Geometry Processing (Labs 6 - 8)

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ABSTRACT

This set of labs covered Voronoi diagrams, Power Diagrams, semi-optimal transport, and simulating fluid dynamics using a mix of the previous labs.

Lab 6

The sixth lab was focused on ultimately being construct **Voronoi diagrams**. Intuitively, what we are doing is taking a set of points $\mathscr{D} = \{x_1, \cdots, x_n\}$ and assigning a convex polygon to each x_i so that for every point in the polygon, the closest point in $\mathscr{D}$ is x_i. To do this, I implemented the **Voronoï Parallel Linear Enumeration** algorithm, which relies on the **Sutherland-Hodgman polygon clipping** algorithm.

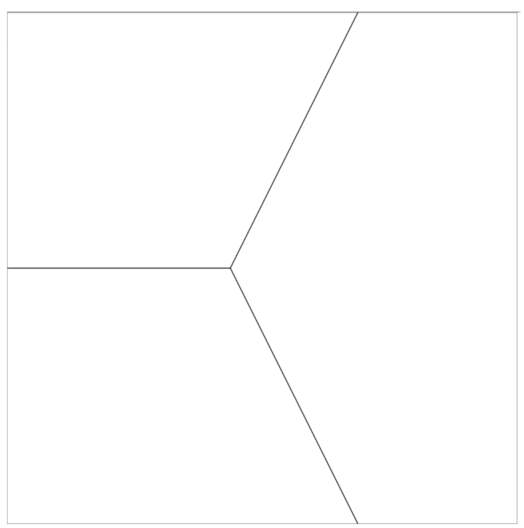

Fig. 1: Simple Voronoi diagram. $\mathscr{D} = \{(0,0)^T, (2,1)^T, (0,2)^T\}$. Running time: 2.6e-05 seconds

Figure 1 consists of a Voronoi diagram for a set of points $\mathscr{D} = \{(0,0)^T, (2,1)^T, (0,2)^T\}$

Lab 7

One of the first tasks of Lab 7 was to extend **Voronoi diagrams** to construct **Power Diagrams**, which add weights to control the size of each cell. In Fig 2, we see a Voronoi diagram for the dataset $\mathscr{D} = \{(i, j)^T\}_{1\leq i,j\leq 9}$. In Fig 3, we see the same dataset but with the added weights, $W = \{(i+1)(j+1)\}_{1\leq i,j\leq 9}$.

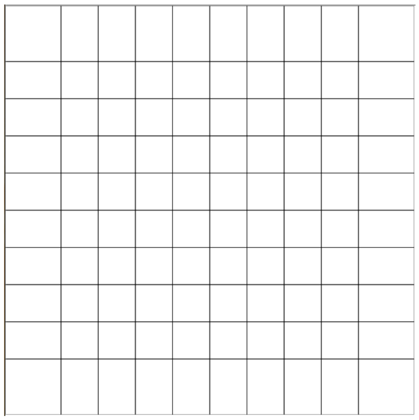

Fig. 2: Voronoi diagram for $\mathscr{D}$. Running time: 0.0068 seconds

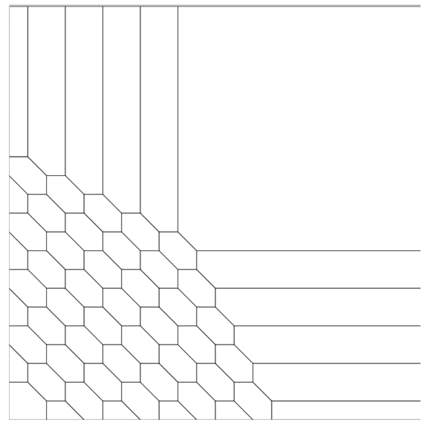

Fig. 3: Power diagram for $\mathscr{D}$ with weights W. Running time: 0.006 seconds

In the second part of Lab 7, I implemented an optimization of the weights using **semi-optimal transport**.

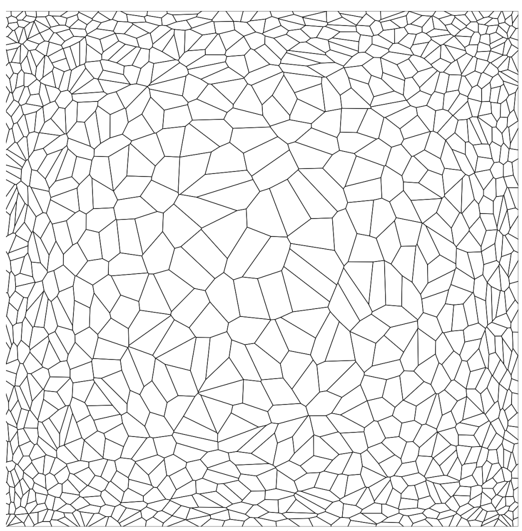

Fig. 4: Power-diagram optimized with semi-discrete optimal transport so that the cell associated to a site at position x_i has an area of $\exp(5||x_i - C||)$ where C is the center of this unit square (f = 1). Running time: 184 seconds

Lab 8

Lab 8 saw the use of the previous labs in order to simulate fluid dynamics. To do this, I implemented the Gallouet-Mérigot incompressible Euler scheme which enabled me to generate a simulation of falling water. Since we cannot include gifs in pdfs, figure 5 is one of the frames in the gif that you can find on my Github.

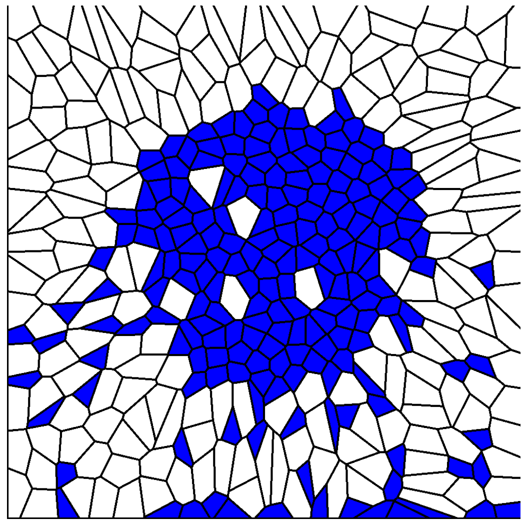

Fig. 5: One of the frames in the simulation. Cells containing water are in blue. 200 fluid and 200 air cells. Running time to create 20 frames: $\approx$ 5 minutes