Homework for Linear Algebra (XXVI)

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Exercise 1. Is the (or *a* for that matter) singular value decomposition of a matrix *unique*?

Exercise 2. Find the eigenvalues and the singular values of

$$A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}.$$

Exercise 3. Compute the singular value decomposition of the Fibonacci matrix

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}.$$

Exercise 4. Suppose $\boldsymbol{u}_1, \ldots, \boldsymbol{u}_n$ and $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n$ are orthonormal bases for $\mathbb{R}^n$. Construct the matrix $A = U\Sigma V^T$ that transforms each $\boldsymbol{v}_j$ into $\boldsymbol{u}_j$ to give $A\boldsymbol{v}_1 = \boldsymbol{u}_1, \ldots, A\boldsymbol{v}_n = \boldsymbol{u}_n$.

Exercise 5. Let S be a symmetric matrix. Prove that S is positive semidefinite, i.e., the eigenvalues of S are all nonnegative if and only if for all $\boldsymbol{x} \in \mathbb{R}^n$ we have

$$\boldsymbol{x}^T S \boldsymbol{x} \geqslant 0.$$