## Lecture 2 Univariate Portfolio Analysis

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## **Capital Asset Pricing Model**

- ▶ The central prediction of the CAPM (Sharpe (1964), Lintner (1965) and Black (1972), SLB hereafter) is that the market portfolio of invested wealth is mean-variance efficient which implies that
  - ightharpoonup Expected returns on securities are a positive linear function of their market  $eta_{
    m S}$
  - Market  $\beta s$  suffice to describe the cross-section of expected returns
- $\blacktriangleright$  Banz (1981) found that the average returns are too high on small (low ME) stocks and too low on large stocks given their  $\beta$  estimates where ME denotes the market equity (a stock's price times shares outstanding).

#### **Basics**

- Market equity (ME or size) is a measure of a company's value which is calculated by multiplying the current stock price by the total number of outstanding shares.
- Shares outstanding refer to a company's stock currently held by all its shareholders
  - ► These include share blocks held by institutional investors and restricted shares owned by the company's officers and insiders.
  - A company's number of shares outstanding is not static and may fluctuate wildly over time.

# **Portfolio Analysis**

## **Portfolio Analysis**

- Portfolio analysis is a nonparametric method to examine the cross-sectional relation between two or more variables.
- In emperical asset pricing, it is used to examine the abitility of one or more variables to predict furture stock returns.
- ▶ The portfolio analysis procedure can be roughly describe as follows:
  - 1. Divide the sample of stocks into portfolios by some variable(s) (X, sorted variable) at period t
  - 2. Calculate the average value of the outcome variable Y within each portfolio for each period t
  - 3. Examine variation in these average values of *Y* across the different portfolios.

## **Univariate Portfolio Analysis**

- The univariate portfolio analysis has only one sort variable X.
- The objective of the analysis is to assess the cross-sectional relation between X and the outcome variable Y.
- ► The univariate portfolio analysis procedure has four steps:
  - Calculate the breakpoints that will be used to divide the sample into portfolios
  - 2. Use these breakpoints to form the portfolios.
  - 3. Calculate the average value of the outcome variable Y within each portfolio for each period t
  - 4. Examine variation in these average values of *Y* across the different portfolios.

- ▶ The breakpoints for period *t* are determined by percentiles of the time *t* cross-sectional distribution of the sort variable *X*.
- Let  $n_P$  be the number of portfolios to be formed each time period as  $n_P$ , then the number of breakpoints that need to be calculated each period is  $n_P 1$ , the kth breakpoint for period t is  $B_{k,t}$ .
- Let  $p_k$  be the percentile that determines the kth breakpoint, then  $B_{k,t}$  for period t is the  $p_k$ th percentile of the values of X across all entities in the sample for which X is available in period t:

$$B_{k,t} = \mathsf{Pctl}_{p_k} (\{X_t\})$$

where Pctl  $_p(Z)$  is the p th percentile of the set Z and  $\{X_t\}$  represents the set valid values of the sort variable X across all entities i in the sample in time period t.

- ▶ In some cases, breakpoints are calculated using only a subset of the entities that are in the sample for the given period t.
- For example, sometimes we form breakpoints using only stocks that trade on the New York Stock Exchange, and then use those breakpoints to sort all stocks in the sample (including stocks that trade on other exchanges) into portfolios.
- ► There is a tradeoff between the number of entities in each portfolio against the dispersion of the sort variable among the portfolios:
  - Having a large number of entities in each portfolio increases the accuracy of estimate of the true mean value for each portfolio
  - ► The more entities grouped into each portfolio, the less dispersion in the sort variable *X* among the portfolios.

- The following table presents breakpoints for  $\beta$ -sorted portfolios. Each year t, the first  $(B_{1,t})$ , second  $(B_{2,t})$ , third  $(B_{3,})$ , fourth  $(B_{4,t})$ , fifth  $(B_{5,t})$ , and sixth  $(B_{6,t})$  breakpoints for portfolios sorted on  $\beta$  are calculated as the 10 th, 20 th, 40th, 60th, 80th, and 90 th percentiles, respectively, of the cross-sectional distribution of  $\beta$ ..
- Univariate Breakpoints for  $\beta$ -Sorted Portfolios (Part I)

| t    | $B_{1,t}$ | $B_{2,t}$ | $B_{3,t}$ | $B_{4,t}$ | $B_{5,t}$ | $B_{6,t}$ |
|------|-----------|-----------|-----------|-----------|-----------|-----------|
| 1988 | -0.05     | 0.07      | 0.29      | 0.51      | 0.86      | 1.11      |
| 1989 | -0.11     | 0.05      | 0.29      | 0.54      | 0.89      | 1.17      |
| 1990 | -0.06     | 0.10      | 0.37      | 0.68      | 1.07      | 1.37      |
| 1991 | -0.09     | 0.09      | 0.38      | 0.67      | 1.05      | 1.33      |
| 1992 | -0.22     | 0.08      | 0.42      | 0.77      | 1.23      | 1.66      |
| 1993 | -0.21     | 0.10      | 0.44      | 0.73      | 1.18      | 1.55      |
| 1994 | -0.05     | 0.19      | 0.52      | 0.81      | 1.19      | 1.56      |
| 1995 | -0.17     | 0.10      | 0.42      | 0.71      | 1.16      | 1.65      |
| 1996 | 0.00      | 0.19      | 0.46      | 0.73      | 1.14      | 1.52      |
| 1997 | -0.00     | 0.15      | 0.36      | 0.59      | 0.89      | 1.15      |
| 1998 | 0.13      | 0.28      | 0.54      | 0.79      | 1.13      | 1.38      |
| 1999 | -0.06     | 0.06      | 0.24      | 0.42      | 0.70      | 0.99      |
|      |           |           |           |           |           |           |

► Univariate Breakpoints for β-Sorted Portfolios (Part II)

| t    | $B_{1,t}$ | $B_{2,t}$ | $B_{3,t}$ | $B_{4,t}$ | $B_{5,t}$ | $B_{6,t}$ |
|------|-----------|-----------|-----------|-----------|-----------|-----------|
| 2000 | 0.03      | 0.14      | 0.37      | 0.63      | 1.22      | 1.79      |
| 2001 | 0.05      | 0.18      | 0.46      | 0.76      | 1.23      | 1.75      |
| 2002 | 0.04      | 0.17      | 0.48      | 0.75      | 1.06      | 1.37      |
| 2003 | 0.05      | 0.22      | 0.54      | 0.83      | 1.16      | 1.46      |
| 2004 | 0.14      | 0.40      | 0.81      | 1.16      | 1.56      | 1.96      |
| 2005 | 0.09      | 0.33      | 0.80      | 1.14      | 1.49      | 1.74      |
| 2006 | 0.10      | 0.35      | 0.82      | 1.21      | 1.64      | 1.94      |
| 2007 | 0.13      | 0.33      | 0.75      | 1.04      | 1.33      | 1.54      |
| 2008 | 0.16      | 0.38      | 0.74      | 1.02      | 1.31      | 1.54      |
| 2009 | 0.24      | 0.45      | 0.84      | 1.23      | 1.70      | 2.06      |
| 2010 | 0.29      | 0.56      | 0.92      | 1.19      | 1.49      | 1.72      |
| 2011 | 0.26      | 0.56      | 0.99      | 1.25      | 1.52      | 1.73      |
| 2012 | 0.28      | 0.55      | 0.91      | 1.18      | 1.48      | 1.75      |

## Univariate Portfolio Analysis - Portfolio Formation

Let  $B_{0,t} = -\infty$ ,  $B_{n_p,t} = \infty$  and  $P_{k,t}$  be the set of entities in the kth portfolio formed at the end of period t, then

$$P_{k,t} = \{i \mid B_{k-1,t} \le X_{i,t} \le B_{k,t}\}$$

for  $k \in \{1, 2, ..., n_P\}$ .

- Once the breakpoints are calculated, they can be applied to any set of entities, whether it is a superset, subset, or the same set as was used to calculate the breakpoints.
- If a given entity has a value of X during time period t that is exactly equal to the k th breakpoint,  $B_{k,t}$ , then this entity is included in both portfolio k and portfolio k+1.

## Univariate Portfolio Analysis - Portfolio Formation

- ▶ The following table presents presents the number of stocks in each of the portfolios formed in each year during the sample period. The column labeled t indicates the year. The subsequent columns, labeled  $n_{k,t}$  for  $k \in \{1, 2, \ldots, 7\}$  present the number of stocks in the k th portfolio.
- Number of Stocks per Portfolio (Part I)

| t    | $n_{1,t}$ | $n_{2,t}$ | $n_{3,t}$ | $n_{4,t}$ | $n_{5,t}$ | $n_{6,t}$ | n <sub>7,t</sub> |
|------|-----------|-----------|-----------|-----------|-----------|-----------|------------------|
| 1988 | 569       | 569       | 1138      | 1138      | 1138      | 569       | 569              |
| 1989 | 552       | 552       | 1104      | 1103      | 1104      | 552       | 552              |
| 1990 | 541       | 541       | 1082      | 1081      | 1082      | 541       | 541              |
| 1991 | 531       | 530       | 1060      | 1061      | 1061      | 529       | 531              |
| 1992 | 539       | 539       | 1078      | 1077      | 1078      | 539       | 539              |
| 1993 | 567       | 567       | 1134      | 1134      | 1134      | 567       | 567              |
| 1994 | 615       | 615       | 1229      | 1230      | 1229      | 615       | 615              |
| 1995 | 629       | 629       | 1257      | 1258      | 1257      | 629       | 629              |
| 1996 | 659       | 658       | 1317      | 1318      | 1316      | 659       | 659              |
| 1997 | 687       | 687       | 1373      | 1373      | 1373      | 687       | 687              |
| 1998 | 661       | 661       | 1321      | 1322      | 1321      | 661       | 661              |
| 1999 | 610       | 610       | 1219      | 1219      | 1219      | 610       | 610              |

## Univariate Portfolio Analysis - Portfolio Formation

► Number of Stocks per Portfolio (Part II)

| t    | $n_{1,t}$ | $n_{2,t}$ | <b>n</b> 3,t | n <sub>4,t</sub> | $n_{5,t}$ | <i>n</i> <sub>6,t</sub> | <b>n</b> 7,t |
|------|-----------|-----------|--------------|------------------|-----------|-------------------------|--------------|
| 2000 | 591       | 590       | 1180         | 1180             | 1180      | 589                     | 591          |
| 2001 | 551       | 551       | 1101         | 1102             | 1101      | 551                     | 551          |
| 2002 | 510       | 510       | 1020         | 1019             | 1020      | 510                     | 510          |
| 2003 | 474       | 474       | 947          | 947              | 947       | 474                     | 474          |
| 2004 | 458       | 457       | 915          | 914              | 915       | 457                     | 458          |
| 2005 | 450       | 449       | 899          | 899              | 898       | 450                     | 450          |
| 2006 | 446       | 445       | 890          | 891              | 890       | 445                     | 446          |
| 2007 | 434       | 433       | 866          | 866              | 866       | 433                     | 434          |
| 2008 | 427       | 426       | 853          | 853              | 852       | 426                     | 427          |
| 2009 | 398       | 398       | 795          | 795              | 795       | 398                     | 398          |
| 2010 | 381       | 380       | 761          | 761              | 761       | 380                     | 381          |
| 2011 | 369       | 368       | 736          | 736              | 736       | 368                     | 369          |
| 2012 | 355       | 354       | 709          | 709              | 709       | 354                     | 355          |

The average value of the outcome variable for portfolio k in period t is defined as

$$\bar{Y}_{k,t} = \frac{\sum_{i \in P_{k,t}} W_{i,t} Y_{i,t}}{\sum_{i \in P_{k,t}} W_{i,t}}$$

for  $k \in \{1, \dots, n_P\}$ , where  $W_{i,t}$  is the weight for each entity in a portfolio.

- The most commonly used weight variable is ME which is denoted as the value-weighted average.
- ▶ If  $W_{i,t} = 1 \quad \forall i, t$ , the portfolios is denoted as equal-weighted portfolios.
- ► For each period *t*, the difference in the average outcome variable between the highest and lowest portfolios is defined to be

$$\bar{Y}_{Diff,t} = \bar{Y}_{n_P,t} - \bar{Y}_{1,t}$$

 $ar{Y}_{Diff,t}$  is the primary value used to detect a cross-sectional relation between the sort variable and the outcome variable which is the main objective of portfolio anlysis.

- Let the stock excess return  $r_{t+1}$  be the outcome variable, the market  $\beta$  be the sort variable.
- At the end of each year t, calculate  $\beta$  for each stock and form seven different portfolios.
- The prices paid for each of the stocks are the closing prices of the last trading day during year t.
- ▶ Hold the portfolios unchanged until the end of year t + 1.
- All portfolios are liquidated at the closing prices on the last trading day of year t + 1.

Table: Univariate Portfolio Equal-Weighted Excess Returns

| t    | t+1  | 1      | 2      | 3      | 4      | 5      | 6      | 7      | 7 – 1  |
|------|------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1988 | 1989 | -0.97  | 1.12   | 2.12   | 6.77   | 3.18   | 9.04   | 8.96   | 9.93   |
| 1989 | 1990 | -30.21 | -29.09 | -28.72 | -29.81 | -27.85 | -26.85 | -25.75 | 4.45   |
| 1990 | 1991 | 56.81  | 28.99  | 36.22  | 40.42  | 54.51  | 64.44  | 66.49  | 9.68   |
| 1991 | 1992 | 55.37  | 30.93  | 29.45  | 21.99  | 19.16  | 15.95  | 20.12  | -35.26 |
| 1992 | 1993 | 36.07  | 27.60  | 22.98  | 24.03  | 19.78  | 15.39  | 7.86   | -28.21 |
| 1993 | 1994 | -4.51  | -4.47  | -5.55  | -4.16  | -5.88  | -10.42 | -4.74  | -0.23  |
| 1994 | 1995 | 28.38  | 21.81  | 24.95  | 29.62  | 26.82  | 23.90  | 37.46  | 9.08   |
| 1995 | 1996 | 21.04  | 16.14  | 18.46  | 14.16  | 12.32  | 11.73  | 9.07   | -11.97 |
| 1996 | 1997 | 22.72  | 39.12  | 28.28  | 20.44  | 18.00  | 6.13   | -7.58  | -30.30 |
| 1997 | 1998 | -6.68  | -9.02  | -7.01  | -10.42 | -6.52  | -6.60  | 0.13   | 6.81   |
| 1998 | 1999 | 14.69  | 10.32  | 19.71  | 23.15  | 38.11  | 61.77  | 93.44  | 78.74  |
| 1999 | 2000 | -11.18 | -4.54  | -0.96  | -1.23  | -3.50  | -10.26 | -31.33 | -20.16 |
| 2000 | 2001 | 37.64  | 28.96  | 27.08  | 28.23  | 22.53  | -1.75  | -22.09 | -59.73 |

Table: Univariate Portfolio Equal-Weighted Excess Returns

| t    | t+1  | 1      | 2      | 3      | 4      | 5      | 6      | 7      | 7 – 1  |
|------|------|--------|--------|--------|--------|--------|--------|--------|--------|
| 2001 | 2002 | 11.60  | 11.08  | -0.27  | -8.09  | -22.56 | -36.19 | -53.83 | -65.43 |
| 2002 | 2003 | 76.69  | 68.50  | 85.65  | 64.78  | 63.70  | 76.56  | 86.90  | 10.22  |
| 2003 | 2004 | 27.56  | 21.07  | 25.82  | 21.34  | 15.75  | 12.89  | -0.10  | -27.65 |
| 2004 | 2005 | 6.06   | 5.52   | 2.97   | 3.40   | 3.11   | -3.15  | -10.24 | -16.31 |
| 2005 | 2006 | 8.85   | 17.13  | 13.32  | 10.62  | 6.87   | 12.98  | 12.26  | 3.41   |
| 2006 | 2007 | -13.52 | -13.06 | -6.97  | -7.65  | -6.57  | -2.60  | -4.19  | 9.33   |
| 2007 | 2008 | -42.54 | -42.30 | -41.36 | -39.18 | -40.75 | -45.34 | -44.46 | -1.92  |
| 2008 | 2009 | 57.06  | 65.73  | 64.76  | 59.46  | 55.19  | 60.47  | 73.60  | 16.55  |
| 2009 | 2010 | 20.31  | 20.19  | 23.14  | 22.98  | 33.39  | 30.27  | 36.77  | 16.46  |
| 2010 | 2011 | -3.87  | -5.50  | -1.04  | -4.24  | -7.11  | -14.11 | -16.48 | -12.61 |
| 2011 | 2012 | 27.95  | 27.11  | 16.22  | 20.33  | 16.10  | 17.83  | 18.05  | -9.89  |

## Univariate Portfolio Analysis - Summarizing the Results

lacktriangle Calculate the time series mean of the the outcome variable,  $ar{Y}_{k,t}$ 

$$\bar{Y}_k = \frac{\sum_{t=1}^T \bar{Y}_{k,t}}{T}.$$

lacktriangle Calculate the time series mean of the the outcome variable,  $ar{Y}_{\!\it Diff,t}$ 

$$ar{Y}_{\textit{Diff}} = rac{\sum_{t=1}^{T} ar{Y}_{\textit{Diff},t}}{T}.$$

- It is important to test whether the time-series mean for each of the portfolios differs from zero.
- ▶ It is also important to examine whether the time-series mean of the difference portfolio is statistically distinguishable from zero:
  - A statistically nonzero mean for the difference portfolio is evidence that, in the average time period, a cross-sectional relation exists between the sort variable and the outcome variable.
- In addition to examining whether the time-series mean for the difference portfolio is statistically distinguishable from zero, researchers frequently examine the average values of Y across the  $n_P$  portfolios  $(\bar{Y}_k, k \in \{1, 2, \dots, n_P\})$  for monotonicity
  - ▶ If a monotonic or near monotonic pattern arises, it is a strong indication that the results of the difference portfolio are not spurious.

Univariate Portfolio Equal-Weighted Excess Returns Summary

|                 | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 7 – 1 |
|-----------------|-------|-------|-------|-------|-------|-------|-------|-------|
| Average         | 16.47 | 13.89 | 14.55 | 12.79 | 11.99 | 10.92 | 10.43 | -6.04 |
| Standard error  | 3.62  | 2.42  | 2.50  | 1.90  | 1.83  | 1.80  | 3.04  | 4.61  |
| t-statistic     | 4.55  | 5.74  | 5.83  | 6.73  | 6.57  | 6.06  | 3.43  | -1.31 |
| <i>p</i> -value | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.20  |

- ► The average excess returns for portfolios 1 through 7 are highly statistically significant, as the corresponding *t*-statistics range from 3.43 for portfolio 7 to 6.73 for portfolio 4.
- ► The average return of the difference portfolio is not statistically distinguishable from zero as the *t*-statistic is −1.31 and the *p*-value is 0.20:
  - The portfolio analysis fails to detect a cross-sectional relation between  $\beta$  and one-year-ahead excess stock returns  $(r_{t+1})$ .

Based on the Capital Asset Pricing Model (CAPM) of Sharpe (1964), the one-factor market model to adjust the portfolio returns for the effect of the overall stock market return is

$$r_{p,t} = \alpha_p + \beta_{MKT}MKT_t + \epsilon_t$$

where  $r_{p,t}$  is the excess return of the portfolio and  $MKT_t$  is the excess return on the market factor mimicking portfolio during the period t.

- For the returns of the securities, we want to examine the average values  $\bar{Y}_{k,t}$ , whether patterns in the average portfolio returns are driven by cross-sectional variation in portfolio sensitivities to systematic risk factors.
- Stated alternatively, we want to examine whether after controlling for sensitivity of the portfolios to systematic risk factors, the patterns in the average portfolio returns persist.

▶ β-Sorted Portfolio Risk-Adjusted Results

| Model  | Coefficient   | 1      | 2      | 3      | 4      | 5      | 6      | 7       | 7 - 1   |
|--------|---------------|--------|--------|--------|--------|--------|--------|---------|---------|
| Excess | Excess        | 16.47  | 13.89  | 14.55  | 12.79  | 11.99  | 10.92  | 10.43   | -6.04   |
| return | return        | (4.55) | (5.74) | (5.83) | (6.73) | (6.57) | (6.06) | (3.43)  | (-1.31) |
| CAPM   | $\alpha_p$    | 8.89   | 6.78   | 6.75   | 5.32   | 3.58   | 0.45   | -2.49   | -11.38  |
|        |               | (1.41) | (1.32) | (1.33) | (1.31) | (1.13) | (0.19) | (-1.11) | (-1.72) |
|        | $\beta_{MKT}$ | 1.02   | 0.96   | 1.05   | 1.01   | 1.13   | 1.41   | 1.74    | 0.72    |
|        |               | (3.30) | (3.05) | (3.80) | (3.75) | (5.20) | (7.63) | (8.70)  | (1.89)  |

- When the returns are adjusted for exposure to the market factor using the CAPM risk model (Model = CAPM), none of the seven portfolios generates abnormal returns that are statistically distinguishable from zero as all t-statistics are substantially less than 2.00 in magnitude.
- The excess returns generated by the portfolios are a manifestation of the portfolios' exposures to the market factor.
- After controlling for market factor, the average abnormal return of each of the portfolios is statistically insignificant.

- All portfolios have a positive and statistically significant sensitivity to the market portfolio.
- The sensitivities are nearly monotonically increasing from portfolios 1 to 7 since the portfolios were formed by sorting on  $\beta$ , which measures stock-level sensitivity to the market portfolio.
- ▶ The abnormal return of the difference portfolio remains statistically insignificant when using the CAPM risk model, and the sensitivity of the difference portfolio to the market portfolio of 0.72 is marginally statistically significant.

- ▶ The size effect refers to the observation that stocks with large market capitalizations (large stocks) tend to have lower returns than stocks with small market capitalizations (small stocks).
- ▶ The primary market capitalization variable, MktCap, therefore, is calculated for stock i in month t as

$$\mathsf{MktCap}_{i,t} = \frac{\mid \mathsf{SHROUT}_{i,t} \times \mathsf{ALTPRC}_{i,t} \mid}{1000}$$

where  $SHROUT_{i,t}$  is the number of shares outstanding at the end of month t, and  $ALTPRC_{i,t}$  is the price of the stock.

- The division by 1000 results in MktCap measuring the market capitalization of the stock in millions of dollars.
- ► The absolute value is taken to account for the fact that CRSP reports a negative price when the reported value is calculated as the average of a bid and ask price.

- Fama and French (1992, 1993, FF hereafter) calculate market capitalization as of the last trading day of June in each year y and hold the value constant for the months from June of that same year y until May of year y+1, in June of year y+1, market capitalization is recalculated.
- ► The benefit of this approach is that the market capitalization measure is not affected by short-term movements in the stock price, which may cause the market capitalization measure to exhibit unwanted time-series correlation with stock returns.
- This measure of market capitalization is defined as

$$\mathsf{MktCap} \ _{i,t}^{\mathit{FF}} = \frac{\mid \ \mathsf{SHROUT} \ _{i, \ \mathsf{June}} \ \times \ \mathsf{ALTPRC} \ _{i, \ \mathsf{June}} \ \mid}{1000}$$

▶ Using this approach, the market capitalization for each stock changes only once per calendar year, in June, and therefore remains constant from any given June through the following May.

The natural log of market capitalization, denoted by Size, are defined as

$$Size_{i,t} = In(MktCap_{i,t})$$

and

$$\mathsf{Size}_{i,t}^{\mathsf{FF}} = \mathsf{In}\left(\mathsf{MktCap}_{i,t}^{\mathsf{FF}}\right).$$

The inflation-adjusted values of the market capitalization variables in 2012 dollars are defined as

$$\mathsf{MktCap}_{i,t}^{\mathit{CPI}} = \mathit{MktCap}_{i,t} \times \frac{\mathit{CPI}_{12/2012}}{\mathit{CPI}_t}$$

and

$$MktCap_{i,t}^{FF,CPI} = MktCap_{i,t}^{FF} \times \frac{CPI_{12/2012}}{CPI_{t}}$$

where  $CPI_{12,2012}$  and  $CPI_t$  are the levels of the CPI index as of the end of December 2012 and the end of month t, respectively.

Univariate Portfolio Analysis: NYSE Breakpoints

|               | Panel A: Portfolio Characteristics |      |       |       |       |       |       |       |       |       |        |  |  |  |
|---------------|------------------------------------|------|-------|-------|-------|-------|-------|-------|-------|-------|--------|--|--|--|
| Sort Variable | Value                              | 1    | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10     |  |  |  |
| MktCap        | MktCap                             | 32   | 126   | 226   | 354   | 538   | 806   | 1225  | 2034  | 4078  | 19,987 |  |  |  |
|               | MktCap <sup>CPI</sup>              | 57   | 211   | 365   | 565   | 854   | 1278  | 1953  | 3239  | 6288  | 29,989 |  |  |  |
|               | % MktCap                           | 1.82 | 1.64  | 1.85  | 2.32  | 2.96  | 3.73  | 5.18  | 7.95  | 13.65 | 58.90  |  |  |  |
|               | % NYSE                             | 7.46 | 28.75 | 41.95 | 50.81 | 59.61 | 69.84 | 77.05 | 82.38 | 88.81 | 92.89  |  |  |  |
|               | n                                  | 2372 | 592   | 383   | 303   | 252   | 211   | 189   | 176   | 162   | 155    |  |  |  |
|               | β                                  | 0.60 | 0.91  | 0.96  | 0.97  | 0.97  | 0.96  | 0.97  | 0.98  | 0.99  | 1.03   |  |  |  |

► The average market capitalization of stocks in each of the portfolios increases from \$32 million for decile portfolio one to nearly \$20 billion for decile portfolio 10.

- ► The row %*MktCap* shows that portfolio one holds only 1.82% of the total value of the market portfolio compared to 58.90% for portfolio 10.
  - ► The highest decile portfolio contains, in the average month, more than half of the total market capitalization.
  - ► The portfolios one through five hold less than 11% of the total market capitalization of all portfolios.
- ▶ The row %NYSE indicates that for the small stock portfolio (decile one), 7.46% of such stocks are NYSE-listed and 92.89% for the the large stock portfolio (decile 10).
- ▶ In the average month, there are 2372 stocks in portfolio one and 155 in portfolio 10 because of the skewed distribution of market capitalization among stocks.

Univariate Portfolio Analysis: NYSE Breakpoints

|               | Panel B: Equal-Weighted Portfolio Returns                                 |        |        |        |        |        |        |        |        |        |         |         |  |  |  |
|---------------|---|--------|--------|--------|--------|--------|--------|--------|--------|--------|---------|---------|--|--|--|
| Sort Variable | Sort Variable Coefficient $1$ $2$ $3$ $4$ $5$ $6$ $7$ $8$ $9$ $10$ $10-1$ |        |        |        |        |        |        |        |        |        |         |         |  |  |  |
| MktCap        | Excess  | 0.91   | 0.63   | 0.73   | 0.71   | 0.71   | 0.66   | 0.62   | 0.61   | 0.51   | 0.42    | -0.49   |  |  |  |
|               | return  | (2.53) | (2.13) | (2.56) | (2.69) | (2.76) | (2.69) | (2.60) | (2.73) | (2.40) | (2.18)  | (-1.92) |  |  |  |
|               | CAPM $\alpha$   | 0.41   | 0.07   | 0.16   | 0.16   | 0.17   | 0.13   | 0.10   | 0.11   | 0.04   | -0.03   | -0.44   |  |  |  |
|               |   | (1.85) | (0.50) | (1.22) | (1.48) | (1.69) | (1.50) | (1.40) | (1.86) | (0.79) | (-0.82) | (-1.79) |  |  |  |

- ► The difference portfolio (10 1) generates an economically large average return of -0.49%, which is marginally statistically significant with t-statistic of -1.92.
- ▶ The risk-adjusted abnormal return of this portfolio relative to the CAPM risk model (CAPM  $\alpha$ ) of -0.44% (t-statistic of -1.79) also indicates an economically important and marginally statistically significant relation between MktCap and future stock returns.

- ▶ The first decile portfolio generates an average monthly excess return and alpha of 0.91% and 0.41%, respectively, whereas portfolio two generates an excess return of only 0.63% per month and monthly alpha of only 0.07%.
- ► The cross-sectional relation between *MktCap* and future stock returns appears to be driven primarily by the high abnormal returns of the smallest stocks in the sample.

Univariate Portfolio Analysis: NYSE Breakpoints

|               | Panel            | C: Value- | Weighted   | Portfolio   | Returns  |   |  |   |  |   |  |
|---------------|------------------|-----------|--|---|--|---|--|---|--|---|--|
|               |                  |           |  |   |  |   |  |   |  |   |  |
| Coefficient   | 1                | 2         | 3  | 4   | 5  | 6   | 7  | 8   | 9  | 10  | 10 - 1   |
| Excess        | 0.65             | 0.63      | 0.73   | 0.71  | 0.71   | 0.65  | 0.62   | 0.62  | 0.51   | 0.39  | -0.27  |
| return        | (1.94)           | (2.13)    | (2.57)   | (2.68)  | (2.76)   | (2.67)  | (2.63)   | (2.77)  | (2.39)   | (2.09)  | (-1.09)  |
| CAPM $\alpha$ | 0.13             | 0.07      | 0.16   | 0.16  | 0.17   | 0.13  | 0.11   | 0.11  | 0.04   | -0.03   | -0.17  |
|               | (0.69)           | (0.50)    | (1.24)   | (1.47)  | (1.69)   | (1.46)  | (1.51)   | (1.97)  | (0.77)   | (-0.79)   | (-0.73)  |
|               | Excess<br>return |           | Coefficient         1         2           Excess         0.65         0.63           return         (1.94)         (2.13)           CAPM α         0.13         0.07 | Coefficient         1         2         3           Excess         0.65         0.63         0.73           return         (1.94)         (2.13)         (2.57)           CAPM α         0.13         0.07         0.16 | Coefficient         1         2         3         4           Excess         0.65         0.63         0.73         0.71           return         (1.94)         (2.13)         (2.57)         (2.68)           CAPM α         0.13         0.07         0.16         0.16 | Coefficient         1         2         3         4         5           Excess         0.65         0.63         0.73         0.71         0.71           return         (1.94)         (2.13)         (2.57)         (2.68)         (2.76)           CAPM α         0.13         0.07         0.16         0.16         0.17 | $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$ | Coefficient         1         2         3         4         5         6         7           Excess         0.65         0.63         0.73         0.71         0.71         0.65         0.62           return         (1.94)         (2.13)         (2.57)         (2.68)         (2.76)         (2.67)         (2.63)           CAPM α         0.13         0.07         0.16         0.16         0.17         0.13         0.11 | Coefficient         1         2         3         4         5         6         7         8           Excess         0.65         0.63         0.73         0.71         0.71         0.65         0.62         0.62           return         (1.94)         (2.13)         (2.57)         (2.68)         (2.76)         (2.67)         (2.63)         (2.77)           CAPM α         0.13         0.07         0.16         0.16         0.17         0.13         0.11         0.11 | Coefficient         1         2         3         4         5         6         7         8         9           Excess         0.65         0.63         0.73         0.71         0.71         0.65         0.62         0.62         0.51           return         (1.94)         (2.13)         (2.57)         (2.68)         (2.76)         (2.67)         (2.63)         (2.77)         (2.39)           CAPM α         0.13         0.07         0.16         0.16         0.17         0.13         0.11         0.11         0.04 | Coefficient         1         2         3         4         5         6         7         8         9         10           Excess         0.65         0.63         0.73         0.71         0.71         0.65         0.62         0.62         0.51         0.39           return         (1.94)         (2.13)         (2.57)         (2.68)         (2.76)         (2.67)         (2.63)         (2.77)         (2.39)         (2.09)           CAPM α         0.13         0.07         0.16         0.16         0.17         0.13         0.11         0.11         0.04         -0.03 |

- The negative relation between market capitalization and future stock returns is not detected.
  - ▶ The average return and CAPM alpha of the MktCap-sorted difference portfolio of -0.27% (t-statistic of -1.09) and -0.17%(t-statistic of =-0.73) are both statistically insignificant.
  - ► The magnitudes of the average return and alpha of the value-weighted 10-1 portfolio are substantially smaller than for the equal-weighted portfolios.

► NYSE/AMEX/NASDAQ Breakpoints

|               | Panel A: Portfolio Characteristics       |      |      |      |       |       |       |       |       |       |        |  |  |  |  |
|---------------|--|------|------|------|-------|-------|-------|-------|-------|-------|--------|--|--|--|--|
| Sort Variable | Sort Variable Value 1 2 3 4 5 6 7 8 9 10 |      |      |      |       |       |       |       |       |       |        |  |  |  |  |
| MktCap        | MktCap                                   | 6    | 16   | 29   | 50    | 84    | 141   | 244   | 458   | 1049  | 8923   |  |  |  |  |
|               | MktCap <sup>CPI</sup>                    | 13   | 31   | 55   | 92    | 149   | 244   | 417   | 771   | 1759  | 13,810 |  |  |  |  |
|               | % MktCap                                 | 0.08 | 0.19 | 0.33 | 0.55  | 0.88  | 1.43  | 2.42  | 4.51  | 10.49 | 79.12  |  |  |  |  |
|               | % NYSE                                   | 1.07 | 3.17 | 7.46 | 13.70 | 21.77 | 31.28 | 41.90 | 54.27 | 70.45 | 87.09  |  |  |  |  |
|               | n  | 480  | 479  | 479  | 479   | 479   | 479   | 479   | 479   | 479   | 480    |  |  |  |  |
|               | $\beta$                                  | 0.42 | 0.51 | 0.60 | 0.73  | 0.83  | 0.90  | 0.94  | 0.95  | 0.94  | 0.99   |  |  |  |  |

- ► The average market capitalization (MktCap) of stocks in the first decile portfolio is only \$6 million or \$13 million when inflation-adjusted to 2012 dollars.
- ► Even though 10% of all stocks are held in portfolio one, these stocks only account for 0.08% of the total stock market capitalization.
- ▶ The entire size effect appears to be driven by a subset of stocks that comprise less than one 10th of 1% of the entire stock market.

NYSE/AMEX/NASDAQ Breakpoints

|        | Panel B: Equal-Weighted Portfolio Returns |        |        |        |        |        |        |        |        |        |        |         |
|--------|---|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|---------|
| Sort   | Coefficient                               | 1      | 2      | 3      | 4      | 5      | 6      | 7      | 8      | 9      | 10     | 10 – 1  |
| MktCap | Excess                                    | 2.08   | 0.64   | 0.53   | 0.56   | 0.59   | 0.64   | 0.66   | 0.67   | 0.63   | 0.49   | -1.59   |
|        | return                                    | (4.75) | (1.70) | (1.55) | (1.68) | (1.89) | (2.23) | (2.38) | (2.60) | (2.68) | (2.38) | (-4.94) |
|        | CAPM $\alpha$                             | 1.63   | 0.16   | 0.04   | 0.04   | 0.05   | 0.09   | 0.10   | 0.12   | 0.12   | 0.02   | -1.62   |
|        |   | (4.95) | (0.65) | (0.19) | (0.19) | (0.30) | (0.64) | (0.83) | (1.29) | (1.57) | (0.47) | (-4.95) |

- ▶ The equal-weighted portfolio analysis detects a stronger negative relation between market capitalization and future stock returns.
- ▶ The result appears to be driven primarily by the first decile portfolio and the returns and alphas of portfolios two through 10 are also similar.

NYSE/AMEX/NASDAQ Breakpoints

| Panel C: Value-Weighted Portfolio Returns |               |        |        |        |        |        |        |        |        |        |         |         |
|---|---------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|---------|---------|
| Sort                                      | Coefficient   | 1      | 2      | 3      | 4      | 5      | 6      | 7      | 8      | 9      | 10      | 10 – 1  |
| MktCap                                    | Excess        | 1.53   | 0.63   | 0.54   | 0.56   | 0.59   | 0.65   | 0.66   | 0.67   | 0.63   | 0.42    | -1.11   |
|   | return        | (3.69) | (1.67) | (1.57) | (1.70) | (1.88) | (2.25) | (2.41) | (2.59) | (2.68) | (2.20)  | (-3.47) |
|   | CAPM $\alpha$ | 1.08   | 0.15   | 0.05   | 0.04   | 0.04   | 0.10   | 0.11   | 0.12   | 0.12   | -0.02   | -1.10   |
|   |               | (3.61) | (0.61) | (0.22) | (0.20) | (0.26) | (0.69) | (0.90) | (1.26) | (1.63) | (-0.85) | (-3.47) |

- When using NYSE/AMEX/NASDAQ breakpoints, even value-weighted portfolios produce a strong negative difference portfolio return and CAPM alpha.
- ▶ The value-weighted 10-1 portfolio generates an economically large and highly statistically significant average return (CAPM alpha) of -1.11% (-1.10%) per month.