3D4 Elastic and Inelastic Buckling Phenomena Lab Report

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Summary

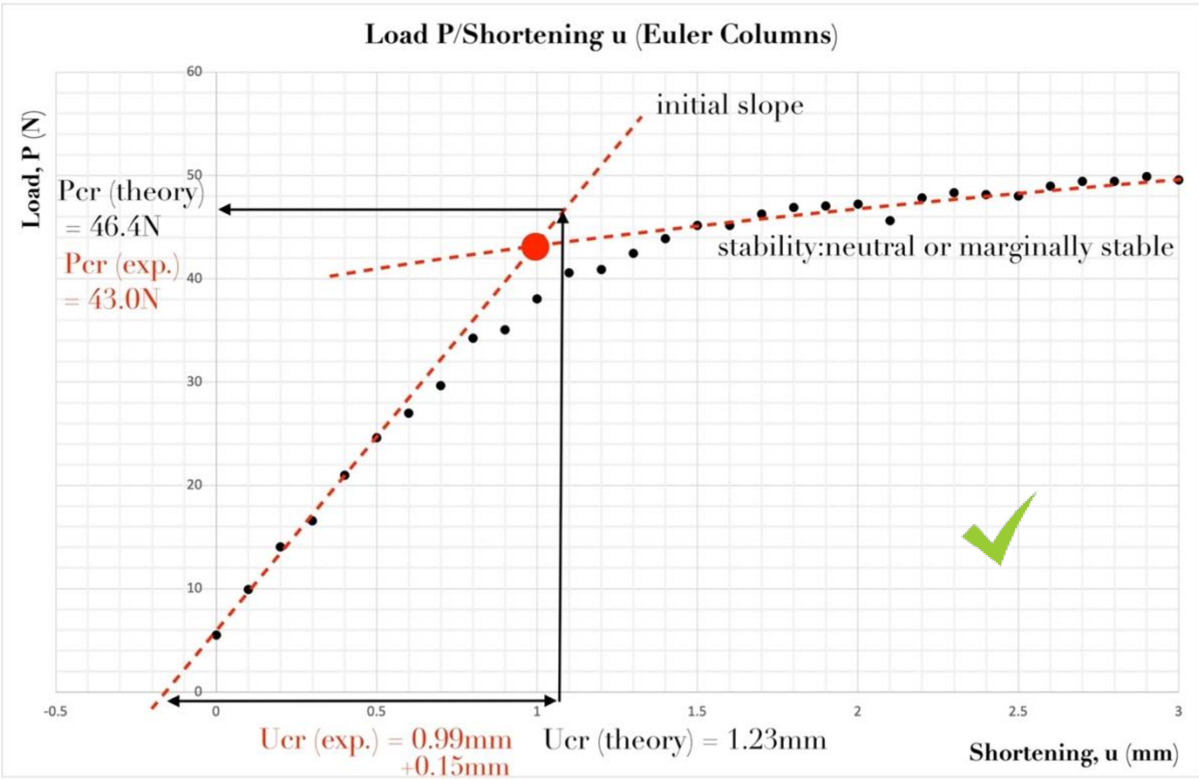
The first part (elastic buckling) of the lab is to perform axial compression tests on silicone rubber specimens of different geometries (Euler columns, box, and cylindrical shell). By plotting their load/shortening length curve, we find the predicted (by the classical buckling theory) and observed values of P_{cr}(axial compressive load at buckling) & u_{cr}(overall shortening at buckling).

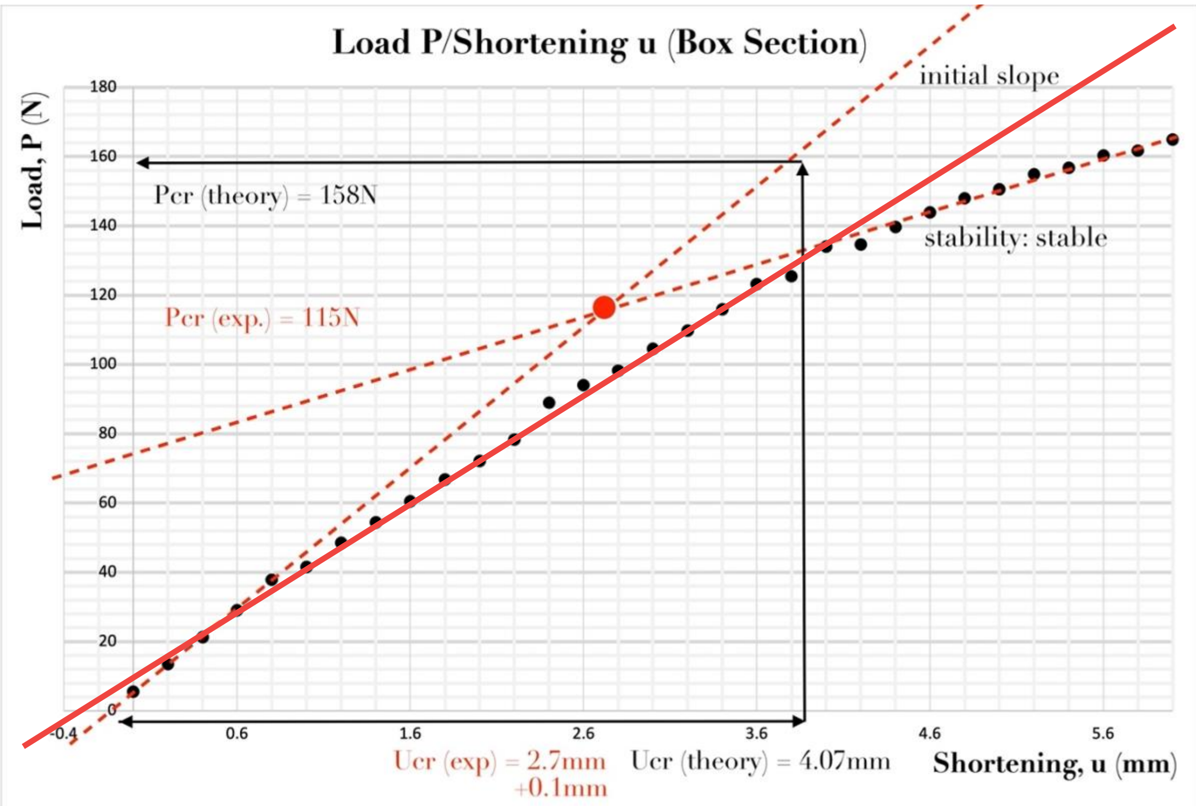
The second part (inelastic buckling) of the lab is to perform axial compression tests on pin-ended aluminium columns of different slenderness. And based on the measured buckling stress of the specimens, we investigate the effectiveness of "tangent modulus formula" in predicting inelastic buckling stress.

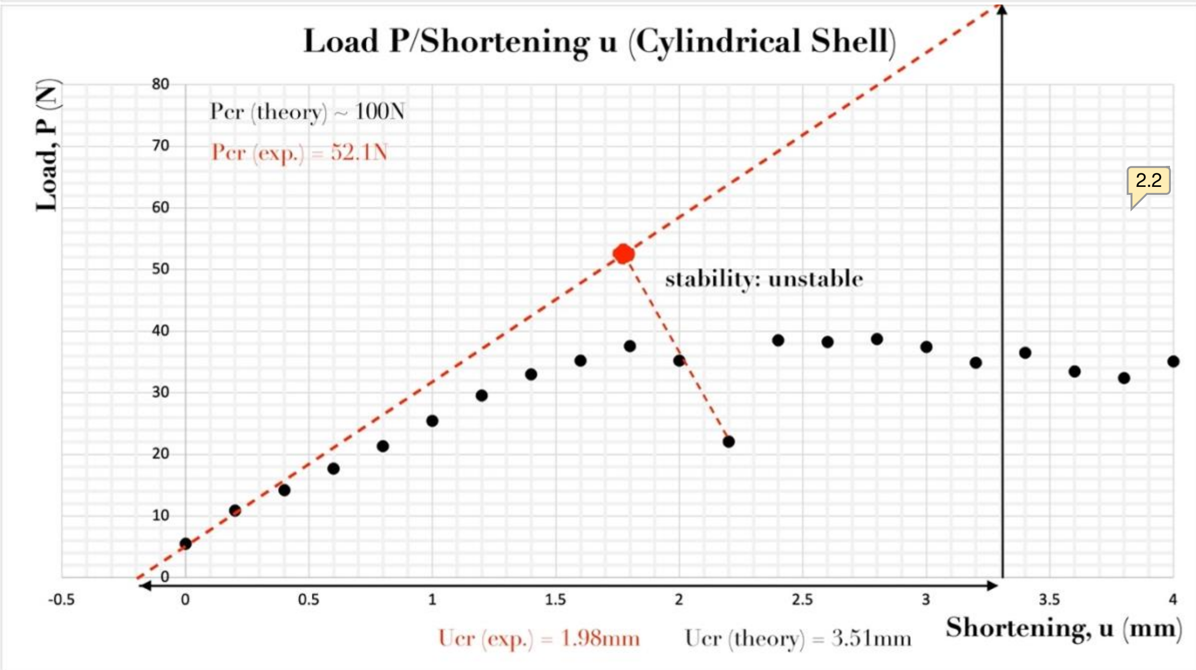
Results and Discussions

Specimen	u_{cr}(theory), mm	P_{cr}(theory), N	u_{cr}(experiment), mm	P_{cr}(experiment), N	Stability
Euler columns	1.23	46.4	1.14	43	~neutral
box	4.07	158.0	2.8	115	stable
cylindrical shell	3.51	~100	1.98	52.1	unstable

Table 1. Readings of axial compression test for the first part (elastic buckling).







Graph 1,2,3. Load/Shortening curves for three specimens of different geometries.

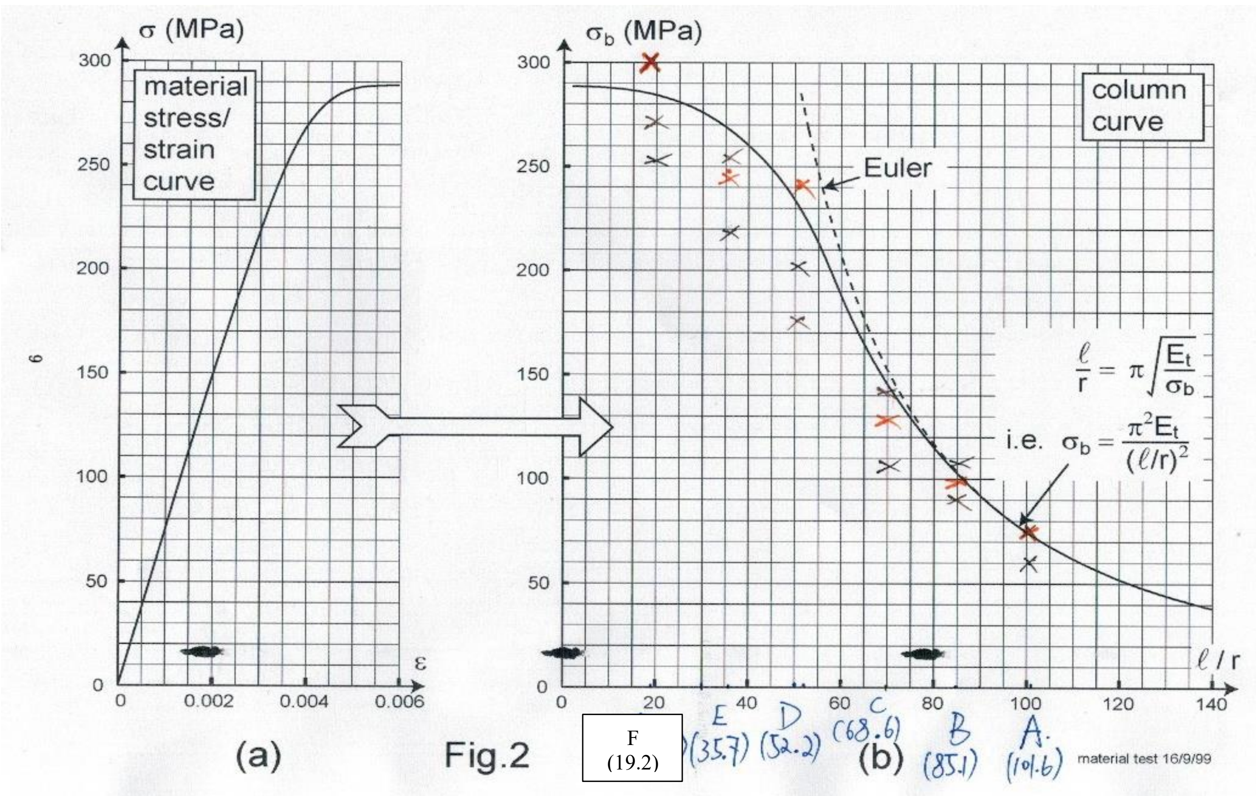
- Among load – deflection curves of all three geometries of specimen, "Euler columns" most closely resembles the pattern of handout Fig. 1(a). (i.e., Before buckling takes place, as the load increases, the curve is a straight slope (primary equilibrium path); after specimen buckles, the curve becomes an almost horizontal straight line (post buckling path))
  The slope of post buckling path for box and cylindrical shell specimens are largely positive (box) and negative (cylindrical shell).

  Unlike the pattern of handout Fig. 1(a), the pre-buckling (primary equilibrium) paths and post buckling paths of our specimens **do not** intersect sharply and form slope discontinuities, instead paths curve and result in smooth intersections at critical loads.

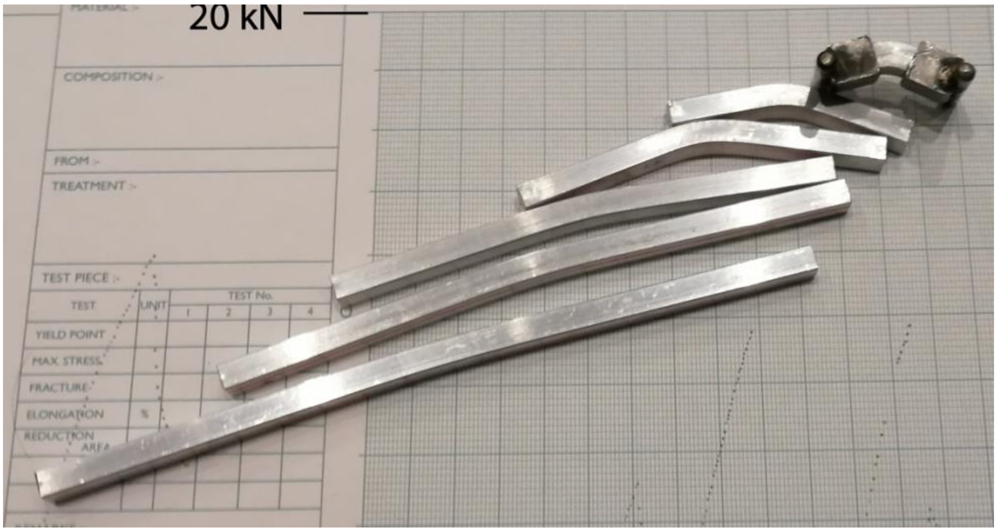
- In the stable buckling case (especially box specimens), at critical load, if a small increase in shortening length u is applied, the structure will resist the disturbance in length and return to u_{cr}. (as indicated by the largely positive slope of the post buckling path)

In the unstable buckling case (cylindrical shell), critical load capacity is sensitive to small initial structural imperfections, and instantaneous snap-through buckling often happen (i.e., at the critical load, small increase in load causes jump in shortening and deformations). Therefore, in terms of designs, a stable post-buckling path is desirable and when designing shell structures, elastic critical loads cannot be trusted.

- For Euler columns: P_{cr} (theory) is slightly less than P_{cr} (exp.).
  For box specimens: P_{cr} (theory) is clearly less than P_{cr} (exp.).
  For cylindrical shell: P_{cr} (theory) is significantly less than P_{cr} (exp.), only about half of the value predicated by the classical theory.
  Therefore, in terms of designs, critical load values predicted by the classical theory can give dangerous overestimations of the strength of structures.



Graph 4. Measured σ_b plotted on column σ_b,$\frac{l}{r}$ curve.
(different batches of Al columns are marked by different colour)



Graph 5. Square bar specimens after unloading and some elastic rebound.

<u>Observation from Graph 4, 5:</u>

- The measured buckling stress in general fit the shape of "tangent modulus formula" predictions on Fig. 2(b), although there are discrepancies.
- Aluminium columns from different batches have different buckling stress at the same $\frac{l}{r}$ values (may related to difference in heat treatments of the aluminium).
- "Tangent modulus formula" works the best with columns of high slenderness ($\frac{l}{r} >$ 80), as shown from specimen groups A, B.
- Columns of higher slenderness A, B show more elastic rebound when unloading, and the buckling processes are stable.
- Specimen groups D, E experienced unstable snap-through buckling during loading (i.e., the bars stayed straight until a no warning loud bang, and sudden large deflection).

<u>Neglected Factors that may negatively affected effectiveness of "tangent modulus formula"</u>

- Different batches of aluminium bars with different heat treatments should have the same Young's modulus, but different yield strength (i.e., Fig 2. (a) stress, strain curve would be different for each batch of specimens.)
  Therefore Fig.2 generated by material test in 1999 is not representative for the 3 batches specimens used in our test.
- "Tangent modulus formula" does not encapsulate the physical nature of inelasticity, its path dependency and irreversibility.

<u>Verify Fig.2 by checking points on the curve</u>

σ_b (MPa)	E_t, tangent from Fig.2 (a)	Calculated $\frac{l}{r}$	$\frac{l}{r}$ from Fig.2 (b)	Agree with Fig. 2 (b) or not
280	20000	26.6	25	Yes
287	5405	13.6	13	Yes
260	40000	39.0	40	Yes
100	66667	81.1	86	~Yes

Table 2. Verify Fig.2 "tangent modulus formula" curve with postulated values of σ_b.

- $\frac{l}{r}$ from Fig.2 (b) agree with our calculated values at those 4 points, however it's worth noting that reading local slope of Fig.2 (a) stress-strain curve is subjective and certainly not very accurate.

$r = \frac{d}{\sqrt{12}}.$

TABLE

Bar cross section d by d where $d =$ 6.31 mm, hence $r =$ 1.82 mm.

					Tangent modulus prediction	Experimental results		
Specimen	length of bar as cut (mm)	length l between pin centers (mm)	$\frac{l}{r}$	σ_b from Fig.2(b) (MPa)	P_b ($\sigma_b \times A$) (N)	P_{max} measured (N)	Remarks	σ_b measured (MPa)
A	180.	185	D=6.31 101.56	73.	2906.6	3000	little inelasticity almost straight	75.3
						3100	after unloading.	77.9
						2400		60.3
B	150	155	D=6.31 85.09	103	4101.1	3900	Apparent elastic	98.0
						3600	rebound when	90.4
						4300	unloading.	108.0
C	120	125	D=6.31 68.62	147	5853.0	5100	More 'bent' than	128.1
						5600	previous 2 samples	140.6
						4200		105.5
D	90	95	D=6.31 52.15	225	8958.6	9600	'Bang' at buckling	241.1
						7000	'Bent' a lot;	175.8
						8100	straighten a little bit at	203.4
E	60	65	D=6.31 35.68	268	10670.7	9700	unloading.	243.6
						10100		253.7
						8700		218.5
F	30	35	D=6.31 19.21	285	11347.6	12000	deformed	301.4
						10800	heavily,	271.2
						10000	lots of plasticity.	251.2

E
little elasticity when unloading,
mostly inelastic, large deformation,
'Bang' when buckling happen.

5.1

Table 3. Readings of axial compression test for "pin-ended columns" (Inelastic buckling).

Index of comments

2.1 if you take this red line as the initial slope you get very good agreement

2.2 The anomalous point at u_cr near 2.2mm is jst a typo in the .csv file. If you watch the video, you will see that there no anomalous result there.

5.1 Good observations