Faculty of Science, Technology, Engineering and Mathematics
M347 Mathematical statistics

M347

TMA 01

Covers Block 1 **See module website for the cut-off date.**

Submitting your assignment

You can submit your TMA either by post or electronically using the online TMA/EMA service. Please read the guidance in the 'Assessment' area of the M347 website.

This TMA

This TMA covers Block 1 of M347. You will probably find it best to answer the questions associated with each unit soon after completing that unit, rather than waiting until you have completed the block.

Each TMA is marked out of 25, with the marks allocated to each part of each question indicated in brackets in the margin. Your overall score for the TMA will be the sum of your marks for all questions in the TMA, which is then converted into a percentage score. Your tutor's reply will give you feedback on how well you answer the TMA. Being a formative assessment, your mark on this TMA counts only towards achieving the required threshold for completion of assignments to be able to pass the module, but does not contribute to your final mark for the module.

Your work should only include your answers to the questions: do not include material which was not asked for in the questions. You should, however, include your working. If you don't, and you make a mistake, then your tutor will not be able to provide any feedback about where you went wrong. Leaving out your working may also cost you marks if it is specifically asked for.

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4.1

This assignment covers Block 1.

Question 1 (Unit 2) – 4 marks

Let X follow the distribution with moment generating function $M_X(t)$. Let $Y = aX + b$ follow the distribution with moment generating function $M_Y(t)$. Show that

$$M_Y(t) = e^{bt} M_X(at),$$

setting out each step in your argument. [4]

Question 2 (Unit 2) – 9 marks

(a) Suppose that X follows the Weibull distribution with pdf

$$f(x) = \beta x^{\beta-1} e^{-x^\beta} \quad \text{on } x > 0,$$

with $\beta > 0$. Show that

$$E(X^r) = \Gamma\left(\frac{r}{\beta} + 1\right).$$

(If you do this using integration by substitution, be sure to check what happens to the limits of integration.) [5]

(b) Using the result of part (a), calculate the numerical value of the variance of the Weibull distribution when $\beta = 1/2$. [4]

Question 3 (Unit 3) – 12 marks

Consider the bivariate distribution for (X, Y) with joint density

$$f(x, y) = \frac{\lambda y^2}{\sqrt{2\pi}} \exp\left\{-(\tfrac{1}{2} + \lambda x)y^2\right\} \quad \text{on } x > 0,\ y \in \mathbb{R}.$$

(a) Show that $f_X(x)$, the marginal density of X, is given by

$$f_X(x) = \frac{\lambda}{2\sqrt{2}(\frac{1}{2} + \lambda x)^{3/2}} \quad \text{on } x > 0.$$

Hints: First, use the fact that the integral over y from $-\infty$ to ∞ of an integrand which is a function solely of y^2 is twice the integral over y from 0 to ∞ of the same integrand (by a symmetry argument). Second, use the substitution $u = \left(\frac{1}{2} + \lambda x\right) y^2$, being sure to check what happens to the limits of integration. Additionally, note that the gamma function (Subsection 4.3 of Unit 2) is relevant. [7]

(b) Using the result of part (a), obtain the conditional density of $Y \mid X = x$, that is, $f_{Y|X}(y|x)$. Write your answer carefully in the form of a constant times the density core. [3]

(c) The joint density in the question was derived in Exercise 3.10(b) in Subsection 3.2 from the marginal distribution of Y and the conditional distribution of $X \mid Y = y$. Given just those marginal and conditional densities, what alternative approach could you have used to obtain the core of $f_{Y|X}(y|x)$, and why? [There is no need to carry out any further calculations.] [2]