Homework 1

Note: The due date is February 1 (Thursday). The goal of this homework is for you to get started with R/RStudio and to work through some problems on probability calculations that will be common in the course. Any questions about R/RStudio, email the teaching assistant. All problems below will carry equal weight.

The homework should be submitted as one PDF file. But it could be the PDF produced from an R-Markdown file, MS Word file, etc., or some combination thereof. I leave it up to you in what format you submit your homework.

Problem 1. Find 2 univariate time series in different fields online and do the following for each of the two series:

(a) Indicate the exact source for the time series data.

(b) Output the first 20 elements $x_1, \ldots, x_{20}$ of the series in `R`.

(c) Produce a time plot of the series in `R`, after transforming the series into a time series object.

(d) Discuss briefly possible objectives for analyzing the time series.

Problem 2. Consider a time series model X_t, $t \in \mathbb{Z}$, defined by

$$X_t = 2Z_t + Z_{t-1},$$

where $\{Z_t\}$ is an IID noise with the common probability distribution $\mathbb{P}(Z_t = 2) = 1/3$ and $\mathbb{P}(Z_t = -1) = 2/3$. Do the following:

(a) Produce two different realizations x_t, $t = 1, \ldots, T$, of the model of length $T = 100$; Include the R code;

(b) Compute theoretically $\mathbb{E}X_t$ and $\mathbb{E}(X_t^2)$; Compare these quantities with $\frac{1}{T}\sum_{t=1}^{T} x_t$ and $\frac{1}{T}\sum_{t=1}^{T} x_t^2$ for the two realizations above; Include the R code;

(c) Compute theoretically $\mathbb{E}(X_t X_{t-1})$ and $\mathrm{Corr}(X_t, X_{t-1})$; Compare these quantities with $\frac{1}{T-1}\sum_{t=2}^{T} x_t x_{t-1}$ and $\mathrm{cor}(v_1, v_2)$ with $v_1 = (x_2, \ldots, x_T)$ and $v_2 = (x_1, \ldots, x_{T-1})$, for the two realizations above; Include the R code;

(d) What are the possible values of X_t and the probabilities that X_t takes these values? Compare your answers from the two realizations above.

Problem 3. Come up with your own time series model and repeat parts (a)-(c) of Problem 2 for the model. Your model should have at least a trend or a periodic component, and incorporate IID noise in some way.

Problem 4. Consider the random walk

$$X_t = X_{t-1} + Z_t,$$

for $t = 1, 2, \ldots$, and $x_0 = 0$, where $\{Z_t\}$ is IID noise with $\mathcal{N}(1, 2^2)$.

(a) Show that the model can be written as the random walk with drift, i.e., $X_t = t + \sum_{s=1}^{t} W_s$, where $\{W_t\}$ is IID noise with $\mathcal{N}(0, 2^2)$.

(b) Produce two different realizations x_t, $t = 1, \ldots, T$, of the model of length $T = 100$; Include the R code;

(c) Compute theoretically $\mathbb{E}X_t$ and $\mathbb{E}(X_t^2)$ at $t = 100$; Write the R code to compute these quantities (Note: Do not plug in the number into the theoretical result).

(d) Compute theoretically $\mathbb{E}(X_t X_{t-1})$ and $\mathrm{Corr}(X_t, X_{t-1})$ at $t = 100$; Write the R code to compute these quantities (Note: Do not plug in the number into the theoretical result).