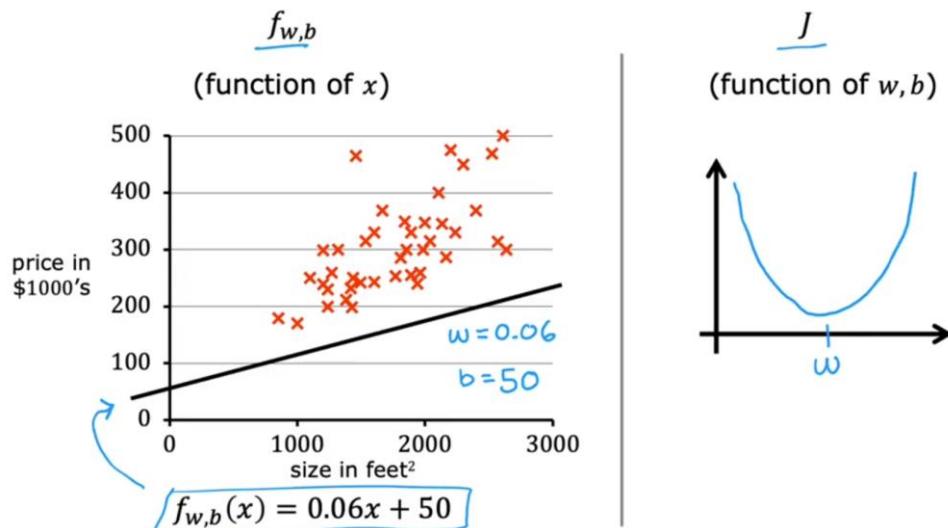
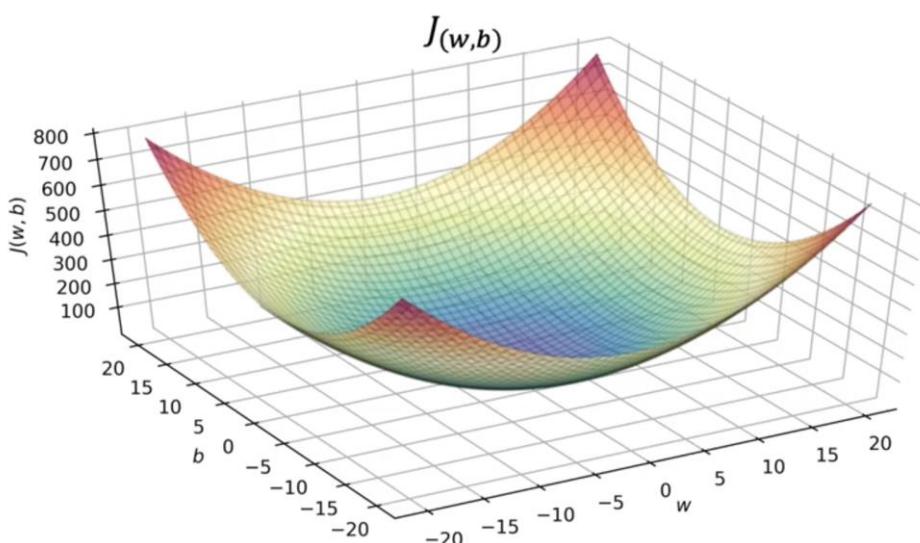


## Visualisation de la fonction de coût

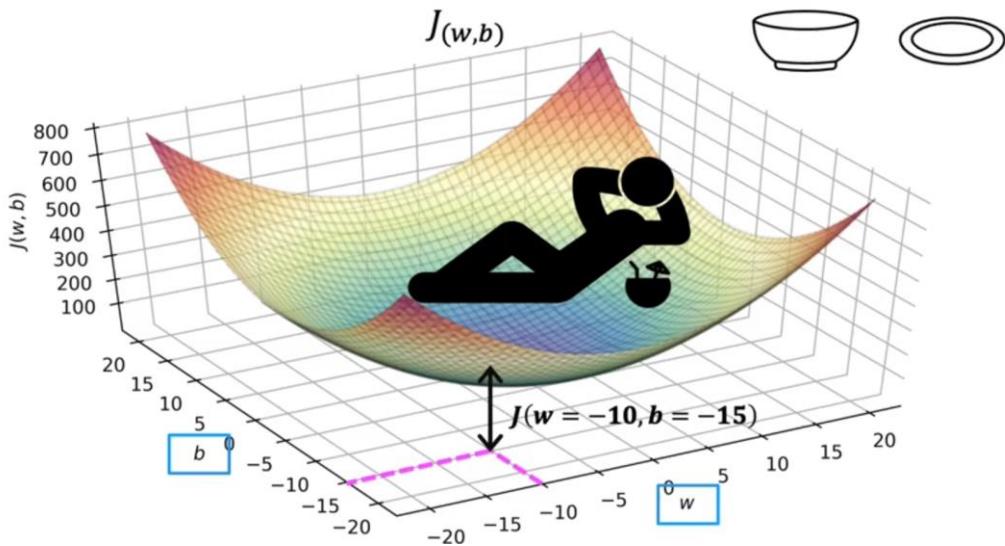
In the last video, we had temporarily set  $b$  to zero in order to simplify the visualizations. Now, let's go back to the original model with both parameters  $w$  and  $b$  without setting  $b$  to be equal to 0. Same as last time, we want to get a visual understanding of the model function,  $f$  of  $x$ , shown here on the left, and how it relates to the cost function  $J$  of  $w, b$ , shown here on the right. Here's a training set of house sizes and prices. Let's say you pick one possible function of  $x$ , like this one. Here, I've set  $w$  to 0.06 and  $b$  to 50.  $f$  of  $x$  is 0.06 times  $x$  plus 50. Note that this is not a particularly good model for this training set, is actually a pretty bad model. It seems to consistently underestimate housing prices. Given these values for  $w$  and  $b$  let's look at what the cost function  $J$  of  $w$  and  $b$  may look like. Recall what we saw last time was when you had only  $w$ , because we temporarily set  $b$  to zero to simplify things, but then we had come up with a plot of the cost function that look like this as a function of  $w$  only. When we had only one parameter,  $w$ , the cost function had this U-shaped curve, shaped a bit like a soup bowl.



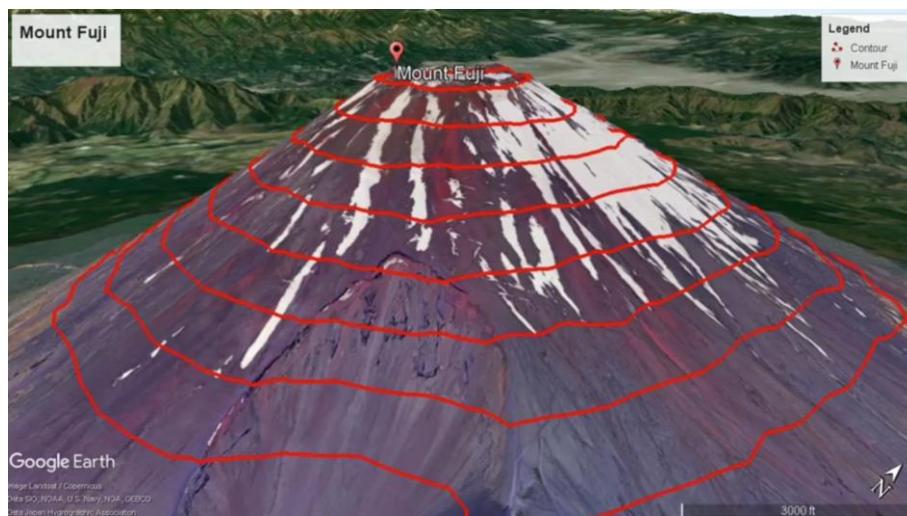
Now, in this housing price example that we have on this slide, we have two parameters,  $w$  and  $b$ . The plots becomes a little more complex. It turns out that the cost function also has a similar shape like a soup bowl, except in three dimensions instead of two. In fact, depending on your training set, the cost function will look something like this.



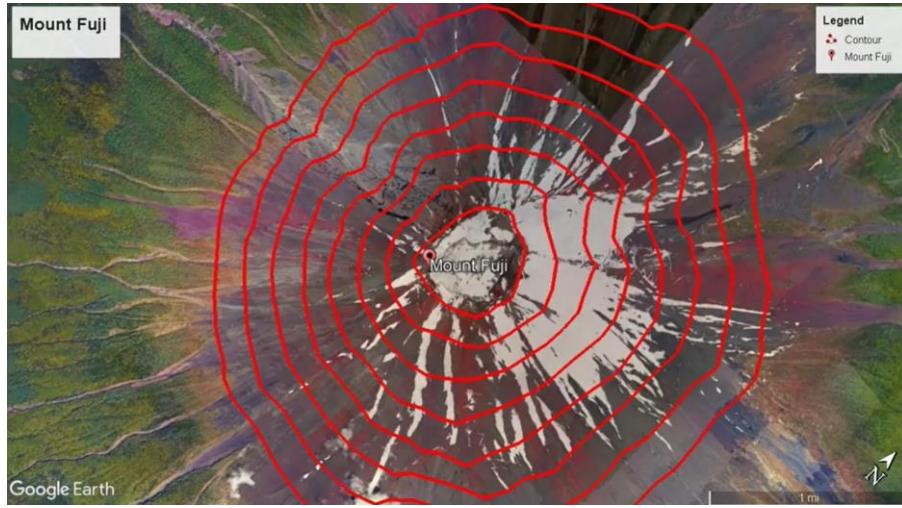
What you see here is a 3D-surface plot where the axes are labeled  $w$  and  $b$ . As you vary  $w$  and  $b$ , which are the two parameters of the model, you get different values for the cost function  $J$  of  $w$ , and  $b$ . This is a lot like the U-shaped curve you saw in the last video, except instead of having one parameter  $w$  as input for the  $J$ , you now have two parameters,  $w$  and  $b$  as inputs into this soup bowl or this hammock-shaped function  $J$ . I just want to point out that any single point on this surface represents some particular choice of  $w$  and  $b$ . For example, if  $w$  was minus 10 and  $b$  was minus 15, then the height of the surface above this point is the value of  $J$  when  $w$  is minus 10 and  $b$  is minus 15.



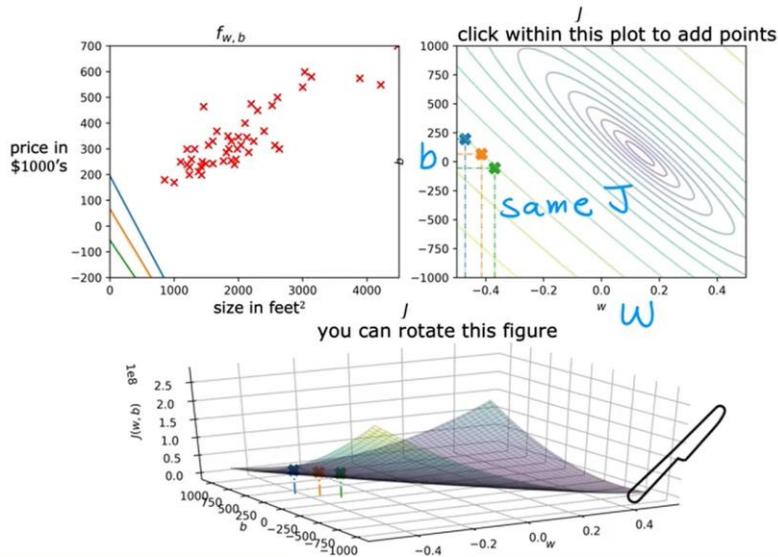
Now, in order to look even more closely at specific points, there's another way of plotting the cost function  $J$  that would be useful for visualization, which is, rather than using these 3D-surface plots, I like to take this exact same function  $J$ . I'm not changing the function  $J$  at all and plot it using something called a contour plot. If you've ever seen a topographical map showing how high different mountains are, the contours in a topographical map are basically horizontal slices of the landscape of say, a mountain. This image is of Mount Fuji in Japan.



If you fly directly above the mountain, that's what this contour map looks like. It shows all the points, they're at the same height for different heights.



At the bottom of this slide is a 3D-surface plot of the cost function  $J$ . I know it doesn't look very bowl-shaped, but it is actually a bowl just very stretched out, which is why it looks like that. In an optional lab, that is shortly to follow, you will be able to see this in 3D and spin around the surface yourself and it'll look more obviously bowl-shaped there. Next, here on the upper right is a contour plot of this exact same cost function as that shown at the bottom. The two axes on this contour plots are  $b$ , on the vertical axis, and  $w$  on the horizontal axis. What each of these ovals, also called ellipses, shows, is the center points on the 3D surface which are at the exact same height.

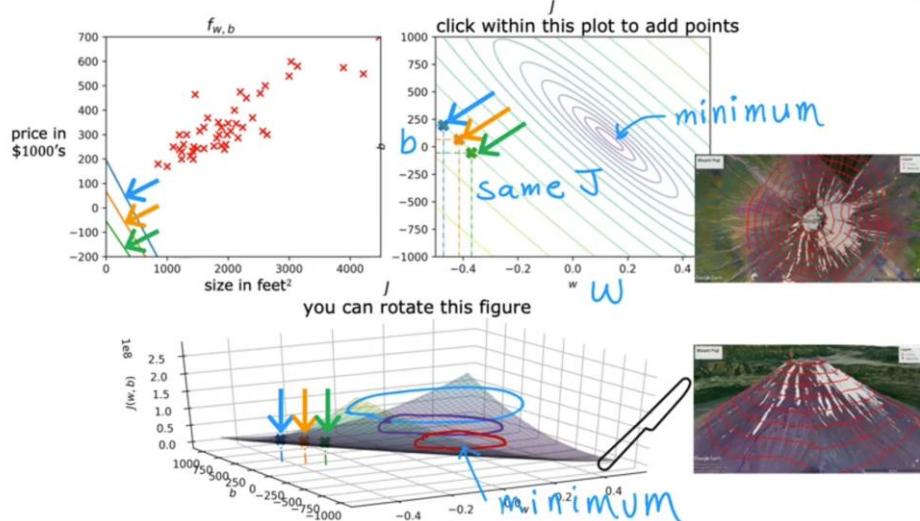


In other words, the set of points which have the same value for the cost function  $J$ . To get the contour plots, you take the 3D surface at the bottom and you use a knife to slice it horizontally. You take horizontal slices of that 3D surface and get all the points, they're at the same height. Therefore, each horizontal slice ends up being shown as one of these ellipses or one of these ovals. Concretely, if you take that point, and that point, and that point, all of these three points have the same value for the cost function  $J$ , even though they have different values for  $w$  and  $b$ .

In the figure on the upper left, you see also that these three points correspond to different functions,  $f$ , all three of which are actually pretty bad for predicting housing prices in this case.

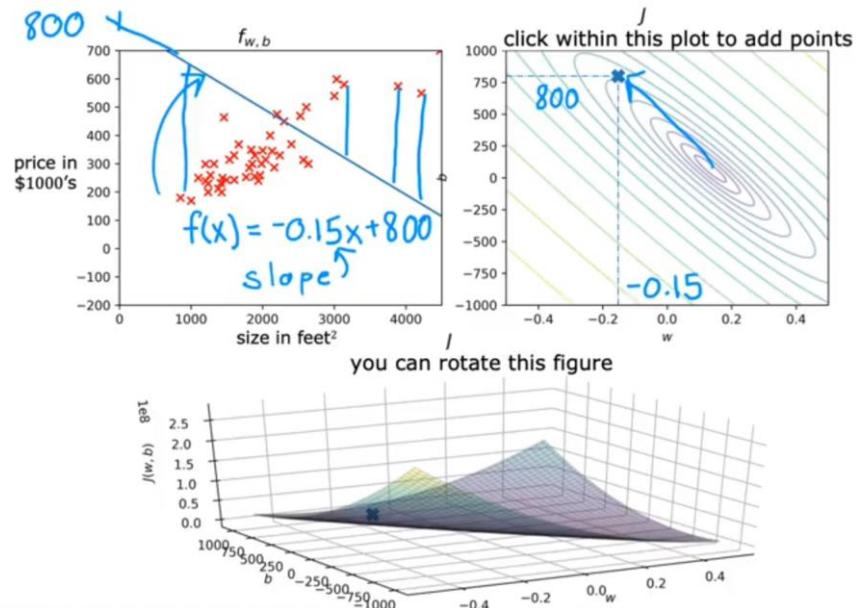
Now, the bottom of the bowl, where the cost function  $J$  is at a minimum, is this point right here, at the center of this concentric ovals. If you haven't seen contour plots much before, I'd like you to imagine, if you will, that you are flying high up above the bowl in an airplane or in a rocket ship, and you're

looking straight down at it. That is as if you set your computer monitor flat on your desk facing up and the bowl shape is coming directly out of your screen, rising above your desk. Imagine that the bowl shape grows out of your computer screen lying flat like that, so that each of these ovals have the same height above your screen and the minimum of the bowl is right down there in the center of the smallest oval.



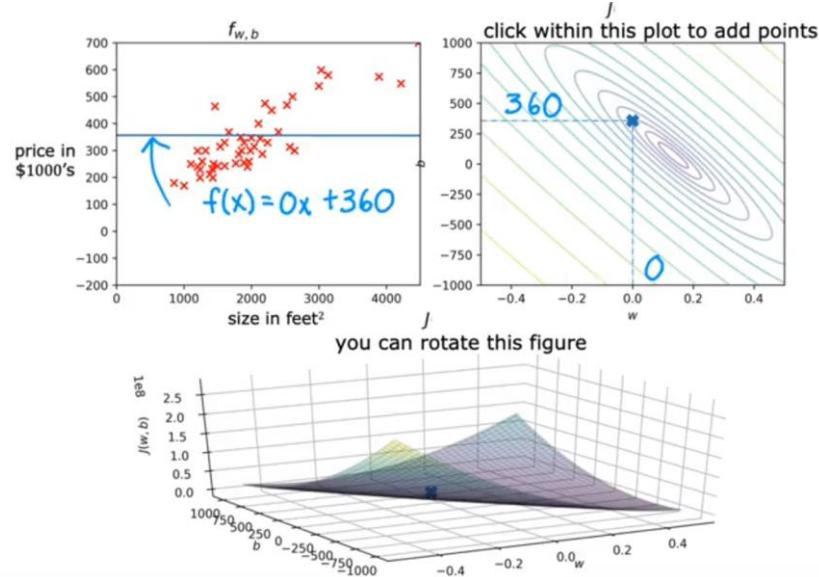
It turns out that the contour plots are a convenient way to visualize the 3D cost function  $J$ , but in a way, there's plotted in just 2D. In this video, you saw how the 3D bowl-shaped surface plot can also be visualized as a contour plot. Using this visualization too

## Exemples de visualisation

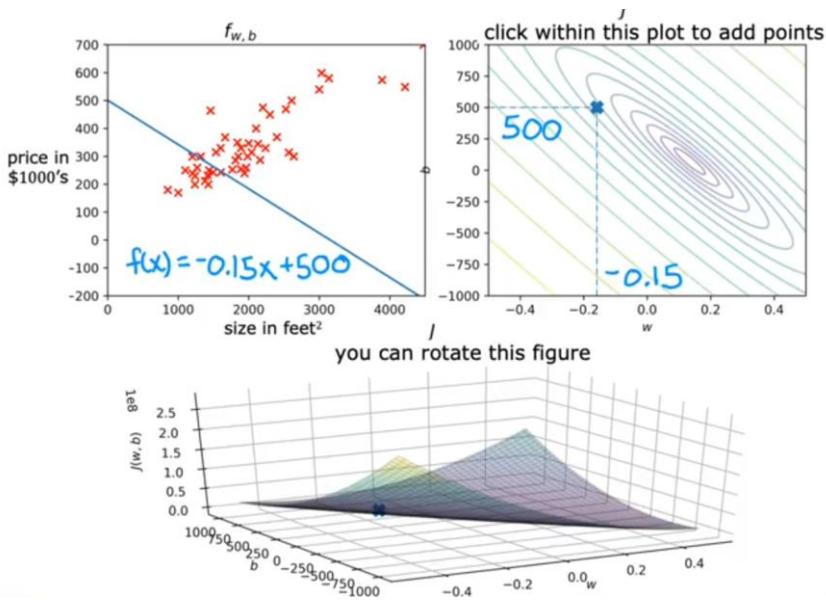


Let's look at some more visualizations of  $w$  and  $b$ . Here's one example. Over here, you have a particular point on the graph  $j$ . For this point,  $w$  equals about negative 0.15 and  $b$  equals about 800. This point corresponds to one pair of values for  $w$  and  $b$  that use a particular cost  $j$ . In fact, this booklet pair of values for  $w$  and  $b$  corresponds to this function  $f$  of  $x$ , which is this line you can see on the left. This line

intersects the vertical axis at 800 because  $b$  equals 800 and the slope of the line is negative 0.15, because  $w$  equals negative 0.15. Now, if you look at the data points in the training set, you may notice that this line is not a good fit to the data. For this function  $f$  of  $x$ , with these values of  $w$  and  $b$ , many of the predictions for the value of  $y$  are quite far from the actual target value of  $y$  that is in the training data. Because this line is not a good fit, if you look at the graph of  $j$ , the cost of this line is out here, which is pretty far from the minimum. There's a pretty high cost because this choice of  $w$  and  $b$  is just not that good a fit to the training set.

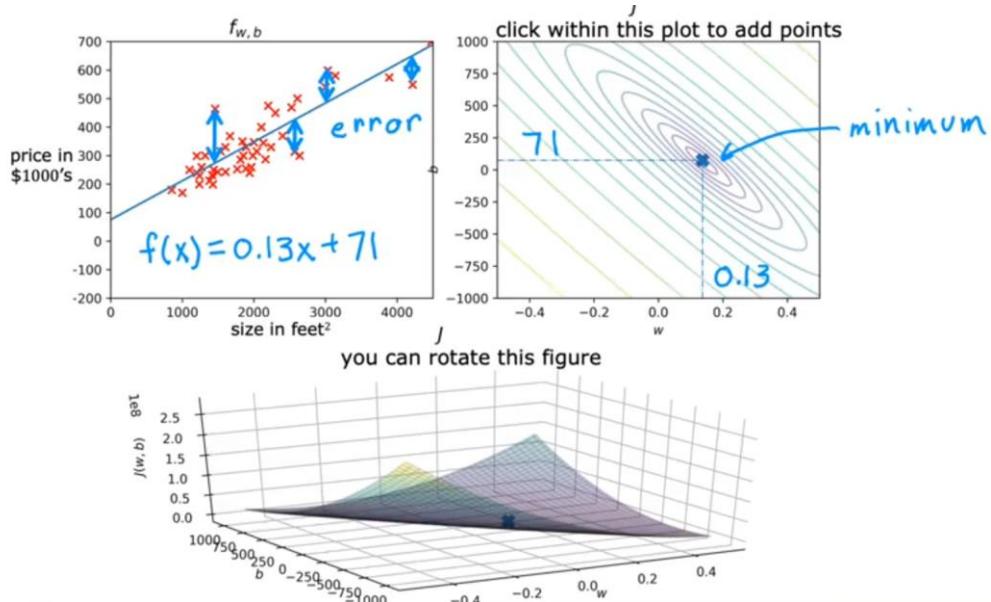


Now, let's look at another example with a different choice of  $w$  and  $b$ . Now, here's another function that is still not a great fit for the data, but maybe slightly less bad. This points here represents the cost for this booklet pair of  $w$  and  $b$  that creates that line. The value of  $w$  is equal to 0 and the value  $b$  is about 500. This pair of parameters corresponds to this function, which is a flat line, because  $f$  of  $x$  equals 0 times  $x$  plus 500. I hope that makes sense.



Let's look at yet another example. Here's one more choice for  $w$  and  $b$ , and with these values, you end up with this line  $f$  of  $x$ . Again, not a great fit to the data, is actually further away from the minimum

compared to the previous example. Remember that the minimum is at the center of that smallest ellipse.



Last example, if you look at  $f$  of  $x$  on the left, this looks like a pretty good fit to the training set. You can see on the right, this point representing the cost is very close to the center of the smaller ellipse, it's not quite exactly the minimum, but it's pretty close. For this value of  $w$  and  $b$ , you get to this line,  $f$  of  $x$ . You can see that if you measure the vertical distances between the data points and the predicted values on the straight line, you'd get the error for each data point. The sum of squared errors for all of these data points is pretty close to the minimum possible sum of squared errors among all possible straight line fits. I hope that by looking at these figures, you can get a better sense of how different choices of the parameters affect the line  $f$  of  $x$  and how this corresponds to different values for the cost  $j$ , and hopefully you can see how the better fit lines correspond to points on the graph of  $j$  that are closer to the minimum possible cost for this cost function  $j$  of  $w$  and  $b$ .