Missing Data Reconstruction Using Gaussian Mixture Models for Fingerprint Images

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ABSTRACT

One of the most important areas in fingerprint biometrics is matching partial fingerprints in fingerprint databases. Recently, significant progress has been made in designing fingerprint identification systems for missing fingerprint information. However, the precise reconstruction of fingerprint images still remains challenging due to the complexity and the ill-posed nature of the problem. This paper presents an algorithm for reconstructing the missing data from the fingerprint image(s). Experimental results illustrate the performance of the proposed method. The offered system can be used for fingerprint image identification and can be automated, as well as extended to numerous other security applications such as postmortem fingerprints, forensic science, investigations, artificial intelligence, robotics, all-access control, and financial security, as well as for the verification of firearm purchasers, driver license applicants, etc.
Keywords: Fingerprints, Gaussian mixture models, regression trees, reconstruction, missing data**.**

1. INTRODUCTION

Methods for reconstruction missing data are important step in various scientific and engineering applications, including fingerprint identification. Ridge structure characteristics and their correlation are used for determining the how any fingerprint is different from other. Biometrics are used in modern security systems including access control [37][50][51] and bio cryptography[37] [52][53][54][55]. Wide range applications in fingerprint biometrics needs matching of partial fingerprints and filling out the missing data fingerprints in the database. In detecting this data, the major problems includes insufficient minutiae and other components like core and delta that needs to be improved [56][37]. Earlier, some methods to solve this partial fingerprint problem using core-based alignment techniques and maximum feature extraction methods before they are matched. The input fingerprint image is usually poor in quality that makes it difficult to reconstruct the ridge line .Even, noise and deformation play an important role in false ridge line structures. This makes it difficult to achieve reliable fingerprint ridge line structures from poor quality images. This problem is studied completely and is still worked upon to improve the quality of image [56][37].

There are two key missing data reconstruction methods: the deterministic method (missing data are interpolated deterministically) and probabilistic reconstruction method (missing data are interpolated probabilistically). The deterministic method includes spline interpolation method. The probabilistic method has been developed in the last few years. The probabilistic interpolation method includes the inpainting procedure.

In the state of art algorithms, in order to achieve the ridge line structures, the ridge line structure is first enhanced by applying filtering approach to the original images and generate lines through binarization methods and thinning processes. Some techniques extract the ridge lines directly from gray scale images. The enhancement method proposed by O'Gorman and Nickerson is based on the convolution of the image with a filtered image [41] [57]. Hong Lin and A.K. Jain proposed that Gabor functions that can be used in enhancing the fingerprint image by taking fingerprint ridge orientation and ridge frequency as filtering parameters[41] [58]. M. T. Leung[41] [59] implemented a neural network based approach that detects the minutiae by applying a multilayer perception method to analyze the output of a rank of Gabor's filters convolved with the gray scale image A Laplacian operator and a dynamic threshold were newly introduced by Moayer and Fu [60][41] which can applied in a binarization . Some skeletonization algorithms have been managed for generating a skeleton of the fingerprint ridge lines. Instead of using a generalized thinning method, the

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concept of interpolation has been implemented in reconstruction mechanism. Nowadays, Gaussian Mixture Model (GMM) is used in variety of applications. GMM is a probabilistic model that assumes all the data points are generated from a mixture of a finite number of Gaussian distributions with unknown parameters.
The paper proposes a new technique to reconstruct the ridge and minutiae line accurately in fingerprinting images. The method is based on combine Gaussian Mixture Models and regression trees concepts. We first formulate a GMM and then use this model to generate a regression trees. This approach does not change any of the original fingerprint gray scale and reconstructs the data without changing the components from the original pixel values. In addition, a feedback technique implemented in this algorithm which imputes all the missing data in a loop. The remainder of the paper is organized as follows. We discuss the introduction with a given state of art algorithms in Section 2, and present the reconstruction algorithm for a GMM in Section 3. Computer Simulation results are presented in Section 4. Section 5 concludes the paper.

2. STATE OF ART ALGORITHMS

This section shows the state of art algorithms that are already been developed for the reconstruction of missing data in the fingerprint images. It includes Improved Smooth Extension, Transform based reconstruction which majorly uses the discrete cosine transform, the various kinds of matching algorithm, partial differentiation equation using impainting techniques and the sample tracing algorithm Gaussian Mixture Models algorithm is the new algorithm that has been proposed in this algorithm.

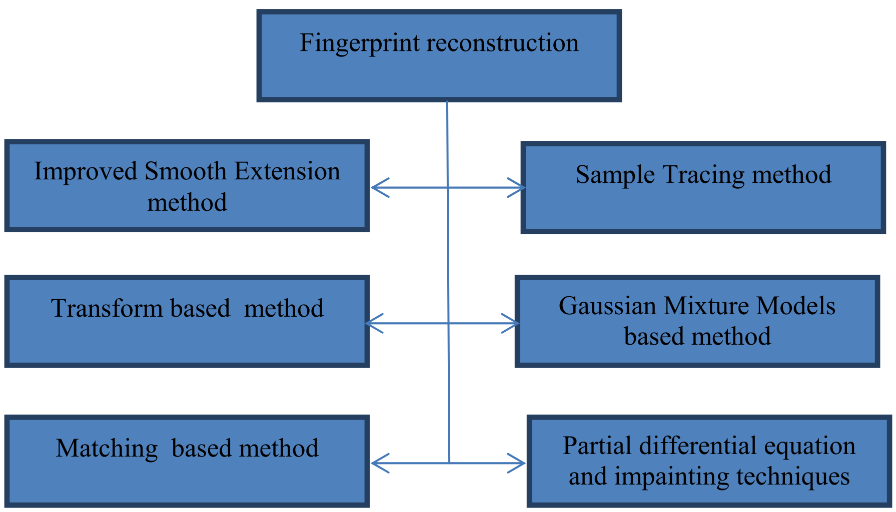

Figure 1: Figure shows various State of Art algorithms for fingerprint image reconstruction

Improved Smooth Extension method [37][44]: This algorithm is used it to reconstruct the missing data by an automated retrieval process using Fourier expansion model by training the phase data every time the coefficients are updated. It can be described as follows:

Transform based method [38][43]: It is used to reconstruct fingerprint orientation field by transforms such as weighted discrete cosine transform (DCT), wavelet transform by adjusting the basis function such as wavelets and sinusoids,
Matching based method [39][47][48]: In this algorithm, dictionary-based fingerprint reconstruction method is proposed which is mainly used reconstruct the minutiae of the orientation field and ridge patterns.
Partial differential equation and inpainting techniques [40][45][46]: Inpainting is the process in which we use information of the surrounding image to fill a part of an image or video. Partial differential equations are used to generating the parameters for the inpainting procedure.

- Ridge-lines are sharpened using automated techniques of diffusion filters. It allows more minutiae and improve latent fingerprint computer matching.

- Linked lists are used for generating a list of curve end points and bifurcations for minutiae.

Sample tracing method [41][49]: It uses the concept of ridge line modelling to reconstruct the missing data which includes. It uses the aspects like continuity and correlation to track the sample of the ridges**.**

- The fingerprint ridge line is assumed as a track of a ridge segment moving along the ridge. It uses the target tracking technique in computer vision to solve this problem
- A new model of fingerprint ridge line segments generated and then tracked along the ridges. To improve the accuracy a feedback technique is used.
- All the generated ridge segments are connected and the missing data is reconstructed in a polyline format.

3. PROPOSED ALGORITHM

In this section, we describe the process of generating models of Gaussian Mixtures which are used to duplicate the finger and then reconstruct the missing data. We present a new application of Gaussian Mixture Models for Fingerprints Image Duplication and analysis.

A Gaussian mixture model is a probabilistic model that assumes all the data points are generated from a mixture of a finite number of Gaussian distributions with unknown parameters.

$$s(\mathbf{x}) = \sum_{k=1}^{K} \pi_i N_k(\mathbf{x}, \mu_k, \Sigma_k)$$

Where $N(x,\mu,\Sigma) = N(x,\theta) = \frac{1}{\Sigma_k \sqrt{2\pi}} e^{\frac{(x-\mu_k)^2}{2\Sigma_k^2}}$

$$\pi_i = \frac{\pi_k N(x \mid \mu_k, \Sigma_k)}{\sum_{j=1}^{k} \pi_j N(x \mid \mu_i \Sigma_i)}$$ where π_i are the weights

$N_k(\mathbf{x}, \mu_k, \Sigma_k)$ is a i-Gaussian distributions component of the mixture model with its own mean μ_k and variance shape Σ_k.

The duplicate fingerprint model of the given original models are obtained using the GMM's concept for duplication [42]

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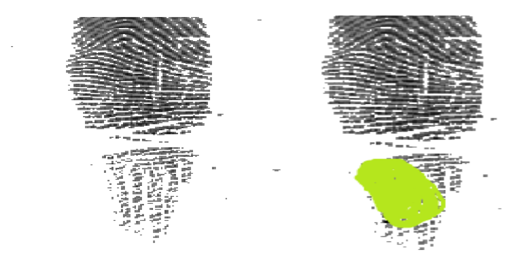
3.1 Proposed algorithm:

The proposed algorithm can be explained as follow step-wise

Step 1: Input a fingerprint image with missing data

Step 2: Duplicate fingerprint the image by applying a Gaussian mixture model [42]



Generated Model Shaded Model

Figure 2: Region of interest for the missing data reconstruction

Step 3: Determine the missing data and color it, for example using green color

Step 4: Estimate the statistical parameters of missing data by using the concept of regression trees.

Step 5: Generate the nodes for the regression tress by using EM algorithm.

The nodes for the regression tress can be found by estimation of new parameters [$S(X,Y \mid \pi)$, $Q(\pi,\pi^{new})$, μ_k^{new}, Σ_k^{new}] for regression trees.

Where S(x) is the Gaussian model . The Q-function is the expected value of the complete data log-likelihood S(X,Y|θ) with respect to Y given X and θ^{new} which can be calculated as follows:

$$Q(\pi,\pi^{new}) = E[\log S(X,Y \mid \pi) \mid X, \pi^{new}, \pi^{new}+1],$$

in which $Y(n) = \frac{\mu_{new}{}^{x_n} e^{-\mu}}{n!}$ [n is the total number of pixels]

$$\mu_k^{new} = \frac{1}{N_k}\sum_{i=1}^{n} \pi_i x_i \ ,$$

$$\Sigma_k^{new} = \frac{1}{N_k}\sum_{i=1}^{n} \pi_i \left(x_i - \mu_k^{new}\right) x_i \left(x_i - \mu_k^{new}\right) \ .$$

For these generated new parameters we are now able to determine the values for spline interpolation.

Step 6: Perform polynomial spline interpolation on this generated regression trees:

$$\frac{1}{x_n - x_{n-1}} k_{n-1} + \frac{2}{x_n - x_{n-1}} k_n = 3\frac{y_n - y_{n-1}}{(x_n - x_{n-1})^2}$$

Where k1 to k_n are the parameters that are generated by differentiating the GMM i.e. s(x) , x_n are the models of the original GMM and y_n are the parameters of the generated model for the missing data.

Step 7: Reconstructed image is obtained

The below table shows the values that are calculated for the GMM for image 1.When we keep the mean of the image model as constant and then find the interpolation constant k .This can be further used to calculate the parameters of the EM algorithm, Πnew(weights,)Σnew(variance) which is seen the table on the next page.

mean	covariance	K	x_n	x_{n-1}	Π	Πnew	y_{n-1}	y_n	Σ	Σnew
		K1	1.021	1	0.21	12.82	0.05	9.74	0.00	9.83
	c = 13	K2	1.024	6.16	0.22	12.82	2.17	13.00	0.34	13.00
		K3	0.32	13.00	0.00	6.38	0.05	7.51	0.00	6.47
		K1	1.021	1	0.22	13.89	0.05	7.07	0.00	6.04
$\mu_0 = 15$	c = 15	K2	1.027	7.44	0.23	13.88	2.20	14.98	0.34	14.99
		K3	0.24	15.00	0.00	7.08	0.13	8.87	0.00	7.47
		K1	1.021	1	0.21	13.21	0.05	3.66	0.00	1.83
	c = 18	K2	1.026	9.08	0.21	13.11	2.29	17.66	0.36	17.73
		K3	0.28	17.76	0.02	7.80	0.37	11.18	0.00	8.95
		K1	1.025	1	0.24	17.70	0.01	14.15	0.00	14.37
	c = 18	K2	1.027	8.88	0.24	17.70	1.78	18.00	0.33	18.00
		K3	0.36	18.00	0.00	8.86	0.00	9.55	0.00	8.94
		K1	1.084	1	0.23	19.06	0.01	10.11	0.00	8.99
$\mu_0 = 21$	c = 21	K2	1.045	10.32	0.23	19.04	1.80	20.97	0.33	20.98

		K3	0.24	20.96	0.00	9.96	0.03	11.46	0.00	10.47
	c = 24	K1	1.078	1	0.23	18.43	0.01	6.22	0.00	4.09
		K2	1.056	11.96	0.23	18.32	1.84	23.72	0.33	23.79
		K3	0.52	23.68	0.01	10.88	0.10	13.60	0.00	11.92
	c = 24	K1	1.021	1	0.24	23.88	0.00	22.48	0.00	22.81
		K2	1.064	12.20	0.24	23.88	1.22	24.00	0.32	24.00
		K3	0.40	24.00	0.00	11.92	0.00	11.69	0.00	11.93
$\mu_0 = 31$	c = 31	K1	1.021	1	0.24	27.52	0.00	15.32	0.00	14.01
		K2	1.027	15.28	0.24	27.46	1.23	30.95	0.31	30.97
		K3	0.20	31.00	0.00	14.85	0.00	15.72	0.00	15.43
	c = 38	K1	1.065	1	0.23	24.21	0.00	6.69	0.00	3.92
		K2	1.018	19.60	0.22	23.79	1.29	36.90	0.32	37.03

Table 1: Various parameters for GMM and interpolation method for reconstruction of missing data

4. COMPUTER SIMULATIONS

In this section we generate the GMMs for the reconstruction of missing data and study the differences between them using Images Similarity Measure[11]. There are many image similarity measures to compare two images [11,25-28]. In this article, we use Structural Similarity Image Measure [11].

$$SSIM = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)}$$

Where

$\mu_k^n = \frac{1}{N_k}\sum_{i=1}^{n} x_i$ is the mean of pixel values of the image model.

$\sigma_x = (\frac{1}{N-1}\sum_{i-1}^{N}(x_i - \mu_x)^2)^{0.5}$ is Standard deviation of pixel values of the image model

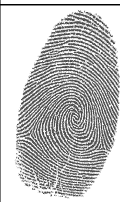
$\sigma_{xy} = \frac{(2\sigma_x\sigma_y + C_2)}{(\sigma_x^2 + \sigma_y^2 + C_2)}$ is the Luminance comparison of pixel values of the image model

The below table shows the comparison study of the various state of art algorithms and the proposed new algorithm to duplicate the image models. It we have generated a total of 10 image models including both state of art algorithms and newly developed algorithms. We use the SSIM index to compare the results of all the image models. We can see that the newly developed algorithm gives better results.

Table 2: The below table shows various GMMs generated [42]:

No.	Algorithm	Image1	Image2	Image3	Image4

1	Original Image	 Q=1	 Q=1	 Q=1	 Q=1
2	Gaussian Mixture Model	 Q=0.3411	 Q=0.3756	 Q=0.3445	 Q=0.3718
3	Generalized Gaussian Mixture Model	 Q=0.6475	 Q=0.6487	 Q=0.6437	 Q=0.6472
4	Generalized Gaussian Mixture Model using Bayesian learning	 Q=0.7692	 Q=0.7641	 Q=0.7664	 Q=0.7656
5	Texture Synthesis	 Q=0.5486	 Q=0.4474	 Q=0.5479	 Q=0.4452
6	Genetic Algorithm	 Q=0.8612	 Q=0.8695	 Q=0.8671	 Q=0.8645

7	Improved Adaptive Algorithm	 Q=0.8154	 Q=0.8612	 Q=0.8173	 Q=0.8629
8	New Algorithm based on Generalized Gaussian and Finite Bayesian learning	 Q=0.8645	 Q=0.8748	 Q=0.8626	 Q=0.8738
9	Image Enhancement Algorithm for Gaussian Mixture Models and Finite Bayesian Model (GMMFB)	 Q=0.8745	 Q=0.8687	 Q=0.8756	 Q=0.8646
10	New Algorithm Based on Improved Adaptive, genetic Algorithm and Finite Bayesian learning (GMMFBE)	 Q=0.9279	 Q=0.9249	 Q=0.9274	 Q=0.9287

Now that we have obtained the Gaussian mixture Model for the above given input images. We consider the best image model as the input to regression trees to reconstruct the missing the data. We follow all steps that are mentioned in the proposed algorithm and impute the missing data. The below table shows the original image, the missing data image and the reconstructed image.

It includes the following:

1. Original image model(obtained for GMM)
2. Region interest(the shaded region that we need to reconstruct the data)
3. Reconstructed image

Table 3: Missing data reconstruction for the given input GMM

1.			
	Original Image Model 1	Original Image Model 2	Original Image Model 3
2.			
	Missing data Image 1	Missing data Image 2	Missing data Image 3
3.			
	Missing data reconstructed Image 1	Missing data reconstructed Image 2	Missing data reconstructed Image 3

Table 4: Differences between Original image and GMM

	Algorithm	Original Image	GMM	Differences
1.	Gaussian Mixture Model	 Q=1	 Q=0.3411	 Q=0.3229
2.	Generalized Gaussian Mixture Model	 Q=1	 Q=0.6475	 Q=0.3845

3.	Generalized Gaussian Mixture Model using Bayesian learning	 Q=1	 Q=0.7692	 Q=0.4589
4.	Texture Synthesis	 Q=1	 Q=0.5486	 Q=0.3149
5.	Genetic Algorithm	 Q=1	 Q=0.8612	 Q=0.4739
6.	Improved Adaptive Algorithm	 Q=1	 Q=0.8154	 Q=0.4589
7.	New Algorithm based on Generalized Gaussian and Finite Bayesian learning	 Q=1	 Q=0.8645	 Q=0.4699
8.	Image Enhancement Algorithm for Gaussian Mixture Models and Finite Bayesian Model	 Q=1	 Q=0.8745	 Q=0.2697

9.	New Algorithm Based on Improved Adaptive, genetic Algorithm and Finite Bayesian learning	 Q=1	 Q=9279	 Q=0.2271

Below we are comparing our results with the various state of art algorithms and newly generated algorithms for missing data reconstruction. We compare the results using the SSIM index, Entropy and the mean square error.

Table 5: Comparison of models using various measures

Sr. No.	Algorithm	SSIM Index				Entropy			
	Original Image	1	1	1	1	845.74	756.25	894.26	486.84
1	Gaussian Mixture Models, (Ming-Hsuan Yang,1999)	0.3411	0.3756	0.3445	0.3411	255.21	178.74	259	963.17
2	Generalized Gaussian Mixture Models, (Tee-Won Lee,2005)	0.6475	0.6487	0.6473	0.6475	454.12	369.76	454.12	245.21
3	Finite Bayesian learning for Gaussian Mixture Models. Nicola Greggio,2010	0.7692	0.7641	0.7671	0.7678	645.17	487.32	461.7	463.12
4	Texture Synthesis, Andrea Rau,2010	0.5496	0.4474	0.5463	0.5456	245.12	278.96	269.12	663.17
5	Improved Adaptive Algorithm, Vahid Majidnezhad,2013	0.8692	0.8294	0.8664	0.8694	687.32	475.32	671.32	271.12
6	Genetic Algorithm, Vahid Majidnezhad,2013	0.8397	0.8695	0.8356	0.8342	643.21	574.39	618.21	644.32
7	GMMBF	0.8645	0.8675	0.8663	0.8676	685.12	573.12	693.12	647.21
8	GMMBFE	0.9249	0.9252	0.9259	0.9296	723.69	627.12	769.69	697.12
9	Regression Trees	0.9456	0.9377	0.9351	0.9412	684.855	564.29	631.28	453.54
10	GMM Regression	0.9356	0.9325	0.9245	0.9307	543.55	566.86	586.22	412.93

Table 6: Comparison of models using various measures

Sr. No.	Algorithm	Mean square error			
	Original Image	0	0	0	0
1	Gaussian Mixture Models, (Ming-Hsuan Yang,1999)	875.23	841.23	818.23	863.23
2	Generalized Gaussian Mixture Models, (Tee-Won Lee,2005)	797.39	752.39	753.39	755.39
3	Finite Bayesian learning for Gaussian Mixture Models. Nicola Greggio,2010	744.43	737.43	797.43	771.43

4	Texture Synthesis, Andrea Rau,2010	612.48	656.48	674.48	656.48
5	Improved Adaptive Algorithm, Vahid Majidnezhad,2013	455.78	494.78	486.78	479.78
6	Genetic Algorithm, Vahid 2013	574.53	571.53	543.53	578.53
7	GMMBF	318.54	369.54	396.54	394.54
8	GMMFBE	219.64	245.64	275.64	222.64
9	Regression Trees	354.94	235.69	214.69	734.89
10	GMM Regression	148.55	258.55	632.96	654.07

The above tables show the comparison of the reconstructed images based on GMM using SSIM index, Entropy and mean square error. The computer simulation results show that the developed method has the following advantages:

1) It provides an overall reconstruction of missing data more accurately and best fits the original degraded image.
2) It tracks the ridges more efficiently than any matching or smoothing technique.
3) It detects the minutiae more smoothly.

5. CONCLUSION

We have developed a combination of Gaussian Mixture Models, Regression trees and spline interpolation based method for reconstruction of missing data in fingerprints images and its analysis. A prominent advantage of the proposed approach gives a more precise ridge line model without changing the components of the original image. We get the best fit statistical model for the finger print images which can be further used for determining ridge line structure and minutiae. The simulation result demonstrates that the proposed algorithm is capable of generating sharp clean ridges while keeping the actual structure of ridge endings. We obtain the best results for the GMM reconstruction as compared to the state of art algorithm Zhou Wei 37], Liu Manhua [38], Cao Kai.[39] , Rahmes Mark [40], Qi Yaxuan[41] which gives the best SSIM index about 0.9345, Entropy as 543.55 ,Mean Square Error as 148.55. The proposed system can be used for fingerprint image identification and can be made automated and extended to numerous other security application such as postmortems, forensics, investigation of crimes, artificial intelligence, robotics, as access control, financial security, and verification of firearm purchasers and driver license applicants etc.

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