lambda x \\ \begin{cases} 2x + y &= \lambda x \\ 4x + 2y &= \lambda y \end{cases} \\ \begin{cases} \lambda_1 &= 4 \\ \lambda_2 &= 0 \end{cases} \\ A^T A &= \begin{bmatrix} 20 & 10 \\ 10 & 5 \end{bmatrix} \end{aligned}$$

Exercise 5.

sufficiency

$\forall \lambda$ is an eigenvalue with a corresponding eigenvector $v : Sv = \lambda v, \lambda v^T v = v^T Sv$

$$\lambda = \frac{v^T Sv}{v^T v} \geq 0$$

So it's semidefinite.

necessity

$$S = \lambda_1 v_1 v_1^T + \cdots + \lambda_n v_n v_n^T (\forall i \in [n] : \lambda_i \geq 0)$$

Then

$$x^T S x = \sum_{i \in [n]} \lambda_i x^T v_i v_i^T x = \sum_{i \in [n]} \lambda_i (v_i^T x)^2 \geq 0$$