# Interpretation

## Lecture 12

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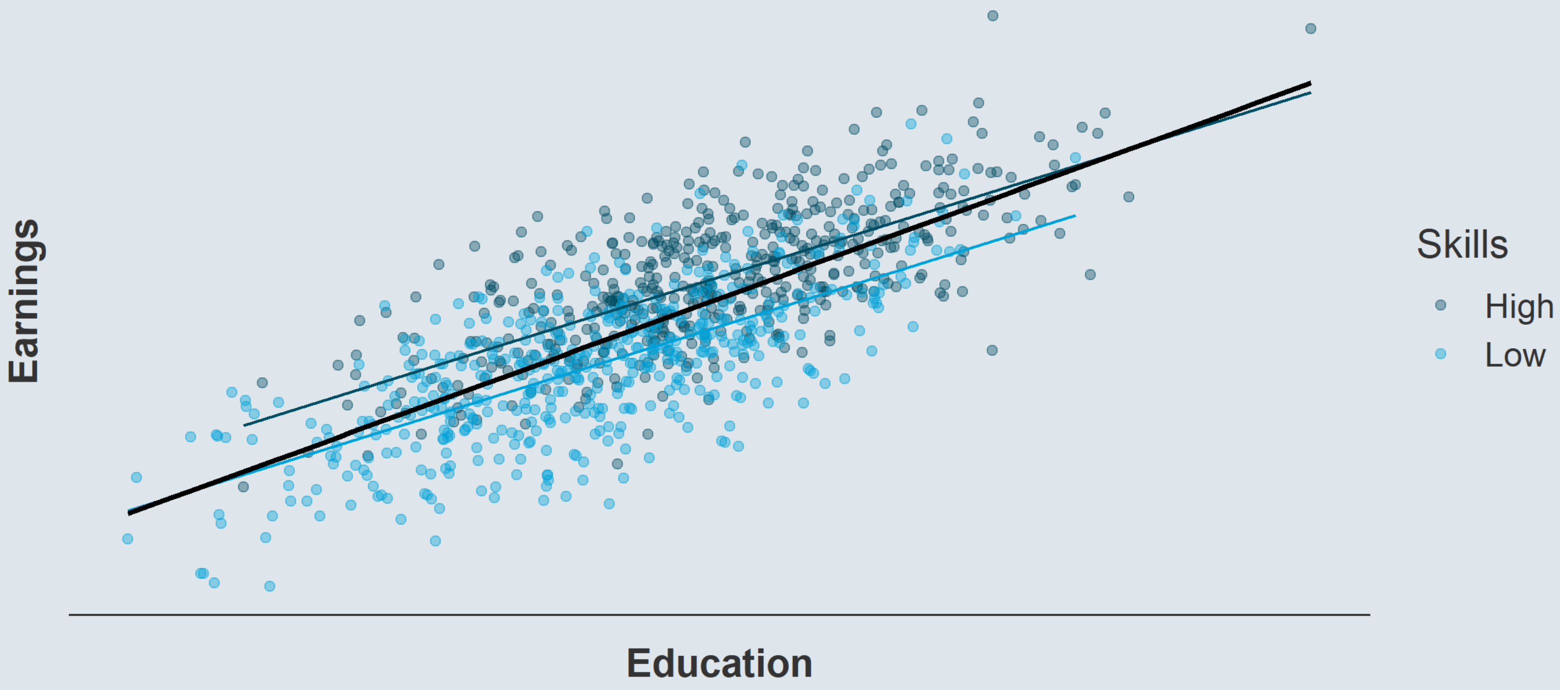
CPES 2 - Fall 2022

# Quick reminder

### Omitted variable bias

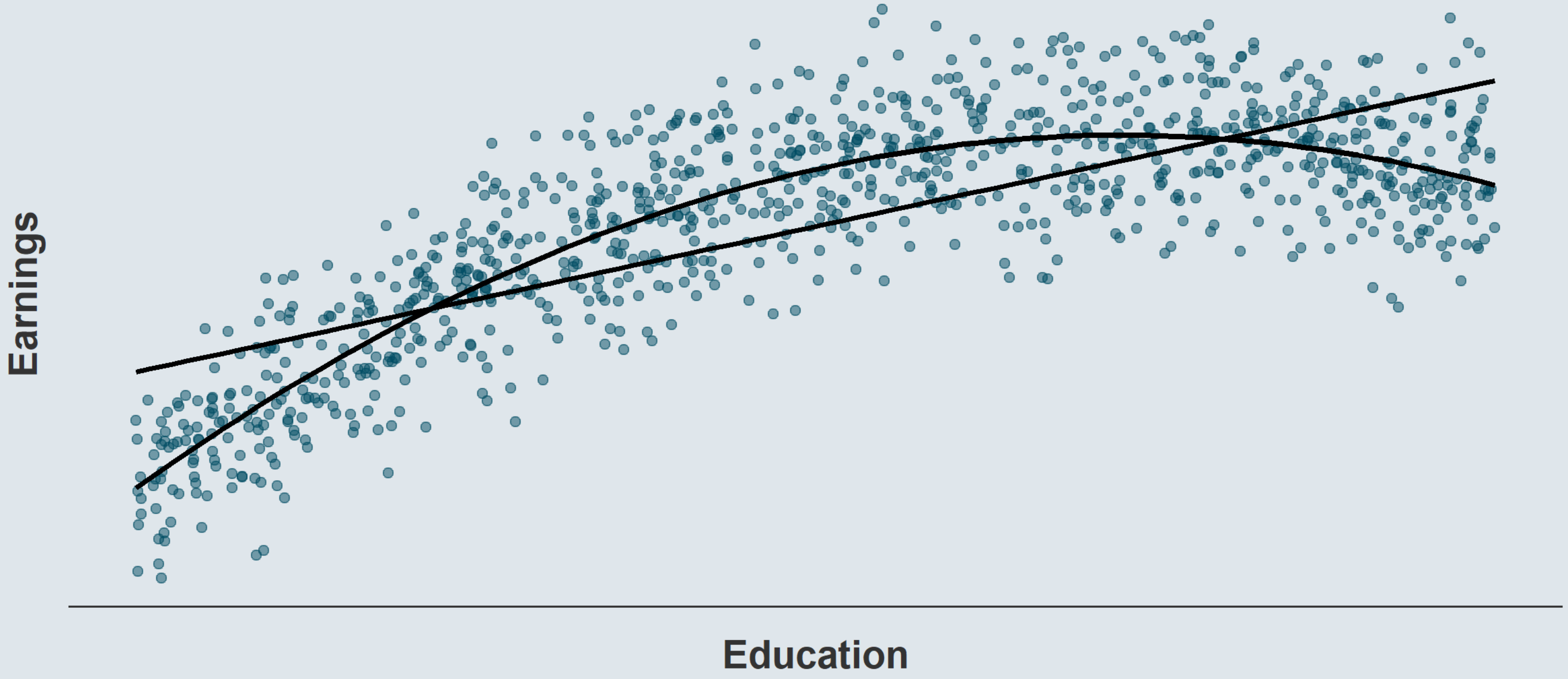
- If a third **variable** is correlated with both $x$ and $y$, it would **bias the relationship**
  - We must then **control** for such variables
  - And if we can't we must acknowledge that our estimate is not causal with ***'ceteris paribus'***



# Quick reminder

### Functional form

- Not capturing the **right functional** form correctly might also lead to biased estimations:
  - Polynomial order, interactions, logs, discretization matter
  - **Visualizing the relationship** is key



# Quick reminder

**Selection bias**

- **Self-selection** is also a common threat to causality

- What is the impact of going to a better neighborhood on your children outcomes?
    - We cannot just regress children outcomes on a mobility dummy
    - Individuals who move may be different from those who stay: **self-selection issue**
    - Here it is not that the sample is not representative of the population, but that **the outcomes of those who stayed are different from the outcomes those who moved would have had, if they had stayed**

**Simultaneity**

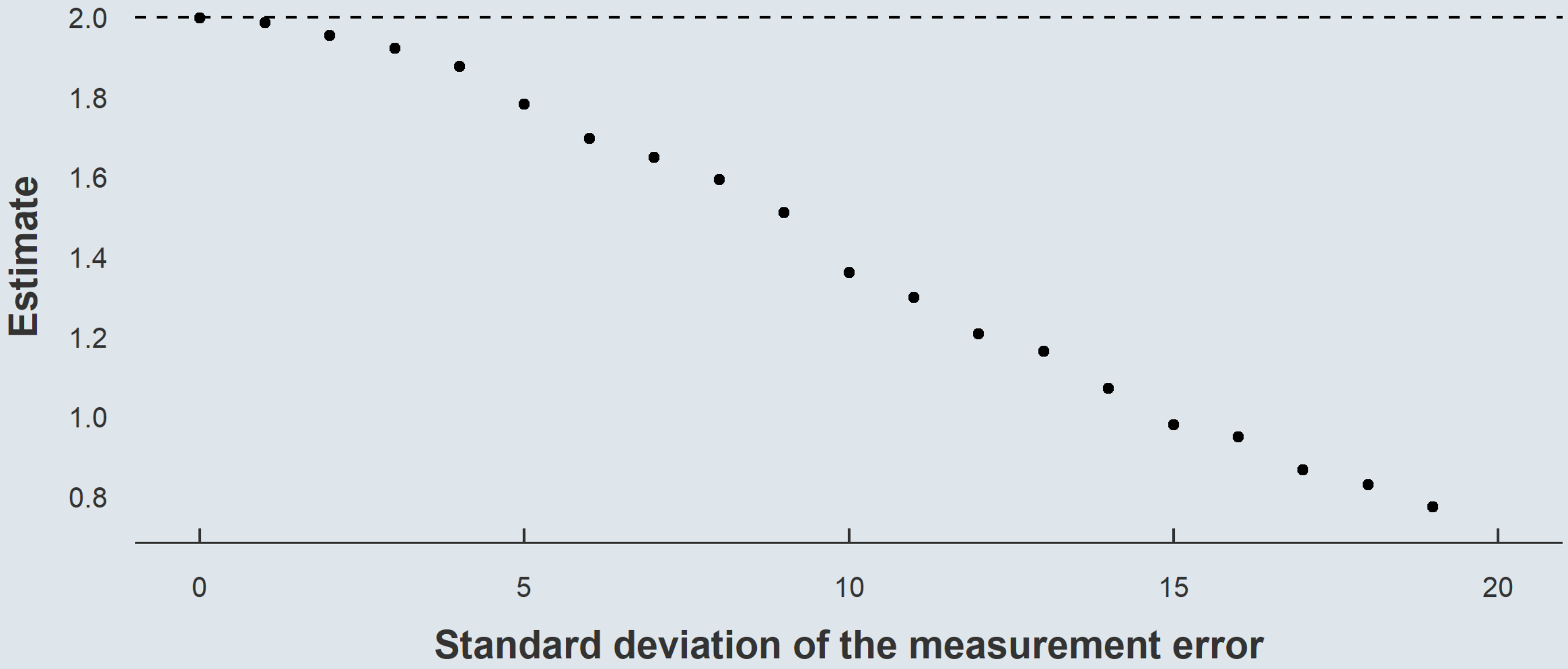
- Consider the relationship between **crime** rate and **police coverage** intensity

- What is the **direction of the relationship?**
    - We cannot just regress crime rate on police intensity
    - It's likely that more crime would cause a positive response in police activity
    - And also that police activity would deter crime

# Quick reminder

### Measurement error

- **Measurement error** in the independent variable also induces a bias
  - The resulting estimation would mechanically be **downward biased**
  - The **noisier** the measure, the **larger the bias**



# Quick reminder

**Randomized Controlled Trials**

- A Randomized Controlled Trial (RCT) is a type of experiment in which the thing we want to know the impact of (called the treatment) is **randomly allocated** in the population
  - The two **groups** would then have the same characteristics on expectation, and would be **comparable**
  - It is a way to obtain **causality** from randomness

- RCTs are very **powerful tools** to sort out issues of:
  - Omitted variables
  - Selection bias
  - Simultaneity

- But RCTs are **not immune** to every problem:
  - The sample must be representative and large enough
  - Participants should comply with their treatment status
  - Independent variables must not be noisy measures of the variable of interest
  - ...

# Today: Interpretation

# Today: Interpretation

# 1. Point estimates

### 1.1. Continuous variables

- In this first part, we're going to consider the **relationship** between:
  - The **income level** of young parents
  - The **health** of their **newborn**

- Consider first the following specification of the two variables:
  - A continuous measure of **annual household income in euros**
  - A continuous measure of **birth weight in grams**

$$\text{Birth weight}_i = \alpha + \beta \times \text{Household income}_i + \varepsilon_i$$

```
lm(birth_weight ~ household_income, data)$coefficients
```

```
##      (Intercept) household_income
##     3.134528e+03     2.213871e-03
```

**➜ How would you interpret $\hat{\beta}$ here?** *(Note that* e+03 *and* e-03 *mean* $\times 10^3$ *and* $\times 10^{-3}$*)*

# 1. Point estimates

### 1.1. Continuous variables

- When both $x$ and $y$ are continuous, the **general** template for the **interpretation** of $\hat{\beta}$ is:

*"Everything else equal, a 1 [unit] increase in [x] is associated with an [in/de]crease of [beta] [units] in [y] on average."*

- So in our case the **adequate interpretation** would be:

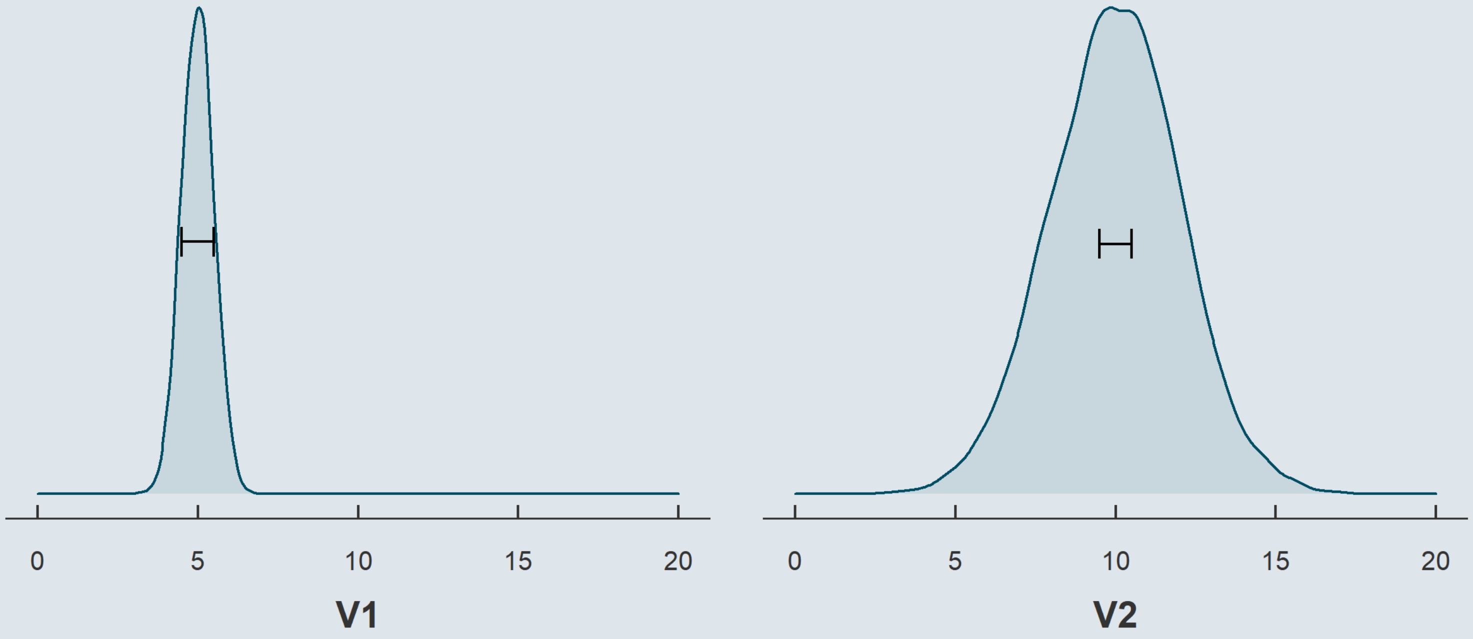
*"Everything else equal, a **1 euro increase in annual household income** is associated with an **increase of 0.002 gram in newborn birth weight** on average."*

- But it would be even better to **interpret** the results for a **meaningful variation** of $x$
  - For an annual household income, a **1 euro variation** is not really meaningful
  - 1 euro increase **→** 0.002 gram increase $\Leftrightarrow$ **1,000 euro increase →** 2 gram increase

# 1. Point estimates

## 1.1. Continuous variables

- A common way to obtain a coefficient for a **meaningful variation** of $x$ is to **standardize** $x$
  - If we divide $x$ by $\text{SD}(x)$, the 1 **unit increase** in $\frac{x}{\text{SD}(x)}$ is equivalent to an $\text{SD}(x)$ **increase** in $x$
  - An $\text{SD}(x)$ change in $x$ is meaningful: it's low if $x$ is very concentrated and high if $x$ is highly spread out



# 1. Point estimates

## 1.1. Continuous variables

- A common way to obtain a coefficient for a **meaningful variation** of $x$ is to **standardize** $x$
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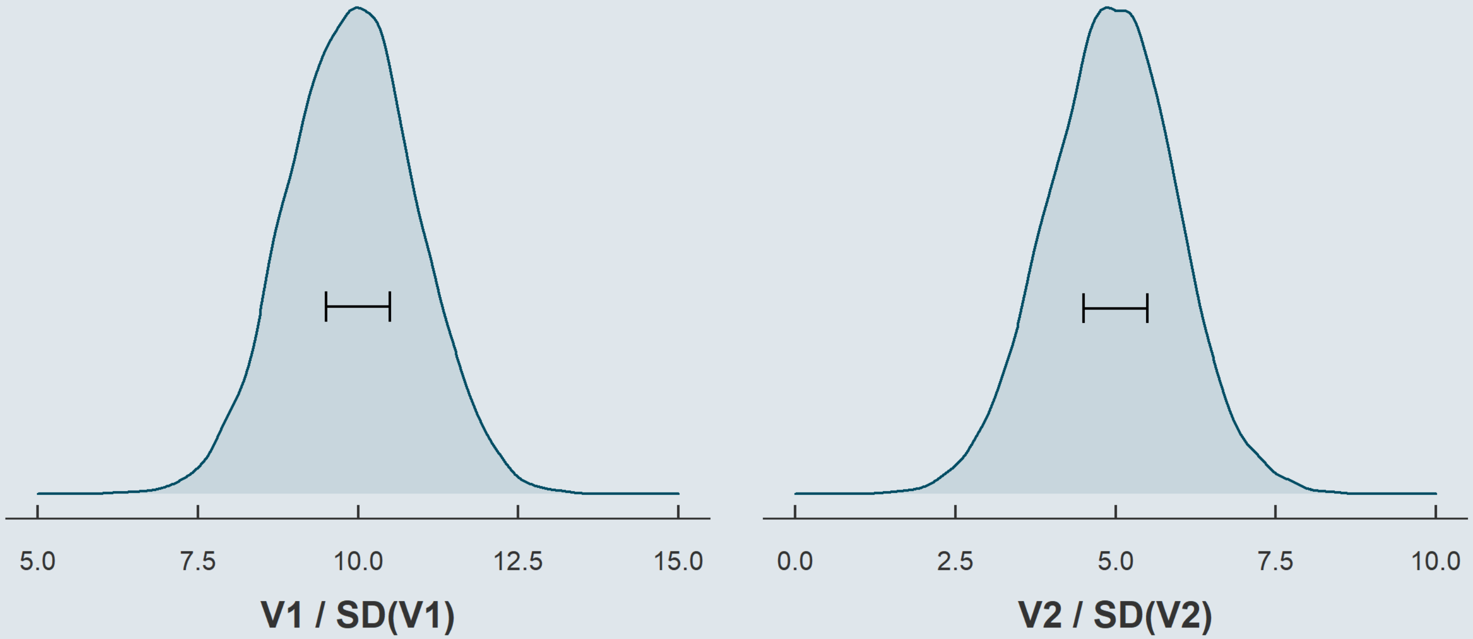


# 1. Point estimates

### 1.1. Continuous variables

- Note that if you **standardize both** $x$ **and** $y$, the resulting $\hat{\beta}$ equals the **correlation** between $x$ and $y$
  - To show that, let's first rewrite the formula of the beta coefficient:

$$\hat{\beta} = \frac{\text{Cov}(x,\, y)}{\text{Var}(x)} = \frac{\text{Cov}(x,\, y)}{\text{SD}(x) \times \text{SD}(x)}$$

$$\hat{\beta} = \frac{\text{Cov}(x,\, y)}{\text{SD}(x) \times \text{SD}(x)} \times \frac{\text{SD}(y)}{\text{SD}(y)}$$

$$\hat{\beta} = \frac{\text{Cov}(x,\, y)}{\text{SD}(x) \times \text{SD}(y)} \times \frac{\text{SD}(y)}{\text{SD}(x)}$$

$$\hat{\beta} = \text{Cor}(x,\, y) \times \frac{\text{SD}(y)}{\text{SD}(x)}$$

# 1. Point estimates

## 1.1. Continuous variables

- Starting with the previous expression, the $\hat{\beta}$ **coefficient with the standardized variables** writes:

$$\hat{\beta} = \frac{\text{Cov}\left(\frac{x}{\text{SD}(x)}, \frac{y}{\text{SD}(y)}\right)}{\text{SD}\left(\frac{x}{\text{SD}(x)}\right) \times \text{SD}\left(\frac{y}{\text{SD}(y)}\right)} \times \frac{\text{SD}\left(\frac{y}{\text{SD}(y)}\right)}{\text{SD}\left(\frac{x}{\text{SD}(x)}\right)}$$

- But by construction, the standard deviation of a standardized variable is 1:

$$\hat{\beta} = \frac{\text{Cov}\left(\frac{x}{\text{SD}(x)}, \frac{y}{\text{SD}(y)}\right)}{1 \times 1} \times \frac{1}{1}$$

$$\hat{\beta} = \text{Cov}\left(\frac{x}{\text{SD}(x)}, \frac{y}{\text{SD}(y)}\right)$$

$$\hat{\beta} = \frac{\text{Cov}(x,\, y)}{\text{SD}(x) \times \text{SD}(y)} = \text{Cor}(x, y)$$

- Learn the cheatsheet on moments properties:

$$\text{Var}(aX) = a^2\text{Var}(X)$$

# 1. Point estimates

### 1.2. Discrete variables

- Consider the following specification of the two variables:
  - A categorical variable for **annual household income divided in terciles**
  - Still a continuous measure of **birth weight in grams**

$$\text{Birth weight}_i = \alpha + \beta_1 \text{T2}_i + \beta_2 \text{T3}_i + \varepsilon_i$$

- Recall that when including a **categorical variable** in a regression, a **reference category** must be **omitted**

```
lm(birth_weight ~ income_tercile, data)$coefficients
```

```
##       (Intercept) income_tercileT2 income_tercileT3
##        3112.83162         88.24778        222.65414
```

**→ How would you interpret $\hat{\beta}_1$ and $\hat{\beta}_2$ here?**

# 1. Point estimates

### 1.2. Discrete variables

- With a discrete $x$, the interpretation of the coefficient must be **relative to the reference category:**

*"Everything else equal, belonging to the [x category] is associated with a [beta] [unit] [higher/lower] average [y] relative to the [reference category]."*

- So in our case, the **adequate interpretations** would be:

*"Everything else equal, belonging to the **second income tercile** is associated with a **88 grams higher average birth weight** relative to the **first income tercile.**"*

*"Everything else equal, belonging to the **third income tercile** is associated with a **223 grams higher average birth weight** relative to the **first income tercile.**"*

- And the **intercept** is the **average birth weight** for newborns to parents in the **first income tercile**

# 1. Point estimates

### 1.2. Discrete variables

- Consider now the following specification of the two variables:
  - A continuous measure of **annual household income in euros**
  - A **binary variable** taking the value 1 if **the newborn is underweight** and 0 otherwise

$$\text{Underweight}_i = \alpha + \beta \times \text{Household income}_i + \varepsilon_i$$

```
lm(underweight ~ household_income, data)$coefficients
```

```
##      (Intercept) household_income
##     5.214013e-02    -4.084787e-07
```

**➜ How would you interpret $\hat{\beta}$ here?**

**➜ And would you consider its magnitude high?**

# 1. Point estimates

## 1.2. Discrete variables

- With a **binary $y$ variable**, the coefficient must be interpreted in **percentage points:**

*"Everything else equal, a 1 [unit] increase in [x] is associated with a [beta $\times$ 100] percentage point [in/de]crease in the probability that [y equals 1] on average."*

- So in our case, the **adequate interpretation** would be:

*"Everything else equal, a **1 euro increase in annual household income** is associated with a **0.00004 percentage point decrease in the probability that the newborn is underweight** on average."*

- Here the **interpretation** would be more **meaningful:**
  - For a **1,000 euro** increase **→** 0.04 percentage point decrease
  - Compared to the **typical probability** to have an underweight newborn

# 1. Point estimates

### 1.2. Discrete variables

- The mean of a dummy variable corresponds to the share of 1s:

```
mean(data$underweight)
```

```
## [1] 0.037
```

- We can also compute the probability that $y = 1$ for the average $x$ with our estimated coefficients:

```
5.214013e-02 + mean(data$household_income) * -4.084787e-07
```

```
## [1] 0.03700001
```

For the **average household**, a **1,000 euro increase** in annual income would be associated with a **0.0004 / 0.037** $\approx$ **1% decrease in the probability** that the newborn is **underweight**

# 1. Point estimates

### 1.2. Discrete variables

- Finally if **both** the $y$ and the $x$ variables are **discrete**, the coefficient must be interpreted:
  - In **percentage points**
  - **Relative** to the reference category

$$\text{Underweight}_i = \alpha + \beta_1 \text{T2}_i + \beta_2 \text{T3}_i + \varepsilon_i$$

```
lm(underweight ~ income_tercile, data)$coefficients
```

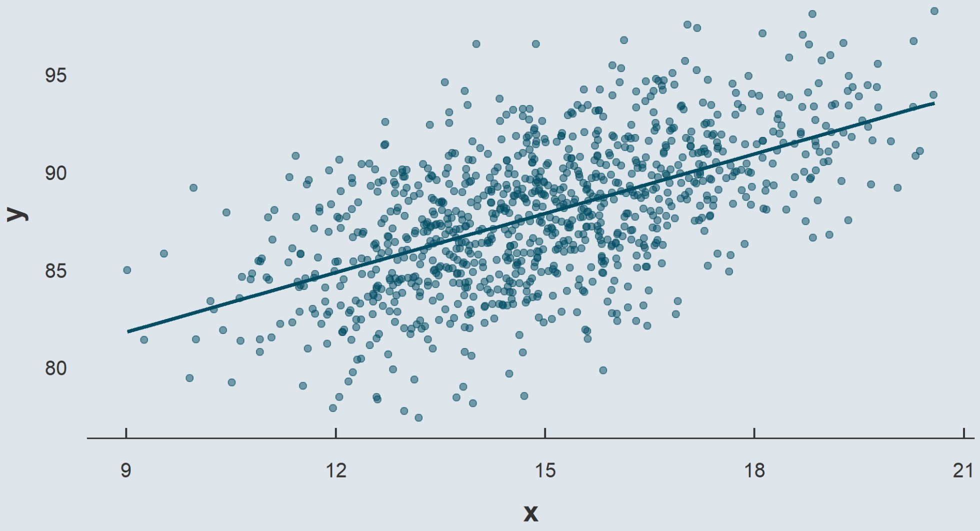
```
##       (Intercept) income_tercileT2 income_tercileT3
##        0.07207207      -0.03903904      -0.06608405
```

*"Everything else equal, belonging to the* ***second income tercile*** *is associated with a* ***3.9 percentage point lower probability that the newborn is underweight*** *relative to the* ***first income tercile.****"*

# 1. Point estimates

## 1.3. Log vs. level

- Consider now the following hypothetical relationship:




- The slope tells us by how many **units** the $y$ variable would increase for a **1 unit** increase in $x$

- But often times in Economics we're interested in the elasticity between the two variables:
  - What is the expected **percentage change** in $y$ for a **one percent increase** in $x$?

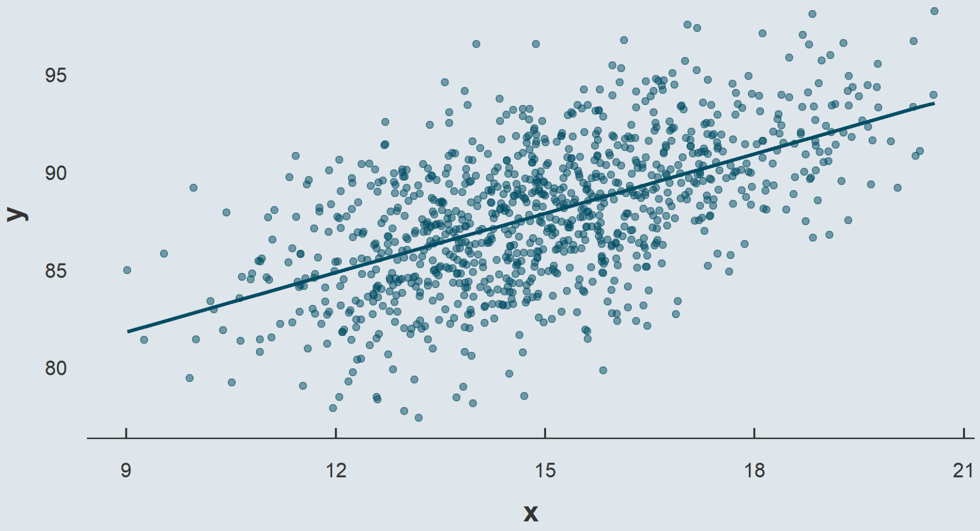
*➜ The log transformation can be used to easily get an approximation of that*

# 1. Point estimates

## 1.3. Log vs. level

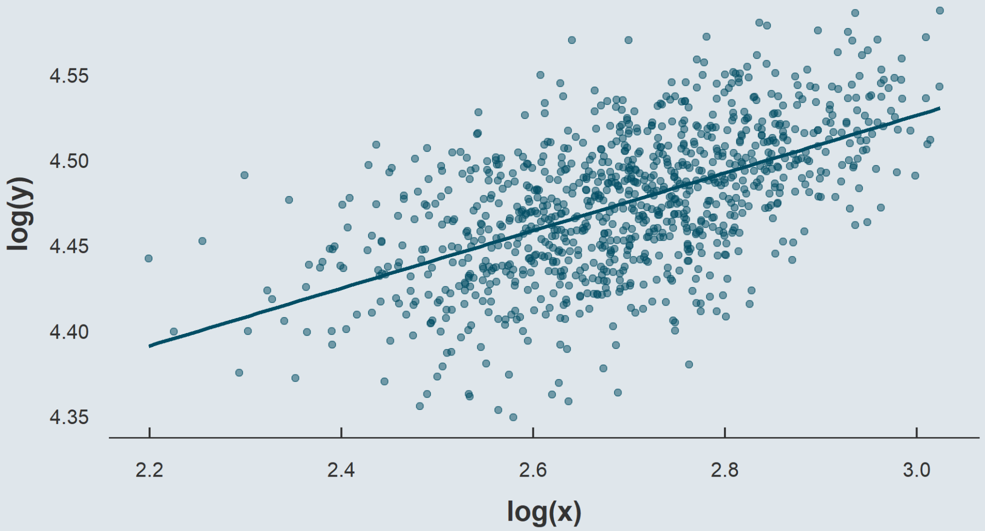
- Instead of considering

$$y_i = \alpha_{lvl} + \beta_{lvl} x_i + \varepsilon_i$$




- We consider

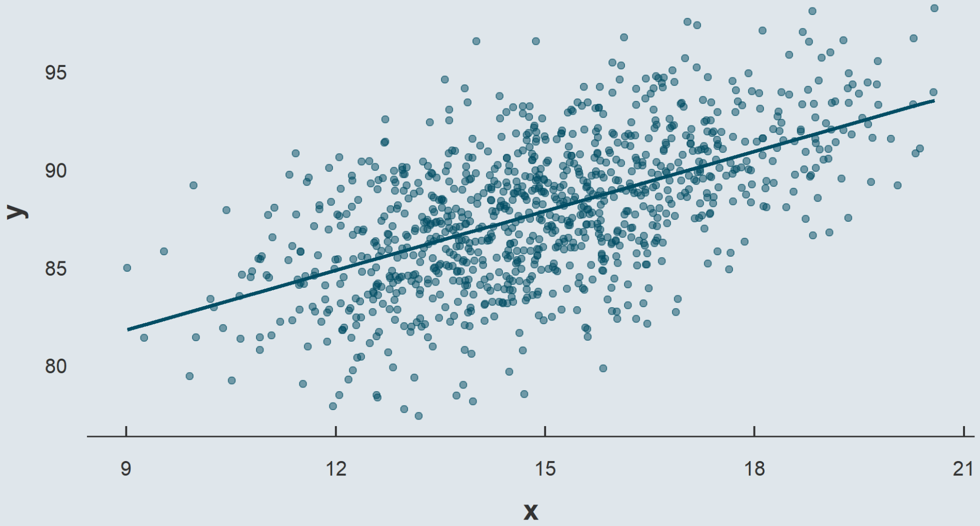
$$\log(y_i) = \alpha_{log} + \beta_{log} \log(x_i) + \varepsilon_i$$



# 1. Point estimates

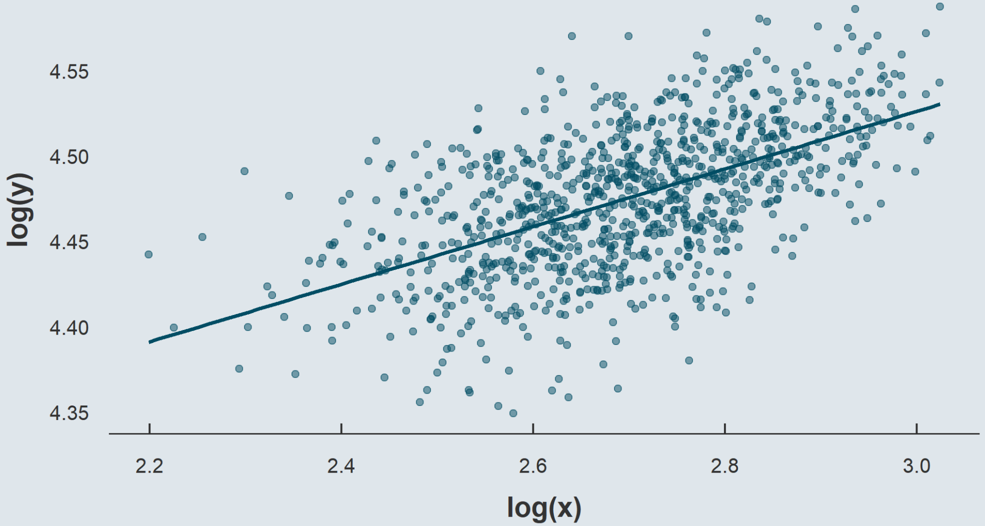
## 1.3. Log vs. level

$\widehat{\beta_{lvl}} = 1.0121933$




$$(15 \div 100) \times \widehat{\beta_{lvl}} \approx (15 \div 100) \times 1.0121933$$
$$\approx 0.0151829$$

$\widehat{\beta_{log}} = 0.16875$




$$0.0151829 \div 90 = 0.0001687$$
$$\approx \beta_{log}\%$$

# 1. Point estimates

### 1.3. Log vs. level

- Thus the interpretation differs depending on whether variables are in log or in level:
  - When variables are in **level** we should interpret the coefficients in terms of **unit** increase
  - When variables are in **log** we should interpret the coefficients in terms of **percentage** increase

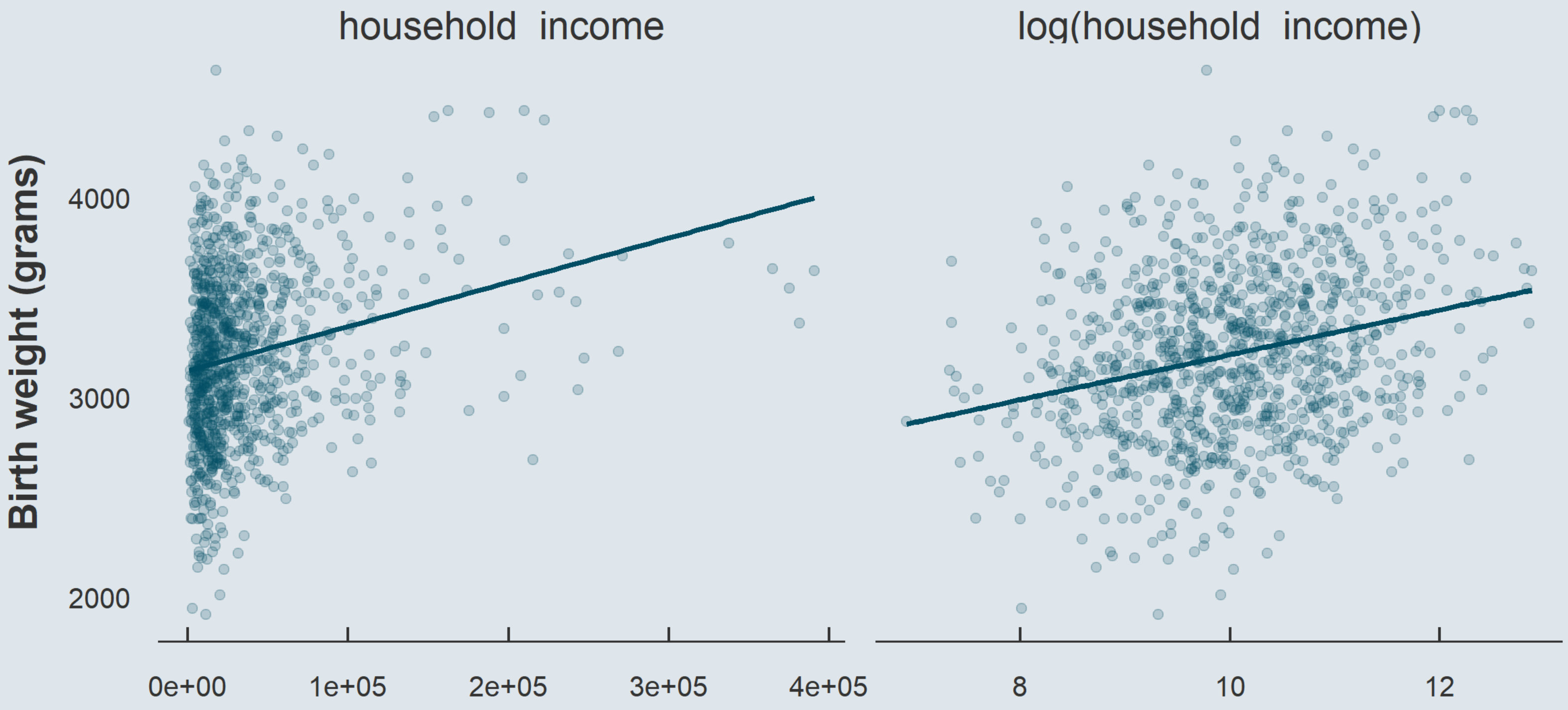
Interpretation of the regression coefficient

| | **y** | **log(y)** |
|---|---|---|
| **x** | $\hat{\beta}$ is the unit increase in $y$ due to a 1 unit increase in $x$ | $\hat{\beta} \times 100$ is the % increase in $y$ due to a 1 unit increase in $x$ |
| **log(x)** | $\hat{\beta} \div 100$ is the unit increase in $y$ due to a 1% increase in $x$ | $\hat{\beta}$ is the % increase in $y$ due to a 1% increase in $x$ |

# 1. Point estimates

## 1.3. Log vs. level

- Let's give it a try with our example on household income and birth weight
  - We've already seen that because income is log-normally distributed, it should be included in log



# 1. Point estimates

### 1.3. Log vs. level

- So what would be your interpretation of the slope estimated from the following regression?

$$\text{Birth weight}_i = \alpha + \beta \log(\text{Household income}_i) + \varepsilon$$

```
lm(birth_weight ~ log(household_income), data)$coefficients
```

```
##           (Intercept) log(household_income)
##             2091.2323              112.3234
```

- With a continuous $y$ **in level** and a **logged** $x$ variable, the template would be:

*"Everything else equal, a 1 percent increase in [x] is associated with a [beta/100] [unit] [in/de]crease in [y] on average."*

- So in our case, the **adequate interpretation** would be:

*"Everything else equal, a **1 percent increase in annual household income** is associated with a **1.12 grams increase in the birth weight of the newborn** on average."*

# Overview

# Overview

# 2. Practice interpretation

**➜ Let's practice coefficient interpration with randomly generated relationships:**

# Overview

# Overview

# 3. Regression tables

### 3.1. Layout

- **So far** we've been **used to** regression results **displayed this way:**

```
lm(birth_weight ~ household_income, data)$coefficients
```

```
##      (Intercept) household_income
##     3.134528e+03     2.213871e-03
```

- Or with the more exhaustive **summary()** coefficients output:

```
summary(lm(birth_weight ~ household_income, data))$coefficients
```

```
##                      Estimate   Std. Error    t value     Pr(>|t|)
## (Intercept)      3.134528e+03 1.656840e+01 189.187165 0.000000e+00
## household_income 2.213871e-03 2.808507e-04   7.882732 8.355367e-15
```

*→ But in **formal** reports and academic papers, the **layout** of regression tables is **a bit different***

# 3. Regression tables

## 3.1. Layout

| | *Dependent variable:* | |
|---|---|---|
| | Birth weight | |
| | (1) | (2) |
| Household income | $0.002^{***}$ | $0.002^{***}$ |
| | (0.0003) | (0.0003) |
| Girl (ref: Boy) | | $-135.218^{***}$ |
| | | (34.838) |
| Constant | $3,134.528^{***}$ | $3,246.365^{***}$ |
| | (16.568) | (34.257) |
| Observations | 1,000 | 963 |
| *Note:* | $^{*}$p<0.1; $^{**}$p<0.05; $^{***}$p<0.01 | |

Regression tables often contain multiple regressions:

- With **one regression in each column**
  - Regression models are numbered
  - Dependent variable mentioned above
- And one variable in **each row**
  - With the **point estimate**
  - And a **precision measure** below
- **General info** on each model **at the bottom**
- A **symbology** for the **p-value** testing whether the coefficient is significantly different from 0 or not

# 3. Regression tables

## 3.1. Layout

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| | (1) | (2) |
| Household income | $0.002^{***}$ | $0.002^{***}$ |
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It makes it easy to compare the different models:

- We can **add controls progressively**
  - Check the **stability** of the main **coefficient**

→ *If it gets significantly closer to 0 it might indicate that the raw relationship was fallaciously driven by a confounding factor*

- And **compare general statistics**
  - N is lower in the second regression
  - It means that there are missing values
  - Could this induce a selection bias?

# 3. Regression tables

## 3.2. Reported significance

| | *Dependent variable:* | |
|---|---|---|
| | Birth weight | |
| | (1) | (2) |
| Household income | $0.002^{***}$ | $0.002^{***}$ |
| | (0.0003) | (0.0003) |
| Girl (ref: Boy) | | $-135.218^{***}$ |
| | | (34.838) |
| Constant | $3,134.528^{***}$ | $3,246.365^{***}$ |
| | (16.568) | (34.257) |
| Observations | 1,000 | 963 |
| *Note:* | $^{*}$p<0.1; $^{**}$p<0.05; $^{***}$p<0.01 | |

It makes it easy to compare the different models:

- The **evolution** of the **significance** matters as well
  - The main coefficient should stay significant
- But don't rely too much on the **symbology**
  - Thresholds are **not always the same**
  - **Sometimes** there are **none**
- Instead, keep in mind this **rule of thumb:**

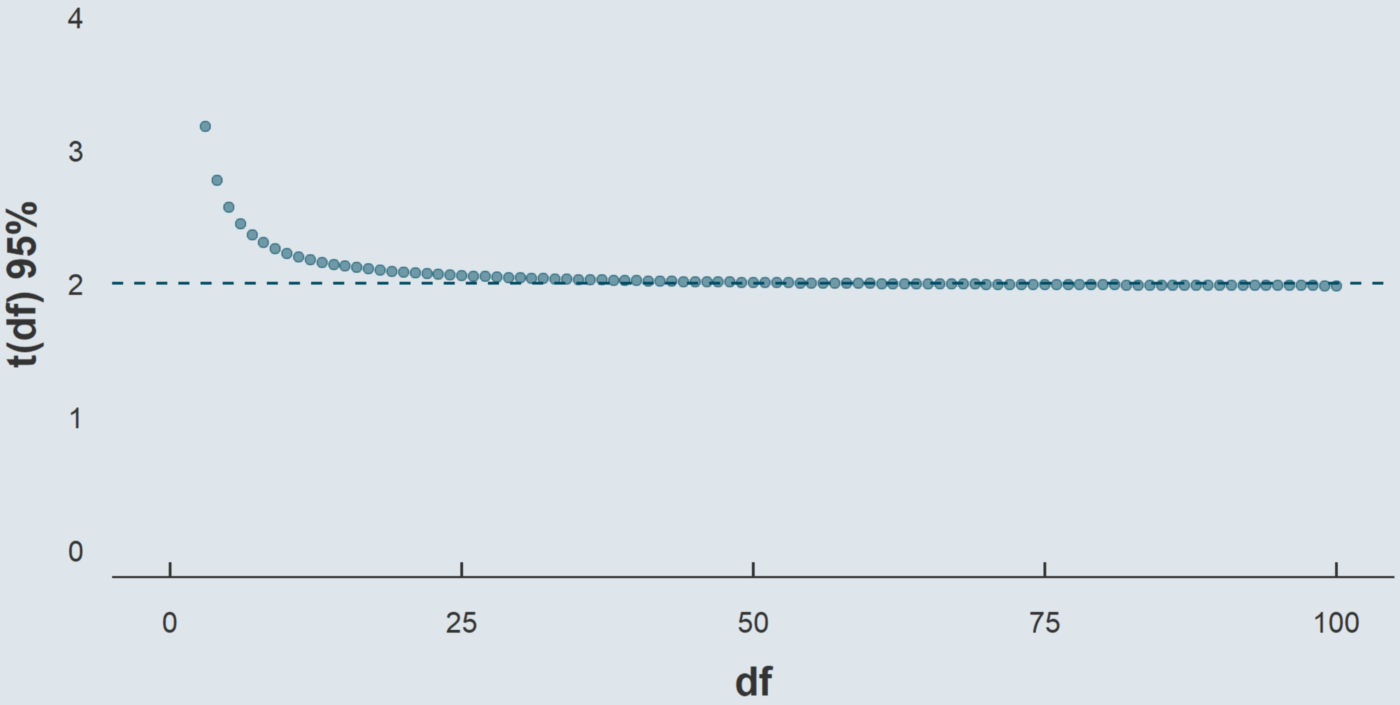
***➜ A coefficient $\approx$ twice larger than its standard error has a p-value of $\approx$ 5%***

# 3. Regression tables

### 3.2. Reported significance

- Remember the formula for the **confidence interval:**
  - We can **fix** the **confidence level** $1 - \alpha$ to 95% and check **how** $t$ **varies with** $\mathrm{df}$

$$\hat{\beta} \pm t(\mathrm{df})_{1-\frac{\alpha}{2}} \times \mathrm{se}(\hat{\beta})$$



# 3. Regression tables

### 3.2. Reported significance

- **As soon as** you have **about 20 observations more than** you have **parameters** to estimate:
  - The $t$ **value** gets very **close to 2**
  - And as $\mathrm{df}$ increases it quickly converges to $\approx$ 2

- The coefficient is statistically significant if the lower bound of its (absolute) confidence interval is larger than 0
  - Which is an easy calculation if we **approximate the $t$ value by 2**
  - A reasonable approximation for a back of the envelope calculation unless there are very few observations

- The *(absolute)* lower bound of the CI writes:

$$|\hat{\beta}| - t(\mathrm{df})_{1-\frac{\alpha}{2}} \times \mathrm{se}(\hat{\beta})$$

$$|\hat{\beta}| - 2 \times \mathrm{se}(\hat{\beta}) > 0$$

$$|\hat{\beta}| > 2 \times \mathrm{se}(\hat{\beta})$$

*So if the **coefficient** is clearly more than **twice larger** than it's **standard error**, it must be **statistically significant** at the **5% significance** level*

➜ But sometimes the p-value or the confidence interval is reported instead of the standard error

# 3. Regression tables

## 3.2. Reported significance

| | *Dependent variable:* | |
|---|---|---|
| | Birth weight | |
| | (1) | (2) |
| Household income | $0.002^{***}$ | $0.002^{***}$ |
| | p = 0.000 | p = 0.000 |
| Girl (ref: Boy) | | $-135.218^{***}$ |
| | | p = 0.0002 |
| Constant | $3,134.528^{***}$ | $3,246.365^{***}$ |
| | p = 0.000 | p = 0.000 |
| Observations | 1,000 | 963 |
| *Note:* | $^{*}$p<0.1; $^{**}$p<0.05; $^{***}$p<0.01 | |

| | *Dependent variable:* | |
|---|---|---|
| | Birth weight | |
| | (1) | (2) |
| Household income | $0.002^{***}$ | $0.002^{***}$ |
| | (0.002, 0.003) | (0.002, 0.003) |
| Girl (ref: Boy) | | $-135.218^{***}$ |
| | | (-203.500, -66.936) |
| Constant | $3,134.528^{***}$ | $3,246.365^{***}$ |
| | (3,102.055, 3,167.002) | (3,179.223, 3,313.507) |
| Observations | 1,000 | 963 |
| *Note:* | $^{*}$p<0.1; $^{**}$p<0.05; $^{***}$p<0.01 | |

# 3. Regression tables

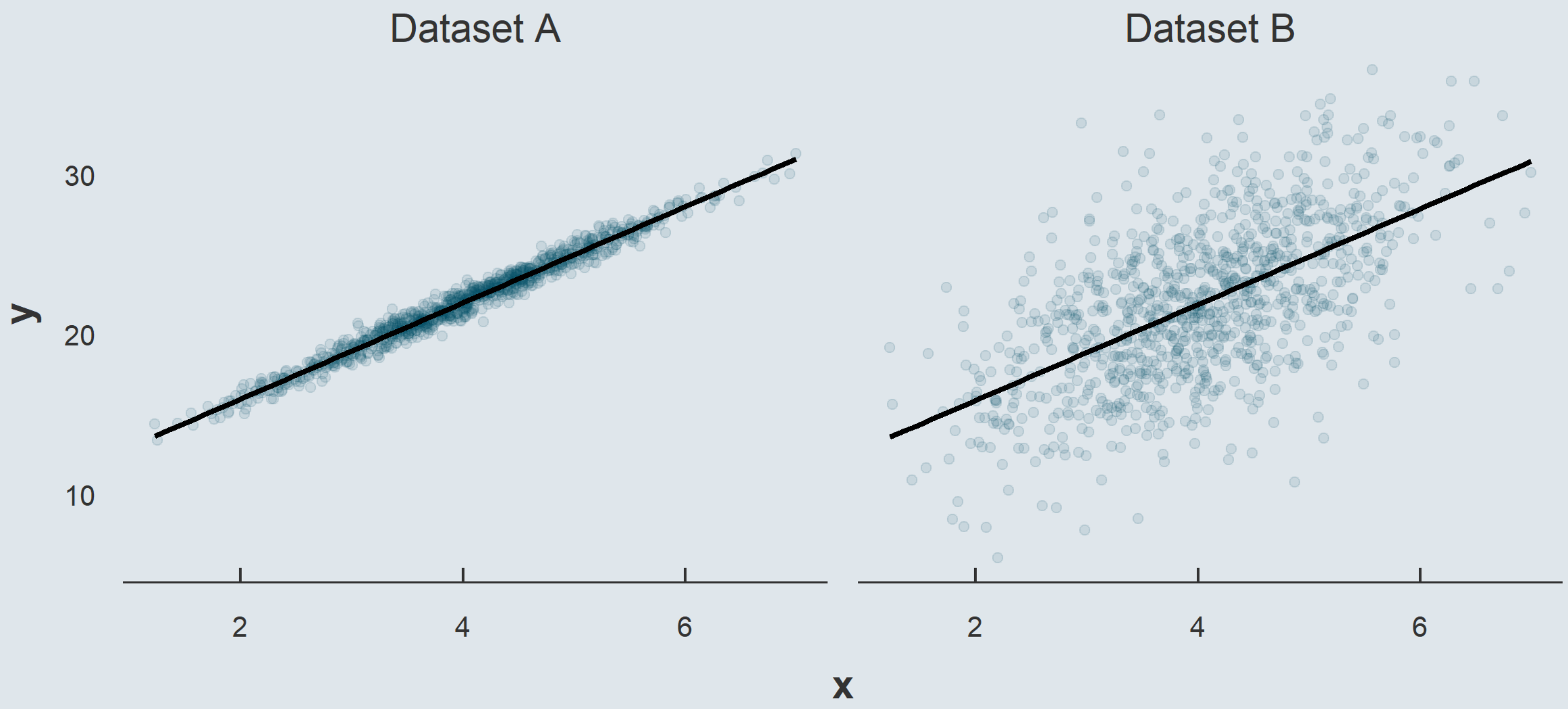
### 3.3. R squared

- In **regression tables**, the $\mathbf{R^2}$ of the model is **always reported** below the number of observations
  - The $R^2$ captures how well the **model fits the data**
  - 

# 3. Regression tables

### 3.3. R squared

- In **regression tables**, the **$R^2$** of the model is **always reported** below the number of observations
  - The $R^2$ captures how well the **model fits the data**
  - The model has a **good fit (high $R^2$)** on dataset A but a **poor fit (low $R^2$)** on dataset B



# 3. Regression tables

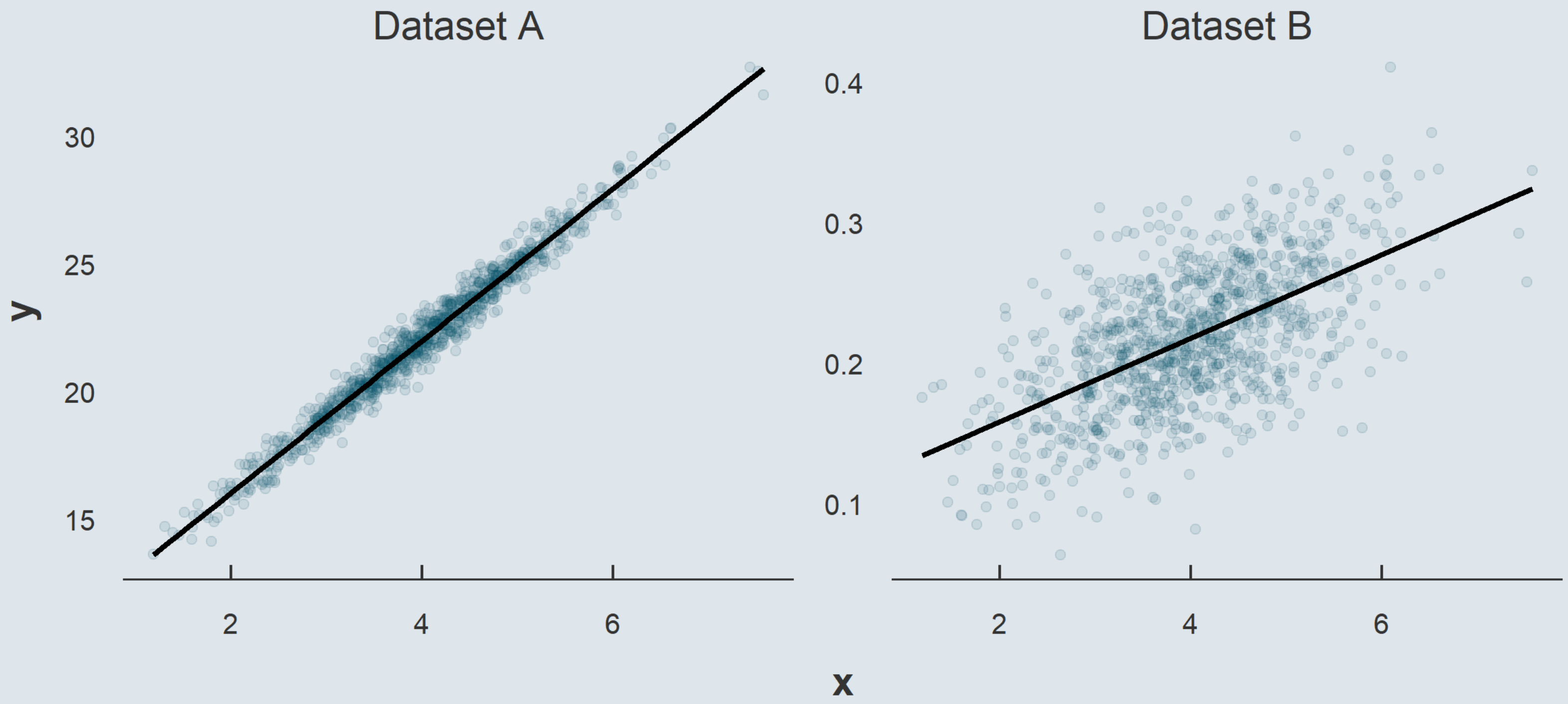
### 3.3. R squared

- The **standard error** already gives an idea on the goodness of the fit, but it is expressed in the **same unit as** $y$
  - So we **cannot compare** two different models based on that statistic
  -

# 3. Regression tables

### 3.3. R squared

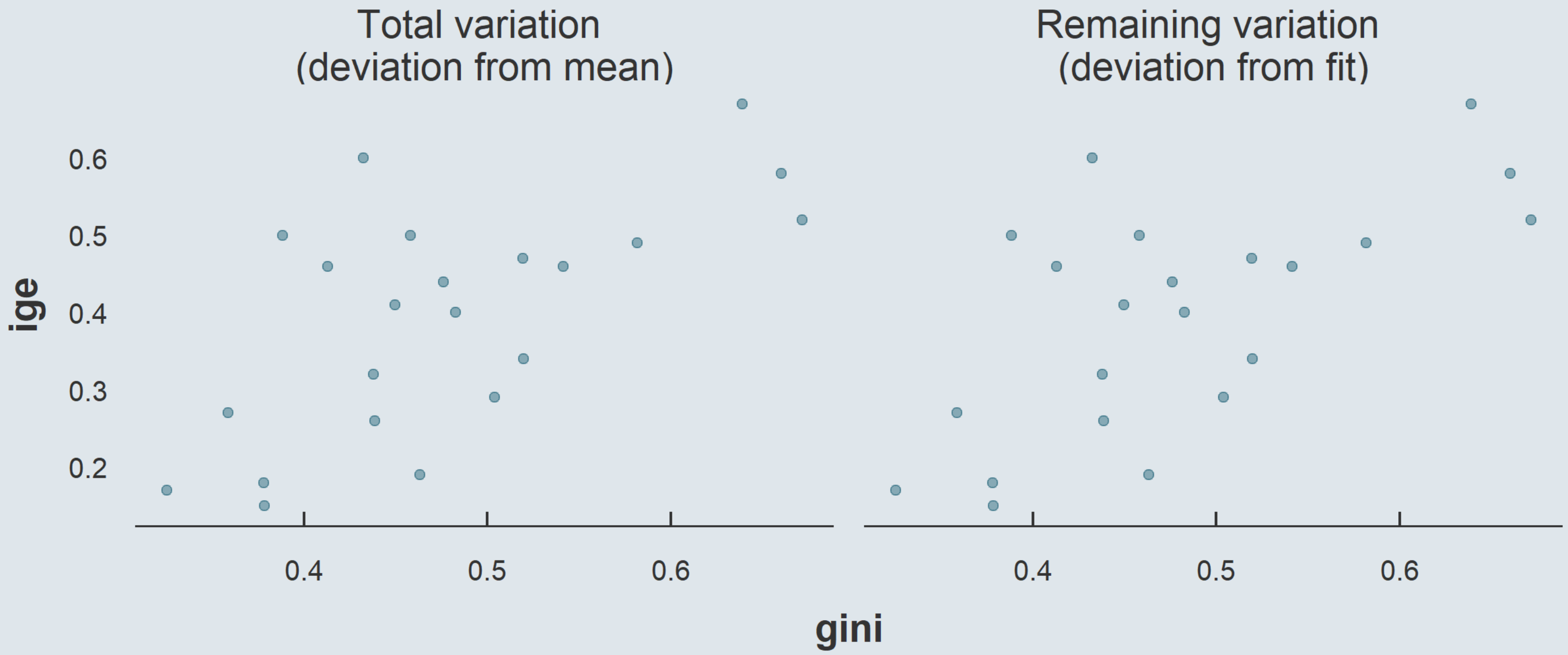
- The **standard error** already gives an idea on the goodness of the fit, but it is expressed in the **same unit as** $y$
  - So we **cannot compare** two different models based on that statistic
  - The standard error of the slope would be larger on dataset A than on dataset B



# 3. Regression tables

### 3.3. R squared

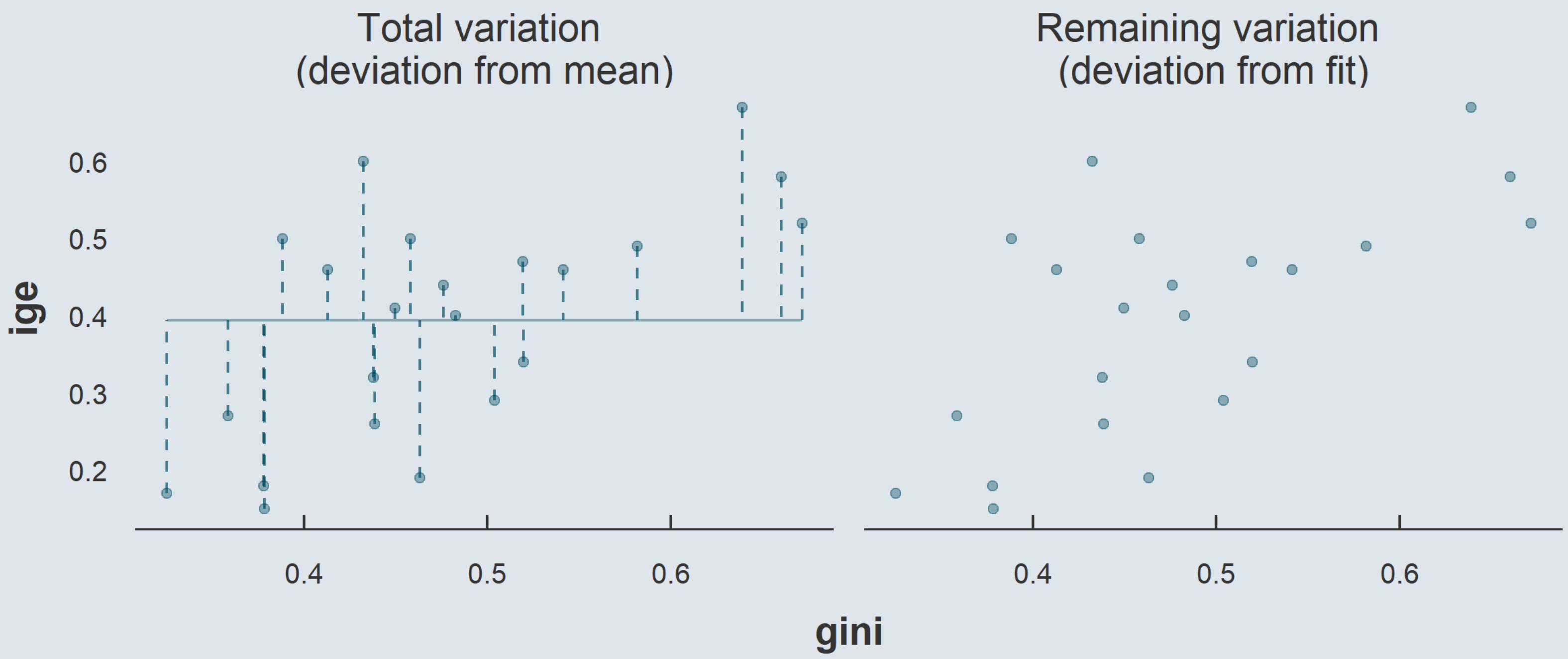
- The $\mathbf{R^2}$ captures the **goodness of fit** as the **percentage** of the $y$ variation captured by the model, from:
    -
    -



# 3. Regression tables

### 3.3. R squared

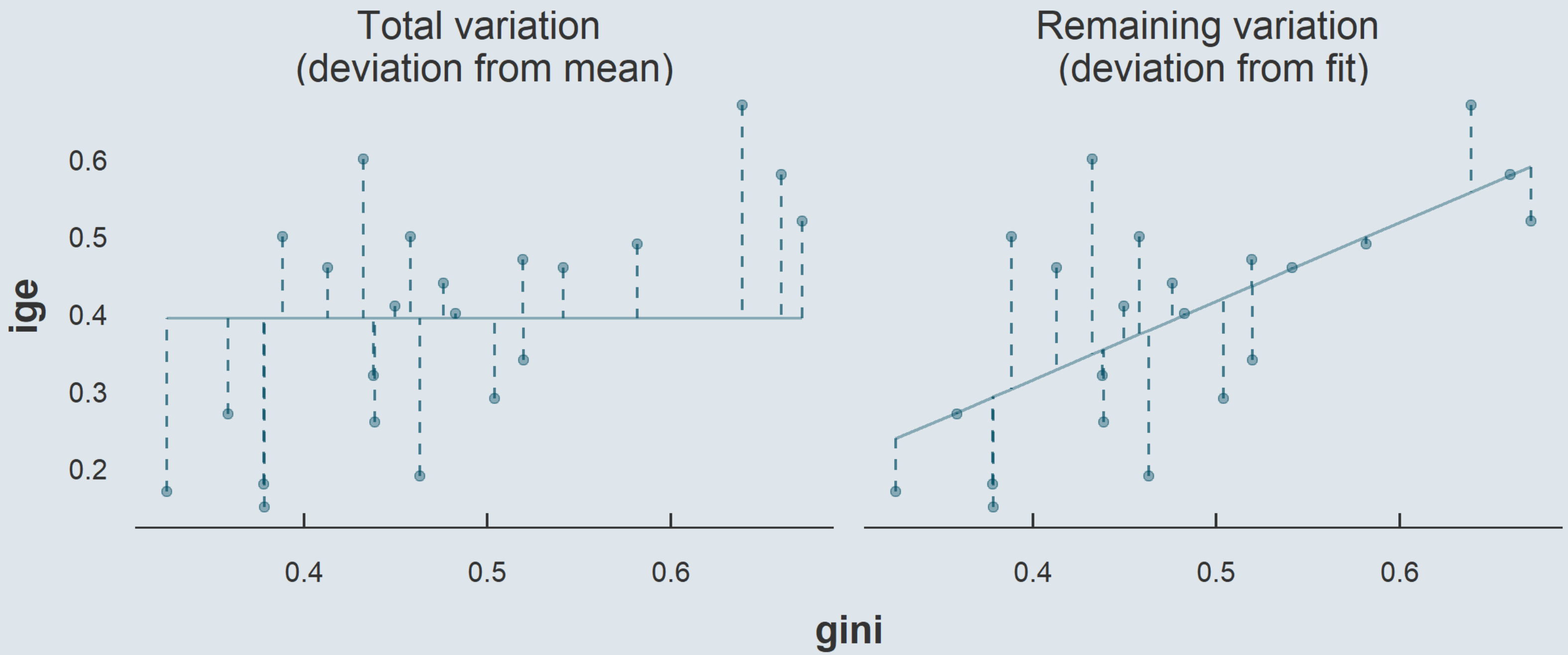
- The **R²** captures the **goodness of fit** as the **percentage** of the $y$ variation captured by the model, from:
  - The **total variation** of the y variable (its variance $\sum_{i=1}^{n}(y_i - \bar{y})^2$)
  - 



# 3. Regression tables

### 3.3. R squared

- The **$R^2$** captures the **goodness of fit** as the **percentage** of the $y$ variation captured by the model, from:
  - The **total variation** of the y variable (its variance $\sum_{i=1}^{n}(y_i - \bar{y})^2$)
  - The **remaining variation** of the y variable once its modeled (the sum of squared residuals $\sum_{i=1}^{n}\hat{\varepsilon}_i{}^2$)



# 3. Regression tables

### 3.3. R squared

- We can then obtain a proper formula from the following reasoning

$$\text{Total variation} = \text{Explained variation} + \text{Remaining variation}$$

$$\frac{\text{Explained variation}}{\text{Total variation}} = 1 - \frac{\text{Remaining variation}}{\text{Total variation}}$$

$$\frac{\text{Explained variation}}{\text{Total variation}} = 1 - \frac{\sum_{i=1}^{n} \hat{\varepsilon_i}^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2} \equiv \text{R}^2$$

- Because all the terms are sums of squares, we usually talk about:
  - **Total Sum of Squares** (TSS)
  - **Explained Sum of Squares** (ESS)
  - **Residual Sum of Squares** (RSS)

# 3. Regression tables

### 3.3. R squared

- Note that the **TSS** is actually the **variance of $y$:**
  - So the **$R^2$** is interpreted as the **share of the variance of $y$** which is **explained** by the model
  - And as such, the $R^2$ is always comprised **between 0 and 1**

$$\text{R}^2 = 1 - \frac{\sum_{i=1}^n \hat{\varepsilon_i}^2}{\sum_{i=1}^n (y_i - \bar{y})^2} = \frac{\text{Explained variation}}{\text{Total variation}}$$

- An undesirable property of the **$R^2$** is that it **mechanically increases** with the number of **dependent variables**
  - Such that with many variables the $R^2$ tends to overestimate the goodness of the fit
  - This is why you will sometimes see some **Adjusted $R^2$**

$$\text{Adjusted R}^2 = 1 - \frac{(1 - \text{R}^2)(n-1)}{n - \#\text{parameters}}$$

# Overview

**1. Point estimates ✓**
- 1.1. Continuous variables
- 1.2. Discrete variables
- 1.3. Log vs. level

**2. Practice interpretation ✓**

**3. Regression tables ✓**
- 3.1. Layout
- 3.2. Reported significance
- 3.3. R squared

**4. Wrap up!**

# 4. Wrap up!

**Standard interpretations**

- When both $x$ and $y$ are continuous, the **general** template for the **interpretation** of $\hat{\beta}$ is:

  *"Everything else equal, a 1 [unit] increase in [x] is associated with an [in/de]crease of [beta] [units] in [y] on average."*

- With a discrete $x$, the interpretation of the coefficient must be **relative to the reference category:**

  *"Everything else equal, belonging to the [x category] is associated with a [beta] [unit] [higher/lower] average [y] relative to the [reference category]."*

- With a **binary $y$ variable**, the coefficient must be interpreted in **percentage points:**

  *"Everything else equal, a 1 [unit] increase in [x] is associated with a [beta $\times$ 100] percentage point [in/de]crease in the probability that [y equals 1] on average."*

# 4. Wrap up!

### Interpretations with variable transformation

**Standardization**

- To standardize a variable is to **divide it by its SD**
  - The variation of a standardized variable should not be **interpreted** in units but **in SD**
  - For instance if $x$ and $y$ are continuous and $x$ is standardized, the interpretation becomes:

*"Everything else equal, a 1* ***standard deviation*** *increase in [x] is associated with an [in/de]crease of [beta] [units] in [y] on average."*

- If both $x$ and $y$ are standardized, the slope is the correlation coefficient between $x$ and $y$

**Log-transformation**

- The log transformation allows to interpret the coefficient in percentage:

Interpretation of the regression coefficient

| | **y** | **log(y)** |
|---|---|---|
| **x** | $\hat{\beta}$ is the unit increase in $y$ due to a 1 unit increase in $x$ | $\hat{\beta} \times 100$ is the % increase in $y$ due to a 1 unit increase in $x$ |
| **log(x)** | $\hat{\beta} \div 100$ is the unit increase in $y$ due to a 1% increase in $x$ | $\hat{\beta}$ is the % increase in $y$ due to a 1% increase in $x$ |

# 4. Wrap up!

## Regression table layout

| | Birth weight | |
|---|---|---|
| | (1) | (2) |
| Household income | 0.002*** | 0.002*** |
| | (0.0003) | (0.0003) |
| Girl (ref: Boy) | | -135.218*** |
| | | (34.838) |
| Constant | 3,134.528*** | 3,246.365*** |
| | (16.568) | (34.257) |
| Observations | 1,000 | 963 |
| $R^2$ | 0.059 | 0.074 |
| *Note:* | *p<0.1; **p<0.05; ***p<0.01 | |

Regression tables often contain multiple regressions:

- With **one regression in each column**
- And one variable in **each row**
  - With the **point estimate**
  - And a **precision measure** below
- **General info** on each model **at the bottom**
  - Number of observations
  - $\text{R}^2 = 1 - \frac{\sum_{i=1}^{n} \hat{\varepsilon}_i^{\,2}}{\sum_{i=1}^{n}(y_i - \bar{y})^2}$
- A **symbology** for the **p-value** testing whether the coefficient is significantly different from 0 or not