Radiation Physics

A. Basics of Radiation

A.1. Cross section

- The number of interactions detected can be calculated as:

$$N_{\text{detector}} = I \cdot n \cdot x \cdot \epsilon \cdot \sigma_{\text{total}}, \tag{1}$$

where:
- $I = \frac{dN_{\text{incident}}}{dt}$ is the incident particle flux (rate of incident particles),
- $n = \frac{dN_{\text{scatter}}}{dV} = \frac{N_{AV}}{A}\rho$ is the number density of scatterers, with:

$$\rho = \text{Material density}, \quad N_{AV} = \text{Avogadro's number},$$

$$A = \text{atomic weight of the material.}$$

- $\epsilon = \epsilon_{\text{intrinsic}} \cdot \epsilon_{\text{geo}}$ is the detection efficiency, consisting of:

$$\epsilon_{\text{geo}} = \frac{S}{d^2},$$

where S is the sensitive area of the detector and d is the distance between the detector and the interaction point. - σ_{total} is the total cross section, which can be expressed as an integral over the solid angle:

$$\sigma_{\text{total}} = \int \frac{d\sigma(\theta, E)}{d\Omega}\, d\Omega,$$

where $\frac{d\sigma(\theta,E)}{d\Omega}$ represents the differential cross section as a function of angle θ and energy E.

- Assume an interaction: $A(a, b)B$
  - I_a = current of incident particles a; number of particles per unit time
  - N_T = number of target nuclei per unit area
  - R_b = number of detected particles b per unit time (reaction rate)

$$R_b \propto I_a N_T$$

Define the **cross section** σ:

$$\sigma = \frac{R_b}{I_a N_T}$$

The cross section σ has the dimension of **area**. The standard unit for cross section is the *barn*:

$$1\ \text{barn} = 10^{-24}\ \text{cm}^2 = 100\ \text{fm}^2$$

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$$I = I_0 e^{-\sigma n x} = I_0 e^{-\mu x}, \tag{2}$$

where I_0 is the initial intensity of the incident radiation, x is the thickness of the material, and μ is the linear attenuation coefficient, defined as $\mu = \sigma n$.

- For a mixture or compound, the mass attenuation coefficient $\left(\frac{\mu}{\rho}\right)$ can be expressed as:

$$\left(\frac{\mu}{\rho}\right) = \sum w_i \left(\frac{\mu}{\rho}\right)_i, \tag{3}$$

A.2. Lab coordinate and Center-Of-Mass coordinate

- **Angle Transformation:**

$$\tan(\theta_{\text{lab}}) = \frac{\sin(\theta_{\text{cm}})}{\gamma + \cos(\theta_{\text{cm}})}$$

- **Solid Angle Transformation:**

$$d\Omega_{\text{lab}} = \frac{|1 + \gamma\cos(\theta_{\text{cm}})|}{(1 + \gamma^2 + 2\gamma\cos(\theta_{\text{cm}}))^{3/2}}\, d\Omega_{\text{cm}}$$

- **Energy Transformation:**

$$E_{\text{cm}} = \frac{M_a}{M_a + M_A} E_{\text{lab}}$$

- **Definition of γ:**

$$\gamma = \sqrt{\frac{M_a M_b}{M_A M_B} \cdot \frac{E_{\text{cm}}}{E_{\text{cm}} + Q}} \approx \frac{M_a}{M_A}$$

B. Interaction of Photon and matter

B.1. Photoelectric effect

- **Moseley's Law**:

$$E_n = Rhc\frac{(Z - s_n)^2}{n^2}$$

where:
  - R is the Rydberg constant,
  - Z is the atomic number of the element,
  - s_n is the screening constant that accounts for the shielding effect of inner electrons,
- **Photoelectron Kinetic Energy**:

$$K_{e^-} = E_\gamma - B_n$$

- **Cross-Section Dependence**:

$$\sigma \propto Z^4 \cdot E_\gamma^{-3}$$

B.2. Scattering

- **Rayleigh Scattering**
  - Type: Elastic scattering, Coherent
  - Interaction Mechanism: Interaction of photons with bound electrons in an atom without energy transfer. It occurs when the incident photon's wavelength is much larger than the size of the particle.
  - Differential Cross Section:

$$\frac{d\sigma}{d\Omega} \propto r_0^2 \left(\frac{1 + \cos^2\theta}{2}\right) \cdot \left[F\left(\frac{\sin(\theta/2)}{\lambda}, Z\right)\right]^2 \cdot 2\pi\sin\theta$$

where $F\left(\frac{\sin(\theta/2)}{\lambda}, Z\right)$ is the atomic form factor that depends on the scattering angle and atomic number Z. This factor accounts for the constructive interference of scattered waves, leading to coherent scattering.

- **Thomson Scattering**
  - Type: Elastic scattering, Coherent
  - Interaction Mechanism: Scattering of low-energy photons by free or loosely bound electrons. It occurs in the limit of low photon energy, where the electron remains non-relativistic.
  - Differential Cross Section:

$$\frac{d\sigma}{d\Omega} = r_0^2 \left(\frac{1 + \cos^2\theta}{2}\right)$$

This describes the scattering of electromagnetic waves off free electrons, assuming no change in photon energy.

- **Compton Scattering**
  - Type: Inelastic scattering, Incoherent
  - Interaction Mechanism: Photon-electron interaction causing an energy transfer from the photon to the electron, resulting in a lower-energy scattered photon and a recoiling electron. It is significant at higher photon energies where energy transfer cannot be ignored.
  - Differential Cross Section: The differential cross-section for Compton scattering is given by the Klein–Nishina formula:

$$\frac{d\sigma}{d\Omega} = r_0^2 \cdot \frac{1 + \cos^2\theta}{2} \cdot \left[\frac{1}{1 + \alpha(1 - \cos\theta)}\right]^2 \cdot$$

$$\left[1 + \frac{\alpha^2(1 - \cos\theta)^2}{(1 + \cos^2\theta)(1 + \alpha(1 - \cos\theta))}\right]$$

where $\alpha = \frac{E_\gamma}{m_e c^2}$ is the normalized photon energy. This formula reflects the dependence of the cross-section on the scattering angle θ and the photon's initial energy E_γ. The energy of the scattered photon E'_γ is given by:

$$E'_\gamma = \frac{E_\gamma}{1 + \frac{E_\gamma}{m_e c^2}(1 - \cos\theta)}$$

The Compton shift, which represents the wavelength change due to scattering, is expressed as:

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$

B.3. Pair/Triple production

- **Threshold Energy**:
  - For **Pair Production**, the photon energy must exceed the combined rest mass energy of an electron-positron pair:

$$h\nu_{\text{threshold}} = 2m_e c^2 \approx 1.022\ \text{MeV}$$

  - For **Triple Production**, involving an additional electron, the threshold energy is:

$$h\nu_{\text{threshold}} = 4m_e c^2 \approx 2.044\ \text{MeV}$$

- **Cross-Section Approximation**: The cross-section σ for pair and triple production processes is approximately proportional to the square of the atomic number Z and the logarithm of the incident photon energy E_γ:

$$\sigma \approx Z^2 \ln(E_\gamma + C)$$

where C is a constant that depends on the material properties and interaction parameters. This logarithmic dependence indicates that the cross-section increases with increasing photon energy but at a decreasing rate.

B.4. Photon-Nuclear Interaction

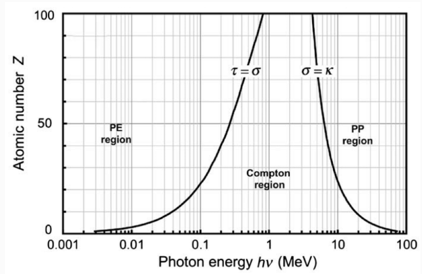
- **Description**: Photon-nuclear interactions occur when high-energy photons (typically above a few MeV) interact with a nucleus, causing nuclear reactions. These interactions can result in the emission of particles such as neutrons, protons, or even alpha particles. The most common photon-nuclear processes include the *photo-neutron* and *photo-proton* reactions.
- **Photonuclear Reactions**: The general form of photonuclear reactions can be represented as:

$$\gamma + {}^A_Z X \rightarrow {}^{A-1}_Z X + n$$

or

$$\gamma + {}^A_Z X \rightarrow {}^{A-1}_{Z-1} Y + p$$

where:
  - γ represents the incident high-energy photon,
  - ${}^A_Z X$ is the target nucleus with atomic number Z and mass number A,
  - n denotes the emitted neutron, and p denotes the emitted proton,
  - ${}^{A-1}_Z X$ and ${}^{A-1}_{Z-1} Y$ are the resulting daughter nuclei.



C. Interaction of charged particle and matter

C.1. Stopping Power and Collision Types

- **Stopping Power**:
  - **Collision Stopping Power**: This represents the energy loss due to inelastic collisions with atomic electrons. These collisions lead to ionization and excitation of atoms within the material.
  - **Radiation Stopping Power**: This accounts for energy loss due to the emission of bremsstrahlung radiation, which occurs when a charged particle is decelerated by the electric field of a nucleus.

$$S = \frac{dE}{dx} = S_{\text{rad}} + S_{\text{col}}$$

- **Collision Types**:
  - **Close/Hard Collisions**: $b \approx a$
  - **Far/Soft Collisions**: $b \gg a$
  - **Radiation/Bremsstrahlung Collisions**: $b \ll a$

C.2. Stopping Power Concepts

- **Bohr Classical Formula**: A model describing the energy loss of charged particles due to Coulomb interactions with atomic electrons. The formula gives an approximation for the stopping power at lower energies.

$$S_{\text{col}} = 4\pi \frac{Z N_A}{A}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \frac{z^2}{m_e v^2} \ln\frac{b_{\max}}{b_{\min}}$$

$$S_{\text{col}} = 2\pi \frac{Z N_A}{A}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \frac{z^2}{m_e v^2} \ln\frac{2mv^2}{I}$$

- **Bethe Equation**: The Bethe formula describes the stopping power of charged particles moving through matter. It is given by:

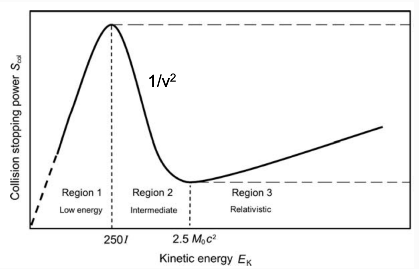
$$S_{\text{col}} = S^{\text{hard}} + S^{\text{soft}}$$

$$= 4\pi \frac{Z N_A}{A}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \frac{z^2}{m_e c^2 \beta^2}\left\{\ln\frac{2m_e c^2}{I} + \ln\frac{\beta^2}{1 - \beta^2} - \beta^2\right\}$$

where I is the mean excitation potential.

- **Fano Correction**: A correction term applied to the Bethe equation to account for deviations from the classical assumptions in the stopping power calculation, particularly for electrons and positrons.

$$4\pi \frac{N_A}{A}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \frac{z^2 Z}{m_e c^2 \beta^2}\left\{\ln\frac{2m_e c^2}{I} + \ln\frac{\beta^2}{1 - \beta^2} - \beta^2 - \frac{C}{Z} - \delta\right\}$$



- **Mass Stopping Power**: The stopping power normalized by the density of the absorbing material, for compounds:

$$\frac{1}{\rho}\frac{dE}{dx} = \sum_i w_i \frac{1}{\rho_i}\left(\frac{dE}{dx}\right)_i$$

$$w_i = (\alpha_i A_i) \Big/ \sum_i \alpha_i A_i$$

- **CSDA Range**: The Continuous Slowing Down Approximation (CSDA) range is the total distance a charged particle travels in a material as it loses energy continuously. It is calculated by integrating the reciprocal of the stopping power:

$$R_{\text{CSDA}} = \int_0^{(E_K)_0} \frac{dE}{S_{\text{tot}}(E)} = \int_{E_0}^0 \frac{dx}{dE}\, dE = \int_{E_0}^0 \frac{1}{(dE/dx)}\, dE [\text{cm}]$$

or

$$R_{m\_CSDA} = \int_{E_0}^0 \frac{d\xi}{dE}\, dE = \int_{E_0}^0 \frac{1}{(dE/d\xi)}\, dE \quad [\text{g/cm}^2]$$

- **Substitution Rule**: For two different materials with charges Z_1, Z_2, and masses m_1, m_2, the range can be related as:

$$\left(\frac{dE}{d\xi}\right)\Big|_{(E_2)} = \frac{z_2^2}{z_1^2}\left(\frac{dE}{d\xi}\right)\Big|_{(E_1)}$$

$$R_2(E_2) = \frac{m_2 Z_1^2}{m_1 Z_2^2} R_1(E_1), \quad \text{where} \quad E_1 = E_2 \frac{m_1}{m_2}$$

- **Bragg-Kleeman Rule**:

$$\frac{(R_1/\rho_1)}{(R_2/\rho_2)} = \frac{\sqrt{A_1}}{\sqrt{A_2}}$$

- **Critical Energy** (E_c): The critical energy is the energy at which the energy loss due to radiation (bremsstrahlung) becomes equal to the energy loss due to collisions. It is given by:

$$\frac{(dE/dx)_{\text{rad}}}{(dE/dx)_{\text{col}}} \approx \frac{E_k Z}{800}, (E_k)_{\text{critical}} = \frac{800}{Z}$$

- **Radiation Yield** (Y): The fraction of the total energy loss due to radiation as opposed to collisions. For electrons, the yield is approximated by:

$$Y = \frac{E_{\text{rad}}}{E_{\text{total}}} \approx \frac{6\times10^{-4}ZT}{1+6\times10^{-4}ZT}$$

where Z is the atomic number of the material and T is the kinetic energy of the particle.

D. Neutron physics

D.1. Energy Classifications of Neutrons

Neutrons can be classified based on their energy levels into several categories:

- **Slow (Cold) Neutrons**: Energy Range: $0 - 0.005$ eV
- **Thermal Neutrons**: Energy Range: $0.005 - 0.5$ eV
- **Epithermal Neutrons**: Energy Range: $0.5 - 1000$ eV
- **Intermediate Neutrons**: Energy Range: $1 - 100$ keV
- **Fast Neutrons**: Energy Range: $0.1 - 10$ MeV

Neutrons also interact differently with target nuclei based on their mass number A:

- **Light Nuclei**: Mass Number: $A < 25$
- **Medium Nuclei**: Mass Number: $25 < A < 80$
- **Heavy Nuclei**: Mass Number: $A > 80$

D.2. Neutron Sources

Neutrons can be sourced from various processes, including:

- **Fission**: Neutrons are produced during the fission of heavy nuclei.
- **Particle Accelerators**: High-energy particle collisions can produce neutrons.
- **(α,n) Reactions**: Neutrons are emitted when alpha particles interact with certain materials.
- **(γ,n) Reactions**: Neutrons are produced when gamma rays interact with certain nuclei.

D.3. Neutron Reactions

Neutrons can undergo various reactions, primarily classified into scattering and absorption:

- **Scattering**:
  - **Elastic Scattering (n, n)**:

$$\Delta E_{K\max} = (E_K)_i \frac{4m_n M}{(m_n+M)^2}$$

Kinetic energy of the scattered neutron

$$(E_K)_f = (E_K)_i - \Delta E_{K\max} = (E_K)_i (\frac{M-m_n}{M+m_n})^2$$

For certain angle :$\Delta E_K = (E_K)_i \frac{4m_n M}{(m_n+M)^2}\cos^2\theta$. The average energy transfer is given by:

$$\langle \Delta E \rangle = \frac{\Delta E_{K\max}}{2}$$

The average kinetic energy attained by the scattered neutron is

$$(E_K)_f = (E_K)_i - \langle \Delta E \rangle = (E_K)_i \frac{m_n^2+M^2}{(m_n+M)^2}$$

  - **Inelastic Scattering (n, n' γ)**: In this reaction, a neutron is scattered and energy is transferred to the nucleus, resulting in gamma radiation.

- **Absorption**:
  - **Fission**: Absorption of a neutron can lead to the fission of heavy nuclei.
  - **Radiative Capture (n,γ)**: A neutron is captured by a nucleus, emitting gamma radiation.
  - Other reactions include:(n, p); (n, d); (n, α); (n, np); (n, 2n); (n, 3n); (n, f)
  - Compound nucleus formation $a + A \to C^* \to B + b$

$$mc^2 + Mc^2 + T = M_{CN}c^2 + E_{k=4}$$

  - Partial decay lifetimes of compound nucleus states

$$\Gamma = \Gamma_{n,n} + \Gamma_{n,n'\gamma} + \Gamma_{n,\gamma} + \Gamma_{n,p} + \cdots$$

Relative probability of radiative capture is therefore $\frac{\Gamma_{n,\gamma}}{\Gamma}$
- For light nuclei, $\Gamma_{n,n} \gg \Gamma_{n,\gamma}$
- For heavy nuclei, $\Gamma_{n,\gamma} \gg \Gamma_{n,n}$

Neutron width increases with energy of resonance: $\Gamma_{n,n} = \Gamma_{n,0}\sqrt{E}$

  - The formula for the Breit-Wigner cross section

$$\sigma_{CN} = \pi\lambda^2(2\ell-1)\frac{\Gamma_a\Gamma}{(E-E_R)^2 + \left(\frac{\Gamma}{2}\right)^2}$$

The decay in to entity b is expressed:

$$\sigma_{ab} = \sigma_{CN}\frac{\Gamma_b}{\Gamma}$$

The energy-dependent cross-sections for compound elastic scattering is given by:

$$\sigma_{n,n} = \pi\lambda^2 \frac{\Gamma_{n,n}^2}{(E-E_R)^2 + \left(\frac{\Gamma}{2}\right)^2}$$

Cross sections for radiative capture:

$$\sigma_{n,\gamma} = \pi\lambda^2 \frac{\Gamma_{n,n}\Gamma_{n,\gamma}}{(E-E_R)^2 + \left(\frac{\Gamma}{2}\right)^2}$$

  - When $E = E_R$, $\sigma_{max} = 4\pi\lambda^2\frac{\Gamma_{n,n}\cdot\Gamma_{n,\gamma}}{\Gamma^2}$,

When $E = E_R \pm \frac{\Gamma}{2}, \sigma = \frac{1}{2}\sigma_{max}$

When $E \to 0, \Gamma \ll E_R$, $\sigma_{n,\gamma} = \pi\lambda_R^2\frac{\Gamma_{n,n}\Gamma_{n,\gamma}}{E_R^2}\frac{1}{\sqrt{E}} \propto \frac{1}{v}$

E. X-ray production

Energy Range	X-ray Type	Typical Application
0.1 - 20 kV	**Soft X-rays**	Microscopy, surface analysis
20 - 150 kV	**Diagnostic X-rays**	Medical imaging, diagnostic
150 - 300 kV	**Orthovoltage X-rays**	Superficial cancer treatment
300 kV – 1 MV	**Intermediate Energy X-rays**	Therapeutic, deeper tissue treatment
> 1 MV	**Megavoltage X-rays**	High-energy cancer treatment

- Bremsstrahlung Radiation Power

$$P = \frac{\mu_0 q^2 a^2}{6\pi c}$$

- Cross-section of Bremsstrahlung

$$\frac{d\sigma}{dE} \propto \frac{Z^2}{m^2}\frac{ln^2 E}{E}$$

- Kramer's law

$$I(E_\gamma) = KZ(T_e - E_\gamma)$$

- Efficiency Estimation

$$\frac{S_{\text{rad}}}{S_{\text{col}}} \approx \frac{E_k Z}{820} \quad (E_k \text{ in MeV})$$

$$P_{\text{deposited}} = IV, P_{\text{radiated}} = 0.9\times10^{-9}ZV^2 I$$

$$\varepsilon = 0.9\times10^{-9}ZV$$

F. Nuclear radiation and radioactive decay

F.1. Nuclear Binding Energy and Q-value

- **Nuclear Binding Energy**:

$$\frac{B}{A} = -(M(A,Z) - (A-Z)m_n - Z(m_p+m_e))$$

$$= -(-a_1 A + a_2 A^{\frac{2}{3}} + a_3\frac{\left(\frac{A}{2}-Z\right)^2}{A} + a_4\frac{Z^2}{A^{\frac{1}{3}}} + a_5\frac{\delta}{A^{3/4}})/A$$

$$M(A,Z) = (A-Z)m_n + Z(m_p+m_e) - B$$

- **Q-value**:

$$Q = \sum_{i,before} M_i c^2 - \sum_{i,after} M_i c^2 = \sum_{i,after} B_i - \sum_{i,before} B_i$$

$$= \sum_{i,before}\Delta_i - \sum_{i,after}\Delta_i -$$

where $\Delta[\text{MeV}] = M(A,Z)c^2 - Am_{\text{amu}}c^2$.
- If $Q > 0$, it indicates an exothermic reaction (energy release).
- If $Q < 0$, it indicates an endothermic reaction (energy absorption).

F.2. Decay Type

- **Alpha Decay (α decay)**:

$$P \to D + \text{He}, \quad Q = \Delta_P - \Delta_D - \Delta_{\text{He}}$$

- **Beta Decay (β decay)**:
  - For β^- decay:

$$P \to D + e^- + \bar{\nu}, \quad Q = \Delta_P - \Delta_D$$

  - For β^+ decay:

$$P \to D + e^+ + \nu, \quad Q = \Delta_P - \Delta_D - 2m_e c^2$$

  - For electron capture:

$$P + e^- \to D + \nu, \quad Q = \Delta_P - \Delta_D - BE(e^-)$$

- **Gamma Decay (γ decay)**:

$$X^m \to X + \gamma$$

F.3. Radioactivity

- **Activity**:

$$A = \lambda N$$

- **Exponential Decay**:

$$N(t) = N_0 e^{-\lambda t}$$

- **Mean Life**

$$\tau = \frac{1}{\lambda} = \frac{T}{\ln 2}$$

- **Special Activity**

$$\text{SA} = \frac{6.02\times10^{23}\lambda}{M} = \frac{4.17\times10^{23}}{MT}.$$

F.4. Serial Radioactive Decay

- **Decay Chain**: A radioactive nuclide decays into another nuclide, forming a decay chain, represented as:

$$N_1 \xrightarrow{\lambda_1} N_2 \xrightarrow{\lambda_2} N_3$$

- **Equilibrium States**:
  - **Secular Equilibrium**: $T_1 \gg T_2$,

$$A_2 = A_1(1 - e^{-\lambda_2 t}) + A_{20}e^{-\lambda_2 t}$$

  - **Transient Equilibrium**: $T_1 \geq T_2$,

$$N_2 = \frac{\lambda_1 N_{10}}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t}), A_2 = \frac{\lambda_2 A_1}{\lambda_2 - \lambda_1}$$

  - **No Equilibrium**: $T_1 < T_2$,

$$N_2 = \frac{\lambda_1 N_{10}}{\lambda_1 - \lambda_2}(e^{-\lambda_2 t} - e^{-\lambda_1 t})$$

G. Counting Statistics and Error Analysis

G.1. General Aspects on Radiation Measurement and Data

- **Fundamental Data:** The core data in radiation measurement are counts, which serve as indicators of radiation events.
- **Types of Errors:**
  - **Systematic Errors (Bias)**: These impact accuracy due to fixed biases, e.g., calibration errors.
  - **Random Errors (Precision)**: Variability arising from factors like electronic noise, minimized by large samples.
- **Accuracy vs Precision:** Accuracy reflects closeness to the true value, while precision indicates measurement repeatability.

G.2. Statistical Methodology

- **Descriptive Statistics**: Central tendency measures (mean, median, mode) and variability (variance, standard deviation).
- **Inferential Statistics**: Used to draw conclusions about population parameters, employing sample data.

G.3. Statistical Models for Counting Statistics

- **Binomial Distribution**: Suitable for processes with two outcomes, defined by the probability of success p and number of trials n.
- **Poisson Distribution**: Used when the event probability is low and trials are numerous, approximates binomial for rare events, e.g., radiation counting.
- **Gaussian (Normal) Distribution**: Approximates distribution with a large sample size or high event mean.

G.4. Error Estimation and Propagation

- **Single Measurement Error**: Variance for a Poisson-distributed process is $\sigma^2 = \mu$.
- **Multiple Measurements**:

$$SE = \frac{\sigma}{\sqrt{N}}$$

- **Error Propagation Formula**:

$$\sigma_u = \sqrt{\left(\frac{\partial u}{\partial x}\sigma_x\right)^2 + \left(\frac{\partial u}{\partial y}\sigma_y\right)^2 + \cdots}$$

- **Chi-Square Test**: Used to test the "goodness of fit" of observed data to expected distributions.

G.5. Applications of Counting Statistics

- **Limits of Detectability**:
  - **ROC Curves**: Analyzes binary decision accuracy using true positive/negative rates.
  - **Minimum Detectable Amount (MDA)**: Defines the smallest reliably detectable signal.
- **Pulse Time Interval Statistics**: Examines the intervals between pulses to interpret random decay event distributions.

	Time intervals	**Counts**
Distribution Function	Erlang: $I_n(t) = \frac{(rt)^{(n-1)}e^{-rt}}{(n-1)!}$	Poisson: $P(n;rt) = \frac{(rt)^n e^{-rt}}{n!}$
Type	Continuous	Discrete
Variable	Time (t)	Count number (n)
Mean	$\tau_n = \frac{n}{r}$	$\bar{n} = rt$
Variance	$\sigma_E^2 = \frac{\tau_n^2}{n} = \frac{n}{r^2}$	$\sigma_P^2 = \bar{n} = rt$

H. General Properties of detector

H.1. Detector working mode

- **Pulse Mode**
  - **Case 1: Small RC Time Constant** ($t \ll \tau_c$)

$$V(t) = R \times i(t)$$

Here, $i(t)$ represents the instantaneous current at time t, and R is the resistance. This is the fast response case, capturing rapid changes in current.

  - **Case 2: Large RC Time Constant** ($t \gg \tau_c$)

$$V_{\max} = \frac{Q}{C}$$

where Q is the charge collected during the pulse and C is the capacitance. In this case, the circuit integrates the current over time.

- ➡ **Current Mode**
  The total current can be expressed as:

$$I(t) = I_0 \pm \sigma_I(t), \quad I_0 = rQ = r\frac{E}{wq_e}$$

The mean squared value of the current fluctuations is given by:

$$\overline{\sigma_I^2}(t) = \frac{1}{T}\int_{t-T}^{t}\left[I(t') - I_0\right]^2 dt' = \frac{1}{T}\int_{t-T}^{t}\sigma_I^2(t')\,dt'$$

where T is the measurement time interval. This equation represents the time-averaged variance in current fluctuations.
From Poisson statistics, the standard deviation in the number of events n over time T is:

$$\sigma_n = \sqrt{n} = \sqrt{rT}$$

where $n = rT$ is the mean number of events in the time interval T. Thus, the fractional standard deviation in current is:

$$\frac{\overline{\sigma_I(t)}}{I_0} = \frac{\sigma_n}{n} = \frac{1}{\sqrt{rT}}$$

- ➡ **Mean Square Voltage (MSV) Mode**

$$\overline{\sigma_I^2}(t) = \frac{I_0^2}{rT} = \frac{(rQ)^2}{rT} = \frac{rQ^2}{T}$$

H.2. Key concepts on detector properties

- ➡ Pulse height spectra
  - ➡➡ Differential PHS:

$$N = \int_{H_1}^{H_2}\frac{dN}{dH}dH$$

  - ➡➡ Integral PHS:

$$N(H*) = \int_{H*}^{\infty}\frac{dN}{dH}dH$$

- ➡ Energy resolution:

$$\text{FWHM} = 2.35\sigma, R = \frac{\text{FWHM}}{N} = \frac{2.35}{\sqrt{N}}$$

- ➡ The Fano factor, F

$$F \equiv \frac{\text{observed variance in } N}{\text{Poisson predicted variance}(= N)}$$

Statistical resolution:

$$R|_{\text{statistical limit}} = 2.35\sqrt{\frac{F}{N}}$$

- ➡ Detector efficiency:

$$N_{obs} = N_{abs}\cdot\varepsilon_{tot} = N_{abs}\cdot W(\Theta,\Phi)\cdot\varepsilon_{geo}\cdot\varepsilon_{intrinsic}\cdot\varepsilon_{electronic}$$

- ➡ Dead time Assume: n = true event rate, m = measured events rate, T = average period between true events (=1/n), τ = system dead time
  - ➡➡ Nonparalyzable Model: fixed dead time

$$m = \frac{n}{1+n\tau}, n = \frac{m}{1-m\tau}$$

  - ➡➡ Paralyzable Model: extendable dead time

$$m = ne^{-n\tau}, n = -\frac{W(-m\tau)}{\tau}$$

Dead Time Measurement: two source method

$$n_{12} + n_b = n_1 + n_2$$

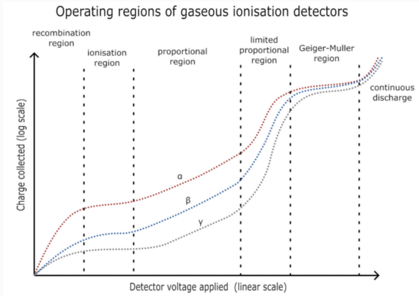
  - ➡➡ Nonparalyzable Model:

$$\tau = \frac{m_1 m_2 - \sqrt{m_1 m_2(m_{12}-m_1)(m_{12}-m_2)}}{m_1 m_2 m_{12}}$$

  - ➡➡ Paralyzable Model:

$$\tau = \frac{2m_{12}}{(m_1+m_2)^2}\ln\left(\frac{m_1+m_2}{m_{12}}\right)$$

I. Gas-filled Detector



- Ionization region: ionization chambers
- Proportional region: proportional counters
- Geiger-Mueller region: Geiger counters

I.1. Ionization Chambers

- ➡ Average number of ion pairs:

$$\langle n_T\rangle = \frac{L\cdot\left\langle\frac{dE}{dx}\right\rangle_i}{W_i}$$

- ➡ The drift velocity of charge in a gas

$$v = \mu\frac{\mathcal{E}}{p}$$

- ➡ - No internal gain (gain $\sim$ 1).
  - Slow drift velocity ($\nu_e \sim 10^3 m/s, \nu_{ion} \sim 1m/s$).
  - Charge collection time is too slow to count individual pulses.
  - Poor timing properties.
- ➡ Exposure: defined as the amount of ionization charge per unit mass of air

$$X \equiv \frac{Q}{M}$$

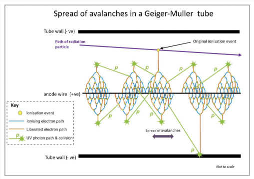
Convert exposure to dose $D = X \times 33.7$[Gy]

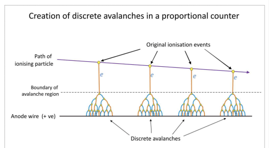
I.2. Proportional Counters

- ➡

$$\mathcal{E} = \frac{V}{r\ln(b/a)}$$

a = anode wire radius, b = cathode inner radius.
- ➡ nobel gas (such as Ar) + "quench gas" (such as CH4)





I.3. Geiger-Mueller Tubes(G-M counters)

- ➡ Chain avalanche leads to Geiger discharge
  – Primary Geiger discharge
  – Secondary Geiger discharge(UV photons)
- ➡ A cloud of positive ions surrounds the anode, with the effect of reducing the electric field intensity, leads the Geiger discharge termination – called Quenching effect.

Detector Type	Avalanche amplification	Good timing?	Energy information	Use as monitor
Ion chamber	none	no, ~10^{-2} s	none	yes
Prop. counter	10^3 - 10^5	yes, ~10^{-6} s	yes	no
G-M tube	~10^{10}	yes, ~10^{-6} s	none	yes

J. Semi-conductor Detector

J.1. Advantages of semiconductor detectors

- ➡ Semiconductor with moderate bandgap (1-2 eV)
- ➡ Thermal energy = 1/40 eV
- ➡ Energy to create e/h pair (signal quanta) w 3.6 eV (c.f gas: w = 35 eV)
- ➡ High charge carrier yield
- ➡ Better energy resolution and large signal
- ➡ High density and Z (comparing to gas)
- ➡ High carrier mobility: Fast (< 30 ns)

J.2. Basic properties of semiconductor

Majority of charge carriers provided by donors (impurities; doping)
– n-type:majority carriers are electrons(pentavalent dopants)
– p-type:majority carriers are positive holes(trivalent dopants)
• Pentavalent dopants (electron donors): P, As, Sb, ...
[5th electron only weakly bound; easily excited into conduction band]
• Trivalent dopants (electron acceptors): Al, B, Ga, In, ...
[One unsaturated binding; easily excepts valence electron leaving hole]
- ➡ PN Junction: depletion depth

$$d \approx \sqrt{\frac{2\varepsilon V}{eN}} = \sqrt{2\varepsilon V\mu\rho}, \quad V_{\text{bias}} = \frac{eNd^2}{2\epsilon}, \quad \rho = \frac{1}{e(\mu_n n + \mu_p p)}$$

$$C = \frac{\epsilon A}{d} = A\sqrt{\frac{e\epsilon N}{2V}}$$

- ➡ Noise sources: (ENC= Equivalent Noise Charge)
  Capacitance: ENC $\propto C_d$
  Leakage Current: ENC $\propto \sqrt{I}$
  Thermal Noise: ENC $\propto \sqrt{K_B T/R}$

J.3. Photodiode (PD)

- ➡ Cut-off wavelength vs Energy bandgap

$$\lambda_g[\mu m] = \frac{1.24}{E_{gap}[eV]}$$

- ➡ Photon attenuation

$$I(x) = I_0 e^{-\alpha x}$$

- ➡ Photocurrent and responsivity

$$I_{Ph} = q\left(\frac{P_0}{h\nu}\right)(1-r)(1-e^{-\alpha d}), \quad R = \frac{I_{ph}}{P_o} = \frac{q}{h\nu}(1-r)(1-e^{-\alpha d})$$

- ➡ Quantum Efficiency(external and internal)

$$\eta_e = \frac{I_{ph}/q}{P_o/h\nu} = (1-r)[1-e^{-\alpha d}], \eta_i = \frac{\eta_e}{(1-r)} = 1-e^{-\alpha d} \cong 1$$

- ➡ Avalanche Photodiodes: current gain and responsivity

$$M = \frac{\textit{Multiplied photocurrent}}{\textit{Primary photocurrent}} = \frac{I_M}{I_p}, R_{\text{APD}} = \frac{\eta q}{h\nu}M = R_0 M$$

- ➡ Response Time: transit time, diffuse time and RC time constant

$$t_d = \frac{W}{v_D}, t_{diff} = \frac{l^2}{2D_e}, \tau = R_T C_T$$

Type	PMT	APD	SiPM
Amplification	High (10^6)	Low (~10^2)	High (10^6)
Magnetic field	Sensitive	Not sensitive	Not sensitive
Compactness	bulky	compact	compact
Bias (V)	HV (600~1200)	HV (300-1500)	20-70
S/N ratio	High	Low	High
Time resolution	~1 ns	> 1 ns (2-4 ns)	<1 ns (~200ps)
Electronic readout	Voltage amplifier	Charge sensitive preamplifier	Voltage amplifier

K. Scintillation Detector

K.1. General characteristics of scintillation detector

• High stopping power, density and Z
• Sensitivity to radiation energy • Scintillation efficiency and light yield
• Fast time response • Pulse shape discrimination
• Radiation hardening • Usually expensive
- ➡ Stokes Shift: emitted photons are at longer wavelengths (smaller energies) than the energy gap of the excitation. This allows the scintillation light to propagate through the material. Emitted photons can't be self-absorbed by exciting the material again
- ➡ Light Output and Light Yield

$$L = L_0\left(e^{-t/\tau_d} - e^{-t/\tau_r}\right), Y = \int_0^\infty L(t)dt$$

- ➡ Scintillation Efficiency

$$\eta = \frac{Y\cdot\frac{hc}{\lambda}}{E}$$

K.2. Scintillators

- ➡ Inorganic Scintillators: Intrinsic(self-activated)/Extrinsic(activated)
- ➡ Efficiency for conversion of energy deposit to scintillation light

$$\eta = \beta SQ$$

where β– efficiency of energy conversion, S – efficiency of energy transfer, Q – quantum efficiency of luminescent centers
- ➡ Three mechanisms for scintillation
  - Excitions (bound e-h pair)
  - Defects (interstitial)
  - activators (doped impurities): trace mounts of activators are purposely introduced to create some narrower levels within the forbidden band gap. E.g. NaI(Tl), CsI(Tl)
- ➡ Organic Scintillators:low density; just C and H; photoelectric effect goes as Z^4
- ➡ Molecular states: • Singlet states (spin=0)
  – A series of levels (vibration states) – Gap between S_0 and S_1 is 3-4 eV
  • Triplet states (spin=1)
  Two scintillation processes:
  Fluorescence : $S_1 \to S_0$ [ns] / Phosphorescence: $T_0 \to S_0$ [ms]
- ➡ Light yield and Birk's law:

$$\frac{dL}{dx} = S\frac{dE}{dx}, \frac{dL}{dx} = \frac{S\frac{dE}{dx}}{1+kB\frac{dE}{dx}}$$

K.3. Photomultiplier Tube (PMT)

- ➡ Gain:

$$G = \delta^N = 10^6 - 10^8, \frac{dG}{G} = N\frac{dV_d}{V_d} = N\frac{dV_a}{V_a}$$

- ➡ Causes of dark current include:
  – Thermionic emission from photocathode and dynodes
  – Leakage current between anode and other electrodes
  – Photocurrent by scintillation from glass or electrode supports
  – Field emission current
  – Cosmic rays, radioactivity in glass envelope, radioactivity (gamma) from surroundings (cement)
- ➡ Energy Resolution: $N_e = xp\delta, \sigma_F^2(N_e) = \sigma_F^2(x) + \sigma_F^2(xp) + \sigma_F^2(xp\delta)$

$$\sigma_F^2\left(N_e\right) = \frac{1}{x} + \frac{1}{x}\frac{1-p}{p} + \frac{1}{xp}\frac{1}{(\delta-1)}$$

$$R_E = 2.35\sqrt{\frac{1}{x}\left(1+\frac{1-p}{p}+\frac{1}{p(\delta-1)}\right)}$$

Type		Mechanism: Interaction	Mechanism: Signal	Device	Energy Proportionality	Counting rate Performance	Temporal Information	Position Information
Gas-filled	Pulse mode	Ionization charges	Collect ions	Ion Chamber	Excellent	Low	Poor	Average
Gas-filled	Pulse mode	Ionization charges	Multiply & Collect ions	Prop. Chamber	Very good	↓	Average	Good
Gas-filled	Pulse mode	Ionization charges	Create discharge	GM Counter & Spark chamber	No	↓	Good to Excellent	Excellent*
Semi-conductor	Pulse mode	Ionization charges	Collect ions	Silicon Diode & CZT	Excellent	↓	Average	Excellent
Scintillator	Pulse mode	Excitation light	Convert light into electrons	Scintillation Detector	Acceptable	↓	Excellent	Poor*
Gas-filled	Current mode	Ionization charges	Collect current	Ion Chamber	Radiation Field	High	None	None