

```
import matplotlib.pyplot as plt
import matplotlib.patches as patches

# 设置绘图风格
plt.figure(figsize=(10, 7), dpi=150)
ax = plt.gca()
ax.set_aspect('equal')
plt.axis('off') # 关闭坐标轴

# -----
# 1. 定义节点位置 (Coordinates)
# -----
# 左侧 (Populations)
pos_S = (-3, 1)
pos_R = (-3, -2)

# 右侧 (Internal States)
pos_P = (3, 1)
pos_A = (3, -2)

# 上方/中心 (Signal)
pos_D = (0, 3)

# -----
# 2. 绘制节点 (Nodes)
# -----
def draw_node(pos, label, color, shape='circle', size=0.6):
    x, y = pos
    if shape == 'circle':
        circle = patches.Circle((x, y), radius=size, edgecolor='black')
        ax.add_patch(circle)
    elif shape == 'cloud':
        # 用圆角矩形/椭圆模拟云状/独特形状
        box = patches.FancyBboxPatch((x-0.8, y-0.4), 1.6, 0.8,
                                      boxstyle="round,pad=0.1,rounding_size=0.2",
                                      edgecolor='black', facecolor=color)
        ax.add_patch(box)

    plt.text(x, y, label, ha='center', va='center', fontsize=16, fontweight='bold')

# 绘制具体节点
draw_node(pos_S, '$S$', '#dae8fc') # 浅蓝
draw_node(pos_R, '$R$', '#f8cecc') # 浅红
draw_node(pos_P, '$P$', '#d5e8d4') # 浅绿
draw_node(pos_A, '$A$', '#fff2cc') # 浅黄
draw_node(pos_D, '$D$', '#e1d5e7', shape='cloud') # 浅紫 (信号)

# 标题 (Populations vs Internal States)
plt.text(-3, 2.2, "Populations\n(Diffusive)", ha='center', fontsize=16, fontweight='bold')
```

```

plt.text(3, 2.2, "Internal States\n(Pointwise UDE)", ha='center', to
# -----
# 3. 绘制实线箭头 (Solid Interactions)
# -----
style_arrow = "Simple, tail_width=0.5, head_width=4, head_length=8"

# S -> R (常数迁移 alpha)
arrow_SR = patches.FancyArrowPatch(
    (pos_S[0], pos_S[1]-0.6), (pos_R[0], pos_R[1]+0.6),
    connectionstyle="arc3,rad=-0.2", color="black", arrowstyle=style_
)
ax.add_patch(arrow_SR)
plt.text(-3.6, -0.5, r'$\alpha$', fontsize=12)

# R -> S (受控迁移)
arrow_RS = patches.FancyArrowPatch(
    (pos_R[0], pos_R[1]+0.6), (pos_S[0], pos_S[1]-0.6),
    connectionstyle="arc3,rad=-0.2", color="black", arrowstyle=style_
)
ax.add_patch(arrow_RS)
plt.text(-2.4, -0.5, r'$\xi(1-\phi(D))$', fontsize=11)

# P <-> A (内部状态转换)
# P -> A
arrow_PA = patches.FancyArrowPatch(
    (pos_P[0]-0.2, pos_P[1]-0.6), (pos_A[0]-0.2, pos_A[1]+0.6),
    connectionstyle="arc3,rad=0", color="black", arrowstyle=style_ar
)
ax.add_patch(arrow_PA)

# A -> P
arrow_AP = patches.FancyArrowPatch(
    (pos_A[0]+0.2, pos_A[1]+0.6), (pos_P[0]+0.2, pos_P[1]-0.6),
    connectionstyle="arc3,rad=0", color="black", arrowstyle=style_ar
)
ax.add_patch(arrow_AP)
plt.text(3.3, -0.5, r'$\phi(D)$', fontsize=12)

# -----
# 4. 绘制自环 (Self-loops - Logistic Growth)
# -----
# S 的 Logistic
arc_S = patches.FancyArrowPatch(
    (pos_S[0]-0.5, pos_S[1]+0.3), (pos_S[0]-0.5, pos_S[1]-0.3),
    connectionstyle="arc3,rad=2.5", color="gray", arrowstyle="->", l
)
ax.add_patch(arc_S)
plt.text(-4.8, 1, r'$\lambda_S(1-\frac{S+R}{K})$', fontsize=9, ha='c

# R 的 Logistic
arc_R = patches.FancyArrowPatch(

```

```

        (pos_R[0]-0.5, pos_R[1]+0.3), (pos_R[0]-0.5, pos_R[1]-0.3),
        connectionstyle="arc3,rad=2.5", color="gray", arrowstyle="->", l
    )
ax.add_patch(arc_R)
plt.text(-4.8, -2, r'$\lambda_R(1-\frac{S+R}{K})$', fontsize=9, ha='

# -----
# 5. 绘制虚线调节箭头 (Dashed Signal Regulation)
# -----
style_dashed = "Simple, tail_width=0.5, head_width=4, head_length=8"

# D -> S (抑制 delta(D))
arrow_DS = patches.FancyArrowPatch(
    (pos_D[0]-0.5, pos_D[1]-0.4), (pos_S[0]+0.6, pos_S[1]+0.6),
    connectionstyle="arc3,rad=0.2", color="purple", arrowstyle=style_dashed
)
ax.add_patch(arrow_DS)
# 标注背景设为白色以便看清
t = plt.text(-1.2, 2.0, r'$\delta(D) (-)$', fontsize=10, color='purple')
t.set_bbox(dict(facecolor='white', alpha=0.7, edgecolor='none'))

# D -> (R->S 迁移) 的调节
# 我们指向 R->S 箭头的中间位置大致坐标 (-2.5, -0.5)
arrow_DRS = patches.FancyArrowPatch(
    (pos_D[0], pos_D[1]-0.4), (-2.4, -0.4),
    connectionstyle="arc3,rad=-0.1", color="purple", arrowstyle="->"
)
ax.add_patch(arrow_DRS)
t2 = plt.text(-1.0, 0.5, r'$\phi(D)$', fontsize=10, color='purple')
t2.set_bbox(dict(facecolor='white', alpha=0.7, edgecolor='none'))

# D -> (P <-> A) 的调节
arrow_DPA = patches.FancyArrowPatch(
    (pos_D[0]+0.5, pos_D[1]-0.4), (2.8, -0.5),
    connectionstyle="arc3,rad=-0.2", color="purple", arrowstyle="->"
)
ax.add_patch(arrow_DPA)

# -----
# 6. 绘制反馈 (Feedback A -> R)
# -----
# 从 A 指向 R, 粗虚线
arrow_Feed = patches.FancyArrowPatch(
    (pos_A[0]-0.6, pos_A[1]), (pos_R[0]+0.6, pos_R[1]),
    connectionstyle="arc3,rad=-0.4",
    color="darkred",
    arrowstyle="Simple, tail_width=1, head_width=6, head_length=10",
    linestyle="--",
    zorder=1
)

```

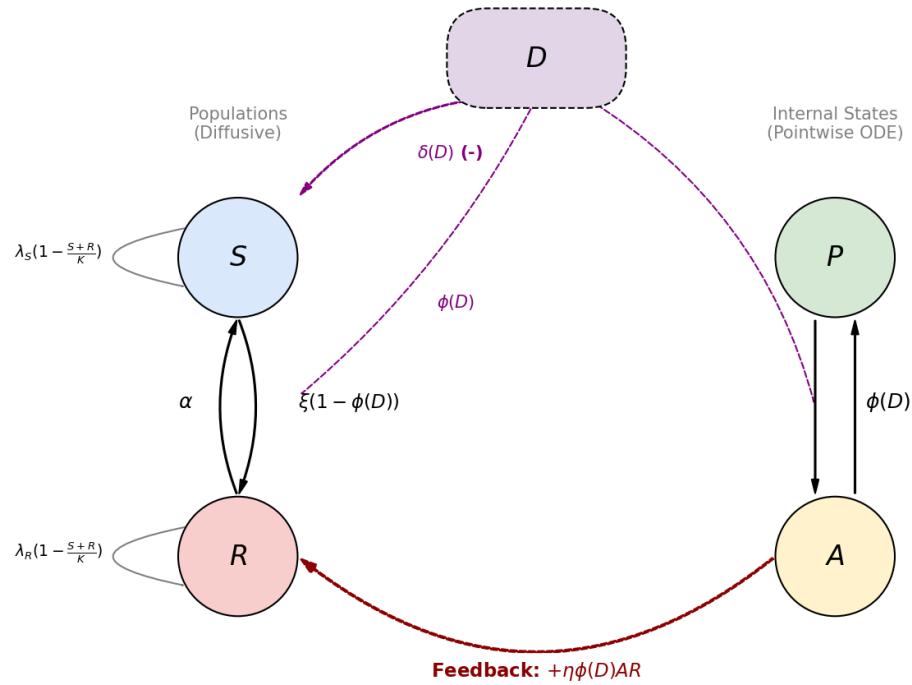
```

ax.add_patch(arrow_Feed)
t3 = plt.text(0, -3.2, r'Feedback: $+\eta\phi(D) A R$', fontsize=11
t3.set_bbox(dict(facecolor='white', alpha=0.8, edgecolor='none'))

# 调整视野范围
plt.xlim(-6, 6)
plt.ylim(-4, 4)

# 保存或显示
plt.tight_layout()
# plt.savefig('model_interaction_diagram.png', dpi=300) # 去掉注释以保
plt.show()

```



开始借助 AI 编写或生成代码。

```
import matplotlib.pyplot as plt
import matplotlib.patches as patches
import numpy as np
from matplotlib.path import Path

# 设置绘图风格
fig, ax = plt.subplots(figsize=(12, 7), dpi=150)
ax.set_aspect('equal')
ax.axis('off')
# 设置坐标轴范围以容纳所有元素
ax.set_xlim(-2, 15)
ax.set_ylim(-3, 10)

# =====
# 1. 宏观尺度区域 (Macro Scale Domain Omega)
# =====

# --- 定义不规则区域边界 ---
theta = np.linspace(0, 2*np.pi, 200)
# 使用几个正弦波叠加生成不规则半径
r = 5 + 0.6 * np.sin(3*theta) + 0.4 * np.cos(5*theta)
domain_center_x, domain_center_y = 4, 4
x_domain = domain_center_x + r * np.cos(theta)
y_domain = domain_center_y + r * np.sin(theta)

domain_path_coords = np.column_stack((x_domain, y_domain))
domain_patch = patches.Polygon(domain_path_coords, closed=True,
                               facecolor='#e6f2ff', # 浅蓝背景
                               edgecolor='#336699', # 深蓝边界
                               linewidth=3, zorder=1)
ax.add_patch(domain_patch)

ax.text(domain_center_x, 9.5, r'Macro Scale: Domain $\Omega$',
```

```

ha='center', fontsize=16, fontweight='bold', color='#336699'

# --- 可视化扩散场 (S, R, D) ---
# 方法: 在区域内散布模糊的彩色点来模拟连续场
np.random.seed(42) # 固定随机种子以保证结果可复现
num_points = 500
# 生成一个边界框内的随机点
px = np.random.uniform(min(x_domain), max(x_domain), num_points)
py = np.random.uniform(min(y_domain), max(y_domain), num_points)

# 使用 Path.contains_points 筛选出在区域内部的点
path = Path(domain_path_coords)
mask = path.contains_points(np.column_stack((px, py)))
px_in, py_in = px[mask], py[mask]

# 绘制代表 S, R, D 的混合场
colors = np.random.choice(['#6baed6', '#fd8d3c', '#a1d99b'], size=l
ax.scatter(px_in, py_in, c=colors, s=150, alpha=0.3, edgecolors='nc

# 添加波浪箭头表示移动/扩散
style_wave = "Simple, tail_width=0.5, head_width=4, head_length=6"
for _ in range(6):
    start_x = domain_center_x + np.random.uniform(-3, 3)
    start_y = domain_center_y + np.random.uniform(-3, 3)
    # 随机方向
    angle = np.random.uniform(0, 2*np.pi)
    length = np.random.uniform(1.5, 2.5)
    end_x = start_x + length * np.cos(angle)
    end_y = start_y + length * np.sin(angle)

    # 确保起点在区域内 (简单近似)
    if path.contains_point((start_x, start_y)):
        arrow = patches.FancyArrowPatch(
            (start_x, start_y), (end_x, end_y),
            connectionstyle=f"arc3,rad={np.random.uniform(-0.3, 0.3
            color="#08519c", arrowstyle='->', lw=2, alpha=0.7, zord
        )
        ax.add_patch(arrow)

# 宏观标签
ax.text(domain_center_x - 3, domain_center_y - 3,
        r'Diffusing Fields $(S, R, D)$',
        fontsize=12, fontstyle='italic', color='#08519c')
ax.text(domain_center_x + 3, domain_center_y + 3,
        r'$-\Delta$ (Global Diffusion)',
        fontsize=14, fontweight='bold', color='darkblue', ha='cente

# =====
# 2. 微观点选择与放大镜效果 (Magnifying Glass)
# =====

```

```

# --- 选取几个点 ---
points = [
    (1.5, 5.5, '$x_1$'),
    (4.5, 3.0, '$x_2$'), # 将要放大的点
    (6.5, 6.5, '$x_3$')
]

for px_val, py_val, label in points:
    ax.plot(px_val, py_val, 'o', color='black', markersize=7, zorder=1)
    ax.text(px_val + 0.3, py_val + 0.3, label, fontsize=12, zorder=1)

# --- 放大镜结构 ---
target_point = points[1] # 选择 x2
tp_coords = (target_point[0], target_point[1])
zoom_center = (12, 4) # 放大视图的中心位置
zoom_radius = 2.8

# 1. 目标点上的小红圈
small_circle = patches.Circle(tp_coords, radius=0.7, fill=False,
                               edgecolor='#d62728', linestyle='--',
                               ax=ax)
ax.add_patch(small_circle)

# 2. 放大的大气泡
big_circle = patches.Circle(zoom_center, radius=zoom_radius,
                             facecolor='white', edgecolor='#d62728',
                             ax=ax)
ax.add_patch(big_circle)

# 3. 连接线 (Zoom lines)
# 计算连接角度
angle_conn = np.arctan2(zoom_center[1] - tp_coords[1], zoom_center[0] - tp_coords[0])
perp_angle = angle_conn + np.pi/2
offset = 0.6 # 连接线的宽度偏移

# 上连接线
con_top = patches.ConnectionPatch(
    xyA=(tp_coords[0] + offset*np.cos(perp_angle), tp_coords[1] + offset*np.sin(perp_angle)),
    xyB=(zoom_center[0] + zoom_radius*np.cos(perp_angle), zoom_center[1] + zoom_radius*np.sin(perp_angle)),
    coordsA="data", coordsB="data", axesA=ax, axesB=ax,
    color='#d62728', linestyle='--', lw=1.5, alpha=0.6, zorder=4
)
# 下连接线
con_bottom = patches.ConnectionPatch(
    xyA=(tp_coords[0] - offset*np.cos(perp_angle), tp_coords[1] - offset*np.sin(perp_angle)),
    xyB=(zoom_center[0] - zoom_radius*np.cos(perp_angle), zoom_center[1] - zoom_radius*np.sin(perp_angle)),
    coordsA="data", coordsB="data", axesA=ax, axesB=ax,
    color='#d62728', linestyle='--', lw=1.5, alpha=0.6, zorder=4
)
ax.add_artist(con_top)
ax.add_artist(con_bottom)

```

```

# =====
# 3. 微观动力学 (Micro Dynamics inside Zoom)
# =====

# P 和 A 节点位置
pos_P_micro = (zoom_center[0] - 1.2, zoom_center[1])
pos_A_micro = (zoom_center[0] + 1.2, zoom_center[1])

# 绘制节点
circle_P = patches.Circle(pos_P_micro, radius=0.6, facecolor="#d5e8f0")
circle_A = patches.Circle(pos_A_micro, radius=0.6, facecolor="#ffff22")
ax.add_patch(circle_P)
ax.add_patch(circle_A)

ax.text(pos_P_micro[0], pos_P_micro[1], '$P(x_2, t)$', ha='center',
        ax.text(pos_A_micro[0], pos_A_micro[1], '$A(x_2, t)$', ha='center',

# 绘制反应箭头 P <-> A
arrow_style_micro = "Simple, tail_width=0.8, head_width=5, head_length=10"
# P -> A
arrow_PA = patches.FancyArrowPatch(
    (pos_P_micro[0] + 0.6, pos_P_micro[1] + 0.2), (pos_A_micro[0] - 0.6, pos_A_micro[1] - 0.2),
    connectionstyle="arc3,rad=-0.3", color="black", arrowstyle=arrow_style_micro)
# A -> P
arrow_AP = patches.FancyArrowPatch(
    (pos_A_micro[0] - 0.6, pos_A_micro[1] - 0.2), (pos_P_micro[0] + 0.6, pos_P_micro[1] + 0.2),
    connectionstyle="arc3,rad=-0.3", color="black", arrowstyle=arrow_style_micro)
ax.add_patch(arrow_PA)
ax.add_patch(arrow_AP)

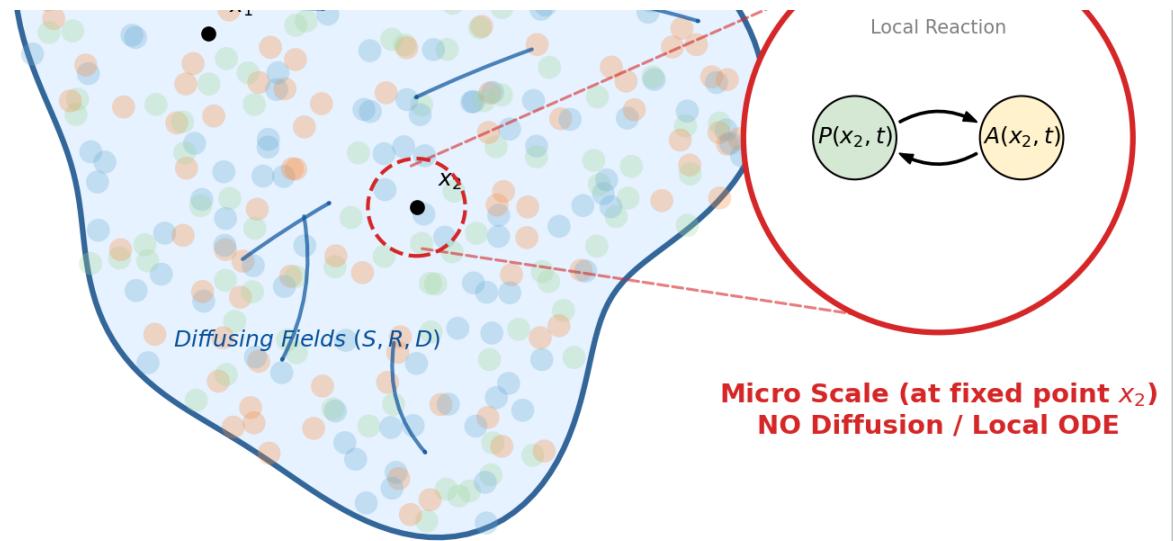
# 微观标签
ax.text(zoom_center[0], zoom_center[1] + 1.5, "Local Reaction", ha='center',
        ax.text(zoom_center[0], zoom_center[1] - 3.5,
                "Macro Scale (at fixed point $x_2$)\nNO Diffusion / Local Convection", ha='center', va='top', fontsize=14, fontweight='bold', color='blue')

# 保存与显示
plt.tight_layout()
# plt.savefig('multiscale_scheme.png', dpi=300, bbox_inches='tight')
plt.show()

```

Macro Scale: Domain Ω





```
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.patches as patches
```

```

# -----
# 1. 参数设置与函数定义
# -----
# 定义 D 的范围 (模拟从 0 到 D_max)
D_max = 1.5
D = np.linspace(0, D_max, 500)

# 参数 (参考之前的论文上下文)
sigma = 5.0           # 控制 phi 的陡峭程度 (开关灵敏度)
delta_0 = 0.3          # delta 的最大饱和值
K_D = 0.2              # delta 的半饱和常数

# 函数定义
# 1. 开关/调节函数 (Sigmoid / Tanh)
phi = np.tanh(sigma * D)

# 2. 抑制/饱和函数 (Michaelis-Menten)
delta = delta_0 * (D / (D + K_D))

# -----
# 2. 绘图初始化
# -----
fig, ax1 = plt.subplots(figsize=(10, 6), dpi=150)

# 设置网格
ax1.grid(True, which='major', linestyle='--', alpha=0.5)
ax1.set_xlabel(r'Signal Strength $D(x,t)$', fontsize=14)
ax1.set_xlim(0, D_max)

# -----
# 3. 绘制左轴 (Phi - 开关行为)
# -----
color_phi = '#1f77b4' # 蓝色
ax1.set_ylabel(r'Switch Function $\phi(D) = \tanh(\sigma D)$', color=color_phi, fontweight='bold')
line1, = ax1.plot(D, phi, color=color_phi, linewidth=2.5, label=r'$\phi(D)$')
ax1.tick_params(axis='y', labelcolor=color_phi)
ax1.set_ylim(0, 1.1) # 留一点空间给标注

# 标注 "ON" 和 "OFF" 状态
ax1.text(0.1, 0.1, r'OFF ($\phi \approx 0$)', color=color_phi, fontweight='bold')
ax1.text(1.2, 1.02, r'ON ($\phi \approx 1$)', color=color_phi, fontweight='bold')

# -----
# 4. 绘制右轴 (Delta - 饱和抑制)
# -----
ax2 = ax1.twinx() # 创建共享 x 轴的第二个 y 轴

color_delta = '#d62728' # 红色
ax2.set_ylabel(r'Inhibition Rate $\delta(D) = \delta_0 \frac{D}{D+K_D}$', color=color_delta, fontweight='bold')
line2, = ax2.plot(D, delta, color=color_delta, linewidth=2.5, label=r'$\delta(D)$')

```

```

ax2.tick_params(axis='y', labelcolor=color_delta)
ax2.set_ylim(0, delta_0 * 1.2)

# 标注饱和值
ax2.axhline(delta_0, color=color_delta, linestyle=':', alpha=0.5)
ax2.text(D_max, delta_0, r' $\delta_{\max}$', va='center', color=color_delta)

# -----
# 5. 高亮区域与时间流向 (关键部分)
# -----

# 阈值区域 (Transition Zone)
# 当 sigma=5, tanh(1) approx 0.76 at D=0.2.
# 我们可以认为 D < 0.2 是快速变化区
threshold_x = 0.4

# 区域 A: 低信号区 (Low D) -> 系统恢复
ax1.axvspan(0, threshold_x, color='green', alpha=0.1)
ax1.text(threshold_x/2, 0.5, "Recovery Phase\n(Low $D$)",
         ha='center', va='center', color='darkgreen', fontsize=12, fontweight='bold')

# 区域 B: 高信号区 (High D) -> 强相互作用
ax1.axvspan(threshold_x, D_max, color='red', alpha=0.1)
ax1.text((threshold_x + D_max)/2, 0.5, "Active Interaction\n(High $D$)",
         ha='center', va='center', color='darkred', fontsize=12, fontweight='bold')

# 添加时间流向箭头 (Time Arrow)
# 因为 D 随时间衰减 ( $\exp(-\gamma t)$ ), 所以时间轴其实是从右向左的
style = "Simple, tail_width=0.5, head_width=4, head_length=8"
arrow = patches.FancyArrowPatch((1.3, 0.8), (0.2, 0.8),
                               connectionstyle="arc3,rad=0", color='black',
                               arrowstyle=style)
ax1.add_patch(arrow)
ax1.text(0.75, 0.82, r'Time Evolution ($D$ decays)', ha='center', fontweight='bold')

# -----
# 6. 图例与美化
# -----

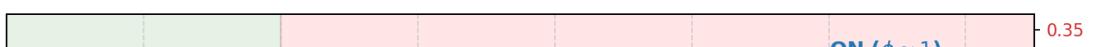
# 合并两个轴的图例
lines = [line1, line2]
labels = [l.get_label() for l in lines]
ax1.legend(lines, labels, loc='lower right', fontsize=12, frameon=True)

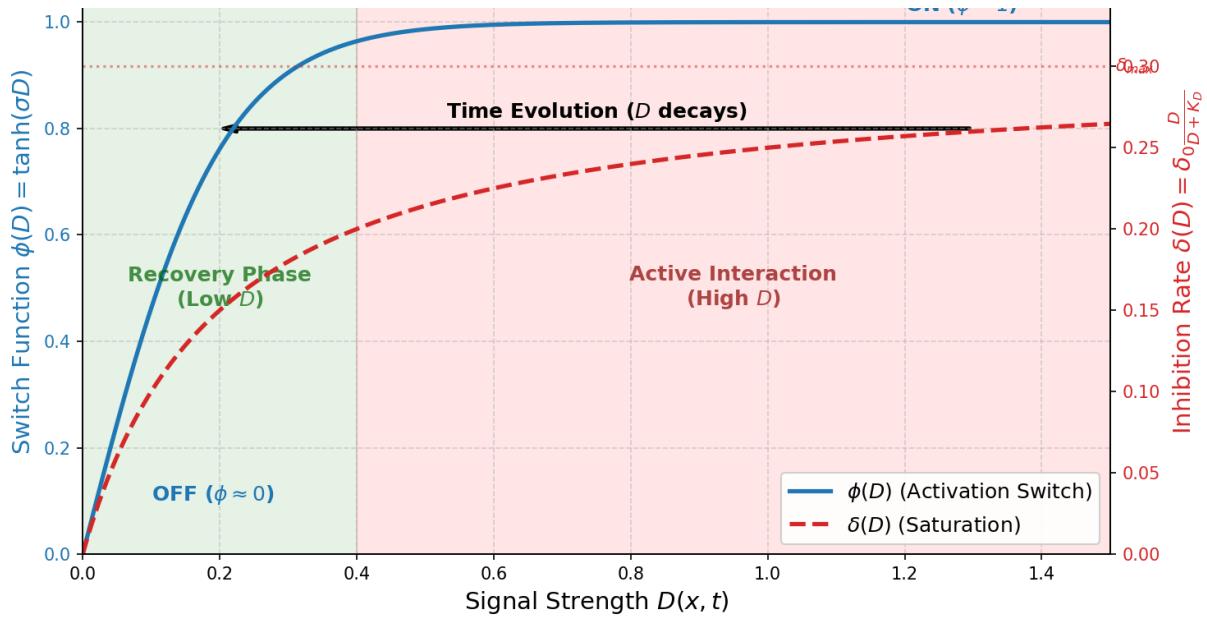
plt.title(r'Coupling Functions: Threshold Effect & Saturation', fontweight='bold')
plt.tight_layout()

# 保存或显示
# plt.savefig('coupling_functions.png', dpi=300)
plt.show()

```

Coupling Functions: Threshold Effect & Saturation





```

import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.axes_grid1 import make_axes_locatable
from scipy.ndimage import gaussian_filter

# =====
# 1. 生成虚拟数据 (Dummy Data Generation)
# =====
# 说明: 这里仅为了演示绘图效果。实际使用时, 请替换为您的模拟数据。
# 数据结构需求: List of 2D numpy arrays [Array_t0, Array_t1, Array_t2]

def generate_pattern_S(seed, time_step):
    np.random.seed(seed)
    noise = np.random.rand(100, 100)
    # 模拟 S 从无序到有序斑图的过程
    if time_step == 0:
        return noise # 初始噪声
    elif time_step == 1:
        # 过渡态: 稍微平滑, 开始出现波纹

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        return gaussian_filter(noise, sigma=2) + np.sin(np.linspace(0, 2 * np.pi, len(noise)))
    else:
        # 稳态: 高度结构化的条纹斑图
        return gaussian_filter(noise, sigma=4) * 2 + np.cos(np.linspace(0, 2 * np.pi, len(noise)))

# --- Row 1 & 2: S and R ---
# S: Prey pattern
S_data = [generate_pattern_S(42, t) for t in range(3)]
# R: Predator pattern (complementary to S)
R_data = [1.2 - d + np.random.normal(0, 0.1, (100, 100)) for d in S_data]

# --- Row 3: D (修正部分: 平滑场 + 扩散 + 衰减) ---
# 物理改进: D 不再是噪点, 而是平滑的浓度场。
np.random.seed(101)
raw_noise = np.random.rand(100, 100)

# t=0: 初始平滑云团 (模拟初始投放或分泌)
# 使用 sigma=3 将随机噪点抹平为连续的场
D_t0 = gaussian_filter(raw_noise, sigma=3)
# 归一化并提升基底, 确保全域有值
D_t0 = (D_t0 - D_t0.min()) / (D_t0.max() - D_t0.min()) + 0.2

# t=trans: 扩散 (变得更模糊) + 衰减 (数值整体变小)
# sigma 增大 (3->5) 模拟扩散项 d_D * Delta D
# 乘以 0.6 模拟衰减项 -gamma * D
D_t1 = gaussian_filter(D_t0, sigma=5) * 0.6

# t=T: 几乎完全消散
# sigma 继续增大 (5->8), 数值大幅衰减 (* 0.1)
D_T = gaussian_filter(D_t0, sigma=8) * 0.1

D_data = [D_t0, D_t1, D_T]

# =====
# 2. 绘图配置
# =====

# 将数据整合到列表: (LaTeX 标题, Colormap, 数据列表)
all_variables = [
    ('$S$ (Prey/Species 1)', 'Oranges', S_data),
    ('$R$ (Predator/Species 2)', 'Blues', R_data),
    ('$D$ (Signal Field)', 'Greens', D_data)
]
time_labels = ['$t=0$ (Initial)', '$t=t_{trans}$ (Pattern Emerging)']

# =====
# 3. 绘图主循环
# =====

fig, axes = plt.subplots(3, 3, figsize=(12, 10), dpi=150)

```

```

plt.subplots_adjust(wspace=0.1, hspace=0.2) # 调整子图间距

for row_idx, (var_name, cmap, snapshots) in enumerate(all_variables):

    # --- 关键步骤：统一该行的量程 ---
    # 必须基于该变量在所有时间点的数据计算 min/max
    # 这样 D 在 t=T 时颜色变浅，才能体现出“衰减”
    vmin = min(d.min() for d in snapshots)
    vmax = max(d.max() for d in snapshots)

    for col_idx, data in enumerate(snapshots):
        ax = axes[row_idx, col_idx]

        # 绘制热力图
        im = ax.imshow(data, cmap=cmap, vmin=vmin, vmax=vmax, origin='lower')

        # 美化：去除坐标轴刻度，保留边框
        ax.set_xticks([])
        ax.set_yticks([])

        # 设置列标题（仅在第一行）
        if row_idx == 0:
            ax.set_title(time_labels[col_idx], fontsize=14, pad=10, weight='bold')

        # 设置行标签（仅在第一列）
        if col_idx == 0:
            ax.set_ylabel(var_name, fontsize=16, fontweight='bold', rotation=90, pad=-10)

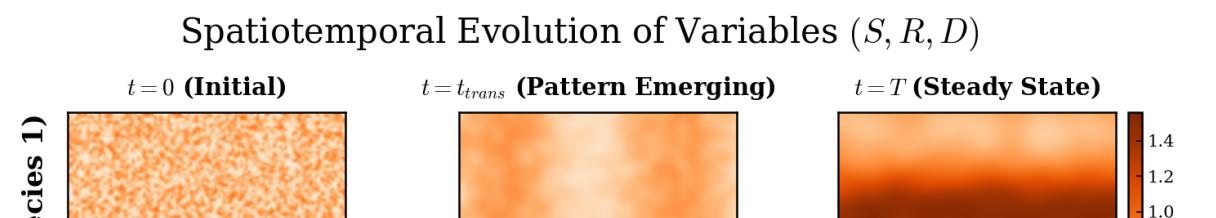
        # --- 添加颜色条（Colorbar）---
        # 放在该行最后一个子图的右侧
        last_ax = axes[row_idx, -1]
        divider = make_axes_locatable(last_ax)
        cax = divider.append_axes("right", size="5%", pad=0.1)

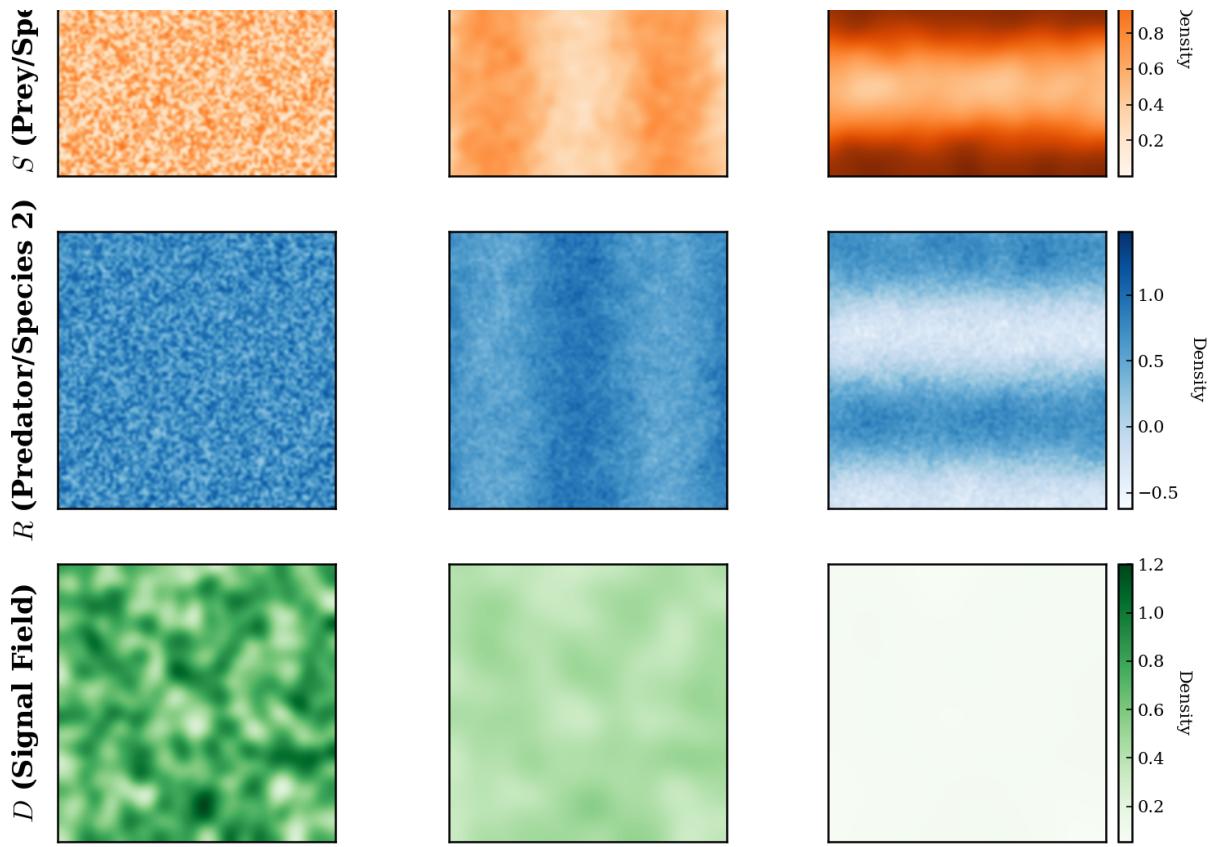
        cbar = fig.colorbar(im, cax=cax)
        cbar.ax.tick_params(labelsize=10)
        # 颜色条标签
        cbar.set_label(f'Density', rotation=270, labelpad=15, fontsize=14)

    # 总标题
    fig.suptitle('Spatiotemporal Evolution of Variables $(S, R, D)$', fontweight='bold', fontsize=18, y=0.95)

    # 保存与显示
    # plt.savefig('simulation_snapshots_corrected.png', bbox_inches='tight')
    plt.show()

```





```
import matplotlib.pyplot as plt
import matplotlib.patches as patches
import numpy as np
```

```

# 设置风格: 使用类似 LaTeX 的字体
plt.rcParams.update({
    "text.usetex": False, # 如果你有本地 LaTeX 环境可设为 True, 否则用 m
    "font.family": "serif",
    "font.serif": ["DejaVu Serif"],
    "mathtext.fontset": "cm", # Computer Modern (LaTeX 风格)
    "font.size": 12
})

def draw_schematic():
    # 创建画布: 宽一些, 以便放下左侧的区域图和右侧的机制图
    fig = plt.figure(figsize=(12, 6), dpi=150)

    # =====
    # 1. 左侧: 空间区域示意图 (Macro Domain)
    # =====
    ax_map = fig.add_axes([0.05, 0.1, 0.45, 0.8]) # [left, bottom,
                                                       top, right]

    # A. Diffusive Layer (PDE) - 背景渐变
    # 创建一个模拟的扩散场 (Gaussian blobs)
    x = np.linspace(0, 10, 100)
    y = np.linspace(0, 10, 100)
    X, Y = np.meshgrid(x, y)
    Z = np.exp(-((X-3)**2 + (Y-3)**2)/4) + 0.8 * np.exp(-((X-7)**2

    # 绘制热图代表浓度 S, R, D
    im = ax_map.imshow(Z, extent=[0, 10, 0, 10], origin='lower', crr
    ax_map.set_title(r"\bf{Spatial\ Domain\ } U$", fontsize=14, pa
    ax_map.set_xlabel("x")
    ax_map.set_ylabel("y")

    # 添加标签说明
    ax_map.text(0.5, 9.2, "Diffusive Layer\n(PDE: $S, R, D$)", colo
        ha='left', va='center', fontweight='bold', fontsize=12, bbox=dic
        facecolor='white', alpha=0.7, edgecolor='black')

    # B. Non-diffusive Layer (ODE) - 离散网格/点
    # 在区域内生成离散点
    grid_x = np.linspace(1, 9, 8)
    grid_y = np.linspace(1, 9, 8)
    GX, GY = np.meshgrid(grid_x, grid_y)

    # 绘制代表 (P, A) 的离散单元
    # 使用散点图, 空心方块代表细胞/定点
    ax_map.scatter(GX, GY, s=150, marker='s', facecolors='none', edg

    # 标注其中一个点
    ax_map.annotate(r"Discrete ODEs ($P, A$)" + "\nNo Diffusion",
                    xy=(GX[2,2], GY[2,2]), xytext=(3, 5),
                    arrowprops=dict(arrowstyle="->", color='darkred')

```

```

        color='darkred', fontsize=10,
        bbox=dict(facecolor='white', alpha=0.8, edgecol

# 美化坐标轴
ax_map.set_xlim(0, 10)
ax_map.set_ylim(0, 10)

# =====
# 2. 右侧: 相互作用机制 (Interaction Topology)
# =====
ax_mech = fig.add_axes([0.55, 0.1, 0.40, 0.8])
ax_mech.set_axis_off() # 隐藏坐标轴
ax_mech.set_title(r"\bf{Coupling\ Mechanism}", fontsize=14)

# 绘制层级框
# PDE Box
pde_box = patches.FancyBboxPatch((0.1, 0.6), 0.8, 0.35, boxstyl
                                 linewidth=2, edgecolor='navy',
                                 ax_mech.add_patch(pde_box)
                                 ax_mech.text(0.15, 0.91, "Diffusive System (PDE)", color='navy'

# ODE Box
ode_box = patches.FancyBboxPatch((0.1, 0.05), 0.8, 0.35, boxsty
                                 linewidth=2, edgecolor='darkre
                                 ax_mech.add_patch(ode_box)
                                 ax_mech.text(0.15, 0.36, "Pointwise System (ODE)", color='darkr

# --- 绘制节点 (Nodes) ---
props_circle = dict(boxstyle="circle,pad=0.3", fc="white", ec="

# S and R
ax_mech.text(0.3, 0.75, r"$S$", ha="center", va="center", size=
ax_mech.text(0.7, 0.75, r"$R$", ha="center", va="center", size=

# D (Signaling Molecule)
ax_mech.text(0.5, 0.65, r"$D$", ha="center", va="center", size=

# P and A
ax_mech.text(0.3, 0.20, r"$P$", ha="center", va="center", size=
ax_mech.text(0.7, 0.20, r"$A$", ha="center", va="center", size=

# --- 绘制箭头 (Arrows) ---
arrow_props = dict(arrowstyle="-|>", mutation_scale=20, lw=1.5,
arrow_bidir = dict(arrowstyle "<|->", mutation_scale=20, lw=1.

# S <-> R interaction
ax_mech.annotate("", xy=(0.35, 0.75), xytext=(0.65, 0.75), arrc
ax_mech.text(0.5, 0.78, "Reaction", ha="center", fontsize=9)

# Production of D (from S/R logic)
# 简化的画法: 从 S/R 连线中间指向 D

```

```

    ax_mech.annotate("", xy=(0.5, 0.69), xytext=(0.5, 0.75), arrowprops=dict(arrowstyle="|->"))

    # P <-> A interaction
    ax_mech.annotate("", xy=(0.35, 0.20), xytext=(0.65, 0.20), arrowprops=dict(arrowstyle="|>"))
    ax_mech.text(0.5, 0.23, "Conversion", ha="center", fontsize=9)

    # --- 关键的耦合箭头 (Coupling Arrow) ---
    # D drives P/A (Downward arrow crossing the boxes)
    # 起点: D node bottom (0.5, 0.61) -> 终点: P/A center (0.5, 0.25)

    # 画一个明显的跨层箭头
    ax_mech.annotate("", xy=(0.5, 0.25), xytext=(0.5, 0.61),
                    arrowprops=dict(arrowstyle="-|>", mutation_scale=10))

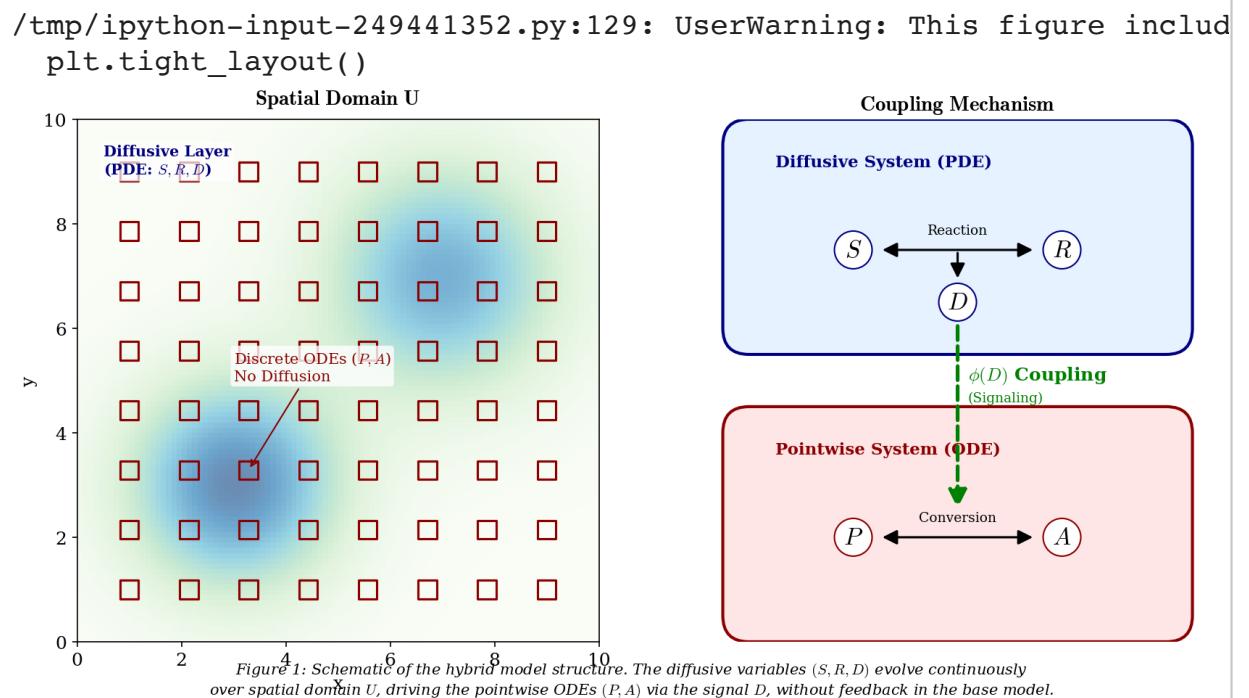
    # 箭头标签
    ax_mech.text(0.52, 0.5, r"$\phi(D)$ Coupling", color='green', fontweight='bold')
    ax_mech.text(0.52, 0.46, "(Signaling)", color='green', fontsize=8)

    # 添加说明文字
    fig.text(0.5, 0.02, "Figure 1: Schematic of the hybrid model structure", ha='center', style='italic')
    fig.text(0.5, 0.01, "over spatial domain $U$, driving the pointwise ODEs $(P, A)$", ha='center', style='italic')
    fig.text(0.5, 0.005, "via the signal $D$ without feedback in the base model.", ha='center', style='italic')

    plt.tight_layout()
    plt.show()

# 运行绘图
if __name__ == "__main__":
    draw_schematic()

```



```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import odeint

# --- 1. 参数设置 (Model Parameters) ---
# 选择一组能够清晰展示动力学的参数
# 确保 alpha, xi > 0 且 K > 0
K = 10.0          # carrying capacity
alpha = 0.6        # transfer rate S -> R
xi = 0.4          # transfer rate R -> S
lambda_S = 1.5    # growth rate of S
lambda_R = 1.0    # growth rate of R

# 计算理论平衡点 (Theorem 3.3)
S_star = (xi * K) / (alpha + xi)
R_star = (alpha * K) / (alpha + xi)
K_star = (S_star, R_star)

# --- 2. 定义 ODE 系统 ---
def reaction_diffusion_ode(state, t):
    S, R = state
    # 逻辑斯蒂项 (S+R 对 K 的竞争)
    logistic = 1 - (S + R) / K
```

```

# 动力学方程
dS = lambda_S * S * logistic - alpha * S + xi * R
dR = lambda_R * R * logistic + alpha * S - xi * R
return [dS, dR]

# 定义用于绘制向量场和 Nullclines 的网格函数
def get_derivatives(S_grid, R_grid):
    logistic = 1 - (S_grid + R_grid) / K
    dS = lambda_S * S_grid * logistic - alpha * S_grid + xi * R_grid
    dR = lambda_R * R_grid * logistic + alpha * S_grid - xi * R_grid
    return dS, dR

# --- 3. 绘图设置 ---
plt.figure(figsize=(8, 7), dpi=150)

# 定义绘图范围
max_val = K * 1.4
S_range = np.linspace(0, max_val, 200)
R_range = np.linspace(0, max_val, 200)
SS, RR = np.meshgrid(S_range, R_range)

# 计算网格上的导数
DS, DR = get_derivatives(SS, RR)

# --- 4. 绘制 Nullclines (零增长线) ---
# 使用 contour 绘制  $\dot{S}=0$  和  $\dot{R}=0$  的等高线, 比解析解更稳健
# S-nullcline (蓝色虚线)
plt.contour(SS, RR, DS, levels=[0], colors='blue', linestyles='--',
# R-nullcline (红色虚线)
plt.contour(SS, RR, DR, levels=[0], colors='red', linestyles='--',

# 添加 Nullcline 标签 (手动添加图例更清晰)
plt.plot([], [], 'b--', label=r'$S$-nullcline ( $\dot{S}=0$ )')
plt.plot([], [], 'r--', label=r'$R$-nullcline ( $\dot{R}=0$ )')

# --- 5. 绘制向量场 (Vector Field) ---
# 降低密度以保持清晰
step = 12
plt.quiver(SS[::step, ::step], RR[::step, ::step],
            DS[::step, ::step], DR[::step, ::step],
            color='gray', alpha=0.4, pivot='mid', scale=None)

# --- 6. 绘制轨迹 (Trajectories) ---
# 定义不同的初始条件, 覆盖第一象限的不同区域
initial_conditions = [
    [1, 1],          # 低密度
    [0.5, 8],        # R 占优
    [8, 0.5],        # S 占优
    [12, 12],        # 过饱和 (超过 K)
    [1, 12],         # 极端偏离
]

```

```

[12, 1]          # 极端偏离
]

t = np.linspace(0, 30, 500)

for i, y0 in enumerate(initial_conditions):
    sol = odeint(reaction_diffusion_ode, y0, t)
    # 画线
    plt.plot(sol[:, 0], sol[:, 1], 'k-', linewidth=1.2, alpha=0.8)
    # 画箭头指示方向 (在轨迹中间位置)
    mid_idx = 30
    plt.arrow(sol[mid_idx, 0], sol[mid_idx, 1],
              sol[mid_idx+1, 0]-sol[mid_idx, 0], sol[mid_idx+1, 1]-
              shape='full', lw=0, length_includes_head=True, head_w_
    # 标记起始点
    plt.plot(y0[0], y0[1], 'k.', markersize=5)

# --- 7. 绘制平衡点 ---
plt.plot(S_star, R_star, 'ko', markersize=8, markerfacecolor='gold'
         label=r'Equilibrium $\mathbb{K}^*$')

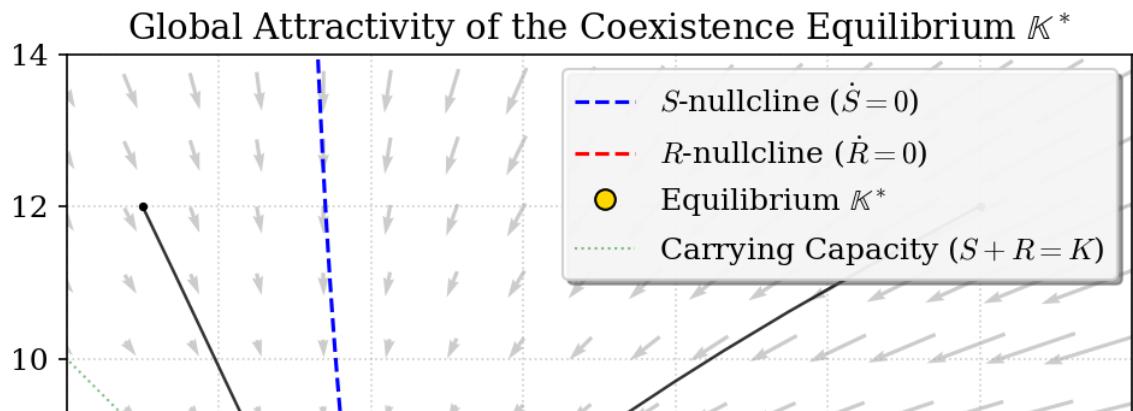
# 标记 Carrying Capacity 线 ( $S+R=K$ ) 辅助理解
# 在平衡点,  $S+R$  必须等于  $K$  (如果  $\alpha/\xi_i$  平衡) 或者由 dynamics 决定
# 在本模型中, 稳态必定满足  $S+R=K$  (Summing  $dS/dt$  and  $dR/dt$  shows  $S+R$  app
plt.plot([K, 0], [0, K], 'g:', linewidth=1, alpha=0.5, label=r'Carri

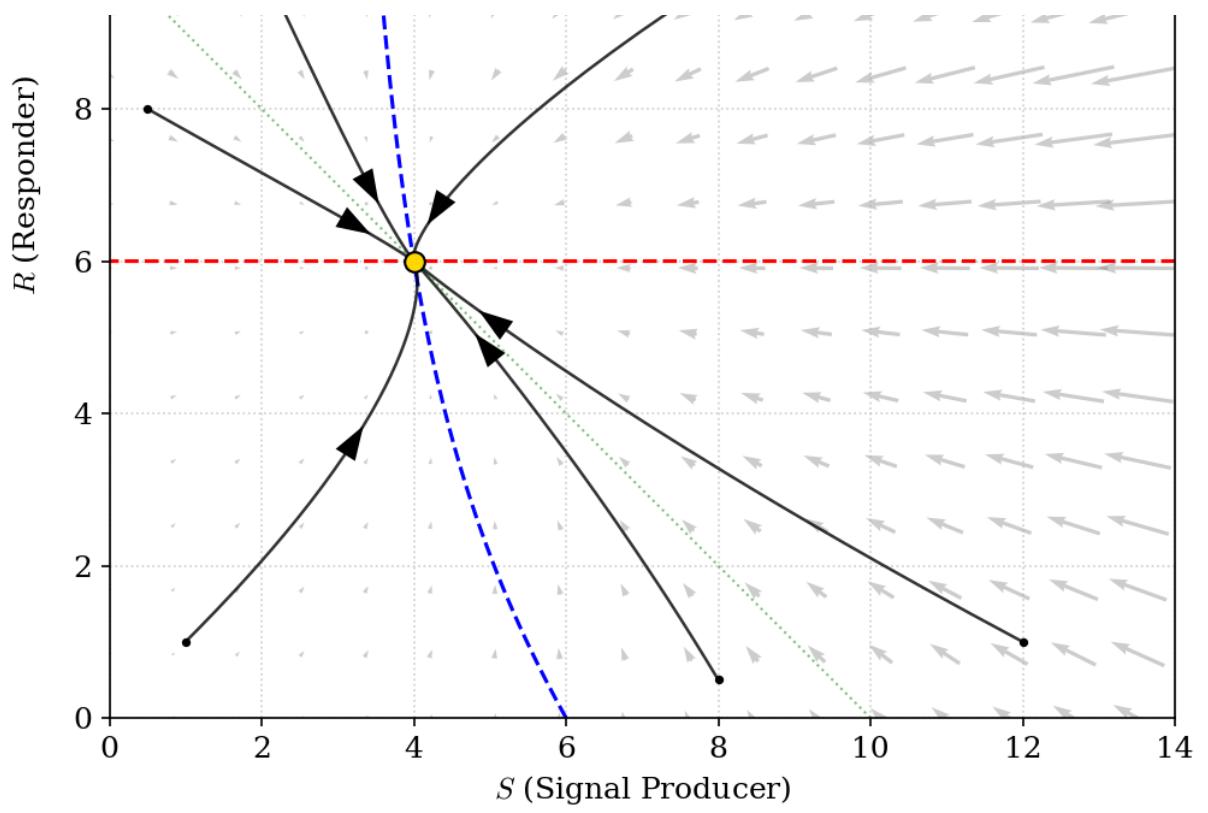
# --- 8. 图像美化 ---
plt.title(r'Global Attractivity of the Coexistence Equilibrium $\mathbb{K}^*$')
plt.xlabel(r'$S$ (Signal Producer)', fontsize=12)
plt.ylabel(r'$R$ (Responder)', fontsize=12)
plt.xlim(0, max_val)
plt.ylim(0, max_val)
plt.grid(True, linestyle=':', alpha=0.6)
plt.legend(loc='upper right', framealpha=0.9, shadow=True)

# 强制坐标轴比例一致, 避免相图扭曲
plt.gca().set_aspect('equal', adjustable='box')

plt.tight_layout()
plt.show()

```





```

import numpy as np
import matplotlib.pyplot as plt

# --- 1. 设置绘图风格 ---
plt.rcParams.update({
    "font.family": "serif",
    "font.size": 12,
    "axes.linewidth": 1.2,
}

```

```

        "xtick.direction": "in",
        "ytick.direction": "in"
    })

def calculate_dispersion_relation():
    # --- 2. 模型参数 (Base Model) ---
    # 使用与 Figure 2 一致的参数
    K = 10.0
    alpha = 0.6
    xi = 0.4
    lambda_S = 1.5
    lambda_R = 1.0

    # 基础模型的扩散系数 (任意 d_S, d_R > 0 都应稳定)
    d_S_base = 1.0
    d_R_base = 1.0

    # 计算稳态 S*, R*
    S_star = (xi * K) / (alpha + xi)
    R_star = (alpha * K) / (alpha + xi)

    # --- 3. 计算雅可比矩阵 (Jacobian at Equilibrium) ---
    # J = [[a, b], [c, d]]
    # 注意: 在稳态处 S+R=K, 因此 logistic 项 (1-(S+R)/K) 为 0, 但其导数不
    # J11 = d(dS/dt)/dS = lambda_S(1-(S+R)/K) + S*(-lambda_S/K) - a

    j11 = -lambda_S * S_star / K - alpha
    j12 = -lambda_S * S_star / K + xi
    j21 = -lambda_R * R_star / K + alpha
    j22 = -lambda_R * R_star / K - xi

    # Base Model 的雅可比矩阵
    J_base = np.array([[j11, j12],
                      [j21, j22]])

    # --- 4. 生成波数范围 ---
    k_vals = np.linspace(0, 6, 200) # 波数 k

    # --- 5. 定义计算最大特征值的函数 ---
    def get_max_eigenvalue(k_array, d_S, d_R, J):
        re_lambdas = []
        for k in k_array:
            k2 = k**2
            # 特征方程: det(lambda*I - (J - k^2*D)) = 0
            # 形式: lambda^2 + A*lambda + B = 0

            # Trace of (J - k^2*D)
            tr = np.trace(J) - k2*(d_S + d_R)

            # Det of (J - k^2*D)
            # M = [[J11-d_S*k2, J12], [J21, J22-d_R*k2]]

```

```

det = (J[0,0] - d_S*k2) * (J[1,1] - d_R*k2) - J[0,1] *

# 求解二次方程
delta = tr**2 - 4*det

if delta >= 0:
    l1 = (tr + np.sqrt(delta)) / 2
    l2 = (tr - np.sqrt(delta)) / 2
    re_lambdas.append(max(l1, l2))
else:
    # 复根, 实部为 tr/2
    re_lambdas.append(tr / 2)
return np.array(re_lambdas)

# 计算 Base Model 曲线 (Solid Line)
lambda_base = get_max_eigenvalue(k_vals, d_S_base, d_R_base, J_)

# --- 6. 构建“不稳定性”对比案例 (Comparison Case) ---
# 为了展示图灵斑图, 我们需要制造 instability:
# 条件1: 抑制剂扩散快 ( $d_R \gg d_S$ )
# 条件2: 激活剂自催化 ( $J_{11} > 0$ )
# 我们这里手动修改参数来模拟 Section 4 可能出现的 Chemotaxis 导致的不稳
d_S_unstable = 0.1
d_R_unstable = 4.0

# 修改雅可比矩阵以模拟 Activator-Inhibitor (仅用于对比展示)
J_unstable = np.array([[0.5, -0.6],
                      [0.6, -0.7]])

lambda_chemotaxis = get_max_eigenvalue(k_vals, d_S_unstable, d_)

# --- 7. 绘图 ---
fig, ax = plt.subplots(figsize=(8, 5), dpi=150)

# 绘制零线 (Critical Threshold)
ax.axhline(0, color='black', linewidth=1, linestyle='-', alpha=1)

# 区域填充 (Unstable Region)
ax.fill_between(k_vals, 0, 2, color='#ffe6e6', alpha=0.5, label='Stable')
ax.fill_between(k_vals, -5, 0, color='#e6ffe6', alpha=0.3, label='Unstable')

# 绘制曲线
# 1. Base Model
ax.plot(k_vals, lambda_base, 'b-', linewidth=2.5,
        label=r'Base Model (Thm 3.4): Stable $\forall k$')

# 2. Chemotaxis Comparison
ax.plot(k_vals, lambda_chemotaxis, 'r--', linewidth=2, dashes=(4, 4),
        label=r'Comparison (Chemotaxis): Turing Instability')

# 标注

```

```

    ax.set_xlabel(r'Wavenumber $k$', fontsize=13)
    ax.set_ylabel(r'Max Real Eigenvalue $\mathrm{Re}(\lambda)$', fc
    ax.set_title(r'Dispersion Relation: Linear Stability Analysis',
                 weight='bold', size=14)

    # 范围限制
    ax.set_ylim(-3, 1.0)
    ax.set_xlim(0, 6)

    # 图例
    ax.legend(loc='lower right', frameon=True, fancybox=False, edge

    # 关键点标注 (Arrow)
    # 找出不稳定曲线的最大值
    max_idx = np.argmax(lambda_chemotaxis)
    max_k = k_vals[max_idx]
    max_val = lambda_chemotaxis[max_idx]

    if max_val > 0:
        ax.annotate('Pattern Formation\n(Unstable Modes)',
                    xy=(max_k, max_val), xytext=(max_k+1, max_val+0.1),
                    arrowprops=dict(arrowstyle='->', color='red'),
                    color='red', fontsize=10, fontweight='bold')

    plt.tight_layout()
    plt.grid(True, linestyle=':', alpha=0.5)
    plt.show()

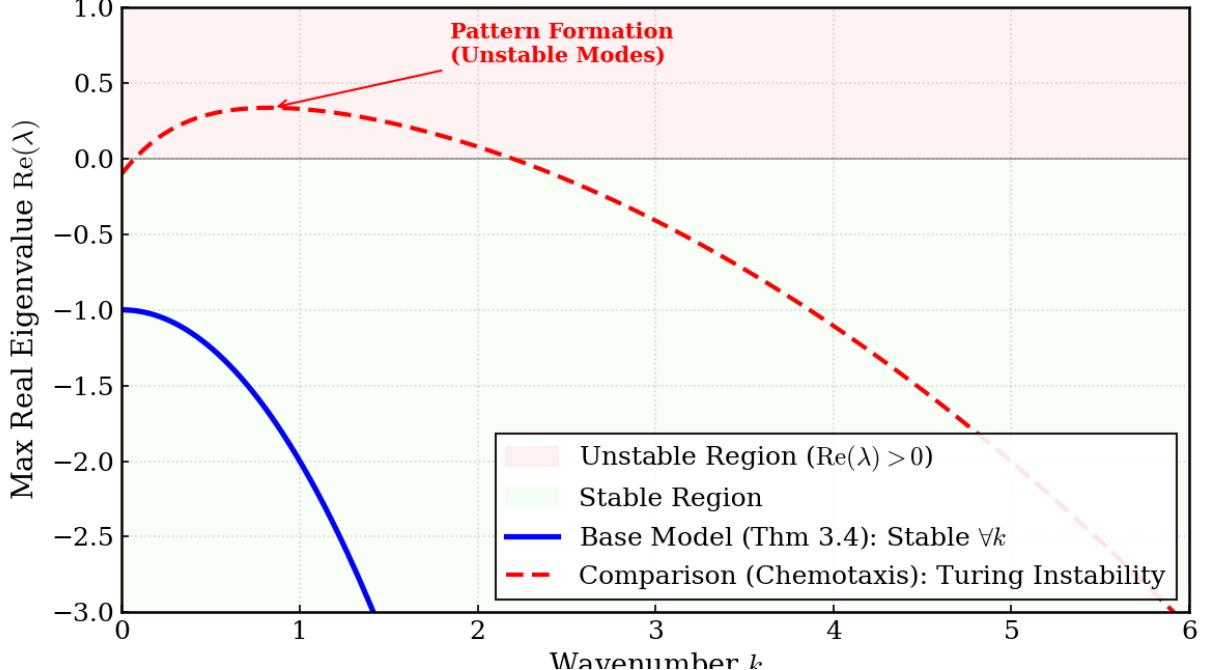
if __name__ == "__main__":
    calculate_dispersion_relation()

```

```

<>:98: SyntaxWarning: invalid escape sequence '\m'
<>:98: SyntaxWarning: invalid escape sequence '\m'
/tmp/ipython-input-1048494908.py:98: SyntaxWarning: invalid escape s
    ax.fill_between(k_vals, 0, 2, color='#ffe6e6', alpha=0.5, label='U
    Dispersion Relation: Linear Stability Analysis

```



```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import solve_ivp

# --- 1. 设置绘图参数 ---
plt.rcParams.update({
    "font.family": "serif",
    "font.size": 12,
    "axes.titlesize": 14,
    "axes.labelsize": 12
})

# --- 2. 模型参数 ---
L = 20.0          # 空间长度
N = 100           # 空间网格点数
dx = L / N
x = np.linspace(0, L, N)

# 时间跨度
T_end = 15.0
t_eval = np.linspace(0, T_end, 200)

# 物理参数
d_S = 1.0          # S 的扩散系数
d_R = 1.0          # R 的扩散系数
d_D = 2.0          # D 的扩散系数 (通常信号分子扩散较快)
mu_D = 0.5         # D 的自然衰减率 (导致 D -> 0)

# 反应速率
```

```

K = 5.0           # 承载量
alpha = 0.5       # S -> R
xi = 0.5          # R -> S
beta = 1.0         # P/A 转化率
theta = 1.0        # P/A 转化率

# 信号函数 phi(D) = D / (1+D)
def phi(D_val):
    return D_val / (1.0 + D_val + 1e-6) # 1e-6 防止分母为0

# --- 3. 定义 PDE 系统 (离散化) ---
def pde_system(t, y):
    # y 是一个展平的一维数组, 包含 [S, R, D, P, A]
    # 拆分数组
    S = y[0:N]
    R = y[N:2*N]
    D = y[2*N:3*N]
    P = y[3*N:4*N]
    A = y[4*N:5*N]

    # 1. 计算拉普拉斯算子 (离散二阶导数)
    # 使用 Neumann 边界条件 (零通量: u[-1]=u[1])
    def laplacian(u):
        d2u = np.zeros_like(u)
        d2u[1:-1] = (u[2:] - 2*u[1:-1] + u[:-2]) / dx**2
        d2u[0]     = 2*(u[1] - u[0]) / dx**2 # Left BC
        d2u[-1]    = 2*(u[-2] - u[-1]) / dx**2 # Right BC
        return d2u

    Delta_S = laplacian(S)
    Delta_R = laplacian(R)
    Delta_D = laplacian(D)

    # 2. 计算反应项
    # Phi(D) 耦合
    phi_D = phi(D)

    # Logistic term
    logistic = 1.0 - (S + R) / K

    # Dynamics for S, R (Base model structure)
    # 假设 D 影响 R->S 的转化率 (或类似机制)
    dS_dt = d_S * Delta_S + S * logistic - alpha * S + xi * (1 - phi_D) * D
    dR_dt = d_R * Delta_R + R * logistic + alpha * S - xi * (1 - phi_D) * D

    # Dynamics for D (Decay assumption from Section 3.3)
    # 假设 D 有初始分布但逐渐衰减 (No production or production < decay)
    dD_dt = d_D * Delta_D - mu_D * D

    # Dynamics for P, A (ODE pointwise, Driven by D)
    dP_dt = beta * (1 - phi_D) * A - theta * phi_D * P

```

```

dA_dt = -dP_dt # 显式保证守恒结构

# 重新拼接
return np.concatenate([dS_dt, dR_dt, dD_dt, dP_dt, dA_dt])

# --- 4. 初始条件 (Initial Conditions) ---
np.random.seed(42)

# S, R: 稳态附近的随机扰动
S0 = 2.5 * np.ones(N) + 0.5 * np.random.randn(N)
R0 = 2.5 * np.ones(N) + 0.5 * np.random.randn(N)

# D: 初始高斯脉冲 (模拟局部信号释放)
D0 = 3.0 * np.exp(-(x - L/2)**2 / 2.0)

# P, A: 随机分布
P0 = 1.0 + 0.2 * np.random.rand(N)
A0 = 1.0 + 0.2 * np.random.rand(N)

# 确保正性 (截断负值, 虽然后续演化会保持正性)
y0 = np.concatenate([np.abs(S0), np.abs(R0), np.abs(D0), np.abs(P0)])

# --- 5. 求解 ---
print("Running simulation (this may take a few seconds)...")
sol = solve_ivp(pde_system, [0, T_end], y0, t_eval=t_eval, method='RK45')
print("Done.")

# 提取结果
res = sol.y
S_res = res[0:N, :]
R_res = res[N:2*N, :]
D_res = res[2*N:3*N, :]
P_res = res[3*N:4*N, :]
A_res = res[4*N:5*N, :]
Total_PA = P_res + A_res

# --- 6. 绘图 (Waterfall / Heatmaps) ---
fig, axes = plt.subplots(3, 1, figsize=(10, 10), sharex=True)

# Plot 1: S(x,t) - Smoothing Effect
# 使用 'viridis' 或 'plasma'
im1 = axes[0].imshow(S_res, aspect='auto', origin='lower',
                      extent=[0, T_end, 0, L], cmap='YlGnBu')
axes[0].set_ylabel('Space $x$')
axes[0].set_title(r'(a) Smoothing of $S(x,t)$: Verification of Stability')
cb1 = plt.colorbar(im1, ax=axes[0])
cb1.set_label('Concentration $S$')

# Plot 2: D(x,t) - Decay Effect
im2 = axes[1].imshow(D_res, aspect='auto', origin='lower',
                      extent=[0, T_end, 0, L], cmap='Reds')

```

```

axes[1].set_ylabel('Space $x$')
axes[1].set_title(r'(b) Decay of Signal $D(x,t)$: Assumption for ODE Reduction')
cb2 = plt.colorbar(im2, ax=axes[1])
cb2.set_label('Concentration $D$')

# Plot 3: P(x,t) + A(x,t) – Conservation
# 这个图应该是垂直的条纹, 颜色随时间不变
im3 = axes[2].imshow(Total_PA, aspect='auto', origin='lower',
                      extent=[0, T_end, 0, L], cmap='coolwarm')
axes[2].set_ylabel('Space $x$')
axes[2].set_xlabel('Time $t$')
axes[2].set_title(r'(c) Pointwise Conservation of $P(x,t)+A(x,t)$')
cb3 = plt.colorbar(im3, ax=axes[2])
cb3.set_label('Sum $P+A$')

# 添加文字说明
plt.figtext(0.5, 0.02, "Figure 4: Spatio-temporal evolution. (a) Diffusion smooths out initial perturbations in S. (b) The signal D decays exponentially to zero. (c) The sum P+A remains constant in time at each spatial point.", ha='center', fontsize=10, style='italic')

plt.tight_layout(rect=[0, 0.05, 1, 1]) # 为底部的文字留出空间
plt.show()

```

Running simulation (this may take a few seconds)...

Done.

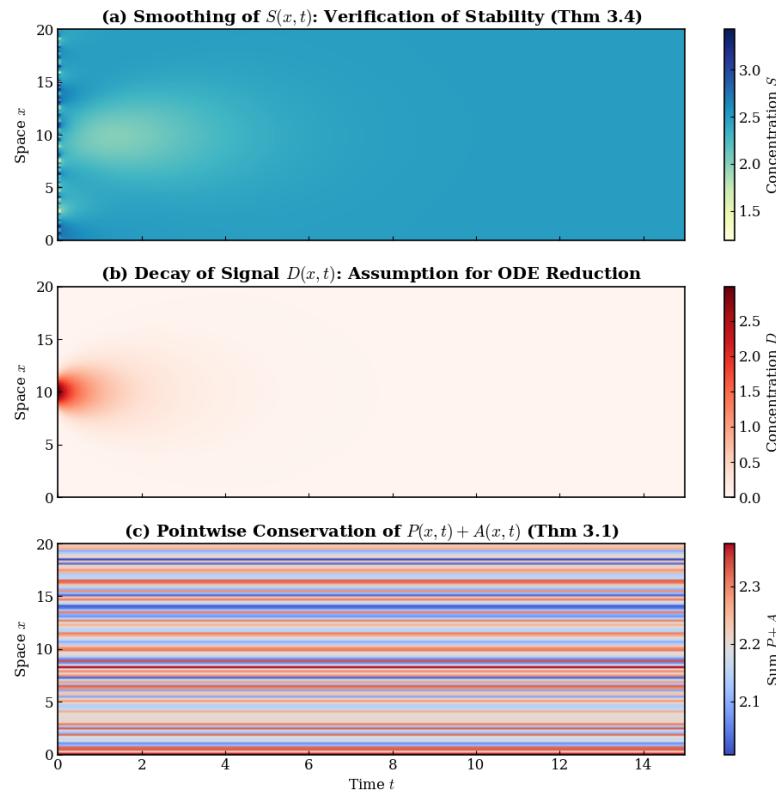


Figure 4: Spatio-temporal evolution. (a) Diffusion smooths out initial perturbations in S . (b) The signal D decays exponentially to zero. (c) The sum $P+A$ remains constant in time at each spatial point.

