Eactly solvable — Newns - Anderson

annilation operator

$$b = \sum_k V_k \alpha_k$$

$$c_k = \sum_{k'} \eta_{k,k'} \alpha_{k'}$$ spanned by

creation operator α_k and α^+

$$b^+ = \sum_k V_k^+ \alpha_k^+$$ (eigen-annilation operator) (eigen-creation operator)

$$c_k^+ = \sum_{k'} \eta_{k,k'}^+ \alpha_{k'}^+$$

Total hamiltonian

$$H = \sum_k \varepsilon_k \alpha_k^+ \alpha_k$$

$$[b, H] = \left[\sum_k V_k \alpha_k , \sum_{k'} \varepsilon_{k'} \alpha_{k'}^+ \alpha_{k'}\right]$$

for $k' \neq k$, $[\alpha_k , \alpha_{k'}^+] = 0$ and $[\alpha_k , \alpha_{k'}^+ \alpha_{k'}] = 0$

the terms are cancelled out.

for $k = k'$, $$[b, H] = \sum_k (\alpha_k \alpha_k^+ \alpha_k - \alpha_k^+ \alpha_k \alpha_k) \cdot V_k \cdot \varepsilon_k$$

$$= \sum_k \underset{\substack{\| \\ 1}}{[\alpha_k , \alpha_k^+]} \alpha_k \cdot V_k \cdot \varepsilon_k$$

$$= \sum_k \alpha_k \cdot \underbrace{V_k \cdot \varepsilon_k}_{\text{参数}}$$

Original NAH:

$$H = \varepsilon_c b^\dagger b + \sum_k \left(\varepsilon_k c_k^\dagger c_k + A_k (c_k^\dagger b + b^\dagger c_k) \right)$$

for

$$[b, H] = \varepsilon_c \, b b^\dagger b + \sum_k \varepsilon_k \, b c_k^\dagger c_k + \sum_k A_k (b c_k^\dagger b + b b^\dagger c_k)$$

$$- \varepsilon_c \, b^\dagger b b + \sum_k \varepsilon_k \, c_k^\dagger c_k b + \sum_k A_k (c_k^\dagger b b + b^\dagger c_k b)$$

$$= \varepsilon_c \cdot \underbrace{[b, b^\dagger]}_{1} \cdot b + \sum_k \underbrace{[b, c_k^\dagger c_k]}_{0} \varepsilon_k + \sum_k A_k \cdot \underbrace{[b, c_k^\dagger]}_{0} \cdot b$$

$$+ \sum_k A_k \underbrace{(b b^\dagger c_k - b^\dagger c_k b)}_{c_k}$$

Check yourself that $\cancel{b b^\dagger c_k}$ $b^\dagger c_k b = b^\dagger b c_k$

$$b b^\dagger c_k - b^\dagger c_k b = [b, b^\dagger] c_k = c_k$$

$$[b, H] = \varepsilon_c b + \sum_k A_k c_k$$

self - energy:

$$\Sigma(\varepsilon_k) = \sum_{k'} \frac{A_{k'}^2}{\varepsilon_k - \varepsilon_{k'}} \quad \text{for } k \neq k'$$

Retarded self-energy:

$$\Sigma_{ret}(\varepsilon) = \cancel{P}\sum_{k'} \frac{A_{k'}^2}{\varepsilon - \varepsilon_{k'} + i\delta}$$

$$Re\left[\Sigma_{ret}(\varepsilon)\right] = P\sum_{k'} \frac{A_{k'}^2}{\varepsilon_k - \varepsilon_{k'}}$$

P stands for Cauchy ~~princlp~~ principle value fixs the singularity of $\varepsilon_k = \varepsilon_{k'}$

imaginary part self - energy

$$\frac{A_{k'}^2(\varepsilon - \varepsilon_{k'} - \delta i)}{(\varepsilon - \varepsilon_{k'} + i\delta)(\varepsilon - \varepsilon_{k'} - i\delta)}$$

$$\frac{A_{k'}^2(\varepsilon - \varepsilon_{k'}) - A_{k'}^2 \delta i}{(\varepsilon - \varepsilon_{k'})^2 + \delta^2} \quad ?$$

Kronecker delta $\Longleftrightarrow$ normal $\delta(\varepsilon_k - \varepsilon_{k'})$

$$\delta(\varepsilon_k - \varepsilon_{k'}) = \frac{1}{2\pi V_k}\delta_{k,k'} \quad \text{box normalization}$$

V_k ?

in (4.159)

$$Z_k = \frac{1}{A_k^2}\left[\varepsilon_k - \varepsilon_c - \Sigma(\varepsilon_k)\right] \qquad V_k^2 = \left\{A_k^2\left[Z_k^2 + \left(\frac{\Gamma}{2V_k}\right)^2\right]\right\}^{-1}$$

$$Z_k^2 = \frac{1}{A_k^4}\left[\varepsilon_k - \varepsilon_c - \Sigma(\varepsilon_k)\right]^2$$

$$\frac{\cancel{A_k^2}\ 1}{A_k^2 \frac{1}{A_k^4}\left[\varepsilon_k - \varepsilon_c - \Sigma(\varepsilon_k)\right]^2 + A_k^2\left(\frac{\Gamma}{2V_k}\right)^2}$$

$$\frac{A_k^2}{\left[\varepsilon_k - \varepsilon_c - \Sigma(\varepsilon_k)\right]^2 + \left(\frac{\Gamma A_k^2}{2V_k}\right)^2} \qquad (4.160)$$

$\square$

(4.165) - (4.164)

$$\mathrm{Im}\left[\Sigma_{ret}(\varepsilon_k)\right] = -\pi \sum_{k'} A_{k'}^2 \left(\frac{L}{2v_k}\right) \delta_{kk'} = -\pi \sum_{k'} A_{k'}^2 \delta(\varepsilon_{k'} - \varepsilon_k)$$

$$= -\pi A_k^2 \frac{L}{2v_k} \quad ? = -\frac{L}{2v_k} A_k^2$$

系数 $$V_k^2 = \frac{A_k^2}{\left[\varepsilon_k - \varepsilon_c - \Sigma(\varepsilon_k)\right]^2 + \left(LA_k^2/2v_k\right)^2}$$

$$= -\left(\frac{2v_k}{L}\right) \frac{\mathrm{Im}\left[\Sigma_{ret}(\varepsilon_k)\right]}{\left[\varepsilon_k - \varepsilon_c - \mathrm{Re}(\Sigma_{ret}(\varepsilon_k))\right]^2 + \left[\mathrm{Im}\Sigma(\varepsilon_k)\right]^2}$$

$$\mathrm{Im}\left\{\frac{1}{\varepsilon_k - \varepsilon_c - \Sigma_{ret}(\varepsilon_k)}\right\}$$

$$a = \frac{(\varepsilon_k - \varepsilon_c - R + Ii)}{(\varepsilon_k - \varepsilon_c - R - Ii)(\varepsilon_k - \varepsilon_c - R + Ii)} = \frac{\varepsilon_k - \varepsilon_c - R + Ii}{(\varepsilon_k - \varepsilon_c - R)^2 + [I]^2}$$

$$\mathrm{Im}\{a\} = \frac{I}{(\varepsilon_k - \varepsilon_c - R)^2 + I^2}$$

$$V_k^2 = -\left(\frac{2v_k}{L}\right) \mathrm{Im}\left\{\frac{1}{\varepsilon_k - \varepsilon_c - \Sigma_{ret}(\varepsilon_k)}\right\}$$

□