A Assignment

Questions Theory & Assignment

For the evaluation, please hand in by 08/11/2024:

- Your response to the theory questions below
- A summary of your solution of the assignment, with some plots
- The source code of your solution

All that on about 2..3 pages only, keep it compact!

Questions Theory:

1. Write down the conservation law for the angular momentum ru_φ. What are the source and sink terms?
2. Choose a control volume containing the rotor disk, and write down the integral form of the conservation equation.
3. Assume a constant angular velocity Ω inside the wake deficit region of the Jensen/Frandsen wake model $0 \leq r \leq D_W(x)$ (no rotation for $r > D_W(x)$), with $u_\varphi = \Omega r$. Derive the formula for $\Omega(x)$ from the integral form using $D_W(x)$ as a radius of the control volume.

Assignment: Aerodynamic forcing of the fore-aft bending of a wind turbine due to turbulence

The total thrust force on a wind turbine rotor moving with velocity $\dot{x}(t)$ can be approximated using the thrust coefficient:

$$F_{Thrust}(t) = C_t \frac{\rho}{2}\left(u_{Wind} - \dot{x}(t)\right)^2 A_{Rotor}$$

$$= C_t \frac{\rho}{2} \int_0^{2\pi} \int_0^R \left(u_{Wind} - \dot{x}(t)\right)^2 r \mathrm{d}r \mathrm{d}\varphi$$

Filling in an environmental model for the wind speed based on a shear profile and a turbulence model with linearized transport

$$u_{Wind}(x, y, z, t) = U_{Shear}(z) + u_{Turb}(x - Ut, y, z)$$

What value should you use for C_t? According to Blade Element – Momentum Theory, the relationship to the axial induction factor is

$$C_t = 4(1 - a)a$$

Assignment: Aerodynamic forcing of the fore-aft bending of a wind turbine due to turbulence

Thrust coefficient

$$C_t = \frac{F_{Thrust}}{\frac{\rho}{2} U_\infty^2 A_{Rotor}}$$

The following terms are obtained:

$$F_{Thrust}(t)$$

$$= C_t \frac{\rho}{2} \left[\int_0^{2\pi} \int_0^R U_{Shear}^2 \mathrm{r dr d}\varphi + \int_0^{2\pi} \int_0^R u_{Turb}^2 \mathrm{r dr d}\varphi + \dot{x}^2 A_{Rotor} \right.$$

$$- 2\dot{x} \int_0^{2\pi} \int_0^R U_{Shear} \mathrm{r dr d}\varphi - 2\dot{x} \int_0^{2\pi} \int_0^R u_{Turb} \mathrm{r dr d}\varphi$$

$$\left. + 2 \int_0^{2\pi} \int_0^R U_{Shear} u_{Turb} \mathrm{r dr d}\varphi \right]$$

05/12/2023

Assignment: Aerodynamic forcing of the fore-aft bending of a wind turbine due to turbulence

Analysing the terms:

- $\int_0^{2\pi}\int_0^R u_{Turb} r\mathrm{d}r\mathrm{d}\varphi = 0$ should average out to zero →to confirm!
- $\dot{x}^2 A_{Rotor}$ is a non-linear term → neglected to linearise the problem, to confirm!
- $\mathrm{C}_t \rho \dot{x} \int_0^{2\pi}\int_0^R U_{Shear} r\mathrm{d}r\mathrm{d}\varphi$ is linear in $\dot{x}$, therefore

$$\gamma = \mathrm{C}_t \rho \int_0^{2\pi}\int_0^R U_{Shear} r\mathrm{d}r\mathrm{d}\varphi$$

  acts as a damping constant to the motion

That leaves for the forcing of the tower vibrations:

$$F_{RHS}(t) = C_t \frac{\rho}{2}\int_0^{2\pi}\int_0^R (U_{Shear} + u_{Turb})^2 r\mathrm{d}r\mathrm{d}\varphi$$

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Coordinate transform:

$$x = x$$
$$y = r\cos\varphi$$
$$z = z_{Hub} + r\sin\varphi$$
$$r = \sqrt{y^2 + (z - z_{Hub})^2}$$
$$\varphi = \arctan\frac{z - z_{Hub}}{y}$$

Model parameters:

- Hub height z_{Hub}, in m
- Mass of the RNA M (Rotor-Nacelle Assembly), in kg
- Spring constant of the tower K, in N/m=kg/s^2

Eigenfrequency of the tower:

$$f = \sqrt{\frac{M}{K}}$$

2nd order dynamic system of the nacelle motion:

$$M\ddot{x}(t) + \gamma\dot{x}(t) + Kx(t) = F_{RHS}(t)$$

with:

- Damping constant $\gamma = \mathrm{C}_t\rho\int_0^{2\pi}\int_0^R U_{Shear}\mathrm{r}\mathrm{d}\mathrm{r}\mathrm{d}\varphi$
- Forcing/excitation

$$F_{RHS}(t) = C_t\frac{\rho}{2}\int_0^{2\pi}\int_0^R (U_{Shear} + u_{Turb})^2\mathrm{r}\mathrm{d}\mathrm{r}\mathrm{d}\varphi$$

Assignment: Aerodynamic forcing of the fore-aft bending of a wind turbine due to turbulence

Programming tasks:

- Write a function for $U_{Shear}(z)$ (power law or log law, your choice)
- Implement the isotropic Mann model according to the Matlab example, to compute all three velocity components of $u_{Turb}(x,y,z)$, $v_{Turb}(x,y,z)$ and $w_{Turb}(x,y,z)$
- Write a function that approximates the integral numerically by a sum:

$$\int_0^{2\pi}\int_0^R f(r,\varphi) r \mathrm{d}r \mathrm{d}\varphi \cong \sum_{n=1}^{N} w_n f(r_n,\varphi_n)$$

- The quadrature nodes $\tilde{r}_n$ and as well as the quadrature weights $\widetilde{w}_n$ are given to you as a table for a unit disk ($R = 1$), the scaled nodes and weights for any rotor radius R are obtained by scaling $r_n = R\tilde{r}_n$ and $w_n = R^2\widetilde{w}_n$, also write a function for that and the transform to cartesian coordinates!
- Write a function that interpolates u_{Turb} at arbitrary x, y, z

Assignment: Aerodynamic forcing of the fore-aft bending of a wind turbine due to turbulence

Spatial resolution turbulent velocity component:

$$\Delta x = \frac{6000\mathrm{m}}{8192} = 0.7324\mathrm{m}$$

$$\Delta y = \Delta z = \frac{384\mathrm{m}}{64} = 6\mathrm{m}$$

Preparational tasks:

- Assume a transport velocity of $U = 10\,\frac{\mathrm{m}}{\mathrm{s}}$, and generate u_{Turb} for a grid $L_x = 600\mathrm{s} * 10\,\frac{\mathrm{m}}{\mathrm{s}} = 6000\mathrm{m}$ long, $L_y = L_z = 384\mathrm{m}$ wide and high; the turbulence field should be centered around $y = 0$ and $z = 192\mathrm{m}$ (shifted $L_z/2$ upwards) to fully cover the rotor disk
- Use an integral length scale of $L = 35.4\mathrm{m}$ as a parameter for the von Kármán spectral tensor
- Use $N_x = 8192$ and $N_y = N_z = 64$ or $N_y = N_z = 128$ (the latter only if the amount of memory in your computer is large enough)

Using this wind field, you now have data for 8192 time steps spaced by $\Delta t = \frac{6000\mathrm{m}}{8192 \cdot 10\frac{\mathrm{m}}{\mathrm{s}}} = 0.07324s$, for which you can integrate the forcing term $F_{Thrust}(t)$ per time step, the damping constant γ does not depend on time

Assignment: Aerodynamic forcing of the fore-aft bending of a wind turbine due to turbulence

100m rotor:
- $z_{Hub} = 100\mathrm{m}$
- $M = 1.4 \cdot 10^5 \mathrm{kg}$
- $K = 2.5 \cdot 10^6 \frac{\mathrm{kg}}{\mathrm{s}^2}$

200m rotor:
- $z_{Hub} = 150\mathrm{m}$
- $M = 6.5 \cdot 10^5 \mathrm{kg}$
- $K = 2.0 \cdot 10^6 \frac{\mathrm{kg}}{\mathrm{s}^2}$

300m rotor:
- $z_{Hub} = 200\mathrm{m}$
- $M = 1.9 \cdot 10^6 \mathrm{kg}$
- $K = 1.8 \cdot 10^6 \frac{\mathrm{kg}}{\mathrm{s}^2}$

From previous steps:
- Compute the forcing time series $F_{RHS}(t)$ at all 8192 time steps for a given rotor radius R
- Compute the damping constant γ for a given rotor radius R and hub height z_{Hub}

Solve the 2nd order dynamical system:
- Fourier-transform the equation

$$-\omega^2 M \hat{x}(\omega) + \mathrm{i}\omega\gamma\hat{x}(\omega) + K\hat{x}(\omega) = \hat{f}(\omega)$$

- Solve the system in Fourier space

$$\hat{x}(\omega) = \frac{1}{-\omega^2 M + \mathrm{i}\omega\gamma + K}\hat{f}(\omega)$$

- Apply the inverse Fourier transform to obtain $x(t)$ for a given set of parameters (you might get very a small imaginary part due to numerical errors in the Fourier transform)
- Check that the two terms that were neglected are indeed small!

Assignment: Aerodynamic forcing of the fore-aft bending of a wind turbine due to turbulence

Try to answer the following questions:

- Plot the three power spectral densities; how does the fore-aft movement change when moving to larger rotors?
- Plot the three motion time series; what can you say about amplitudes and oscillations of the time series?
- Compute the time series of the bending moments at the tower bottom

Quadrature over the Unit Disk, 57 nodes

weight w	y	z	r	phi
0.114983341799986	0	0	0	0
0.042666281539387	-0.886967663163937	0	0.886967663163937	3.141592653589793
0.042666281539387	0	-0.886967663163937	0.886967663163937	-1.570796326794897
0.042666281539387	0	0.886967663163937	0.886967663163937	1.570796326794897
0.042666281539387	0.886967663163937	0	0.886967663163937	0
0.087938325357146	-0.424633903743234	0	0.424633903743234	3.141592653589793
0.087938325357146	0	-0.424633903743234	0.424633903743234	-1.570796326794897
0.087938325357146	0	0.424633903743234	0.424633903743234	1.570796326794897
0.087938325357146	0.424633903743234	0	0.424633903743234	0
0.076206570461793	-0.694469023080834	0	0.694469023080834	3.141592653589793
0.076206570461793	0	-0.694469023080834	0.694469023080834	-1.570796326794897
0.076206570461793	0	0.694469023080834	0.694469023080834	1.570796326794897
0.076206570461793	0.694469023080834	0	0.694469023080834	0
0.019156522218856	-0.687853540826993	-0.687853540826993	0.972771806363888	-2.356194490192345
0.019156522218856	-0.687853540826993	0.687853540826993	0.972771806363888	2.356194490192345
0.019156522218856	0.687853540826993	-0.687853540826993	0.972771806363888	-0.785398163397448
0.019156522218856	0.687853540826993	0.687853540826993	0.972771806363888	0.785398163397448
0.062085722273139	-0.596647677814557	-0.596647677814557	0.843787237923759	-2.356194490192345
0.062085722273139	-0.596647677814557	0.596647677814557	0.843787237923759	2.356194490192345
0.062085722273139	0.596647677814557	-0.596647677814557	0.843787237923759	-0.785398163397448
0.062085722273139	0.596647677814557	0.596647677814557	0.843787237923759	0.785398163397448
0.095664962820418	-0.235622520915308	-0.235622520915308	0.333220564678967	-2.356194490192345
0.095664962820418	-0.235622520915308	0.235622520915308	0.333220564678967	2.356194490192345
0.095664962820418	0.235622520915308	-0.235622520915308	0.333220564678967	-0.785398163397448
0.095664962820418	0.235622520915308	0.235622520915308	0.333220564678967	0.785398163397448
0.085162533604289	-0.548940255237015	-0.312947548883440	0.631879396856577	-2.623453175107159
0.085162533604289	-0.548940255237015	0.312947548883440	0.631879396856577	2.623453175107159
0.085162533604289	-0.312947548883440	-0.548940255237015	0.631879396856577	-2.088935805277532
0.085162533604289	-0.312947548883440	0.548940255237015	0.631879396856577	2.088935805277532
0.085162533604289	0.312947548883440	-0.548940255237015	0.631879396856577	-1.052656848312262
0.085162533604289	0.312947548883440	0.548940255237015	0.631879396856577	1.052656848312262
0.085162533604289	0.548940255237015	-0.312947548883440	0.631879396856577	-0.518139478482635
0.085162533604289	0.548940255237015	0.312947548883440	0.631879396856577	0.518139478482635
0.020201237989565	-0.961212285046179	-0.173857450886836	0.976808819653347	-2.962654139766104
0.020201237989565	-0.961212285046179	0.173857450886836	0.976808819653347	2.962654139766104
0.020201237989565	-0.173857450886836	-0.961212285046179	0.976808819653347	-1.749734840618586
0.020201237989565	-0.173857450886836	0.961212285046179	0.976808819653347	1.749734840618586
0.020201237989565	0.173857450886836	-0.961212285046179	0.976808819653347	-1.391857812971208
0.020201237989565	0.173857450886836	0.961212285046179	0.976808819653347	1.391857812971208
0.020201237989565	0.961212285046179	-0.173857450886836	0.976808819653347	-0.178938513823689
0.020201237989565	0.961212285046179	0.173857450886836	0.976808819653347	0.178938513823689
0.056834571713157	-0.790354875311486	-0.305387322252147	0.847302924296247	-2.772871497753526
0.056834571713157	-0.790354875311486	0.305387322252147	0.847302924296247	2.772871497753526
0.056834571713157	-0.305387322252147	-0.790354875311486	0.847302924296247	-1.939517482631164
0.056834571713157	-0.305387322252147	0.790354875311486	0.847302924296247	1.939517482631164
0.056834571713157	0.305387322252147	-0.790354875311486	0.847302924296247	-1.202075170958629
0.056834571713157	0.305387322252147	0.790354875311486	0.847302924296247	1.202075170958629
0.056834571713157	0.790354875311486	-0.305387322252147	0.847302924296247	-0.368721155836267
0.056834571713157	0.790354875311486	0.305387322252147	0.847302924296247	0.368721155836267
0.024268628331346	-0.849372904096328	-0.462700565982938	0.967226004599732	-2.642784840531044
0.024268628331346	-0.849372904096328	0.462700565982938	0.967226004599732	2.642784840531044
0.024268628331346	-0.462700565982938	-0.849372904096328	0.967226004599732	-2.069604139853646
0.024268628331346	-0.462700565982938	0.849372904096328	0.967226004599732	2.069604139853646
0.024268628331346	0.462700565982938	-0.849372904096328	0.967226004599732	-1.071988513736147
0.024268628331346	0.462700565982938	0.849372904096328	0.967226004599732	1.071988513736147
0.024268628331346	0.849372904096328	-0.462700565982938	0.967226004599732	-0.498807813058749
0.024268628331346	0.849372904096328	0.462700565982938	0.967226004599732	0.498807813058749

Quadrature over the Unit Disk, 57 nodes

Cools & Kim: *A survey of known and new cubature formulas for the unit disk.* 2000 https://www.cs.kuleuven.be/publicaties/rapporten/tw/TW300.pdf