Homework 1

Note: The due date is February 1 (Thursday). The goal of this homework is for you to get started with R/RStudio and to work through some problems on probability calculations that will be common in the course. Any questions about R/RStudio, email the teaching assistant. All problems below will carry equal weight.

The homework should be submitted as one PDF file. But it could be the PDF produced from an R-Markdown file, MS Word file, etc., or some combination thereof. I leave it up to you in what format you submit your homework.

Problem 1. Find 2 univariate time series in different fields online and do the following for each of the two series:

(a) Indicate the exact source for the time series data.

(b) Output the first 20 elements $x_1, \dots, x_{20}$ of the series in `R`.

(c) Produce a time plot of the series in `R`, after transforming the series into a time series object.

(d) Discuss briefly possible objectives for analyzing the time series.

Problem 2. Consider a time series model X_t, $t \in \mathbb{Z}$, defined by

$$X_t = 2Z_t + Z_{t-1},$$

where $\{Z_t\}$ is an IID noise with the common probability distribution $\mathbb{P}(Z_t = 2) = 1/3$ and $\mathbb{P}(Z_t = -1) = 2/3$. Do the following:

(a) Produce two different realizations x_t, $t = 1, \dots, T$, of the model of length $T = 100$; Include the R code;

(b) Compute theoretically $\mathbb{E}X_t$ and $\mathbb{E}(X_t^2)$; Compare these quantities with $\frac{1}{T}\sum_{t=1}^{T} x_t$ and $\frac{1}{T}\sum_{t=1}^{T} x_t^2$ for the two realizations above; Include the R code;

(c) Compute theoretically $\mathbb{E}(X_t X_{t-1})$ and $\mathrm{Corr}(X_t, X_{t-1})$; Compare these quantities with $\frac{1}{T-1}\sum_{t=2}^{T} x_t x_{t-1}$ and $\mathrm{cor}(v_1, v_2)$ with $v_1 = (x_2, \dots, x_T)$ and $v_2 = (x_1, \dots, x_{T-1})$, for the two realizations above; Include the R code;

(d) What are the possible values of X_t and the probabilities that X_t takes these values? Compare your answers from the two realizations above.

Problem 3. Come up with your own time series model and repeat parts (a)-(c) of Problem 2 for the model. Your model should have at least a trend or a periodic component, and incorporate IID noise in some way.

Problem 4. Consider the random walk

$$X_t = X_{t-1} + Z_t,$$

for $t = 1, 2, \ldots$, and $x_0 = 0$, where $\{Z_t\}$ is IID noise with $\mathcal{N}(1, 2^2)$.

(a) Show that the model can be written as the random walk with drift, i.e., $X_t = t + \sum_{s=1}^{t} W_s$, where $\{W_t\}$ is IID noise with $\mathcal{N}(0, 2^2)$.

(b) Produce two different realizations x_t, $t = 1, \ldots, T$, of the model of length $T = 100$; Include the R code;

(c) Compute theoretically $\mathbb{E}X_t$ and $\mathbb{E}(X_t^2)$ at $t = 100$; Write the R code to compute these quantities (Note: Do not plug in the number into the theoretical result).

(d) Compute theoretically $\mathbb{E}(X_t X_{t-1})$ and $\mathrm{Corr}(X_t, X_{t-1})$ at $t = 100$; Write the R code to compute these quantities (Note: Do not plug in the number into the theoretical result).

$\boxed{2}$ $X_t = 2Z_t + Z_{t-1}$

$$Z_t \begin{cases} 2 & \text{prob } 1/3 \\ -1 & \text{prob } 2/3 \end{cases}$$

a) See R code section

b) $\mathbb{E}(Z_t) = 2\frac{1}{3} + (-1)\frac{2}{3} = 0$

$$\mathrm{Var}(Z_t) = \frac{1}{3}(2^2) + \frac{2}{3}(-1)^2 = \frac{4}{3} + \frac{2}{3} = \frac{6}{3} = 2$$

$$\mathbb{E}(X_t) = 2\,\mathbb{E}(Z_t) + \mathbb{E}(Z_{t-1}) = 0$$

$$\mathbb{E}(X_t^2) = \mathrm{Var}(X_t) = 4\,\mathrm{Var}(Z_t) + \mathrm{Var}(Z_{t-1})$$

$$= 4(2) + 2 = 10$$

c)

$$\mathbb{E}(X_t X_{t-1}) = \mathbb{E}\left[(2Z_t + Z_{t-1})(2Z_{t-1} + Z_{t-2})\right]$$

$$= \mathbb{E}\left[4Z_tZ_{t-1} + 2Z_tZ_{t-2} + 2Z_{t-1}^2 + Z_{t-1}Z_{t-2}\right]$$

$$= \underbrace{4\,\mathbb{E}(Z_t)\,\mathbb{E}(Z_{t-1})}_{0} + \underbrace{2\,\mathbb{E}(Z_t)\,\mathbb{E}(Z_{t-2})}_{0} + \underbrace{2\,\mathbb{E}(Z_{t-1}^2)}_{2\cdot 2} + \underbrace{\mathbb{E}(Z_{t-1})\mathbb{E}(Z_{t-2})}_{0}$$

$$= 4$$

$$\mathrm{Corr}(X_t, X_{t-1}) = \frac{\mathbb{E}(X_tX_{t-1}) - \mathbb{E}(X_t)\,\mathbb{E}(X_{t-1})}{\sqrt{\mathbb{E}(X_t^2) - [\mathbb{E}(X_t)]^2}\sqrt{\mathbb{E}(X_{t-1}^2) - [\mathbb{E}(X_{t-1})]^2}} = \frac{4}{\sqrt{10}\sqrt{10}} = \frac{2}{5}$$

d)

$$X_t \begin{cases} 6 & \text{prob } 1/9 = \left(\frac{1}{3}\cdot\frac{1}{3}\right) \\ 3 & \text{prob } 2/9 = \left(\frac{1}{3}\cdot\frac{2}{3}\right) \\ 0 & \text{prob } 2/9 = \left(1/3 \cdot 2/3\right) \\ -3 & \text{prob } 4/9 = \left(2/3 \cdot 2/3\right) \end{cases}$$

3

$X_t = \alpha t + Z_t$ and $Z_t \sim N(0, \sigma_z^2)$ iid noise, $t = 1, 2, \dots, T$, $\alpha \in \mathbb{R}$

a) See R code section

** For the R section, I will use $T = 100$, $\alpha = 2$, $\sigma_z^2 = 2$ (instead of the generalization developed here) **

b) $\mathbb{E}(X_t) = \mathbb{E}(\alpha t) + \mathbb{E}(Z_t) = \alpha t$

$$\mathbb{E}(X_t^2) = \mathbb{E}\left[(\alpha t + Z_t)^2\right] = \mathbb{E}\left[(\alpha t)^2\right] + \mathbb{E}\left[2\alpha t Z_t\right] + \mathbb{E}\left[Z_t^2\right]$$

$$= (\alpha t)^2 + \sigma_z^2$$

c) $\mathbb{E}(X_t X_{t-1}) = \mathbb{E}\left[(\alpha t + Z_t)(\alpha(t-1) + Z_{t-1})\right] = \mathbb{E}\left[(\alpha t + Z_t)(\alpha t - \alpha + Z_{t-1})\right]$

$$= \mathbb{E}\left[(\alpha t)^2 - \alpha^2 t + \alpha t Z_{t-1} + \alpha t Z_t - \alpha Z_t + Z_t Z_{t-1}\right]$$

$$= (\alpha t)^2 - \alpha^2 t = \alpha^2 t(t-1)$$

$$\mathrm{Corr}(X_t, X_{t-1}) = \frac{\mathbb{E}(X_t X_{t-1}) - \mathbb{E}(X_t)\,\mathbb{E}(X_{t-1})}{\sqrt{\mathbb{E}(X_t^2) - [\mathbb{E}(X_t)]^2}\sqrt{\mathbb{E}(X_{t-1}^2) - [\mathbb{E}(X_{t-1})]^2}}$$

$$= \frac{\alpha^2 t(t-1) - (\alpha t)[\alpha(t-1)]}{\sqrt{(\alpha t)^2 + \sigma_z^2 - (\alpha t)^2}\sqrt{[\alpha(t-1)]^2 + \sigma_z^2 - [\alpha(t-1)]^2}}$$

$$= \frac{\alpha^2 t(t-1) - (\alpha t)[\alpha(t-1)]}{\sigma_z^2}$$

$\boxed{4}$ $X_t = X_{t-1} + Z_t$ with $Z_t \sim N(1, 2^2)$, $X_0 = 0$

a) Let $\{W_t\}$ iid noise $N(0, 2^2)$

$$X_t = X_{t-1} + Z_t = X_{t-2} + Z_{t-1} + Z_t \ (\ldots \text{so on} \ldots)$$

$$= \sum_{s=1}^{t} Z_s = \sum_{s=1}^{t} \Big(\overbrace{Z_s - \mathbb{E}(Z_s)}^{W_t} + \overbrace{\mathbb{E}(Z_s)}^{1} \Big)$$

$$= \sum_{s=1}^{t} W_s + t$$

b) See R code section

c) $\mathbb{E}(X_t) = t + \mathbb{E}\left(\sum_{s=1}^{t} W_s\right) = t + \sum_{s=1}^{t} \mathbb{E}(W_s) = \underline{t}$

$$\mathbb{E}(X_t^2) = \mathbb{E}\left[\left(X_{t-1} + Z_t\right)^2\right] = \mathbb{E}\left\{\left[\left(t + \sum_{s=1}^{t} W_s\right)\right]^2\right\}$$

$$= \mathbb{E}\left[t^2 + 2t \sum_{s=1}^{t} W_s + \left(\sum_{s=1}^{t} W_s\right)^2\right]$$

$$= t^2 + 2t\, \mathbb{E}\left(\sum_{s=1}^{t} W_s\right) + \mathbb{E}\left[\left(\sum_{s=1}^{t} W_s\right)^2\right]$$

$$= t^2 + \mathbb{E}\left[\left(\sum_{s=1}^{t} W_s\right)^2\right]$$

$$= t^2 + \mathbb{E}\left[\sum_{s=1}^{t} W_s^2 + 2 \sum_{j=1}^{t} \sum_{i=1}^{j-1} W_i W_j\right]$$

$$= t^2 + \left[\underbrace{\sum_{s=1}^{t} \mathbb{E}(W_s^2)}_{t \cdot 4} + 2 \sum_{j=1}^{t} \sum_{i=1}^{j-1} \underbrace{\mathbb{E}(W_i W_j)}_{0} \right]$$

$$= t^2 + 4t = \underline{t(t+4)}$$

d) $$\mathbb{E}(X_t X_{t-1}) = \mathbb{E}\left[\left(t + \sum_{s=1}^{t} W_s\right)\left(t-1 + \sum_{s=1}^{t-1} W_s\right)\right]$$

$$= \mathbb{E}\left[X_{t-1} X_{t-2} + X_{t-1} Z_{t-1} + Z_t X_{t-2} + Z_t Z_{t-1}\right]$$

$$= \mathbb{E}\left[t(t-1) + t \sum_{s=1}^{t-1} W_s + (t-1) \sum_{s=1}^{t} W_s + \sum_{s=1}^{t} W_s \sum_{s=1}^{t-1} W_s\right]$$

$$= t(t-1) + t \sum_{s=1}^{t-1} \mathbb{E}(W_s) + (t-1) \sum_{s=1}^{t} \mathbb{E}(W_s) + \mathbb{E}\left(W_t \left(\sum_{s=1}^{t-1} W_s\right)^2\right)$$

$$= t(t-1) + \mathbb{E}(W_t) \cdot \mathbb{E}\left(\sum_{s=1}^{t-1} W_s\right)^2 = \underline{t(t-1)}$$

$$\operatorname{Corr}(X_t X_{t-1}) = \frac{\mathbb{E}(X_t X_{t-1}) - \mathbb{E}(X_t)\,\mathbb{E}(X_{t-1})}{\sqrt{\mathbb{E}(X_t^2) - [\mathbb{E}(X_t)]^2}\sqrt{\mathbb{E}(X_{t-1}^2) - [\mathbb{E}(X_{t-1})]^2}}$$

$$= \frac{t(t-1) - t(t-1)}{\sqrt{t(t+4) - t^2}\sqrt{(t-1)(t+3) - (t-1)^2}}$$

$$= 0$$

ORIE5550 - HW1 - lc2234

Luis Alonso Cendra Villalobos (lc2234)

2024-02-01

install.packages(“tinytex”)

Problem 1

Find 2 univariate time series in different fields online and do the following for each of the two series:

(a) Indicate the exact source for the time series data.

Source of data1: World Bank. URL: https://data.worldbank.org/indicator/SE.PRM.CMPT.FE.ZS?locations=1W&start=1973&view=chart

Source of data2: Yahoo! Finance. URL: https://finance.yahoo.com/quote/AAPL/history?period1=1548720000&period2=1706486400&interval=1mo&filter=history&frequency=1mo&includeAdjustedClose=true

(b) Output the first 20 elements x1,…,x20 of the series in R.

```
#Set working directory
setwd("/Users/alons/OneDrive - Cornell University/Cornell University/Spring 2024/ORIE 5550/ORIE5550_Homework1")
#extract data from CSV
data1 <- read.csv("ORIE5550_DataHW1.csv")
data1$Year <- as.Date(data1$Year)
data2 <- read.csv("ORIE5550_Data2HW1.csv")
data2$Date <- as.Date(data2$Date)


#transform data to Time Series
time_series_data1 <- ts(data1$Value, start = min(data1$Year), frequency = 1)
time_series_data2 <- ts(data2$Value, start = min(data2$Date), frequency = 12)

# Output the first 20 values
first_20_values1 <- head(time_series_data1, 20)
print(first_20_values1)
```

```
##  [1] 70.67788 69.07636 68.12719 68.04817 71.75885 73.93382 73.02761 73.40257
##  [9] 73.72074 73.15771 75.41759 75.92915 76.64542 76.75523 77.51466 77.79708
## [17] 77.87164 77.33327 77.51179 76.97076
```

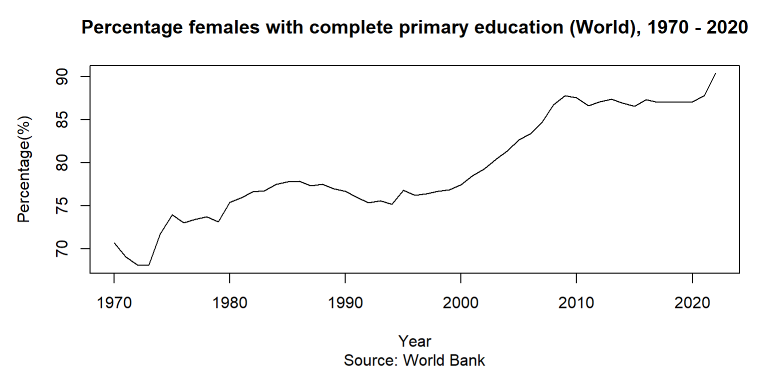
```
first_20_values2 <- head(time_series_data2, 20)
print(first_20_values2)
```

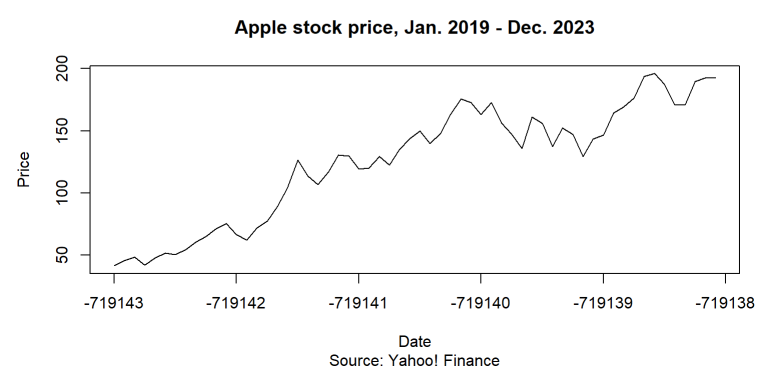
```
##  [1]  41.54781  45.77450  48.35781  42.18869  47.87880  51.53647  50.49627
##  [8]  54.38640  60.40613  64.89605  71.52080  75.38364  66.57903  62.08135
## [15]  71.72715  77.62061  89.30191 104.04848 126.35438 113.60417
```

(c) Produce a time plot of the series in R, after transforming the series into a time series object.

```
par(mfrow=c(2,1))
plot.ts(time_series_data1, ylab='Percentage(%)', xlab='Year',
        main = "Percentage females with complete primary education (World), 1970 - 2020",
        sub = "Source: World Bank")


plot.ts(time_series_data2, ylab='Price', xlab='Date',
        main = "Apple stock price, Jan. 2019 - Dec. 2023",
        sub = "Source: Yahoo! Finance")
```





(d) Discuss briefly possible objectives for analyzing the time series.

The percentage of females with complete primary education can be analyzed to study the factors that are involved in the literacy rate of women worldwide, we can regress the series against others like investment in education, inclusion of women to the education system, gender equality policies/decisions, among others. The Apple stock price can be examined to determine factors that can forecast the stock price in the future with the objective of making investment decisions, for example.

Problem 2

(a) Produce two different realizations xt, t = 1,…,T, of the model of length T = 100; Include the R code;

```
# Set the length of the time series
length_of_series <- 100

# Initialize vectors to store the time series values
X_1 <- numeric(length_of_series)
Y_1 <- numeric(length_of_series)
X_2 <- numeric(length_of_series)
Y_2 <- numeric(length_of_series)
Z_1 <- numeric(length_of_series)
Z_2 <- numeric(length_of_series)


# Set the initial value of Z
Z_1[1] <- sample(c(2, -1), size = 1, prob = c(1/3, 2/3)) #realization 1 of Z[1]
Z_2[1] <- sample(c(2, -1), size = 1, prob = c(1/3, 2/3)) #realization 2 of Z[1]

# Generate the time series realizations for Z_t, X_t, and X_t * X_(t-1)
for (t in 2:length_of_series) {
  Z_1[t] <- sample(c(2, -1), size = 1, prob = c(1/3, 2/3))
  Z_2[t] <- sample(c(2, -1), size = 1, prob = c(1/3, 2/3))

  # Realization 1 of X_t and Y = X_t * X_(t-1)
  X_1[t] <- 2 * Z_1[t] + Z_1[t - 1]
  Y_1[t] <- X_1[t] * X_1[t - 1]

  # Realization 2 of X_t and Y = X_t * X_(t-1)
  X_2[t] <- 2 * Z_2[t] + Z_2[t - 1]
  Y_2[t] <- X_2[t] * X_2[t - 1]

}

# Calculate the mean of Z
mean_Z_1 <- mean(Z_1) #mean of realization 1
mean_Z_2 <- mean(Z_2) #mean of realization 2

# Calculate the mean of Z^2
mean_Z_1_squared <- mean(Z_1^2) #mean of realization 1 squared
mean_Z_2_squared <- mean(Z_2^2) #mean of realization 2 squared

# Display the results
print(paste("Mean of Z_1:", mean_Z_1))
```

```
## [1] "Mean of Z_1: -0.04"
```

```
print(paste("Mean of Z_1^2:", mean_Z_1_squared))
```

```
## [1] "Mean of Z_1^2: 1.96"
```

```
print(paste("Mean of Z_1:", mean_Z_2))
```

```
## [1] "Mean of Z_1: -0.25"
```

```
print(paste("Mean of Z_1^2:", mean_Z_2_squared))
```

```
## [1] "Mean of Z_1^2: 1.75"
```

(b) Compute theoretically EXt and E(X2 t); Compare these quantities with the two realizations above; Include the R code.

```
# Calculate the mean of X_t
mean_X_1 <- mean(X_1)
mean_X_2 <- mean(X_2)

# Display the results
print(paste("Mean of X_1:", mean_X_1))
```

```
## [1] "Mean of X_1: -0.09"
```

```
print(paste("Mean of X_2:", mean_X_2))
```

```
## [1] "Mean of X_2: -0.72"
```

```
# Calculate the mean of X_t squared
mean_X_1_squared <- mean(X_1^2)
mean_X_2_squared <- mean(X_2^2)

# Display the results
print(paste("Mean of X_1^2:", mean_X_1_squared))
```

```
## [1] "Mean of X_1^2: 10.71"
```

```
print(paste("Mean of X_2^2:", mean_X_2_squared))
```

```
## [1] "Mean of X_2^2: 9.18"
```

(c) Compute theoretically E(XtXt−1) and Corr(Xt,Xt−1); Compare these quantities with the two realizations above; Include the R code;

```
# Calculate the mean of Y = X_t * X_(t-1)
mean_Y_1 <- mean(Y_1)
mean_Y_2 <- mean(Y_2)

# Display the results
print("Let Y = X_t * X_(t-1), then:")
```

```
## [1] "Let Y = X_t * X_(t-1), then:"
```

```
print(paste("Mean of Y_1:", mean_Y_1))
```

```
## [1] "Mean of Y_1: 5.13"
```

```
print(paste("Mean of Y_2:", mean_Y_2))
```

```
## [1] "Mean of Y_2: 4.23"
```

```
# Correlations

V_11 <- X_1[2:length_of_series]
V_12 <- X_1[1:(length_of_series - 1)]

V_21 <- X_2[2:length_of_series]
V_22 <- X_2[1:(length_of_series - 1)]

# Calculate the correlation between V_1 and V_2
correlation_V1_V2_1 <- cor(V_11, V_12)
correlation_V1_V2_2 <- cor(V_21, V_22)

# Display the correlation
print(paste("Correlation between V_1 and V_2 realization 1:", correlation_V1_V2_1))
```

```
## [1] "Correlation between V_1 and V_2 realization 1: 0.48077145322455"
```

```
print(paste("Correlation between V_1 and V_2 realization 2:", correlation_V1_V2_2))
```

```
## [1] "Correlation between V_1 and V_2 realization 2: 0.431867667638816"
```

(d) What are the possible values of Xt and the probabilities that Xt takes these values? Compare your answers from the two realizations above

See handwritten notes.

Problem 3

Come up with your own time series model and repeat parts (a)-(c) of Problem 2 for the model. Your model should have at least a trend or a periodic component, and incorporate IID noise in some way

See Handwritten notes. We assume T=100, alpha = 2, variance = 2.

```
# Set the length of the time series
length_of_series <- 100

# Initialize vectors to store the time series values
X_1 <- numeric(length_of_series)
Y_1 <- numeric(length_of_series)
X_2 <- numeric(length_of_series)
Y_2 <- numeric(length_of_series)
Z_1 <- numeric(length_of_series)
Z_2 <- numeric(length_of_series)


# Create a time series Z_t with elements distributed normally
Z_1 <- rnorm(length_of_series, mean = 0, sd = sqrt(2))
Z_2 <- rnorm(length_of_series, mean = 0, sd = sqrt(2))

X_1[1] <- 2 + Z_1[1]
X_2[1] <- 2 + Z_2[1]

# Generate the time series realizations for Z_t, X_t, and X_t * X_(t-1)
for (t in 2:length_of_series) {

  # Realization 1 of X_t and Y = X_t * X_(t-1)
  X_1[t] <- 2 * t + Z_1[t]
  Y_1[t] <- X_1[t] * X_1[t - 1]

  # Realization 2 of X_t and Y = X_t * X_(t-1)
  X_2[t] <- 2 * t + Z_2[t]
  Y_2[t] <- X_2[t] * X_2[t - 1]

}


# Calculate the mean of X_t
mean_X_1 <- mean(X_1)
mean_X_2 <- mean(X_2)

# Display the results
print(paste("Mean of X_1:", mean_X_1))
```

```
## [1] "Mean of X_1: 101.137590232438"
```

```
print(paste("Mean of X_2:", mean_X_2))
```

```
## [1] "Mean of X_2: 101.015028634881"
```

```
# Calculate the mean of X_t squared
mean_X_1_squared <- mean(X_1^2)
mean_X_2_squared <- mean(X_2^2)

# Display the results
print(paste("Mean of X_1^2:", mean_X_1_squared))
```

```
## [1] "Mean of X_1^2: 13553.4612728173"
```

```
print(paste("Mean of X_2^2:", mean_X_2_squared))
```

```
## [1] "Mean of X_2^2: 13552.037203831"
```

```
# Calculate the mean of Y = X_t * X_(t-1)
mean_Y_1 <- mean(Y_1)
mean_Y_2 <- mean(Y_2)

# Display the results
print("Let Y = X_t * X_(t-1), then:")
```

```
## [1] "Let Y = X_t * X_(t-1), then:"
```

```
print(paste("Mean of Y_1:", mean_Y_1))
```

```
## [1] "Mean of Y_1: 13348.6767750323"
```

```
print(paste("Mean of Y_2:", mean_Y_2))
```

```
## [1] "Mean of Y_2: 13348.6614269418"
```

```
# Correlations
V_11 <- X_1[2:length_of_series]
V_12 <- X_1[1:(length_of_series - 1)]
V_21 <- X_2[2:length_of_series]
V_22 <- X_2[1:(length_of_series - 1)]

# Calculate the correlation between V_1 and V_2
correlation_V1_V2_1 <- cor(V_11, V_12)
correlation_V1_V2_2 <- cor(V_21, V_22)

# Display the correlation
print(paste("Correlation between V_1 and V_2 realization 1:", correlation_V1_V2_1))
```

```
## [1] "Correlation between V_1 and V_2 realization 1: 0.999444408187673"
```

```
print(paste("Correlation between V_1 and V_2 realization 2:", correlation_V1_V2_2))
```

```
## [1] "Correlation between V_1 and V_2 realization 2: 0.999150932010411"
```

Problem 4

(a)

See Handwritten notes.

(b) Produce two different realizations xt, t = 1,…,T, of the model of length T = 100; Include the R code;

```
# Set the parameters
mean_value <- 1
variance_value <- 4
TT <- 100
num_simulations <- 10000

# Initialize array to store values at T=100 from Monte Carlo
X_values_at_T_1 <- numeric(num_simulations)
X_values_at_T_2 <- numeric(num_simulations)
Y_values_at_T_1 <- numeric(num_simulations)
Y_values_at_T_2 <- numeric(num_simulations)

# Perform simulations
for (sim in 1:num_simulations) {
  # Generate random variable time series Z_t white noise Normal(1,4)
  Z_1_t <- rnorm(TT, mean = mean_value, sd = sqrt(variance_value))
  Z_2_t <- rnorm(TT, mean = mean_value, sd = sqrt(variance_value))

  # Initialize X_t with X_0 = 0 for the 2 realizations
  X_1_t <- numeric(TT)
  X_2_t <- numeric(TT)
  Y_1 <- numeric(TT)
  Y_2 <- numeric(TT)
  X_1_t[1] <- 0
  X_2_t[1] <- 0

  # Generate AR(1) model for both realizations
  for (t in 2:TT) {
    X_1_t[t] <- X_1_t[t-1] + Z_1_t[t]
    X_2_t[t] <- X_2_t[t-1] + Z_2_t[t]

    # Realizations of Y = X_t * X_(t-1)
    Y_1[t] <- X_1_t[t] * X_1_t[t - 1]
    Y_2[t] <- X_2_t[t] * X_2_t[t - 1]

  }
  # Store the last element of X_t in values_at_T for both realizations
  X_values_at_T_1[sim] <- X_1_t[TT]
  X_values_at_T_2[sim] <- X_2_t[TT]
  Y_values_at_T_1[sim] <- Y_1[TT]
  Y_values_at_T_2[sim] <- Y_2[TT]
}
```

(c) Compute theoretically EXt and E(X2 t) at t = 100; Write the R code to compute these quantities (Note: Do not plug in the number into the theoretical result).

```
# Calculate the mean of the realizations
X_mean_1 <- mean(X_values_at_T_1) #mean of realization 1
X_mean_2 <- mean(X_values_at_T_2) #mean of realization 2

# Calculate the mean of Z^2
X_mean_1_squared <- mean(X_values_at_T_1^2) #mean of realization 1 squared
X_mean_2_squared <- mean(X_values_at_T_2^2) #mean of realization 2 squared

# Display the results
print(paste("Mean of realization 1 of X:", X_mean_1))
```

```
## [1] "Mean of realization 1 of X: 99.095224496095"
```

```
print(paste("Mean of realization 1 of X squared:", X_mean_1_squared))
```

```
## [1] "Mean of realization 1 of X squared: 10218.7946025257"
```

```
print(paste("Mean of realization 2 of X:", X_mean_2))
```

```
## [1] "Mean of realization 2 of X: 98.7155213597477"
```

```
print(paste("Mean of realization 2 of X squared:", X_mean_2_squared))
```

```
## [1] "Mean of realization 2 of X squared: 10131.1417835615"
```

(d) Compute theoretically E(Xt * Xt−1) and Corr(Xt,Xt−1) at t = 100; Write the R code to compute these quantities (Note: Do not plug in the number into the theoretical result).

```
# Calculate the mean of Y = X_t * X_(t-1)
mean_Y_1 <- mean(Y_values_at_T_1)
mean_Y_2 <- mean(Y_values_at_T_1)

# Display the results
print("Let Y = X_t * X_(t-1), then:")
```

```
## [1] "Let Y = X_t * X_(t-1), then:"
```

```
print(paste("Mean of realization 1 of Y:", mean_Y_1))
```

```
## [1] "Mean of realization 1 of Y: 10118.4295853896"
```

```
print(paste("Mean of realization 2 of Y:", mean_Y_2))
```

```
## [1] "Mean of realization 2 of Y: 10118.4295853896"
```

```
# Correlations
V_11 <- X_values_at_T_1[2:length_of_series]
V_12 <- X_values_at_T_1[1:(length_of_series - 1)]

V_21 <- X_values_at_T_2[2:length_of_series]
V_22 <- X_values_at_T_2[1:(length_of_series - 1)]

# Calculate the correlation between V_1 and V_2
correlation_V1_V2_1 <- cor(V_11, V_12)
correlation_V1_V2_2 <- cor(V_21, V_22)

# Display the correlation
print(paste("Correlation between V_1 and V_2 realization 1:", correlation_V1_V2_1))
```

```
## [1] "Correlation between V_1 and V_2 realization 1: -0.0337713177447808"
```

```
print(paste("Correlation between V_1 and V_2 realization 2:", correlation_V1_V2_2))
```

```
## [1] "Correlation between V_1 and V_2 realization 2: -0.0107623813319924"
```