Contents

1 Introduction & Fundamentals of Medical Imaging

1.1 Major Medical Imaging Modalities

1.1.1 X-ray Imaging

X-ray imaging is based on the differential attenuation of X-ray photons as they pass through tissues. It includes both traditional film-based radiography and modern digital radiography.

- **Radiography (Screen/Film)**: Uses X-ray attenuation to provide anatomical information with a spatial resolution of approximately 0.1 mm.
- **Digital Radiography (DR)**: Offers similar anatomical insights with slightly lower resolution (0.2–0.5 mm) but enhanced digital processing capabilities.
- **Computed Tomography (CT)**: Provides cross-sectional (3D) anatomical images with spatial resolution around 0.5 mm. It is widely used in neurology, cardiology, pulmonology, and oncology.

1.1.2 Ultrasound Imaging

Ultrasound imaging employs high-frequency sound waves and measures the reflected signals to construct images.

- **B-mode and Doppler Ultrasound**: Provide both anatomical and physiological information based on wave reflection. Typical spatial resolution is 1 mm.
- **Applications**: Widely used in obstetrics, cardiology, urology, and gastroenterology.
- **Advantages**: Inexpensive, real-time, and non-invasive; however, image quality is typically lower compared to CT or MRI.

1.1.3 Nuclear Medicine Imaging

This modality visualizes the distribution of radioactively labeled substances within the body.

- **SPECT (Single Photon Emission Computed Tomography)**: Uses gamma rays emitted from isotopes to provide functional imaging with 10 mm spatial resolution.
- **PET (Positron Emission Tomography)**: Detects annihilation gamma rays from positron emitters (e.g., ^{11}C, ^{18}F) to provide functional and biochemical imaging with 5–8 mm resolution.
- **Comparison with CT**:
  - CT: Transmission imaging, high photon flux, anatomical.
  - SPECT/PET: Emission imaging, low photon flux, functional.

1.1.4 Magnetic Resonance Imaging (MRI)

MRI utilizes radiofrequency (RF) signals generated by nuclear magnetic resonance (NMR) of protons.

- **Contrast Mechanisms**: Proton density, T_1 and T_2 relaxation times.
- **Applications**: Neurological, cardiovascular, musculoskeletal, oncological imaging.
- **Resolution**: Sub-millimeter spatial resolution; provides excellent soft tissue contrast.

1.1.5 Emerging Imaging Modalities

New modalities are being developed to address limitations of traditional imaging and to explore novel contrast mechanisms.

- **Optical Imaging**: Uses visible or infrared light; provides anatomical information with 10 mm resolution.
- **Thermography**: Captures infrared radiation corresponding to thermal patterns; useful in physiological assessments.
- **Electrical Impedance Tomography (EIT)**: Maps conductivity/resistivity; functional resolution 10 mm.

1.2 Historical Development of Medical Imaging

1.2.1 Early Developments

- **1895**: Discovery of X-rays by Wilhelm Röntgen, initiating the field of radiographic imaging.
- **1940s–50s**: Foundational developments in ultrasound and gamma camera technologies.

1.2.2 Technological Milestones

- **1960s**: Emergence of CT and ultrasound imaging revolutionized diagnostics.
- **1970s**: Introduction of clinical X-ray CT and SPECT.
- **1980s**: Development of MRI and PET for advanced soft tissue and functional imaging.
- **1990s**: Hybrid systems such as SPECT/CT and PET/CT.
- **2000s–present**: Emphasis on molecular imaging and multimodal approaches (e.g., PET/MRI).

1.3 Trends and Future Directions

1.3.1 Healthcare Paradigm Shift

Modern imaging is increasingly driven by preventive care strategies emphasizing early detection, diagnosis, and treatment. This shift is reinforced by health policies such as Medicare.

1.3.2 Molecular Imaging

Molecular imaging (MI) aims to detect cellular and molecular-level abnormalities using targeted biomarkers, providing insights at the gene and protein level.

1.3.3 Technological Innovations

- **Multimodal Imaging**: Integration of modalities (e.g., PET/CT, PET/MRI) for comprehensive diagnosis.
- **Advanced Detectors and Sensors**: Improved sensitivity and specificity.
- **Quantitative Imaging**: From organ-level to molecular-level imaging.

1.3.4 Applications Beyond Diagnosis

Medical imaging now plays a critical role in:

- Therapy planning and monitoring (e.g., treatment planning system in radiation therapy).
- Drug development and evaluation.
- Personalized medicine.

1.4 What is an Image?

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1.4.1 Image as a Mapping Function

An image is a mapping of a 3D object into a 2D spatial domain using a projection function. It represents the spatial characteristics of the object as viewed by an imaging system, eventually perceived by human vision. The entire process involves analog-to-digital and then digital-to-analog conversion.

1.4.2 Why Digital Images?

Digital images allow for:

- Efficient storage and transmission
- Ease of processing and analysis
- Enhanced integration with computer-based systems

1.4.3 How Computers View Images

Computers represent a digital image as a matrix $M \times N$ of pixel values. Each pixel is a sampled value from a continuous function, with an associated quantized intensity. In 3D imaging, each unit is a voxel.

1.5 Digitization of Images

1.5.1 Sampling and Quantization

- **Sampling**: Discretizing spatial coordinates (x, y) into a grid
- **Quantization**: Discretizing amplitude (intensity) values into a finite set $[0, L-1]$

1.5.2 Sampling Grid and Function

Sampling uses a uniform rectilinear grid determined by the detector system. Only sampled grid points are retained from the continuous image function.

$$g_{(\Delta x, \Delta y)}(x, y) = \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \delta(x - m\Delta x, y - n\Delta y)$$

1.5.3 Quantization and ADC

Analog-to-Digital Converters (ADC) transform continuous amplitude into k-bit discrete levels. Quantization affects image fidelity and introduces quantization error.

Discrete intensity interval $[0, L-1]$, $L = 2^k$
L: intensity-level
k: Number of bits of the quantizer (bit per pixel, bpp)

1.6 Image Size and Storage

Image size is calculated as:

$$\text{Image Size} = M \times N \times bpp$$

where M, N are image dimensions and bpp is bits per pixel.

1.7 Types of Digital Images

1.7.1 Binary Images

Pixel values: 0 (black) or 1 (white)

1.7.2 Grayscale Images

8 bpp, 256 levels of gray

1.7.3 Colour Images

- 16 bpp: Basic color format
- 24 bpp: True color (8 bits each for RGB)

1.7.4 Colour Schemes

- **RGB (Red, Green, Blue)**: Additive color model used in digital displays such as monitors and cameras.
- **CMYK (Cyan, Magenta, Yellow, Black)**: Subtractive color model used in color printing.
- **HSV (Hue, Saturation, Value)**: Alternative representation of RGB often used in image analysis and computer vision.

Color Space Conversions:

- **RGB to CMYK** (assuming R, G, B in [0,1]):

$$K = 1 - \max(R, G, B)$$
$$C = \frac{1 - R - K}{1 - K}, \quad M = \frac{1 - G - K}{1 - K}, \quad Y = \frac{1 - B - K}{1 - K}$$

(if $K = 1$, then $C = M = Y = 0$)

- **RGB to HSV** (R, G, B in [0,1]):

$$\text{max} = \max(R, G, B), \quad \text{min} = \min(R, G, B), \quad \Delta = \text{max} - \text{min}$$

$$V = \text{max}, \quad S = \begin{cases} 0 & \text{if max} = 0 \\ \dfrac{\Delta}{\text{max}} & \text{otherwise} \end{cases}$$

$$H = \begin{cases} 0 & \text{if } \Delta = 0 \\ 60^\circ \times \left(\dfrac{G - B}{\Delta} \bmod 6 \right) & \text{if max} = R \\ 60^\circ \times \left(\dfrac{B - R}{\Delta} + 2 \right) & \text{if max} = G \\ 60^\circ \times \left(\dfrac{R - G}{\Delta} + 4 \right) & \text{if max} = B \end{cases}$$

1.7.5 Colour Depth

Colour depth, also known as bit depth or bits per pixel (bpp), determines the number of distinct colours that can be represented in an image. It defines the precision with which colour information is stored and displayed.

General Definition: For a colour depth of k bits, the total number of possible colours is 2^k. In grayscale images, the value 0 typically denotes black, and the maximum value $(2^k - 1)$ denotes white.

- **8-bit Colour Format (Grayscale / Pseudo-color):**
  - Represents $2^8 = 256$ shades of grey.
  - Pixel values range from 0 (black) to 255 (white).
  - A value of 127 corresponds to mid-level grey.
  - Sometimes referred to as pseudo-colour when false colours are mapped to intensity levels.
- **16-bit Colour Format:**
  - Provides $2^{16} = 65,536$ levels of intensity.
  - Typically used to represent colour images with greater depth or for scientific imaging.
  - The 16 bits are distributed among Red, Green, and Blue components, often in a 5-6-5 format:

$$\text{Red (5 bits)} + \text{Green (6 bits)} + \text{Blue (5 bits)}$$

  - This allows for 32 levels of red and blue, and 64 levels of green, resulting in a total of 65,536 colours.
- **24-bit Colour Format (True Colour):**
  - Represents over 16 million colours: $2^{24} = 16,777,216$.
  - Allocates 8 bits each for Red, Green, and Blue channels.
  - Commonly referred to as "true colour" as it can closely approximate human colour perception.
  - Often stored in a 32-bit format, where the extra 8 bits may be used for an alpha channel (transparency) or padding.

1.8 Imaging System Modelling

1.8.1 Signal and System Framework

Imaging systems transform an input signal (object) into an output signal (image). This can be generalized as:

$$f(x, y) \to \mathcal{H} \to g(x, y)$$

1.8.2 System Characteristics

- **Linear shift-invariant (LSI)** systems:

  A system is said to be **linear** if it satisfies the principle of superposition:

$$\mathcal{H}[w_1 f_1 + w_2 f_2] = w_1 \mathcal{H}[f_1] + w_2 \mathcal{H}[f_2]$$

  for all signals f_1, f_2 and weights w_1, w_2.

A linear system also satisfies the **superposition integral**:

$$g(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x,y;\xi,\eta) f(\xi,\eta)\, d\xi d\eta$$

A system is **shift-invariant** if a shift in the input results in an identical shift in the output:

$$g(x - x_0, y - y_0) = \mathcal{H}[f(x - x_0, y - y_0)]$$

for every shift (x_0, y_0).

A **linear shift-invariant (LSI)** system thus satisfies:

$$h(x,y;\xi,\eta) \to h(x - \xi, y - \eta)$$

- **Convolution Integral:**

  For a 2D LSI system, the output $g(x,y)$ is given by the convolution of the input $f(x,y)$ with the system's point spread function (PSF) $h(x,y)$:

  $$g(x,y) = (f * h)(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi,\eta)\, h(x - \xi, y - \eta)\, d\xi d\eta$$

  This is known as the **convolution integral**, and it plays a central role in describing imaging systems under the LSI assumption.

- **Point Spread Function (PSF)**: The response of an imaging system to a point source (i.e., impulse input). In an LSI system, the PSF fully characterizes the system and can be used in convolution to compute the system output.

1.9 Image Quality Metrics

- Physical Measures
  - Contrast
  - Spatial Resolution (via LSF, PSF, FWHM)
  - Noise
  - MTF (modulation transfer function) SNR (signal-to-noise ratio) NPS (noise power spectrum)
- Clinical Measures
  - Contrast-detail analysis
  - ROC analysis
  - Sensitivity and specificity

1.10 Noise and Diagnostic Accuracy

1.10.1 Sources of Noise

- Quantum noise
- Electronic noise
- Sampling noise
- Anatomical noise

1.10.2 Diagnostic Accuracy Terms

- **TP, TN, FP, FN**
- **Sensitivity** = TP / (TP + FN)
- **Specificity** = TN / (TN + FP)

1.10.3 ROC Analysis

Receiver Operating Characteristic (ROC) analysis assesses diagnostic performance as a function of decision threshold. The area under the ROC curve (AUC) quantifies accuracy.

2 Fundamentals of Digital Image Processing

2.1 Image Enhancement in the Spatial Domain

Image enhancement in the spatial domain, also known as the pixel domain, involves direct manipulation of the pixel values of an image. The primary methods are intensity processing and spatial filtering.

2.1.1 Intensity Transformation

Intensity transformations operate on single pixels of an image. The transformation is a function T that maps an input intensity r to an output intensity s:

$$s = T(r)$$

This is also referred to as a point operation.

Basic Intensity Transformation Functions There are three fundamental types of gray level transformations:

- **Linear Transformations:**
  - **Image Negatives:** For a gray level range of $[0, L-1]$, the negative transformation is given by:

$$s = L - 1 - r$$

- **Logarithmic Transformations:** The log transformation is defined as:

$$s = c \log(1 + r)$$

  where c is a constant and $r \geq 0$. This transformation compresses the dynamic range of the image.

- **Power-Law (Gamma) Transformations:** The gamma transformation is given by:

$$s = cr^{\gamma}$$

  where c and γ are positive constants. This is useful for mapping a narrow range of dark or bright input values to a wider range of output values.

Piecewise-Linear Transformations These transformations use multiple linear functions to map the intensity values.

- **Contrast Stretching:** This enhances the contrast by stretching the range of intensity values.
- **Gray-level Slicing:** This highlights a specific range of gray levels in an image.

Histogram Processing The histogram of an image represents the statistical distribution of its intensity data.

- **Histogram Equalization:** This is a technique to obtain a uniform histogram for the output image. The core idea is to find a transformation function $T(r)$ such that the probability density function (PDF) of the transformed image is uniform. This is often achieved by using the cumulative distribution function (CDF) of the image intensities.

2.1.2 Spatial Filtering

Spatial filtering involves considering a pixel and its neighbors to compute the new pixel value. This is achieved by using a filter, also known as a mask or kernel.

Correlation and Convolution Spatial filtering is performed by convolving a filter kernel $w(x, y)$ with an image $f(x, y)$.

- **Convolution** of an image $f(x, y)$ of size $M \times N$ with a filter $w(s, t)$ of size $m \times n$ is defined as:

$$g(x, y) = w(x, y) * f(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x - s, y - t)$$

  where $a = (m-1)/2$ and $b = (n-1)/2$.

- **Correlation** is a similar operation, but the kernel is not flipped:

$$w(x, y) \star f(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x + s, y + t)$$

  If the kernel is symmetric, convolution and correlation yield the same result.

Linear Filters

- **Smoothing Filters (Low-pass):** These are used for blurring and noise reduction. Examples include the Box filter and the Gaussian filter.
- **Sharpening Filters (High-pass):** These filters are based on spatial derivatives to enhance edges and fine details. The second-order derivative, or Laplacian, is commonly used. The Laplacian of a 2D function $f(x, y)$ is:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x, y)$$

2.2 Image Enhancement in the Frequency Domain

This approach involves transforming the image into the frequency domain, performing filtering, and then transforming it back to the spatial domain.

2.2.1 Fourier Transform (FT)

The Fourier Transform decomposes a function into its constituent frequencies.

- **Fundamental Principle:** Any function can be represented as a sum (or integral) of sines and cosines of different frequencies.

2D Discrete Fourier Transform (DFT) For a digital image $f(x, y)$ of size $M \times N$, the 2D DFT is:

$$F(u, v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi(ux/M + vy/N)}$$

for $u = 0, 1, ..., M-1$ and $v = 0, 1, ..., N-1$. The variables u and v are the spatial frequencies.

The inverse 2D DFT is:

$$f(x, y) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u, v) e^{j2\pi(ux/M + vy/N)}$$

The Fourier Spectrum The FT $F(u, v)$ is generally a complex function:

$$F(u, v) = R(u, v) + jI(u, v) = |F(u, v)| e^{j\phi(u,v)}$$

- **Magnitude (Fourier Spectrum):** $|F(u, v)| = \sqrt{R^2(u, v) + I^2(u, v)}$
- **Phase Angle:** $\phi(u, v) = \tan^{-1}\left(\frac{I(u, v)}{R(u, v)}\right)$
- **Power Spectrum:** $P(u, v) = |F(u, v)|^2 = R^2(u, v) + I^2(u, v)$

The low frequencies are located near the center of the Fourier spectrum and correspond to the general intensity levels in the image. High frequencies are at the edges and represent details and noise.

2.2.2 Filtering in the Fourier Domain

The Convolution Theorem states that convolution in the spatial domain is equivalent to multiplication in the frequency domain, and vice versa.

$$f(x, y) * h(x, y) \Leftrightarrow F(u, v) H(u, v)$$
$$f(x, y) h(x, y) \Leftrightarrow F(u, v) * H(u, v)$$

This allows for efficient filtering by transforming the image and a filter kernel to the frequency domain, performing element-wise multiplication, and then applying the inverse FT.

Low Pass (LP) Filters Low-pass filters are used to attenuate high frequencies, resulting in a smoothing effect.

- **Ideal Low-Pass Filter (ILPF):** This filter sharply cuts off all frequencies beyond a specified cutoff frequency, D_0. Its transfer function is:

$$H(u, v) = \begin{cases} 1, & \text{if } D(u, v) \leq D_0 \\ 0, & \text{if } D(u, v) > D_0 \end{cases}$$

  where $D(u, v)$ is the distance from the frequency origin. The sharp cutoff can produce ringing artifacts.

- **Butterworth Low-Pass Filter (BLPF):** This filter provides a smoother transition between the passband and stopband. Its transfer function is:
$$H(u,v) = \frac{1}{1 + [D(u,v)/D_0]^{2n}}$$
where n is the order of the filter. Higher orders result in a sharper cutoff, approaching the ideal filter.

- **Gaussian Low-Pass Filter (GLPF):** The GLPF has a very smooth transition, which helps to avoid ringing artifacts completely. Its transfer function is:
$$H(u,v) = e^{-D^2(u,v)/(2D_0^2)}$$
The inverse Fourier transform of a Gaussian is also a Gaussian, ensuring no ringing.

High Pass (HP) Filters High-pass filters are used to sharpen images by attenuating low-frequency components while passing high-frequency components. A high-pass filter can be constructed from its corresponding low-pass filter (H_{lp}) using the relation:
$$H_{hp}(u,v) = 1 - H_{lp}(u,v)$$

- **Ideal High-Pass Filter (IHPF):** The IHPF completely rejects all frequencies inside a circle of radius D_0 from the origin and passes all frequencies outside this circle without attenuation. This sharp cutoff can lead to significant ringing artifacts. Its transfer function is:
$$H(u,v) = \begin{cases} 0, & \text{if } D(u,v) \leq D_0 \\ 1, & \text{if } D(u,v) > D_0 \end{cases}$$
where $D(u,v)$ is the distance from the frequency origin.

- **Butterworth High-Pass Filter (BHPF):**The Butterworth high-pass filter provides a smoother transition from the stopband to the passband compared to the ideal filter, which reduces ringing artifacts. The transfer function is given by:
$$H(u,v) = \frac{1}{1 + \left[\frac{D_0}{D(u,v)}\right]^{2n}}$$
where n is the order of the filter.

- **Gaussian High-Pass Filter (GHPF):**The Gaussian high-pass filter offers the smoothest transition and completely avoids ringing artifacts. Its transfer function is derived from the corresponding GLPF:
$$H(u,v) = 1 - e^{-D^2(u,v)/(2D_0^2)}$$

2.3 Nyquist-Shannon Sampling Theorem

The selection of an appropriate matrix size for digital imaging is guided by the Nyquist-Shannon sampling theorem. This fundamental principle ensures that the spatial details of the object being imaged are captured without significant loss or distortion (aliasing).

- The sampling frequency, which is the reciprocal of the pixel size ($1/\Delta x$), must be at least twice the maximum frequency (k_{max}) present in the object's signal.

- This implies that the sampling pixel size (Δx) must satisfy the following condition:
$$\Delta x \leq \frac{1}{2k_{max}}$$
In medical imaging, k_{max} is often determined by the intrinsic spatial resolution of the detector system.

2.4 A Practical Rule of Thumb

To avoid a significant loss of spatial resolution, a common rule of thumb relates the pixel size to the desired Full Width at Half Maximum (FWHM) of the system's resolution:
$$\Delta x \leq \frac{\text{FWHM}}{3}$$
This ensures that the resolution is adequately sampled.

2.5 Required Matrix Size

Based on this rule, the minimum required matrix size for a given field of view (FOV) can be calculated as follows:
$$\text{matrix size} \geq \frac{\text{field size used}}{\text{pixel size}} = \frac{\text{field size used}}{\text{FWHM}/3}$$

3 Core Physics of X-Ray Production & Tube Technology

3.1 Overview of X-ray Imaging in Physics

X-ray medical imaging has evolved significantly since Roentgen's discovery in 1895, leading to various diagnostic techniques.

3.1.1 Evolution and Major Techniques

- **1895**: Röntgen discovers X-rays.
- **1896**: First medical X-ray.
- **1913**: Coolidge invents the thermionic emission tube.
- **1937**: First practical rotating anode tubes.
- **1971**: Hounsfield develops the first CT prototype.
- **1980s**: Breast screening mammography emerges.
- **1990s**: Emergence of digital X-ray imaging and Multi-slice Spiral CT.
- **2000s-2010s**: Progression toward Spectral CT.

Major X-ray Imaging Techniques:

- **X-ray Radiography**: Produces a 2D projection image, fixed source and detector, no depth information.
- **X-ray CT (Computed Tomography)**: Creates slice and 3D images; source and detector rotate around the patient; offers depth information but at a much higher dose.

Other X-ray Imaging Techniques:

- **X-ray Fluoroscopy**: Real-time continuous imaging, used for studying contrast agent passage (e.g., GI/GU tract).
- **Digital Mammography & Tomosynthesis**: Uses lower X-ray energies for finer resolution breast imaging.
- **Interventional Radiology (IR)**: Minimally invasive, image-guided procedures (e.g., angiography, biopsy guidance, stent placement).

3.1.2 Fundamental Questions in X-ray Imaging

- Physical properties utilized for X-ray image encoding ($\mu(x, y)$).
- Methodology and configuration for X-ray image formation.
- X-ray beam generation and detection.
- Imaging system instrumentation and performance characterization.
- Image quality characterization.
- CT dose assessment.

3.1.3 X-Rays as an Imaging Probe

- **Key Features**: High penetration power, ionizing radiation, real-time imaging capability, versatility, non-invasive.
- **Applications**: Diagnostic imaging, interventional procedures, therapeutic applications, orthopedics, dental imaging.
- **Advantages**: Speed, cost-effectiveness, widespread availability.
- **Limitations**: Radiation exposure, limited soft-tissue contrast.

3.1.4 X-Ray Imaging Principle & Modeling

X-ray imaging is a non-invasive methodology involving an X-ray source, attenuation by the object, and detectors. The imaging process can be generalized as: **Input** $\rightarrow$ **System operator** $\rightarrow$ **Output**.

- **Input signal**: $\mu(x, y)$ (attenuation distribution of the object).
- **Output signal**: $g(y)$ (measured projection).
- **Imaging process**: Integration over the x variable.

The physical principle is **Lambert-Beer's Law**: $I = I_0 e^{-\mu x}$. The measured signal g is defined as $g \equiv \mu x = \ln(I_0/I)$. For inhomogeneous matter, this extends to a **line integral** or **ray-sum**:

$$g(y) \equiv \ln\left(\frac{I_0}{I}\right) = \sum_{i=1}^{n} \mu_i \cdot \Delta x_i = \int_a^b \mu(x, y) \cdot dx$$

3.1.5 Understanding and Implications of the Imaging Equation

- The image is a mapping function of the object's μ distribution.
- The measured projection is a ray-sum or line integral. A 2D object $\mu(x, y)$ becomes a 1D projection $g(y)$, leading to **loss of depth information** as μ is coupled with depth (Δx).
- To decouple depth dependence and get accurate $\mu(x, y)$, **tomographic methodology (Computed Tomography)** is required.

3.1.6 Implied Assumptions from the Equation

- **Beam straight-line assumption**: Assumes parallel straight-line beam geometry, though actual X-ray beams are diverging. Scatter can affect this assumption.
- **Beam energy assumption**: Assumes a mono-energetic beam (E unchanging), but actual X-ray beams are poly-energetic. This leads to **beam hardening** effects. Only beam intensity is typically measured, implying integral detection of energy.

3.1.7 Key Questions about $\mu(x, y)$

- Physics meaning of $\mu(x, y)$ and difference from cross-section (σ).
- How $\mu(x, y)$ is characterized and determined.
- Physics attributed to $\mu(x, y)$.
- Factors affecting $\mu(x, y)$ measurements.

3.1.8 Big Picture of X-ray Imaging

The overall process involves X-ray production, interaction with the patient, X-ray detection, and image quality assessment, leading to various clinical applications like radiography, interventional radiology, and CT.

3.2 Brief on Radiation Interaction

This section covers the general picture and quantification of photon interactions, focusing on attenuation.

3.2.1 Radiation Interactions with Matter: Photon vs. Charged Particle

- **X/γ Photons**: Interactions are characterized by **probability**. Types: Rayleigh scattering, photoelectric effect, Compton scattering, pair production. Quantified by **Cross-section (σ)** and **Attenuation Coefficient (μ)**.
- **Charged Particles**: Interactions are characterized by **certainty**, involving continuous energy loss (slowing down). Types: Ionization, excitation, Bremsstrahlung, Cerenkov radiation. Quantified by **Stopping Power (dE/dx)**.

3.2.2 Physics Pictures for Radiation Interactions

Interactions depend on whether the radiation interacts with orbital electrons or the nucleus.

- **X/γ Photons**:
  - *Orbital Electron*: Rayleigh scattering (tightly bound), Photoelectric effect (tightly bound), Thomson scattering (loosely bound), Compton scattering (loosely bound), Triplet production (Coulomb field).
  - *Nucleus*: Photodisintegration (strong force), Pair production (Coulomb field).
- **Charged Particles**:
  - *Orbital Electron (Collisional loss)*: Soft collision (excitation), Hard collision (ionization), Positron annihilation.
  - *Nucleus*: Radiation loss (Bremsstrahlung), Absorption (secondary particles).
- **Neutrons**: Primarily interact with the nucleus via scattering and absorption.

3.2.3 Quantification and Characterization

- **Photon/Neutron Interactions**:
  - **Attenuation Coefficient**: μ or μ/ρ. $I = I_0 e^{-\mu x}$.
  - Total mass attenuation coefficient: $\frac{\mu}{\rho} = \frac{1}{\rho}(\tau + \sigma_R + \sigma_C + \kappa)$.
  - Dependence: Photon energy ($h\nu$), medium properties (Z, ρ).
    - Photoelectric (τ) $\propto Z^5/E_\gamma^{7/2}$
    - Compton (σ_C) $\propto Z/E_\gamma$
    - Pair Production (κ) $\propto Z^2 \log(E_\gamma)$ for $E_\gamma \geq 1.02$ MeV
- **Charged Particle Interactions**:
  - **Stopping Power**: $-dE/dx$ or $-\frac{1}{\rho}dE/dx$.
  - Total stopping power: $(-\frac{1}{\rho}dE/dx)_{\text{tot}} = (-\frac{1}{\rho}dE/dx)_{\text{col}} + (-\frac{1}{\rho}dE/dx)_{\text{rad}}$.
  - Dependence: Particle type/charge (z), particle energy (β), medium properties (Z, ρ, A, I).
  - **CSDA Range**: Continuous-slowing-down-approximation range.
  - **Bragg Peak**: Point of maximum ionization for heavy charged particles.

3.2.4 Characteristics of Radiation Interactions

- **Charged Particle Interaction**: Certain, dominated by Coulomb interaction, continuous energy loss (slowing down), includes ionization, excitation, Bremsstrahlung, and Cerenkov radiation.
- **Photon Interaction**: Probabilistic (likelihood), elastic (scattering) or inelastic (absorption), transfers energy to medium, generates secondary radiation. Quantified by "probability."

3.2.5 How to Study Photon Interaction?

- Starting from individual photon interactions: Identify types of physical processes (interaction object, photon fate, energy exchange, by-products).
- Quantification: Use **cross-section** (σ).
- Characterize each physical process.
- Extend for photon flux (beam) interactions: Use **attenuation coefficient** (μ).

3.2.6 Beam Interaction Quantization

For a beam of intensity I_0 interacting with a thin target:

$$-dI = I \cdot (n_T \sigma) \cdot dx$$

Leading to:

$$I = I_0 e^{-n_T \sigma x} = I_0 e^{-\Sigma x} = I_0 e^{-\mu x}$$

Where:

- $\Sigma = \mu$: Macroscopic cross-section or linear attenuation coefficient ([cm^{-1}]).
- n_T: Number density of target atoms per unit volume (#/cm^3). $n_T = \frac{\rho \cdot N_{av}}{A}$.
- σ: Microscopic physical quantity, cross-section ([cm^2] or barn).

3.2.7 Differential and Total Cross Sections

- **Differential Cross-Section ($\frac{d\sigma}{d\Omega}$)**: Measure of probability per unit solid angle for scattered particles. Unit: area/steradian.
- **Total Cross-Section (σ_T)**: Integral of the differential cross-section over all solid angles. $\sigma_T = \int \sigma(\theta) \cdot d\Omega$.
- **Doubly Differential Cross-Section ($\frac{d^2\sigma}{d\Omega dE}$)**: If scattered particle's energy can be measured. Unit: $[\text{mbarn}/(\text{sr} \cdot \text{MeV})]$.

3.2.8 Understanding the Cross-section (σ)

- σ measures the **likelihood (probability)** of an interaction/collision between two particles.
- It's a **microscopic physics quantity**, independent of experiments, only depends on intrinsic physics.
- It's **strongly energy dependent**. $\sigma_E \propto \pi\lambda^2 \rho_P(E) X(E, A)_l$.
- It is **not necessarily the geometric "area."**

3.2.9 Classification of Photon Interactions

Photon interactions are classified into scattering (σ_{sc}) and absorption (σ_{ab}), contributing to total attenuation (σ_{tot}).

- **Scattering**:
  - **Coherent (Rayleigh) Scattering (σ_R)**: Photon changes direction but not energy. Interacts with bound electrons. $\propto 1/E$, $\propto Z^2$.
  - **Incoherent (Compton) Scattering (σ_C)**: Photon interacts with loosely bound electron, loses some energy, changes direction. $\sim$constant for 10-100keV, $\propto 1/E$ over 100keV. $\propto Z^0$.
- **Absorption**:
  - **Photoelectric Effect (τ)**: Photon completely absorbed by bound electron, ejecting it. $\propto 1/E^3$, $\propto Z^3$.
  - **Pair Production (κ)**: High energy photon (> 1.02 MeV) interacts with nucleus, producing electron-positron pair. $\propto Z$.

The **mass attenuation coefficient**: $\frac{\mu}{\rho} = \frac{1}{\rho}(\tau + \sigma_R + \sigma_C + \kappa)$.

3.2.10 Cross section vs. Attenuation Coefficient

- σ: Probability per unit area for a photon to interact with an atom (microscopic, unit: barn).
- μ: Probability per unit path-length for a photon to interact with bulk media (macroscopic, unit: $1/\text{cm}$).
- **Relation**: $\mu = n_T \sigma$.

3.2.11 Physics Attribute of μ

The relative importance of interaction modes varies with photon energy and atomic number.

- Photoelectric effect dominates at low energies and high Z.
- Compton scattering dominates in the diagnostic range (50-150 keV).
- Pair production dominates at very high energies (> 1.02 MeV).

3.2.12 Factors Affecting Attenuation

$\mu(\vec{r}; \rho, Z, E)$.

- ρ, Z: Tissue-specific (electron density).
- E: X-ray beam energy.

Diagnostic X-ray imaging maps $\mu(\vec{r})$, which is strongly energy-dependent $\mu(\vec{r}, E)$. Technological evolution aims to improve mapping and measurement of $\mu(\vec{r}, E)$ (e.g., Spectral CT).

3.2.13 How to Determine μ?

Attenuation is photon removal from a beam by absorption and scattering.

$$N(x) = N_0 e^{-\mu(x)\cdot\Delta x}$$

- **Linear attenuation coefficient (μ)**: $1/\text{cm}$.
- **Mass attenuation coefficient ($\mu_m = \mu/\rho$)**: cm^2/g.

3.2.14 How to Measure μ? Scatter Effect on μ

- **Narrow beam attenuation**: No scatter reaches detector, measures true μ.
- **Broad beam attenuation**: Includes scatter reaching detector, underestimates μ.
- **Scatter Build-up Factor (B)**: Accounts for secondary photons scattered into the detector.

$$I_B(x) = B \cdot I_N(x) = I_0 B e^{-\mu x} = I_0 e^{-\mu_{eff} x}$$

$$\mu_{eff} = \mu - \frac{\ln(B)}{x}$$

3.2.15 Half-Value Layer (HVL)

The thickness of an absorber that attenuates the original intensity to 50%.

- Indication of beam quality.
- Valid for all beam types (mono-energetic, poly-energetic).
- Higher HVL means more penetrating beam, lower attenuation coefficient.

$$HVL = \frac{\ln(2)}{\mu} = \frac{0.693}{\mu}$$

3.2.16 Poly-energetic Attenuation & Beam Hardening

- **Poly-energetic Attenuation**: Shows as a curved line on a semi-log graph, straightening with increased attenuation, as lower energy photons are preferentially absorbed.
- **Beam Hardening**: Occurs when low-energy photons are absorbed, increasing the mean beam energy. This causes the measured μ to be underestimated. Filtration is a solution to minimize its effect.

3.2.17 Dominant Interactions in Tissue

- **Up to 50 keV**: Photoelectric effect.
- **50 – 150 keV**: Photoelectric and Compton.
- **0.15 – 2 MeV**: Compton.
- **2 – 10 MeV**: Compton and Pair Production.
- **Above 10 MeV**: Pair Production.

3.3 X-ray Production & Tube Technology

This section details the working principle, main components, and factors affecting the X-ray beam within an X-ray tube.

3.3.1 Work Principle of X-ray Tube

A simplified X-ray tube shows the core idea: electrons are generated, accelerated, and then rapidly decelerated upon striking a target to produce X-rays.

3.3.2 Typical Structure and Main Components of X-ray Tube

- **Cathode** (Negative Terminal, Electron Production):
  - **Filament**: Tungsten wire heated by current ($\sim$ 10A) for thermionic emission of electrons. Tube current is controlled by filament current. Tungsten's high melting point (3370°C) and minimal deposition on glass are key.
  - **Focusing Cup**: Uses electric field lines to focus emitted electrons into a narrow beam toward the anode.
  - **Dual Filaments**: Typically switchable (long/short) to control focal spot size. Long filament: higher current, large focal spot, lower resolution. Short filament: lower current, small focal spot, higher resolution.
- **Anode** (Positive Terminal, Target):
  - **Target**: High-Z material (e.g., Tungsten) for efficient X-ray production, supported by high heat conductivity materials (e.g., copper). Molybdenum and Rhodium are used for softer X-rays in mammography.
  - **Rotating Surface**: Rotates at 3k-10k revolutions/minute to distribute heat, preventing damage. Resides in vacuum ($\sim 10^{-6}$ torr) and is thermally decoupled from the motor.
  - **Tilted Angle**: Target is angled to maximize exposed area for heat distribution, minimize heat per unit area, and optimize the effective focal spot size (improving resolution). Bremsstrahlung is emitted at $\sim 90^{\circ}$ for low-energy electrons.
- **Other Components**:
  - **Window**: Exit point for X-rays.
  - **Glass Envelope**: Maintains vacuum for electron flow.
  - **Insulating Oil**: Electrical insulation and cooling.
  - **Filter**: Shapes X-ray spectrum.
  - **Housing**: Encloses tube, oil, provides protection.
  - **Accelerating Voltage (20-150 kVp)**: High potential difference accelerating electrons.

3.3.3 Characteristics of Anode & Limit Factors

- **Power**: $P = V \times I$ (watts).
- **Energy**: $E = V \times I \times s$ (joules).
- **Heat Unit (HU)**: Commonly used measure of heat load.
- **Heat Load**: Determined by voltage, current, and exposure time. Heat (J) $= w \times$ kVp $\times$ mAs, where w is the waveform factor.
- **Anode heating and cooling curve charts**: Used to manage heat dissipation and necessary cooling time between exposures.

3.3.4 Anode Angle, Focal Spot Size and FOV

- **Effective focal spot size** $=$ (actual focal spot size) $\times \sin\theta =$ (beam width) $\times \tan\theta$. A smaller effective focal spot (achieved with smaller anode angles) increases image resolution.
- **Field of View (FOV)** $= 2 \times$ (source-to-patient-distance) $\times \tan\theta$.
- There's a trade-off between small effective focal spot (high resolution) and large FOV/power loading (requiring a large actual focal area).

3.3.5 Anode Heel Effect

- An inhomogeneous X-ray intensity across the beam field, with higher intensity on the cathode side and lower on the anode side.
- **Cause**: Self-attenuation of X-rays within the anode target material, as X-rays emerging near the anode side travel through more target material.
- **Result**: Loss of intensity on the anode side of the X-ray field of view.

3.3.6 Beam Filter

Used to reduce low-energy photons, increasing penetrating photons (more information) and decreasing patient dose (especially skin dose).

- **Inherent Filtration**: Always present (target, glass envelope, oil, window).
- **Additional Filtration**: Removable materials (e.g., Al, Cu) to shape the spectrum.
- **ICRP Advice (Total Filtration in Al equivalent)**:
  - < 50 kV: 0.5 mm Al
  - 50 kV $< V < 70$ kV: 1.5 mm Al
  - > 70 kV: 2.5 mm Al

3.3.7 Composite Filter

- Made of two materials (e.g., aluminum and a metal like copper).
- The material with the **highest Z faces the X-ray tube** for maximum attenuation and to produce characteristic X-rays.
- Aluminum absorbs the low-energy characteristic X-rays from the high-Z metal, further shaping the spectrum.
- Used to optimize the X-ray spectrum, e.g., K-edge filtration in mammography.

3.3.8 Accelerating Voltage

- Generated by transformers from AC power.
- **Rectification**: Converts AC to DC voltage using diode circuits (half-wave, full-wave) or three-phase power (6-pulse, 12-pulse) to reduce voltage ripple. Constant potential (CP) operation provides almost no ripple.
- **Effect of Rectification on X-ray Spectrum**: Increased mean photon energy, increased X-ray output, shorter exposure times, lower patient dose.

3.4 Factors Affecting X-ray Beam

The X-ray tube output is described by its quality, quantity, and exposure.

3.4.1 Factors affecting X-ray Beam

- **X-ray Quantity (Number of Photons)**:
  - $\propto Z_{target} \times \text{kV}^2 \times \text{mAs}$
  - Affected by: Target Z, Tube voltage (kV), Tube current (mA), Exposure time (s), Beam filtration, Generator waveform.
- **X-ray Quality (Average Energy of Photons)**:
  - Affected by: Target Z, Tube voltage (kVp), Beam filtration, Generator waveform.
- **Exposure**: Nearly proportional to the energy fluence of the beam, depending on both quantity and quality.

3.4.2 Specific Factor Impacts on X-ray Spectrum

- **Target Z**: Increasing Z increases quantity and characteristic energy.
- **Tube Voltage (kV)**: Increasing kV increases quantity ($\propto \text{kV}^2$) and quality (spectrum shifts to higher energy, characteristic lines appear).
- **Tube Current & Exposure Time (mAs)**: Increasing mAs increases quantity proportionally, but does not change quality (characteristic, average, min/max energies remain the same).
- **Beam Filtration**: Decreases quantity (fewer low-energy photons) but increases quality (higher average energy).
- **Waveform of Current**: More uniform current (better rectification) increases average energy (quality) and quantity, with the same maximum keV.

3.5 New Type of X-ray Tube Technology

Advances include:

- **Liquid-metal jet anode**: For improved heat dissipation and higher power loads.
- **Carbon-nanotube field emission X-ray source**: Explores field emission for potentially smaller, more precise, and rapidly switchable X-ray sources, distinct from thermionic emission.

3.6 Therapeutic X-rays (LINAC)

For therapeutic applications requiring much higher energies than diagnostic tubes:

- Generated by **Linear Accelerators (LINACs)**.
- **Components**: Electron gun, magnetron (RF source), accelerating waveguide, bending magnets, target (X-ray source) & flattening filter, ion chamber, collimators.

4 X-Ray Radiography & Detector Technology

X-ray radiography acquires two-dimensional projection images by measuring the transmitted photon fluence through a patient. The recorded signal is the logarithm of the ratio between incident and transmitted intensities, providing information on line integrals of the linear attenuation coefficient $\mu(\mathbf{r})$. This modality underpins diagnostic imaging in radiology, with applications from chest radiographs to interventional fluoroscopy.

4.1 Imaging Equation

For a monoenergetic, point source and ideal detector, the Beer–Lambert law yields:

$$I(x, y) = I_0 \, e^{-\int_a^b \mu(x,y,z)\,\mathrm{d}z},$$

where

- I_0 is the incident intensity,
- $I(x, y)$ is the detected intensity at detector coordinate (x, y),
- $\mu(x, y, z)$ is the spatially varying linear attenuation coefficient,
- the integration is along the ray path in z-direction from a to b.

Taking the negative logarithm gives the projection or sinogram:

$$g(x, y) \equiv -\ln \frac{I(x, y)}{I_0} = \int_a^b \mu(x, y, z)\,\mathrm{d}z.$$

4.2 Polyenergetic Beam and Beam Hardening

Real X-ray tubes emit a continuous bremsstrahlung spectrum $S_0(E)$ plus discrete characteristic lines. The detected signal becomes:

$$I = \int_0^{E_{\max}} S_0(E) \, e^{-\int \mu(E,\mathbf{r})\,\mathrm{d}z}\,\mathrm{d}E.$$

Beam hardening arises as low-energy photons are preferentially absorbed, yielding an effective energy that increases with path length, causing cupping artifacts and nonlinear attenuation.

4.3 Inverse-Square and Obliquity Effects

For a point source at distance R_0 from the detector center, the photon fluence obeys inverse-square law:

$$I(r) = I_0 \frac{R_0^2}{(R_0 + d(r))^2},$$

where $d(r)$ is the additional path to an off-axis detector element.

Photon fluence is modified by both distance falloff and detector orientation:

- **Inverse-Square Law**: The net flux of photons decreases as $1/r^2$, where r is the distance from the point source.
- **Obliquity Effect**: Away from the detector center, rays strike at an angle θ, reducing fluence per unit area by a factor $\cos\theta$.
- **Combined Effect**: Multiplying both factors yields

$$I(r, \theta) = I_0 \frac{\cos^3\theta}{r^2}.$$

- **Small-Angle Approximation**: For small θ, $\cos^3\theta \approx 1$, so the effect can often be neglected.

4.4 Source Size and Geometric Unsharpness

Finite focal spot size f produces penumbra blur:

$$U = f \frac{\mathrm{OID}}{\mathrm{SOD}},$$

where

- SOD is source-to-object distance,
- OID is object-to-image distance.

4.5 Scatter and Contrast Degradation

Scattered photons degrade contrast. Define:

$$\begin{aligned} \text{Primary } P &= \text{unscattered photons}, \\ \text{Scatter } S &= \text{photons deviated by Compton interactions}. \end{aligned}$$

The scatter-to-primary ratio (SPR) and scatter fraction (SF) are:

$$\text{SPR} = \frac{S}{P}, \quad \text{SF} = \frac{S}{P+S}.$$

Anti-scatter grids improve contrast by absorbing oblique scatter, at the cost of increased patient dose. Grid performance is quantified by grid ratio $r = \dfrac{h}{D}$ (height over interspace width).

4.6 Screen-Film Systems

Screen-film systems use intensifying screens (e.g., Gd_2O_2S:Tb) to convert X-rays to visible light, exposing silver-halide emulsion. Key parameters:

- **Speed (Sensitivity)**: reciprocal of exposure for OD = 1,
- **Contrast (Gamma)**: slope of H–D curve in linear region,
- **Latitude**: exposure range over which OD is useful.

The characteristic curve (OD vs. log exposure) is typically sigmoid with toe, straight-line, and shoulder regions.

4.7 Computed Radiography (CR)

CR uses photostimulable phosphors (BaFBr:Eu^{2+}) in cassettes. Workflow:

1. X-ray exposure traps electrons in F-centers.
2. Red-laser scan releases trapped electrons, emitting blue light.
3. Photomultiplier tubes convert light to digital signal.
4. Plate is erased with white light to clear residual trap.

CR metrics: sampling pitch (laser spot spacing), modulation transfer function (MTF) limited by light spread, detective quantum efficiency (DQE).

4.8 Digital Radiography (DR)

DR detectors offer direct digital readout, divided into:

Indirect DR: Scintillator (CsI:Tl or Gd_2O_2S) converts X-rays to light, which is detected by an amorphous silicon (a-Si) thin-film transistor (TFT) array. Light spread in scintillator reduces MTF.

Direct DR: Photoconductor (amorphous Se) directly converts X-rays to charge, collected by TFTs. Benefits include near-unity fill factor and high spatial resolution (MTF > 0.7 at Nyquist).

Key performance metrics for all digital detectors:

- **DQE**(f): Detective quantum efficiency as function of spatial frequency:

$$\text{DQE}(f) = \frac{\text{MTF}(f)^2}{\text{NNPS}(f) \times (q_0)},$$

  where NNPS is normalized noise power spectrum and q_0 is incident quanta.

- **MTF**(f): Modulation transfer function, the Fourier transform of point-spread function.
- **Dynamic Range**: ratio of maximum to minimum detectable signal, typically $>$ 10,000:1 for DR.
- **Noise Characteristics**: quantum noise, electronic noise, fixed-pattern noise.

5 X-Ray Mammography

5.1 Breast Composition and X-Ray Attenuation

Breast parenchyma comprises adipose (fat) and glandular tissue, each with distinct mass energy-absorption coefficients μ_{en}/ρ. Primary elements (C, H, O, N) yield soft-tissue attenuation; malignant lesions often increase local density. Attenuation follows Lambert–Beer law:

$$I = I_0 \exp(-\mu t)$$

Contrast C between lesion and background:

$$C = \frac{I_b - I_l}{I_b} = 1 - \exp[-(\mu_l - \mu_b)t]$$

which diminishes at higher photon energies.

5.2 X-Ray Tube and Spectrum

Radiographic tubes for mammography optimize low-energy output. Key parameters:

- Peak tube voltage: 25–35 kV (to maximize contrast while limiting penetration).
- Tube current: up to 100 mA for large focal spot; 25 mA for magnification.

5.2.1 Target Materials and Filtration

Targets: Mo (Z=42), Rh (Z=45), W (Z=74). Filters of same or adjacent material (e.g., 0.03 mm Mo, 0.025 mm Rh) preferentially absorb low-energy photons below K-edges, hardening the beam.

5.2.2 Characteristic and Bremsstrahlung Emission

Bremsstrahlung spectrum is continuous up to $E_{\max} = eV_p$. Characteristic lines arise when electrons fill K-shell vacancies:

$$E_{K\alpha} \approx Z^2 R \left(1 - \frac{1}{2^2}\right), \quad E_{K\beta} \approx Z^2 R \left(1 - \frac{1}{3^2}\right)$$

For Mo: K_α = 17.5 keV, K_β = 19.6 keV; for Rh: K_α = 20.2 keV, K_β = 22.7 keV.

5.3 Imaging Geometry and Spatial Resolution

5.3.1 Focal Spot and Unsharpness

Focal spot size f dictates geometric blur U:

$$U = f \times \frac{\text{OID}}{\text{SOD}}.$$

Typical f: 0.3 mm (contact), 0.1 mm (magnification). Compression paddle reduces OID, lowering U and breast thickness.

5.3.2 Magnification Techniques

Magnification factor M:

$$M = \frac{\text{SID}}{\text{SOD}}.$$

With SID = 65 cm, OID = 30 cm, $M \approx 1.5$–1.8. Magnification enhances small detail (e.g., microcalcifications) at cost of increased dose and reduced exposure latitude.

5.4 Beam Quality and Half-Value Layer (HVL)

Beam quality quantified by HVL (aluminium):

$$\text{HVL} = \frac{\ln 2}{\mu_{\text{Al}}}.$$

Typical HVL in mammography: 0.3–0.45 mm Al. Higher kVp or high-Z targets (W) produce higher HVL.

5.5 Radiation Dose and Mean Glandular Dose (MGD)

Mean Glandular Dose is computed as:

$$\mathrm{MGD} = K_{\mathrm{ESAK}} \times g(t) \times c(t,p) \times s(T/F),$$

where:

- K_{ESAK}: entrance surface air kerma without backscatter, measured with ion chamber.
- $g(t)$: thickness-dependent conversion factor (Dance *et al.* 2000).
- $c(t,p)$: glandularity correction (50% standard breast).
- $s(T/F)$: spectral correction for target/filter combination (e.g., Mo/Mo = 1.000, Mo/Rh = 1.017, Rh/Rh = 1.061).

Recommended: MGD $\leq$ 2 mGy for 4.2 cm, 50% glandularity; legal limit $\leq$ 3 mGy.

5.6 Detector Technologies

5.6.1 Screen-Film Systems

Conventional: silver halide film + intensifying screen; latitude 25:1, spatial resolution 15–20 LP/mm.

5.6.2 Digital Detectors

Indirect Detection Scintillator (CsI or Gd_2O_2S) converts x-rays to light, then amorphous-Si TFT array digitizes; pixel size 70–150 μm, fill factor 30–70%.

Direct Detection Amorphous-Se photoconductor on TFT array directly converts x-rays to charge; higher resolution, lower scatter.

5.7 Digital Breast Tomosynthesis (DBT)

DBT acquires N projections over $\pm$7.5–15° sweep, reconstructs slices via filtered backprojection or iterative algorithms. Key metrics:

- Sensitivity increase of 20–30% in dense breasts.
- Specificity increase of 15–25%, reducing recall rates.
- Dose 2–3$\times$ that of 2D; synthesized 2D images reduce additional dose by up to 50%.

5.8 Quality Assurance and Phantom Testing

Use ACR mammography phantom (PMMA) containing fibers (0.18–0.75 mm), specks (0.3–0.6 mm), and masses (2–5 mm). Acceptable performance:

- Resolution: $\geq$10 line pairs per mm.
- Contrast-detail: all specks and most fibers visible.
- Field congruence: light vs. x-ray field $\leq$1% SID (edge), $\leq$2% overall.
- Compression paddle: no visible edges; reproducible thickness $\pm$2 mm.

5.9 Advanced Applications

5.9.1 Contrast-Enhanced Mammography

Dual-energy subtraction with iodinated contrast enhances lesion conspicuity.

5.9.2 Dedicated Breast CT

360° spiral acquisition; true volumetric imaging at 5 mGy; resolution 200 μm.

5.9.3 Phase-Contrast and Spectral Imaging

Grating-based interferometry and photon-counting detectors enable material decomposition and edge enhancement.

5.10 Comparison of X-Ray Based Imaging Modalities

Different X-ray imaging modalities are engineered to meet specific clinical requirements. The design choices for each system represent a balance between diagnostic objectives, patient demographics, radiation risk, and technical performance. The fundamental differences can be categorised by their clinical considerations and their underlying engineering specifications.

5.11 Clinical and General Considerations

The intended use of an imaging modality dictates its operational parameters and the associated risks. Table 1 summarises these high-level differences.

Table 1: Comparison of Clinical and General Requirements for X-Ray Modalities.

Consideration	**General X-ray**	**Mammography**	**Fluoroscopy**	**Computed Tomography (CT)**
Population	All people	Well women ¿ 40 years	Limited, typically sick patients	Mostly adult, with increasing use
Risk	Very small (low dose)	Small (stochastic risk)	Variable, potential for tissue effects from cumulative dose	Moderate, cumulative dose is a concern
Features Imaged	All structures of moderate size	Small objects, microcalcifications, low contrast features	Dynamic processes, injected contrast media, clear objects	All bodily tissues, full volume, excellent low-contrast resolution
Time	Very short, static views	Fairly fixed, 1 to 4 seconds per view	Variable, can extend for minutes to hours	Moderate, up to many seconds for a scan
Dose	Low, variable, sensitivity limited	Focused, low but highly controlled, high local sensitivity	Can be high due to long exposure times	High, full volume irradiation with high sensitivity

5.12 Engineering and Technical Requirements

To achieve the clinical goals outlined above, each modality is engineered with distinct hardware and software specifications. These technical differences are summarised in Table 2.

Table 2: Comparison of Engineering and Technical Requirements.

Parameter	**General X-ray**	**Mammography**	**Fluoroscopy**	**Computed Tomography (CT)**
Power Output	Moderate	Low	Low but continuous	High
Focal Spot	Variable	Small (for high spatial resolution)	Large (to handle heat from long exposures)	Moderate
Filters	Used to reduce dose (harden beam)	Used to shape spectrum for optimal contrast (e.g., Mo, Rh)	Used to reduce dose (e.g., Cu)	Used to shape beam profile (e.g., bowtie filters)
Radiation Output	High rate, short bursts	Low rate, limited by low energy spectra	Moderate, sustained output	High, varied for patient size
Energy	Variable, adjusted for anatomical site	Low (e.g., 20-35 kVp) to maximise photoelectric contrast	Automatically varied by system to maintain image quality	High and constant (e.g., 100-140 kVp), contrast dominated by electron density

6 X-Ray Fluoroscopy & Interventional Radiology

X-ray fluoroscopy is a dynamic imaging modality that provides real-time, two-dimensional projection images of internal structures. By acquiring a sequence of images at a specified frame rate, it allows for the visualisation of physiological motion and the guidance of minimally invasive procedures. This capability is fundamental to interventional radiology, where it is used for a vast range of diagnostic and therapeutic procedures.

6.1 Imaging Principles and Technology

The fundamental principle of fluoroscopy involves the continuous or pulsed emission of X-rays transmitted through the patient and captured by a detector.

6.1.1 Image Intensifier (II)

A key component in traditional fluoroscopic systems is the image intensifier. It converts the incident low-intensity X-ray signal into a high-intensity light image. The process involves several stages:

1. **Input Phosphor:** Made of Caesium Iodide (CsI), it absorbs X-ray photons and converts their energy into visible light photons.
2. **Photocathode:** Bonded to the input phosphor, it absorbs the light photons and releases a proportionate number of electrons via the photoelectric effect.
3. **Electron Optics:** The electrons are accelerated across a vacuum by a high potential difference (e.g., 25,000 Volts) and focused by electrodes onto the output phosphor.
4. **Output Phosphor:** A smaller screen that converts the kinetic energy of the incident electrons back into a bright visible light image, which is then captured by a camera (e.g., CCD/CMOS).

The signal amplification, or brightness gain, is a product of both flux gain (from electron acceleration) and minification gain (from focusing the electron beam onto a smaller area).

6.1.2 Pulsed Fluoroscopy

Modern systems predominantly use pulsed fluoroscopy to manage radiation dose. Instead of a continuous stream, X-rays are emitted in short bursts (pulses). The last captured image is held and displayed on the monitor between pulses (*last image hold*). This technique significantly reduces the cumulative patient dose, with the trade-off that fast motion can appear "jerky" at lower pulse rates.

6.1.3 Digital Subtraction Angiography (DSA)

DSA is a crucial technique in vascular imaging that isolates contrast-filled vessels from surrounding anatomy. The process involves subtracting a pre-contrast "mask" image from subsequent images acquired after the injection of iodinated contrast media. For a given pixel coordinate (x, y), the resulting subtracted image I_{DSA} is generated as:

$$I_{DSA}(x, y) = I_{live}(x, y) - I_{mask}(x, y) \tag{1}$$

where I_{live} is the image containing contrast and I_{mask} is the pre-contrast image. This eliminates static background structures, providing an unobstructed view of the vasculature.

6.2 System Geometry and Operation

The most common fluoroscopic configuration is the C-arm, which provides flexible positioning around the patient.

6.2.1 DICOM Angle Conventions

To standardise the description of imaging views, the DICOM standard defines primary and secondary angle conventions for the C-arm:

- **Primary Angle (Rotation):** Describes the rotation of the C-arm to the patient's left or right.
  - **LAO:** Left Anterior Oblique.
  - **RAO:** Right Anterior Oblique.
- **Secondary Angle (Angulation):** Describes the tilt of the C-arm towards the patient's head or feet.
  - **CRA:** Cranial.
  - **CAU:** Caudal.

6.3 Radiation Dose Management and Metrics

The extended duration of interventional procedures presents a significant risk of tissue effects (deterministic effects). Consequently, dose management and monitoring are central concerns.

6.3.1 Dose Control Mechanisms

Modern fluoroscopy systems employ **Automatic Dose Rate Control (ADRC)**, which automatically adjusts exposure parameters (kVp, mA, pulse width) to maintain a consistent signal at the detector, thereby optimising image quality and dose based on patient thickness. Furthermore, added **copper (Cu) filtration** is used to ”harden” the X-ray beam by removing low-energy photons that increase patient skin dose without contributing to the image.

6.3.2 Key Dose Metrics

Two primary metrics are used to quantify radiation output:

- **Incident Air Kerma (IAK):** Measured in Gray (Gy), it quantifies the kerma (Kinetic Energy Released per unit MAss) at a specific Interventional Reference Point (IRP) in air. It is used to estimate the peak skin dose (PSD) to the patient.
- **Kerma Area Product (KAP) or Dose Area Product (DAP):** Measured in $\text{Gy} \cdot \text{cm}^2$, it is the product of the air kerma and the area of the X-ray beam. KAP is a useful proxy for the total radiation energy delivered and has the advantage of being largely independent of the distance from the X-ray source.

6.3.3 Reference Point Definitions for Kerma

The location for measuring IAK is standardised by the IEC as shown in Table 3.

Table 3: Reference Point Location for Kerma Measurement (IEC 2010).

Fluoroscopic Device Type	Reference Point Location
C-arm (with isocenter)	15 cm from the isocenter toward the X-ray source along the beam axis.
X-ray tube under tabletop	1 cm above the tabletop.
X-ray tube over tabletop	30 cm above the tabletop with the beam-limiting device positioned as close as possible to the measurement point.

7 Computed Tomography (CT) Methodology and Technology

7.1 CT Concept and Methodology

CT is an imaging modality that reconstructs a 2D cross-sectional image (slice) of a 3D object from a collection of 1D projections acquired at different angles. This process, rooted in tomography (from Greek "tomos" for slice and "graphia" for to write/draw), aims to map and measure the attenuation coefficient (μ) of an object as accurately as possible while minimizing radiation dose.

7.1.1 Principle and Physical Basis

The physical principle underlying X-ray imaging, including CT, is described by Lambert-Beer's Law, which relates the attenuation of X-rays to the properties of the material they pass through.

$$I = I_0 e^{-\mu x}$$

Where:

- I is the transmitted X-ray intensity.
- I_0 is the incident X-ray intensity.
- μ is the linear attenuation coefficient of the material.
- x is the thickness of the material.

In CT, the measured quantity, often referred to as the "ray-sum" or line integral, is the natural logarithm of the ratio of incident to transmitted intensity:

$$g(y) \equiv \ln\left(\frac{I_0}{I}\right) = \sum_{i=1}^{n} \mu_i \cdot \Delta x_i = \int_a^b \mu(x,y) \cdot dx$$

This line integral encapsulates the fundamental challenge of conventional radiography: the coupling of μ with depth, leading to a loss of depth information and soft-tissue contrast. CT overcomes this limitation by decoupling the depth dependence.

CT numbers, or Hounsfield Units (HU), quantify the attenuation coefficients of tissues relative to water. The formula for CT number is:

$$HU = k\frac{\mu_t - \mu_w}{\mu_w}$$

Where μ_t is the linear attenuation coefficient of the tissue and μ_w is the linear attenuation coefficient of water. The constant k is typically 1000 for diagnostic CT. This allows for standardized grayscale representation of tissue densities, with water defined as 0 HU.

7.1.2 CT Methodology: Projection Description

- **Projection (View)**: A collection of ray-sums acquired along a specific direction (angle θ).
- **Sinogram**: A 2D representation of the acquired projection data, where one axis represents the projection angle (θ) and the other represents the transverse position (t) of the ray within each projection. For a point object, its trajectory in the sinogram traces a sine wave.
- **Radon Transformation (RT)**: A fundamental mathematical transform that maps a 2D function $f(x,y)$ (representing the object's attenuation coefficients) to its set of 1D line integrals (projections). For parallel beam geometry, the Radon Transform $P(\theta,t)$ of a 2D object function $f(x,y)$ is given by:

  $$P(\theta,t) = \iint_{-\infty}^{\infty} f(x,y)\delta(x\cos\theta + y\sin\theta - t)dxdy$$

  where t is the perpendicular distance from the origin to the line of integration, and θ is the angle of the normal to the line with respect to the x-axis. The Radon Transform is a linear transform and is periodic in θ with a period of 2π. For parallel projection geometry, an angular range of 180° contains complete object information, as $P(\theta,t) = P(\theta+\pi,-t)$.
- **Fourier Slice Theorem (Central Slice Theorem)**: This theorem provides the mathematical cornerstone for many CT reconstruction algorithms by establishing a crucial link between the Radon domain (projections) and the 2D Fourier domain of the object. It states that the 1D Fourier Transform of a projection profile $P(\theta,t)$ at a given angle θ is equal to a 1D slice (line) through the 2D Fourier Transform $F(u,v)$ of the original object function $f(x,y)$ at the same angle θ, passing through the origin. Mathematically:

  $$S_\theta(\omega) = \mathcal{F}_1\{P(\theta,t)\} = F(\omega\cos\theta, \omega\sin\theta)$$

  where $S_\theta(\omega)$ is the 1D Fourier Transform of $P(\theta,t)$ with respect to t, and $F(u,v)$ is the 2D Fourier Transform of $f(x,y)$. This theorem implies that by collecting projections from all angles, the entire 2D Fourier space of the object can be sampled, theoretically allowing for reconstruction via an inverse 2D Fourier transform.

7.1.3 CT Methodology: Image Reconstruction

While direct inverse 2D Fourier transform is theoretically possible based on the Fourier Slice Theorem, practical limitations due to data sampling issues (specifically, an oversampling of low frequencies and sparse sampling of high frequencies in the Fourier domain when using angular and radial sampling patterns) lead to blurred reconstructed images. Therefore, more sophisticated algorithms are employed.

- **Simple Backprojection (SBP)**: This method is conceptually straightforward. It involves taking each 1D projection and "smearing" or "back-projecting" its values uniformly back across the image plane along the original ray paths. The final image is then the sum of all these back-projected views.
  - **Blurring Effect**: SBP alone does not accurately reconstruct the original image. It inherently introduces a characteristic $1/r$ blurring artifact (where r is the radial distance from a point source) in the reconstructed image. This blurring arises because SBP corresponds to a convolution of the true image with the $1/r$ function, which, in the Fourier domain, corresponds to an overweighting of low-frequency components relative to high-frequency components (the Fourier transform of $1/r$ is proportional to $1/|\omega|$).
- **Filtered Backprojection (FBP)**: FBP is the most widely used analytical reconstruction algorithm in clinical CT. It addresses the $1/r$ blurring inherent in SBP by applying a frequency-domain filter, specifically a "Ramp filter" (or its modified versions), to each projection *before* backprojection.
  - **Ramp Filtering**: The Ramp filter (often denoted as $|\omega|$ or $R(\omega)$ in the frequency domain) acts as a high-pass filter. When multiplied by the Fourier transform of the projection (which has the $1/|\omega|$ characteristic due to the Radon Transform's relationship with the Fourier Transform), it effectively compensates for the $1/r$ blurring.
  - **Modified Ramp Filters**: In practice, an ideal Ramp filter can amplify high-frequency noise significantly. Therefore, modified Ramp filters are often used, which incorporate smoothing functions (e.g., Hann, Hamming, Shepp-Logan, Butterworth filters) to attenuate high-frequency noise while preserving image detail. These modifications balance noise reduction with spatial resolution.

  The general FBP algorithm implementation involves:

  1. For each projection angle θ:
     - Acquire the 1D projection data $P(\theta, t)$.
     - Compute the 1D Fourier Transform of the projection: $\mathcal{F}_1\{P(\theta, t)\}$.
     - Apply the frequency domain filter (e.g., multiply by a modified Ramp filter $H(\omega)$): $S^*_\theta(\omega) = \mathcal{F}_1\{P(\theta, t)\} \cdot H(\omega)$.
     - Compute the inverse 1D Fourier Transform of the filtered projection: $P^*(\theta, t) = \mathcal{F}_1^{-1}\{S^*_\theta(\omega)\}$. This yields the filtered projection data.
     - Back-project the filtered data $P^*(\theta, t)$ across the image plane.
  2. Sum all backprojected images from all angles to obtain the final reconstructed image $f(x, y)$.

  The ramp-filtering step can also be conceptualized as a convolution in the spatial domain with the inverse Fourier transform of the Ramp filter. **Limitations of FBP**: While efficient and widely used, FBP is an analytical algorithm that assumes ideal data acquisition. It can be problematic with noisy projection data and when projection data are affected by other physical degrading effects (e.g., beam hardening, metal artifacts), as it does not explicitly model these phenomena.
- **Fan Beam Reconstruction**: The standard FBP algorithm, developed for parallel beam geometry, cannot be directly applied to fan beam geometry (where X-rays diverge from a point source). Reconstruction methods for fan beam data include:
  - **Rebinning**: Re-sampling the fan beam data to approximate parallel beam data, allowing the application of parallel-beam FBP algorithms. This involves interpolation and can introduce artifacts.
  - **Direct Inversion Formulas**: Reformulation of the inverse Radon transform that directly handles fan beam geometry, often involving specific weighting functions before backprojection (e.g., the convolution-backprojection algorithm adapted for fan beams).

7.1.4 Factors Affecting Reconstructed Image Quality

Several factors significantly influence the quality of the reconstructed CT image, often posing trade-offs in system design and scan protocols:

- **Angular Sampling**: The number of projections acquired (N_θ). Increasing N_θ enhances image resolution and reduces streak artifacts. According to the Nyquist-Shannon sampling theorem, to avoid aliasing artifacts, the number of angles should be $\geq \pi N_p/2$ for 180° scans, where N_p is the number of detector elements. A rule of thumb for good image quality is that the number of views should be $\geq \pi \times$ (number of pixels across the object)$/2$.
- **Ray Sampling (Spatial Sampling within a Projection)**: Determined by the number and size of individual detector elements (N_r). Finer ray sampling (smaller detector elements, more rays) increases spatial resolution.

- **Frequency Filters**: The choice and characteristics of the reconstruction filter (e.g., Ramp, smoothing filters) are crucial. Filters with a higher cutoff frequency (sharper filters) enhance spatial resolution but increase image noise. Smoother filters reduce noise but can lead to a loss of detail.
- **Noise**: Random fluctuations in the detected X-ray signal. Noise is a fundamental limiting factor in image quality and is often quantified by the standard deviation of pixel values in a uniform region. It is primarily influenced by the number of detected photons (related to radiation dose) and the quantum detection efficiency of the detectors. Higher noise can obscure fine details and reduce contrast.
- **Artifacts**: Various phenomena can introduce distortions or false information into the CT image, including:
  - **Beam Hardening**: Differential attenuation of low-energy versus high-energy X-rays, leading to "cupping" artifacts and streaks.
  - **Metal Artifacts**: Severe streaks and shadows caused by high-attenuating metallic objects.
  - **Motion Artifacts**: Blurring or ghosting caused by patient movement during the scan.
  - **Partial Volume Artifacts**: Occur when a voxel contains multiple tissue types, leading to an averaged attenuation value.

7.2 CT Technology Evolution

CT technology has undergone significant advancements since its inception, progressing through several generations marked by improvements in scanning geometry, detector technology, and reconstruction algorithms.

7.2.1 Computed Axial Tomography (CAT)

Early forms of tomography, like Computed Axial Tomography (CAT), developed in 1933, involved moving the X-ray tube and detector system in a synchronized manner to "focus" on a specific plane or section of the patient. Points on the focal plane appear in focus, while structures outside this plane are blurred due to the motion. This approach offered lower radiation doses and costs compared to early CT, finding applications in areas like dental panoramic imaging. Tomosynthesis, a more advanced application of limited angle tomography, extends this concept to limited 3D reconstruction from a small number of projections, using "shift and add" principles to computationally reconstruct any slice through the object and remove overlying/underlying structures.

7.2.2 Conventional CT (Generations 1-7)

The historical evolution of conventional CT scanners is typically categorized into several generations, primarily differentiated by their X-ray beam and detector configurations, and their mechanical scanning motions:

Table 4: Evolution of CT Scanner Generations

Generation	X-ray Beam	Detector Array	Scanning Motion	Scan Time (Typical)	Key Features/Limitations
1st Gen	Pencil beam	Single detector	Translate-Rotate	Several minutes per slice	Very slow, only for head scans
2nd Gen	Narrow fan beam	Linear array (approx. 30)	Translate-Rotate	20-60 seconds per slice	Faster than 1G, still translation
3rd Gen	Wide fan beam	Curved array (many detectors)	Rotate-Rotate (continuous)	1-10 seconds per slice	Slip ring introduced, continuous rotation
4th Gen	Wide fan beam	Stationary ring (full circle)	Source Rotate only	1-10 seconds per slice	Reduced motion artifacts, uncollimated detectors
5th Gen (EBCT/EBT)	Electron beam	Stationary target ring	Electron beam deflection (no mechanical motion)	50-100 ms per slice	Extremely fast (cardiac), high cost, limited spatial resolution
6th Gen (Single-slice Spiral/Helical CT)	Fan beam	3rd/4th Gen	Continuous rotation + table translation	30-60 seconds per volume	Volumetric data, reduced breath-hold
7th Gen (Multi-slice Spiral/Helical CT)	Cone beam	Multiple detector rows	Continuous rotation + table translation	¡1-20 seconds per volume	Significantly faster, thinner slices, true 3D acquisition

- **Slip-Ring Technology**: A pivotal innovation (introduced in 3rd generation CT) comprising circular electrical conductive rings and brushes that transmit electrical energy and data across a moving interface. This enables continuous gantry rotation, eliminating the need for cables to unwind, significantly reducing scan times and enabling helical scanning.
- **Collimators and Beam Shaping Filters**:
  - **Collimators**: Located between the X-ray tube and the patient, they shape the X-ray beam to define slice thickness and reduce scatter radiation, which improves image contrast and reduces patient dose.
  - **Beam Shaping (Bowtie) Filters**: These filters are placed in the X-ray beam path, typically after the tube. They are designed to be thicker at the edges and thinner in the center. This geometry compensates for the typically cylindrical shape of the human body, resulting in more uniform X-ray intensity reaching the detectors and reducing dose to peripheral tissues (especially skin dose).
- **CT Detector Technology**: Evolution from gas (xenon) ionization chambers (now largely obsolete due to lower quantum detection efficiency and longer after-glow) to solid-state detectors (SSDs). Modern SSDs typically utilize scintillation crystals (e.g., Cadmium Tungstate, Gadolinium Oxysulfide, Cesium Iodide) coupled to photodiodes. These detectors offer high quantum detection efficiency (95-100%), fast response times, and minimal after-glow. Current CT scanners feature 2D detector arrays, typically curved in the axial plane and rectangular along the longitudinal (z-axis), with pixel sizes ranging from 0.5 to 2 mm and up to 320 detector rows along the z-axis.
- **CT Scanning Modes**:
  - **Axial Scanning ("Step and Shoot")**: The gantry rotates to acquire data for a single slice, stops, the patient table moves to the next position, and the process repeats. This method is suitable for high-resolution static imaging.
  - **Spiral (Helical) Scanning**: The gantry rotates continuously while the patient table simultaneously moves through the gantry bore. This creates a continuous helical X-ray path around the patient, enabling rapid acquisition of volumetric data.
- **Pitch Determination**: In helical scanning, pitch is a critical parameter that quantifies the relationship between table movement and X-ray beam width.
  - For single-slice CT: The pitch P is defined as the table travel (T) per 360° gantry rotation divided by the nominal X-ray beam width (W):
$$P = \frac{T}{W}$$
  - For multi-row detector CT: The pitch P is defined as the table travel (T) per 360° gantry rotation divided by the total width of the active detector rows (nD, where n is the number of active rows and D is the width of a single detector element):
$$P = \frac{T}{nD}$$

  A pitch of 1 indicates contiguous scanning without overlap or gaps. A pitch > 1 results in faster scans but undersampling along the z-axis. A pitch < 1 leads to overlapping scans, improved z-axis sampling, but increased scan time and radiation dose.
- **Multi-slice CT (MSCT)**: A significant advancement (7th generation onwards) that utilizes multiple rows of detectors along the z-axis, allowing simultaneous acquisition of multiple slices. This drastically reduces scan times, improves z-axis resolution by enabling thinner slices, and facilitates true 3D imaging. Detector array designs for MSCT include linear, hybrid, and adaptive arrays, differing in the uniform or varying width of their detector elements across rows.
- **Helical Interpolation and Reconstruction**: Due to the helical path of data acquisition, direct application of 2D FBP is not possible. Sophisticated interpolation techniques (e.g., 360° Linear Interpolation, 180° Linear Interpolation, or more advanced algorithms like N-interpolation) are applied to estimate projection data at specific axial positions for reconstruction. For wider beam geometries used in modern MSCT, 3D Cone-Beam Reconstruction algorithms (e.g., Feldkamp-Davis-Kress (FDK) algorithm, or more exact algorithms for spiral CT) are employed to handle the diverging X-ray beam geometry.

7.2.3 Emerging CT Technology

The field of CT continues to advance with novel approaches aiming to enhance diagnostic capabilities and patient safety, pushing the boundaries of spatial and temporal resolution and material characterization.

- **Spectral CT (Multi-energy CT)**: This technology moves beyond conventional CT's single energy-integrated measurement to exploit the energy-dependent nature of X-ray attenuation.

  - **Principle**: Different materials exhibit distinct attenuation coefficients at varying X-ray energies (photoelectric effect dominates at lower energies, Compton scattering at higher energies). Spectral CT takes advantage of the information contained within each energy bin of the incident X-ray spectrum. The linear attenuation coefficient $\mu(E)$ can be expressed as a linear combination of independent basis functions (e.g., photoelectric absorption and Compton scattering basis functions), allowing for decomposition and quantification of different materials.
  - **Dual Energy CT (DECT)**: A practical implementation of spectral CT that analyzes two X-ray spectra acquired at two different average photon energies (e.g., switching kVp during the scan, using dual X-ray tubes at different kVp, or utilizing dual-layer/sandwich detectors). DECT enables material differentiation (e.g., bone removal, iodine mapping, fat quantification) and provides virtual monoenergetic images, which can improve contrast and reduce artifacts.

- **Photon Counting Detector (PCD) CT**: Represents a significant paradigm shift from conventional energy-integrating detectors.
  - **Work Principle**: Unlike energy-integrating detectors that sum up the total energy deposited by multiple photons over a time interval, PCDs count individual X-ray photons that hit each detector pixel and simultaneously measure their energy. This direct counting and energy discrimination provide superior signal-to-noise ratio and eliminate electronic noise and spectral distortions (e.g., beam hardening).
  - **Advantages**: PCDs offer inherent spectral information (allowing for "always on" spectral imaging without additional scans), potentially higher spatial resolution (due to smaller pixel sizes and elimination of scintillator spreading), and improved dose efficiency (by rejecting low-energy noise and more precisely counting photons).
  - **Instrumentation Challenges**: High X-ray flux in CT ($> 10^9$ counts per second per mm^2) presents significant technical challenges. Key issues include:
    * *Pulse Pile-up*: Multiple photons arriving almost simultaneously can be counted as a single, higher-energy photon, leading to spectral distortion and count rate limitations.
    * *Charge Sharing & Cross-talk*: Charge generated by a photon interaction in one pixel might spread to adjacent pixels, leading to miscounts or energy misassignment.
    * *Compton Scattering*: Scatter within the patient and detector can degrade image quality and spectral accuracy.
    * *Polarization and Long-term Reliability*: Issues related to semiconductor material properties under high flux.
    * *Data Rate Capability*: The need to process billions of events per second requires extremely fast readout ASICs.
  - **Detector Architectures**: Common types include hybrid pixel detectors (semiconductor sensor and readout ASIC are separate but connected via bump bonds) and monolithic CMOS pixel detectors (sensor and readout integrated into a single chip). Each has trade-offs in fill-factor, cost, and complexity.

- **Quanta Image Sensor (QIS)**: An emerging sensor technology that aims to count every single photon that hits the sensor, recording its location and precise arrival time. With gigapixel capabilities, QIS holds immense potential for ultra-high spatial resolution and ultra-low light performance in various computational imaging applications, including future generations of CT.

- **Machine Learning for Tomographic Imaging**: Machine learning and artificial intelligence (AI) algorithms are increasingly being integrated into CT workflows. Applications include:
  - **Image Reconstruction**: Learning-based reconstruction methods (e.g., deep learning reconstruction) can potentially outperform conventional FBP in noise reduction, artifact suppression, and image quality at lower doses.
  - **Image Processing and Enhancement**: Noise reduction, de-blurring, and artifact correction.
  - **Image Analysis and Quantification**: Automated lesion detection, segmentation, and quantitative measurements.
  - **Dose Optimization and Protocol Design**: Intelligent systems for personalized dose management.

  These advancements promise to further improve diagnostic accuracy, reduce radiation dose, and enhance the efficiency of CT examinations.

8 X-Ray CT Dosimetry and Image Quality

8.1 CT Concept and Methodology

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  $$S_\theta(\omega) = \mathcal{F}_1\{P(\theta,t)\} = F(\omega\cos\theta, \omega\sin\theta)$$

  where $S_\theta(\omega)$ is the 1D Fourier Transform of $P(\theta,t)$ with respect to t, and $F(u,v)$ is the 2D Fourier Transform of $f(x,y)$. This theoretical foundation suggests that by collecting projections from all angles, the entire 2D Fourier space of the object can be sampled, theoretically allowing for reconstruction via an inverse 2D Fourier transform.

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  The general FBP algorithm implementation involves:

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     - Compute the inverse 1D Fourier Transform of the filtered projection: $P^*(\theta, t) = \mathcal{F}_1^{-1}\{S^*_\theta(\omega)\}$. This yields the filtered projection data.
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- **CT Scanning Modes**:
  - **Axial Scanning ("Step and Shoot")**: The gantry rotates to acquire data for a single slice, stops, the patient table moves to the next position, and the process repeats. This method is suitable for high-resolution static imaging.
  - **Spiral (Helical) Scanning**: The gantry rotates continuously while the patient table simultaneously moves through the gantry bore. This creates a continuous helical X-ray path around the patient, enabling rapid acquisition of volumetric data.
- **Pitch Determination**: In helical scanning, pitch is a critical parameter that quantifies the relationship between table movement and X-ray beam width.
  - For single-slice CT: The pitch P is defined as the table travel (T) per 360° gantry rotation divided by the nominal X-ray beam width (W):

$$P = \frac{T}{W}$$

– For multi-row detector CT: The pitch P is defined as the table travel (T) per 360° gantry rotation divided by the total width of the active detector rows (nD, where n is the number of active rows and D is the width of a single detector element):

$$P = \frac{T}{nD}$$

A pitch of 1 indicates contiguous scanning without overlap or gaps. A pitch > 1 results in faster scans but undersampling along the z-axis. A pitch < 1 leads to overlapping scans, improved z-axis sampling, but increased scan time and radiation dose.

- **Multi-slice CT (MSCT)**: A significant advancement (7th generation onwards) that utilizes multiple rows of detectors along the z-axis, allowing simultaneous acquisition of multiple slices. This drastically reduces scan times, improves z-axis resolution by enabling thinner slices, and facilitates true 3D imaging. Detector array designs for MSCT include linear, hybrid, and adaptive arrays, differing in the uniform or varying width of their detector elements across rows.
- **Helical Interpolation and Reconstruction**: Due to the helical path of data acquisition, direct application of 2D FBP is not possible. Sophisticated interpolation techniques (e.g., 360° Linear Interpolation, 180° Linear Interpolation, or more advanced algorithms like N-interpolation) are applied to estimate projection data at specific axial positions for reconstruction. For wider beam geometries used in modern MSCT, 3D Cone-Beam Reconstruction algorithms (e.g., Feldkamp-Davis-Kress (FDK) algorithm, or more exact algorithms for spiral CT) are employed to handle the diverging X-ray beam geometry.

8.2.2 Emerging CT Technology

The field of CT continues to advance with novel approaches aiming to enhance diagnostic capabilities and patient safety, pushing the boundaries of spatial and temporal resolution and material characterization.

- **Spectral CT (Multi-energy CT)**: This technology moves beyond conventional CT's single energy-integrated measurement to exploit the energy-dependent nature of X-ray attenuation.
  - **Principle**: Different materials exhibit distinct attenuation coefficients at varying X-ray energies (photoelectric effect dominates at lower energies, Compton scattering at higher energies). Spectral CT takes advantage of the information contained within each energy bin of the incident X-ray spectrum. The linear attenuation coefficient $\mu(E)$ can be expressed as a linear combination of independent basis functions (e.g., photoelectric absorption and Compton scattering basis functions), allowing for decomposition and quantification of different materials.
  - **Dual Energy CT (DECT)**: A practical implementation of spectral CT that analyzes two X-ray spectra acquired at two different average photon energies (e.g., switching kVp during the scan, using dual X-ray tubes at different kVp, or utilizing dual-layer/sandwich detectors). DECT enables material differentiation (e.g., bone removal, iodine mapping, fat quantification) and provides virtual monoenergetic images, which can improve contrast and reduce artifacts.
- **Photon Counting Detector (PCD) CT**: Represents a significant paradigm shift from conventional energy-integrating detectors.
  - **Work Principle**: Unlike energy-integrating detectors that sum up the total energy deposited by multiple photons over a time interval, PCDs count individual X-ray photons that hit each detector pixel and simultaneously measure their energy. This direct counting and energy discrimination provide superior signal-to-noise ratio and eliminate electronic noise and spectral distortions (e.g., beam hardening).
  - **Advantages**: PCDs offer inherent spectral information (allowing for "always on" spectral imaging without additional scans), potentially higher spatial resolution (due to smaller pixel sizes and elimination of scintillator spreading), and improved dose efficiency (by rejecting low-energy noise and more precisely counting photons).
  - **Instrumentation Challenges**: High X-ray flux in CT ($> 10^9$ counts per second per mm^2) presents significant technical challenges. Key issues include:
    - *Pulse Pile-up*: Multiple photons arriving almost simultaneously can be counted as a single, higher-energy photon, leading to spectral distortion and count rate limitations.
    - *Charge Sharing & Cross-talk*: Charge generated by a photon interaction in one pixel might spread to adjacent pixels, leading to miscounts or energy misassignment.
    - *Compton Scattering*: Scatter within the patient and detector can degrade image quality and spectral accuracy.
    - *Polarization and Long-term Reliability*: Issues related to semiconductor material properties under high flux.
    - *Data Rate Capability*: The need to process billions of events per second requires extremely fast readout ASICs.
  - **Detector Architectures**: Common types include hybrid pixel detectors (semiconductor sensor and readout ASIC are separate but connected via bump bonds) and monolithic CMOS pixel detectors (sensor and readout integrated into a single chip). Each has trade-offs in fill-factor, cost, and complexity.

- **Quanta Image Sensor (QIS)**: An emerging sensor technology that aims to count every single photon that hits the sensor, recording its location and precise arrival time. With gigapixel capabilities, QIS holds immense potential for ultra-high spatial resolution and ultra-low light performance in various computational imaging applications, including future generations of CT.
- **Machine Learning for Tomographic Imaging**: Machine learning and artificial intelligence (AI) algorithms are increasingly being integrated into CT workflows. Applications include:
  - **Image Reconstruction**: Learning-based reconstruction methods (e.g., deep learning reconstruction) can potentially outperform conventional FBP in noise reduction, artifact suppression, and image quality at lower doses.
  - **Image Processing and Enhancement**: Noise reduction, de-blurring, and artifact correction.
  - **Image Analysis and Quantification**: Automated lesion detection, segmentation, and quantitative measurements.
  - **Dose Optimization and Protocol Design**: Intelligent systems for personalized dose management.

  These advancements promise to further improve diagnostic accuracy, reduce radiation dose, and enhance the efficiency of CT examinations.

9 CT Dosimetry and Image Quality Characterization

This section details the critical balance between CT image quality and patient radiation dose, along with comprehensive methods for characterizing and controlling CT image quality and identifying common artifacts.

9.1 Image Quality vs. Dose

There is an inherent trade-off between CT image quality and patient radiation dose. Improving image quality typically involves increasing the number of detected photons, which directly correlates with higher patient dose. This relationship is quantitatively described by:

$$D \propto \frac{SNR^2}{\Delta^3 T}$$

Where D is the radiation dose, SNR is the signal-to-noise ratio, Δ is the pixel size (FoV/matrix size), and T is the slice thickness. This highlights that increasing SNR (for better image quality) results in a squared increase in dose. Given the increasing trend in CT usage and public concern about potential long-term cancer risks, dose optimization is paramount.

9.1.1 Factors Affecting CT Dose & Image Quality

Both dose and image quality are influenced by a multitude of factors:

- **Primary Factors (directly influence dose and image quality):**
  - **Tube Current (mA) and Exposure Time (s) / mAs**: Directly proportional to the number of X-ray photons produced. Higher mAs increases SNR and reduces noise but increases dose.
  - **Tube Voltage (kVp)**: Affects X-ray beam energy and penetration. Higher kVp generally reduces image contrast (especially for iodine/bone) but increases signal, improving SNR for a given mAs and reducing beam hardening. Dose increases non-linearly with kVp (approx. kVp^{2-3}).
  - **Pitch Factor**: In helical CT, higher pitch reduces scan time and dose (by reducing overlaps) but can decrease image quality due to undersampling.
  - **Scan Acquisition Type**: Axial vs. helical vs. prospective gating, etc.
- **Secondary Factors (affect dose/quality via scanner settings):**
  - **Scan Field-of-View (SFOV)**: The area from which projection data are collected.
  - **Display Field-of-View (DFOV)**: The area displayed in the reconstructed image.
  - **Beam Collimation**: Determines the slice thickness and number of active detector rows.
  - **Reconstructed Slice Width & Interval**: Influences partial volume effects and volume coverage.
  - **Reconstruction Algorithms**: Filters (kernels) and iterative reconstruction methods impact noise and resolution.
- **Other Factors (patient- or system-dependent):**
  - **Patient Size/Attenuation**: Larger patients require higher doses for comparable image quality.
  - **Scan Length**: Longer scans result in higher cumulative dose. **Patient Motion**: Causes artifacts that degrade image quality, potentially requiring repeat scans and increased dose.
  - **Geometry and Detector Efficiency**: System design characteristics affecting photon detection.

9.2 CT Dosimetry Metrics

To quantify patient dose and scanner output, specific dose descriptors and metrics are used:

- **Absorbed Dose in Tissue (D_T)**: Physical dose, measured in milliGray (mGy).
- **Dose Equivalent in Tissue (H_T)**: Accounts for the biological effectiveness of different radiation types, measured in milliSievert (mSv).
- **Effective Dose (E_T)**: A weighted sum of dose equivalents to various organs, representing the overall stochastic risk to the patient, measured in mSv. This is often compared to natural background radiation (e.g., 3 mSv annual average).

9.2.1 Common CT Dose Descriptors

- **Computed Tomography Dose Index (CTDI)**: A measure of the X-ray tube output and absorbed dose delivered in a standardized phantom.
  - **CTDI$_{100}$**: Measured over a 100mm long pencil ion chamber in specific CT phantoms (head and body).
  - **CTDI$_W$ (Weighted CTDI)**: Combines central and peripheral dose measurements in the phantom to represent the average dose across the phantom slice:

$$CTDI_W = \frac{1}{3}CTDI_{centre} + \frac{2}{3}CTDI_{periphery}$$

  - **CTDI$_{\text{VOL}}$ (Volumetric CTDI)**: Accounts for the helical pitch, providing a measure of the average dose for a complete scan volume.

$$CTDI_{\text{VOL}} = \frac{CTDI_W}{\text{Pitch}}$$

    Units for CTDI metrics are mGy.

- **Dose Length Product (DLP)**: A measure of the total radiation energy imparted to the patient for an entire CT examination, combining the dose per slice with the scan length.

$$DLP = CTDI_{\text{VOL}} \times \text{Scan length}$$

  Units for DLP are mGy·cm. DLP is a better indicator of total energy deposited than CTDI alone.

- **Relationship between DLP and Effective Dose**: A reasonable approximation of effective dose (E) can be obtained by multiplying DLP with a conversion factor (k), which is specific to the scanned body part and patient age.

$$E = DLP \times k$$

9.2.2 Limitations of CTDI

- CTDI is primarily a measure of the **scanner's radiation output** under specific conditions (standard phantoms and ion chambers), not the actual patient dose.
- It does **not account for patient size or anatomy**. A CTDI$_{\text{VOL}}$ of 20 mGy for a large patient results in a much lower actual dose than the same CTDI$_{\text{VOL}}$ for a small patient.
- Therefore, using CTDI directly to estimate patient dose is problematic without significant correction factors.
- **Size-Specific Dose Estimate (SSDE)**: To address CTDI's limitations, SSDE was introduced. It uses patient-specific anatomical information (e.g., water-equivalent diameter derived from the scout image) and applies conversion factors to CTDI$_{\text{VOL}}$ to provide a more accurate estimate of individual patient dose.

9.3 CT Dose Optimization and Management

Dose optimization in CT is guided by the **ALARA (As Low As Reasonably Achievable)** principle. Strategies include:

- **Tube Current Modulation (TCM) / Automatic Exposure Control (AEC)**: These systems automatically adjust the tube current (mA) during the scan based on the patient's attenuation profile (derived from scout images), ensuring adequate image quality while minimizing dose. This includes z-axis modulation (along the length) and angular modulation (around the circumference).
- **Selection of Dose-Efficient Tube Potential (kVp)**: Adjusting kVp based on clinical indication and patient size.
- **Iterative Reconstruction (IR) Methods**: These advanced reconstruction algorithms iteratively refine the image by modeling the noise and physics of the imaging system. They can significantly reduce image noise or allow for substantial dose reduction (e.g., 30-50% or more) for the same image quality compared to FBP.
- **Multi-detector Row Arrays**: Enable faster volumetric scanning, reducing motion artifacts and breath-hold times.
- **Increased Spiral Pitch**: Reduces scan time and dose.
- **Beam Shaping Filters**: Ensure uniform dose distribution.
- **Solid State Scintillating Detectors**: Higher detection efficiency.
- **Electronic Circuits with Lower Background Noise**: Improves signal detection.
- **CT Dose Reporting**: Increasingly required (e.g., California SB-1237) to record and report CT doses, promote accreditation, and track potential "overdosing" or cumulative patient dose.

9.4 Image Quality Characterization for X-ray CT

Image quality is quantified using various descriptors to ensure diagnostic efficacy.

9.4.1 CT Spatial Resolution

Spatial resolution describes the ability to distinguish small objects or details within an image. In CT, there are two main types:

- **Transaxial Resolution (In-plane Resolution)**: Measured in line pairs per millimeter (lp/mm), typically 5-8 lp/mm. It describes resolution axially across the patient (x-y plane).
  - **Limiting Spatial Resolution (LSR)**: The highest spatial frequency that can be visualized.
  - **Factors Affecting Transaxial Resolution**:
    - *Scanner Factors (Hardware)*: Beam focal spot size (smaller is better, flying focal spot technology doubles effective projections), detector element size (smaller is better, but reduces efficiency), and detector design properties (e.g., quarter-ray detector offset for improved sampling).
    - *Scan & Reconstruction Parameters*: Number of projections (angular sampling), reconstruction filter (sharp kernels give higher resolution but more noise), pixel size ($d = \text{FOV}/N$, where N is matrix size), and highest spatial frequency ($f_{max} = 1/(2d)$).
  - **Dynamic or Flying Focal Spot Technology**: Rapidly oscillates the focal spot position on the anode during gantry rotation, effectively doubling the number of projections without increasing mechanical rotation speed, thereby improving spatial resolution.
- **Z-sensitivity (Longitudinal Resolution)**: Measured in millimeters (mm), typically 0.5-10 mm. It refers to the effective imaged slice width along the length of the patient (z-direction).
  - **Factors Affecting Z-sensitivity**: Detector slice thickness (thinner slices improve z-sensitivity), pitch factor (pitch ¡ 1 (overlapping slices) improves z-sensitivity), and focal spot (finer spot improves z-sensitivity).
  - **Importance of Slice Thickness**: Thinner slices improve resolution and reduce partial volume effects, enabling isotropic scanning (where voxel dimensions are equal in all directions), which allows for better multi-planar reformatting and volume rendering. However, thinner slices also lead to increased image noise for the same dose.

9.4.2 Image Contrast

Image Contrast (C_i) describes the ability to differentiate distinct objects or tissues within an image. It depends on:

- **Object Contrast (C_o)**: The inherent difference in linear attenuation coefficients between the object and its background.
- **Contrast Sensitivity of Imaging System**: Determined by the **Modulation Transfer Function (MTF)**.

$$C_i = C_o \cdot \text{MTF}$$

Factors Influencing CT Image Contrast:

- **Noise**: Higher noise reduces contrast differentiation.
- **Tube Current (mAs)**: Lower mAs leads to more noise, thus worse contrast.
- **Inherent Tissue Properties**: Fundamental differences in μ of adjacent tissues.
- **Beam Kilovoltage (kVp)**: Higher beam energy generally reduces image contrast by decreasing the dominance of the photoelectric effect.
- **Patient Scatter**: Reduces image contrast by adding a "fog" to the image.
- **Use of Contrast Media**: Enhances contrast between specific tissues (e.g., blood vessels, tumors) and surrounding structures.

9.4.3 Image Noise

Noise describes the uncertainty or random fluctuations in the recorded imaging signal. It is typically measured as the standard deviation of CT numbers in a uniform region of interest. **Main Sources of Noise**:

- **Source-Related Noise**:
  - *Quantum Noise (Photon Starvation)*: The most dominant source, arising from the statistical fluctuation in the number of X-ray photons reaching the detector (Poisson statistics). Noise is inversely proportional to the square root of the number of photons ($\sigma \propto 1/\sqrt{N}$).
  - *Scatter*: Scattered photons contribute to noise by adding unwanted signal.

- **Detector Noise**: Electronic noise, dark current, and imperfections in pixels.
- **Reconstruction Process Noise**: Introduced by the algorithms, especially high-frequency filters.

Factors Affecting Noise:

- **Number of Photons (mAs)**: Higher mAs reduces quantum noise ($noise^2 \propto 1/mAs$).
- **Energy of Photons (kVp)**: Higher kVp can reduce noise by increasing penetration.
- **Slice Thickness**: Thinner slices have fewer photons per voxel, increasing noise ($noise^2 \propto 1/\text{slice thickness}$).
- **Collimation**: Affects scatter.
- **Helical Speed/Pitch**: Affects the number of photons per projection.
- **Helical Interpolation**: Can introduce noise or artifacts. **Detector Efficiency**: Higher efficiency reduces noise.
- **Pixel Size**: Larger pixel sizes integrate over more photons, reducing noise (e.g., quadrupling pixel area halves noise standard deviation).
- **Reconstruction Filter**: Sharper filters increase noise.

There is an inherent relationship: $\sigma^2 \propto \frac{f^3}{zD}$ where σ is image noise, f is spatial resolution, z is slice thickness, and D is dose. This highlights the interconnectedness of these factors.

9.4.4 Combined Performance Measures

Beyond basic descriptors (Contrast, Resolution, Noise), combined performance measures provide a more holistic view of system performance:

- **Modulation Transfer Function (MTF)**: Expresses how well an imaging system transfers object contrast from the input to the output as a function of spatial frequency. It describes the system's ability to preserve object details across different sizes.

  $$MTF(f) = \frac{M_{\text{output}}(f)}{M_{\text{input}}(f)}$$

  Where M is the modulation (contrast) at a given spatial frequency f. The MTF of a system is the product of the MTFs of its individual components. It can be derived from the Fourier Transform of the Point Spread Function (PSF) or Line Spread Function (LSF).

- **Signal-to-Noise Ratio (SNR)**: Quantifies the strength of the signal relative to the background noise, indicating the ability to distinguish an object from noise.

  $$SNR = \frac{\text{Signal}}{\text{Noise}} = \frac{N}{\sqrt{N}} = \sqrt{N}$$

  Where N is the number of detected X-ray photons. Doubling the SNR requires quadrupling the dose.

- **Wiener Spectrum (WS) or Noise Power Spectrum (NPS)**: Characterizes the spatial frequency distribution of noise in an image. It describes noise power as a function of spatial frequency.

- **Contrast-to-Noise Ratio (CNR)**: Similar to SNR but specifically focuses on the detectability of a low-contrast object against its background.

  $$CNR = \frac{I_1 - I_2}{\sigma_{\text{noise}}}$$

  Where I_1 and I_2 are the intensities of the object and background, respectively, and σ_{noise} is the noise standard deviation. CNR is highly sensitive to scattered radiation.

- **Detective Quantum Efficiency (DQE)**: A fundamental measure of detector performance, describing how efficiently an imaging system converts incident X-ray photons into a useful image signal, relative to an ideal detector. It quantifies the degradation of SNR from the input to the output of the detection process.

  $$DQE = \frac{SNR^2_{\text{out}}}{SNR^2_{\text{in}}}$$

  DQE is a frequency-dependent metric that combines the effects of MTF and noise (NPS):

  $$DQE(\nu) = k\frac{[MTF(\nu)]^2}{NPS(\nu) \cdot \text{Exposure}}$$

  A higher DQE indicates better dose efficiency and SNR preservation, aiding detection, especially at low-to-mid frequencies.

9.5 CT Image Quality Control and Management

Regular Quality Assurance (QA) and Quality Control (QC) programs are essential to ensure consistent CT image quality and optimize dose.

9.5.1 CT QA and Performance Tests

CT QA involves routine performance tests using specialized phantoms (e.g., Catphan 600) to assess various image quality parameters:

- **Noise and Field Uniformity**: Measured by placing ROIs in uniform phantom sections. CT number uniformity requires differences between peripheral and central regions to be small (< 8 HU), often affected by beam hardening.
- **CT Number Linearity**: Assesses the accuracy of CT numbers for different materials, evaluated using sensitometry modules.
- **Low Contrast Detectability (Low Contrast Resolution, LCR)**: The ability to detect structures with small differences in HU from their background. It is primarily limited by image noise. Tested using phantoms with subtle contrast differences.
- **Spatial Resolution (High Contrast Resolution)**: The ability to resolve small, high-contrast objects (e.g., line pairs, small beads). Tested using high-resolution modules or by measuring the system's PSF/MTF.
- **Slice Sensitivity Profile (SSP)**: Evaluates the actual width of the imaged slice, crucial for understanding z-sensitivity. Measured by scanning a thin ramp or bead.
- **CTDI Measurement**: To quantify scanner output.

9.5.2 CT Artifacts

Artifacts are distortions or errors in the CT image that do not represent true anatomical features. They degrade image quality and can obscure pathology. Sources are broadly categorized:

- **Physics-Based Artifacts**: Related to the physical interactions of X-rays with the patient or detector.
  - **Beam Hardening**: Occurs because lower-energy photons in the polychromatic X-ray beam are preferentially absorbed, leaving a "harder" (higher mean energy) beam. This leads to cupping artifacts (darker center in uniform objects) and streaks between dense objects (e.g., bone). Solutions include pre-patient filters and bowtie filters.
  - **Partial Volume Artifacts (PVE)**: Occur when a voxel contains a mixture of different tissue types, resulting in an averaged CT number that doesn't accurately represent either tissue. This only reduces apparent attenuation and is common with thick slices. Solutions include thinner slices.
  - **Scatter Artifacts**: Compton scattering adds a uniform "fog" to the image, reducing contrast and increasing noise.
  - **Aliasing Artifacts**: Occur when sampling frequencies (angular or radial) are too low, leading to "ringing" at sharp edges or streak artifacts.
- **Patient-Based Artifacts**: Related to the patient's characteristics or movements.
  - **Metallic Artifacts**: Severe streaking and shadowing caused by highly attenuating metal objects (e.g., implants, dental fillings). Solutions include gantry tilt, increased kVp (with dose implications), or metal artifact reduction (MAR) software.
  - **Motion Artifacts**: Blurring, streaking, or ghosting caused by patient movement (e.g., breathing, cardiac motion, swallowing). Solutions include shorter scan times, breath-holds, or cardiac gating.
- **Scanner-Based Artifacts**: Resulting from hardware malfunction or calibration issues.
  - **Ring Artifacts**: Concentric rings in the image, typically caused by a single malfunctioning or miscalibrated detector element. More visible in uniform objects.
- **Helical-Based Artifacts**: Specific to spiral scanning due to interpolation of helical data.
  - **Pitch Artifacts**: Streaks or "venetian blind" appearance at high pitch values due to undersampling.

Artifacts are primarily caused by inconsistencies in the acquired projection data. Thorough Quality Control routines are crucial for their avoidance and management.

10 MRI Physics: Signal Generation & Characteristics

10.1 Key Characteristics of MRI

- **Non-ionizing Radiation**: Unlike X-rays, CT, and SPECT/PET, MRI does not use ionizing radiation, making it safer for repeated examinations.
- **Superior Soft Tissue Contrast**: MRI provides exceptional contrast between different soft tissues, aiding in the diagnosis of various pathologies.
- **Tunability of Image Contrast**: The ability to manipulate rich physical properties of tissues allows for elegant control over image contrasts, such as T1-weighted (T1W), T2-weighted (T2W), and Proton Density-weighted (PDW) images.

10.2 Synopsis of MRI Operation

1. **Magnetization (M)**: The subject is placed in a strong, static magnetic field (B_0), causing the atomic nuclei with a net magnetic moment to align.
2. **Resonance (R)**: Radiofrequency (RF) waves are transmitted into the subject at the Larmor frequency, exciting the aligned nuclei. When the transmitter is turned off, the excited nuclei emit RF signals (the MR signal).
3. **Imaging (I)**: The strength of the magnetic field is slightly modulated over space using gradient coils, allowing for spatial localization of the emitted signals and subsequent image formation.

10.3 MRI System Components

A typical MRI scanner comprises several coils:

- **Main Field (B_0) Coils**: Generate the strong, constant, and homogeneous static magnetic field (e.g., 1.5T, 3.0T).
- **RF Coils (Radiofrequency Coils)**:
  - **RF Transmission Coil (B_1^+)**: Transmits RF pulses at the Larmor frequency to excite spins.
  - **Receiver Coils**: Detect the weak RF signals emitted by the subject.
- **Gradient Field Coils**: Produce spatially varying magnetic fields for image encoding.
- **Shimming Coils**: Improve the homogeneity of the B_0 field.
- **Shielding Coils**: Reduce the stray magnetic field outside the scanner.

10.4 Milestones in MRI Development

- **1946**: Nuclear Magnetic Resonance (NMR) phenomenon discovered by Felix Bloch and Edward Mills Purcell (Nobel Prize in Physics, 1952).
- **1973**: Magnetic Resonance Imaging (MRI) technology developed by Paul Lauterbur and Peter Mansfield (Nobel Prize in Medicine, 2003).
- **Early 1980s**: Clinical MRI systems become a major medical imaging modality.
- **1993**: Functional MRI (fMRI) in humans.

10.5 Origin of the MR Signal

The MR signal originates from atomic nuclei with a non-zero net spin, primarily hydrogen (^{1}H) protons in the human body (abundant in water and fat). These spinning charged nuclei possess a magnetic moment ($\vec{\mu}$).

10.5.1 Gyromagnetic Ratio (γ)

The magnetic moment ($\vec{\mu}$) is directly proportional to the nuclear spin angular momentum ($\vec{J}$), with the constant of proportionality being the gyromagnetic ratio (γ). For ^{1}H, $\gamma/2\pi = 42.58$ MHz/Tesla.

$$\vec{\mu} = \gamma \vec{J}$$

10.5.2 Precession in a Magnetic Field

When placed in a static magnetic field (B_0), these magnetic moments experience a torque ($\vec{\tau} = \vec{\mu} \times \vec{B_0}$), causing them to precess around the B_0 axis, similar to a spinning toy top. This precessional motion occurs at the Larmor frequency (f_0).

$$f_0 = \frac{\gamma}{2\pi} B_0$$

or in angular frequency:

$$\omega_0 = \gamma B_0$$

10.5.3 Quantum Mechanical Picture: Zeeman Splitting

According to quantum mechanics, in a magnetic field, the energy states of nuclear spins split (Zeeman splitting). For a spin-1/2 proton, there are two energy states: spin-up (lower energy, aligned with B_0) and spin-down (higher energy, anti-aligned with B_0). The energy gap (ΔE) between these states is:

$$\Delta E = \gamma \hbar B_0$$

where $\hbar$ is the reduced Planck constant.

10.5.4 Boltzmann Distribution and Net Magnetization (M)

At thermal equilibrium, there is a slight excess of protons in the lower energy (spin-up) state, following the Boltzmann distribution. This small population difference creates a bulk (net) magnetization vector ($\vec{M}$) that is aligned with the B_0 field.

$$\vec{M} \propto \rho \frac{B_0}{T}$$

where ρ is the spin density and T is the temperature. This net magnetization is static and undetectable without perturbation.

To measure the bulk magnetization, it must be perturbed into an alternating magnetic field that can induce a current in a receiver coil (Faraday's Law of Induction).

10.6 RF Excitation (Resonance)

A radiofrequency (RF) field ($\vec{B_1}$) is applied perpendicular to $\vec{B_0}$ at the Larmor frequency ($\omega_{rf} = \omega_0$). This causes the net magnetization vector ($\vec{M}$) to rotate away from the $\vec{B_0}$ axis, creating a transverse magnetization component ($\vec{M_{xy}}$) and reducing the longitudinal magnetization ($\vec{M_z}$). This process is known as **excitation** or **resonance**. The angle to which $\vec{M}$ is rotated is called the **flip angle** (θ), which is proportional to the amplitude and duration (τ) of the RF pulse.

$$\theta = \gamma B_1 \tau$$

A 90° pulse rotates $\vec{M}$ entirely into the transverse plane ($\vec{M_{xy}}$), while a 180° pulse inverts $\vec{M_z}$.

10.7 Relaxation Processes

After the RF pulse is turned off, the system relaxes back to equilibrium, involving two simultaneous processes:

10.7.1 T2 Relaxation (Transverse Relaxation)

- Characterizes the decay of the transverse magnetization (M_{xy}) due to loss of phase coherence among precessing spins.
- The signal amplitude decreases due to dephasing of spins.
- Caused by:
  1. **Spin-spin interactions**: Irreversible dephasing due to microscopic magnetic field inhomogeneities caused by neighboring nuclei (intrinsic T_2).
  2. **Static magnetic field inhomogeneities**: Reversible dephasing due to imperfections in the main magnetic field and differences in magnetic susceptibility of tissues (contributes to T_2^*).
- The observed transverse magnetization decay is described by T_2^*:

  $$M_{xy}(t) = M_{xy,0} e^{-t/T_2^*}$$

  where $M_{xy,0}$ is the initial transverse magnetization.
- T_2^* is typically shorter than T_2 ($T_2^* < T_2$) because it includes both intrinsic and static field inhomogeneity effects. The relationship is:

  $$\frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T_2'}$$

  where T_2' is the relaxation component due to static field inhomogeneities. Note that the original lecture slide used $\frac{1}{T_2'} = \gamma \Delta B$, which represents the dephasing rate due to field inhomogeneities.

10.7.2 T1 Relaxation (Longitudinal Relaxation)

- Characterizes the recovery of the longitudinal magnetization (M_z) back to its equilibrium value (M_0).
- Caused by **spin-lattice interactions**: Energy transfer from the excited spins to the surrounding molecular lattice.
- After a 90° pulse, the longitudinal magnetization recovery is:

$$M_z(t) = M_0(1 - e^{-t/T_1})$$

- After a 180° inversion pulse, the longitudinal magnetization recovery is:

$$M_z(t) = M_0(1 - 2e^{-t/T_1})$$

- T_1 is typically longer than T_2 ($T_1 > T_2$) because energy transfer is a slower process than dephasing.
- T_1 and T_2 values are tissue-specific and influenced by molecular motion, size, and interactions.

10.8 MR Signals and Pulse Sequences

A pulse sequence is a precisely timed series of RF pulses and magnetic field gradients that manipulate the magnetization to produce desired signal contrasts and image encoding. Key parameters include:

- **Time of Repetition (TR)**: Time between successive excitation pulses. Affects T1 contrast.
- **Time of Echo (TE)**: Time between the excitation pulse and the peak of the echo signal. Affects T2 contrast.
- **Flip Angle** (α): Angle to which $\vec{M}$ is rotated by the RF pulse. Influences signal strength and contrast, especially in GRE sequences.
- **Time of Inversion (TI)**: Used in Inversion Recovery sequences; time between an initial 180° inversion pulse and a 90° readout pulse.

The general signal strength equation for a spin echo sequence is approximately:

$$S \propto \rho \cdot \left(1 - e^{-\mathrm{TR}/T_1}\right) \cdot e^{-\mathrm{TE}/T_2}$$

where ρ is proton density. This equation highlights how ρ, T_1, and T_2 are biological factors, while TR and TE are technical parameters controlled by the pulse sequence.

10.8.1 Free Induction Decay (FID)

- **Formation**: Produced immediately after a single RF excitation pulse (e.g., 90° pulse). The transverse magnetization precesses and induces a signal in the receiver coil.
- **Signal Characteristics**: The signal oscillates at the Larmor frequency and its amplitude decays exponentially with a T_2^* time constant.

$$S_{\mathrm{FID}}(t) \propto M_{xy,0} e^{-t/T_2^*} e^{i\omega_0 t}$$

- **Disadvantages**:
  - Signal decays very quickly due to T_2^* effects (including static field inhomogeneities).
  - The receiver cannot acquire data immediately after RF transmission due to receiver blanking, leading to significant signal loss.
  - The signal is reflective of T_2^*, which is highly sensitive to field inhomogeneities, rather than the intrinsic T_2.

10.8.2 Spin Echo (SE)

- **Formation**: Generated by a 90° RF excitation pulse followed by a 180° RF refocusing pulse at time $\mathrm{TE}/2$. The 180° pulse inverts the phase of the precessing spins, causing those that dephased due to static field inhomogeneities to rephase and form an echo signal at time TE.
- **Signal Characteristics**: The spin echo peak occurs at time TE, and its amplitude is primarily modulated by T_2.

$$S_{\mathrm{SE}}(t) \propto \rho \cdot \left(1 - e^{-\mathrm{TR}/T_1}\right) \cdot e^{-\mathrm{TE}/T_2} \cdot e^{i\omega_0 t}$$

- **Advantages**:

- Compensates for dephasing due to static field inhomogeneities, providing a signal that reflects the intrinsic T_2 of the tissue.
- Leads to higher signal-to-noise ratio (SNR) and contrast-to-noise ratio (CNR) compared to FID for T_2-weighted imaging.
- Can generate multiple echoes (Multi-Echo SE) to measure T_2 more accurately.

- **Disadvantages**:
  - Longer acquisition times due to the need for longer repetition times (TR) and the time taken for the 180° refocusing pulse.
  - Higher RF power deposition due to the 180° pulse.

10.8.3 Gradient Echo (GRE)

- **Formation**: Uses a magnetic field gradient to intentionally dephase spins after an initial RF excitation (often with a small flip angle). A reverse gradient is then applied to rephase the spins and generate an echo signal. Unlike SE, no 180° RF pulse is used for refocusing.
- **Signal Characteristics**: The gradient echo peak occurs when the dephasing from the first gradient is perfectly compensated by the rephasing from the second. The signal is T_2^* weighted.

$$S_{\text{GRE}}(t) \propto \rho \cdot M_z(\text{before RF}) \cdot \sin(\alpha) \cdot e^{-\text{TE}/T_2^*} \cdot e^{i\omega_0 t}$$

  where $M_z(\text{before RF})$ depends on TR and T_1.

- **Advantages**:
  - Faster acquisition times due to shorter TR values and variable (often small) flip angles.
  - Lower RF power deposition.
  - Can be sensitive to susceptibility differences (e.g., blood oxygenation level dependent (BOLD) effect in fMRI).

- **Disadvantages**:
  - More susceptible to magnetic field inhomogeneities (i.e., T_2^* weighting) because it only rephases dephasing from applied gradients, not from intrinsic T_2 effects or static field inhomogeneities.
  - Prone to artifacts near air-tissue interfaces or metal implants.

10.8.4 Comparison of FID, Spin Echo, and Gradient Echo Signals

Table 5: Comparison of FID, Spin Echo, and Gradient Echo Signals

Feature	Free Induction Decay (FID)	Spin Echo (SE)	Gradient Echo (GRE)
Excitation Pulse(s)	Single RF pulse (90°)	90° RF pulse followed by 180° RF refocusing pulse	Single RF pulse (variable flip angle, often small)
Refocusing Mechanism	None (natural dephasing)	180° RF pulse	Bipolar magnetic field gradient
Signal Decay/Weighting	T_2^* weighted	T_2 weighted (rephases static inhomogeneities)	T_2^* weighted (sensitive to static inhomogeneities)
Sensitivity to B_0 Inhomogeneities	High	Low (refocused)	High (not refocused by RF)
Signal Amplitude	Decays rapidly	Higher at TE (refocused)	Varies with flip angle, generally lower than SE for same TE/TR
Acquisition Speed	Fastest (theoretically, but practical issues limit use)	Slower (due to 180° pulse and longer TR)	Faster (due to small flip angles, short TR, and no 180° pulse)
RF Power Deposition	Low	High	Low
Applications	Fundamental NMR studies, initial signal observation	Most conventional T2-weighted imaging, accurate T_2 measurement	Fast imaging, T1-weighted imaging, fMRI (BOLD), perfusion imaging

10.9 Contrast Weighting

By adjusting TR and TE, different tissue properties can be emphasized:

- **T1-weighted (T1W) Images**:
  - **Parameters**: Short TR, Short TE.
  - **Mechanism**: Short TR allows tissues with short T_1 (e.g., fat) to recover their longitudinal magnetization faster, appearing bright. Tissues with long T_1 (e.g., CSF) appear dark. Short TE minimizes T2 effects.
- **T2-weighted (T2W) Images**:
  - **Parameters**: Long TR, Long TE.
  - **Mechanism**: Long TR minimizes T1 contrast. Long TE allows tissues with long T_2 (e.g., fluid/edema) to retain transverse magnetization longer, appearing bright. Tissues with short T_2 (e.g., muscle) appear dark.
- **Proton Density-weighted (PDW) Images**:
  - **Parameters**: Long TR, Short TE.
  - **Mechanism**: Long TR minimizes T1 contrast, and short TE minimizes T2 contrast, allowing the signal to primarily reflect the proton density.

10.9.1 Inversion Recovery (IR) Sequence

- **Formation**: Begins with a 180° inversion pulse, followed by a variable inversion time (TI), and then a 90° readout pulse (often as part of a spin echo sequence).
- **Purpose**: To null the signal from specific tissues by choosing a TI such that the tissue's longitudinal magnetization crosses zero at the time of the 90° readout pulse.
- **Examples**:
  - **STIR (Short Tau Inversion Recovery)**: Short TI (e.g., 130 ms) to suppress fat signal.
  - **FLAIR (Fluid Attenuated Inversion Recovery)**: Long TI (e.g., 2500 ms) to suppress fluid (CSF) signal, useful for detecting periventricular white matter lesions.

10.9.2 Gradient Echo Sequences (Advanced)

- Often use small flip angles to allow for very short TR values.
- Can be "spoiled" (e.g., FLASH - Fast Low Angle SHot) to destroy any residual transverse magnetization, making them primarily T1-weighted.
- **Steady-State Free Precession (SSFP)**: Maintains a steady, residual transverse magnetization between cycles, useful for dynamic imaging like cardiac function assessment.

11 MRI Techniques & Technology

11.1 The Theory of MR Image Formation

11.1.1 Basic Idea

The fundamental principle of MRI relies on the interaction of nuclear spins, particularly hydrogen protons (^{1}H), with strong magnetic fields and radiofrequency (RF) pulses. The process can be summarized as:

$$\mu_i \rightarrow \mathbf{M} \rightarrow \mathbf{M}_{xy} \rightarrow S(t) \rightarrow S(\mathbf{k}) \rightarrow I(\mathbf{r})$$

Where:

- μ_i: Microscopic magnetic moment of individual nuclei.
- $\mathbf{M}$: Bulk (net) magnetization established in the main magnetic field (B_0).
- $\mathbf{M}_{xy}$: Transverse magnetization, generated by tipping $\mathbf{M}$ into the transverse plane.
- $S(t)$: Time-dependent resonance signal (Free Induction Decay - FID) detected by receiver coils.
- $S(\mathbf{k})$: Spatially encoded k-space resonance signal.
- $I(\mathbf{r})$: Spatial image reconstructed from k-space data.

Key Steps Learned So Far:

1. **Main Field** B_0: A static, homogeneous, strong (0.5-10 T) magnetic field establishes a steady (equilibrium) state of magnetization. This aligns ^{1}H nuclear spins with B_0 through spin precession at the Larmor frequency ($\omega_0 = \gamma B_0$), creating a bulk magnetization ($\mathbf{M}$) along the B_0 direction.
2. **RF-field** $B_1(t)$ **on**: A radiofrequency field, $B_1(t)$, applied perpendicular to B_0 with a frequency $\omega_{rf} = \gamma B_0$ (short pulse, ~ 1 ms), establishes an excited (unstable) state. This tips the bulk magnetization ($\mathbf{M}$) into the transverse plane as $\mathbf{M}_{xy}$ and synchronizes the spins into phase coherence (resonance).
3. **RF-field** $B_1(t)$ **off**: Excitation is followed by relaxation, generating an MR signal, $S(t)$, known as the FID signal. This involves:
    - Transverse (M_{xy}) decay: Spin-spin relaxation, characterized by T_2 relaxation.
    - Longitudinal (M_z) recovery: Spin-lattice relaxation, characterized by T_1 relaxation.

11.1.2 Tomographic Approach - Spatial Encoding

A single sinusoidal MR signal at the Larmor frequency ω_0 lacks spatial location information. To obtain bulk magnetization density distributions in individual spatial locations (voxels), spatial encoding is crucial.

General Idea for Spatial Encoding: Generate spatially varying magnetic fields so that spins at different locations precess at frequencies unique to their location. This allows for location selection based on frequency, and excitation of the desired spatial location using an RF pulse to acquire its MR signal. The Larmor frequency $\omega_0 = \gamma B$ is dependent on the local magnetic field B.

The MR signal $S(t)$ contains encoded frequency (ω) and phase (θ) information, which can be utilized to determine in-plane location. The time-dependent resonance signal is given by:

$$S(t) = Ae^{-t/T_2^*}e^{i\omega_0 t}$$

Where T_2^* accounts for both T_2 decay and magnetic field inhomogeneities.

Image formation in MRI uses a tomographic approach, sampling data from the frequency domain. This is achieved using gradient fields (G_x, G_y, G_z).

Gradient Fields and Their Actions: Gradient magnetic fields change the main magnetic field in a controlled, predictable pattern, making it non-homogeneous. They change the strength, but not the direction, of the main field.

In MRI, gradient coils primarily generate additional magnetic fields along the Z-axis (the direction of the main magnetic field B_0). These gradient fields introduce a linear spatial variation to the Z-component of the total magnetic field.

The total magnetic field in the Z-direction at a given spatial position $\mathbf{r} = (x, y, z)$ can be expressed as:

$$B_z(x, y, z) = B_0 + G_x x + G_y y + G_z z$$

Where:

- B_0 is the strength of the homogeneous main magnetic field.
- G_x, G_y, G_z are the **gradient strengths** (or gradient amplitudes) along the x, y, and z axes, respectively.

These gradient strengths represent the rate of change of the B_z field along each direction:

$$G_x = \frac{\partial B_z}{\partial x}, \quad G_y = \frac{\partial B_z}{\partial y}, \quad G_z = \frac{\partial B_z}{\partial z}$$

The units for gradients are typically **mT/m** (millitesla per meter).

This spatially varying magnetic field causes protons at different locations to precess at unique Larmor frequencies, which is fundamental for spatial encoding in MRI.

Spatial Encoding Techniques:

1. **Slice Selection (z-axis)**: A slice selection gradient (G_{SSG}) is applied along an axis perpendicular to the desired slice. Simultaneously, a shaped RF pulse with specific frequency components ($F_c \pm \Delta F/2$) is applied. Only protons precessing at resonant frequencies within a narrow range around the center frequency (ω_0) are excited.

   $$B(Z) = B_0 + G_Z \cdot Z$$

   $$\omega(Z) = \gamma B(Z) = \gamma(B_0 + G_Z \cdot Z)$$

   The center frequency of the RF pulse determines the slice location (F_c), and the bandwidth of the RF pulse (ΔF) determines the slice thickness (Δz):

   $$F_c = \gamma(B_0 + z_{slice} \cdot G_{SSG})$$

   $$\Delta F = \gamma \cdot G_{SSG} \cdot \Delta z$$

   Slice thickness can be controlled by:

   (a) Changing the slope of the slice selection gradient (G_{SSG}).

   (b) Changing the bandwidth (ΔF) of the RF excitation pulse.

   Slice location is primarily controlled by manipulating the gradient.

2. **Phase Encoding (y-axis)**: A phase encoding gradient (G_{PE}) is applied perpendicular to the frequency encoding gradient, along the "logical" y-axis, after slice selection but before readout. Different locations along this axis acquire different phase shifts, encoding spatial information into the phase component of the signal. The strength and duration of G_{PE} are varied for each line of k-space.

3. **Frequency Encoding (x-axis)**: The frequency encoding gradient (G_{FE}), also known as the readout gradient, is applied during the echo (signal readout) along the "logical" x-axis. This causes protons at different locations along this axis to precess at different frequencies, encoding spatial information into the frequency component of the signal.

11.1.3 K-space Filling

K-space is a mathematical space where raw MR data is stored before image reconstruction. Each point in k-space contains information about the spatial frequencies of the image.

Core Concept: K-space coordinate is the time integral of the gradient waveform.

$$k_x = \frac{\gamma}{2\pi} \int_0^t G_x(t')dt'$$

$$k_y = \frac{\gamma}{2\pi} \int_0^t G_y(t')dt'$$

Data measurements ($S(t)$) occur during the frequency encoding gradient. The spatial image $I(\mathbf{r})$ is obtained by applying an Inverse Fast Fourier Transform (IFFT) to the k-space data $S(\mathbf{k})$:

$$I(\mathbf{r}) = \mathcal{F}^{-1}\{S(\mathbf{k})\}$$

where $\mathbf{r} = (x, y)$ and $\mathbf{k} = (k_x, k_y)$. The relationship between the image and k-space is expressed by the Fourier Transform:

$$S(k_x, k_y) = \iint f(x, y)e^{-i2\pi(k_x x + k_y y)}dxdy$$

Where $f(x, y)$ represents the transverse magnetization density, which contributes to the signal. The signal measured is related to the proton density and relaxation times:

$$S(t) \propto \iint \rho(x, y)(1 - e^{-TR/T_1})e^{-TE/T_2^*}e^{i\gamma G_x x t}dxdy$$

This equation highlights how the signal is a function of position, influenced by proton density $\rho(x, y)$, T_1 relaxation (via TR), and T_2^* relaxation (via TE and gradient effects).

K-space Properties:

- The center of k-space primarily contributes to image contrast and general structure (low spatial frequencies).
- The outer regions of k-space contain high spatial frequency information, which determines the image resolution.
- K-space is point symmetric.

Relationship between Sampling and Image Parameters:

- **Field-of-View (FOV)**: Determined by the sampling density in k-space.

$$\text{FOV}_x = \frac{1}{\Delta k_x}$$

$$\text{FOV}_y = \frac{1}{\Delta k_y}$$

- **Spatial Resolution (Δx, Δy)**: Determined by the extent of k-space filled.

$$\Delta x = \frac{1}{K_x^{max}}$$

$$\Delta y = \frac{1}{K_y^{max}}$$

  where K_x^{max} and K_y^{max} are the maximum k-space values sampled.

Scan Duration Calculation: For a standard 2D image acquisition:

$$\text{Scan Time} = TR \times N_{PE} \times NEX$$

Where:

- TR: Repetition Time (time between successive RF pulses for the same slice).
- N_{PE}: Number of phase encoding steps (equivalent to the matrix size in the phase encoding direction).
- NEX: Number of excitations (or averages).

For multi-slice acquisitions where not all slices can be acquired within one TR:

$$\text{Scan Time} = TR \times N_{PE} \times NEX \times N_{packages}$$

Where $N_{packages}$ is the number of groups of slices acquired in multiple TRs.

11.2 Advanced Imaging Techniques

11.2.1 Multi-Slice Acquisition Technique

To acquire a ”stack” of adjacent slices efficiently, multi-slice acquisition techniques are employed. Instead of acquiring data for each slice sequentially, which significantly increases scan duration, data for multiple slices can be acquired during each TR period. This often involves interleaved slice acquisition to reduce ”cross-talk” between adjacent slices.

11.2.2 Scan Time Reducing Techniques

Reducing scan time is crucial for patient comfort and throughput. Methods focus on decreasing the number of TRs needed to fill k-space or acquiring data more efficiently.

1. **Measuring Only Part of K-space**:
   - **Partial k-space (Half k-space) Filling**: Only a portion of k-space (e.g., just over half) is measured. The unmeasured points are estimated by calculating the complex conjugate of the measured points, symmetric through the center of k-space.
   - **Zero-filling**: If outer k-space data (high spatial frequencies) are estimated to be zero, this can increase apparent resolution but sacrifices SNR and can lead to Gibbs Ringing artifact.
   - **Keyhole Sampling**: For fast dynamics, only the central part of k-space is sampled repeatedly, while the outer parts are updated less frequently.
2. **Measuring More Than One Line in K-space Per TR**:
   - **Turbo Spin Echo (TSE) / Fast Spin Echo (FSE)**: Uses a train of refocusing 180° pulses to generate multiple echoes per excitation. Each echo is used to fill a different k-space line. This significantly reduces scan duration by a ”turbo factor” (number of echoes per excitation).

Table 6: Turbo Spin Echo (TSE) Summary

Advantages	Disadvantages
Reduction of acquisition time	Short T2 and long echo train $\rightarrow$ blurring
Excellent T2-weighted images	High SAR values
Elimination of T1 contamination by long TR	Bright fat signal

- **Single-shot Echo Planar Imaging (EPI)**: The fastest MRI sequence. Fills k-space in a "zig-zag" pattern using alternating positive/negative readout gradients after a single RF excitation. Commonly used for fMRI and diffusion imaging. Can suffer from geometric distortions due to long gradient echo trains.

3. **Measuring More Than One Line in K-space Simultaneously with Different Receiver Loops (Parallel Imaging)**: Uses multiple receiver coils with distinct spatial sensitivities to acquire multiple k-space lines concurrently, accelerating image acquisition. This can lead to image quality problems (e.g., artifacts, reduced SNR) if not properly managed.

11.2.3 Diffusion Imaging (Flow Imaging)

Diffusion Weighted Imaging (DWI) measures the random Brownian motion of water molecules within tissue. This provides insight into cellularity (e.g., tumors), cell swelling (e.g., ischemia), and edema.

Signal Attenuation Model: Diffusion Weighted Imaging (DWI) measures the random Brownian motion of water molecules within tissue. This provides insight into cellularity (e.g., tumors), cell swelling (e.g., ischemia), and edema.

Signal Attenuation Model: The signal in DWI is attenuated by T2 decay and diffusion. The diffusion-weighted signal (S) relates to the signal without diffusion weighting (S_0) by the following equation:

$$S = S_0 e^{-b \cdot D}$$

Where:

- S: Diffusion-weighted signal.
- S_0: Signal at $b = 0$ (no diffusion weighting).
- b: Diffusion sensitivity (b-value). This dimensionless value quantifies the strength and duration of the diffusion-sensitizing gradients. Higher b-values mean stronger diffusion weighting.
- D: Apparent Diffusion Coefficient (ADC), which is a measure of the magnitude of diffusion.

ADC maps reflect the actual diffusion values and are independent of T2 weighting.

The b-value formula: The b-value is a crucial parameter determined by the characteristics of the applied diffusion gradient pulses:

$$b = (\gamma G \delta)^2 \left(\Delta - \frac{\delta}{3} \right)$$

Where:

- γ: The **gyromagnetic ratio** of the nucleus being imaged (for protons, approximately 42.57 MHz/T).
- G: The **amplitude (strength)** of the diffusion-sensitizing gradient pulses (in T/m).
- δ: The **duration** of the individual diffusion gradient pulses (in seconds).
- Δ: The **time between the leading edges** of the two diffusion gradient pulses (in seconds), also known as the diffusion time or observation time.

Manipulating these parameters allows for varying the sensitivity of the MR signal to water diffusion.

Types of Diffusion:

- **Free diffusion**: No restriction to movement (e.g., CSF).
- **Restricted diffusion**: Movement is hindered (e.g., highly cellular tumors, acute stroke).
- **Isotropic diffusion**: Diffusion is equal in all directions (e.g., gray matter).
- **Anisotropic diffusion**: Diffusion is preferential along certain directions (e.g., white matter tracts).

Diffusion Tensor Imaging (DTI): Exploits the directional properties of anisotropic diffusion. It measures a diffusion tensor, which is a 3×3 matrix describing the diffusivity in three dimensions, allowing for the tracking of brain fibers. **Types of Diffusion:**

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11.2.4 3D MRI (Volume Imaging)

Instead of acquiring individual 2D slices, 3D MRI acquires a volume. This involves two phase encoding directions (y, z) and one frequency encoding direction (x).

- **Key Characteristics**:
  - Very short TR possible for T1-weighted imaging.
  - High SNR, enabling high resolution (e.g., 1mm isotropic voxels).
- **Advantages**: Single acquisition for multiple planes, beneficial for tumor studies or where multiple reformatting planes are needed.
- **Common Sequences**: 3D TFE (Philips), SPGR (GE), MP-RAGE, FLASH (Siemens).

11.3 Image Quality Factors and Control

MR image quality is judged by clinical efficacy and is affected by intrinsic (tissue characteristics) and extrinsic (user-selectable) factors.

11.3.1 Factors Affecting MR Image Quality

Intrinsic Factors:

- Proton density (PD or N(H))
- T_1 relaxation time
- T_2 relaxation time
- Magnetic susceptibility
- Flow phenomena

Extrinsic (User-selectable) Factors:

- Repetition time (TR)
- Echo time (TE)
- Field-of-view (FOV)
- Matrix size
- Number of signal averages (NEX)
- Magnetic field strength (B_0)
- Properties of the detection coil(s)
- Magnetic field inhomogeneity (also contributes to T_2^*)

Spatial Resolution: MRI resolution is not wavelength-dependent. Typical in-plane resolution is 0.5-1 mm, and out-of-plane is 2-10 mm. Higher resolution is achieved by using stronger gradients and more phase-encoding steps. Trade-offs include loss of signal intensity and increased acquisition time.

SNR: Signal-to-Noise Ratio: SNR is a major limitation for MRI.

$$SNR \propto \frac{\text{Average Signal}}{\sigma_{\text{noise}}}$$

Where σ_{noise} is the standard deviation of Gaussian noise. SNR increases with the square root of the number of averages (NEX):

$$SNR \propto \sqrt{NEX}$$

Signal is highly dependent on the object, pulse sequence, and parameters chosen. Noise is primarily thermal and electronic, with thermal noise dominating and depending on coil type and patient size.

11.3.2 Artifacts

- **Tissue-related artifacts**:
  - **Chemical Shift Artifacts**: Occur because ^{1}H nuclei in fat and water precess at slightly different frequencies due to differing chemical environments. This shift increases with field strength, leading to misregistration of fat and water signals.
  - **Susceptibility Artifacts**: Caused by differences in magnetic susceptibility between adjacent tissues (e.g., air-tissue interfaces, metallic implants), leading to local field inhomogeneities and signal distortion/loss.
- **Machine & technique dependent artifacts**: Main field inhomogeneity, gradient field problems, RF coil/pulse artifacts, aliasing ("wrap-around").
- **Motion-related artifacts**: Patient movement during acquisition, leading to blurring or ghosting.

11.3.3 QA of MRI

Quality Assurance (QA) of MRI systems is routinely performed using phantoms like the ACR MRI Phantom. Key parameters checked include:

- Magnetic field stability (center frequency)
- Signal-to-noise ratio (SNR)
- Geometric accuracy (distortion)
- Image intensity uniformity
- High-contrast spatial resolution
- Low-contrast object detectability
- Percent-signal ghosting

11.4 Emerging New MRI Technology

11.4.1 New/Novel Magnet Designs

- **Higher Fields (e.g., 7T and above)**:
  - Utilize superconducting wire technology (solenoid superconducting magnets).
  - **Advantages**: Higher SNR ($\propto B_0$), improved spatial resolution, better contrast (e.g., T2* contrast for hemosiderin).
  - **Disadvantages**: Increased SAR ($\propto \omega_0^2 \propto B_0^2$), higher cost, more complex shielding, increased susceptibility artifacts.
- **Low Fields (e.g., 0.2T - 1.2T)**:
  - Use permanent and resistive magnets (e.g., C-shaped, dipolar electromagnet designs).
  - **Advantages**: Open geometry (favorable for interventional radiology), cheaper to buy/shield/operate, reduced susceptibility artifacts, lower SAR.
  - **Ultralow-field MRI (0.064T)**: Focus on mobility for bedside MRI (e.g., Hyperfine portable MRI).

11.4.2 Faster and More Powerful Gradient Subsystems

Enable faster switching times and higher gradient strengths, contributing to shorter scan times and improved resolution.

11.4.3 Multi-Modality Imaging Systems

Integration of MRI with other imaging modalities to provide complementary information, often for simultaneous acquisition or treatment guidance.

- **MRI/PET(SPECT)**: Simultaneous acquisition of anatomical (MR) and functional/metabolic (PET) data (e.g., Biograph mMR).
- **CT/MRI**: Potential for combined anatomical imaging, especially in radiotherapy planning (e.g., CT-MRI for more accurate treatment planning).
- **MRI-LINAC**: Integration of MRI with a Linear Accelerator for real-time MRI-guided radiation therapy, allowing for precise tumor tracking and adaptation during treatment (iMRI).

- **Omni-Tomography**: Conceptual framework for integrating multiple imaging modalities (e.g., CT/MRI/PET/SPECT) for comprehensive, simultaneous data acquisition.
- **Polarized Nuclear Imaging (PNI) in MR Framework**: Emerging concept for simultaneous emission-transmission tomography using polarized nuclei (e.g., ^{131m}Xe, ^{127m}Xe, ^{79m}Kr), though clinical practicality is still limited by tracer characteristics.

11.5 Brain MRI Sequence Image Characteristics Comparison

Table 7: Brain MRI Sequence Image Characteristics Comparison

Sequence Type	Grey Matter (GM)	White Matter (WM)	Fat	Cerebrospinal Fluid (CSF)	Primary Clinical Applications/Features
T1-weighted (T1W)	Mid-grey / Grey	Bright White / White	Bright White / White	Dark / Low Signal	Excellent for anatomical detail; useful for evaluating mass effects, hemorrhage.
T2-weighted (T2W)	Light Grey / Grey	Dark Grey / Grey	Intermediate Signal	Bright White / White	Highly sensitive to pathology with increased water content (edema, inflammation, demyelination, stroke).
T1 FLAIR	Mid-grey / Grey	Bright White / White	Bright White / White	Dark / Low Signal (Suppressed)	Improves lesion visibility near CSF (e.g., leptomeningeal disease, MS plaques) by suppressing bright CSF.
T2 FLAIR	Light Grey / Grey	Dark Grey / Grey	Intermediate Signal	Dark / Low Signal (Suppressed)	Most common sequence for detecting brain lesions (e.g., ischemic changes, white matter lesions, tumors, edema, epilepsy foci) by nulling CSF signal.
STIR	Variable / Grey	Variable / Grey	Dark / Low Signal (Suppressed)	Bright White / White	Primarily used for fat suppression; good for bone marrow edema, musculoskeletal pathology. Less common for brain parenchyma.
PDW	Grey / Intermediate	Bright Grey / Bright	Intermediate Signal	Bright White / White	Shows good contrast between grey and white matter; useful for evaluating cartilage (less common for primary brain lesions).

- **T1-weighted (T1W)**
  - **Reflects:** Primarily the **T1 relaxation time** of tissues.
  - **CSF:** Appears **dark/black** (long T1).
  - **Fat:** Appears **bright/white** (short T1).
  - **White Matter:** Appears **brighter** than grey matter (shorter T1 than grey matter due to myelin).
  - **Grey Matter:** Appears **mid-grey**.
  - **Primary Use:** Excellent for detailed **anatomical evaluation** (e.g., assessing brain morphology, tumor mass effect, hemorrhage).
- **T2-weighted (T2W)**
  - **Reflects:** Primarily the **T2 relaxation time** of tissues.
  - **CSF:** Appears **bright/white** (long T2).
  - **Fat:** Appears **intermediate signal**.
  - **White Matter:** Appears **darker** than grey matter (shorter T2 than grey matter).
  - **Grey Matter:** Appears **light grey**.
  - **Primary Use:** Highly sensitive for detecting **pathology with increased water content** (e.g., edema, inflammation, demyelination, stroke).
- **T1 FLAIR (Fluid-Attenuated Inversion Recovery)**
  - **Mechanism:** A specialized T1W sequence that uses an **inversion recovery pulse to suppress the signal from free water**.
  - **CSF:** Appears **dark/black** (suppressed).
  - **Fat:** Appears **bright/white**.
  - **White Matter & Grey Matter:** Contrast is similar to standard T1W.

- **Primary Use:** Useful for visualizing **lesions adjacent to CSF spaces** (e.g., multiple sclerosis plaques, leptomeningeal disease) by eliminating interfering bright CSF signal.

- **T2 FLAIR**
  - **Mechanism:** An inversion recovery sequence that **suppresses free water signal while retaining T2-weighted characteristics**.
  - **CSF:** Appears **dark/black** (suppressed).
  - **Fat:** Appears **intermediate signal**.
  - **White Matter:** Appears **darker** than grey matter.
  - **Grey Matter:** Appears **light grey**.
  - **Primary Use:** One of the most commonly used sequences for detecting **subtle brain lesions** (e.g., ischemic changes, white matter hyperintensities, tumors, areas of edema, epileptic foci) as it highlights pathology without CSF masking.

- **STIR (Short Tau Inversion Recovery)**
  - **Mechanism:** An inversion recovery sequence specifically designed to **suppress fat signal** by nulling it at a specific inversion time (tau).
  - **Fat:** Appears **dark/black** (suppressed).
  - **CSF:** Appears **bright/white**.
  - **White Matter & Grey Matter:** Signal can be variable.
  - **Primary Use:** Very useful for detecting **bone marrow edema** and lesions in areas with high fat content (e.g., spine, musculoskeletal injuries). Less common for brain parenchyma itself.

- **Proton Density-weighted (PDW)**
  - **Mechanism:** Aims to highlight differences in **proton density** by minimizing T1 and T2 effects (typically using a long TR and short TE).
  - **CSF:** Appears **bright/white**.
  - **Fat:** Appears **intermediate signal**.
  - **White Matter:** Appears **slightly brighter** than grey matter (due to higher proton density).
  - **Grey Matter:** Appears **intermediate signal**.
  - **Primary Use:** Provides **good contrast between grey and white matter**; sometimes used for subtle brain lesions, though more common for joint imaging (e.g., cartilage).

12 Ultrasound Imaging Physics

12.1 Introduction to Ultrasound (US) Imaging

12.1.1 Overview and Clinical Context

Ultrasound imaging is a powerful and widely used medical diagnostic modality, consistently ranking among the most prescribed imaging techniques globally. Its roots trace back to Lord Rayleigh's "The Theory of Sound" in 1877 and Pierre Curie's discovery of the piezoelectric effect in 1880, which was fundamental to generating and detecting ultrasonic waves. Early applications included submarine detection during WWI and industrial non-destructive testing. Clinical use began in 1942 for brain tumor localization, evolving into greyscale imaging in the 1950s and widespread real-time adoption by the 1980s. Modern advancements include sophisticated 3D and 4D ultrasound, pushing its use beyond diagnostics into image-guided procedures.

Main Applications and Characteristics US imaging finds extensive use in areas like Obstetrics/Gynecology (OB/Gyn), Cardiology, Vascular studies, and General Imaging (e.g., kidney, liver).

- **Advantages**:
  - **Non-ionizing Radiation**: Unlike X-rays or CT, US is radiation-free, making it safe for repeated use, especially in pregnant patients and children.
  - **Real-time Capability**: Offers immediate visualization of dynamic physiological processes, like heart motion or blood flow.
  - **Cost-Effective**: Generally more affordable than other advanced imaging modalities.
  - **Portability**: Many US scanners are compact, enabling convenient bedside examinations.
- **Disadvantages**:
  - **Lower Spatial Resolution**: Typically less detailed than CT or MRI.
  - **Limited Penetration**: High-frequency sound waves attenuate quickly, restricting imaging depth, particularly through bone or in obese patients.
  - **Artifacts**: Susceptible to various artifacts that can complicate image interpretation.
  - **Noisy Images**: Images can appear grainy due to factors like speckle.

12.1.2 Fundamental Working Principle

At its core, ultrasound imaging mirrors echolocation used by bats. An ultrasound transducer emits high-frequency sound waves into the body and then listens for the returning echoes. By precisely measuring the time it takes for these echoes to return and their intensity, the system constructs a detailed map of internal structures and boundaries. Modern systems employ arrays of transducer elements, working in concert to create these images.

12.2 Physics of Ultrasound: Acoustic Wave Characteristics

12.2.1 Defining Acoustic Waves

An acoustic wave is a mechanical, longitudinal pressure wave. It propagates by causing particles in the medium to oscillate back and forth, parallel to the wave's direction of travel. This creates sequential regions of compression and rarefaction. A key distinction from electromagnetic (EM) waves (like X-rays or MRI's RF waves) is that acoustic waves inherently require a material medium (solid, liquid, or gas) for their propagation, as they rely on the physical displacement of particles.

Comparison with Electromagnetic (EM) Waves

- **Acoustic Wave**:
  - **Energy Type**: Mechanical (pressure and particle motion).
  - **Wave Type**: Longitudinal (particles oscillate parallel to propagation).
  - **Medium Requirement**: Requires a material medium.
  - **Frequency Range**: Medical ultrasound typically falls between 1 to 20 MHz.
- **Electromagnetic (EM) Wave**:
  - **Energy Type**: Electromagnetic (oscillating electric and magnetic fields).
  - **Wave Type**: Transverse (fields oscillate perpendicular to propagation). **Medium Requirement**: Does not require a material medium (travels through vacuum).

12.2.2 Fundamental Acoustic Parameters

Wavelength (λ), Frequency (f), and Speed (c)

- **Wavelength (λ)**: The spatial extent of one wave cycle. It's inversely proportional to frequency.
- **Frequency (f)**: The number of cycles per second (Hz). For medical US, this is typically in MegaHertz (MHz).
- **Speed of Sound (c)**: The velocity at which the wave propagates. This is a property of the medium, determined by its density (ρ) and compressibility (κ).

$$c = \lambda f = \frac{1}{\sqrt{\kappa \rho}}$$

For soft tissue, $c \approx 1540$ m/s. For example, a 5 MHz US beam in soft tissue has a wavelength of $\lambda = 1540/(5 \times 10^6) \approx 0.308$ mm. This relationship highlights a crucial trade-off: higher frequency leads to shorter wavelength and thus better spatial resolution, but it also results in greater attenuation and shallower penetration.

Acoustic Pressure (p), Particle Velocity (v), and Acoustic Impedance (Z)

- **Acoustic Pressure (p)**: The localized change in pressure due to the wave. Diagnostic US pulses can generate pressure amplitudes around 1 MPa.
- **Particle Displacement Velocity (v)**: The oscillatory velocity of the medium's particles, distinct from the wave propagation speed.
- **Acoustic Impedance (Z)**: A medium's resistance to sound propagation, defined as the product of its density and the speed of sound within it.

$$Z = \rho c$$

Its unit is Rayls (kg/(m^{2}s)). Acoustic impedance is vital for determining reflection and transmission at tissue interfaces.

Acoustic Energy and Intensity (I) **Acoustic Intensity** (I) quantifies the average rate of energy flow per unit area (W/m^2 or W/cm^2). It relates to acoustic pressure and impedance by:

$$I = \frac{p^2}{2\rho c} = \frac{p^2}{2Z}$$

Intensity is frequently expressed on a decibel (dB) scale to manage wide dynamic ranges:

$$\text{Intensity in dB} = 10 \log_{10} \left(\frac{I_1}{I_2} \right)$$

For pressure, the ratio is $20 \log_{10}(p_1/p_2)$. For instance, an echo with an intensity of 0.001 W/cm^2 from an initial 10 W/cm^2 pulse represents a -40 dB loss.

12.2.3 Acoustic Wave Propagation Equation

The fundamental behavior of acoustic waves is governed by the wave equation. For acoustic pressure $p(x, y, z, t)$, it is:

$$\nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0$$

For a plane wave propagating along the z-axis, this simplifies to:

$$\frac{\partial^2 p}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0$$

Solutions describe forward- and backward-traveling waves, with harmonic plane waves taking the form $p(z, t) = p_0 \cos(\omega t - kz)$.

12.3 Acoustic Interaction with Tissue

As an ultrasound beam traverses tissue, its energy is modified through reflection, refraction, scattering, and absorption, which collectively cause attenuation.

12.3.1 Boundary Interactions: Reflection and Refraction

Reflection (Echo) Reflection is the primary mechanism for ultrasound image formation, occurring at interfaces between media with different acoustic impedances. The Intensity Reflection Coefficient (R_i) quantifies the reflected fraction of incident intensity at normal incidence:

$$R_i = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2$$

A larger difference in impedances (Z_1, Z_2) leads to stronger reflection. For example, the high reflection at soft tissue/air interfaces (nearly 99.9%) severely limits imaging beyond air-filled structures. This is why a water-based gel is used to couple the transducer to the skin. The remaining intensity is transmitted, with a transmission coefficient $T_i = 1 - R_i$.

Refraction Refraction is the bending of the ultrasound beam when it crosses an interface at an oblique angle and the speed of sound changes. This phenomenon follows Snell's Law:

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{c_1}{c_2}$$

Refraction can lead to image distortions if the system's reconstruction algorithms don't account for it.

12.3.2 Inside Tissue Interactions: Scattering and Absorption

Scattering Scattering is the diffusion of ultrasound energy in multiple directions. It happens when the wave encounters small inhomogeneities (relative to wavelength) within the tissue, like red blood cells, or rough interfaces. Scattering increases with a larger acoustic impedance mismatch and a smaller scatterer size relative to the wavelength.

Absorption Absorption is the conversion of ultrasound's mechanical energy into heat within the tissue. This is the dominant energy loss mechanism in diagnostic ultrasound, accounting for 80-90% of total attenuation. Absorption is influenced by frequency, tissue viscosity, and the medium's relaxation time. Higher frequencies are absorbed more rapidly.

12.3.3 Attenuation

Attenuation is the overall reduction in ultrasound intensity as it propagates through tissue, encompassing all loss mechanisms. It's described by the attenuation coefficient (μ), which is highly frequency-dependent. The intensity $I(x)$ at depth x is:

$$I(x) = I_0 e^{-\mu x}$$

The coefficient μ is often reported in dB/(cm·MHz) and is generally proportional to frequency (f), approximately $\mu \propto f$. This means higher frequency waves attenuate faster, leading to shallower penetration but better resolution. The Half-Value Thickness (HVT) is the tissue depth that reduces intensity by 50% (or -3 dB).

12.4 US Image Formation Principle and Hardware

12.4.1 The Pulse-Echo Principle

Ultrasound imaging fundamentally relies on the pulse-echo principle: the transducer emits a short pulse, and the time it takes for the echo to return determines the distance to the reflector. The distance (d) is calculated as:

$$d = \frac{ct}{2}$$

where c is the speed of sound and t is the time-of-flight.

Pulse Characteristics and Resolution

- **Pulse Duration (τ)**: The temporal length of the pulse (e.g., 1-2 μs), determined by the number of cycles (N) and frequency (f): $\tau = N/f$.
- **Spatial Pulse Length (SPL)**: The physical length of the pulse in the medium:

$$\text{SPL} = c\tau = \frac{cN}{f}$$

- **Axial Spatial Resolution (FWHM)**: The ability to distinguish objects along the beam direction. It's half of the SPL:

$$\text{FWHM}_{\text{axial}} = \frac{1}{2}\text{SPL} = \frac{cN}{2f}$$

  Higher frequency (f) and **fewer cycles (N)** (i.e., shorter pulse duration) improve axial resolution.

- **Pulse Repetition Frequency (PRF)**: The number of pulses per second (e.g., 2-4 kHz). It dictates the maximum unambiguous penetration depth (D_{max}):

$$\text{PRF}_{\text{max}} = \frac{c}{2D_{\text{max}}}$$

  A higher PRF implies less time for echoes to return, thus reducing maximum penetration.

12.4.2 Ultrasound Transducers

Transducers are the heart of an ultrasound system, converting electrical energy into mechanical (sound) waves and vice-versa, leveraging the piezoelectric effect.

Transducer Construction

- **Piezoelectric Element**: The active component, typically made of Lead Zirconate Titanate (PZT). It generates ultrasound when voltage is applied (transmitter mode) and produces electrical signals when detecting echoes (receiver mode). The element's thickness (L) determines its resonant frequency (f_0):

$$f_0 = \frac{c_{\text{PZT}}}{2L}$$

- **Matching Layer(s)**: Placed between the piezoelectric element and tissue to minimize impedance mismatch and maximize energy transfer. Optimal thickness is $\lambda/4$ within the layer.
- **Backing/Damping Block**: Located behind the piezoelectric element, it absorbs backward-directed energy and quickly dampens the crystal's vibrations. This heavy damping yields a low Q-factor ($Q = f_0/\Delta f$) and a broad bandwidth (Δf), which are crucial for generating short pulses and achieving good axial resolution.

Ultrasound Beam Properties and Focusing

- **Beam Profile (Near and Far Field)**: The beam has a relatively collimated near field (Fresnel Zone), whose length (Z_{NF}) for a circular transducer of radius a is $Z_{NF} = a^2/\lambda$. Beyond this is the diverging far field (Fraunhofer Zone).
- **Focusing**: To improve lateral resolution, the beam can be focused using concave crystals, acoustic mirrors, or lenses. The focal zone is where the beam is narrowest, providing the sharpest image.

12.4.3 Transducer Arrays and Beamforming

Modern ultrasound systems use arrays of piezoelectric elements (e.g., 128 to 512 elements) for advanced beam control.

Types of Transducer Arrays

- **Linear Arrays**: Elements arranged linearly; a subset is sequentially activated to form a rectangular field of view.
- **Phased Arrays**: All elements are activated simultaneously with precise time delays. These delays electronically steer (change direction) and focus the ultrasound beam, enabling dynamic beam control and diverse imaging geometries (e.g., sector images for cardiology).

12.4.4 Image Data Acquisition Modes

Ultrasound signals are presented in various display modes:

A-Mode (Amplitude Mode) Displays echo amplitude versus depth along a single scan line. Useful for 1D distance measurements but lacks anatomical shape information. PRF is limited by the maximum imaging depth L_{max}: $PRF = c/(2L_{max})$.

B-Mode (Brightness Mode) The most common mode for anatomical imaging. It converts A-mode amplitudes into brightness-modulated dots. By sweeping the beam (mechanically or electronically), a 2D grayscale image is constructed, where brighter dots represent stronger reflections.

M-Mode (Motion Mode) Shows the movement of reflecting surfaces along a single scan line over time. Primarily used in cardiac imaging for high-temporal resolution assessment of heart valve and chamber wall motion.

3D/4D Ultrasound **3D ultrasound** involves acquiring and reconstructing a volume of data, typically by sweeping a 2D plane through tissue. 4D ultrasound adds the dimension of real-time movement to 3D imaging, allowing for dynamic volumetric visualization (e.g., real-time fetal movements).

12.5 Image Quality Measures and Factors

12.5.1 Spatial Resolution

The ability to distinguish closely spaced objects in an image.

Axial (Longitudinal) Resolution Resolution along the beam direction. It is determined by the spatial pulse length (SPL): Axial Resolution $= \frac{1}{2}$SPL. It improves with higher frequency and shorter pulse duration, and is generally independent of depth.

Lateral (Azimuthal) Resolution Resolution perpendicular to the beam direction (within the image plane). It's determined by beam width and is depth-dependent, being optimal in the focal zone. Lateral resolution is typically worse than axial resolution.

Elevation Resolution (Slice Thickness) Resolution perpendicular to the imaging plane. It's determined by the beam thickness in this dimension, often fixed by physical lenses on 2D array transducers, and is usually the weakest resolution measure.

12.5.2 Temporal Resolution

The ability to display events occurring at different times as separate images, measured in frames per second (fps). It's determined by the frame rate:

$$\text{Frame Rate} = \frac{c}{2 \times N_{\text{lines}} \times D}$$

where N_{lines} is the number of scan lines and D is the maximum penetration depth. High frame rates are essential for real-time imaging of fast-moving structures. Temporal resolution decreases with deeper imaging, more scan lines, or more focal zones.

12.5.3 Image Quality Control and Artifacts

Quality Assurance (QA) Measures Regular QA ensures optimal system performance. Key checks include:

- **Dead Zone**: The superficial region close to the transducer where no usable echoes are obtained.
- **Resolution Testing**: Using phantoms to assess axial and lateral resolution.
- **Calibration**: Verifying the accuracy of distance measurements.
- **Time Gain Compensation (TGC)**: Adjusting receiver gain with depth to counteract signal attenuation.

Understanding Ultrasound Artifacts Artifacts are false echoes or misrepresentations in the image, arising when the system's underlying assumptions (e.g., straight-line travel, constant velocity, single reflection) are violated.

Artifacts Associated with Beam Characteristics

- **Beam Width Artifacts**: Caused by echoes from objects outside the narrow main beam, leading to a wider perceived structure or "filling in" of fluid-filled spaces.
- **Side Lobes and Grating Lobes**: Off-axis energy emissions that create false echoes. Apodization (varying element excitation/amplification) is used to suppress these.

Artifacts Associated with Multiple Echoes (Reverberations) Occur when sound bounces between strong reflectors, generating multiple, equally spaced echoes that appear deeper than the original reflector.

- **Reverberation Artifact**: Appears as parallel bright lines, gradually weakening with depth (e.g., from the anterior bladder wall or bowel gas).
- **Comet Tail Artifact**: A dense, triangular bright band stemming from highly reflective objects (e.g., metal, calcifications).
- **Ring Down Artifact**: A continuous bright line, often from gas bubbles or resonant structures.
- **Mirror Image Artifact**: A duplicated image of a structure, appearing deeper, caused by sound reflecting off a strong "mirror" interface (e.g., diaphragm) before returning to the transducer.

Artifacts Associated with Velocity Errors

- **Speed Displacement Artifact**: Occurs if the assumed speed of sound (1540 m/s) differs from the actual speed in the tissue, leading to incorrect depth display.
- **Refraction Artifact**: Causes misplacement or distortion due to beam bending at interfaces where sound speed changes.

Artifacts Associated with Attenuation Errors

- **Acoustic Shadowing**: A dark region behind highly attenuating or strongly reflective structures (e.g., bone, gallstones, air), indicating no signal from that area.
- **Acoustic Enhancement (Posterior Enhancement)**: A bright region behind weakly attenuating structures (e.g., fluid-filled cysts). More sound passes through the fluid, making structures behind it appear brighter due to the system's fixed TGC. This is a valuable sign of fluid.

Other Artifacts

- **Speckle**: A grainy texture from the interference of scattered echoes, which affects contrast.
- **Edge Effect Shadows**: Dark lines along the curved edges of structures, caused by refraction and beam divergence.

13 Doppler Ultrasound and Advanced US Techniques

13.1 Concepts and Principles of Doppler Ultrasound

Doppler ultrasound utilizes the Doppler effect to detect and quantify motion, primarily blood flow, within the body.

13.1.1 Interaction of Ultrasound with Blood

Ultrasound interacts with flowing blood primarily through scattering and absorption.

Properties of Human Blood Flow Human blood flow exhibits complex characteristics:

- Spatially and temporally variant.
- Occurs in vessels with diverse geometric dimensions, curves, and branches.

Composition of Blood and Scattering Blood is a complex fluid composed of:

- **Liquid Plasma**
- **Erythrocytes (Red Blood Cells)**: Concentration $\approx 5 \times 10^6$ mm^{-3} (hematocrit $\approx 45\%$).
- **Leukocytes (White Blood Cells)**: Concentration $\approx 7.5 \times 10^3$ mm^{-3}.
- **Platelets**: Concentration $\approx 3.5 \times 10^5$ mm^{-3}.

Crucially, the vast numerical and volumetric superiority of red blood cells means that the scattering of the ultrasound beam is mainly due to red blood cells.

Scattering Theory of Ultrasound by Blood Given that the ultrasound wavelength ($\lambda_{\text{ultrasound}}$) is significantly larger than the size of blood cells ($\lambda_{\text{ultrasound}} \gg \text{size}_{\text{cell}}$), individual cells can be treated as point targets. The scattering largely obeys Rayleigh's scattering theory for small particles. Rschevkin's (1963) scattering theory provides the amplitude of the ultrasound wave scattered to an angle θ_s:

$$p_s = p_i \frac{k^2 \tau}{4\pi} \left[\left(\frac{\chi' - \chi}{\chi} \right) + \frac{3(1 - \cos\theta_s)}{1 + 2(\rho'/\rho)} \left(\frac{\rho' - \rho}{\rho} \right) \right]$$

where:

- p_s: scattered pressure amplitude
- p_i: incident pressure amplitude
- k: wave number ($2\pi/\lambda$)
- τ: target volume (e.g., red blood cell volume)
- χ, ρ: bulk modulus and mass density of the surroundings (plasma)
- χ', ρ': bulk modulus and mass density of the target (red blood cell)
- θ_s: scattering angle

Key observations from this theory and empirical studies include:

- The scattering coefficient of US by blood is proportional to f^4, consistent with Rayleigh scattering for small scatterers.
- Scattering of US depends on the hematocrit (HMCT) content. It increases with HMCT up to a maximum for HMCT between 24% and 30%, then decreases as HMCT further increases.
- Attenuation of US in blood is mainly due to absorption. Scattering becomes comparable to absorption only for US frequencies above approximately 15 MHz.

13.1.2 The Doppler Effect and Doppler Shift

The Doppler effect, first described by Johan Christian Doppler in 1842, states that any directional motion between a source of waves and an observer will produce a detectable frequency shift.

Illustrative Experiment (Buys Ballot, 1845) Buys Ballot's experiment with musicians on a train demonstrated that the perceived musical pitch (frequency) increased as the train approached and decreased as it receded from stationary observers, verifying Doppler's theory for sound.

Mathematical Description of Doppler Effect The Doppler effect can be understood as a change in the effective wavelength (for a stationary source and moving observer) or effective frequency (for a moving source and stationary observer). The general formula for the observed frequency f' for a moving source (v_s) and moving observer (v_o) is:

$$f' = f\left(\frac{c \pm v_o}{c \mp v_s}\right)$$

where c is the speed of sound in the medium, and the signs depend on the direction of motion (e.g., '+' for observer moving towards source, '-' for source moving towards observer).

Applying to Ultrasound Case For Doppler ultrasound, we consider two main cases:

- **Case 1: Stationary Source (Transducer), Moving Listener (Red Blood Cell)**: The red blood cell "sees" an incident frequency (f_i) given by:

  $$f_i = f_0\left(\frac{c + v\cos\theta}{c}\right)$$

  where f_0 is the original transmit frequency, v is the blood velocity, and θ is the Doppler angle between the beam and flow direction.

- **Case 2: Moving Source (Red Blood Cell), Stationary Listener (Transducer)**: The transducer "sees" a scattered frequency (f_s) from the moving red blood cell given by:

  $$f_s = f_i\left(\frac{c}{c - v\cos\theta}\right)$$

  Substituting f_i from Case 1 into Case 2:

  $$f_s = f_0\left(\frac{c + v\cos\theta}{c}\right)\left(\frac{c}{c - v\cos\theta}\right) = f_0\left(\frac{c + v\cos\theta}{c - v\cos\theta}\right)$$

Doppler Frequency Shift (f_D) The Doppler frequency shift (f_D) is the difference between the received and transmitted frequencies:

$$f_D = f_s - f_0$$

Assuming $c \gg v$, the Doppler Equation for blood velocity (v) is derived:

$$f_D = f_0\left(\frac{c + v\cos\theta}{c - v\cos\theta}\right) - f_0 = f_0\left(\frac{2v\cos\theta}{c - v\cos\theta}\right) \approx \frac{2f_0 v\cos\theta}{c}$$

Thus, the blood velocity can be determined by:

$$v = \frac{f_D c}{2f_0\cos\theta}$$

Doppler ultrasound imaging precisely utilizes this Doppler shift to determine blood flow velocity (both speed and direction).

Doppler Angle Effect The Doppler angle (θ) is crucial for accurate velocity estimation.

- **Preferred Angle**: Doppler angles between 30° and 60° are preferred.
- **High Angles ($\theta > 60^\circ$)**: The $\cos\theta$ term approaches zero, making the Doppler shift very small. Minor errors in angle measurement can lead to large errors in estimated velocity. For example, a 3° error at 80° can result in a 42.5% error in speed.
- **Low Angles ($\theta < 20^\circ$)**: Can lead to issues like refraction and critical angle aliasing effects in pulsed Doppler.

The Doppler shift for blood flow typically falls within the audible range (15 Hz - 20 kHz), allowing for audible interpretation in some Doppler modes.

Positive/Negative Doppler Shift

- **Positive Doppler Shift**: $f_s > f_0$. Indicates that the source reflecting sound waves is moving **towards** the emitting source. Depicted as **red** in color flow Doppler and above the baseline in spectral Doppler.
- **Negative Doppler Shift**: $f_s < f_0$. Indicates that the source reflecting sound waves is moving **away from** the emitting source. Depicted as **blue** in color flow Doppler and below the baseline in spectral Doppler.

13.2 Blood Flow Velocity Measurement and Imaging Methods

Doppler ultrasound operates in various modes to measure and visualize blood flow.

13.2.1 Continuous Wave (CW) Doppler

Principle and Operation CW Doppler uses separate, dedicated transducer elements for continuous transmission and reception of ultrasound waves. The overlapping area of the transmit and receive beams defines the region of flow detection. Signals from the continuously transmitting and receiving transducers are mixed and demodulated to extract the Doppler shift signal.

Features

- **Simplicity and Cost-Effectiveness**: Oldest and simplest technique (developed by 1957), often found in dedicated handheld devices.
- **High Velocity Measurement**: Can accurately measure very high blood flow velocities without aliasing.
- **No Range Resolution**: Cannot differentiate between structures at different depths within the overlapping beam, making it impossible to determine the exact depth of the flow.
- **Audible Interpretation**: The Doppler shift is in the audible range, allowing direct interpretation of flow characteristics (pitch indicates velocity, harshness indicates turbulence) by the physician without imaging.

Doppler Signal Extraction (Demodulation) The Doppler processor extracts the Doppler frequency ($\omega_D = 2\pi f_D$) from the received signal $S(t) = A\cos(\omega_0 + \omega_D)t$ by mixing it with the original transmit frequency ω_0 (using $\cos\omega_0 t$ and $\sin\omega_0 t$ as reference signals). This yields in-phase (I) and quadrature (Q) components:

$$I(t) = \text{LPF}\{A\cos[(\omega_0+\omega_D)t]\cos(\omega_0 t)\} = \text{LPF}\left\{\frac{A}{2}[\cos(2\omega_0+\omega_D)t + \cos(\omega_D t)]\right\} = \frac{A}{2}\cos(\omega_D t)$$

$$Q(t) = \text{LPF}\{A\cos[(\omega_0+\omega_D)t]\sin(\omega_0 t)\} = \text{LPF}\left\{\frac{A}{2}[\sin(2\omega_0+\omega_D)t + \sin(\omega_D t)]\right\} = \frac{A}{2}\sin(\omega_D t)$$

The Low Pass Filters (LPF) remove the high-frequency $2\omega_0$ terms, leaving only components dependent on the Doppler shift frequency. These I and Q signals can be represented as a complex number, and their phase and amplitude can be analyzed.

13.2.2 Pulsed Wave (PW) Doppler

Principle and Operation PW Doppler combines the Doppler frequency measurement capabilities with range discrimination (range gating), a feature from pulse-echo imaging. The same transducer acts as both transmitter and receiver. It transmits short ultrasound pulses at a constant Pulse Repetition Frequency (PRF) and then listens for echoes. Range gating allows the system to accept echoes only from a user-selected specific depth (sample volume).

Features

- **Range Resolution**: Can measure velocities at a specific, user-defined location.
- **Duplex Scanning**: Often combined with B-mode imaging, where the Doppler sample volume is superimposed on the anatomical image. The transducer array interleaves B-mode and PW Doppler acquisitions.
- **Velocity Limitation (Aliasing)**: Limited maximum velocity that can be accurately measured due to the Nyquist limit.

PW Doppler Signal Sampling and Nyquist Limit Each individual Doppler pulse provides only a sample of the shifted frequencies (as a phase change). Repeated echoes from the selected sample volume are analyzed to build up the Doppler signal. The sampling frequency is controlled by the PRF. The Nyquist Limit states that the sampling frequency (PRF) must be at least twice the highest frequency present in the input signal to accurately represent it. For Doppler, this means:

$$\text{PRF} \geq 2f_{D,\text{max}}$$

If the maximum Doppler shift ($f_{D,\text{max}}$) exceeds half the PRF (the Nyquist limit), aliasing occurs, where velocities are incorrectly displayed as lower and "wrap around" to the opposite direction. The maximum detectable velocity (v_{max}) in PW Doppler is:

$$v_{\text{max}} = \frac{\text{PRF}\cdot c}{4f_0\cos\theta}$$

Since PRF is also limited by the maximum imaging depth (D_{max}) as $\text{PRF} = c/(2D_{\text{max}})$, there's a trade-off between maximum depth and maximum measurable velocity.

Aliasing Correction Aliasing (appearing as an abrupt termination and "wrap-around" of the spectral waveform) can be corrected by:

- **Increasing the PRF**: This raises the Nyquist limit.
- **Adjusting the Baseline (Shifting Zero Line)**: Allows more of the aliased signal to be displayed correctly, but doesn't eliminate aliasing itself.
- **Decreasing the Transmit Frequency (f_0)**: Reduces f_D for a given velocity.
- **Changing the Doppler Angle (θ)**: Increase θ to decrease $\cos\theta$ and thus f_D.

13.2.3 Types of PW Doppler Displays

The extracted Doppler information is processed using a Fourier Transform (FFT) to decompose the complex signal into its constituent frequencies (Doppler shifts), which correspond to velocities.

Spectral Doppler

- **Display**: A plot of Doppler shift frequency (or velocity) versus time. The x-axis represents time, and the y-axis represents velocity. Velocities towards the transducer are plotted above the baseline (positive), and velocities away are below (negative).
- **Information**: Provides a range of velocities present within the sample volume at any given time, reflecting the full velocity profile (e.g., laminar flow showing a range of speeds). The brightness of the display indicates the amount of energy (number of red blood cells) at a given frequency/velocity.
- **Indices**: Quantitative indices are derived from the spectral Doppler waveform, particularly the maximum frequency shift envelope for one cardiac cycle:
  - **Peak Systolic Velocity (S)**
  - **End Diastolic Velocity (D)**
  - **Temporal Average Velocity (A)**
  - **Resistive Index (RI)**: $RI = (S - D)/S$. Indicates downstream resistance.
  - **Pulsatility Index (PI)**: $PI = (S - A)/S$. Reflects pulsatility of flow.

Color Flow Doppler (CFD)

- **Principle**: Overlays a color-encoded velocity map onto a grayscale B-mode image within a user-defined region of interest. It typically estimates the mean velocity within each pixel.
- **Color Encoding**: Arbitrary colors represent direction and magnitude:
  - **Red**: Flow moving **towards** the transducer (positive Doppler shift).
  - **Blue**: Flow moving **away from** the transducer (negative Doppler shift).
  - Varying shades/brightness of red/blue indicate increasing velocity magnitude.
  - A central **black/dark band** indicates zero or very low calculated velocity.
- **Limitations**: The pulse frame rate for color Doppler is lower than B-mode, affecting real-time performance, especially with larger color boxes. It's also susceptible to aliasing (color "wrap-around").

Power Doppler (Energy Doppler)

- **Principle**: Maps only the amplitude (power or intensity) of the Doppler signal, not its direction or velocity. All movement, regardless of phase, contributes to the amplitude. This highlights the quantity of blood flow.
- **Advantages**:
  - **Less Angle Dependent**: Much less susceptible to the Doppler angle.
  - **High Sensitivity**: Can detect very low flow rates.
  - **No Aliasing**: Since it doesn't display velocity or direction, aliasing is not an issue.
- **Disadvantages**:
  - **Qualitative**: Provides no information on flow direction or specific velocity.
  - **Motion Artifacts**: Highly sensitive to tissue motion, which can create strong artifacts.

13.2.4 Flow States and Artifacts in Doppler Ultrasound

Flow States Blood flow within vessels can exhibit various states:

- **Plug Flow**: Uniform velocity across the vessel lumen (e.g., in large arteries).
- **Laminar Flow**: Parabolic velocity profile, fastest in the center and slowest near the walls.
- **Jet Stream**: Localized, high-velocity flow through a narrow opening (e.g., stenotic valve).
- **Turbulence**: Chaotic, non-laminar flow, often distal to stenoses, producing spectral broadening.

Doppler Artifacts

- **Aliasing**: The most common artifact in PW and Color Doppler, occurring when the Doppler shift frequency exceeds the Nyquist limit (PRF/2). Appears as "wrap-around" of the spectrum or color reversal.
- **Spectral Broadening**: The "smearing" of the spectral display, indicating a wider range of velocities than ideal. Caused by:
  - Turbulent flow.
  - Beam divergence (multi-angle insonation).
  - Increased sample volume size.
  - Pulsing effects.
- **Doppler Angle Errors**: Inaccurate estimation of velocity if the Doppler angle is not precisely measured, especially at angles greater than 60°.
- **Wall Filters**: Electronic filters remove low-frequency signals, which are typically from low-velocity structures like vessel walls. If set too high, they can filter out genuine low-velocity blood flow signals.

13.3 Advanced Ultrasound Imaging Techniques

13.3.1 Tissue Harmonic Imaging (THI)

Physics Phenomenon: Origin of Ultrasound Harmonics Harmonics are integer multiples of a fundamental frequency (e.g., 2nd harmonic is $2f_0$, 3rd harmonic is $3f_0$).

- Ultrasound harmonics arise from the non-linear propagation of sound in tissue.
- As a high-intensity ultrasound pulse propagates, its waveform distorts (peaks travel slightly faster than troughs), generating new frequencies that are integer multiples (harmonics) of the fundamental transmit frequency. The 1st harmonic ($2f_0$) is of greatest interest.

Working Principle and Features of THI

- Modern transducers have a wide bandwidth, enabling them to transmit at a fundamental frequency (f_0) and receive echoes at its harmonic frequencies (e.g., $2f_0$).
- The harmonic intensity is depth-dependent: small at the skin level, increasing with depth to a maximum before tissue attenuation causes it to decrease.
- Advantages of THI:
  - **Reduced Artifacts**: Harmonics are generated deeper in the tissue, containing fewer reverberation artifacts that typically arise from shallow structures near the transducer.
  - **Improved Spatial Resolution**: Harmonic echoes are less subject to distortion from non-linear propagation and have better lateral resolution.
  - **Improved Visualization**: Particularly beneficial for visualizing endocardial borders in cardiac imaging (ventricles appear dark, tissue appears bright) and fluid-filled structures (reduced reverberation).
  - Useful in obese patients and for measuring carotid artery wall thickness.
  - **Higher Frequency Benefits**: While harmonics are higher frequencies, they are generated deeper, meaning they only travel one way back to the transducer, somewhat mitigating the increased attenuation.

13.3.2 Contrast-Enhanced Ultrasound (CEUS)

Idea of Contrast Agents Traditional ultrasound imaging relies on acoustic impedance differences. Contrast agents are introduced to provide additional harmonic signals and significantly enhance image contrast.

Microbubble Contrast Agents

- Composition: Encapsulated gas microbubbles (e.g., perfluorocarbon gas core with an albumin or lipid shell). Free gas bubbles are not suitable as they dissolve too quickly.
- Mechanism: These microbubbles resonate non-linearly when insonated by ultrasound, particularly at low mechanical index (MI) values. This non-linear oscillation generates strong harmonic signals that are orders of magnitude greater (signal enhancement of ≈ 1000-fold) than those from tissue.
- **Pressure Response of Microbubbles**:
  - < 0.05 MPa: Bubble resonates linearly.
  - ≈ 0.1 MPa: Bubble oscillates non-linearly, producing higher harmonics.
  - > 1 MPa: Bubble may be destroyed (rupture).
- **Applications**:
  - **Microvasculature Imaging**: Visualizing small blood vessels, which are difficult to see with conventional US.
  - **Molecularly Targeted Microbubbles**: Under development for specific diagnostic or therapeutic applications ("theranostics").
  - **Drug Delivery**: Microbubbles can be engineered to rupture under specific ultrasound conditions, releasing encapsulated drugs at a target site (e.g., for brain tumors).

13.3.3 Ultrasound Safety

Ultrasound is generally considered safe when used appropriately. Safety guidelines are based on controlling potential thermal and mechanical bioeffects.

Bioeffects and Indices

- **Local Heating (Thermal Index - TI)**:
  - **Indication**: Estimates the risk of local tissue heating.
  - **Formula**: TI $=$ power emitted$/$(power required to increase temperature by 1°C).
  - **Guidelines**: TI values between 0 and 1.0 are generally considered safe. Lower thresholds apply for febrile patients, fetal scanning, and eye examinations. TI should never exceed 3.0 in fetal scanning.
- **Cavitation (Mechanical Index - MI)**:
  - **Indication**: Estimates the risk of cavitation, where pressure changes cause microbubbles in a liquid to expand and collapse.
  - **Formula**: MI $=$ peak negative pressure$/\sqrt{\text{ultrasound frequency}}$.
  - **Guidelines**: MI < 0.7 for general use, MI < 0.5 for fetal scanning. MI > 0.7 should never be used with ultrasound contrast agents due to increased risk of microbubble rupture.
  - **Increased Risk of Cavitation**: Higher risk in gas-containing structures (bowel, lung), with low-frequency pulses (longer wavelengths), higher power/intensity, or with ultrasound contrast agents.

Other Potential Complications Beyond heating and cavitation, there is a theoretical risk of mechanical damage to cell membranes, particularly at very high intensities or with prolonged exposure.