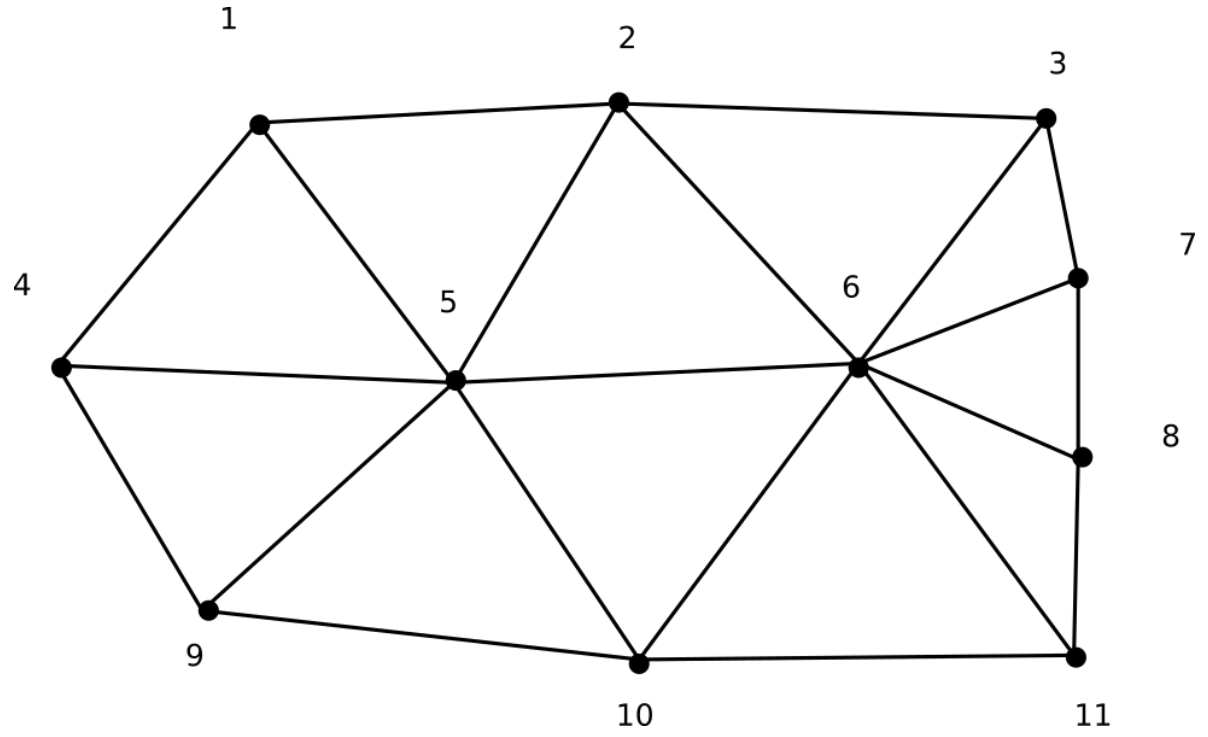
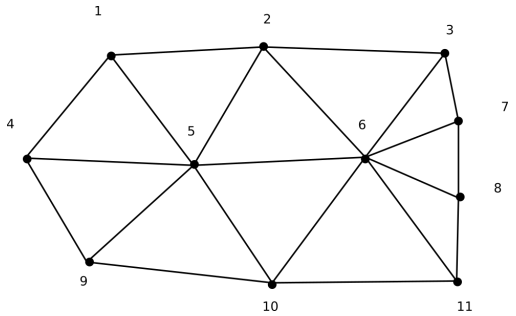
Example mesh

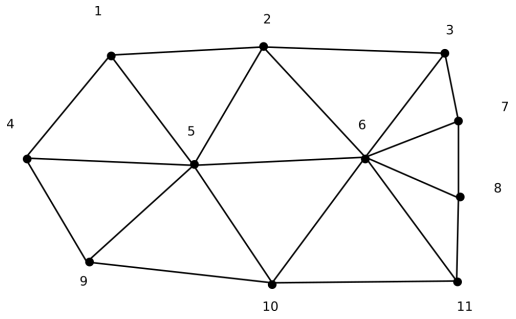


Problem



- Value of a scalar funciton at each node or vertex of the graph is denoted by f_i where i is the vertex number
- The value is a known quantity along the boundary
- The value is unknown at 5 and 6
- Values at 5 and 6 are to be determined by some mathematical relationship, in this case, “the value at each interior node is equal to the average value of its neighbors”

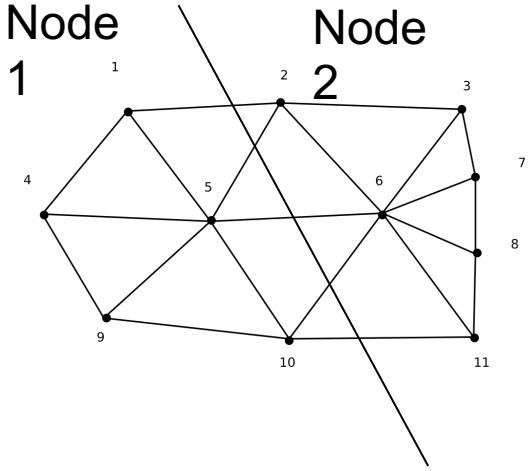
Problem



$$f_5 = (f_1 + f_2 + f_4 + f_6 + f_9 + f_10)/6$$
$$f_6 = (f_2 + f_3 + f_5 + f_7 + f_8 + f_10 + f_11)/7$$

$$\left(\begin{bmatrix} 6 & 0 \\ 0 & 7 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right) \begin{bmatrix} f_5 \\ f_6 \end{bmatrix} = \begin{bmatrix} f_1 + f_2 + f_4 + f_6 + f_9 + f_{10} \\ f_2 + f_3 + f_5 + f_7 + f_8 + f_{10} + f_{11}) \end{bmatrix}$$

Decomposition



$$\left(\begin{bmatrix} 6 & 0 \\ 0 & 7 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\right)\begin{bmatrix} f_5 \\ f_6 \end{bmatrix} = \begin{bmatrix} f_1 + f_2 + f_4 + f_6 + f_9 + f_{10} \\ f_2 + f_3 + f_5 + f_7 + f_8 + f_{10} + f_{11}) \end{bmatrix}$$