混淆矩陣
Confusion Matrix

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1

不平衡資料（Imbalanced Data）

$$y = (0,0,0,0,0,0,0,0,0,1)$$

$$\hat{y} = (\cancel{1,1,1},0,0,0,0,0,0,1)$$

Score= 0.7

$$\hat{y} = (0,0,0,0,0,0,0,0,\cancel{1,0})$$

Score= 0.8

$$Score = \frac{\sum_{i=1}^{m} I(y_i = \hat{y}_i)}{m}$$

$$\hat{y} = (0,0,0,0,0,0,0,0,0,\cancel{0})$$

Score= 0.9

2

混淆矩陣（Confusion Matrix）

		預測	
		Not 1	1
實際	Not 1	True Negative	False Positive
	1	False Negative	True Positive

3

不平衡資料（Imbalanced Data）

$$y = (0, 0, 0, 0, 0, 0, 0, 0, 0, 1)$$

$$\hat{y} = (1, 1, 1, 0, 0, 0, 0, 0, 0, 1)$$

Score= 0.7

		預測	
		0	1
實際	0	6	3
	1	0	1

$$\hat{y} = (0, 0, 0, 0, 0, 0, 0, 0, 1, 0)$$

Score= 0.8

		預測	
		0	1
實際	0	8	1
	1	1	0

4

- Precision
  - $Precision = \frac{TP}{TP+FP}$
- Recall (True Positive Rate, Sensitivity)
  - $Recall = \frac{TP}{TP+FN}$
- F1 score
  - $F_1 = \frac{2}{\frac{1}{Precision}+\frac{1}{Recall}} = \frac{TP}{TP+\frac{FN+FP}{2}}$

		預測	
		Not 1	1
實際	Not 1	True Negative	False Positive
	1	False Negative	True Positive

5

- Precision
  - $Precision = \frac{TP}{TP+FP}$
- Recall (True Positive Rate, Sensitivity)
  - $Recall = \frac{TP}{TP+FN}$
- F1 score
  - $F_1 = \frac{2}{\frac{1}{Precision}+\frac{1}{Recall}} = \frac{TP}{TP+\frac{FN+FP}{2}}$

		預測	
		Not 1	1
實際	Not 1	True Negative	False Positive
	1	False Negative	True Positive

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6

- Precision
  - $Precision = \frac{TP}{TP+FP}$
- Recall (True Positive Rate, Sensitivity)
  - $Recall = \frac{TP}{TP+FN}$
- F1 score
  - $F_1 = \frac{2}{\frac{1}{Precision}+\frac{1}{Recall}} = \frac{TP}{TP+\frac{FN+FP}{2}}$

		預測	
		Not 1	1
實際	Not 1	True Negative	False Positive
	1	False Negative	True Positive

7

$$y = (0,0,0,0,0,0,0,0,0,1)$$

$$\hat{y} = (1,1,1,0,0,0,0,0,0,1)$$

Score= 0.7

		預測	
		0	1
實際	0	6	3
	1	0	1

$Precision = \frac{1}{1+3} = 0.25$

$$\hat{y} = (0,0,0,0,0,0,0,0,1,0)$$

Score= 0.8

		預測	
		0	1
實際	0	8	1
	1	1	0

$Precision = \frac{0}{0+1} = 0$

8

$$y = (0,0,0,0,0,0,0,0,0,1)$$

$\hat{y} = (1,1,1,0,0,0,0,0,0,1)$

Score= 0.7

實際 \ 預測	0	1
0	6	3
1	0	1

$Precision = \frac{1}{1+3} = 0.25$

$Recall = \frac{1}{1+0} = 1$

$\hat{y} = (0,0,0,0,0,0,0,0,1,0)$

Score= 0.8

實際 \ 預測	0	1
0	8	1
1	1	0

$Precision = \frac{0}{0+1} = 0$

$Recall = \frac{0}{0+1} = 0$

9

$$y = (0,0,0,0,0,0,0,0,0,1)$$

$\hat{y} = (1,1,1,0,0,0,0,0,0,1)$

Score= 0.7

實際 \ 預測	0	1
0	6	3
1	0	1

$Precision = \frac{1}{1+3} = 0.25$

$Recall = \frac{1}{1+0} = 1$

$F_1 = \frac{1}{1+\frac{0+3}{2}} = 0.4$

$\hat{y} = (0,0,0,0,0,0,0,0,1,0)$

Score= 0.8

實際 \ 預測	0	1
0	8	1
1	1	0

$Precision = \frac{0}{0+1} = 0$

$Recall = \frac{0}{0+1} = 0$

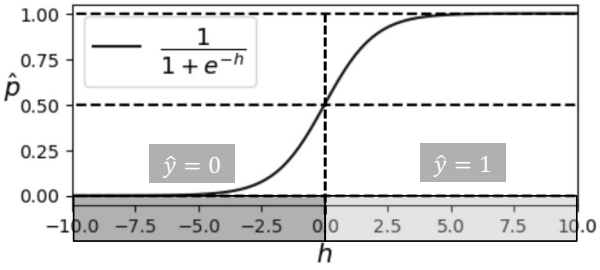
$F_1 = \frac{0}{0+\frac{1+1}{2}} = 0$

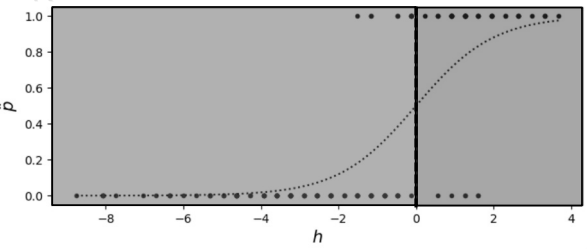
10

Precision/Recall Trade-off

決策界線（ Decision Boundary ）

- $(x_1, \cdots, x_n)\ s.t.\ h = 0$ $\quad h = b + w_1 x_1 + \cdots + w_n x_n$
- $\hat{y} = \begin{cases} 0, & \hat{p} \leq 0.5 \\ 1, & \hat{p} > 0.5 \end{cases}$ ← Threshold $\Rightarrow$ $\hat{y} = \begin{cases} 0, & h \leq 0 \\ 1, & h > 0 \end{cases}$ ← Threshold
- $\hat{p} = \frac{1}{1+e^{-h}}$



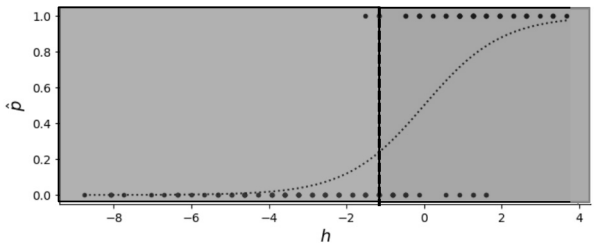


Threshold $h = 0$

預測		
實際	24	4
	4	11

$$Precision = \frac{11}{11+4} = 0.73$$

$$Recall = \frac{11}{11+4} = 0.73$$



Threshold $h = -1$

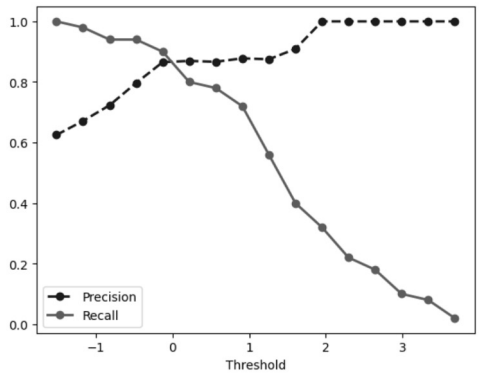
預測		
實際	22	6
	2	13

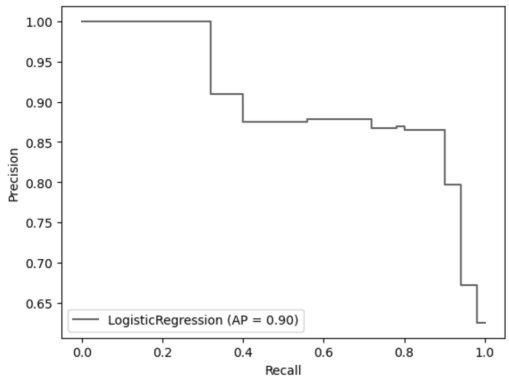
$$Precision = \frac{13}{13+6} = 0.68$$

$$Recall = \frac{13}{13+2} = 0.87$$

13

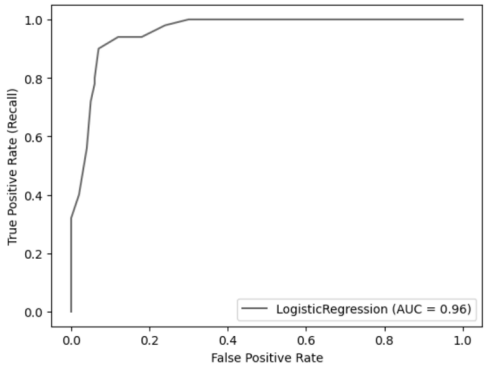
PR （ Preciesion Recall ） Curve





14

ROC（Reciver Operating Characteristic）Curve



		預測	
		Not 1	1
實際	Not 1	TN	FP
	1	FN	TP

$$\text{False Positive Rat} = \frac{FP}{TN+FP}$$

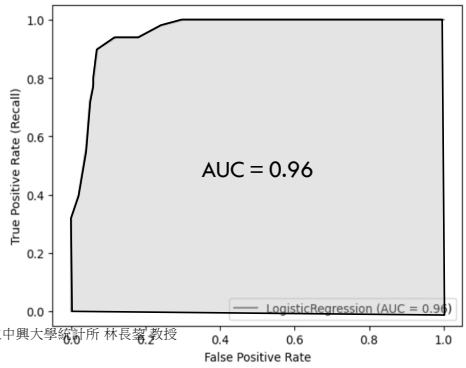
$$\text{True Positive Rat} = \frac{TP}{FN+TP}$$

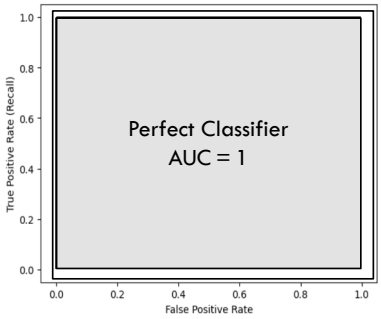
↑ Recall

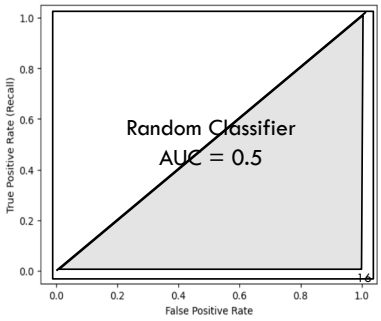
15

AUC（Area Under the Curve）

- 用於比較不同的 Classifiers （models）







16