

Unveil the Mystery behind Musical Evolution

Summary

Throughout the history of music, **musical influence relationships** have played an imperative role in driving the *innovation* and *inheritance* of music. It is known that artists draw inspiration from important musical influencers when creating new masterpieces. Moreover, once new genres emerge, influence still plays an important role, given the diverse influence from multiple sources, leading to variation and novelty. While influencer analysis has witnessed tremendous interests in the recent decade, mainly driven by social networks and the desire to understand dynamic phenomena, the influence is not well understood.

In an attempt to contribute to the understanding of influencing processes in music, and towards resolving the mystery behind musical evolution and revolution, we build up a computational framework which is based on two recently-developed domains: *Network Science* and *Data Science*. The focus of our framework is on **the effect of musical influence on the temporal and spatial development of artist and genre**. The framework consists of multiple components, which are briefly reviewed here. At first, we address the challenge to identify true musical influence. Instead of using artists' interactions based on their average music characteristics, which are not necessarily representative for the artists' overall work, our major hypothesis is that artists are being followed for their most popular work. Accordingly, we redefine the characteristic data of a specific artist to be **the sum of musical characteristics of all music, weighted by the popularity of each music**. An **Artist network** based on the follower-relationship, which is found to exhibit **scale-free properties** and **small-world characteristics**, indicating that a few artists play outstanding roles in the network; or, in terms of network science, are able to *control the network*.

In order to perform further influence analysis, we propose the two additional indexes. The **I-index** captures the influence degree of a specific artist, taking into account *the number of (in)direct influencers and the decaying of influence power with respect to time*. **D-index** captures the common feature of revolutionary artists by *quantifying the ability of specific artist to consolidate or revolutionize the current genre*. Another essential part of our framework is the measurement of music similarity, which is implemented by the **Heterogeneous Euclidean-Overlap Metric (HEOM)**. Another highlight of our study is an insight into the **law of genre development**. To capture the mechanisms behind the growth of genres, we propose the **General Index of Genre Development (GIGD)** based on the *combined effect of I-index, popularity and release frequency of intra-genre artists*.

An analysis of samples from our indexes shows that they do indeed lead to the identification of outstanding influential artists (e.g., Beatles, the Rolling Stones, and Bob Dylan) and genre pioneers (e.g., John Cage, Roy Acuff, and Bob Wills). Moreover, we show the HEOM **unveils the high similarity within genre and low similarity between genre..** Our experimental results surprisingly exhibits **a strong correlation between GIGD and the number of increasing intra-genre artists** at an interval of 10 years, which is essentially the growth rate of genre. Finally, we show that the non-linear t-SNE-Distributed Stochastic Neighbor Embedding method(t-SNE) is able to reduce the music characteristics of the genres to two dimensions, which is helpful to reveal comprehensive music characteristics in two dimension, better than linear method such as Principle Component Analysis. Most importantly, **one of the t-SNE components indicates a strong temporal evolutions**, highlighting the fact that music indeed changes significantly over time. We believe that our computational model and experiments contribute to a better understanding of musical influence.

Keywords: Musical Evolution; Complex Network; Similarity measure; Musical influence;

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1 Introduction and Problem Statement

Throughout the history of music development, *musical influence relationships* have played an imperative role in depicting the inheritance and innovation of music. Not only do artists draw inspiration from their “influencers” and create outstanding masterpieces, brand new genres also emerge due to the prosperity and extension of existing genres. The music influence has already permeated in our life. For instance, a person recommending a new singer to a friend will often mention this singer in terms of who he or she sounds like.

In the past, music influence was a topic of interest and debate among cultural critics and scholars of art history. From a musicological and sociological standpoint, discussing musical influence is surely significant, since it can help us better understand the historical development of genres and the overall evolution of music over time [33].

With the development of data-driven scientific approach over the past decades, more and more researchers begin to tackle this subject in a quantitative way. A variety of computational frameworks and tools to model musical influence have been proposed, including sample-based musical influence networks [7], content-based classification of tracks [10], topic modeling approach [24] and etc.

In this paper, we resort to several modern scientific approaches, especially *Network Science* and *Data Science*, in an attempt to unveil the mystery behind Musical Evolution. Nowadays, network science has been widely utilized and investigated in multiple subjects and fields, for instance controlling the spread of epidemic and building the social network [5]. Given the abundant data regarding artist influence and musical information, we are able to establish a comprehensive model to quantify and measure the influence of previously produced music on new music, and further examine the evolutionary and revolutionary trends of artists and genres.

With the aid of our computational model, we will make a detailed and comprehensive analysis on the following topics, including inter-genre similarity, revolutionary patterns, presence of true influence relationship, genre evolution and etc. The remaining of this paper is organized as follows: In section 2 we review and summarize the previous research on the related topic. Section 3 provides a description of dataset and data preprocessing; Section 4 summarizes the assumptions of our model; Section 5 formally introduces the methodology of our model; Section 6 presents the experimental results and solves the problems one by one. Section 7 summarizes the strength and weakness of our model, and puts forward the direction of future work.

2 Literature Review

To gain deeper insights into the musical evolution, researchers have used a variety of computational frameworks and tools to model musical influence. One of the earliest to research the recognition of musical influence is Nick Collions, who explored the question of influences based on online information seeking and content-based classification of Synth Pop tracks on a small dataset of 364 tracks [10]. Afterwards, Shalit et al. exploited a large-scale machine-learned quantitative model of musical influence, and arrived at a conclusion that musical innovation and musical influence are not monotonically correlated [24]. Nicholas J. Bryan and Ge Wang applied several classic node centralities in network science to the musical influence network to measure the influence between songs, musicians and genres [7].

However, some researchers also found challenge in modeling musical influence in the field of music information retrieval, especially the audio-based approach. There still exists a large gap in extracting high-level properties such as "genre, mood, instrumentation and themes" from audio [24].

Another challenge is due to a lack of precise definition for influence. Morton and Kim found that influence can take on different meanings and be reflected by different interactions [18]. Some interactions are prolonged and directed, for instance "teacher-student relationships" and "familial associations", while others may be as brief as a 10 second segment of a song that the artists happen to hear by chance.

Besides investigating on the similarity relations of music or artists, Felipe Falcão and Nazareno Andrade applied the index of measuring technological change proposed by Russell J. Funk and Jason Owen-Smith [15] to the context of music creation to measure disruption in song similarity network [4]. And they also extend this method to artist influence network, aiming to find disruptive artists [14].

Moreover, some science and technology bloggers also participate in this state-of-the-art topic with the aid of data science. Sejal Dual and Victor Ramirez found themselves indecisive to pick a playlist based on their mood in daily life, thus they exploited the machine learning technique K-means clustering and dimensionality reduction technique Principal Component Analysis (PCA) to build a music recommendation framework[13]. By clustering music into clusters based on audio features, they discovered that these clusters compared with the conventional genres, provide a better description of music.[22]

3 Music from 1920 to 2020: A data-driven analysis

3.1 Data Description

The influence data ("influence_data.csv"), scraped from AllMusic website, contains influencers and followers for 5,854 artists in last 90 years. The AllMusic site defines influencers as "Artists that have had a direct musical influence on, or were an inspiration to, the selected artist, as determined by our music editors" and followers as "Artists who were influenced by the selected artist [3].

The music data ("full_music_data.csv"), obtained from Spotify's API, provides 16 variable entries of musical features together with corresponding artist information. These entries can be divided into three categories, music characteristic, type of vocal and music description. The music characteristic includes danceability, energy, valence, tempo, loudness, mode and key, in which mode and key are categorical attributes and the others are numerical attributes. The type of vocals includes acousticness, instrumentalness, liveness, speechiness and explicit, in which explicit is categorical attribute and the remaining is numerical attributes.

Although we are given the artist data ("data_by_artist.csv"), which contains the mean values of musical characteristic by artist, we modify and reprocess this dataset before using it based on the following reason: The mean values of musical characteristic, which are retrieved from all pieces of music released by the artist, cannot accurately reflect the characteristic of the artist. A possible better candidate can be **the sum of musical characteristics weighted by the popularity of the music**. For each piece of music that an artist released, those with relatively high popularity can better represent the style and characteristic of the artist, since the mass get to know more about this artist through those popular music.

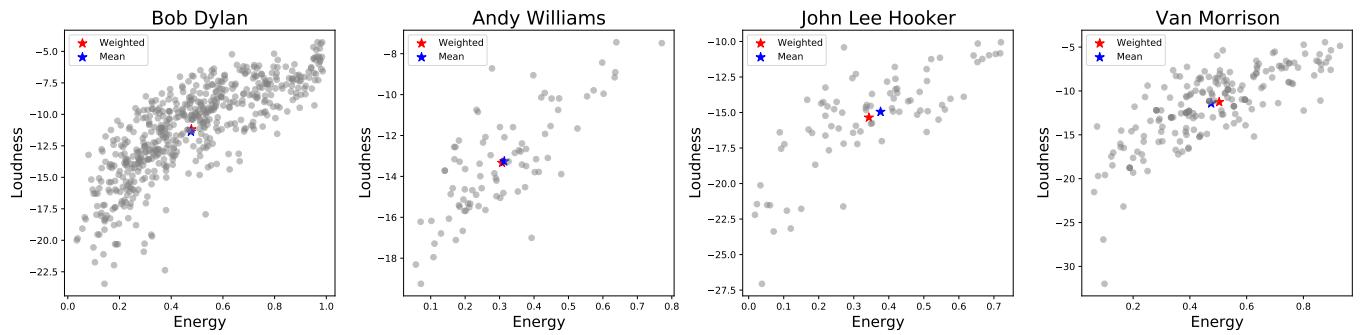


Figure 1: Feasibility test of the sum of musical characteristics weighted by the popularity of the music

To verify the assumption that the weighted sum of musical characteristics are better than the mean value of musical characteristics, we conduct a series of experiments. We randomly select four artists, and for better visualization, we extract any two musical characteristics to describe the musical characteristics of each song.

Through multiple similar experiments, we found that the results are comparable as the figure 1 below. The gray points are the description of all the songs of the specific musician in the two dimensions we have selected. The red star represents the musical characteristics of musicians weighted by popularity and the blue star represents the mean characteristics of artists. We can observe that these two points are very close, which indicates that the characteristics they describe are very similar.

Since this result is contradicted to our original assumption, we look into the data set and find that **the popularity of all the songs of the same artist is similar, which possibly explains the ineffectiveness of the weighted sum approach.**

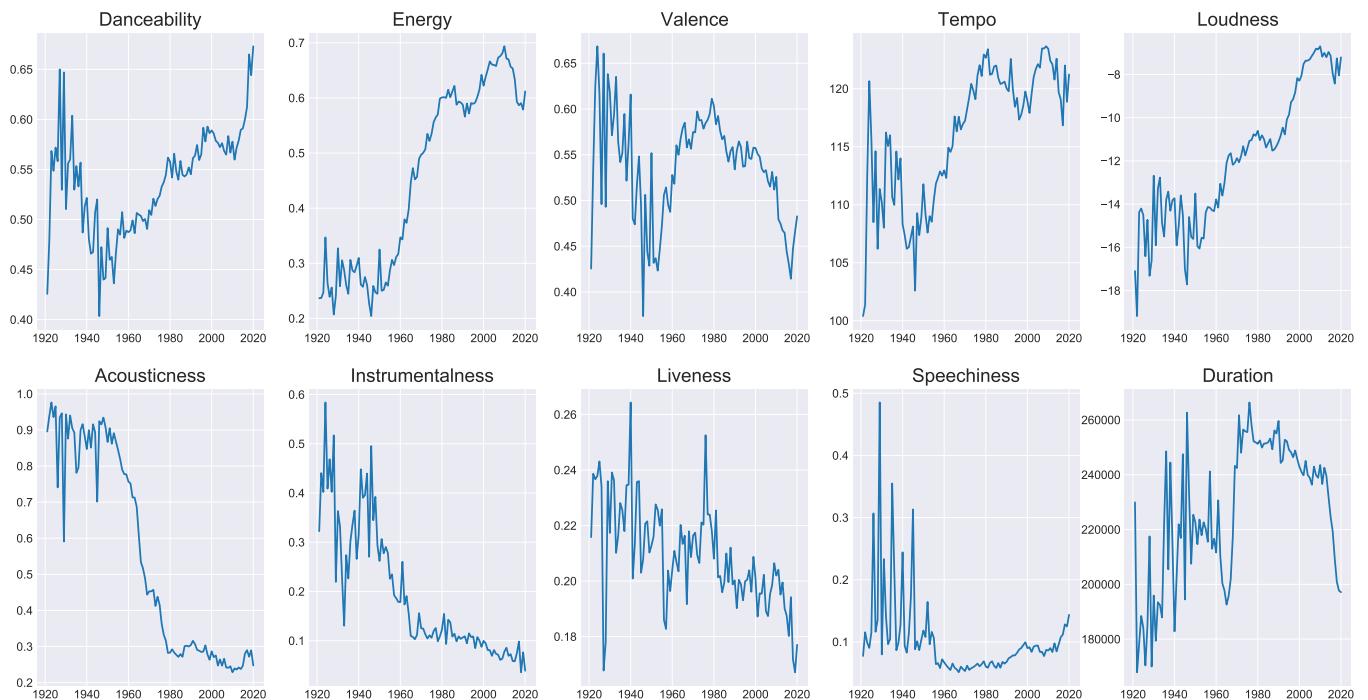


Figure 2: The temporal evolution of musical characteristics, with the abscissa as time and the ordinate as musical characteristics.

The annual music data (data_by_year.csv) is also retrieved from the music data, and it has already served our purpose without further modification.

The temporal evolution of musical characteristics is visualized in Figure 2, with the abscissa as time and the ordinate as musical characteristics. We observe that each of the musical characteristics evolved differently over time, and there is a correlation between some musical characteristics. For instance, the curves of Energy and Loudness are comparable, with a significant increase in both values between 1960 and 2010, which is quite intuitive since people tend to feel that loud music is full of energy. On the contrary, Acousticness was in decline over time, since technically enhanced or electronically amplified music has more power and volume than pure human voice. Instrumentalness has an obvious decreasing trend over time, which indicates that modern musicians are more and more fond of integrating human voice into music, rather than just the performance of instruments. Tempo has a significant upward trend over time, indicating that modern musicians advocate faster music rhythm.

3.2 Prevelence of Genre from 1920 to 2020

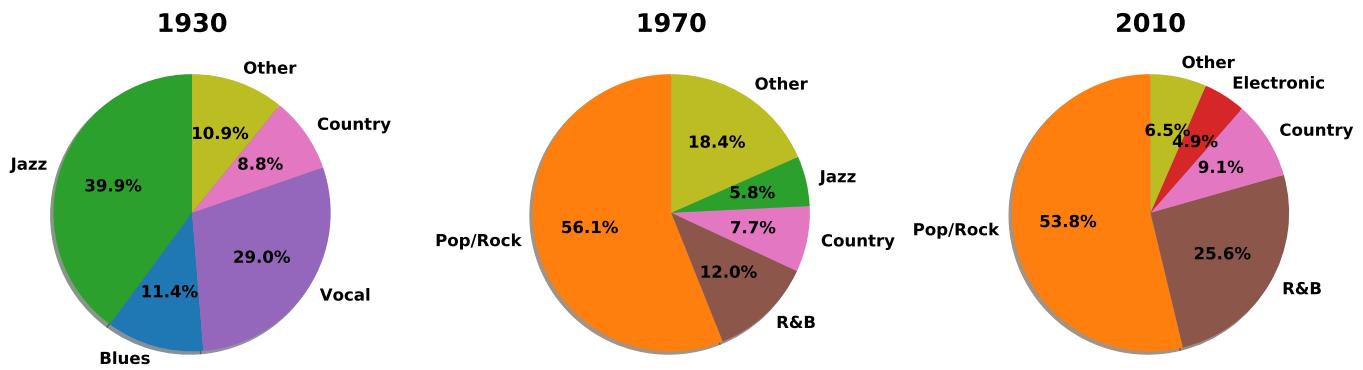


Figure 3: The prevalence of genre throughout history. The porportion of influencer's genre at each time point from 1930 to 2010 are shown in pie charts. This visualization is based on the assumption that the preference of the follower artists can represent the preference of the mass.

To explore the trends and prevelance of genres, we establish a data-driven analysis for artists and genres from 1920 to 2020. In this preliminary analysis, we observe the prevalence of genre throughout history by the music influence data.

In Figure 3, at each time point from 1930 to 2010, the porportion of influencer's genre is visualized in the pie chart. This visualization is based on the following assumption: The greater extent that follower artists are influenced by influencer artists of a specific genre, the more popular this specific genre is to the mass market. Briefly, the preference of the follower artists can represent the preference of the mass market. In the following, we will interprete those pie charts together with the history of American music.

During 1930 to 1940, the porportion of influencer who belongs to Jazz, Country, Vocal and Blue is in dominant position. This conforms to the history of American music well [21]. At this period, Jazz has already developed for 20 years and has been recognized as a major form of musical expression in traditional and popular music, linked by the common bonds of African-American and European-American musical parentage [30]. Meanwhile, Country music and Blues originate together, gather wave of enthusiasm in southern America [28].

During 1950 to 1980, the porportion of influencer who belongs to Pop/Rock significantly rises from 14.1% to 70.8%, and the porportion of the influencer who belongs to the previous prevalence genre (e.g. Jazz, Country, Blue) gradually falls down. This observation again greatly conforms with the reality, when American as well as the world witness the thrive of hip hop music. At that time, hip hop culture spread across America like wildfire, with every major city developing its own unique brand of this powerful and exciting new genre [29].

During 1990 to 2010, the porportion of influencer who belongs to Pop/Rock start to decline, but meanwhile the porportion of influencer who belongs to R&B music gradually arise. This also conforms to

the reality. At that period, R&B lyrical themes often encapsulate the African-American experience of pain and the quest for freedom and joy, as well as triumphs and failures in terms of relationships, economics, and aspirations [25].

4 Assumption and Limitation

In this chapter, we summarize the assumptions used in all our models:

- Influence and music data used in this study can reflect the fact of artist and genre. Since the data used in this study are a subset of larger data sets from AllMusic.com website and Spotify's API, information regarding to artists and genres may be incomplete. We assume the data used in this study to represent the full appearance of artist and genre.
- The greater extent that follower artists are influenced by influencer artists of a specific genre, the more popular this specific genre is to the whole society. Briefly, the preference of the follower artists can represent the preference of the mass.
- The following relationship between artists is fixed. Artists do not increase or decrease their follower-ship.
- The artist's genre is fixed and valid. The musical genre of an artist doesn't change over the course of his career.
- We do not consider the influence of personal relationships between artists on their musical style.
- We believe that the musical features in full_music_data can well summarize the music. Possible errors in audio processing will not be taken into account.

5 Methodology

In this section, we describe all the mathematical models we have developed to analyze the musical influence. In section 5.1, we built a directed network based on the following relationships between artists. And according to the topology of the network, I-index to measure the influence of artists and D-index to measure the revolution of artists are proposed. In section 5.2, we use a mixed-type distance measurement, Heterogeneous Euclidean-Overlat Metric(HEOM), to measure similarity between music or artists. In section 6.3, we propose the General Index of Genre Development(GIGD) to measure the development of musical genres. And we also introduce t-Stochastic Neighbor Embedding(t-SNE) as a dimensionality reduction method to further analysis the music features of genres.

5.1 Artist Network

We live in a very large and complex real world. Many real complex systems can be abstracted into a network. A complex network consists of nodes and the edges between nodes. Nodes are the abstractions of units and individuals in the real world, while edges represent the interactions and influences among nodes [5]. In recent years, complex networks have been used in various complex systems, such as social relations, protein interactions in medicine [6], power grids [20], transportation systems [16] and so on. We can also apply network science to music, abstracting the relationships between musicians and between songs. By studying the topology of the network, we think we can gain a deeper understanding of the interaction between musicians [9].

5.1.1 Artist Network Construction

Based on the data given in the influence_data.csv, we set up the Music Artist Influence Network. In this network, each artist is a node. The following relationship between artists is represented as a directed edge in the network, from followers to influencers. The network has 5603 nodes and 42270 directed edges. The

(a), (b) and (c) of figure 4 show the degree distribution of the artist network. The clustering coefficient of this network is 0.091, and it has the average shortest path of 6.16. Noting that this network has a pretty low average shortest distance and high clustering coefficient, the artist network also shows a certain degree of small-world network characteristics [19]. A small-world network is a type of network in which most of the nodes are not adjacent to each other, but most of the nodes can be reached in a few steps from any other point. This shows that there are a certain number of great artists among all the artists, who have high popularity and are followed by many artists. We also show the genre distribution among these artists in (d) of Figure 4.

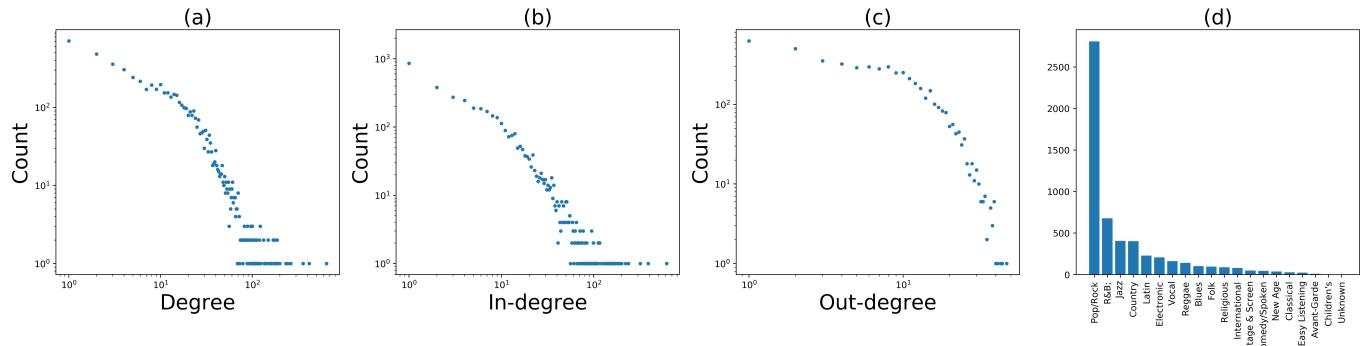


Figure 4: (a),(b),(c) The degree distribution of the artist network. (d) The genre distribution among artists.

5.1.2 Musical influence of artist

Different musicians have different musical influences. For example, apparently The Beatles, who is named among the 20th century's 100 most important people by Time magazine [12], has a higher musical influence than Sarah Ross(an american singer). In general, master artists in the field have a high number of direct and indirect influences. Here, we mainly consider the number of followers a musician has, as well as the number of his second-order followers, that is, followers of his followers, and the number of his multi-order followers. Considering that the earlier a musician emerges, the greater the number of all possible multilevel followers he can acquire, we introduce a decay factor, so that the longer the distance from multi-order followes to the influencer, the less influence he adds to his influencer. Because each artist follows a different number of influencer. And, being directly influenced by one is much stronger than being directly influenced one hundred. Thus, the amount of influence each follower adds to its influencer is related to the number influencer this follower follows. Following this line of thought, we propose an I-index to measure the influence of a musician. The I-index for a node a can be computed by the formula:

$$I_a = \sum \alpha^{d-1} * \sum_{i \in F_d} \frac{1}{|Inf_i|} \quad (1)$$

where α is the decay factor, d is the distance between the influencer a and the follower, F_d is the set of all d th-order followers and $|Inf_i|$ is the number of influencer of i . Our preliminary experimental results show that $\alpha = 0.3$ is a reasonable value.

5.1.3 Revolutionary Artist Measurement

The artist network does not only cover the following relationship information among artists. By studying the topology of the network, we can also find revolutionary artists. A revolutionary artist should have a huge impact on the field and on other artists. The revolutionary artist breaks with established musical styles and understandings, thus differentiating himself from the artists who came before him. When a revolutionary artist appears, subsequent artists follow less of the previous artist, because the revolutionary artist ushered in a new way of thinking. On the contrary, a non-revolutionary artist, i.e., a musician who reinforces an existing style of music, will be followed by subsequent artists as well as previous artist, because they present the same system of art themselves [4].

An indicator proposed by Funk and Owen-Smit [15], known as D-index, makes good use of the above-mentioned intuitive understanding to quantify the consolidating or destabilizing effect of nodes. The D-index for a artist a is defined as:

$$D_a = \frac{n_j - n_i}{n_i + n_j + n_k} \quad (2)$$

where n_i represents the number of artists that follow a and at least one of his influencers, n_j represents the number of artists that follow a but none of his influencers and n_k represents the number of artists that do not follow a but follow at least one of his influencers.

D-index ranges from -1 to 1 . The closer D is to -1 , the more it indicates that the artist has played a consolidating role in the field. In extreme case, for example, $n_j = n_k = 0$, i.e., the crowds that follow the artist and his influencers repeat themselves. On the contrary, the closer D is to 1 , the more revolutionary the artist is. In extreme case, for example, $n_i = n_k = 0$, the artist has completely pioneered a new trend, causing subsequent artists to abandon to follow the previous artists. In Figure 5, we selected two representative artists in the dataset influence_data.csv for visualization, one with high D-index(Shirley Caesar) and the other with low D-index(The Abyssinians). It can be clearly seen from the figure that D-index is related to the intuitive characteristics of revolutionary artists.

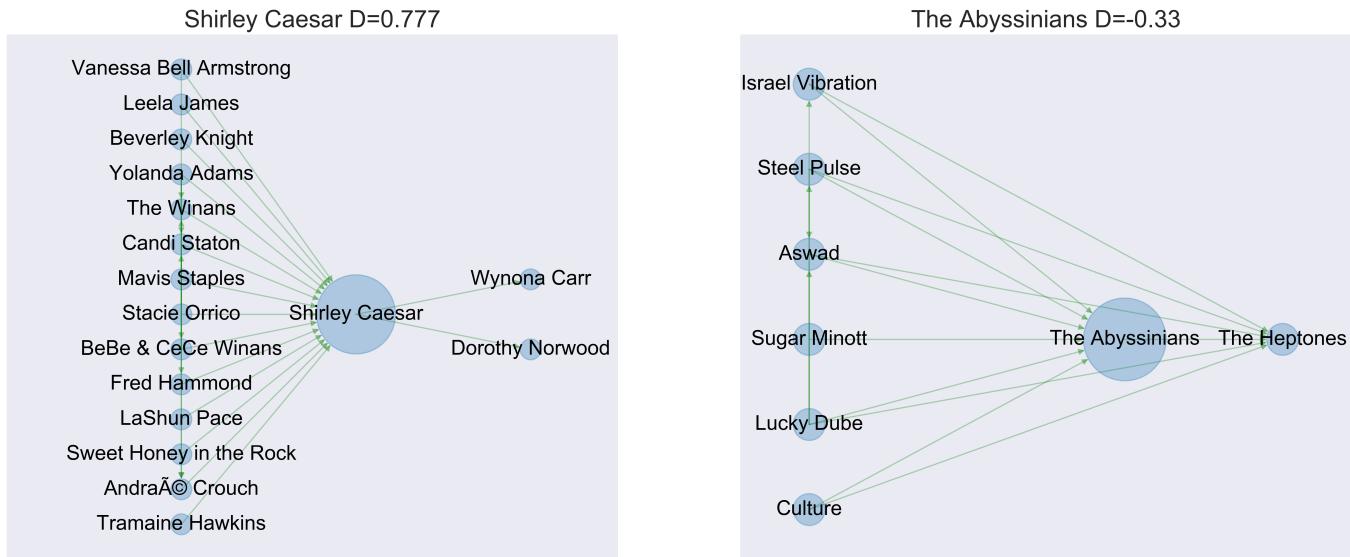


Figure 5: Two representative artists are plotted here. Shirley Caesar(on the left) has pretty high D-index. None of her followers follow her influencer. She has created a disruption on the network, which is revolutionary. The Abyssinians(on the right) has a low D-index, playing a consolidating role in her field.

5.2 Measuring musical similarity

Despite the fact that music evolves over time, the majority of audio and visual characteristics did not vary greatly from one another. Accordingly, given a series of detailed parameters of musical features (e.g. tempo and energy) as well as types of vocals (e.g. acousticness and liveness), some appropriate data science measures can be developed to measure the similarity of music pair. In this section, we resort to a well-established similarity measures, *Heterogeneous Euclidean-Overlap Metric (HEOM)* [31], which performs well in case of missing data and heterogeneous data.

We select the HEOM to measure the similarity among musics or similarity among artists based on the following two reasons:

- *Missing Data*. When comparing the similarity between two artists, some artist characteristic data are missing (NaN). The HEOM is capable of handling missing data.

- *Mixed-type Data.* Given a mixed-type data with both categorical (e.g. mode and key) and numerical data (e.g. energy and valence), most of the popular distance metrics may not function well, thus heterogenous distance function especially HEOM is designed to handle applications with both categorical and numerical data.

The HEOM make use of *overlap* metric for categorical attributes [2] and normalized Euclidean distance [11] for numerical attributes. The distance between two music characteristic values u and v of a given attribute β is given as:

$$d_\beta(u, v) = \begin{cases} 1 & \text{if } u \text{ or } v \text{ is unknown} \\ overlap(u, v) & \text{else if } \beta \text{ is categorical variable} \\ dif_\beta(u, v) & \text{else} \end{cases} \quad (3)$$

where the function *overlap* and dif_β are respectively defined as

$$overlap(u, v) = \begin{cases} 0 & \text{if } u = v \\ 1 & \text{otherwise} \end{cases} \quad (4)$$

$$dif_\beta(u, v) = \frac{|u - v|}{\beta_{max} - \beta_{min}} \quad (5)$$

Through the normalization process, the difference of characteristic values u and v is scaled down to less than one. On top of these definitions, the overall distance between two input vectors (i.e. music object) \mathbf{u} and \mathbf{v} is given by HEOM(\mathbf{u}, \mathbf{v}):

$$HEOM(u, v) = \sqrt{\sum_{\beta} d_{\beta}(u_{\beta}, v_{\beta})^2} \quad (6)$$

5.3 Genre analysis

5.3.1 GIGD: General Index of Genre Development

In this section, we would like to investigate what actually influence the development of a genre. The artists belonging to a specific genre and the influence relationship between artists in this genre form a subnetwork of the artist network, denoted as the genre network. In the following, we develop the *General Index of Genre Development (GIGD)*, to quantify and measure the development of a specific genre.

Regarding of genre development, we should take two primary measures into account:

- *Size* : $|G|(g, t)$. The number of artists contained in the genre network g at time t .
- *Growth Rate* : $V(g, t)$. The number of increasing artists of genre network g in the interval of 10 years at time t .

We are thus curious about what are the indicators/factors of genre growth rate. Through careful considerations, we proposed the following three indices that may explicitly or implicitly influence the growth rate $V(g, t)$:

- *Genre Influence Index (GII)*. GII is the sum of I-index of artists within genre, i.e. $GII(g, t) = \sum_a I_a$, $a \in g$, which reflects the influence power of the whole genre to other artists.
- *Genre Popularity Index (GPI)*. GPI is the sum of popularity of artists within genre, i.e. $GPI(g, t) = \sum_a P_a$, $a \in g$, where P_a denotes the popularity of artist a . GPI indicates the overall popularity of the genre.

- *Genre Updation Index (GUI)*. GUI is the sum of the number of released music of artists within genre, i.e. $GUI(g, t) = \sum_a U_a$, $a \in g$, where U_a denotes the number of released music of artist a (i.e. 'count' in dataset). GUI represents the album release frequency of artists within genre and the cohesive force of the genre.

We follow the well-established statistical approach, Analytic Hierarchy Process (AHP) [23], to determine the weights for each index and finally derive the general index. Based on mathematics and psychology, the AHP is a structured technique for organizing and analyzing complex decisions. After calculation, the weights of three genre index are 0.2764, 0.5954, 0.1283.

Finally, the General Index of Genre Development (GIGD) is defined as the weighted sum of genre index for a specific genre g ,

$$GIGD(g, t) = 0.2764 * GII(g, t) + 0.5954 * GPI(g, t) + 0.1283 * GUI(g, t) \quad (7)$$

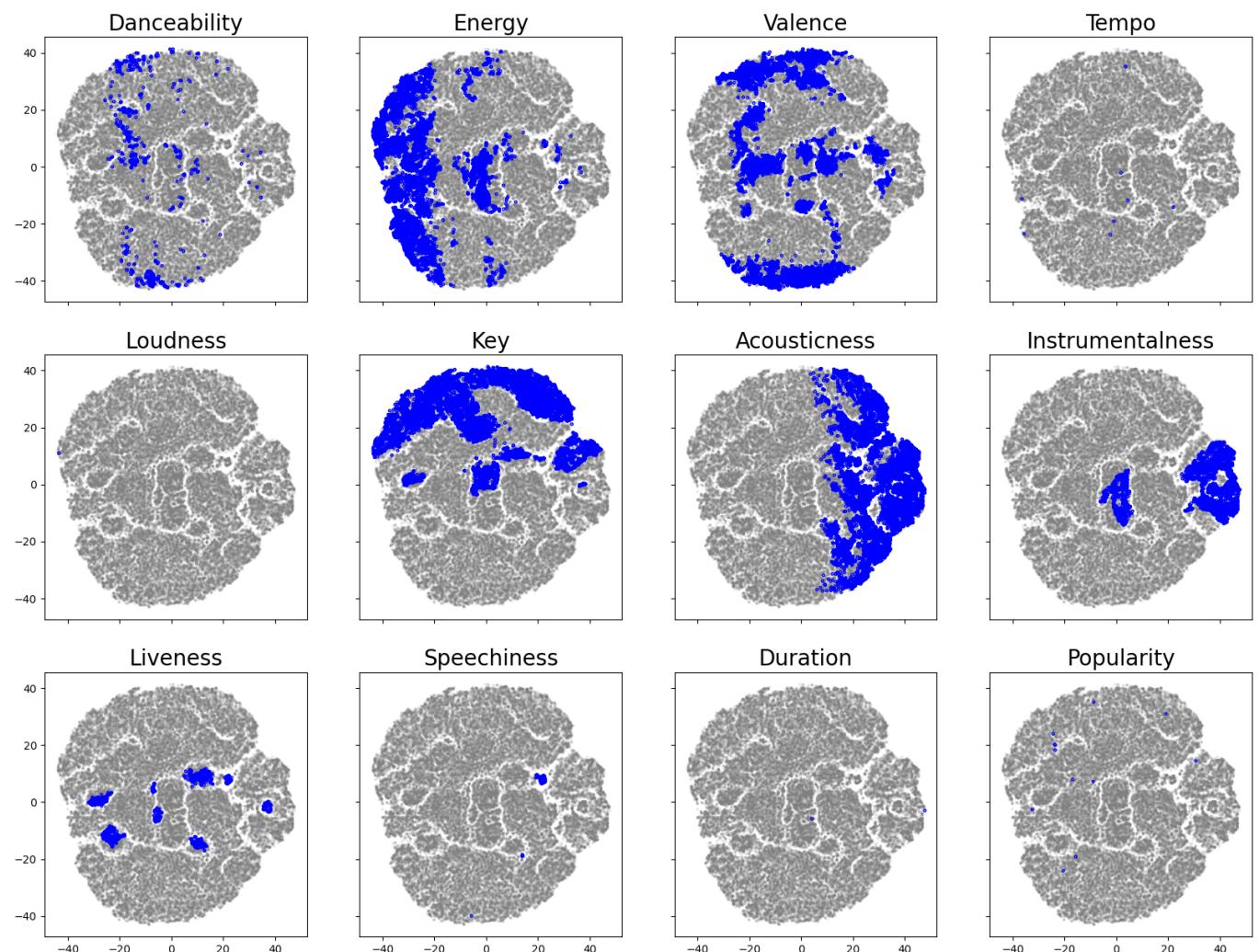


Figure 6: We took the two components after t-SNE dimensionality reduction as the abscissa and ordinate respectively to draw all the songs on the plane graph. In each subgraph, songs with top 90% values of a specific attribute are highlighted in blue. As shown in the figure, the blue dots have a strong aggregation, indicating that the music with similar original attributes still has a strong similarity after processing. In other words, the two dimensions obtained through T-SNE retain the original information of the data very well.

5.3.2 Dimensionality Reduction and Visualization

Since we have too many characteristics of songs, We want to do the data dimensionality reduction processing. t-Stochastic Neighbor Embedding(**t-SNE**) is a good approach to reduce the data dimensionality and visualize classification of the the data [27].

There are four important parameters which should be set in the beginning of the t-SNE.Based on experience, We terminate the optimization process when the iteration more than 1000 or the norm of the gradient less than 10^{-7} .Moreover, the learning rate is 200 and the perplexity is 30.

In order to verified that t-SNE is suitable for the reduction the data dimensionality which contains the music characteristics, we first pick out non-boolean indicators for processing. For each indicator, we do the Deviation standardization. Then, We use Principal Component Analysis(PCA) [32] to initialize the t-SNE so that it can run faster. Finally we get the result which have two dimension. Taking the two columns of the result as the horizontal and vertical coordinates, We draw a scatter plot. The songs with top 90% values of a specific attribute are highlighted in blue.

It is not difficult to find from the figure that the points marked blue show a kind of aggregation. In other words, nodes with higher original musical characteristics still have similarity in the graph after dimensionality reduction. For these musical features, t-SNE retains the original information very well.

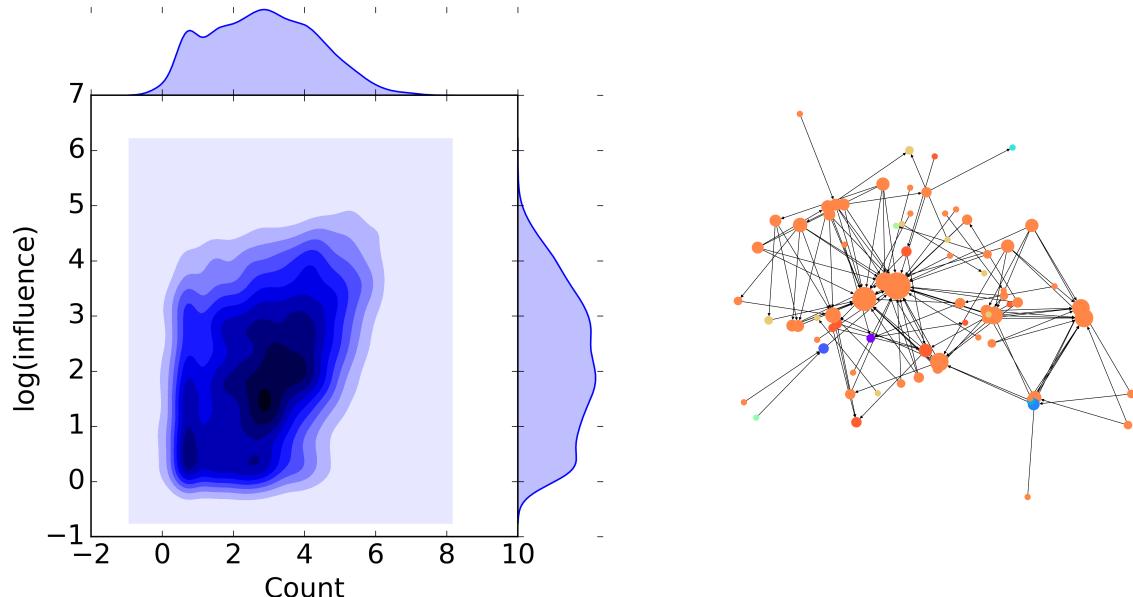


Figure 7: The relation between the number of songs(count) and influence is shown here. The data of influence is processed logarithmically. Figure 8: A subnetwork of artist netwrk. Nodes are colored by their genres and the node size is proportional to their influence.

6 Results

6.1 Artist network analysis

In section 5.1, we construct a directed network to describe the following relation between artists. We also proposed an I-index based on direct and indirect influencers to measure the influence of an artist. Given that an influential artist has a greater chance of being liked by audiences, the more music he is likely to release. Also, if an artist releases only a few songs, chances are he won't be a highly influential artist. A master is never short of good music. A larger body of work is more likely to affect more people. Therefore, we believe that there is a certain relationship between influence and count. In Figure 7, we shows the relation between influence and count. Obviously, the more influential an artist is, the more songs he releases. We can conclude that the influence indicator I-index proposed by us has a certain degree of confidence.

In Figure 8, we visualize a subnetwork of artist network. The nodes are colored by their genres and the node size is proportional to their influence. As we can see from this graph, there is a strong inheritance between the follower genre and the influencer genre. In addition, the direct followers of high-impact musicians were also highly influential.

6.2 Inter-genre and intra-genre similarity of music (artists)

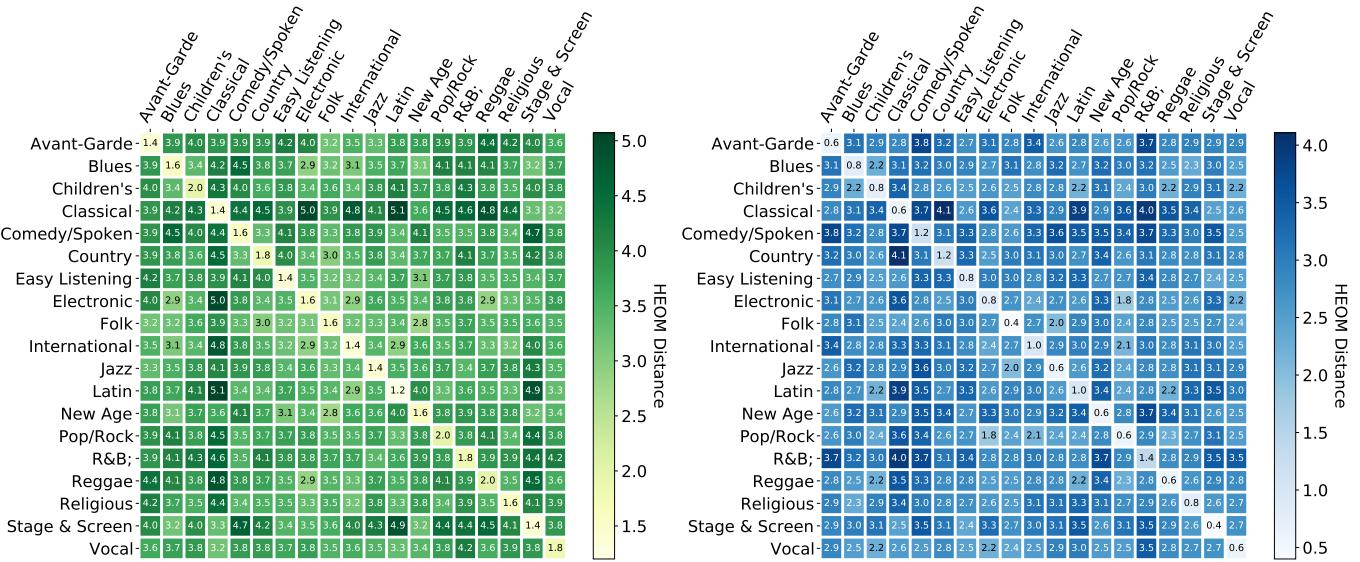


Figure 9: Inter-genre and intra-genre similarity of music (left figure), similarity of artists (right figure). Either of the figure shows pair-wise comparision of music (artists) and record the HEOM distance in each coordinate (i, j) . More specifically, the most popular five pieces of music (artists) in genre i are compared with the most popular five pieces of music (artists) in genre j respectively, and the ultimate distance of (i, j) is the average value of the five comparision results. The distinct light-colored diagonal line unveils the high similarity (i.e. short HEOM distance) within genre and low similarity (i.e. long HEOM distance) between genre.

As is introduced in Section 5.2, the heterogeneous distance function $HEOM$ can handle both continuous and categorical music attributes, which is suitable for our usage. In `full_music_data` and `data_by_artist`, various music characteristics and artist characteristics are provided, in which categorical attributes such as mode and key exist.

In an attempt to evaluate the similarity of music (artist) between and within genre, we conduct pair-wise comparision of music (artist) and measure the HEOM distance by means of heap map. As is shown in Figure 9, with all of the 19 genres listed as x and y coordinates, the distance value of coordinate (i, j) stands for the result of pairwise comparision between genre i and j . More specifically, the most popular five pieces of music (artists) in genre i compare with the most popular five pieces of music (artists) in genre j respectively, and the ultimate distance of (i, j) is the average value of the five comparision results.

The most distinct feature of the music similarity heat map is the distinct light-colored diagonal line, which unveils the high similarity (i.e. short HEOM distance) within genre and low similarity (i.e. long HEOM distance) between genre. Specifically, the HEOM distance of intra-genre musics (artists) never exceeds the upper bound of 2.0, while that of the inter-genre musics (artists) mostly stay at a high level.

Meanwhile, we can also gain observations for specific genre using HEOM distance measure. For instance, Classical music shows large HEOM distance when comparing with Electronic, Latin and Reggae, and Country music are quite similar to Folk and Religious. These observations quite conform to our experience and life.

6.3 Genre-specific analysis and evolution

6.3.1 The trend of evolution

In this section, we will explore the trend of evolution by performing t-SNE. We select Pop/Rock for analysis. We use the t-SNE method to reduce the music characteristics of the genres in the data set to two dimensions. Using these two components(defined as y_1 and y_2), Figure 12 is plotted to show the distribution of the songs of created by artists of Pop/Rock. We can see the points are moving from right to left gradually. It illustrates the evolution of the Pop/Rock in y_1 is quit strong.

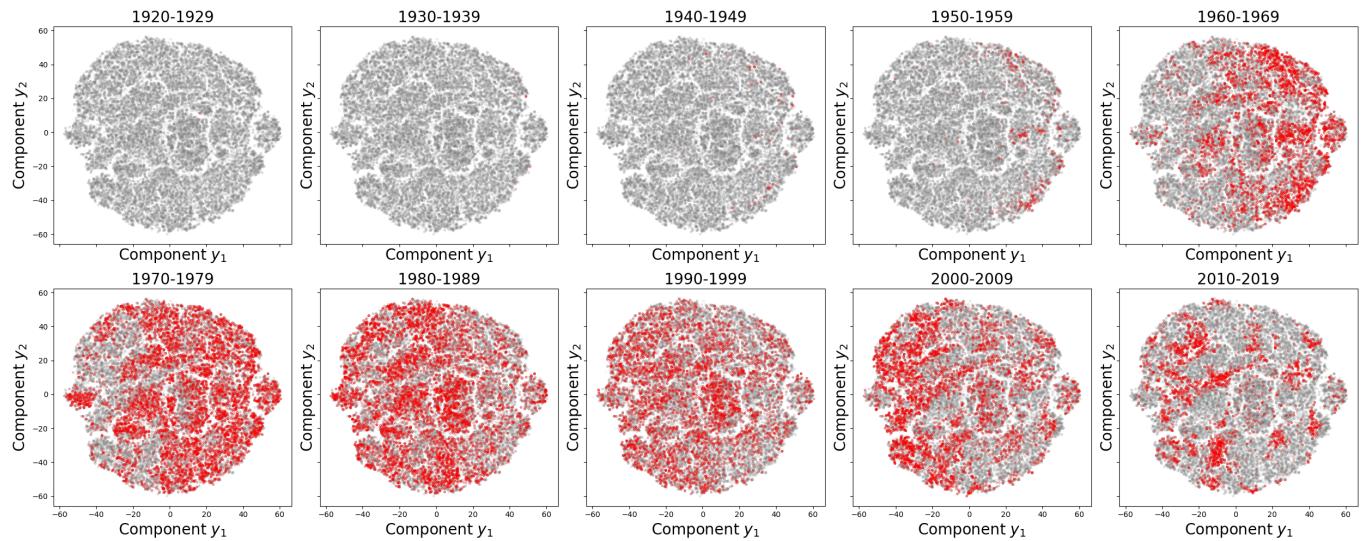


Figure 10: The general evolution of the songs of artists whose genre is Pop/Rock. We can see that pop/rock has changed significantly and regularly over the decades.

6.3.2 The relationship between genres

In order to study the relationship between the various musical genres, we consider this question from two perspectives. First, the following relationship between artists of various genres. Second, consider the musical characteristics of each genre.

From the following relationship of artist network, we mainly consider the number of followers among different genres and the influence of different genres on other genres. The result is shown in Figure 11. In the figure on the left, we show the number of followings between genres. The color of the grid shows how much artists in the genre on the abscissa follow artists in the corresponding genre on the ordinate. But the obvious problem is that the number of follower relationships is affected by the number of artist in genres. It's easy to find that pop/rock has the biggest following relationships, but that's probably just because it has so many more artists than any other genre. In order to solve this problem, we use the influence of influencers in the following relationship as an indicator, and average by number of follower relationship to represent the influence relationship between genres. One of the obvious features of this chart is that Pop/Rock, Jazz, R&B and Blues are greatly influenced by other music genres.

The relationship between genres is not only reflected in the artist network, but also in the characteristics of the music within the genre. t-SNE was performed to explore the relationship between the musical characteristics of the genre. We have selected four genres, which are Pop/Rock, R&B, Jazz and Country, because they appear more often in the data set. First, we use the t-SNE method to reduce the music characteristics of the four genres in the data set to two dimensions. Then, using these two components(defined as y_1 and y_2), Figure 12 is plotted to show the Kernal density distribution of songs in differnet genres. We can notice that there are several kernels in each plot. The distribution of the kernel is different between each plots. For instance, the kenel of Jazz located in the right top of the plot, but the kenel of the R&B lies in the

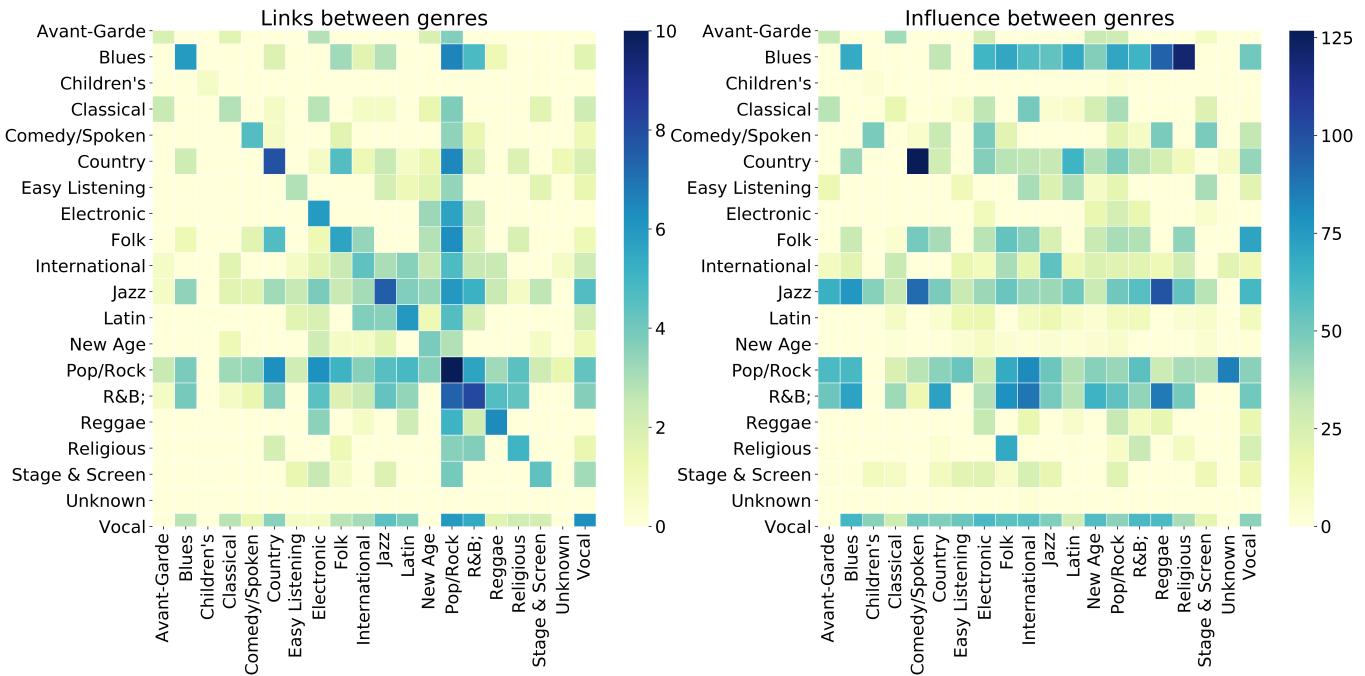


Figure 11: The number of following relationship between genres is shown on the left. The influence between genres is shown on the right. Here the influence is defined by the average influence of all influencers in one genre who are followed by artists in the other genre.

left of the plot. Let's take number of the kernel into consideration. Pop/Rock and Country music has more than one kernel in the plot. It suggests that these two music genres have various characteristics and there are many branches within the genre.

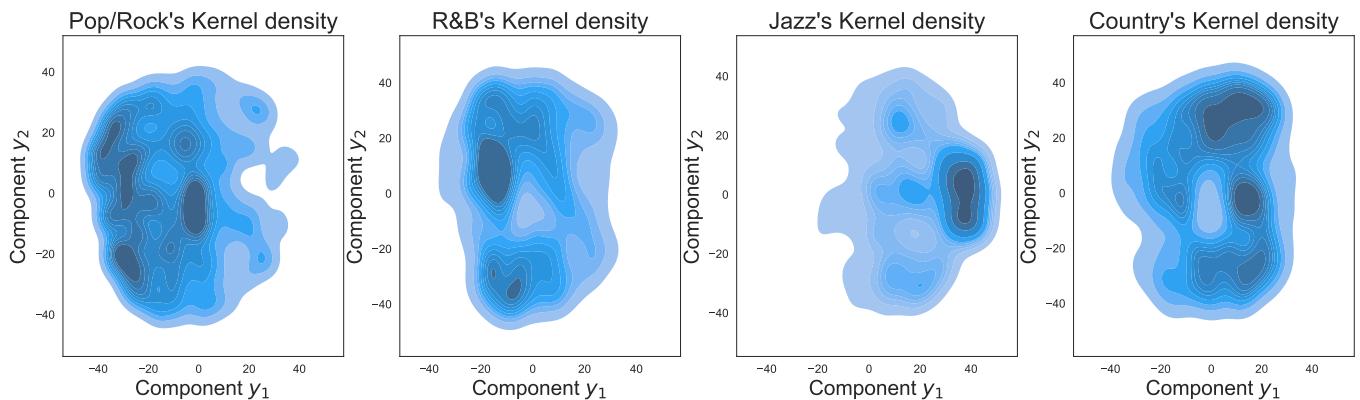


Figure 12: We took the two components after t-SNE dimensionality reduction as the abscissa and ordinate respectively to draw the Kernel density distribution maps of four music genres. From the figure, we can see that the musical characteristics of these four genres are very different.

In order to better explore the connections and differences between genres. We analyze the two components of y_1 and y_2 respectively. Figure 13 are the one-dimensional Kernel density distribution graphs of the four genres. The Kernel density distribution of y_1 is shown on the left. We can clearly see that the Kernel density of the four music genres peaks at different positions. This way we can distinguish the four genres through them. This conclusion is the same as that one we get in the above section 6.3.1. In the right is the Kernel density distribution of y_2 . We can see that most of the images are overlapped, indicating that there is a connection between the various genres.

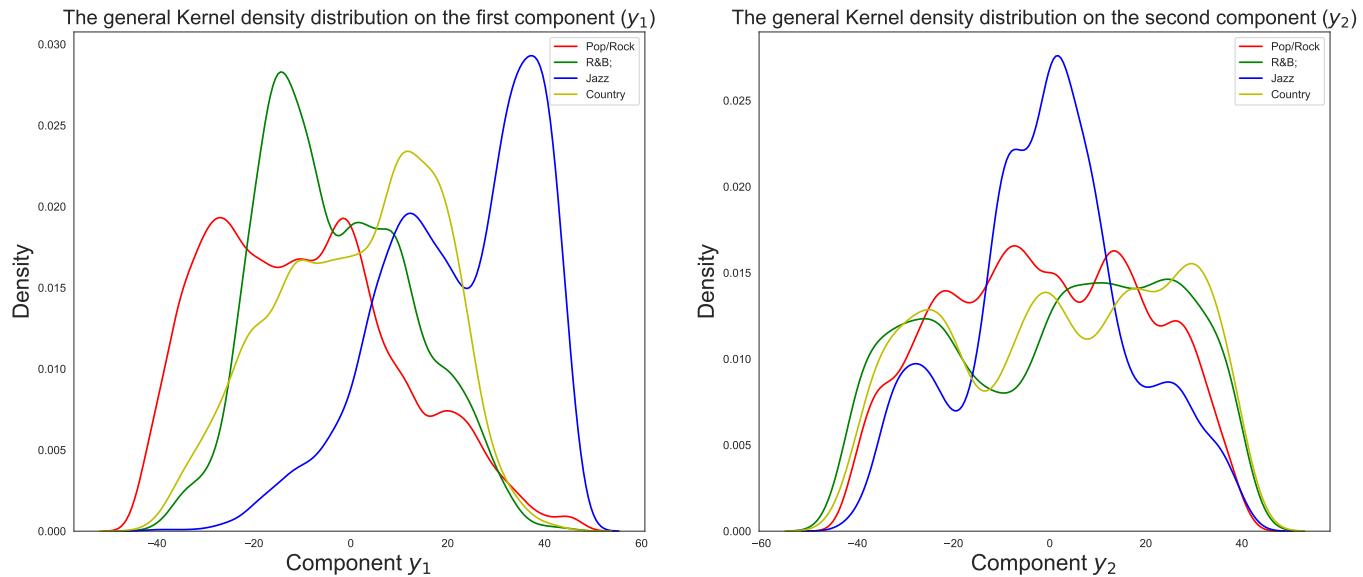


Figure 13: The distribution of two components (y_1, y_2) of the songs in four different genres. For component y_1 , these four genres peak quite differently, indicating that y_1 is a good indicator to distinguish them. For component y_2 only Jazz shows a difference from the other three genres.

6.4 Presence of true influence

In this section, we aim to investigate the presence of true influence between "influencers" and "followers", does this reported "influence" really matter the follower's music characteristics? Naturally, if the 'influencers' do affect the music created by the followers, than similar values of music characteristics can be observed between the pair.

We normalize eight music characteristics $\{\beta_i\}$ by min-max scaling, with the normalized music characteristic denoted by $\{\beta'_i\}$

$$\beta'_i = \frac{\beta_i - \beta_{min}}{\beta_{max} - \beta_{min}} \quad (8)$$

Due to multiple dimensions of attributes that an artist possess, we project each artist into four 2-dimensional plot. Afterwards, we randomly select two artist to be the root node of two subnetworks, one from Pop/Rock genre and the other from Folk genre, and search for their 20 neighbors (i.e. direct followers) to form two directed subgraphs. As is shown in Figure 14, the blue subnetwork stands for a subset of Pop/Rock artists and their mutual influence, while the orange subnetwork stands for a subset of Folk artists and their mutual influence.

Within either subnetwork, each value of the music characteristic of an artist is comparable to that of the others, with the length of range not exceeding 0.4. This fact clearly indicates that 'influencers' do have a strong impact on 'followers', which results in a comparable value of music characteristic of followers. Meanwhile, we can also get observations when comparing the Pop/Rock subnetwork and Folk subnetwork. In figure 14 (a) and (c), the two subnetworks are clearly separated, which reveals the fact that some music characteristics, for instance energy and acousticness, differ a lot between artists in the absence of influence relationship. Thus, music characteristics like energy and acousticness are indeed "contagious", and closely bond the influencer and follower together.

Furthermore, we also want to investigate another question: What is the probability that the follower inherit the genre of the influencer? Here, we again apply the artist network and use the in-degree metric to determine the probability of genre inheritance of artist a ,

$$P_a = \frac{\lambda_a}{d_a^{in}} \quad (9)$$

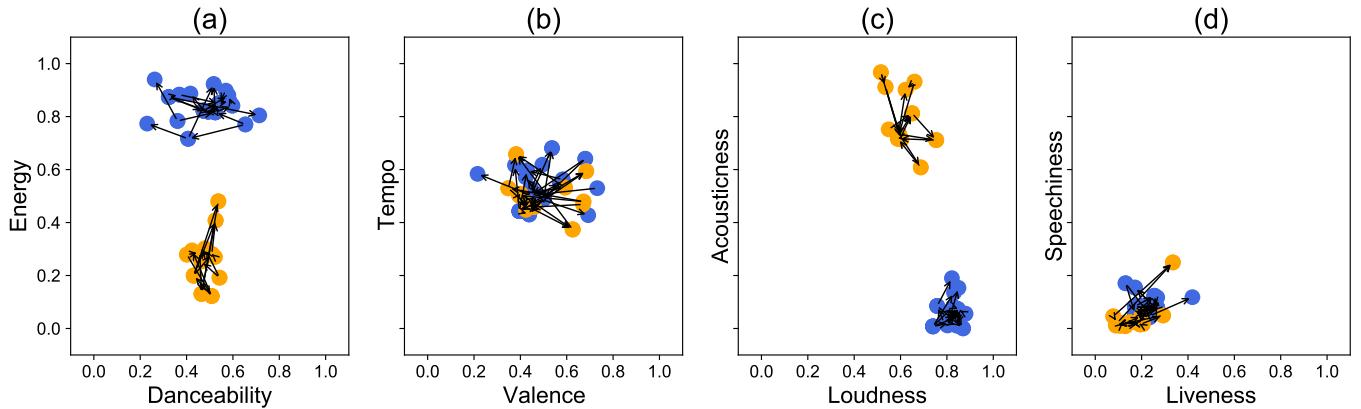


Figure 14: Presence of true influence between influencers and followers. Eight music characteristics are first normalized by min-max rescaling method, and then projected into four 2-dimensional plot. Two artists (from Pop/Rock genre and Folk genre) are chosen to be the root node of two subnetworks. Within either subnetwork, each value of the music characteristic of an artist is comparable to that of the others. The two subnetworks are clearly separated in figure (a) and (c), indicating that some music characteristics, for instance energy and acousticness, differ a lot between artists in the absence of influence relationship.

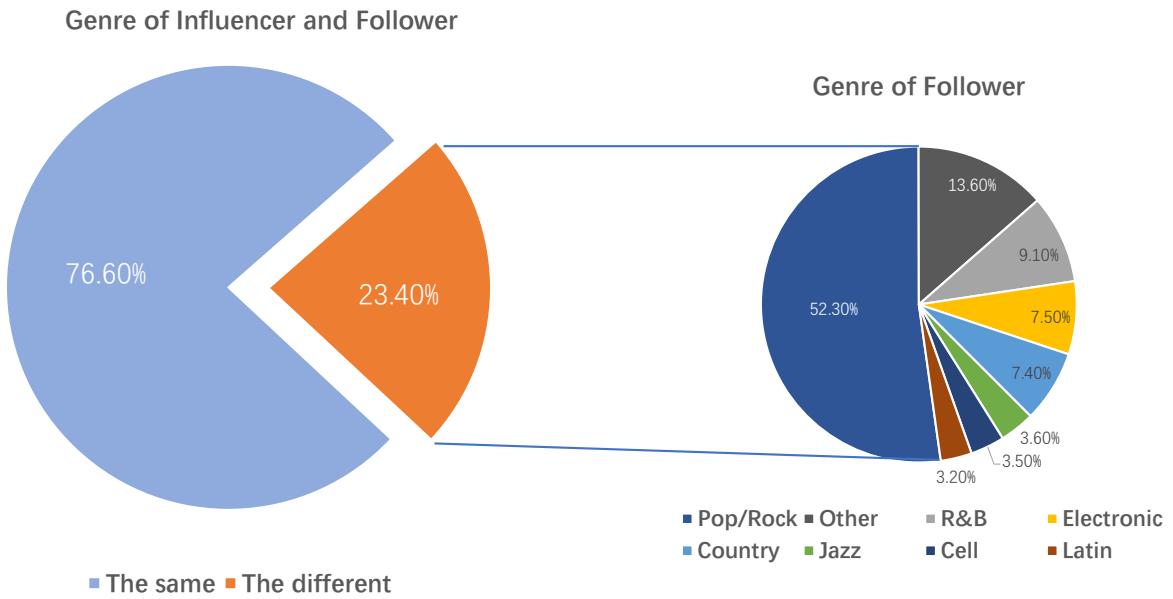


Figure 15: The probability of genre inheritance. The majority of the followers indeed inherit the same genre as the influencer, while 23.4% followers start their career with a new genre. Among these artists, over 50% of them decide to begin their career as a Pop/Rock artist.

where P_a denotes the probability of genre inheritance of artist a , and λ_a denotes the number of influencers that belongs to the same genre as artist a .

As is shown in Figure 15, the majority of the followers indeed inherit the same genre as the influencer, while 23.4% followers start their career with a new genre. Among these artists, over 50% of them decide to begin their career as a Pop/Rock artist. This fact reveals the current trend and situation that pop music and rock music is in dominant position, which is unavoidable. In conclusion, the identified influencers indeed

influence the respective followers, and the majority of followers do start up their career with the same genre as influencers.

6.5 Revolutionary patterns

From Figure 2, we can see that the energy, loudness, acousticness and instrumentalness changed a lot between 1960 and 1980, suggesting that there was probably a revolution in music during this period. In fact, the 1960s are also a time of great social change in living memory [17]. At that time, many young people in Europe and the United States began to adopt values that were very different from those of their parents. At the same time, the music of that period also reflected the new way of thinking of young people. The 1960s also saw the birth of the greatest and most influential band in the history of pop music, the Beatles, who influenced the music and ideas of generations of rock bands since the 1960s, leading the revolution and development of rock music.

In section 5.1.3, we use the D-index to measure how revolutionary a musical artist is. Figure 16(a) shows the distribution of D-index. The D-index of most artists is concentrated around 0, which indicates that most musicians are in the neutral position. As mentioned in the previous article, the closer the d-index number is to 1, the more revolutionary the musician. D-index is positive, indicating that the musician has contributed to the music revolution. Musicians whose D-index is greater than 0 are selected and their distribution over time is shown in Figure 16(b). Consistent with the above conclusion, a large number of revolutionary musicians emerged around the 1960s.

We selected musicians with more than 30 followers and ranked them from largest to smallest according to D-index. Table 1 shows the top 5 musicians. As you can see, musicians with a high D-index are indeed highly revolutionary. John Cage is known as a pioneer of opportunity music, extension technique, and electronic music [8]. Acuff is often credited with moving the genre from its early string band and "hoedown" format to the singer-based format [1]. Bob is also generally considered one of the pioneers of western swing music [26].

name	John Cage	Roy Acuff	Bob Wills	Ernest Tubb	The Mills Brothers
D-index	1	1	1	1	0.921053
Followers	34	50	40	52	36
Genre	Classical	Country	Country	Country	Vocal

Table 1: Musicians who has more than 30 followers ranked by D-index. Artists with a high D-index are indeed famous revolutionary pioneers in history.

6.6 Genre-specific evolution

In this section, we will analyze the influence processes of musical evolution that occurred over time in one specific genre, Electronic genre. The artists belonging to the Electronic genre and the influence relationship between artists in Electronic genre form a subnetwork of the artist network, denoted as Electronic-genre network. There are 208 artists and 372 influence relation in Electronic-genre network.

We are thus curious about how this Electronic-genre network evolves over time. The evolution of influence processes within Electronic-genre network is visualized in Figure 18. The earliest four artists of Electronic genre, which are essentially the "founders", emerges in 1960 and form one influence relationship. With the time passing by, more and more artists are attracted by Electronic genre through being influenced by these "founders" (i.e. influencer). Until 2010, the Electronic-genre network has developed into a dense network with 208 artists.

In section 6.3, we propose the General Index of Genre Development to measure the growth rate of genre network. To test the effectiveness of *GIGD*, we perform experiments on the Electronic-genre network defined in the previous section. The size $|G|(ele, t)$ of the network and the growth rate $V(ele, t)$ of the network versus time series t is visualized in Figure 18(a). $V(ele, t)$ monotonically increases until it reaches the peak value at $t = 1990$, and then monotonically decreases until $t = 2010$. Before examining the

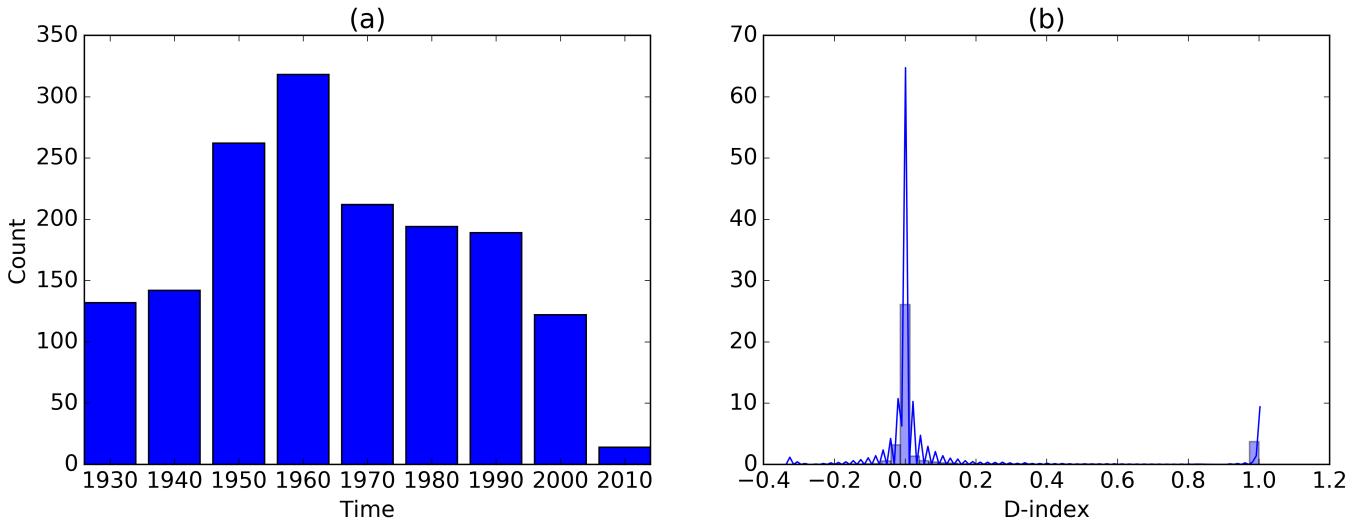


Figure 16: (a) The distribution of artists who has D-index larger than 0. These artists make positive contribution to musical revolution. Around the 1960s, a large number of revolutionary musicians were emerged. (b) D-index distribution. Most artists have D-index very close to 0, indicating that most artist are neutral to musical revolution.

effectiveness of $GIGD(ele, t)$ through comparision with $V(ele, t)$, we normalize both of them by min-max scaling,

$$GIGD'(ele, t) = \frac{GIGD(ele, t) - GIGD_{min}(ele, t)}{GIGD_{max}(ele, t) - GIGD_{min}(ele, t)} \quad (10)$$

$$V'(ele, t) = \frac{V(ele, t) - V_{min}(ele, t)}{V_{max}(ele, t) - V_{min}(ele, t)} \quad (11)$$

The comparision of $GIGD'(ele, t)$ w.r.t time t and $V'(ele, t)$ w.r.t time t is visualized in Figure 18(b). We surprisingly find that the two polylines are very close to each other across the full range of time. To quantitatively describe the correlation of $GIGD'(ele, t)$ and $V'(ele, t)$, we vectorize both of them into vector $\overrightarrow{GIGD'(ele)}$ and $\overrightarrow{V'(ele)}$ and calculate the angle between these two vectors,

$$\cos\langle\overrightarrow{GIGD'(ele)}, \overrightarrow{V'(ele)}\rangle = \frac{\overrightarrow{GIGD'(ele)} \cdot \overrightarrow{V'(ele)}}{|\overrightarrow{GIGD'(ele)}||\overrightarrow{V'(ele)}|} = 0.9979 \quad (12)$$

A closer cosine value to 1 means a smaller angle between two input vectors, and thus indicate a high similarity between the two sets of data. Since the result 0.9979 is rather close to 1, we can verify that our GIGD measure the Growth rate of genre network (and thus genre development) well.

6.7 Explaining evolutionary observations

6.7.1 cultural influence of music

In this section, we will take a closer look at Figure 2.

The minimum point of danceability appeared around 1945. Considering that World War II was underway before 1945, the whole world was immersed in grief, and artists used music to complain about the war. After World War II, the rapid economic development of the United States, artists were more willing to express their love for life, thus the danceability arise rapidly.

Energy and loudness increased dramatically from 1960 to 1980. During this period, many resistance struggles such as the conservative movement appeared in the United States. During this period, artists

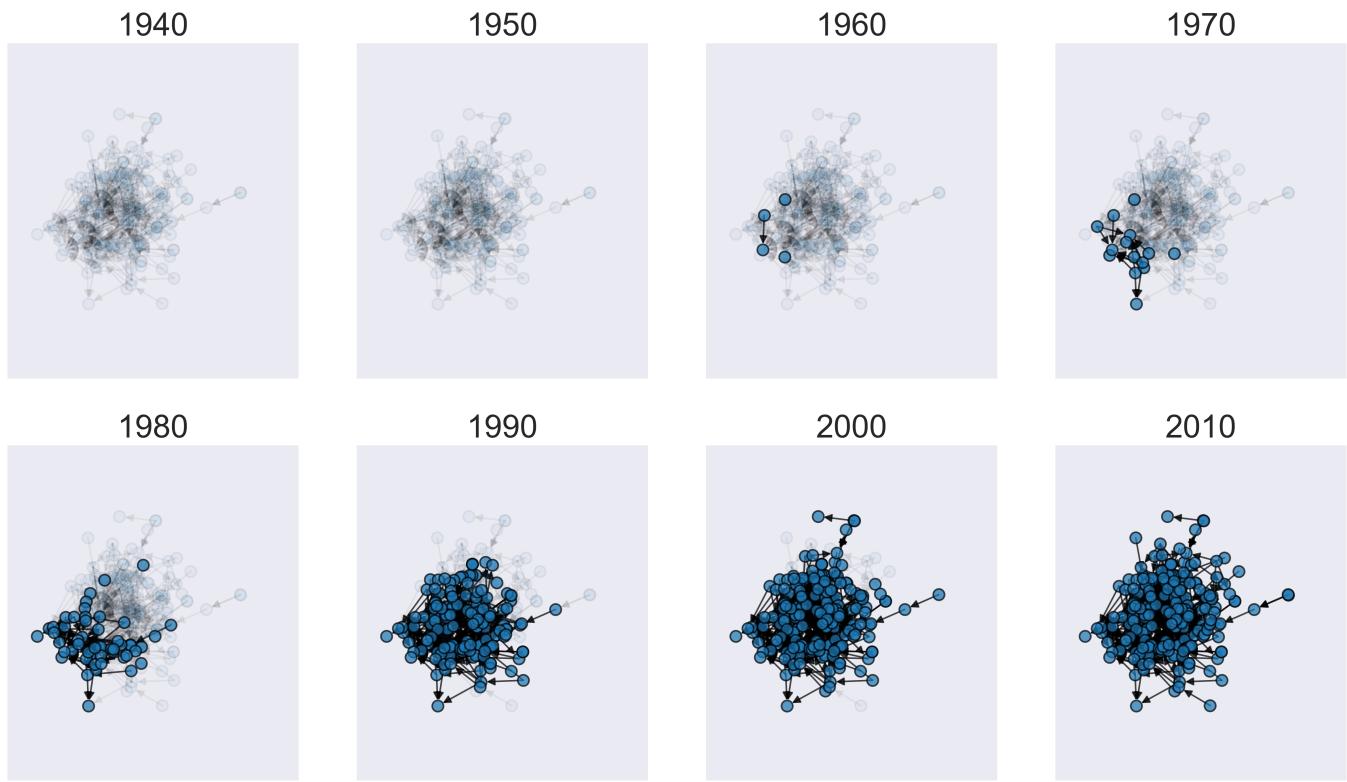


Figure 17: The evolution of influence processes within Electronic-genre network. The earliest four artists of Electronic genre emerged in 1960. With the time passing by, more and more artists are attracted by Electronic genre through being influenced by previous artists.

want to make more voices of resistance through music and guide people to fight against social injustice. For instance, Rock and roll which use high energy to express resistance is rising in this stage.

During this period, music contains fewer and fewer natural human voices and more synthetic human voices such as electronic sounds. The development of American science and technology has led to the integration of more technological elements into music. Artists promote the integration of technology and culture through music.

All in all, music has deep cultural influence over time.

6.7.2 Factors influence network

There are too many factors which influence the network. In this section, we take the Electronic music into consideration. As we can see in Figure 18, the growth speed of lovers of the Electronic music was increasing during the 1900s. Correspondingly, In the twentieth century, the level of science and technology in the United States developed rapidly, especially in the middle and late stages. This reveals that the development of technology has played some role in the development of music.

7 Discussion and Future work

7.1 Strength and Weakness

The strengths and weaknesses of our computational model are summarized as follows:

- Artist network: It can effectively model the complex system behind music influence; The topological structure and metrics of the artist network can be used to discover the influence relation between artists, and can also measure the consolidation or revolution of artists with respect to the field. However, it is

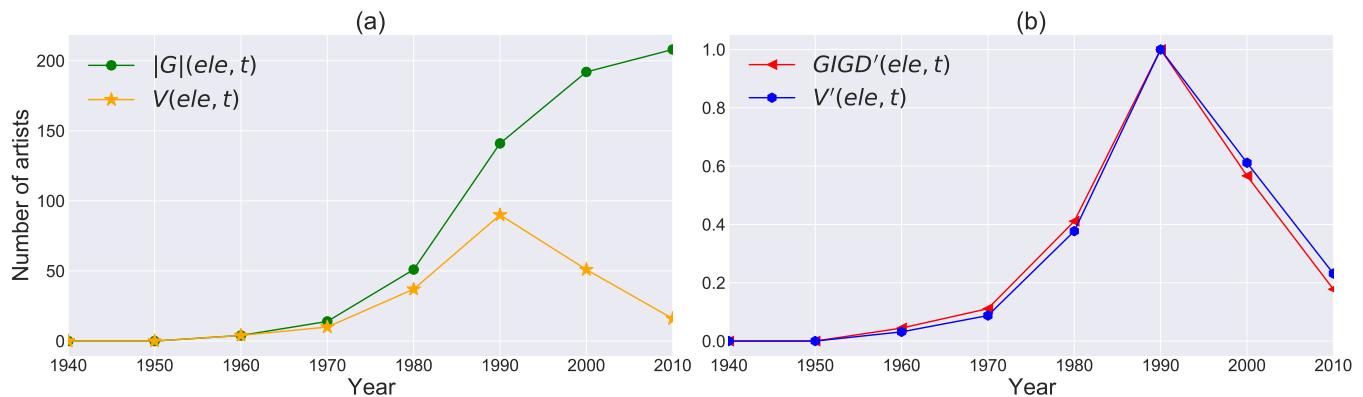


Figure 18: (a) The size of the Electronic-genre network $|G|(ele, t)$ versus time series t and the growth rate of the Electronic-genre network $V(ele, t)$ versus time series t . $V(ele, t)$ monotonically increases until it reaches the peak value at $t = 1990$, and then monotonically decreases until $t = 2010$. (b) The normalized General Index of Genre Development of Electronic-genre network $GIGD'(ele, t)$ versus time series t and the normalized growth rate of Electronic-genre network $V'(ele, t)$ versus time series t . The similarity of the two polylines is 99.79%, which indicates the feasibility and effectiveness of GIGD

a static network which does not consider the evolution of network elements. Similarly, we do not take the dynamic transition of music influence into account due to lack of adequate data.

- Similarity measure: The HEOM can handle both numerical and categorical music characteristic data as well as missing data. We assume that each music characteristic has the same contribution to the music, however, this is not necessarily the case in reality.
- GIGD: It can effectively measure the development (growth rate) of a genre over time. However, the weight determined by AHP is time-invariant. Whether the relationship among popularity, influence and count will change over time is not considered in this study.
- t-SNE: It can reduce the music characteristics of the genres to two dimensions. One of the components can clearly reveal the evolution of genres. However, it can only be used for visualization, and we cannot use the outcome after dimensionality reduction to illustrate the effect of original characteristics.

7.2 Future Work

In future work, we could construct a dynamic network and investigate its temporal evolution. We can also add time dependency to the weight of indicators in GIGD. Moreover, we can consider whether the relationship among popularity, influence and count will change over time.

To further contribute to the research of musical influence, we may build up a recommendation system based on the similarity of audio characteristics through HEOM; In addition, the GIGD can help to assess the development of a genre and guide musicologists to adopt targeted measures.

LETTER

To: ICM Society

Subject: Unveil the Mystery behind Musical Evolution

Date: February 9, 2021

Dear representative of the ICM Society,

Music is undoubtedly an important part of human art, having deeply influenced our life for centuries. Unlike the traditional understanding of music through empiricism, we use a mathematical model to quantify the evolution of music and measure the influence of music on later musicians and their work. At the heart of our model is a network-based influencer detection, which is able to assist an analysis of the relationships between musicians and the underlying characteristics of music.

The advantage of using a network-based mathematical model to analyze musical influence is that it can help to discover otherwise-hidden patterns, as it has been shown by recent research social network influence. By using our proposed method, we can precisely and also intuitively quantify the influence between artists and also between genres. In addition, our method makes clever use of the network topology, giving a quantitative explanation to the revolutionary character, which is a rather fuzzy concept originally.

The data set we processed as part of our study is only a very small part of the music data available in the real life, containing mostly mainstream music. In every corner of the world, however, there will be more kinds of music to discover, with rich characteristics. With the increasing coverage of larger data sets, one can expect many data management and scalability challenges. Powerful analysis algorithms over pairs of music often come with a quadratic (or worse) time complexity. Accordingly, one would need more efficient models and processing techniques, possibly making use of filtering, heuristic grouping, window-based methods etc. Moreover, the entire music influence network might not be interconnected: African indigenous music, for example, is expected to be almost entirely unaffected by traditional Chinese folk music. Therefore, it might be possible to analyze the network in individual chunks. In addition, the proliferation of music genres and artists will greatly increase the diversity and complexity of musical characteristics. Existing musical features may not be representative of the sound information of music. We may need to introduce more types of data, which will further increase the complexity of the task. In the part of measuring the development and evolution of music genres, because some music genres may be relatively close, the GIGD model we designed will also need to be adjusted. The change of similarity of music characteristics under a fixed time difference may be a good indicator to measure.

We think that in the future, when studying the influence of music on culture, one should consider psychological research as well. Different music conveys different emotions to the audience. For instance, we feel joy when we listen to Beethoven's Pastoral Symphony, and sorrow and anger when we hear Chopin's Revolutionary Étude. It is an exciting research direction to quantify the emotion expressed by music through models. Combined with the relevant theories of psychology and anthropology, future research may reveal the interactive relationship between culture and music in a more profound way.

Best wishes,

ICM team members

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