Eactly solvable - Newns - Anderson

annilation operator

$$b = \sum_k V_k \alpha_k$$

$$c_k = \sum_{k'} \eta_{k,k'} \alpha_{k'}$$

spanned by

α_k and $\alpha^\dagger$

(eigen-annilation operator) (eigen-creation operator)

creation operator

$$b^\dagger = \sum_k V_k^\dagger \alpha_k^\dagger$$

$$c_k^\dagger = \sum_{k'} \eta_{k,k'}^\dagger \alpha_{k'}^\dagger$$

Total hamiltonian

$$H = \sum_k \varepsilon_k \alpha_k^\dagger \alpha_k$$

$$[b, H] = \left[\sum_k V_k \alpha_k , \sum_{k'} \varepsilon_{k'} \alpha_{k'}^\dagger \alpha_{k'}\right]$$

for $k' \neq k$, $[\alpha_k, \alpha_{k'}^\dagger] = 0$ and $[\alpha_k, \alpha_{k'}^\dagger \alpha_{k'}] = 0$

the terms are cancelled out.

for $k = k'$, $[b, H] = \sum_k \left(\alpha_k \alpha_k^\dagger \alpha_k - \alpha_k^\dagger \alpha_k \alpha_k\right) \cdot V_k \cdot \varepsilon_k$

$$= \sum_k [\alpha_k, \alpha_k^\dagger] \alpha_k \cdot V_k \cdot \varepsilon_k$$

$$= \sum_k \alpha_k \cdot \underbrace{V_k \cdot \varepsilon_k}_{\text{参数}}$$

Original NAH:

$$H = \varepsilon_c b^\dagger b + \sum_k \left(\varepsilon_k c_k^\dagger c_k + A_k (c_k^\dagger b + b^\dagger c_k) \right)$$

for

$$[b, H] = \varepsilon_c b b^\dagger b + \sum_k \varepsilon_k b c_k^\dagger c_k + \sum_k A_k (b c_k^\dagger b + b b^\dagger c_k)$$

$$-$$

$$\varepsilon_c b^\dagger b b + \sum_k \varepsilon_k c_k^\dagger c_k b + \sum_k A_k (c_k^\dagger b b + b^\dagger c_k b)$$

$$= \varepsilon_c \cdot \underbrace{[b, b^\dagger]}_{1} \cdot b + \sum_k \underbrace{[b, c_k^\dagger c_k]}_{0} \varepsilon_k + \sum_k A_k \cdot \underbrace{[b, c_k^\dagger]}_{0} \cdot b$$

$$+ \sum_k A_k \big(\underbrace{b b^\dagger c_k - b^\dagger c_k b}_{c_k} \big)$$

Check yourself that $\cancel{b b^\dagger c_k}$ $b^\dagger c_k b = b^\dagger b c_k$

$$b b^\dagger c_k - b^\dagger c_k b = [b, b^\dagger] c_k = c_k$$

$$[b, H] = \varepsilon_c b + \sum_k A_k c_k$$