Model Predictive Control for Quadrotor under the Influence of Turbulence Flow

Ruizhe Wang, Yingzhuo Wang, Xueyang Qi

INTRODUCTION/MOTIVATION

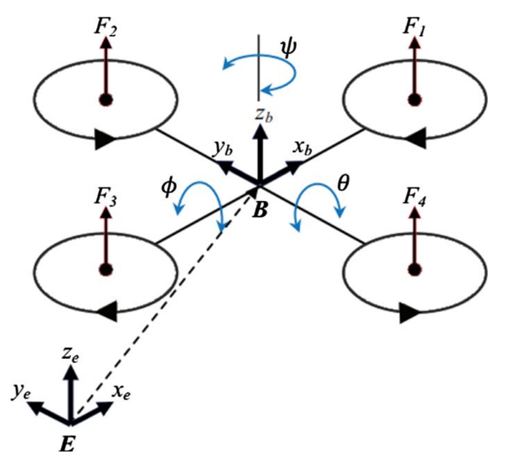
This innovative quadrotor drone project represents a significant advancement in disaster response technology. By incorporating Model Predictive Control (MPC), the drone is capable of highly precise trajectory planning, a critical factor in delivering essential supplies like food and water to areas made inaccessible by natural disasters. The ability to adapt to varying mass loads and navigate through environmental disturbances, especially those modeled by the Von Karman Wind Turbulence Model (Continuous), further enhances its effectiveness. This technology holds great promise for providing urgent aid in challenging conditions, potentially revolutionizing how we respond to natural disasters and ensure timely delivery of critical supplies.



METHODS

- Mathematical Model

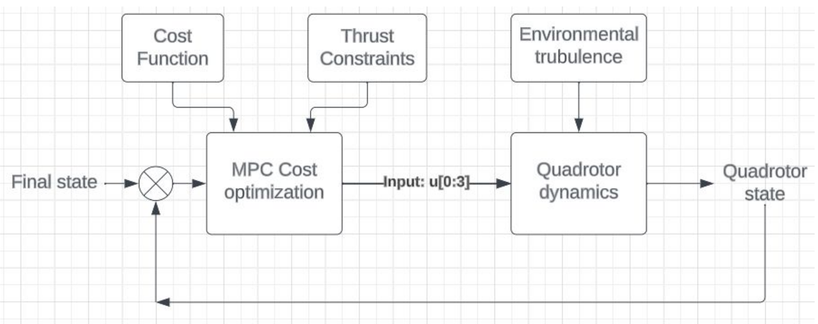
State-space model (in Euler format):



$$X = \begin{bmatrix} q \\ \dot{q} \end{bmatrix} = \begin{bmatrix} x \\ y \\ x \\ \phi \\ \theta \\ \psi \\ \dot{x} \\ \dot{y} \\ \dot{x} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \qquad \dot{X} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \\ \ddot{x} \\ \ddot{y} \\ \ddot{z} \\ \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \\ p + (\sin\phi\tan\theta)q + (\cos\phi\tan\theta)r \\ q\cos\phi - r\sin\phi \\ \frac{\sin\phi}{\cos\theta}q + \frac{\cos\phi}{\cos\theta}r \\ -(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi)\frac{u_1}{m} \\ -(\cos\phi\sin\theta\cos\psi - \sin\phi\sin\psi)\frac{u_1}{m} \\ -(\cos\phi\cos\theta)\frac{u_1}{m} + g \\ \frac{(I_y-I_z)qr+u_2}{I_x} \\ \frac{(I_z-I_x)pr+u_3}{I_y} \\ \frac{(I_x-I_y)pq+u_4}{I_z} \end{bmatrix}$$

$$u = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$$

- Linear MPC



For the non-linear dynamic system, we could linearize it through taking the first order Taylor expansion near the final states.

$$x_{k+1} = Ax_k + Bu_k \qquad A = \frac{\partial f(x,u)}{\partial x}\bigg|_{x=x_{current}}$$

$$B = \frac{\partial f(x,u)}{\partial u}\bigg|_{u=u_{current}}$$

The cost optimization is:

$$\min_{x[k],u[k]} \sum_{k=0}^{N} [(x[k]-x_{final})^T Q (x[k]-x_{final}) + (u[k]-u_{final})^T R (u[k]-u_{final})$$

$$subj.to \quad x_{k+1} = Ax_k + Bu_k$$

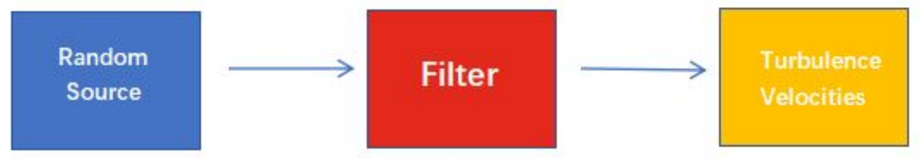
$$x[0] = x_{current}$$

$$u_{final} = \begin{bmatrix} \frac{1}{4}mg & \frac{1}{4}mg & \frac{1}{4}mg & \frac{1}{4}mg \end{bmatrix}^T$$

$$u_{lowerlimit} \le u \le u_{upperlimit}$$

METHODS Continued

- Von Karman Wind Turbulence Model (Military Specification MIL-F-8785C)



Type	Transfer Function
Longitudinal	$H_u(s) = \frac{\sigma_u\sqrt{\frac{2}{\pi V}}L_u\left(1+0.25\left(\frac{L_u}{V}\right)\right)}{1+1.357\left(\frac{L_u}{V}s\right)+0.1987\left(\frac{L_u}{V}s\right)^2}$
Lateral	$H_v(s) = \frac{\sigma_v\sqrt{\frac{1}{\pi V}}L_v\left(1+2.7478\left(\frac{L_v}{V}s\right)+0.3398\left(\frac{L_v}{V}s\right)^2\right)}{1+2.9958\left(\frac{L_v}{V}s\right)+1.9754\left(\frac{L_v}{V}s\right)^2+0.1539\left(\frac{L_v}{V}s\right)^3}$
Directional	$H_w(s) = \frac{\sigma_w\sqrt{\frac{1}{\pi V}}L_w\left(1+2.7478\left(\frac{L_w}{V}s\right)+0.3398\left(\frac{L_w}{V}s\right)^2\right)}{1+2.9958\left(\frac{L_w}{V}s\right)+1.9754\left(\frac{L_w}{V}s\right)^2+0.1539\left(\frac{L_w}{V}s\right)^3}$

For low-altitude model (altitude < 1000 ft.)

$$L_w = h$$

$$L_u = L_v = \frac{h}{(0.177+0.000823h)^{1.2}}$$

$$\sigma_w = 0.1W_{20}$$

$$\frac{\sigma_u}{\sigma_w} = \frac{\sigma_v}{\sigma_w} = \frac{1}{(0.177+0.000823h)^{0.4}}$$

For medium/high altitudes (altitudes > 2000 ft.)

$$L_u = L_v = L_w = 2500ft$$

RESULTS

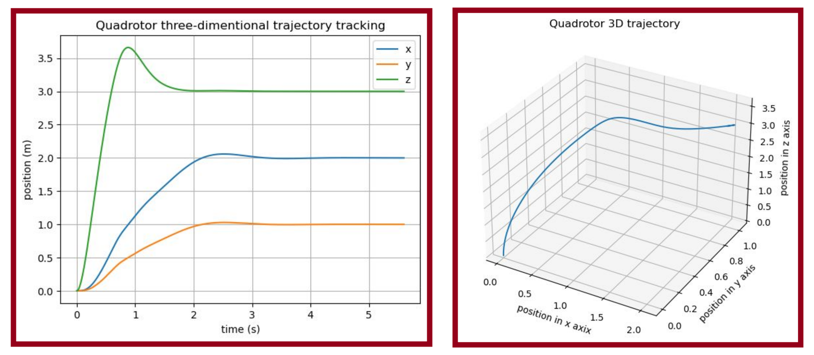


Fig.1 Trajectory of quadrotor without any disturbance.

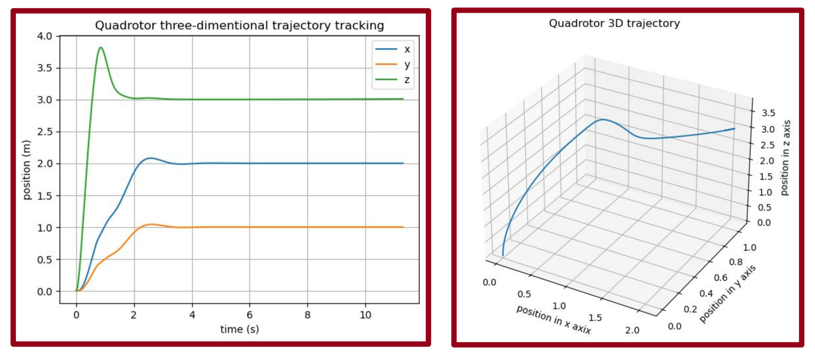


Fig.2 Trajectory of quadrotor with low speed turbulence flow.

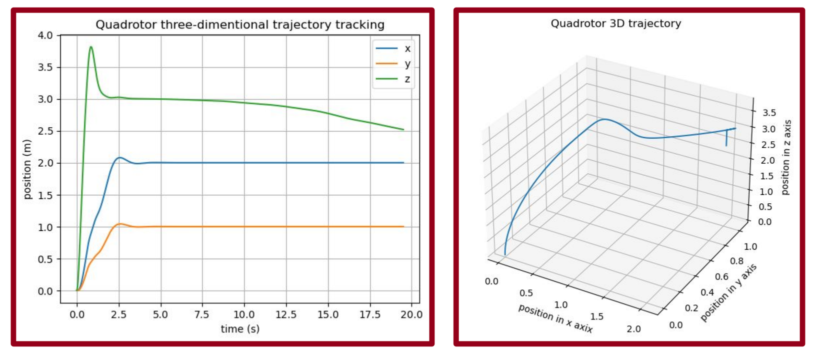


Fig.3 Trajectory of quadrotor high speed turbulence flow.

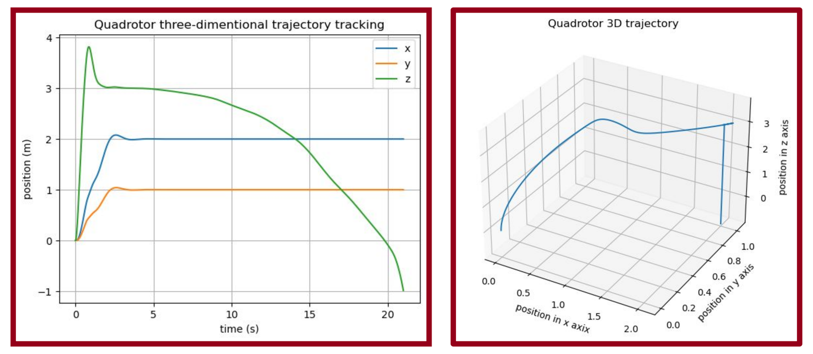


Fig.4 Trajectory of quadrotor extremely high speed turbulence flow.

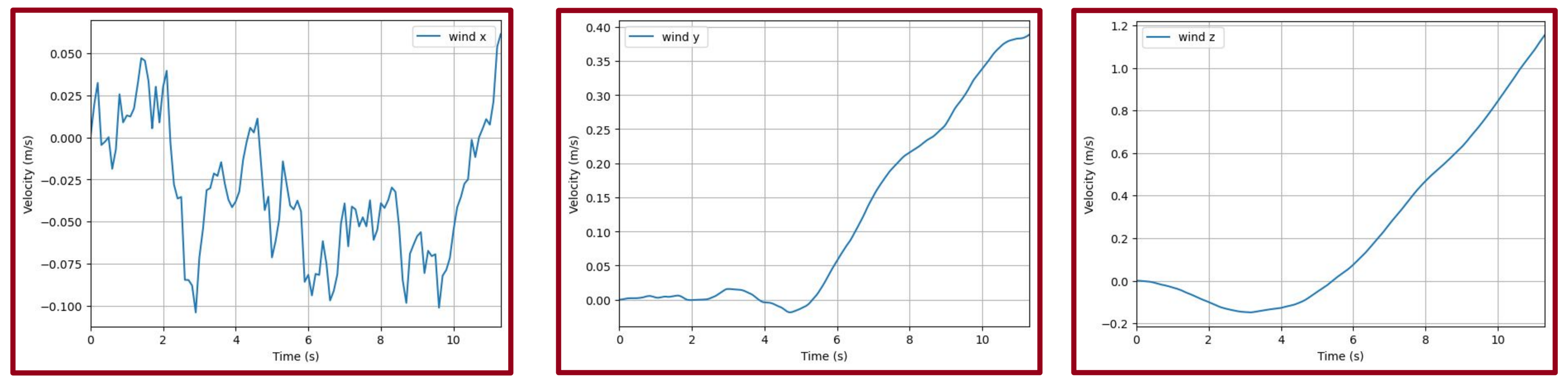


Fig.5 Turbulence flow in low speed.

RESULTS Continued

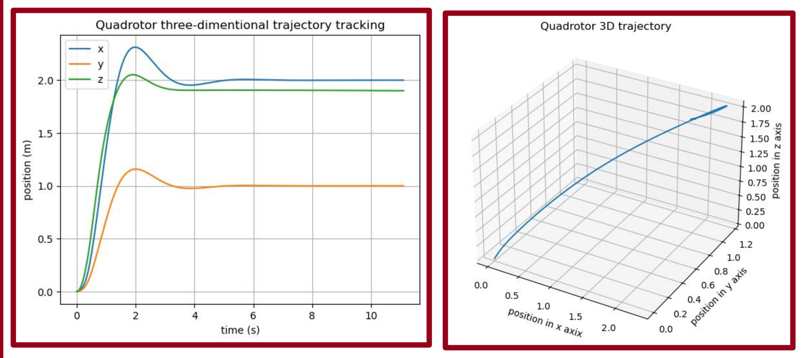


Fig.6 Trajectory of quadrotor with large payload.

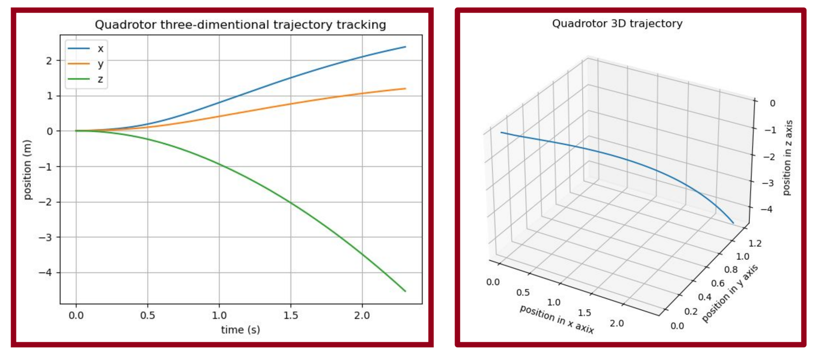


Fig.7 Trajectory of quadrotor with extremely large payload.

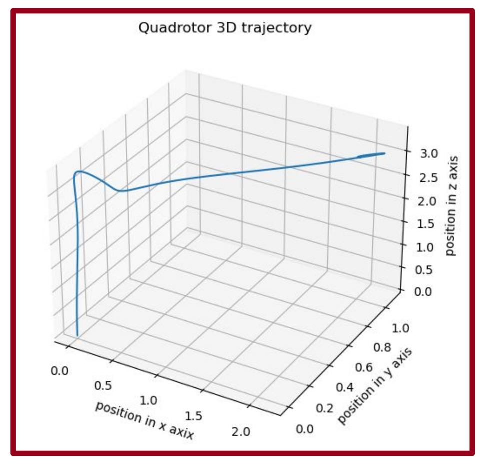
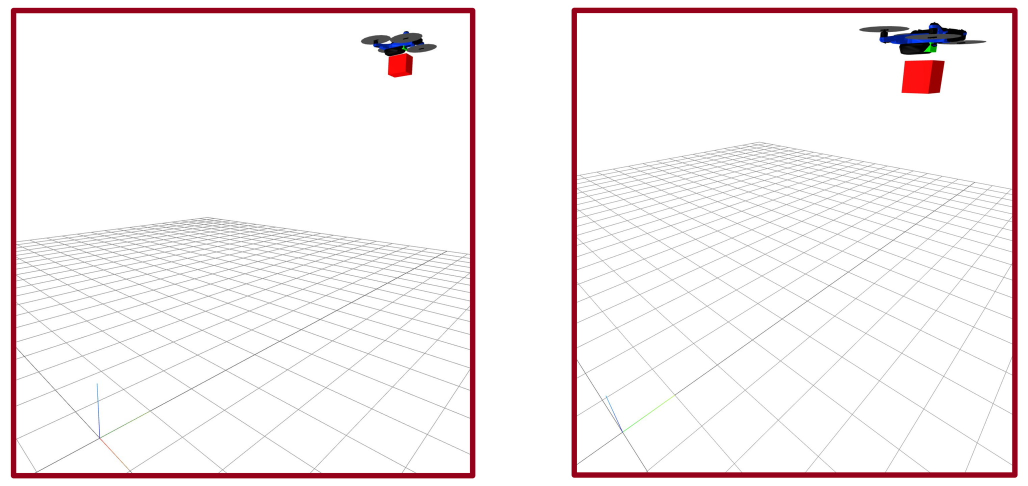


Fig.8 Trajectory of quadrotor with lager magnitude Q value corresponds to z .



a) without any disturbance.

b) with low speed turbulence flow.

Fig.9 Captured image of quadrotor final state in MeshCat 3D viewer.

The attached sets of pictures could first show that is capable of driving the quadrotor to the desired position with reasonable trajectory.

Then, aiming to test the capability of the MPC controller, we performed the following tests:

1. Trying different value in the Q and R: Figure 8 is an example plot of MPC with larger magnitude Q value corresponds to z parameter; in addition, we also tested with other sets of Q & R values and the results turned out that the trajectory changed correspondingly with no distinct final state errors. This test extended to the conditions with turbulence flow and the results turned out to be the same as the counterparts without turbulence disturbance.
2. Turbulence flow anti-interference test: As shown in the Figure 2-4, we conducted simulations under different levels of turbulence flow and finally concluded that our MPC controller was only resistant to low speed turbulence flow. As for those higher speed ones, the quadrotor was blown away from its original trajectory.
3. Payload tolerance test: Under the low speed turbulence flow environment, we further performed testing with different amount payload. In our simulations, the quadrotor could carry maximum objects in 8 times of its own mass. As for objects with weight larger than that range, the quadrotor could not fly off the ground.

FUTURE WORK

In our work, we attached the payload to the quadrotor and we calculated them as a whole. In the future, we could try to separate the payload with the quadrotor so that it can be dropped when it reaches the desired point. In real life, the payload may be attached to the quadrotor by a rope. In addition, our work was not accurate about how the quadrotor with a payload was affected by turbulence flow, so we needed to improve aerodynamics in the system.