3C9 Failure of Engineering Materials Lab Report

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Summary

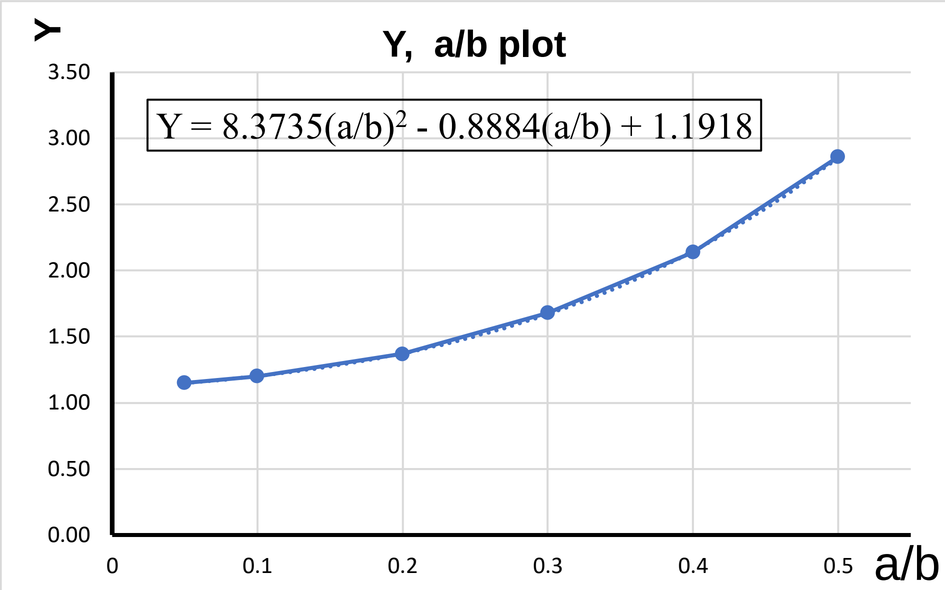
In the first part of the lab, we conduct Instron machine axial tensile testing on strips of material (PMMA and MDPE) specimens which contain edge cracks. Our aim is to
(a) determine in which failure mode (fast fracture or plastic collapse) each material fail;
(**b**) depending on the material failure mode, calculate material properties K_c(fracture toughness) or σ_y(yield stress) for each polymer.
In second part of the lab, we estimated bursting pressure of given PMMA/MDPE pipes using corresponding fast fracture and plastic collapse (plastic limit load) failure modes; we compared our estimation to the design pressures of the pipes to judge their safety; we also compared our estimation to the actual pipe bursting pressure observed in the experiments.

Results and Discussions

(Left) Single edged notched strip specimen used to determine $K_c$and notations.
(Middle) Strip specimen cut from PMMA sheet.
(Right) Strip specimen cut from MDPE pipe.

(Below) Stress intensity factor K= $Y\sigma\sqrt{\pi a}$, where geometric parameter Y is a function of $\frac{a}{b}$. And by plotting Y, $\frac{a}{b}$ variations from the left, the bast fit relationship between Y and $\frac{a}{b}$ is the polynomial curve shown on the right.

a/b	*Y*
0.05	1.15
0.10	1.20
0.20	1.37
0.30	1.68
0.40	2.14
0.50	2.86

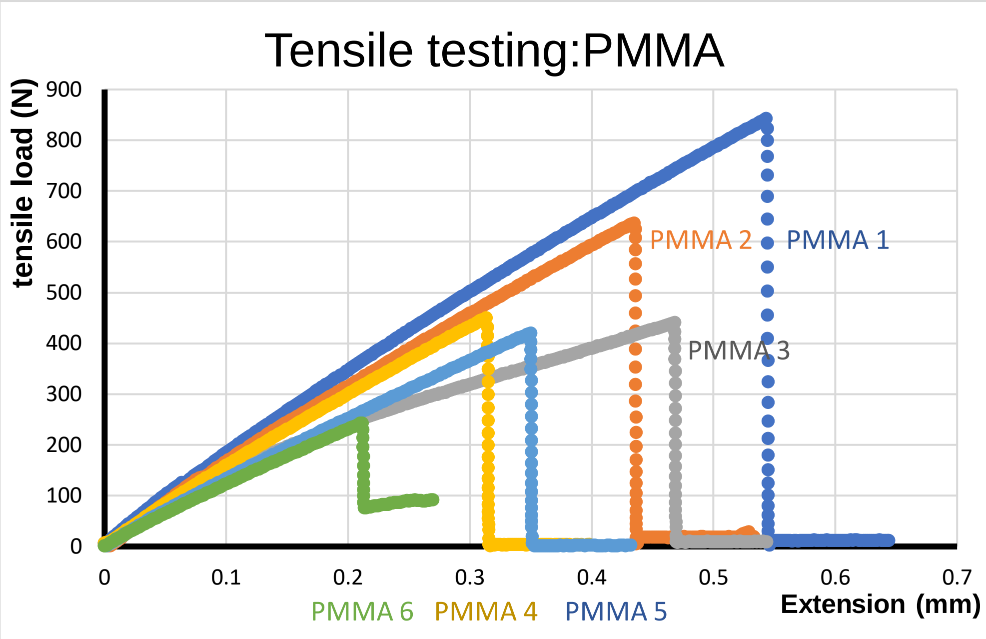


Specimen Label	crack length (mm) **(a)**	specimen width (mm) **(b)**	thickness (mm)	failure extension (mm)	failure load (N)	failure stress σ_f (MPa)	a/b	Y
PMMA 1	4.4	24.68	2.98	0.54428	823.50464	11.19708	0.18	1.29956
PMMA 2	6.15	25.03	2.99	0.43504	637.30713	8.51563	0.25	1.47903
PMMA 3	7.48	25.11	2.98	0.46836	440.80405	5.89091	0.30	1.67020
PMMA 4	9.96	25.2	2.93	0.31332	450.52884	6.10175	0.40	2.14872
PMMA 5	11.8	25.48	3.04	0.34996	420.34708	5.42669	0.46	2.57623
PMMA 6	13.52	25.12	2.85	0.21	242.39944	3.38585	0.54	3.13926
MDPE 1	21.42	27.14	6.01	2.82388	933.3399	5.72210	0.79	5.70650
MDPE 2	14.73	27.32	6.05	5.28656	1786.23315	10.80692	0.54	3.14698
MDPE 3	7.21	27.05	6.05	7.47096	2722.38867	16.63518	0.27	1.54990
MDPE 4	0	27.00	6.00	9.67224	3434.43628	21.20022	0.00	1.19180

Table 1: key specimen dimensions, measured failure stress σ_f, geometric parameter Y

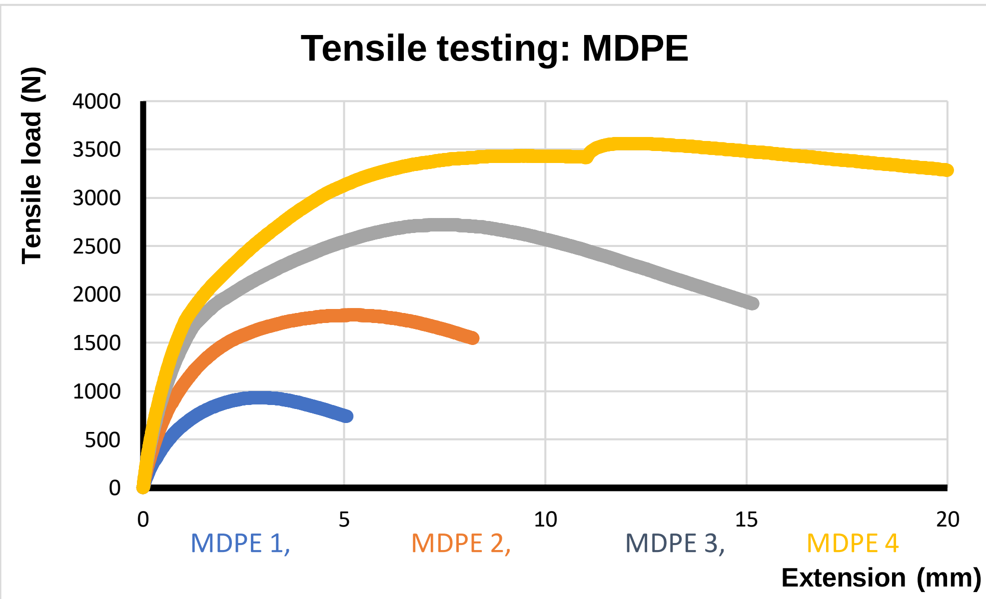
In the experiment, both the load-extension graph of PMMA (shown left) and their behaviours in axial tensile testing (shown right) suggest linear elastic fracture mechanics (fast fracture) applies to PMMA specimens.

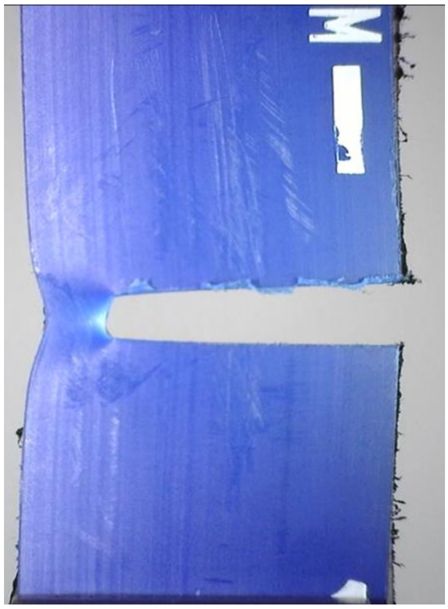
- In the left graph, PMMA displacements increase linearly until sudden failures.
- From Table 1 above, when failures happen, PMMA specimens have very small failure extensions.
- In the right photo, the edge crack in the specimen expands quickly (coincides with sudden plunge in the load-extension graph), lead to its failure.

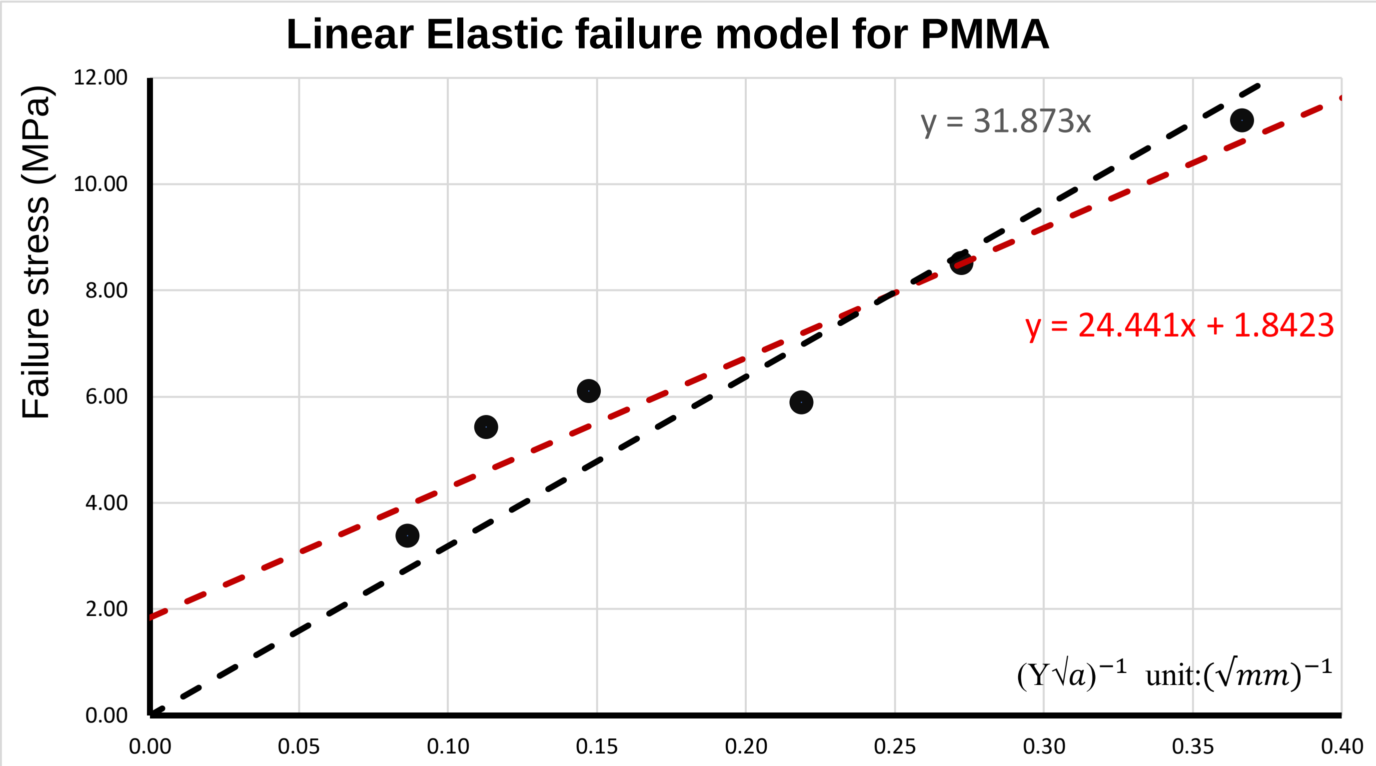


In the experiment, the load-extension graph plotted for MDPE (shown left) and its behaviours in axial tensile testing (shown right) suggest MDPE specimens failed by plastic collapse.

- From Table 1 above, when failures happen (for MDPE 4, failure occurs at the 1st peak in tensile load), MDPE specimens experience very large failure extensions. For MDPE 4, its extension essentially goes unbounded and well beyond the boundary of following graph.
- In the right photo, we can see the white "necking" adjacent to the edge crack, plastic zone has most likely become very large and spread throughout the specimen.
- In the left graph, after the initial linear elastic load-extension growth, the load-extension response become non-linear and irreversible extension increases steadily with the applied load (is illustrated in the experiment by the growing thin "necking" adjacent to the edge crack).







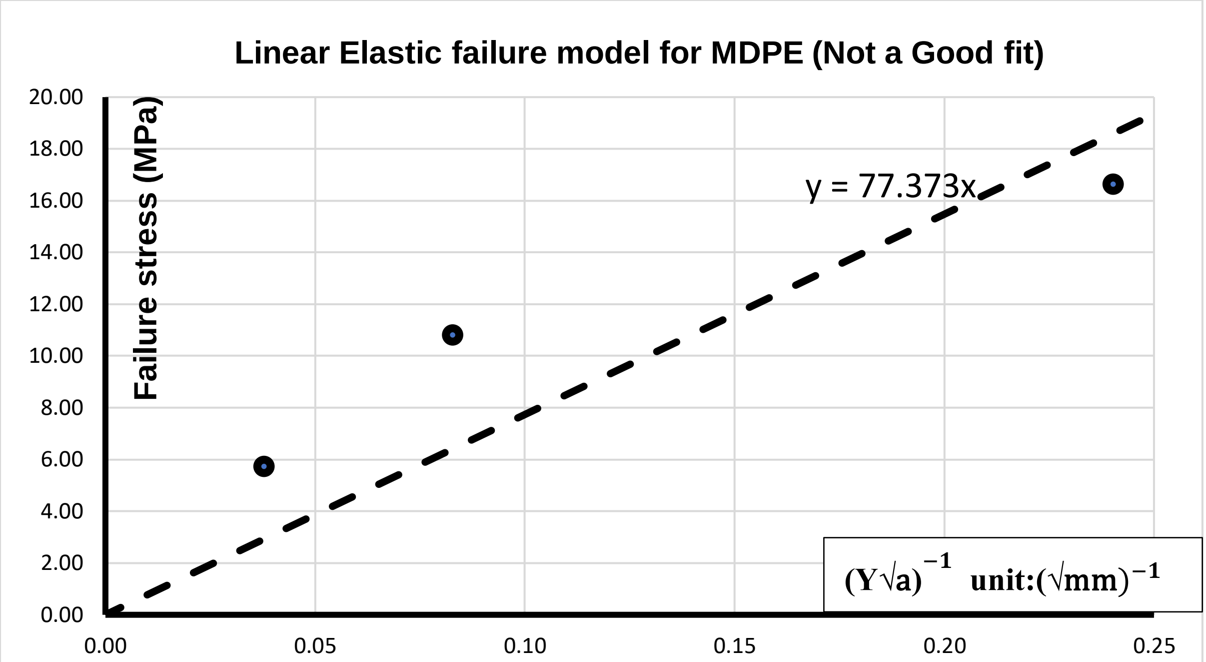
Failure stress σ_f ---- $(Y\sqrt{a})^{-1}$graph for PMMA specimens is plotted as above.

If Linear Elastic fracture mechaics applies for PMMA, we would expect a straight line with slope $K_c/\sqrt{\pi}$ (black line) which goes through the origin. However, the best fit linear relationship between σ_f and $(Y\sqrt{a})^{-1}$given by Excel is the red line σ_f=24.441$(Y\sqrt{a})^{-1}$+1.8423 which doesn't go through the origin.

From the black through origin line:
$K_c/\sqrt{\pi}$ = 31.873 (MPa × $\sqrt{mm}$)
Fracture toughness for PMMA, K_c = 1.786 (MPa × $\sqrt{m}$)
From the red best fit line:
$K_{c,red}/\sqrt{\pi}$ = 24.441 (MPa × $\sqrt{mm}$)
Fracture toughness for PMMA, $K_{c,red}$ = 1.370 (MPa × $\sqrt{m}$)
Both are within the range of expected PMMA fracture toughness (1.08-1.96 MPa× $\sqrt{m}$, subject to loading rate, given by ReaserchGate).

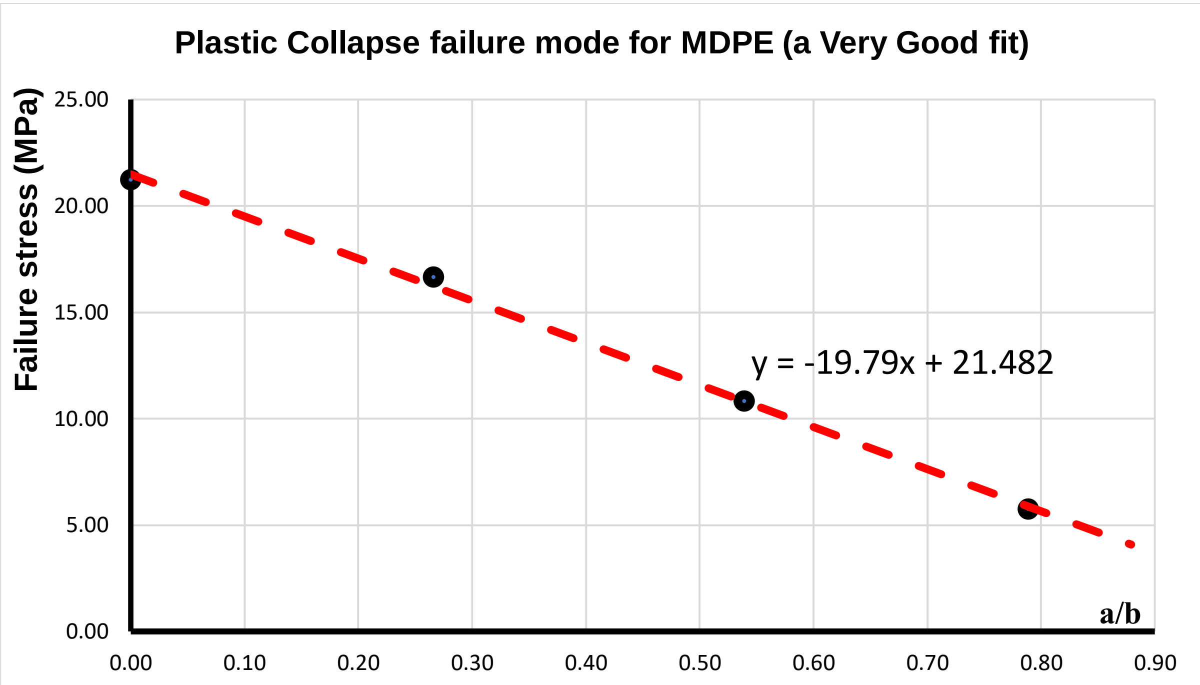
And it can be observed from the plots that the data aren't perfectly located on the red line (best fit) nor the black line (expected when material is perfectly linear elastic). Possible reasons of variations as following:

- Our PMMA specimens are sheets with slots machined using a 0.35mm thick circular saw, which is significant compared with crack length **a**; sawing process was done by hand, so the exact local crack tip conditions vary in all 6 specimens.
- PMMA sheets have low stiffness and may slip slightly during loading at the grips to the Instron machine.
- were gripped to Instron machine by hand, which could result in differences in gripping, and non-perfect grip could result in bending during tensile loading process.
- Displacement reading from the Instron machine is intrinsically inaccurate due to machine's own compliance (minor source of error in polymer loading).



Just for entirety, failure stress σ_f ---- $(Y\sqrt{a})^{-1}$graph for MDPE specimens is plotted as above.

While it can be seen that data doesn't fit well on a straight line that goes through the origin; but the results from only 3 data sets is not conclusive.
But we can say that fractures on MDPE are unlikely resulted by linear elastic failure.

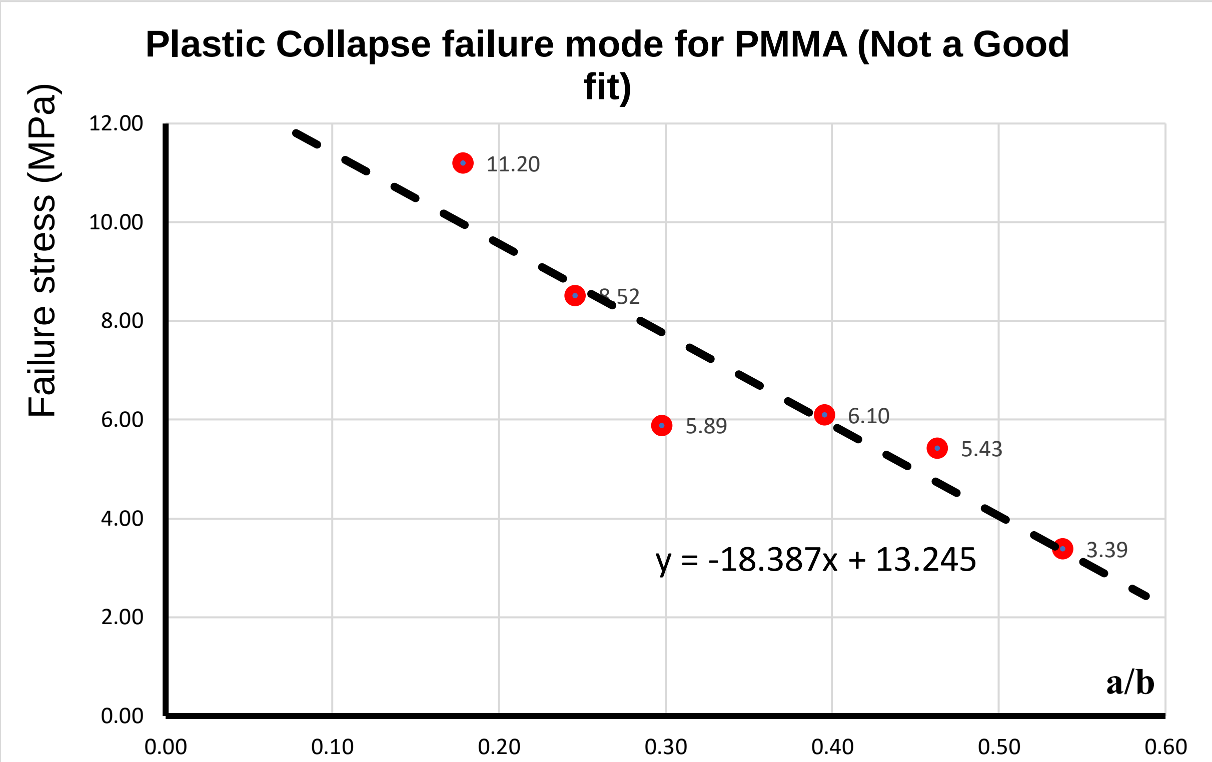


Failure stress σ_f ---- a/b graph for MDPE specimens is plotted as above.

A perfect fit on a line of slope -$\sigma_y$and intercept σ_y. Despite the lack of specimen data, we can say with confidence that MDPE fractures are caused by plastic collapse.
By taking the average:
σ_y=0.5×(21.482+19.79) = 20.64 MPa
which is slightly larger than the typical tensile strength (12.4 – 19.3 MPa) of MDPE given by Wikipedia, possible reasons of the difference with typical range as following:

- MDPE's ductile load-extension response is sensitive to straining rate (Instron machine loading rate); as shown in the load-extension of MDPE 4, an upward kind is caused by increasing of the Instron machine loading rate.

- MDPE specimens were cut from circular pipes, when grip them to the Instron machine by hand, it's impossible to create exact same conditions at the grip (e.g., forces act on the clamps).



Just for entirety, failure stress σ_f ---- a/b graph for PMMA specimens is plotted as above.

As shown, it is a less good fit to a straight slope than the MDPE plot, which convince us more that plastic collapse is not the failure mode resulted in PMMA fractures.

Material	External diameter (mm)	Wall thickness, **t** (mm)	Mid-thickness of wall, **R** (mm)	Yield strength (MPa)	Design pressure, p_{design} (MPa)
PMMA	40	3.0	18.5	80 (given)	3.243
MDPE	90	6.0	42.0	20.64 (previous)	0.737

Finding design pressure through: $\sigma_{hoop} = \frac{p_{design}R}{t} = \frac{\sigma_{yield}}{4}$

In the experiment, PMMA tube failed by linear elastic fracture mechanics, therefore by equating $\sigma = \frac{pR}{t}$ and K = Y $\sigma\sqrt{\pi b}$, we have:

Predicted failure pressure of PMMA: $p_f = \frac{K_c t}{YR\sqrt{\pi b}}$;

Using limit load method from handout calculate Predicted failure pressure p_f of MDPE

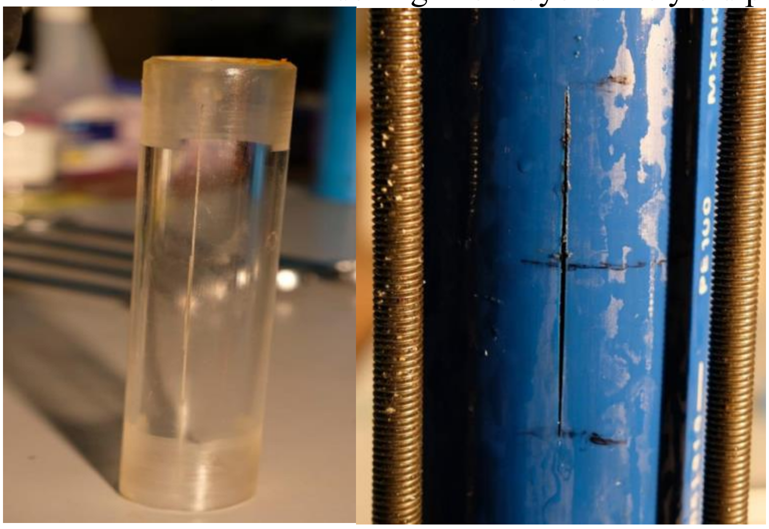
Material	K_c (MPa$\sqrt{m}$)	σ_y (MPa)	λ	Y	Predicted failure pressure, p_f (MPa)	Safe or Not	Experiment pipe bursting pressure, p_{ex} (MPa)
PMMA	1.786	/	2.68	3.016	1.21	$p_{design} > p_f$ **unsafe**	1.25
MDPE	/	20.64	2.52	/	1.07	$p_{design} < p_f$ **safe**	2.10

For PMMA tube, its predicted failure pressure is similar to that observed in the experiment (slight underestimation), quick growth of the sharp axial crack was observed, further proofs PMMA fails by linear elastic fracture mechanics (fast fracture). Possible sources of error:

- In Part 1 tension testing, PMMA specimens are flat sheet, which cannot accurately represent characters of the circular PMMA pipe.
- Initial cracks in the pipe were machined by hand using a saw, which may result variations along the crack.

For MDPE tube, it’s clear from the experiment that MDPE specimen is ductile and failed by plastic collapse. Its predicted failure pressure underpredicted the experiment pipe bursting pressure by **about a half**. Possible sources of error:

- The M(λ) formular used in predicting failure pressure is deduced from “a series of tests on steel tubes”. Unlike steel, polymer like MDPE is sensitive to hydrostatic stress when yielding. And for steel tubes: yield $\approx$ failure, while MDPE strain hardening a lot beyond the yield point.



(Left) PMMA pipe after bursting, notice the sharp crack grown across the pipe height.
(Right) MDPE pipe after bursting.

Conclusions

PMMA specimens failed by linear elastic fracture mechanics (fast fracture);
MDPE specimens failed by plastic collapse (plastic limit load);
We can have a rather good estimation on bursting pressure of pipes made by PMMA from its fracture toughness K;
We **cannot** have a good estimation on bursting pressure of pipes made by MDPE from calculating its plastic limit loads, due to the significant difference in MDPE & steel’s material characters (while relevant empirical formulas we used are based on steel tubes).