10 minutes to explain…

The Schröder-Bernstein Theorem

$$\forall A\,\forall B\,((\exists f \subseteq A \times B\,(\forall x \in A\,\exists!\,y \in B\,(x,y) \in f) \land (\forall y \in B\,!\,x \in A\,\;(x,y) \in f))$$
$$\land\,(\exists g \subseteq B \times A\,(\forall x \in B\,\exists!\,y \in A\,(x,y) \in g) \land (\forall y \in A\,!\,x \in B\,\;(x,y) \in g)))$$
$$\Rightarrow\;\exists h \subseteq A \times B\;\,(\forall x \in A\,\exists!\,y \in B\,(x,y) \in h) \land (\forall y \in B\,\exists!\,x \in A\,\;(x,y) \in h)$$

Three facts about SBT

- ❑ In **set theory**
  About *cardinal numbers* (sizes of sets)

Three facts about SBT

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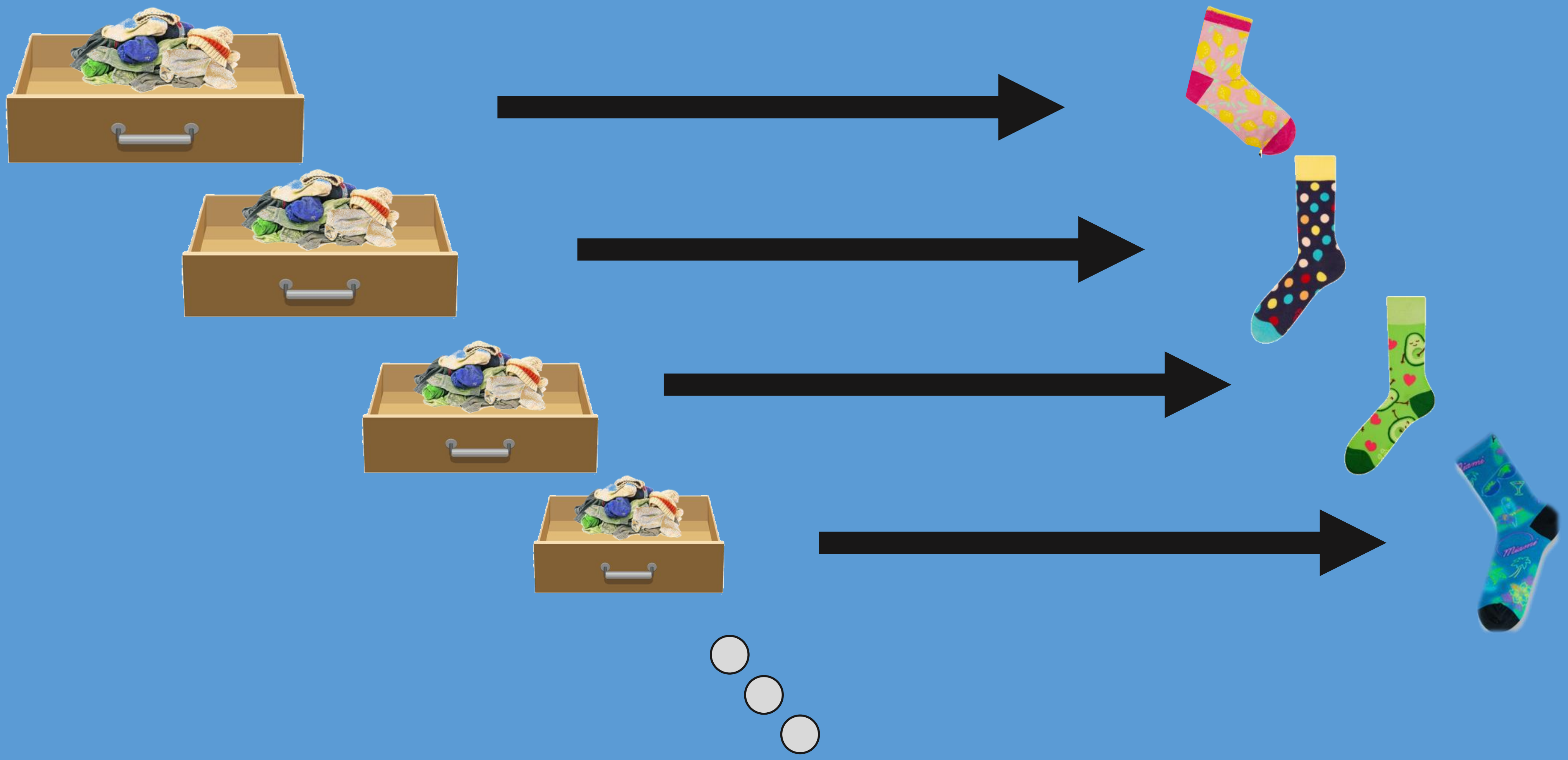
Three facts about SBT

- ❑ In **set theory**
  About *cardinal numbers* (sizes of sets)
- ❑ A property of **antisymmetry**
- ❑ Lots of names : Cantor, Schröder, Bernstein, and so on.

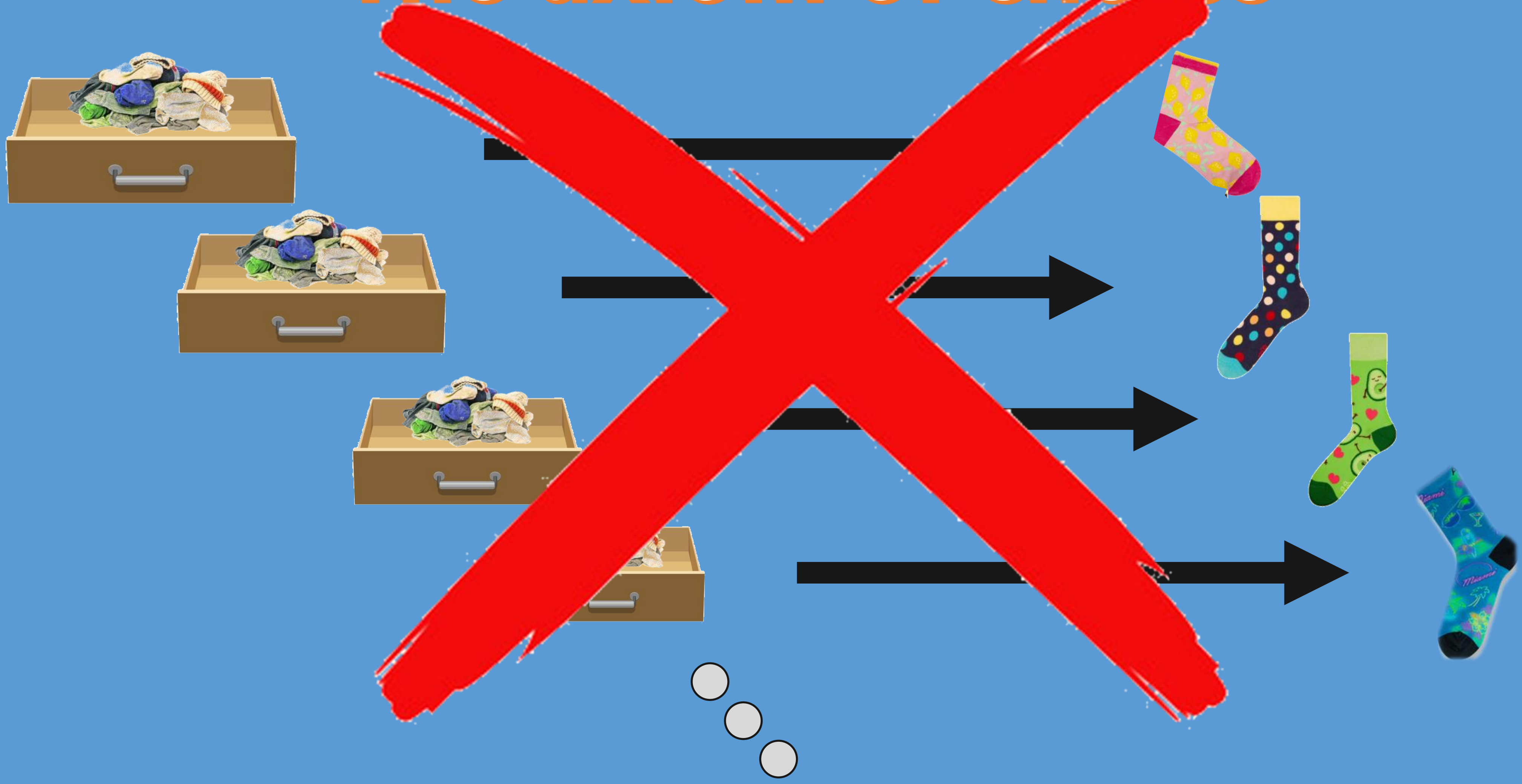
A little bit of history

- 1887 : Georg Cantor (without proof)
- 1895 : 1st proof
- 1896 : Ernst Schröder thinks he has a proof
- 1897 : Felix Bernstein (without AC)
- 1898 : Ernst Schröder is sure to have a proof
- 1902 : the fall of Ernst Schröder
- 1930 : Richard Dedekind… from 1887 (and 1897)

The axiom of choice

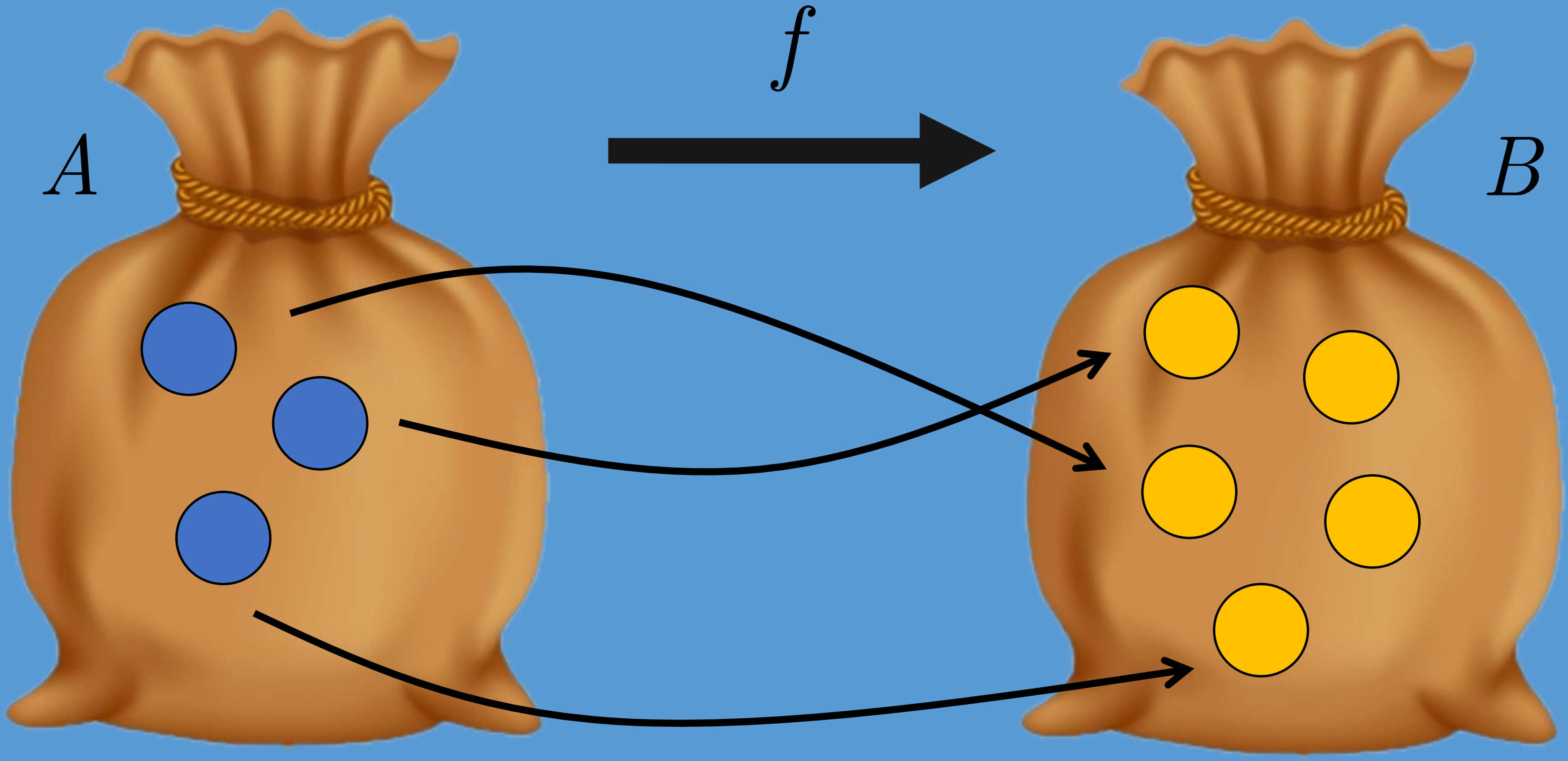


The axiom of choice

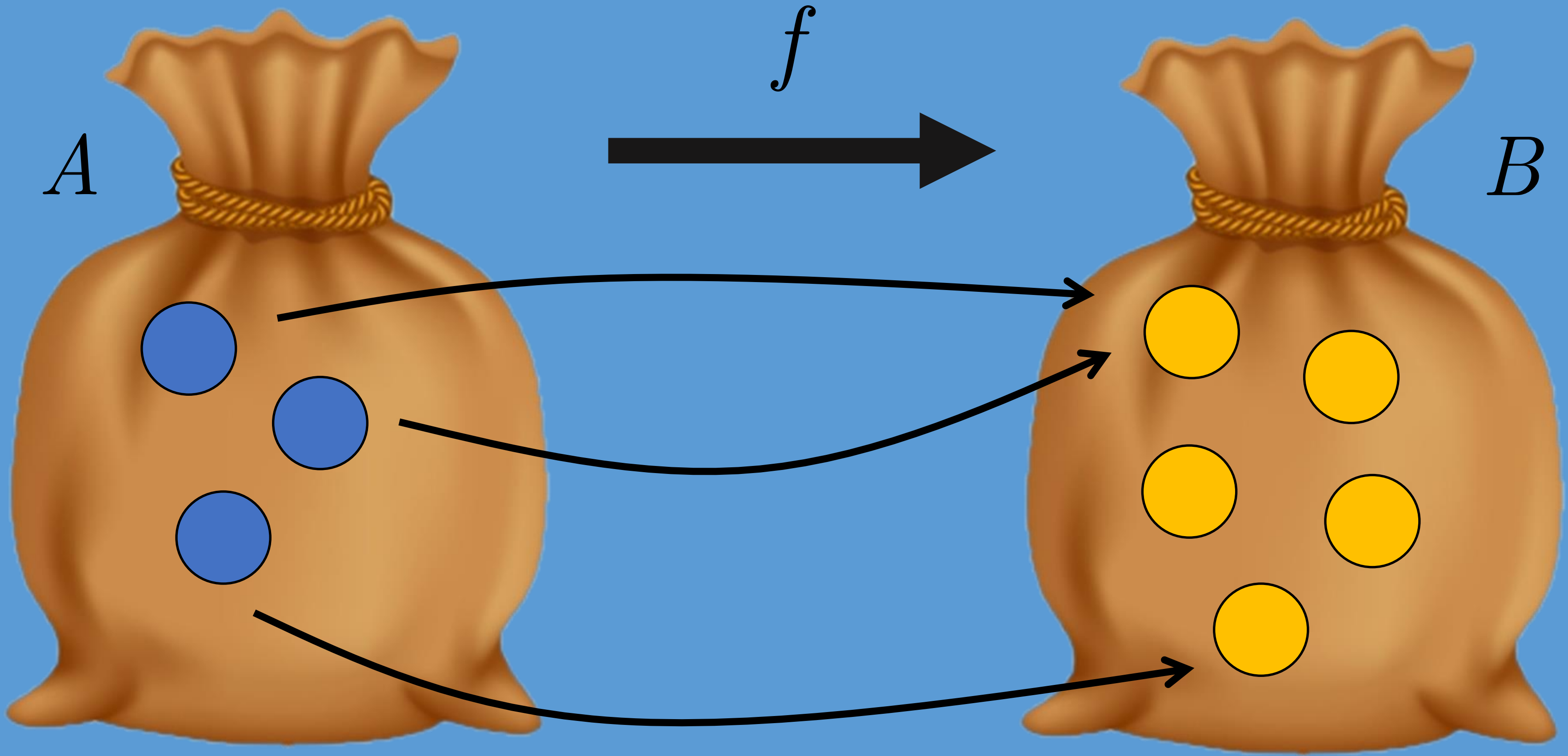


Injective functions

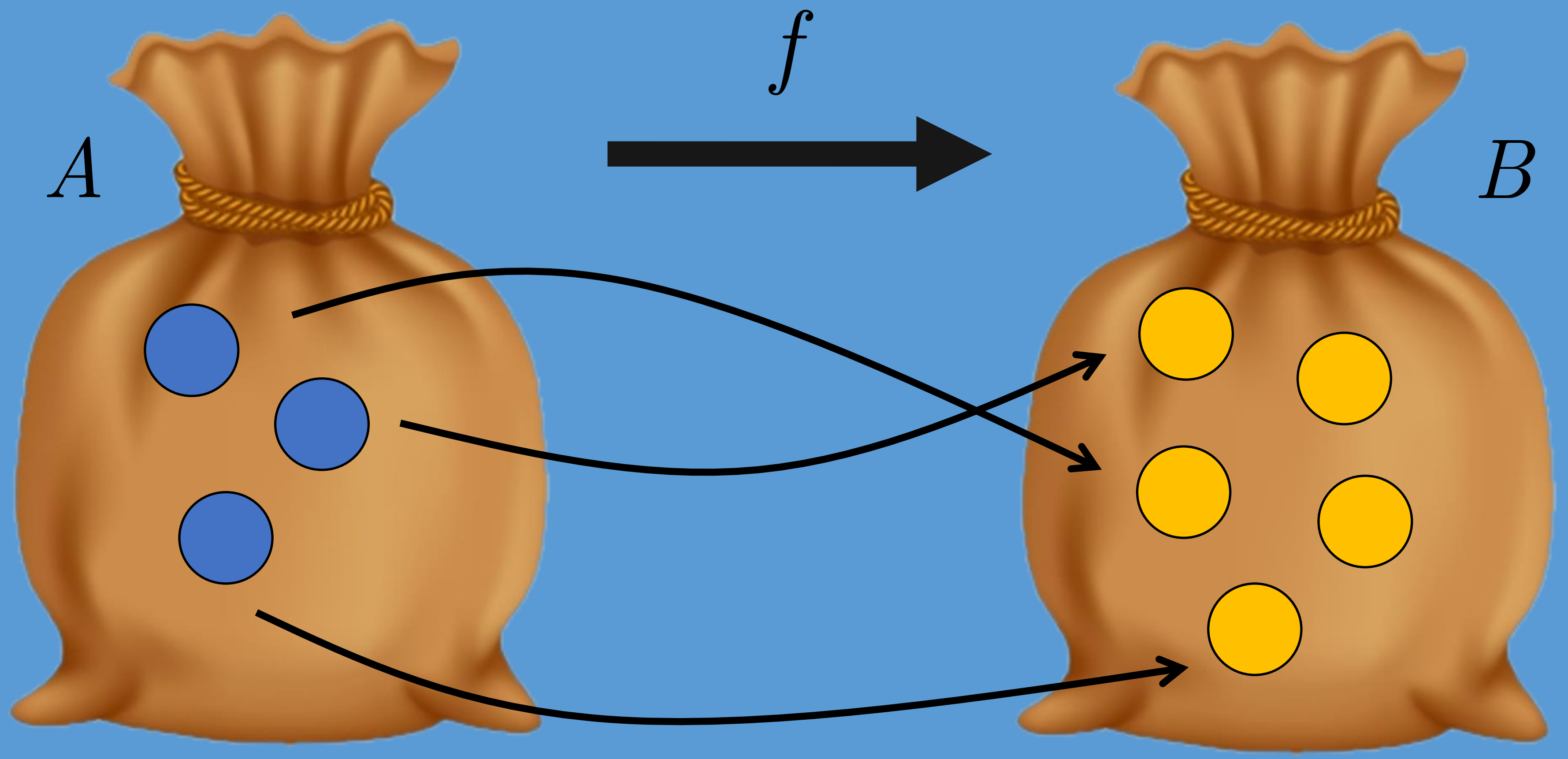
Injective functions



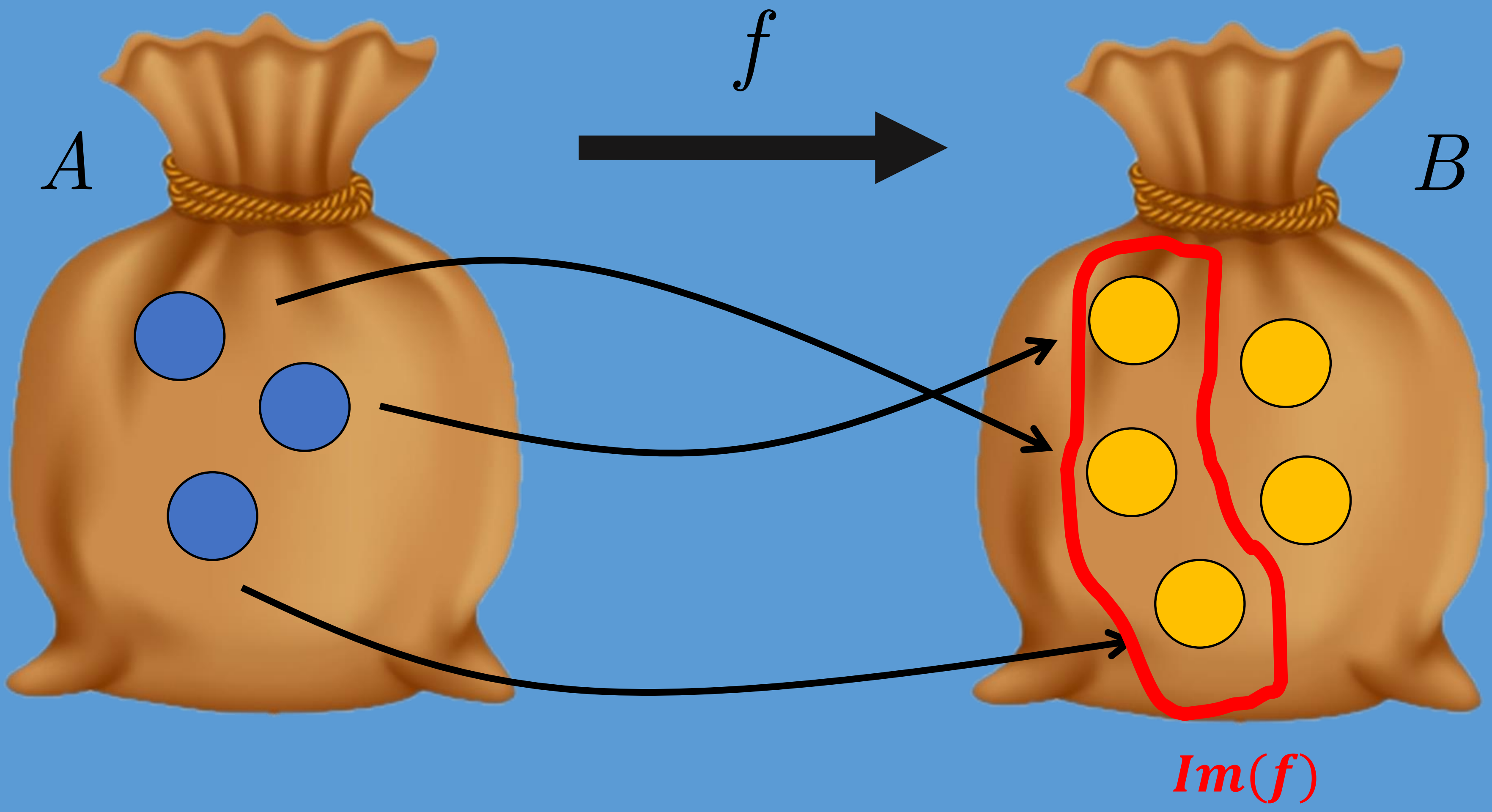
~~Injective~~ functions



Injective functions



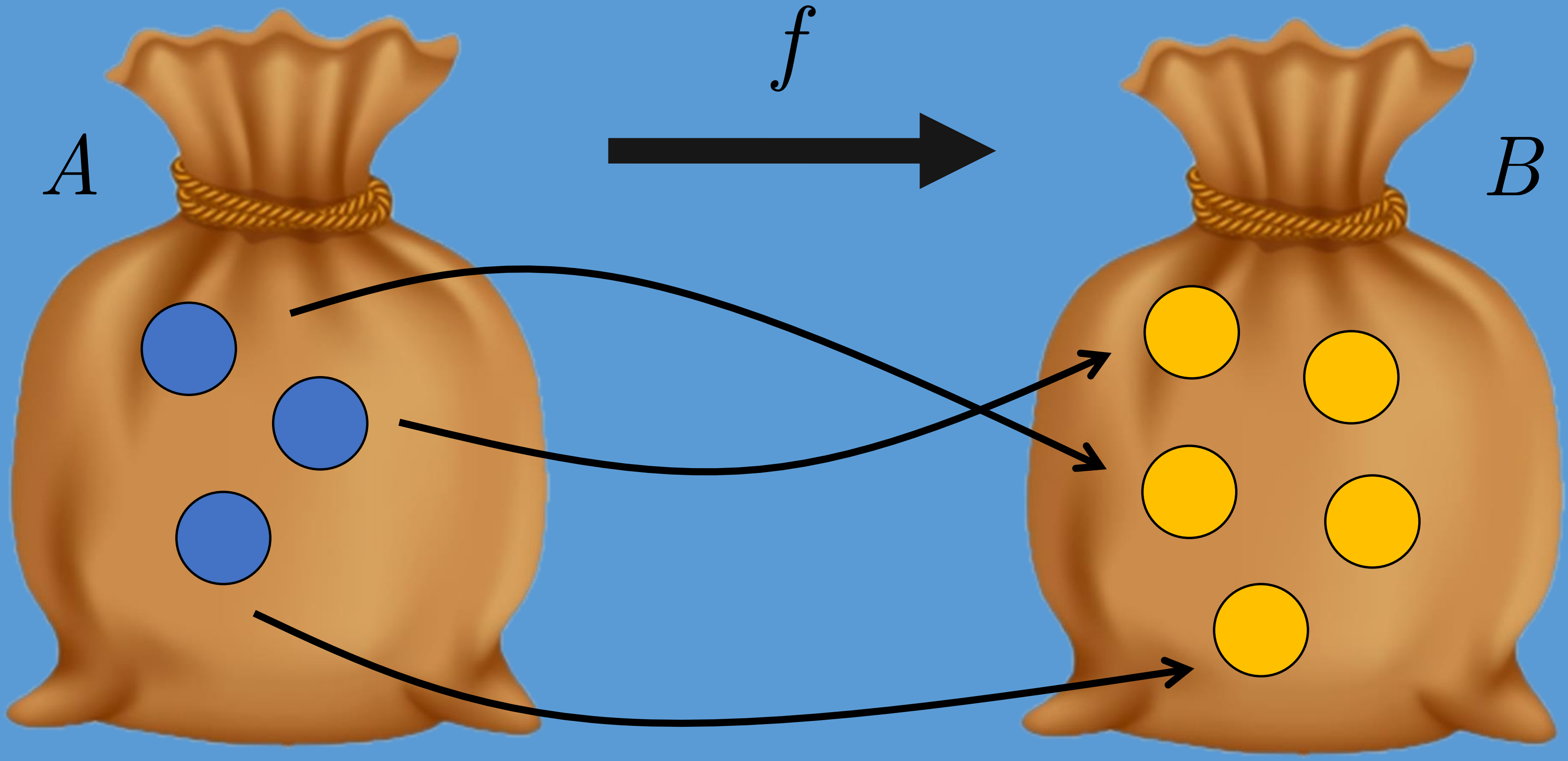
Injective functions



Definition

"Let A,B be two sets. We say that A is smaller than B, if and only if there exists an injective function from A to B"

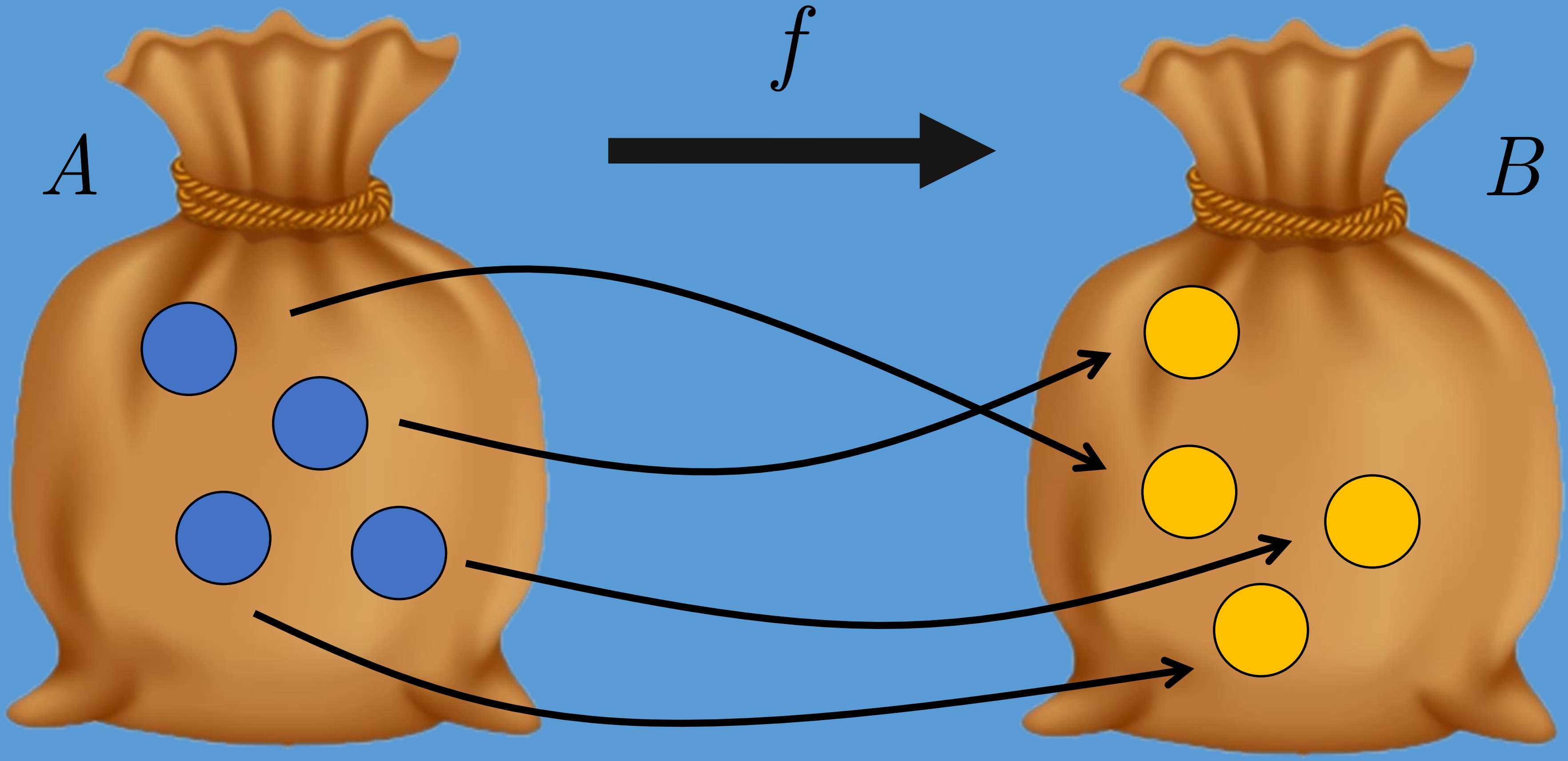
Injective functions



Infinite = not finite

Bijective functions

Bijective functions



Definition

"We say that A is the same size (or cardinal) as B, or that A is equipotent *to B, if and only if there exists a bijective function from A to B, or from B to A, which is equivalent"*

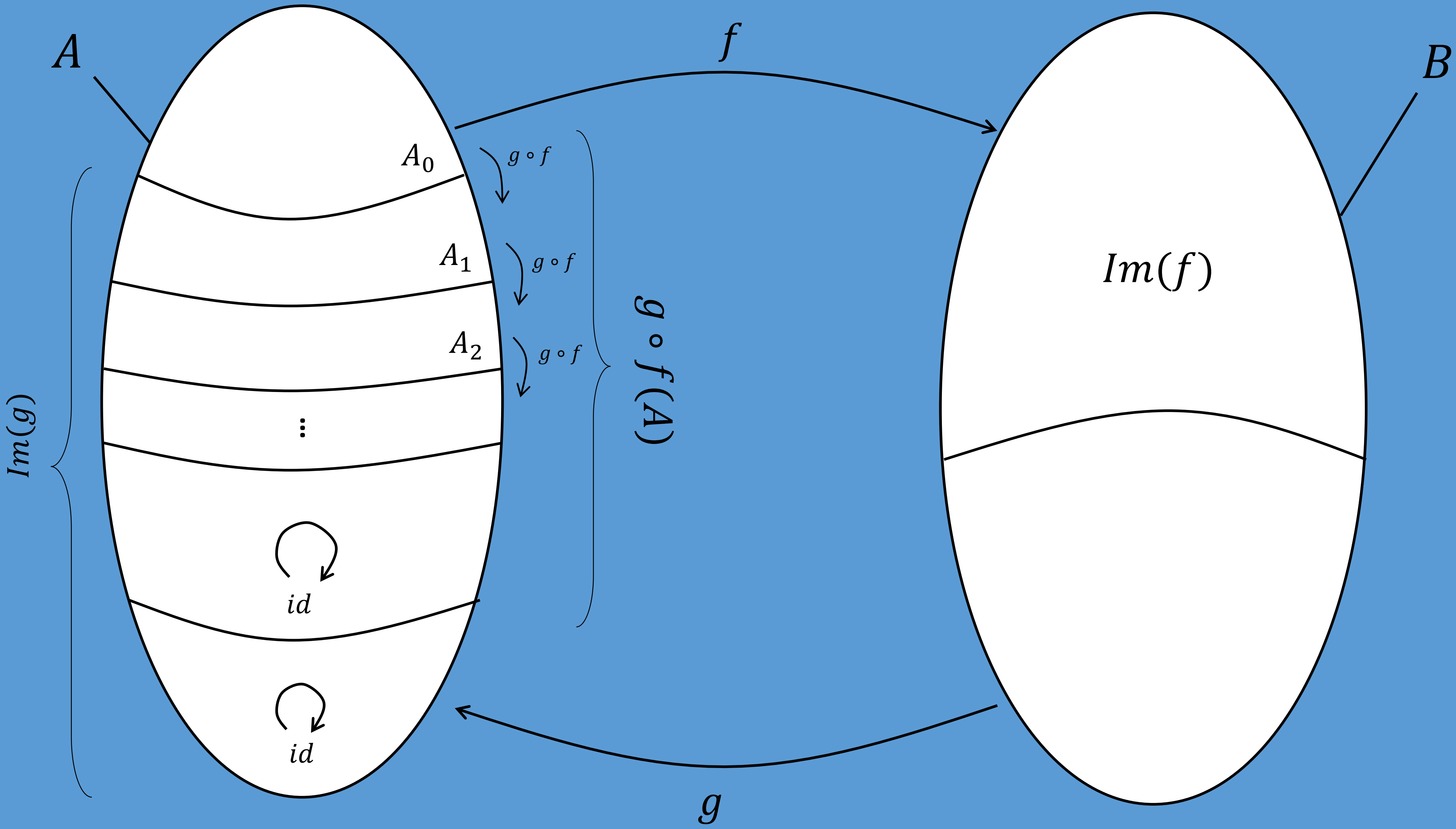
Schröder-Bernstein Theorem

Schröder-Bernstein Theorem

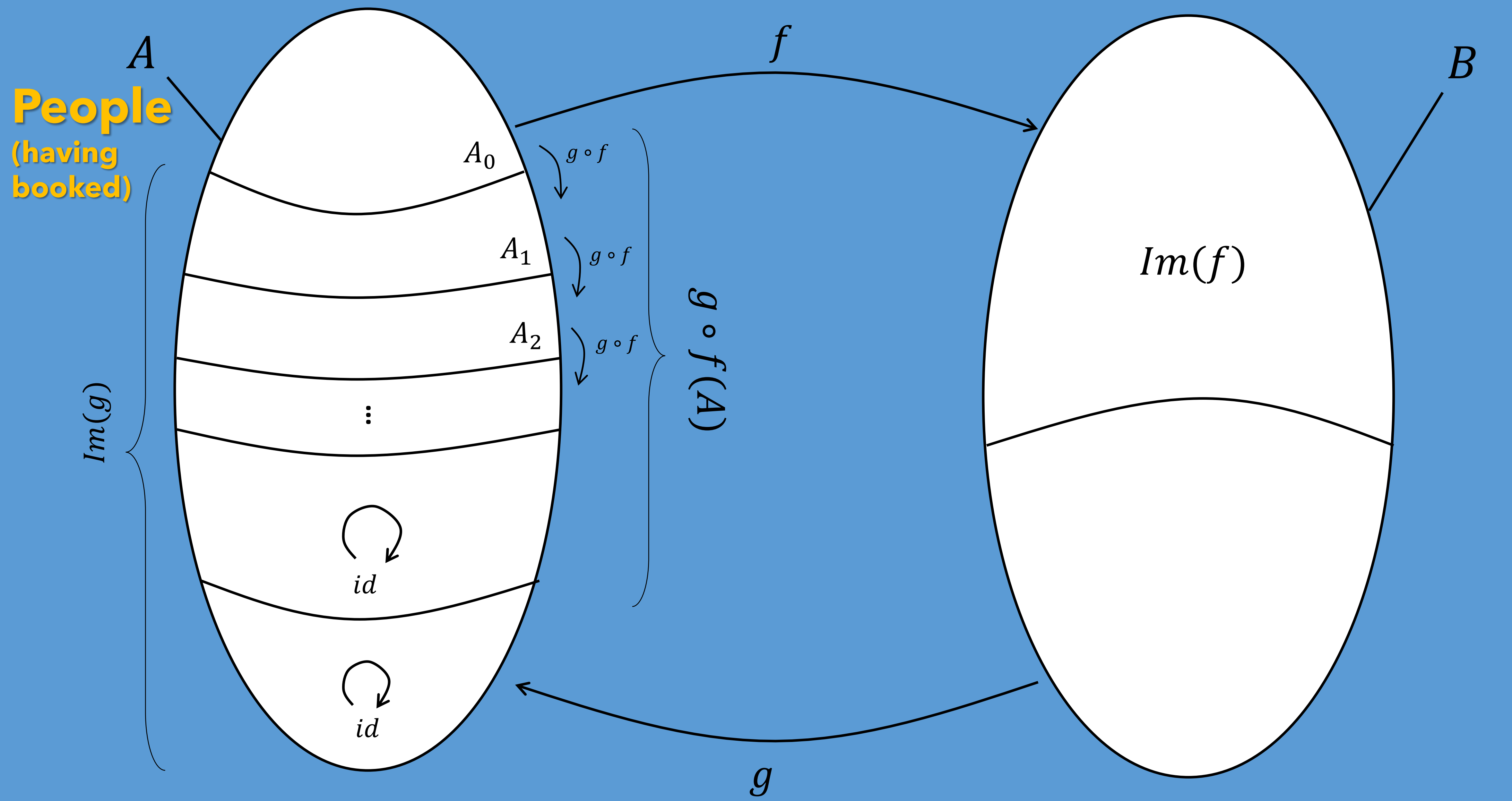
"If A is smaller than B, and B is smaller than A, then A and B are the same size"

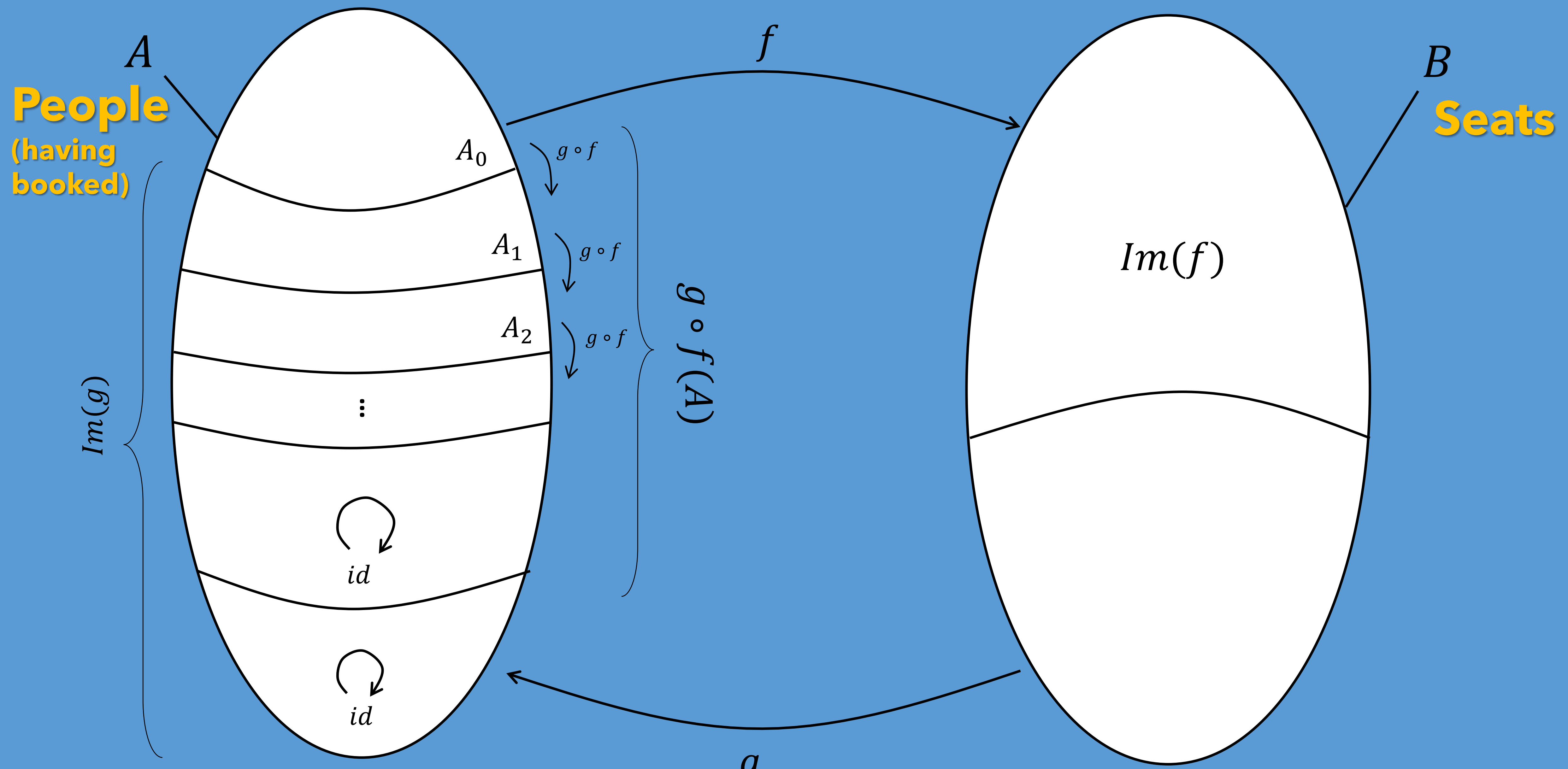
A proof

The movie theater metaphor

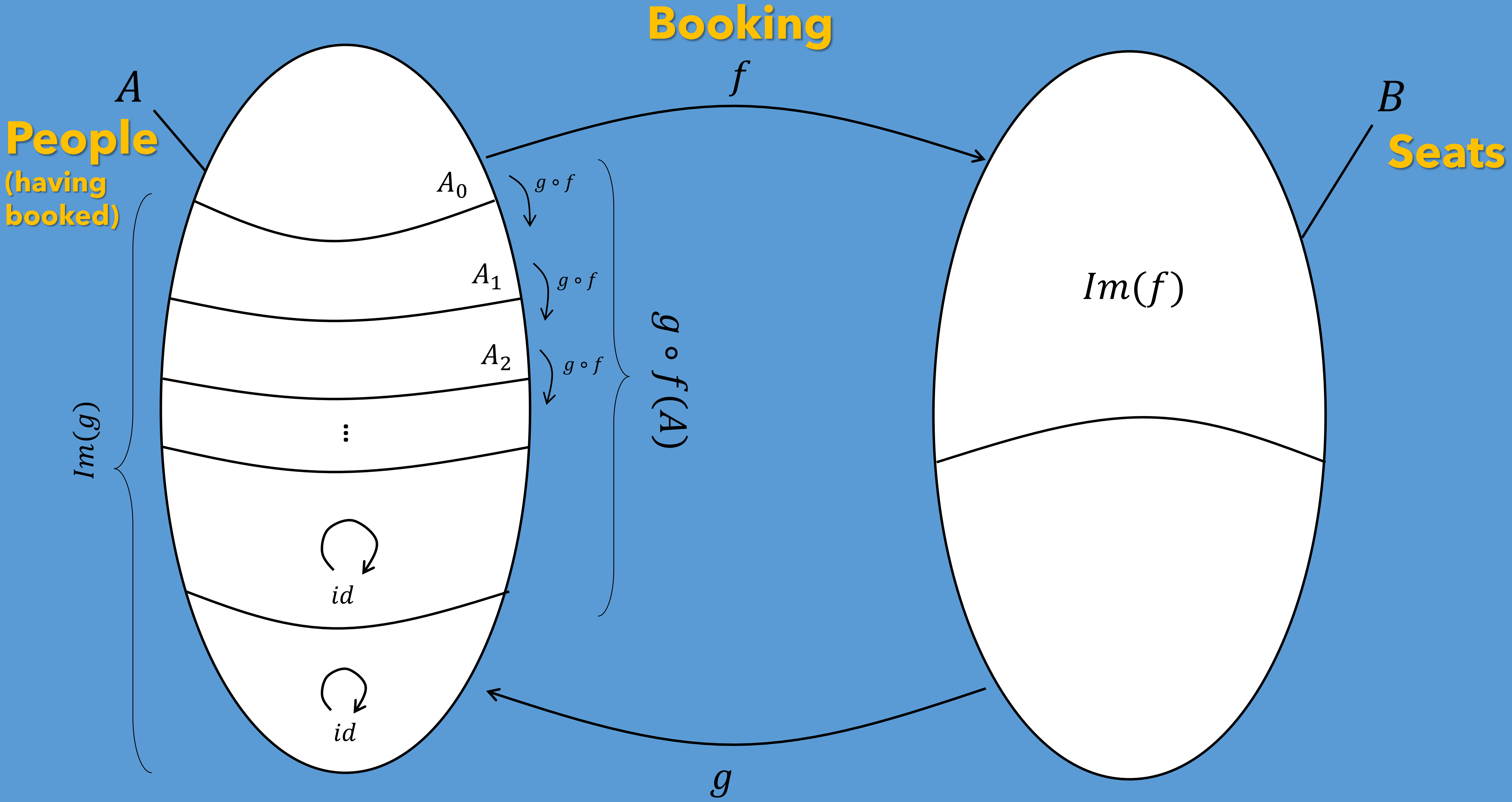


A
B
f
g
A_0
A_1
A_2
$g \circ f$
$g \circ f$
$g \circ f$
$g \circ f(A)$
$Im(g)$
$Im(f)$
id
id

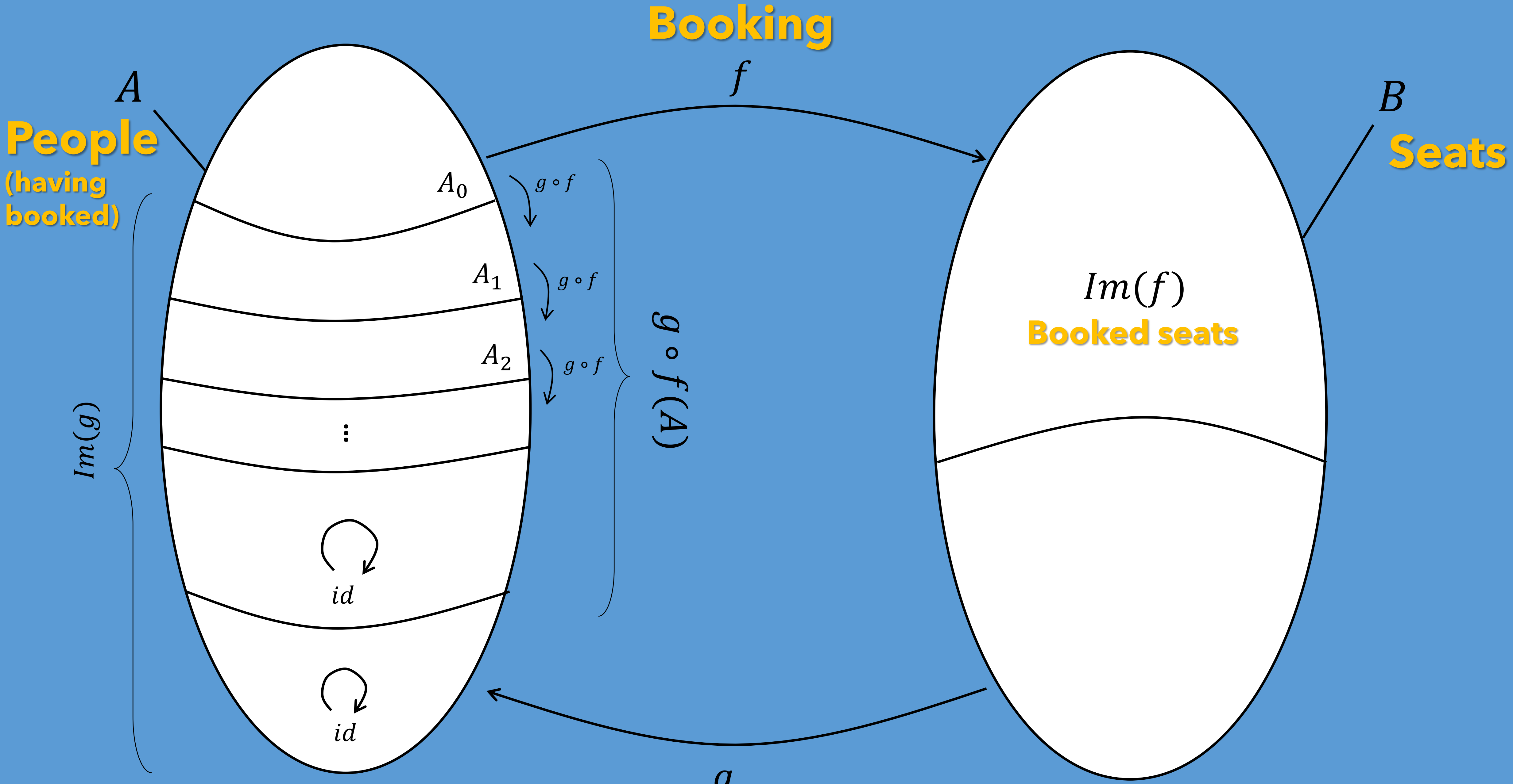

A
People
(having
booked)
A_0
A_1
A_2
$g \circ f$
$g \circ f$
$g \circ f$
$g \circ f(A)$
$Im(g)$
id
id
f
g
B
$Im(f)$



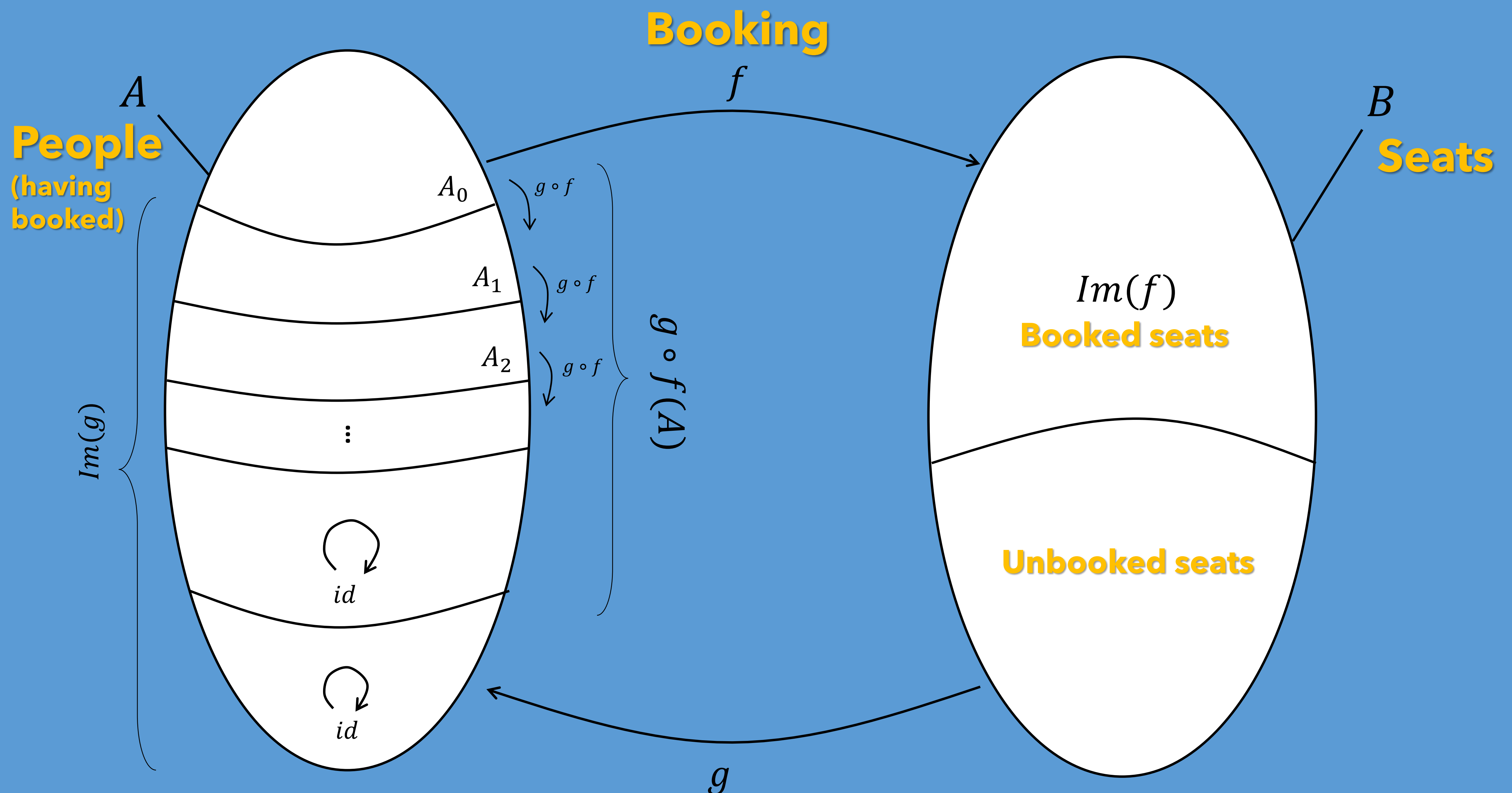
A
People
(having
booked)
f
B
Seats
A_0
$g \circ f$
A_1
$g \circ f$
A_2
$g \circ f$
$\vdots$
$g \circ f(A)$
$Im(f)$
$Im(g)$
id
id
g



Booking
f
A
People
(having
booked)
B
Seats
A_0
A_1
A_2
g ∘ f
g ∘ f
g ∘ f
g ∘ f(A)
Im(g)
Im(f)
id
id
g



Booking
f
A
People
(having booked)
A_0
$g \circ f$
A_1
$g \circ f$
A_2
$g \circ f$
$g \circ f(A)$
$Im(g)$
id
id
B
Seats
$Im(f)$
Booked seats
g



Booking
f
A
People
(having booked)
A_0
$g \circ f$
A_1
$g \circ f$
A_2
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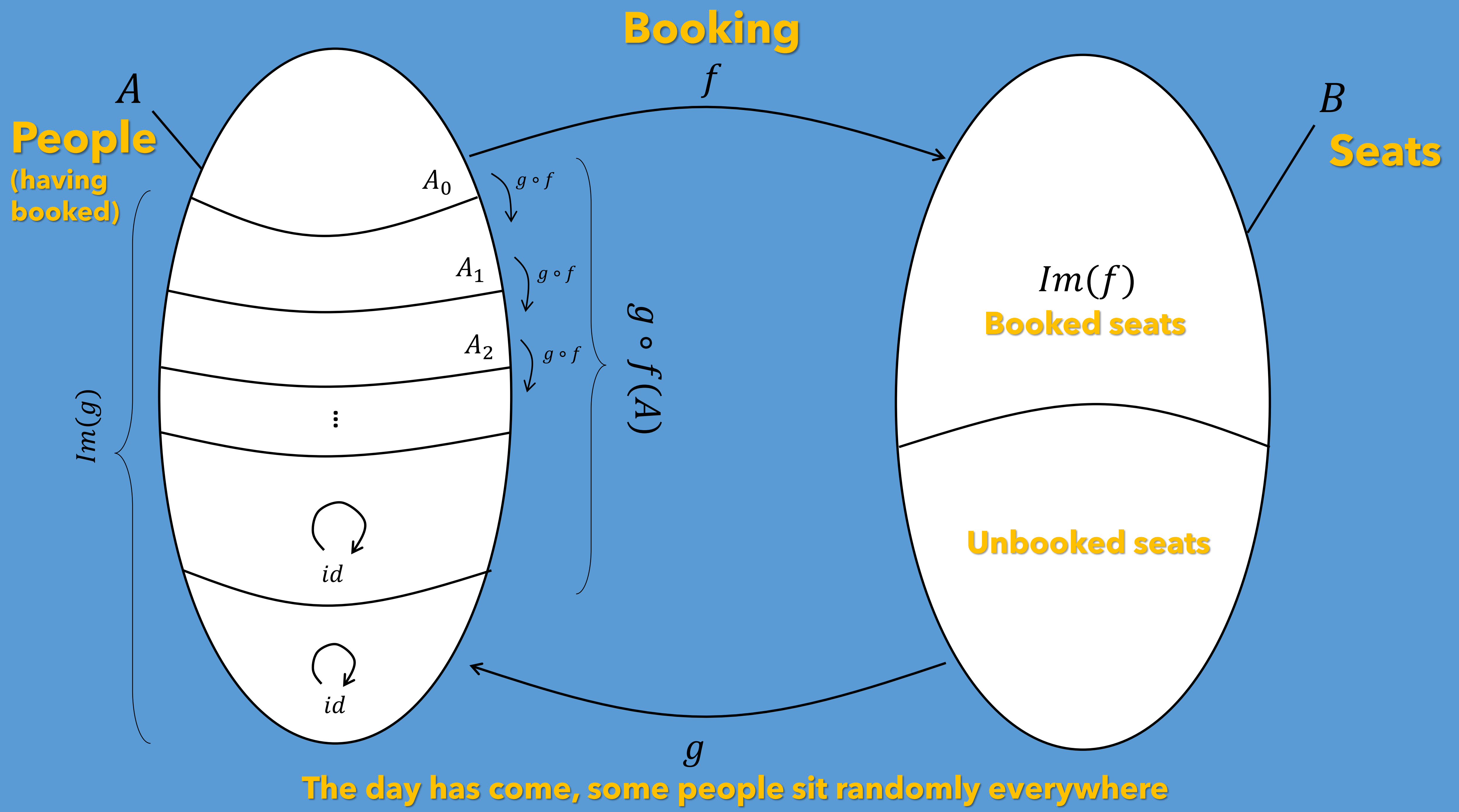
$g \circ f(A)$
$Im(g)$
id
id
B
Seats
$Im(f)$
Booked seats
Unbooked seats
g



Booking
f
A
People
(having
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A_0
$g \circ f$
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Seats
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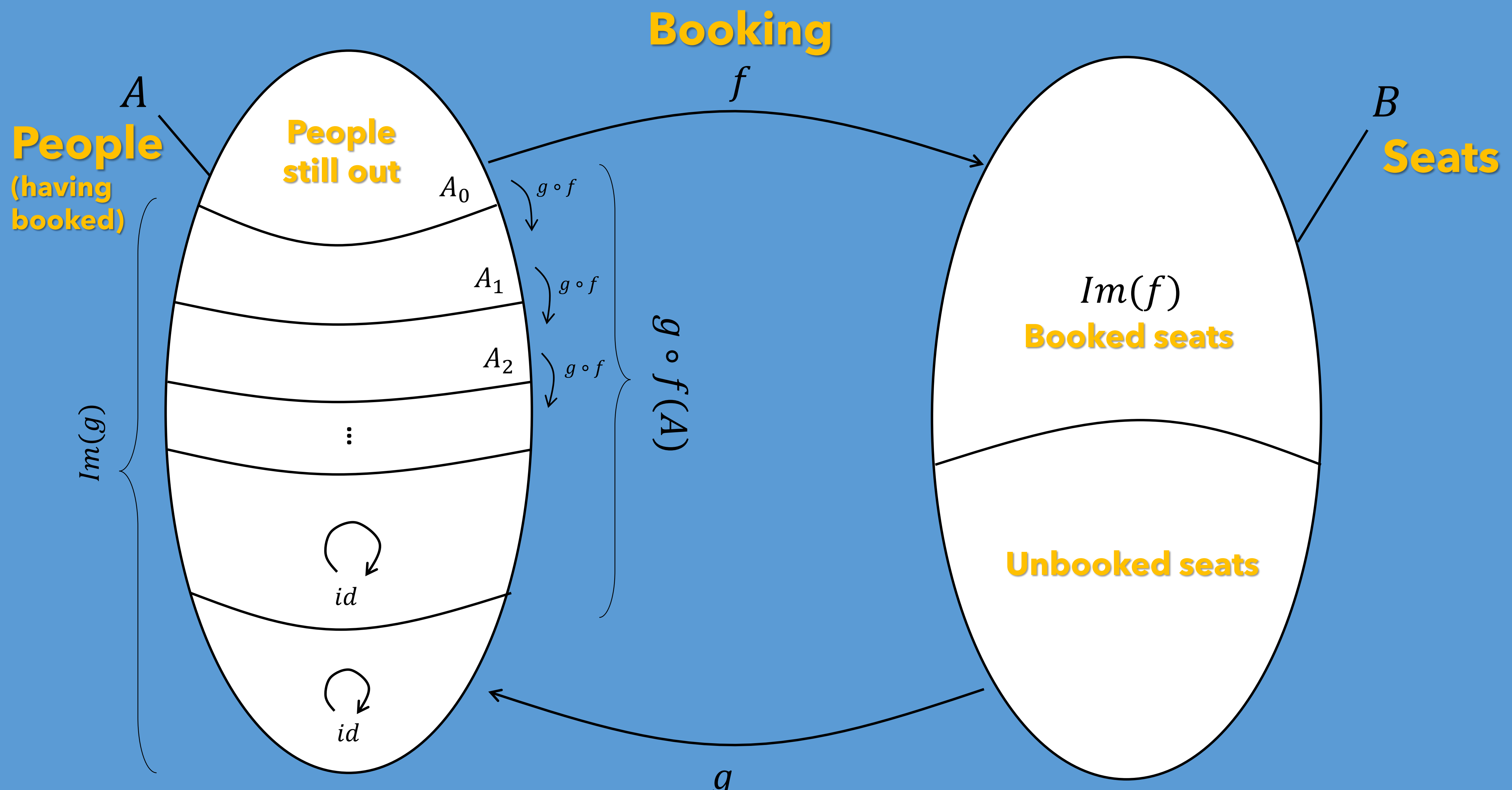
Booked seats
Unbooked seats
g
The day has come, some people sit randomly everywhere



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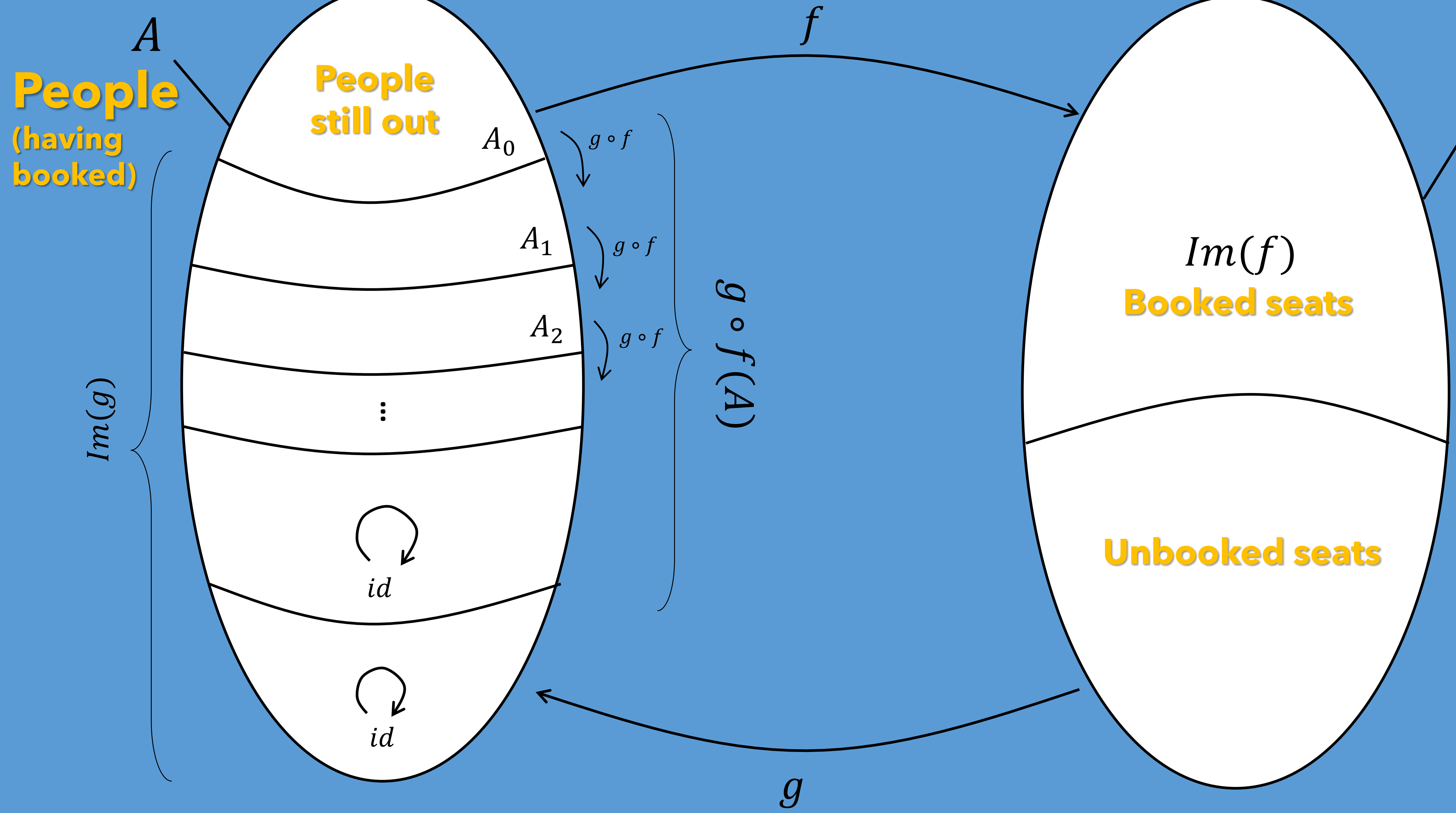
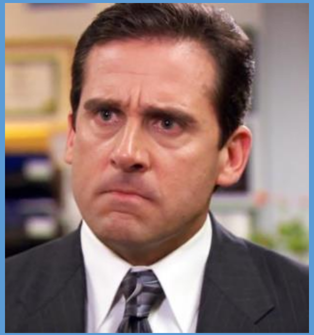
Booking





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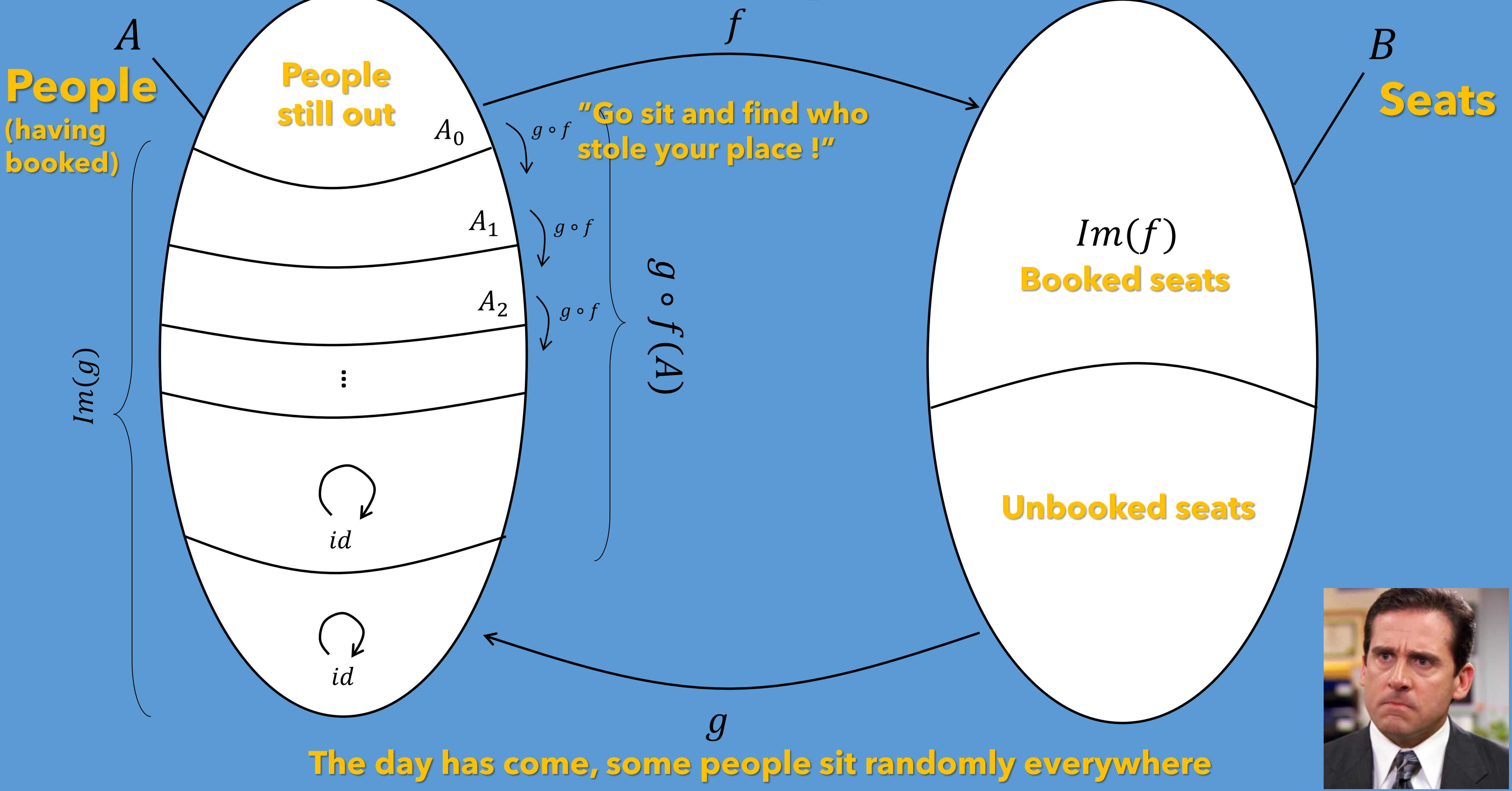

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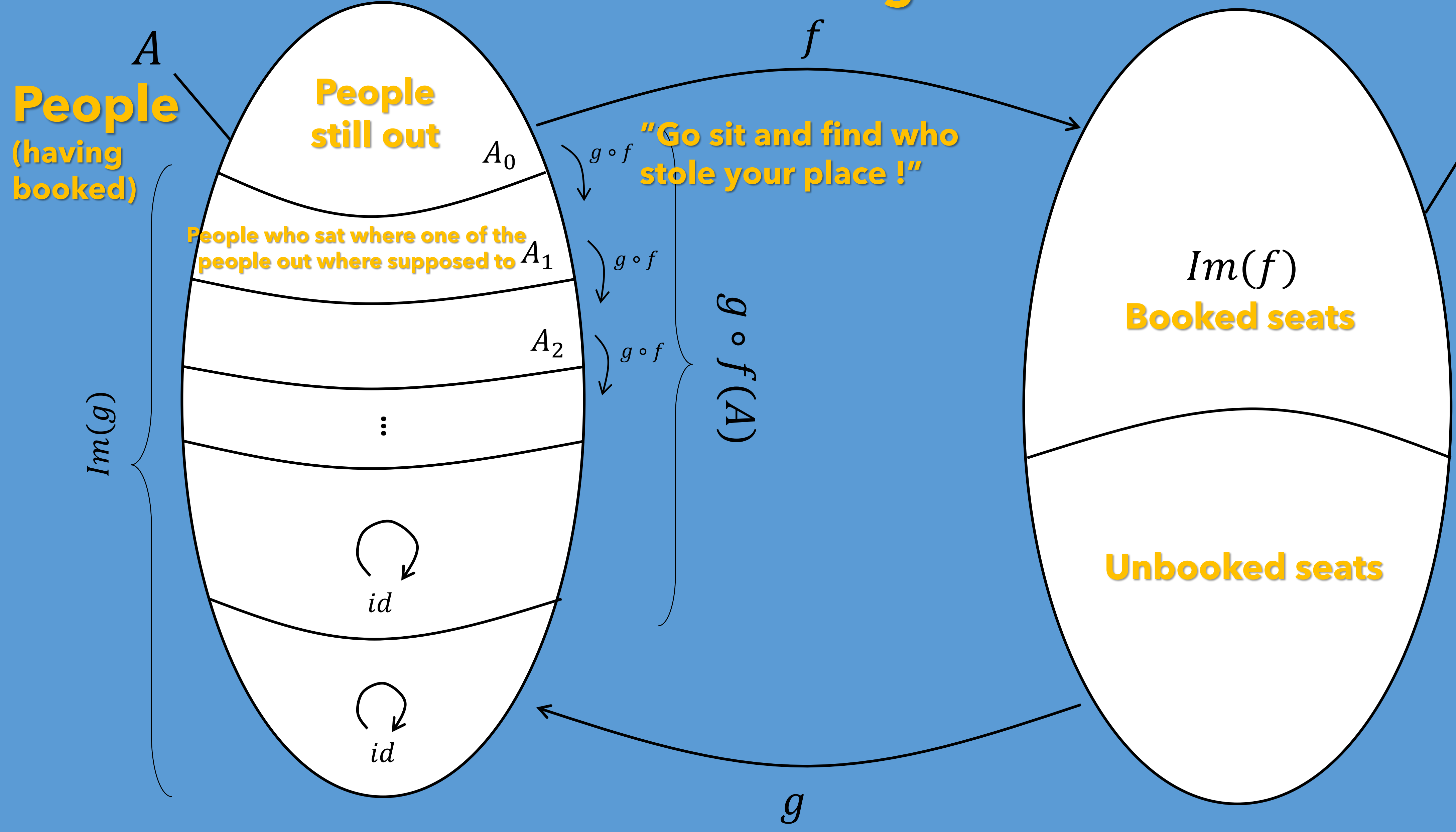

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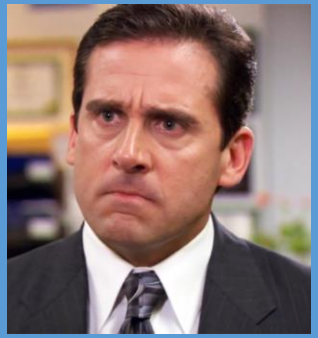
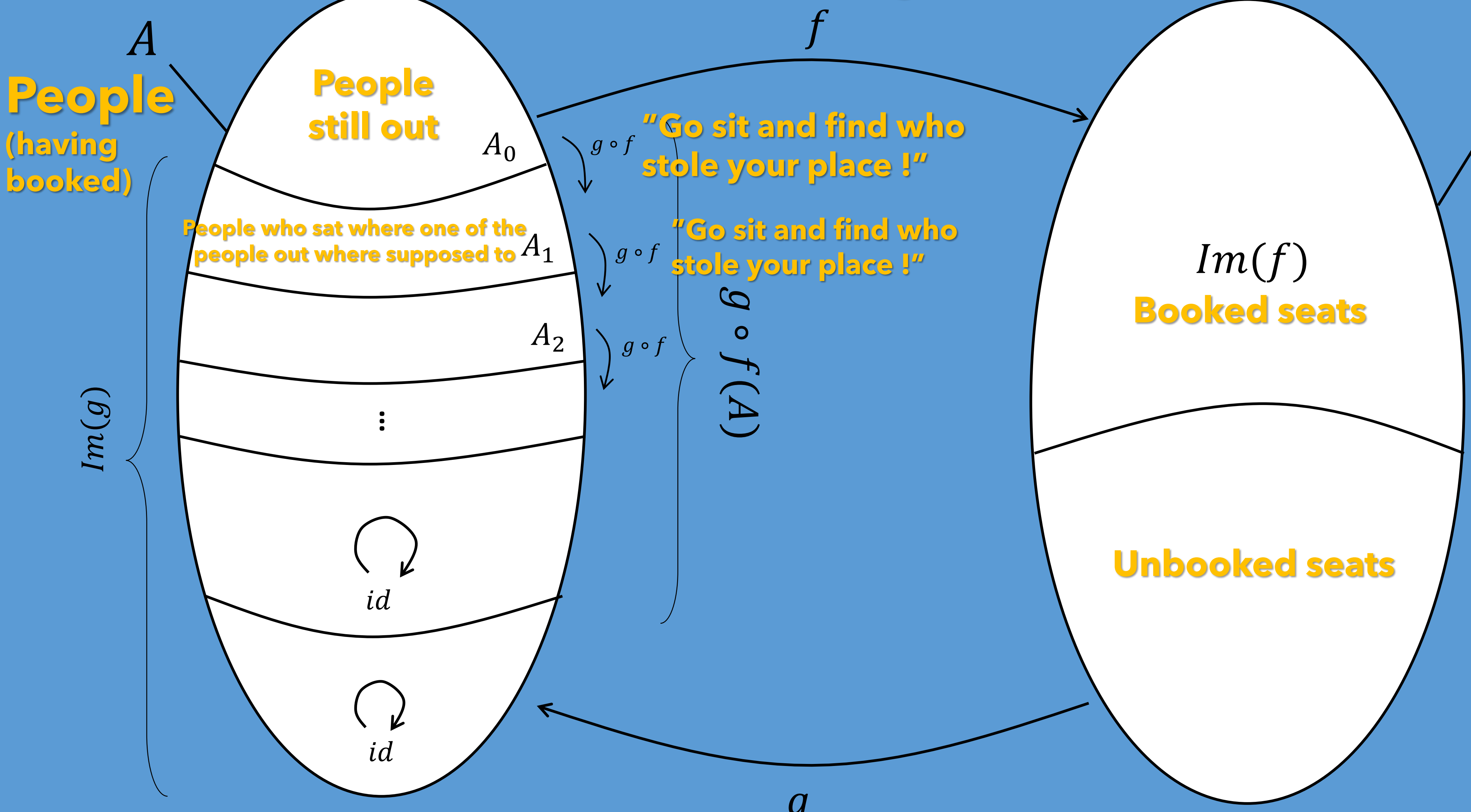
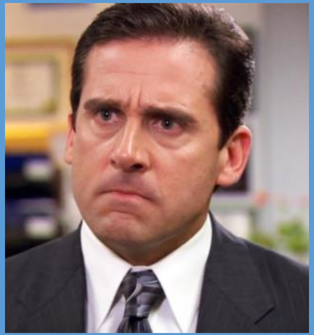


Booking





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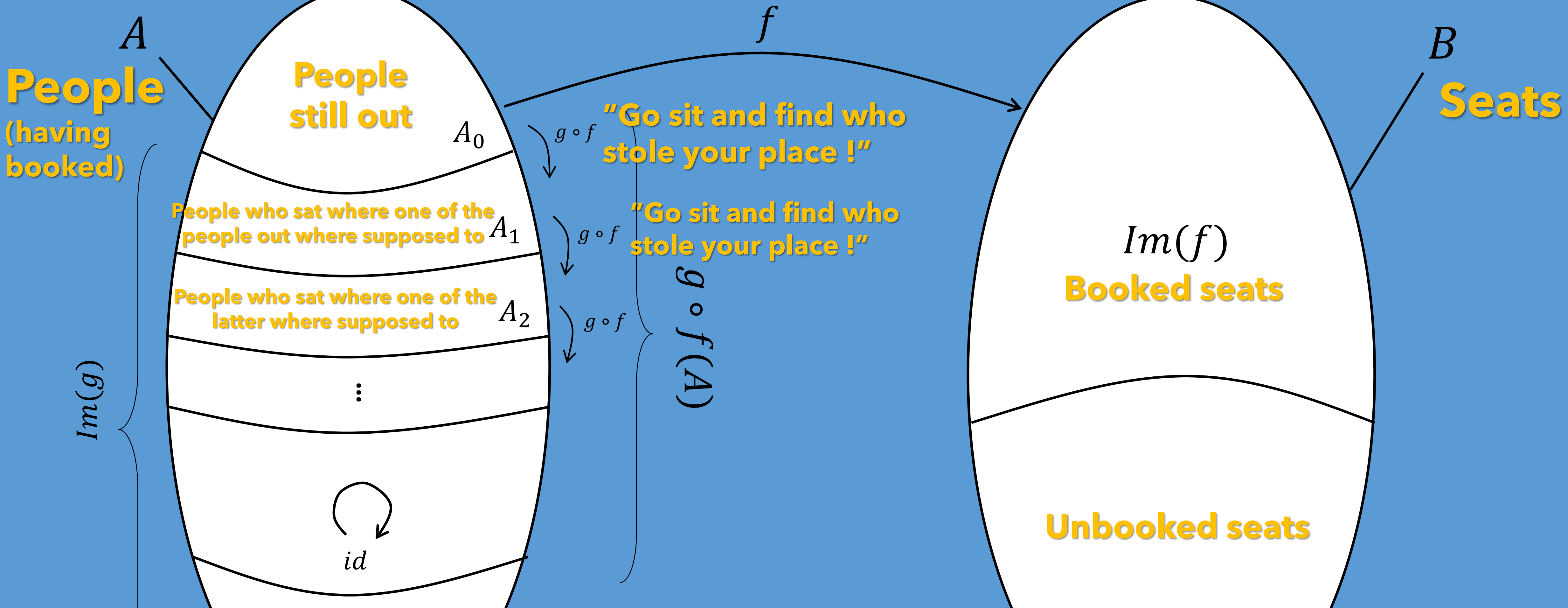

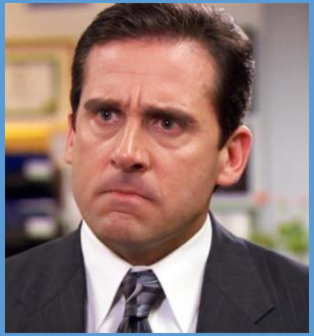

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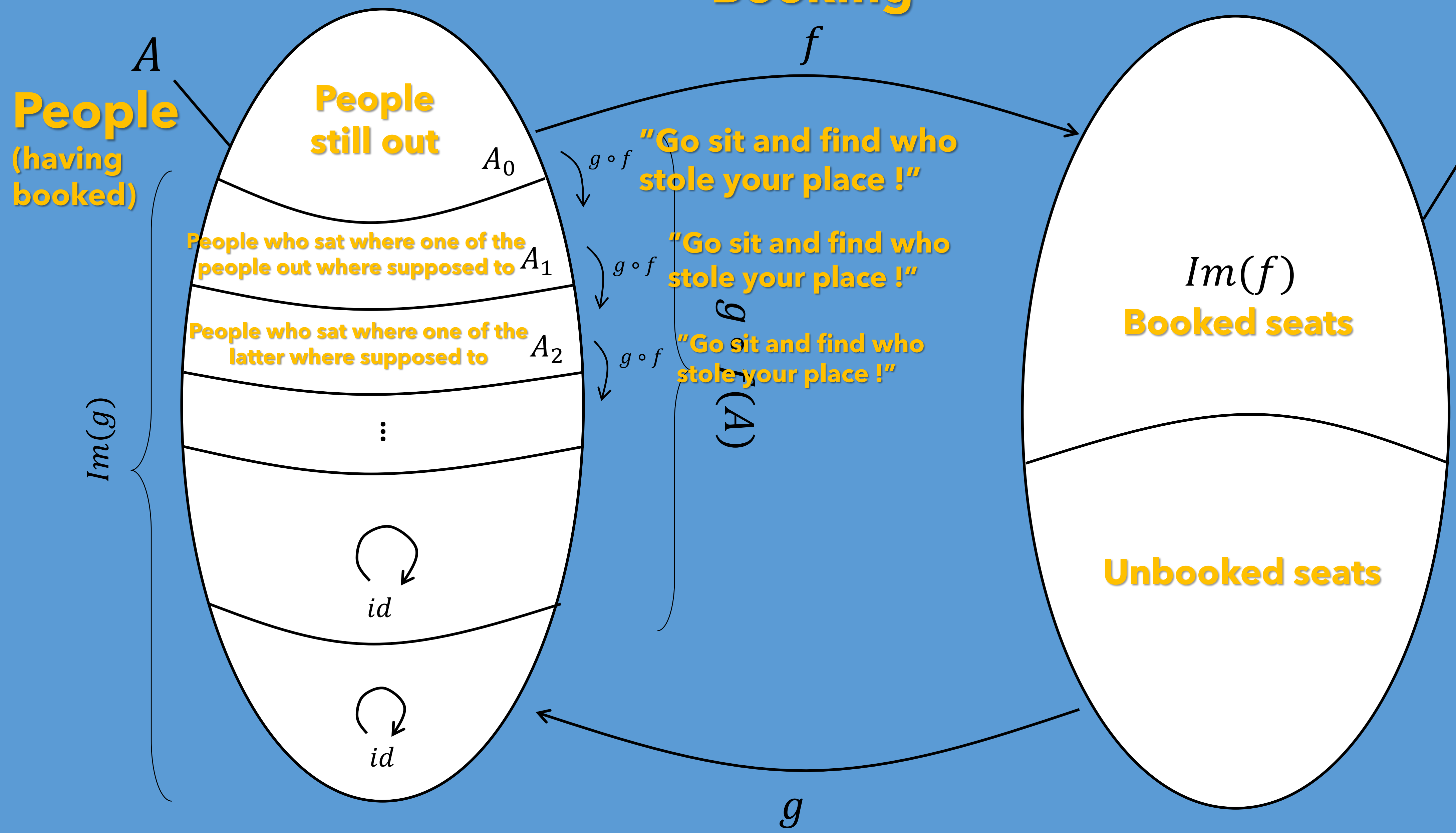

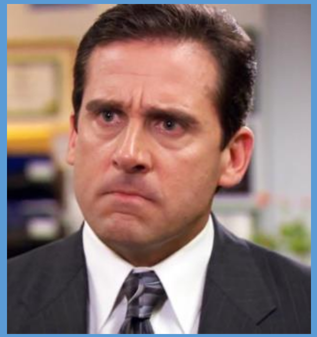

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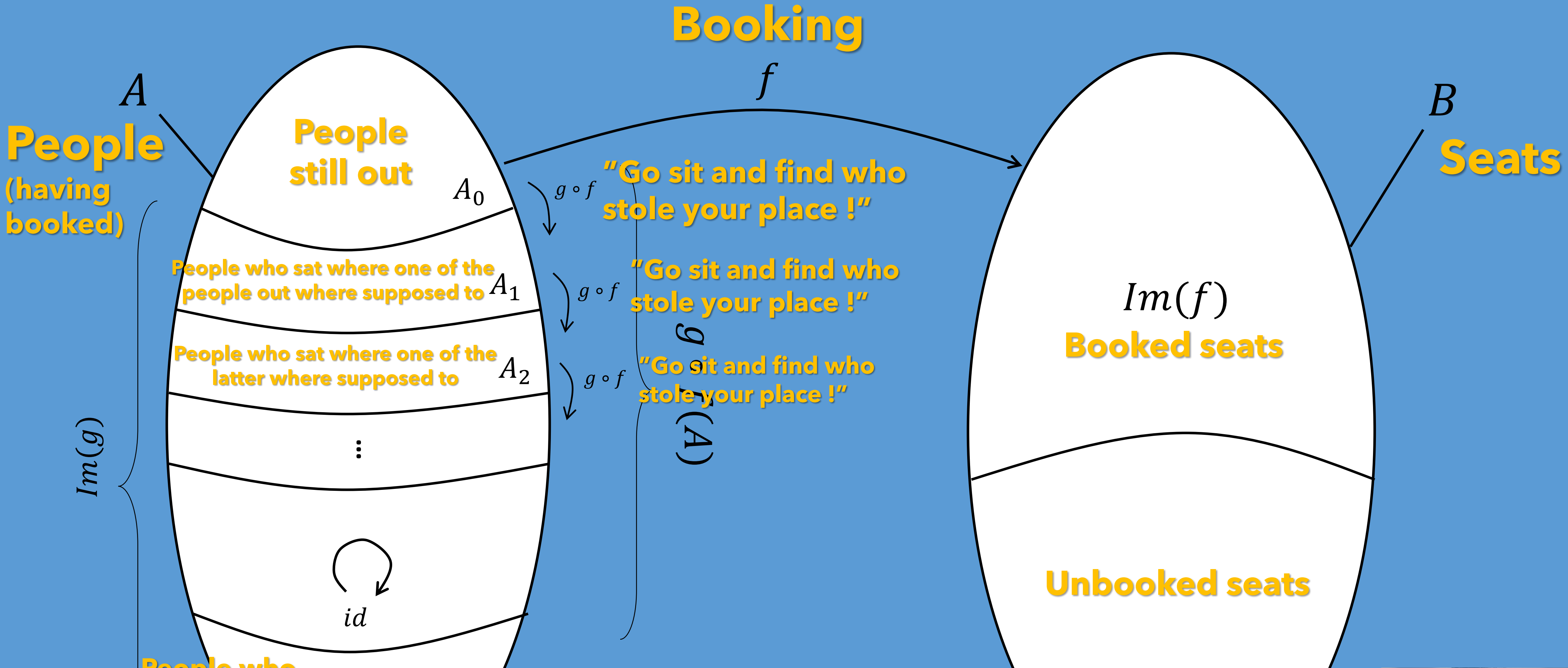

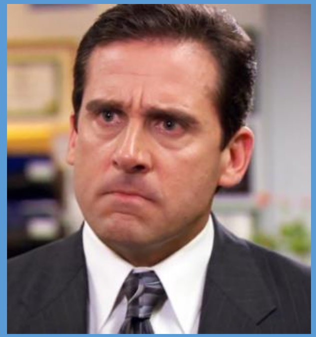

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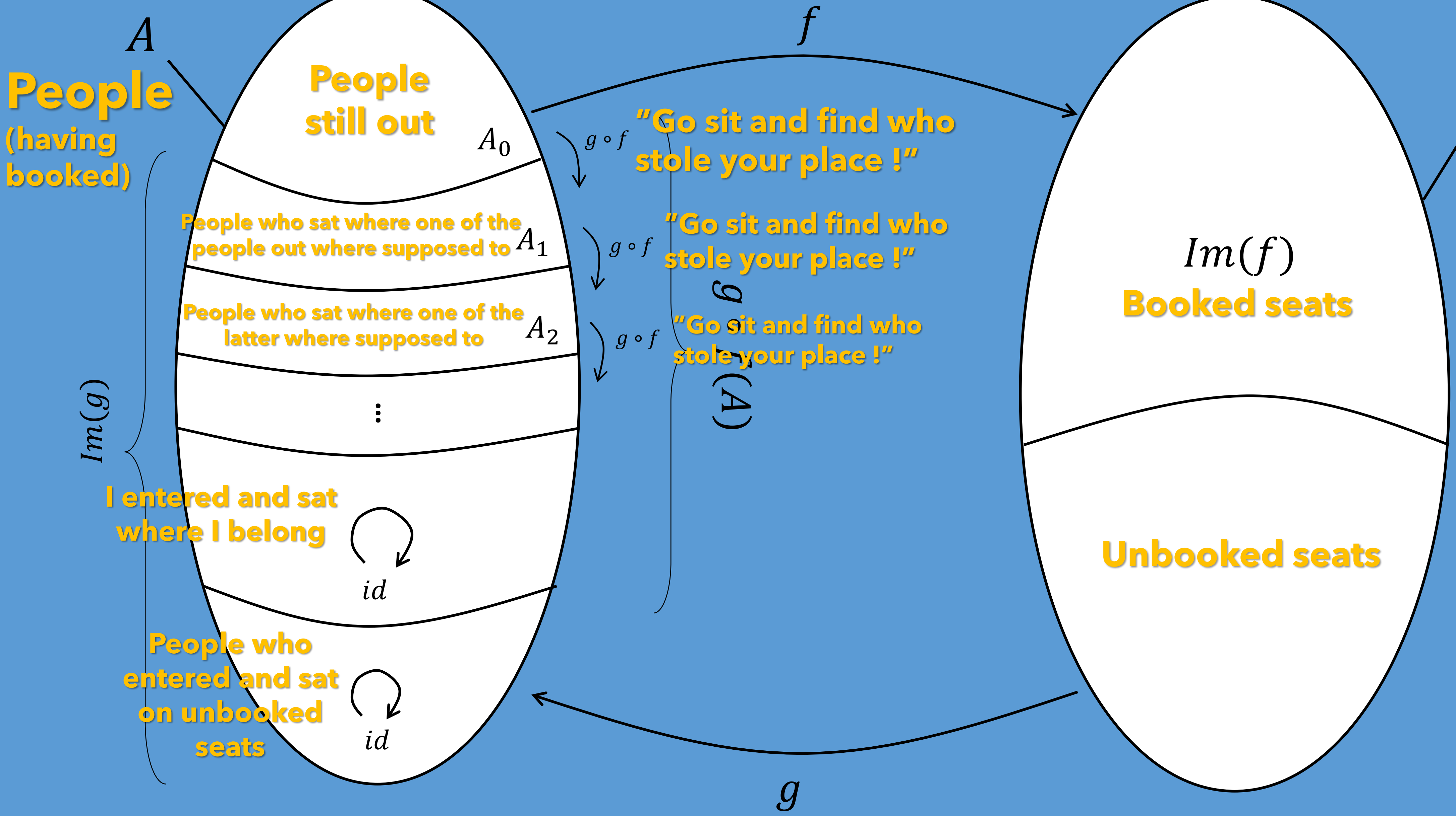

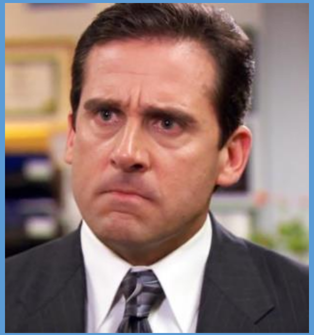

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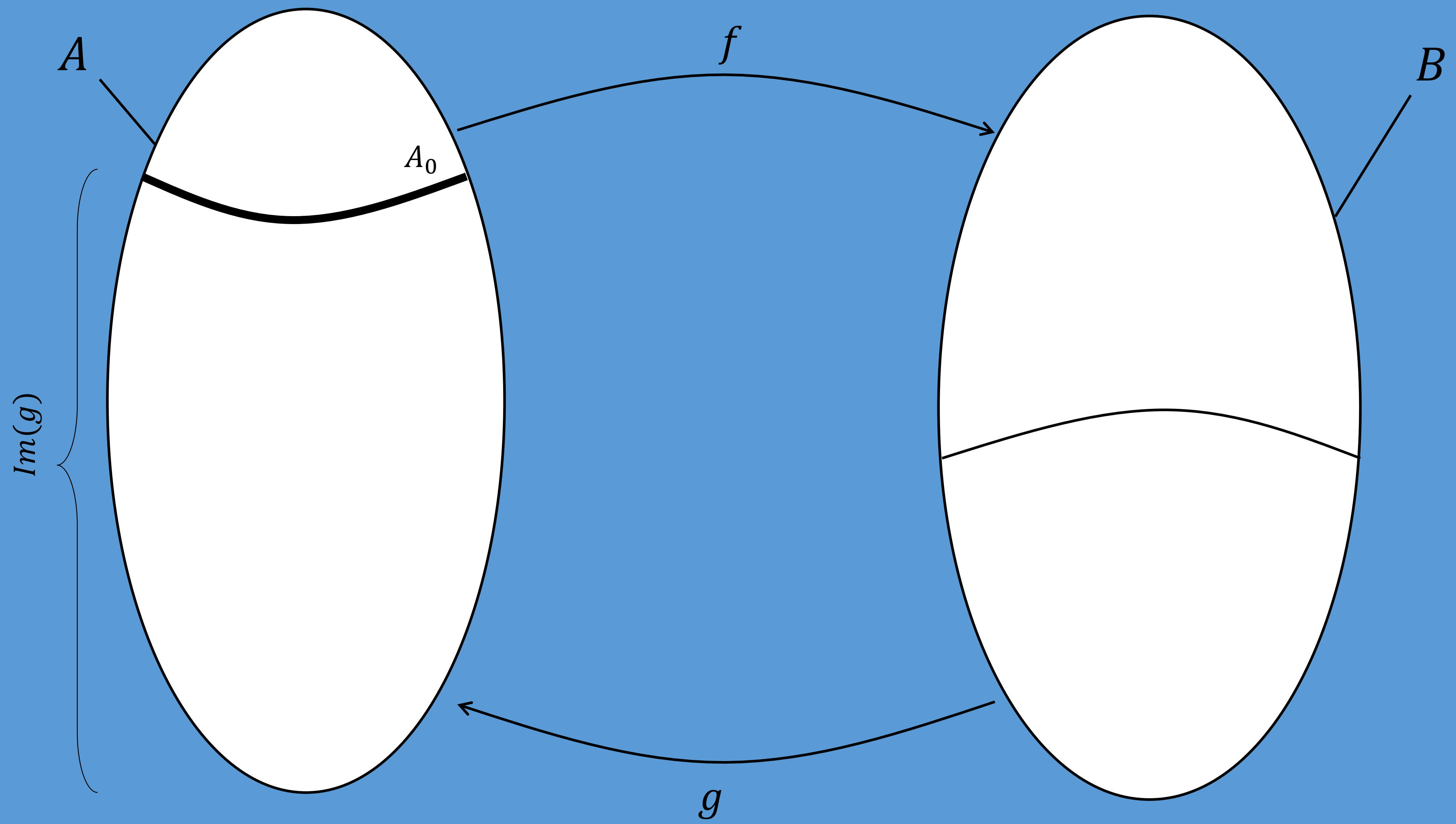
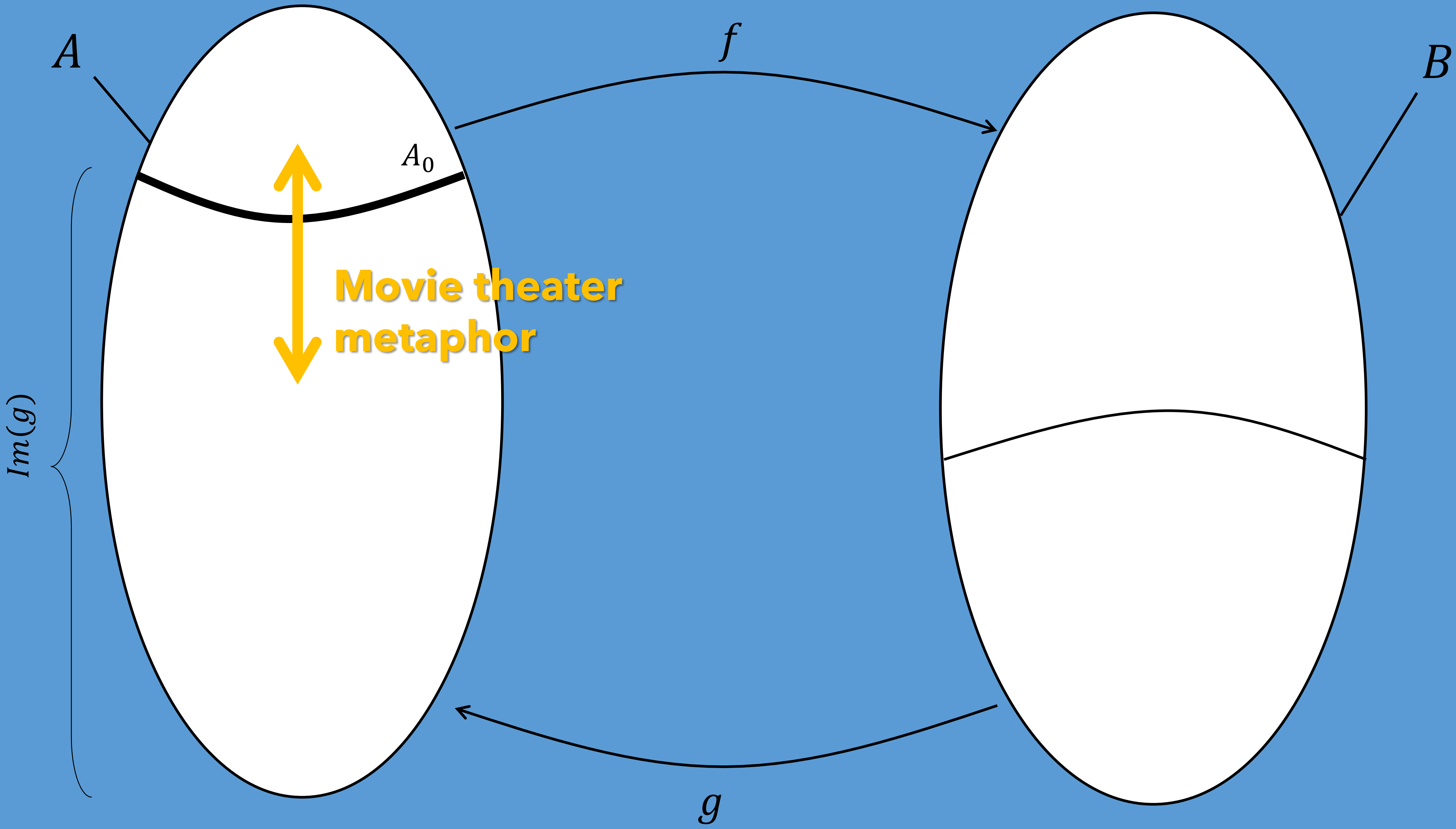
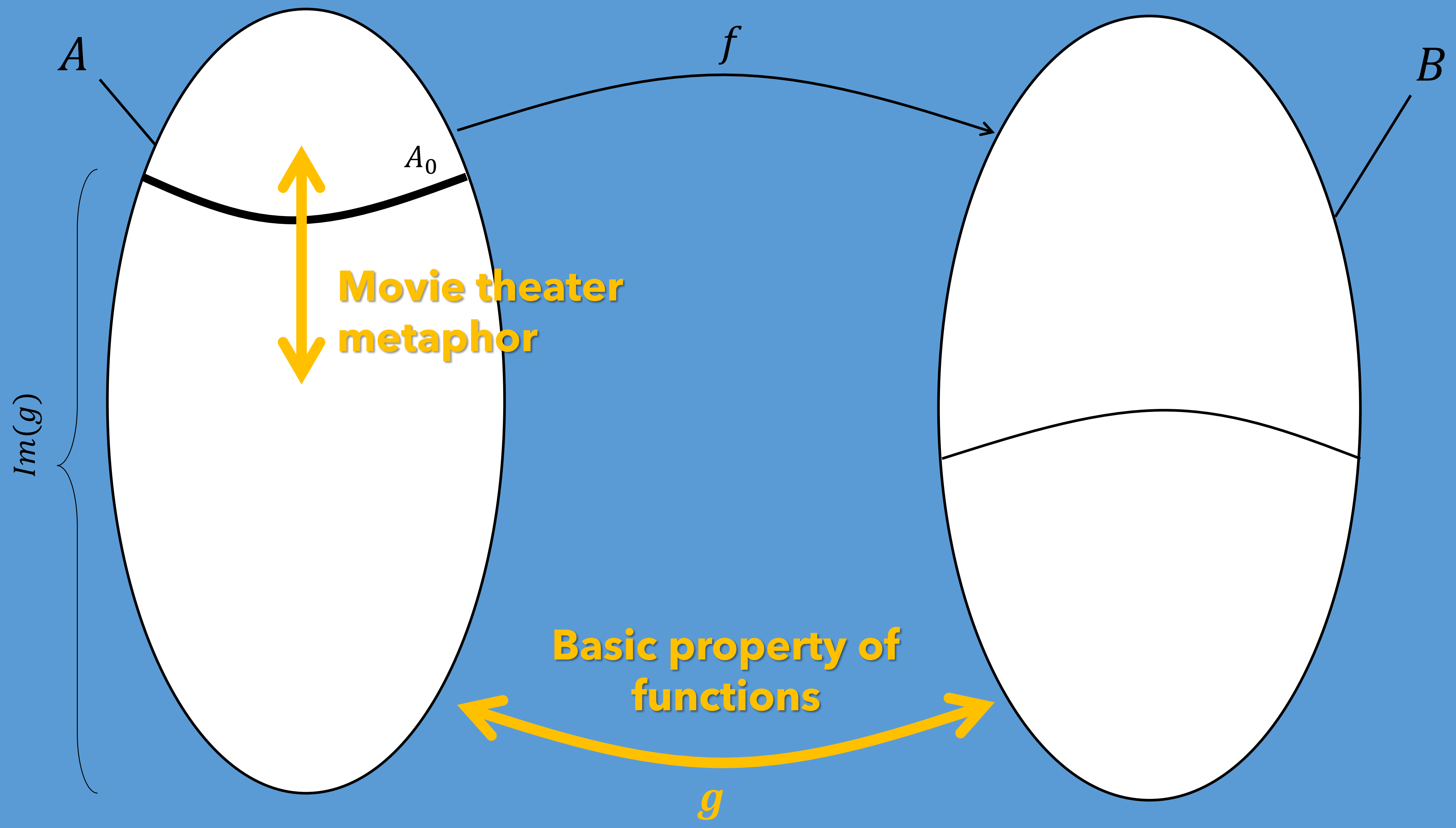


A
A_0
f
B
$Im(g)$
g



A
f
B
A_0
Movie theater metaphor
$Im(g)$
g



A
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Movie theater metaphor
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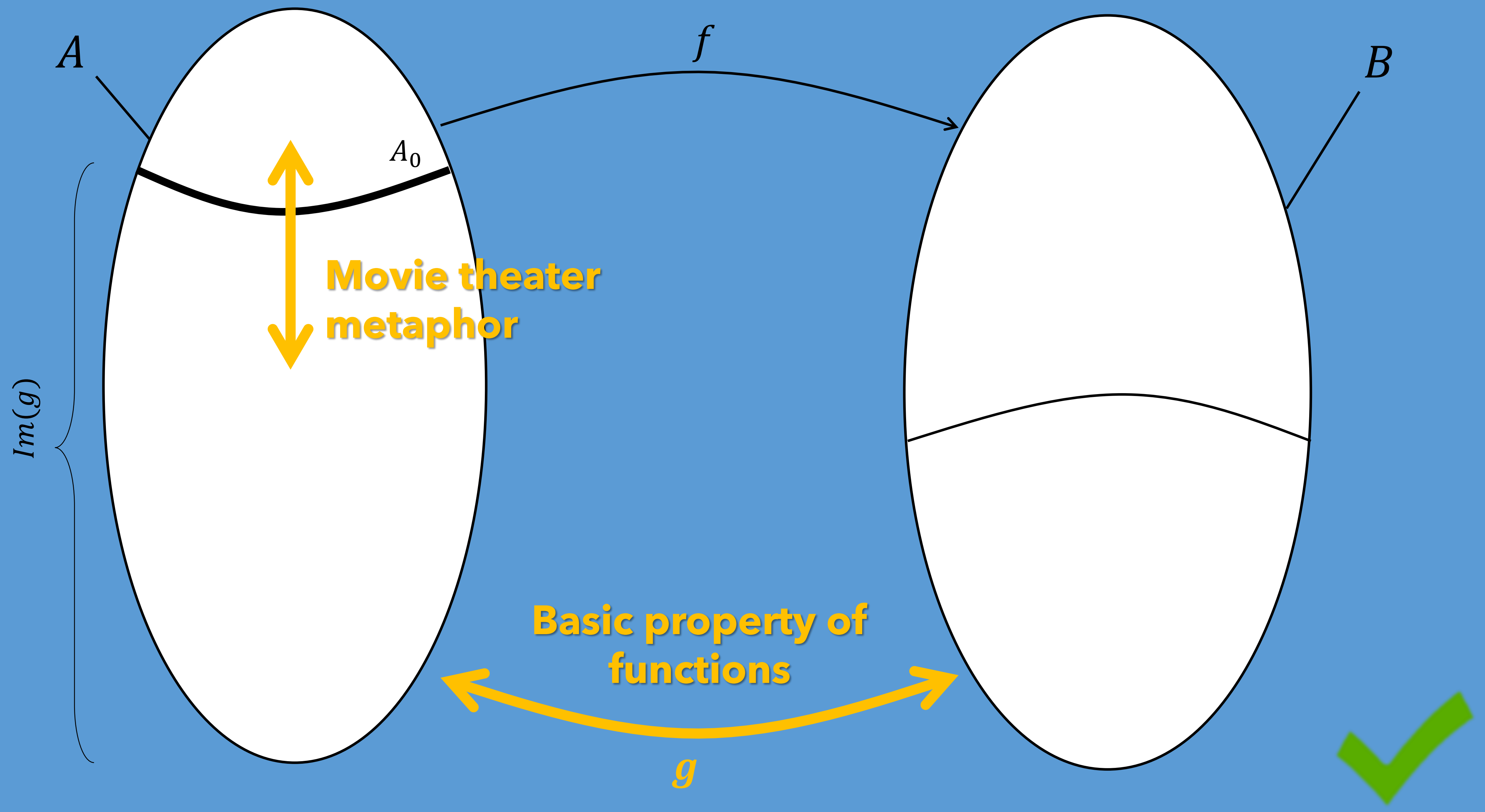
Basic property of functions
g



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Movie theater metaphor
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A conclusion

A conclusion

- Why is it important?

A conclusion

- ❑ Why is it important?
- ❑ An **intuitive property** whose proof is **not trivial**

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- ❑ Generalisation

A conclusion

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- Generalisation