Formulae sheet for Electromagnetism

A. Operator in different coordinates

- Cartesian coordinates:

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right), \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$d\vec{\ell} = dx\hat{x} + dy\hat{y} + dz\hat{z}, dV = dxdydz$$

  - **Gradient:**

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$$

  - **Divergence:**

$$\nabla \cdot \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

  - **Curl:**

$$\nabla \times \vec{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}, \frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}, \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)$$

- Cylindrical coordinates:

$$\nabla = \left(\frac{\partial}{\partial r}, \frac{1}{r}\frac{\partial}{\partial \theta}, \frac{\partial}{\partial z}\right), \nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$$

$$d\vec{\ell} = dr\hat{r} + rd\theta\hat{\theta} + dz\hat{z}$$

$$dV = rdrd\theta dz$$

  - **Gradient:**

$$\nabla f = \left(\frac{\partial f}{\partial r}, \frac{1}{r}\frac{\partial f}{\partial \theta}, \frac{\partial f}{\partial z}\right)$$

  - **Divergence:**

$$\nabla \cdot \vec{F} = \frac{1}{r}\frac{\partial}{\partial r}(rF_r) + \frac{1}{r}\frac{\partial F_\theta}{\partial \theta} + \frac{\partial F_z}{\partial z}$$

  - **Curl:**

$$\nabla \times \vec{F} = \left(\frac{1}{r}\frac{\partial F_z}{\partial \theta} - \frac{\partial F_\theta}{\partial z}, \frac{\partial F_r}{\partial z} - \frac{\partial F_z}{\partial r}, \frac{1}{r}\frac{\partial}{\partial r}(rF_\theta) - \frac{\partial F_r}{\partial \theta}\right)$$

- Spherical coordinates:

$$\nabla = \left(\frac{\partial}{\partial r}, \frac{1}{r}\frac{\partial}{\partial \theta}, \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\right)$$

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial \phi^2}$$

$$d\vec{\ell} = dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi}$$

$$dV = r^2\sin\theta drd\theta d\phi$$

  - **Gradient:**

$$\nabla f = \left(\frac{\partial f}{\partial r}, \frac{1}{r}\frac{\partial f}{\partial \theta}, \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\right)$$

  - **Divergence:**

$$\nabla \cdot \vec{F} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2F_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta F_\theta) + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial \phi}$$

  - **Curl:**

$$\nabla \times \vec{F} = \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial \theta}(F_\phi\sin\theta) - \frac{\partial F_\theta}{\partial \phi}, \frac{1}{r}\left(\frac{\partial F_r}{\partial \phi} - \frac{\partial}{\partial r}(rF_\phi)\right), \frac{1}{r}\left(\frac{\partial F_\theta}{\partial r} - \frac{\partial F_r}{\partial \theta}\right)\right)$$

B. Vector formulae

- $\vec{a}\cdot(\vec{b}\times\vec{c}) = \vec{b}\cdot(\vec{c}\times\vec{a}) = \vec{c}\cdot(\vec{a}\times\vec{b})$
- $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$
- $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d}) = (\vec{a}\cdot\vec{c})(\vec{b}\cdot\vec{d}) - (\vec{a}\cdot\vec{d})(\vec{b}\cdot\vec{c})$
- $\nabla\times\nabla\psi = 0$
- $\nabla\cdot(\nabla\times\vec{a}) = 0$
- $\nabla\times(\nabla\times\vec{a}) = \nabla(\nabla\cdot\vec{a}) - \nabla^2\vec{a}$
- $\nabla\cdot(\psi\vec{a}) = \nabla\psi\cdot\vec{a} + \psi\nabla\cdot\vec{a}$
- $\nabla\times(\psi\vec{a}) = \nabla\psi\times\vec{a} + \psi\nabla\times\vec{a}$
- $\nabla(\vec{a}\cdot\vec{b}) = (\vec{a}\cdot\nabla)\vec{b} + (\vec{b}\cdot\nabla)\vec{a} + \vec{a}\times(\nabla\times\vec{b}) + \vec{b}\times(\nabla\times\vec{a})$
- $\nabla\cdot(\vec{a}\times\vec{b}) = \vec{b}\cdot(\nabla\times\vec{a}) - \vec{a}\cdot(\nabla\times\vec{b}) + (\vec{b}\cdot\nabla)\vec{a} - (\vec{a}\cdot\nabla)\vec{b}$
- $\nabla\times(\vec{a}\times\vec{b}) = (\vec{b}\cdot\nabla)\vec{a} - (\vec{a}\cdot\nabla)\vec{b} + \vec{a}(\nabla\cdot\vec{b}) - \vec{b}(\nabla\cdot\vec{a})$
- $\nabla\cdot\vec{r} = 3$
- $\nabla\times\vec{r} = 0$
- $\nabla\cdot[\hat{r}\cdot f(r)] = \frac{2}{r}f + \frac{\partial f}{\partial r}$
- $\nabla\times[\hat{r}\cdot f(r)] = 0$
- $(\vec{a}\cdot\nabla)\hat{r}f(r) = \frac{f(r)}{r}[\vec{a} - \hat{r}(\vec{a}\cdot\hat{r})] + \hat{r}(\vec{a}\cdot\hat{r})\frac{\partial f}{\partial r}$

C. Theorems from vector calculus

- Divergence Theorem:

$$\int_V \nabla\cdot\vec{A}\,d^3x = \int_S \vec{A}\cdot\hat{n}\,dS$$

$$\int_V \nabla\psi\,d^3x = \int_S \psi\hat{n}\,dS$$

$$\int_V \nabla\times\vec{A}\,d^3x = \int_S \hat{n}\times\vec{A}\,dS$$

- Green's First Identity:

$$\int_V \left(\phi\nabla^2\psi + \nabla\phi\cdot\nabla\psi\right)d^3x = \int_S \phi\hat{n}\cdot\nabla\psi\,dS$$

- Green's Theorem:

$$\int_V \left(\phi\nabla^2\psi - \psi\nabla^2\phi\right)d^3x = \int_S (\phi\nabla\psi - \psi\nabla\phi)\cdot\hat{n}\,dS$$

- Stokes's Theorem:

$$\int_S \left(\nabla\times\vec{A}\right)\cdot\hat{n}\,dS = \oint_C \vec{A}\cdot d\vec{l}$$

$$\int_S \hat{n}\times\nabla\psi\,dS = \oint_C \psi\,d\vec{l}$$

D. Vector calculus with delta-function

- The Dirac delta function $\delta(\vec{r})$ is a distribution that satisfies:

$$\int_{\mathbb{R}^3} \delta(\vec{r})\,d^3r = 1$$

and is zero everywhere except at $\vec{r} = \vec{0}$.
- The sifting property of the Dirac delta function allows us to extract function values:

$$\int_{\mathbb{R}^3} f(\vec{r})\delta(\vec{r} - \vec{r_0})\,d^3r = f(\vec{r_0})$$

- The Laplacian of the Dirac delta function in three dimensions is:

$$\nabla^2\delta(\vec{r}) = -4\pi\delta(\vec{r})$$

- The gradient and Laplacian of the function $\frac{1}{r}$ is:

$$\nabla\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}, \quad \nabla^2\left(\frac{1}{r}\right) = -4\pi\delta(\vec{r})$$

- The divergence of a vector field multiplied by a delta function can be expressed as:

$$\nabla\cdot(f(\vec{r})\delta(\vec{r} - \vec{r_0})) = \delta(\vec{r} - \vec{r_0})\nabla f(\vec{r_0}) + f(\vec{r_0})\nabla\delta(\vec{r} - \vec{r_0})$$

E. Constants

- Permittivity of free space:

$$\epsilon_0 = 8.854\times10^{-12}\ \text{F/m}$$

- Permeability of free space:

$$\mu_0 = 4\pi\times10^{-7}\ \text{H/m}$$

- Speed of light in vacuum:

$$c = 3.00\times10^{8}\ \text{m/s}$$

- Charge of an electron:

$$e = 1.602\times10^{-19}\ \text{C}$$

- Avogadro's number:

$$N_A = 6.022\times10^{23}\ \text{mol}^{-1}$$

F. Electrostatics

- **Coulomb's Law**:

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}\hat{r}$$

- **Electric Field**:

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int\frac{\rho(\vec{r}')(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}dV'$$

or

$$\vec{E} = -\nabla V, \quad V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int\frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|}dV'$$

The electric field is the gradient of the electric potential.
- **Gauss's Law**:

$$\nabla\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$

$$\oint_{\partial V}\vec{E}\cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

The divergence of the electric field is proportional to the charge density.
- **Poisson's Equation**:

$$\nabla^2V = -\frac{\rho}{\epsilon_0}$$

Relates the electric potential to the charge density.
- **Ohm's Law for Current Density**:

$$\vec{J_f} = \sigma\vec{E} = \frac{1}{\rho}\vec{E}$$

- **Polarization**:

$$\vec{P} = \epsilon_0\chi_r\vec{E} = \epsilon_0(\epsilon_r - 1)\vec{E}$$

Describes the polarization density in a medium.
- **Electric Displacement Field**:

$$\vec{D} = \epsilon_0\vec{E} + \vec{P} = \epsilon_r\epsilon_0\vec{E}$$

The electric displacement field accounts for free and bound charges.
- **Surface Bound Charge Density**:

$$\sigma_b = \vec{P}\cdot\hat{n}$$

Surface charge density due to polarization.
- **Volume Bound Charge Density**:

$$\rho_b = -\nabla\cdot\vec{P}$$

Volume charge density due to polarization.
- Capacitance:

$$C = \frac{\Delta Q}{\Delta V}, E = \frac{Q^2}{2C}$$

- The current density J can be expressed as:

$$J = nqv_D$$

where $v_D = -\frac{eE}{m}\langle\tau\rangle$. $\langle\tau\rangle$ is the average time between collisions (mean free time).

G. Magnetostatics

- **Biot-Savart Law**:

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int\frac{Id\vec{\ell}'\times(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$

- **Lorentz Force**:

$$\vec{F} = q\left(\vec{v}\times\vec{B}\right)$$

- **Ampère's Law**:

$$\nabla\times\vec{B} = \mu_0\vec{J}$$

$$\oint_{\partial S}\vec{B}\cdot d\vec{\ell} = \mu_0I_{\text{enc}}$$

In the absence of time-varying electric fields, the curl of the magnetic field is proportional to the current density.
- **Magnetostatic Gauss's Law**:

$$\nabla\cdot\vec{B} = 0$$

- **Relation between Magnetic Field and Magnetization**:

$$\vec{B} = \mu_0\vec{H} + \vec{M}$$

- **Magnetization Current Density**:

$$\vec{J}_m = \nabla\times\vec{M}$$

- Magnetic inductance:

$$L = \frac{\Delta\Phi}{\Delta I}, E = \frac{I^2L}{2}$$

H. Electrodynamics

- Faraday's Law of induction states that a time-varying magnetic field induces a curl in the electric field.

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\mathcal{E} = \oint_{\partial S}\vec{E}\cdot d\vec{\ell} = -\frac{d}{dt}\int_S\vec{B}\cdot d\vec{A}$$

The electromotive force (EMF) around a closed loop ∂S is equal to the negative rate of change of the magnetic flux through the surface S bounded by the loop.
- **Maxwell's Equations**:

$$\nabla\cdot\vec{E} = \frac{\rho_f}{\epsilon_0} \qquad \text{(Gauss's Law)}$$

$$\nabla\cdot\vec{B} = 0 \qquad \text{(Magnetostatic Gauss's Law)}$$

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad \text{(Faraday's Law)}$$

$$\nabla\times\vec{B} = \mu_0\vec{J_f} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t} \qquad \text{(Maxwell-Ampère Law)}$$

- **Poynting Vector**:

$$\vec{S} = \vec{E}\times\vec{H}$$

Represents the energy flux density of an electromagnetic field.
- **Energy Density**:

$$u = \frac{1}{2}\epsilon_0E^2 + \frac{1}{2\mu_0}B^2$$

The energy density of the electromagnetic field.
- **Electromagnetic Wave Equation**:

$$\nabla^2\vec{E} - \mu_0\epsilon_0\frac{\partial^2\vec{E}}{\partial t^2} = 0$$

$$\nabla^2\vec{B} - \mu_0\epsilon_0\frac{\partial^2\vec{B}}{\partial t^2} = 0$$

Describes the propagation of electromagnetic waves in a vacuum.
- **Continuity Equation in Electrodynamics**:

$$\nabla\cdot\vec{J_f} + \frac{\partial\rho_f}{\partial t} = 0$$

Table of Integrals*

Basic Forms

$$\int x^n dx = \frac{1}{n+1}x^{n+1} \tag{1}$$

$$\int \frac{1}{x}dx = \ln|x| \tag{2}$$

$$\int udv = uv - \int vdu \tag{3}$$

$$\int \frac{1}{ax+b}dx = \frac{1}{a}\ln|ax+b| \tag{4}$$

Integrals of Rational Functions

$$\int \frac{1}{(x+a)^2}dx = -\frac{1}{x+a} \tag{5}$$

$$\int (x+a)^n dx = \frac{(x+a)^{n+1}}{n+1}, n \neq -1 \tag{6}$$

$$\int x(x+a)^n dx = \frac{(x+a)^{n+1}((n+1)x-a)}{(n+1)(n+2)} \tag{7}$$

$$\int \frac{1}{1+x^2}dx = \tan^{-1}x \tag{8}$$

$$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a} \tag{9}$$

$$\int \frac{x}{a^2+x^2}dx = \frac{1}{2}\ln|a^2+x^2| \tag{10}$$

$$\int \frac{x^2}{a^2+x^2}dx = x - a\tan^{-1}\frac{x}{a} \tag{11}$$

$$\int \frac{x^3}{a^2+x^2}dx = \frac{1}{2}x^2 - \frac{1}{2}a^2\ln|a^2+x^2| \tag{12}$$

$$\int \frac{1}{ax^2+bx+c}dx = \frac{2}{\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} \tag{13}$$

$$\int \frac{1}{(x+a)(x+b)}dx = \frac{1}{b-a}\ln\frac{a+x}{b+x},\ a \neq b \tag{14}$$

$$\int \frac{x}{(x+a)^2}dx = \frac{a}{a+x} + \ln|a+x| \tag{15}$$

$$\int \frac{x}{ax^2+bx+c}dx = \frac{1}{2a}\ln|ax^2+bx+c| - \frac{b}{a\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} \tag{16}$$

Integrals with Roots

$$\int \sqrt{x-a}dx = \frac{2}{3}(x-a)^{3/2} \tag{17}$$

$$\int \frac{1}{\sqrt{x\pm a}}dx = 2\sqrt{x\pm a} \tag{18}$$

$$\int \frac{1}{\sqrt{a-x}}dx = -2\sqrt{a-x} \tag{19}$$

$$\int x\sqrt{x-a}dx = \frac{2}{3}a(x-a)^{3/2} + \frac{2}{5}(x-a)^{5/2} \tag{20}$$

$$\int \sqrt{ax+b}dx = \left(\frac{2b}{3a}+\frac{2x}{3}\right)\sqrt{ax+b} \tag{21}$$

$$\int (ax+b)^{3/2}dx = \frac{2}{5a}(ax+b)^{5/2} \tag{22}$$

$$\int \frac{x}{\sqrt{x\pm a}}dx = \frac{2}{3}(x\mp 2a)\sqrt{x\pm a} \tag{23}$$

$$\int \sqrt{\frac{x}{a-x}}dx = -\sqrt{x(a-x)} - a\tan^{-1}\frac{\sqrt{x(a-x)}}{x-a} \tag{24}$$

$$\int \sqrt{\frac{x}{a+x}}dx = \sqrt{x(a+x)} - a\ln\left[\sqrt{x}+\sqrt{x+a}\right] \tag{25}$$

$$\int x\sqrt{ax+b}dx = \frac{2}{15a^2}(-2b^2+abx+3a^2x^2)\sqrt{ax+b} \tag{26}$$

$$\int \sqrt{x(ax+b)}dx = \frac{1}{4a^{3/2}}\left[(2ax+b)\sqrt{ax(ax+b)} - b^2\ln\left|a\sqrt{x}+\sqrt{a(ax+b)}\right|\right] \tag{27}$$

$$\int \sqrt{x^3(ax+b)}dx = \left[\frac{b}{12a}-\frac{b^2}{8a^2x}+\frac{x}{3}\right]\sqrt{x^3(ax+b)} + \frac{b^3}{8a^{5/2}}\ln\left|a\sqrt{x}+\sqrt{a(ax+b)}\right| \tag{28}$$

$$\int \sqrt{x^2\pm a^2}dx = \frac{1}{2}x\sqrt{x^2\pm a^2} \pm \frac{1}{2}a^2\ln\left|x+\sqrt{x^2\pm a^2}\right| \tag{29}$$

$$\int \sqrt{a^2-x^2}dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{1}{2}a^2\tan^{-1}\frac{x}{\sqrt{a^2-x^2}} \tag{30}$$

$$\int x\sqrt{x^2\pm a^2}dx = \frac{1}{3}\left(x^2\pm a^2\right)^{3/2} \tag{31}$$

$$\int \frac{1}{\sqrt{x^2\pm a^2}}dx = \ln\left|x+\sqrt{x^2\pm a^2}\right| \tag{32}$$

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\frac{x}{a} \tag{33}$$

$$\int \frac{x}{\sqrt{x^2\pm a^2}}dx = \sqrt{x^2\pm a^2} \tag{34}$$

$$\int \frac{x}{\sqrt{a^2-x^2}}dx = -\sqrt{a^2-x^2} \tag{35}$$

$$\int \frac{x^2}{\sqrt{x^2\pm a^2}}dx = \frac{1}{2}x\sqrt{x^2\pm a^2} \mp \frac{1}{2}a^2\ln\left|x+\sqrt{x^2\pm a^2}\right| \tag{36}$$

$$\int \sqrt{ax^2+bx+c}dx = \frac{b+2ax}{4a}\sqrt{ax^2+bx+c} + \frac{4ac-b^2}{8a^{3/2}}\ln\left|2ax+b+2\sqrt{a(ax^2+bx^+c)}\right| \tag{37}$$

$$\int x\sqrt{ax^2+bx+c} = \frac{1}{48a^{5/2}}\Big(2\sqrt{a}\sqrt{ax^2+bx+c} \times \left(-3b^2+2abx+8a(c+ax^2)\right) +3(b^3-4abc)\ln\left|b+2ax+2\sqrt{a}\sqrt{ax^2+bx+c}\right|\Big) \tag{38}$$

$$\int \frac{1}{\sqrt{ax^2+bx+c}}dx = \frac{1}{\sqrt{a}}\ln\left|2ax+b+2\sqrt{a(ax^2+bx+c)}\right| \tag{39}$$

$$\int \frac{x}{\sqrt{ax^2+bx+c}}dx = \frac{1}{a}\sqrt{ax^2+bx+c} - \frac{b}{2a^{3/2}}\ln\left|2ax+b+2\sqrt{a(ax^2+bx+c)}\right| \tag{40}$$

$$\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}} \tag{41}$$

Integrals with Logarithms

$$\int \ln ax dx = x\ln ax - x \tag{42}$$

$$\int \frac{\ln ax}{x}dx = \frac{1}{2}(\ln ax)^2 \tag{43}$$

$$\int \ln(ax+b)dx = \left(x+\frac{b}{a}\right)\ln(ax+b) - x, a \neq 0 \tag{44}$$

$$\int \ln(x^2+a^2)\,\mathrm{dx} = x\ln(x^2+a^2) + 2a\tan^{-1}\frac{x}{a} - 2x \tag{45}$$

$$\int \ln(x^2-a^2)\,\mathrm{dx} = x\ln(x^2-a^2) + a\ln\frac{x+a}{x-a} - 2x \tag{46}$$

$$\int \ln\left(ax^2+bx+c\right)dx = \frac{1}{a}\sqrt{4ac-b^2}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} - 2x + \left(\frac{b}{2a}+x\right)\ln\left(ax^2+bx+c\right) \tag{47}$$

$$\int x\ln(ax+b)dx = \frac{bx}{2a} - \frac{1}{4}x^2 + \frac{1}{2}\left(x^2-\frac{b^2}{a^2}\right)\ln(ax+b) \tag{48}$$

$$\int x\ln\left(a^2-b^2x^2\right)dx = -\frac{1}{2}x^2 + \frac{1}{2}\left(x^2-\frac{a^2}{b^2}\right)\ln\left(a^2-b^2x^2\right) \tag{49}$$

Integrals with Exponentials

$$\int e^{ax}dx = \frac{1}{a}e^{ax} \tag{50}$$

$$\int \sqrt{x}e^{ax}dx = \frac{1}{a}\sqrt{x}e^{ax} + \frac{i\sqrt{\pi}}{2a^{3/2}}\mathrm{erf}\left(i\sqrt{ax}\right), \quad \text{where } \mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt \tag{51}$$

$$\int xe^x dx = (x-1)e^x \tag{52}$$

$$\int xe^{ax}dx = \left(\frac{x}{a}-\frac{1}{a^2}\right)e^{ax} \tag{53}$$

$$\int x^2e^x dx = \left(x^2-2x+2\right)e^x \tag{54}$$

$$\int x^2e^{ax}dx = \left(\frac{x^2}{a}-\frac{2x}{a^2}+\frac{2}{a^3}\right)e^{ax} \tag{55}$$

$$\int x^3e^x dx = \left(x^3-3x^2+6x-6\right)e^x \tag{56}$$

$$\int x^ne^{ax}\,\mathrm{dx} = \frac{x^ne^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}\,\mathrm{dx} \tag{57}$$

$$\int x^ne^{ax}\,\mathrm{dx} = \frac{(-1)^n}{a^{n+1}}\Gamma[1+n,-ax], \quad \text{where } \Gamma(a,x) = \int_x^\infty t^{a-1}e^{-t}\,\mathrm{dt} \tag{58}$$

$$\int e^{ax^2}\,\mathrm{dx} = -\frac{i\sqrt{\pi}}{2\sqrt{a}}\mathrm{erf}\left(ix\sqrt{a}\right) \tag{59}$$

$$\int e^{-ax^2}\,\mathrm{dx} = \frac{\sqrt{\pi}}{2\sqrt{a}}\mathrm{erf}\left(x\sqrt{a}\right) \tag{60}$$

$$\int xe^{-ax^2}\,\mathrm{dx} = -\frac{1}{2a}e^{-ax^2} \tag{61}$$

$$\int x^2e^{-ax^2}\,\mathrm{dx} = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}\mathrm{erf}(x\sqrt{a}) - \frac{x}{2a}e^{-ax^2} \tag{62}$$

Integrals with Trigonometric Functions

$$\int \sin ax dx = -\frac{1}{a}\cos ax \tag{63}$$

$$\int \sin^2 ax dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \tag{64}$$

$$\int \sin^n ax dx = -\frac{1}{a}\cos ax\ {}_2F_1\left[\frac{1}{2},\frac{1-n}{2},\frac{3}{2},\cos^2 ax\right] \tag{65}$$

$$\int \sin^3 ax dx = -\frac{3\cos ax}{4a} + \frac{\cos 3ax}{12a} \tag{66}$$

$$\int \cos ax dx = \frac{1}{a}\sin ax \tag{67}$$

$$\int \cos^2 ax dx = \frac{x}{2} + \frac{\sin 2ax}{4a} \tag{68}$$

$$\int \cos^p ax dx = -\frac{1}{a(1+p)}\cos^{1+p}ax \times {}_2F_1\left[\frac{1+p}{2},\frac{1}{2},\frac{3+p}{2},\cos^2 ax\right] \tag{69}$$

$$\int \cos^3 ax dx = \frac{3\sin ax}{4a} + \frac{\sin 3ax}{12a} \tag{70}$$

$$\int \cos ax\sin bx dx = \frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)}, a \neq b \tag{71}$$

$$\int \sin^2 ax\cos bx dx = -\frac{\sin[(2a-b)x]}{4(2a-b)} + \frac{\sin bx}{2b} - \frac{\sin[(2a+b)x]}{4(2a+b)} \tag{72}$$

$$\int \sin^2 x\cos x dx = \frac{1}{3}\sin^3 x \tag{73}$$

$$\int \cos^2 ax\sin bx dx = \frac{\cos[(2a-b)x]}{4(2a-b)} - \frac{\cos bx}{2b} - \frac{\cos[(2a+b)x]}{4(2a+b)} \tag{74}$$

$$\int \cos^2 ax\sin ax dx = -\frac{1}{3a}\cos^3 ax \tag{75}$$

$$\int \sin^2 ax\cos^2 bx dx = \frac{x}{4} - \frac{\sin 2ax}{8a} - \frac{\sin[2(a-b)x]}{16(a-b)} + \frac{\sin 2bx}{8b} - \frac{\sin[2(a+b)x]}{16(a+b)} \tag{76}$$

$$\int \sin^2 ax\cos^2 ax dx = \frac{x}{8} - \frac{\sin 4ax}{32a} \tag{77}$$

$$\int \tan ax dx = -\frac{1}{a}\ln\cos ax \tag{78}$$

$$\int \tan^2 ax dx = -x + \frac{1}{a}\tan ax \tag{79}$$

$$\int \tan^n ax dx = \frac{\tan^{n+1}ax}{a(1+n)} \times {}_2F_1\left(\frac{n+1}{2},1,\frac{n+3}{2},-\tan^2 ax\right) \tag{80}$$

$$\int \tan^3 ax dx = \frac{1}{a}\ln\cos ax + \frac{1}{2a}\sec^2 ax \tag{81}$$

$$\int \sec x dx = \ln|\sec x + \tan x| = 2\tanh^{-1}\left(\tan\frac{x}{2}\right) \tag{82}$$

$$\int \sec^2 ax dx = \frac{1}{a}\tan ax \tag{83}$$

$$\int \sec^3 x\,\mathrm{dx} = \frac{1}{2}\sec x\tan x + \frac{1}{2}\ln|\sec x + \tan x| \tag{84}$$

$$\int \sec x\tan x dx = \sec x \tag{85}$$

$$\int \sec^2 x\tan x dx = \frac{1}{2}\sec^2 x \tag{86}$$

$$\int \sec^n x\tan x dx = \frac{1}{n}\sec^n x, n \neq 0 \tag{87}$$

$$\int \csc x dx = \ln\left|\tan\frac{x}{2}\right| = \ln|\csc x - \cot x| + C \tag{88}$$

$$\int \csc^2 ax dx = -\frac{1}{a}\cot ax \tag{89}$$

$$\int \csc^3 x dx = -\frac{1}{2}\cot x\csc x + \frac{1}{2}\ln|\csc x - \cot x| \tag{90}$$

$$\int \csc^n x\cot x dx = -\frac{1}{n}\csc^n x, n \neq 0 \tag{91}$$

$$\int \sec x\csc x dx = \ln|\tan x| \tag{92}$$

Products of Trigonometric Functions and Monomials

$$\int x\cos x dx = \cos x + x\sin x \tag{93}$$

$$\int x\cos ax dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax \tag{94}$$

$$\int x^2\cos x dx = 2x\cos x + \left(x^2-2\right)\sin x \tag{95}$$

$$\int x^2\cos ax dx = \frac{2x\cos ax}{a^2} + \frac{a^2x^2-2}{a^3}\sin ax \tag{96}$$

$$\int x^n cosx dx = -\frac{1}{2}(i)^{n+1}\left[\Gamma(n+1,-ix) +(-1)^n\Gamma(n+1,ix)\right] \tag{97}$$

$$\int x^n cosax dx = \frac{1}{2}(ia)^{1-n}\left[(-1)^n\Gamma(n+1,-iax) -\Gamma(n+1,ixa)\right] \tag{98}$$

$$\int x\sin x dx = -x\cos x + \sin x \tag{99}$$

$$\int x\sin ax dx = -\frac{x\cos ax}{a} + \frac{\sin ax}{a^2} \tag{100}$$

$$\int x^2\sin x dx = \left(2-x^2\right)\cos x + 2x\sin x \tag{101}$$

$$\int x^2\sin ax dx = \frac{2-a^2x^2}{a^3}\cos ax + \frac{2x\sin ax}{a^2} \tag{102}$$

$$\int x^n\sin x dx = -\frac{1}{2}(i)^n\left[\Gamma(n+1,-ix) - (-1)^n\Gamma(n+1,-ix)\right] \tag{103}$$

Products of Trigonometric Functions and Exponentials

$$\int e^x\sin x dx = \frac{1}{2}e^x(\sin x - \cos x) \tag{104}$$

$$\int e^{bx}\sin ax dx = \frac{1}{a^2+b^2}e^{bx}(b\sin ax - a\cos ax) \tag{105}$$

$$\int e^x\cos x dx = \frac{1}{2}e^x(\sin x + \cos x) \tag{106}$$

$$\int e^{bx}\cos ax dx = \frac{1}{a^2+b^2}e^{bx}(a\sin ax + b\cos ax) \tag{107}$$

$$\int xe^x\sin x dx = \frac{1}{2}e^x(\cos x - x\cos x + x\sin x) \tag{108}$$

$$\int xe^x\cos x dx = \frac{1}{2}e^x(x\cos x - \sin x + x\sin x) \tag{109}$$

Integrals of Hyperbolic Functions

$$\int \cosh ax dx = \frac{1}{a}\sinh ax \tag{110}$$

$$\int e^{ax}\cosh bx dx = \begin{cases} \dfrac{e^{ax}}{a^2-b^2}[a\cosh bx - b\sinh bx] & a \neq b \\ \dfrac{e^{2ax}}{4a} + \dfrac{x}{2} & a = b \end{cases} \tag{111}$$

$$\int \sinh ax dx = \frac{1}{a}\cosh ax \tag{112}$$

$$\int e^{ax}\sinh bx dx = \begin{cases} \dfrac{e^{ax}}{a^2-b^2}[-b\cosh bx + a\sinh bx] & a \neq b \\ \dfrac{e^{2ax}}{4a} - \dfrac{x}{2} & a = b \end{cases} \tag{113}$$

$$\int e^{ax}\tanh bx dx = \begin{cases} \dfrac{e^{(a+2b)x}}{(a+2b)}{}_2F_1\left[1+\dfrac{a}{2b},1,2+\dfrac{a}{2b},-e^{2bx}\right] - \dfrac{1}{a}e^{ax}{}_2F_1\left[\dfrac{a}{2b},1,1E,-e^{2bx}\right] & a \neq b \\ \dfrac{e^{ax}-2\tan^{-1}[e^{ax}]}{a} & a = b \end{cases} \tag{114}$$

$$\int \tanh ax\,dx = \frac{1}{a}\ln\cosh ax \tag{115}$$

$$\int \cos ax\cosh bx dx = \frac{1}{a^2+b^2}\left[a\sin ax\cosh bx +b\cos ax\sinh bx\right] \tag{116}$$

$$\int \cos ax\sinh bx dx = \frac{1}{a^2+b^2}\left[b\cos ax\cosh bx+ a\sin ax\sinh bx\right] \tag{117}$$

$$\int \sin ax\cosh bx dx = \frac{1}{a^2+b^2}\left[-a\cos ax\cosh bx+ b\sin ax\sinh bx\right] \tag{118}$$

$$\int \sin ax\sinh bx dx = \frac{1}{a^2+b^2}\left[b\cosh bx\sin ax- a\cos ax\sinh bx\right] \tag{119}$$

$$\int \sinh ax\cosh ax dx = \frac{1}{4a}\left[-2ax + \sinh 2ax\right] \tag{120}$$

$$\int \sinh ax\cosh bx dx = \frac{1}{b^2-a^2}\left[b\cosh bx\sinh ax -a\cosh ax\sinh bx\right] \tag{121}$$