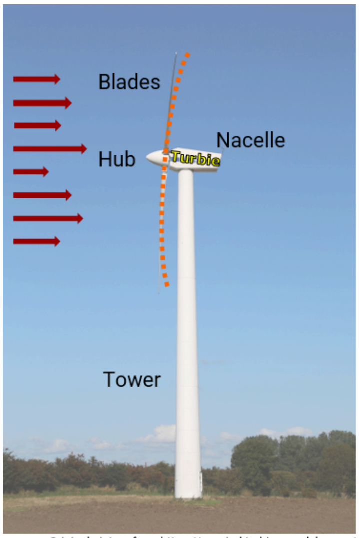
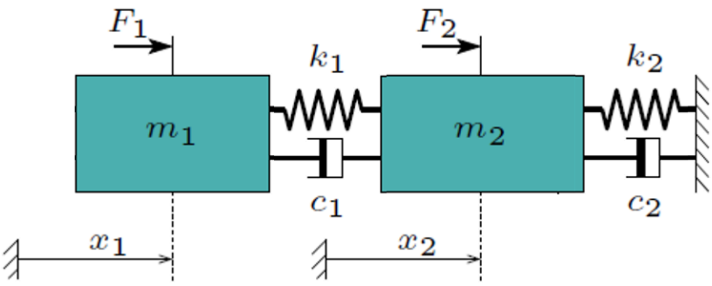
Definition

Turbie is a simple, two-degree-of-freedom (2DOF) system based of the DTU 10 MW Reference Wind Turbine. She is equivalent to the forced 2DOF mass-spring-damper system shown below, which is defined by a mass matrix, stiffness matrix, and damping matrix.



Original picture from https://en.wind-turbine-models.com/



Original sketch from https://i.stack.imgur.com/fRMco.png

Here are two files that you can download with parameters for Turbie: download turbie_parameters.txt and download CT.txt

Mass, stiffness and damping matrices

To derive Turbie's mass, stiffness and damping matrices, we make a few assumptions about our dynamical system:

- The turbine can only move in the fore-aft direction;
- The 3 blade deflections in the fore-aft direction are syncronized, i.e., only collective flapwise deflections.

With these assumptions, we have reduced Turbie to two degrees of freedom: the deflection of the blades in the fore-aft direction and the deflection of the nacelle in the fore-aft direction. The 2 DOFs of the system are therefore defined as

- $x_1(t)$: the deflection of the blades from their undeflected position in the global coordinate system;
- $x_2(t)$: the deflection of the nacelle from its undeflected position in the global coordinate system.

With these DOFs, Turbie is equivalent to a 2DOF mass-spring-damper system, as shown above, where Mass 1 represents the 3 blades and Mass 2 represents the combined effects of the nacelle, hub and tower.

Exercise for the reader! Given the diagram of the 2DOF mass, spring and damper system, derive the equations of motion and give the mass, stiffness and damping matrices.

Answer. The 2DOF mass-spring-damper system has the following system matrices:

$$[M] = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$$

$$[C] = \begin{bmatrix} c_1 & -c_1 \\ -c_1 & c_1 + c_2 \end{bmatrix}$$

$$[K] = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 + k_2 \end{bmatrix}$$

Here is a table of relevant parameter values for Turbie,

Symbol	**Decription**	**Value**
m_b	Mass of a single blade	41 metric tons
m_n	Mass of the nacelle	446 metric tons
m_t	Mass of the tower	628 metric tons
m_h	Mass of the hub	105 metric tons
c_1	Equivalent damping of the blades	4208 N/(m/s)
c_2	Equivalent damping of the nacelle, hub and tower	12730 N/(m/s)
k_1	Equivalent stiffness of the blades	1711000 N/m
k_2	Equivalent stiffness of the nacelle, hub and tower	3278000 N/m
D_{rotor}	Rotor diameter	180 m
ρ	Air density	1.22 kg/m^3

and here is the thrust-coefficient look-up table:

Wind speed (m/s)	**CT (-)**
4.0	0.923
5.0	0.919
6.0	0.904
7.0	0.858

Wind speed (m/s)	CT (-)
8.0	0.814
9.0	0.814
10.0	0.814
11.0	0.814
12.0	0.577
13.0	0.419
14.0	0.323
15.0	0.259
16.0	0.211
17.0	0.175
18.0	0.148
19.0	0.126
20.0	0.109
21.0	0.095
22.0	0.084
23.0	0.074
24.0	0.066
25.0	0.059

Dynamical equations

A wind turbine is, as you might expect, forced by the wind. To accurately model aerodynamics, you should include extra time-dependent variables to include phenomena such as dynamic stall, tower shadow, variable turbine speed, etc. For simplicity, we make the following assumptions:

- No dynamic inflow, dynamic stall, or tower shadow;
- Turbie's thrust coefficient is constant for a simulation and can be calculated from the mean wind speed;
- The only aerodynamic forcing is on the blades;
- No spatial variation of turbulence.

With these assumptions, the aerodynamic forcing on the blades is given by

$$f_{aero}(t) = \frac{1}{2}\,\rho\, C_T\, A\,(u(t) - \dot{x}_1)\,|u(t) - \dot{x}_1|,$$

where A is the rotor area and $u(t)$ is the wind speed at time t. The thrust coefficient C_T is determined from the mean wind speed $U = \overline{u(t)}$ using the look-up table defined above.

The full dynamical equations for Turbie are then given by

$$[M]\ddot{\overline{x}}(t) + [C]\dot{\overline{x}}(t) + [K]\overline{x}(t) = \overline{F}(t)$$

where $\overline{x}(t) = [x_1(t), x_2(t)]^T$ is the state vector and the forcing vector is given by

$$\overline{F}(t) = \begin{bmatrix} f_{aero}(t) \\ 0 \end{bmatrix}.$$