



Inference

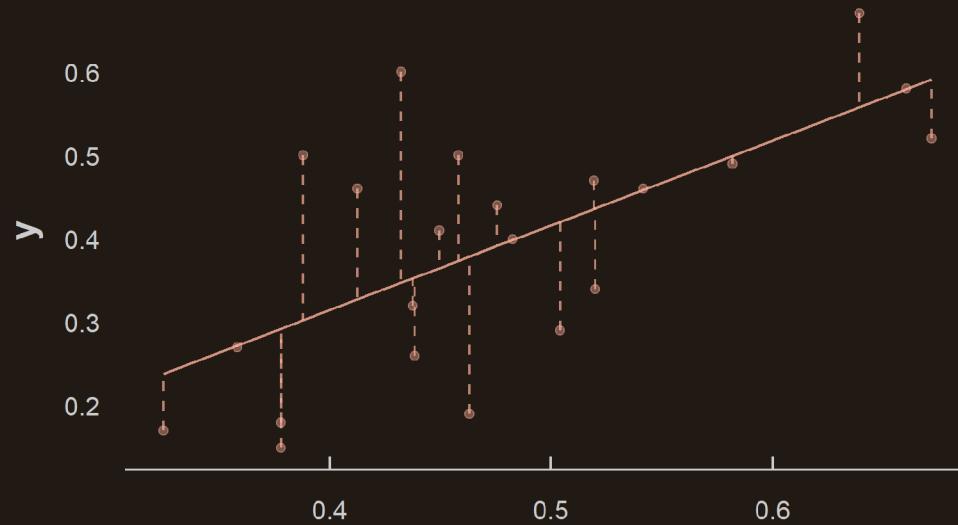
Lecture 10

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CPES 2 - Fall 2022

Quick reminder

1. Regression



```
## 
## Call:
## lm(formula = y ~ x, data = data)
## 
## Coefficients:
## (Intercept)          x
## -0.09129        1.01546
```

- This can be expressed with the **regression equation**:

$$y_i = \hat{\alpha} + \hat{\beta}x_i + \hat{\varepsilon}_i$$

- Where $\hat{\alpha}$ is the **intercept** and $\hat{\beta}$ the **slope** of the **line** $\hat{y}_i = \hat{\alpha} + \hat{\beta}x_i$, and $\hat{\varepsilon}_i$ the **distances** between the points and the line

$$\hat{\beta} = \frac{\text{Cov}(x_i, y_i)}{\text{Var}(x_i)}$$

$$\hat{\alpha} = \bar{y} - \hat{\beta} \times \bar{x}$$

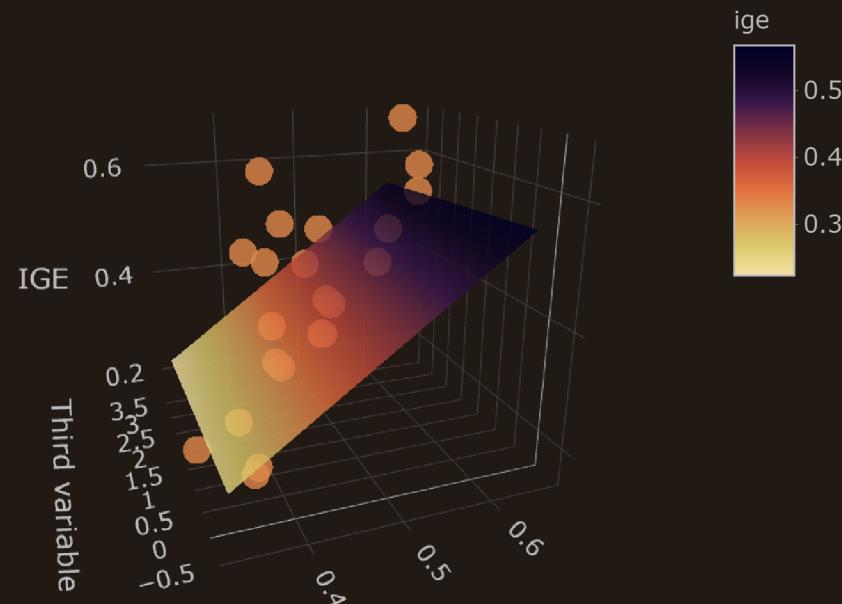
- $\hat{\alpha}$ and $\hat{\beta}$ minimize $\hat{\varepsilon}_i$

Quick reminder

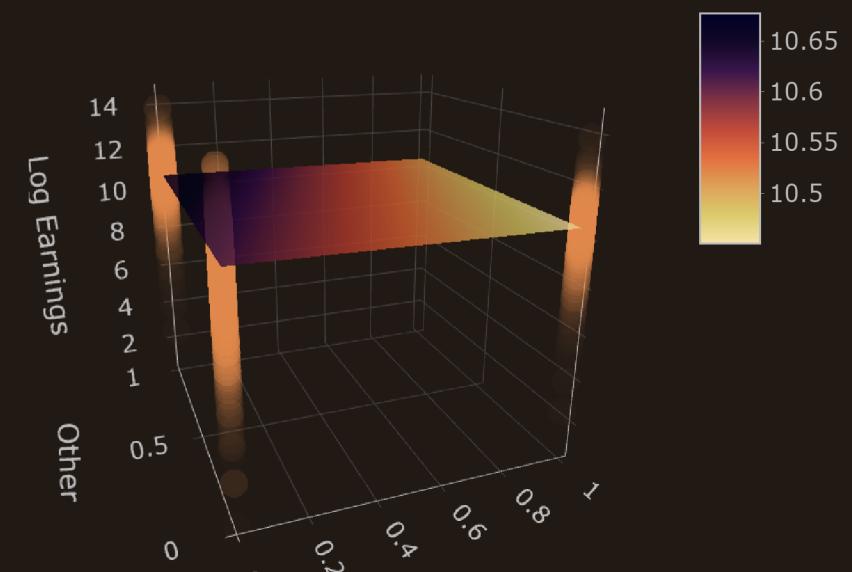
2. Multivariate regressions

- Adding a second independent **variable** in the regression amounts to **fitting a plane** instead of a line
 - Adding a third variable would fit a hyperplane of dimension 3 and so on

Adding a continuous variable



Adding a discrete variable

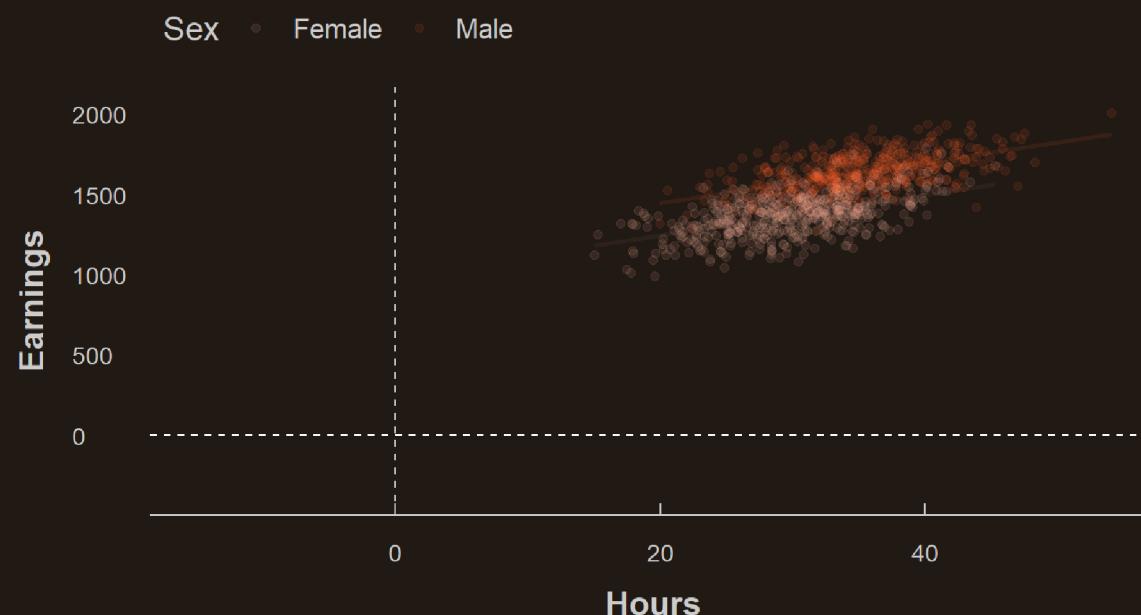


Quick reminder

3. Control variables

- Adding a third variable z **removes** its potential **confounding effect** from the relationship between x and y
 - As we move along the x axis, the **third variable remains constant**

$$\hat{y}_i = \hat{\alpha} + \hat{\beta}_1 x + \hat{\beta}_2 z + \hat{\varepsilon}_i$$

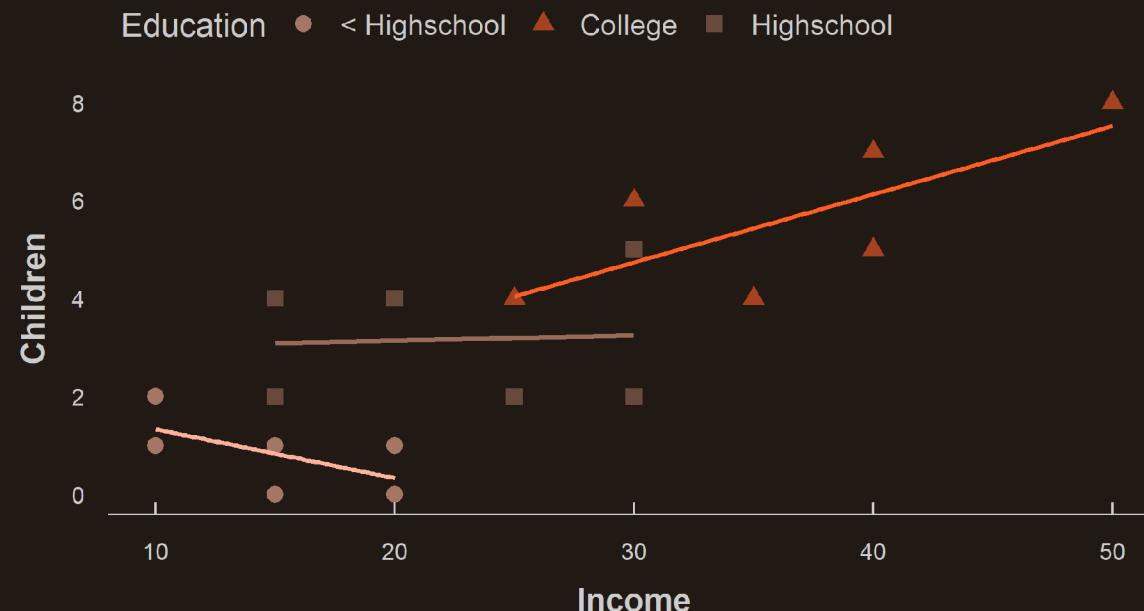


Quick reminder

4. Interactions

- Adding an **interaction** term with z allows to see **how the effect** of x on y **varies** with z
 - If z is **discrete**, it amounts to **regressing** y on x **separately** for each z group

$$\hat{y}_i = \hat{\alpha} + \hat{\beta}_1 x + \hat{\beta}_2 z + \hat{\beta}_3(x \times z) + \hat{\varepsilon}_i$$





Today: Inference

1. Asymptotic inference

- 1.1. Data generating process
- 1.2. Standardization
- 1.3. Confidence interval

2. Exact inference

- 2.1. Standard error
- 2.2. Student-t distribution
- 2.3. Confidence interval

3. Hypothesis testing

- 3.1. P-value
- 3.2. linearHypothesis()

4. Wrap up!



Today: Inference

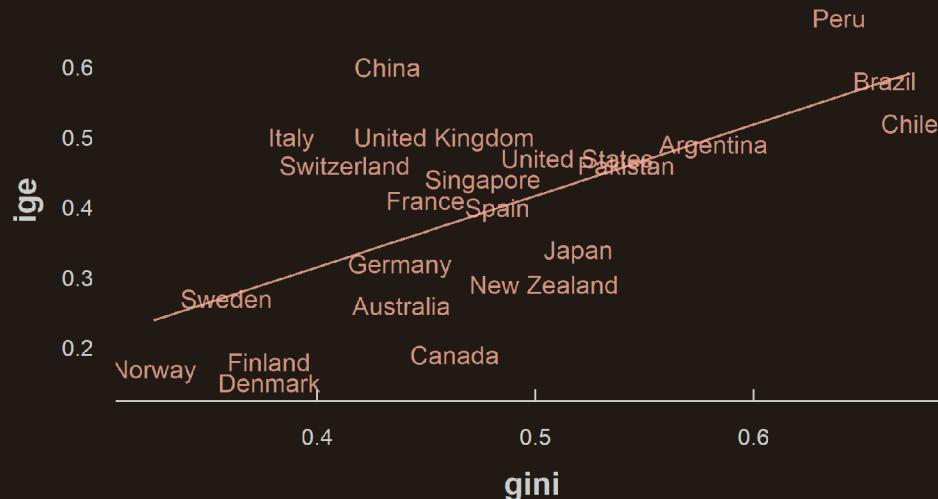
1. Asymptotic inference

- 1.1. Data generating process
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1. Asymptotic inference

1.1. Data generating process

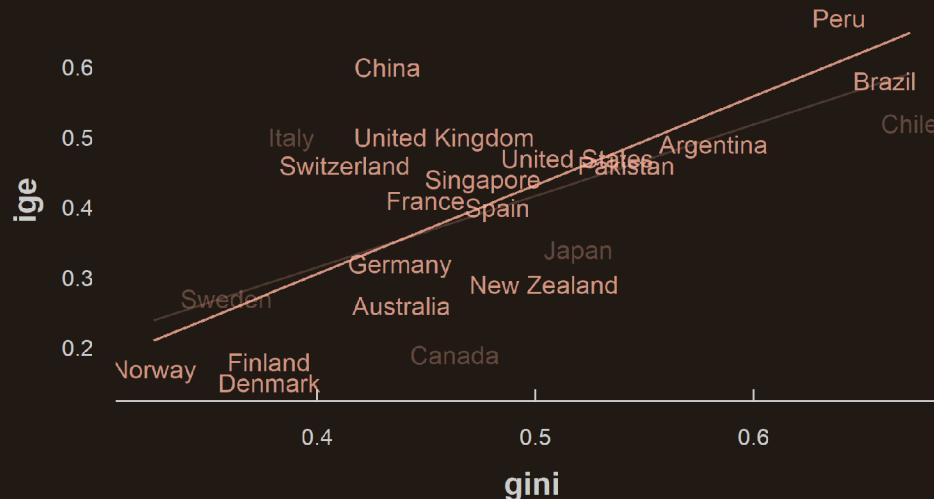
- In Part I of the course, we distinguished the **empirical moments** from the **theoretical moments**
 - Like the *empirical mean* is a **finite-sample estimation** of the *theoretical expected value*
 - The same principle applies to **regression coefficients**
- Take our Great Gatsby Curve for instance
 -
 -



1. Asymptotic inference

1.1. Data generating process

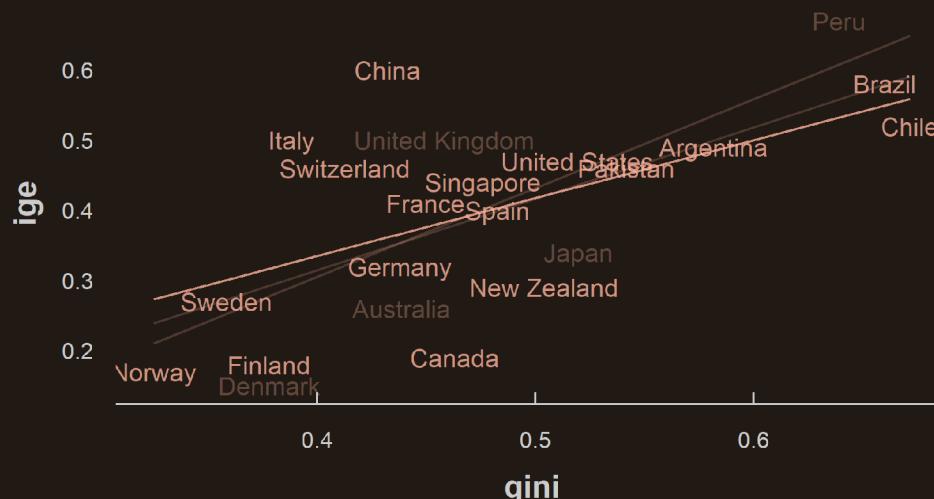
- In Part I of the course, we distinguished the **empirical moments** from the **theoretical moments**
 - Like the *empirical mean* is a **finite-sample estimation** of the *theoretical expected value*
 - The same principle applies to **regression coefficients**
- Take our Great Gatsby Curve for instance
 - Had our **sample** of countries been a bit **different**, our **coefficients** would **not be the same**
 -



1. Asymptotic inference

1.1. Data generating process

- In Part I of the course, we distinguished the **empirical moments** from the **theoretical moments**
 - Like the *empirical mean* is a **finite-sample estimation** of the *theoretical expected value*
 - The same principle applies to **regression coefficients**
- Take our Great Gatsby Curve for instance
 - Had our **sample** of countries been a bit **different**, our **coefficients** would **not be the same**
 - But they would all be **estimations** of a true relationship whose **data-generating process** is **unobserved**



Then how to asses the reliability
of our estimation?



1. Asymptotic inference

1.1. Data generating process

- For **simplicity**, let's work with a relationship whose **DGP is known**
 - Such that we can **understand how estimations** from random samples **behave relative to the DGP**
 - Let's **generate data in R!**
- We can use **functions** that output **random draws from given distributions** whose parameters can be chosen

Normal distribution

→ Sample size, expected value, standard deviation

```
rnorm(n = 10, mean = 100, sd = 5)
```

```
## [1] 103.48482 102.78332 96.55622  
## [4] 96.46252 101.82291 103.84266  
## [7] 99.43827 104.40554 101.99053  
## [10] 96.93987
```

Uniform distribution

→ Sample size, lower bound, upper bound

```
runif(n = 10, min = 4, max = 5)
```

```
## [1] 4.633493 4.213208 4.129372  
## [4] 4.478118 4.924074 4.598761  
## [7] 4.976171 4.731793 4.356727  
## [10] 4.431474
```



1. Asymptotic inference

1.1. Data generating process

- Consider the following **data generating process**:

$$y = -2 + 0.4 \times x + \varepsilon \quad \begin{cases} x \sim \mathcal{N}(4, 25) \\ \varepsilon \sim \mathcal{N}(0, 1) \end{cases}$$

- We can randomly draw **1,000 observations**

```
dt <- tibble(x = rnorm(1000, 4, 5),  
             e = rnorm(1000, 0, 1),  
             y = -2 + (.4 * x) + e)
```

Check the empirical moments:

```
c(mean(dt$x), var(dt$x))
```

```
## [1] 4.127842 26.816044
```

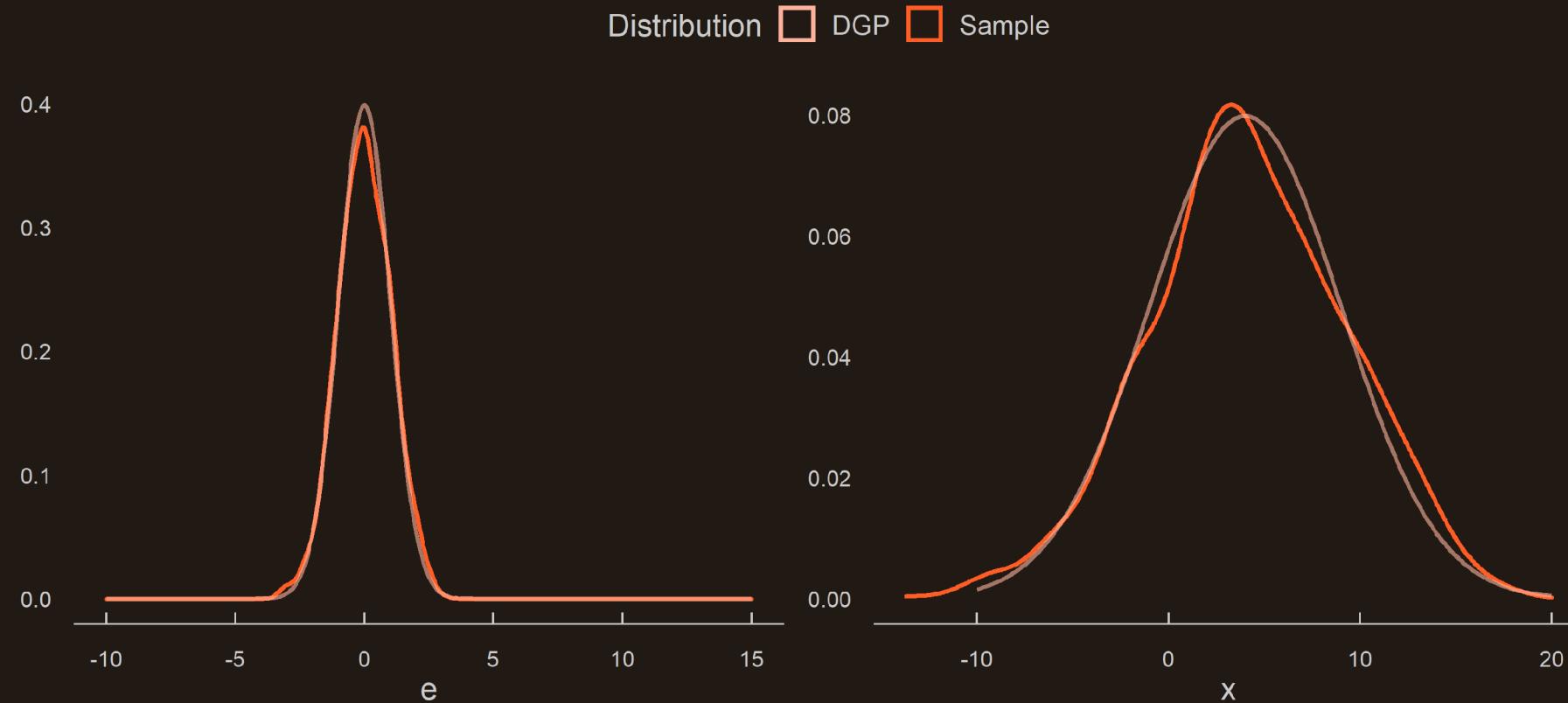
```
c(mean(dt$e), var(dt$e))
```

```
## [1] 0.009459055 1.070444754
```

1. Asymptotic inference

1.1. Data generating process

- Because the randomly drawn **sample** is finite, it **does not match exactly** the features of the DGP:





1. Asymptotic inference

1.1. Data generating process

- Same thing for the **coefficients** of the relationship between x and y :

```
lm(y ~ x, dt)
```

```
##  
## Call:  
## lm(formula = y ~ x, data = dt)  
##  
## Coefficients:  
## (Intercept)          x  
##       -2.0044      0.4033
```

- But what would happen if we were to **redo this operation many times?**
 1. **Draw a random sample** from the DGP
 2. **Compute the slope** of the regression of y on x
 3. Do it many times and **store the coefficients**



1. Asymptotic inference

1.1. Data generating process

- We can **use a loop** to do that:
 -
 -
 -

```
#  
  
for (i in 1:1000) {  
  
#  
#  
#  
  
#  
#  
  
}
```



1. Asymptotic inference

1.1. Data generating process

- We can **use a loop** to do that:
 - First we create an empty vector
 -
 -

```
beta <- c()

for (i in 1:1000) {

#
#
#
#
#



}
```



1. Asymptotic inference

1.1. Data generating process

- We can **use a loop** to do that:
 - First we create an empty vector
 - Then we put the code in a loop
 -

```
beta <- c()

for (i in 1:1000) {

  dt_i <- tibble(x = rnorm(1000, 4, 5),
                  e = rnorm(1000, 0, 1),
                  y = -2 + (.4 * x) + e)

  reg_i <- lm(y ~ x, dt_i)

  #}

}
```



1. Asymptotic inference

1.1. Data generating process

- We can **use a loop** to do that:
 - First we create an empty vector
 - Then we put the code in a loop
 - And we fill the vector at each iteration

```
beta <- c()

for (i in 1:1000) {

  dt_i <- tibble(x = rnorm(1000, 4, 5),
                  e = rnorm(1000, 0, 1),
                  y = -2 + (.4 * x) + e)

  reg_i <- lm(y ~ x, dt_i)

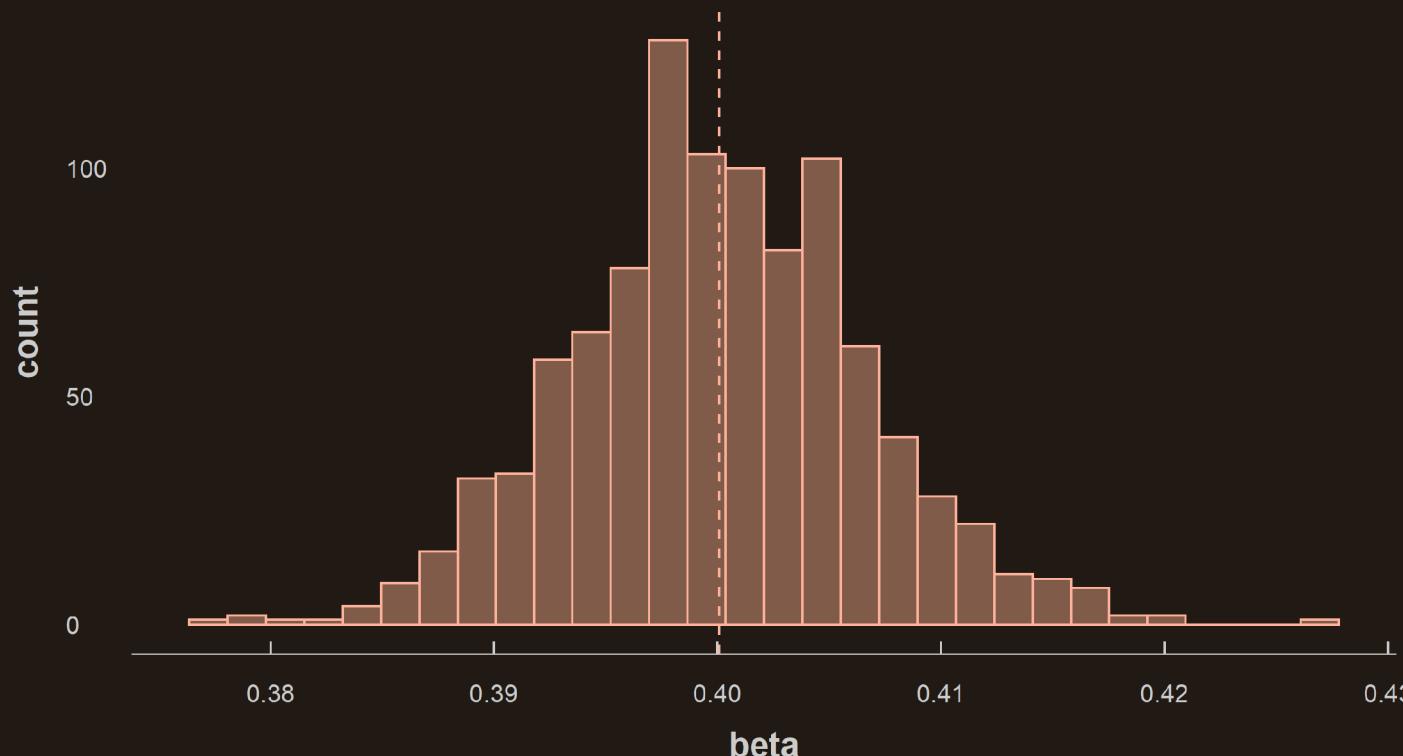
  beta <- c(beta, reg_i$coefficients[2])

}
```

1. Asymptotic inference

1.1. Data generating process

- We now have **1,000 slope coefficients** from 1,000 random samples of the **same DGP**

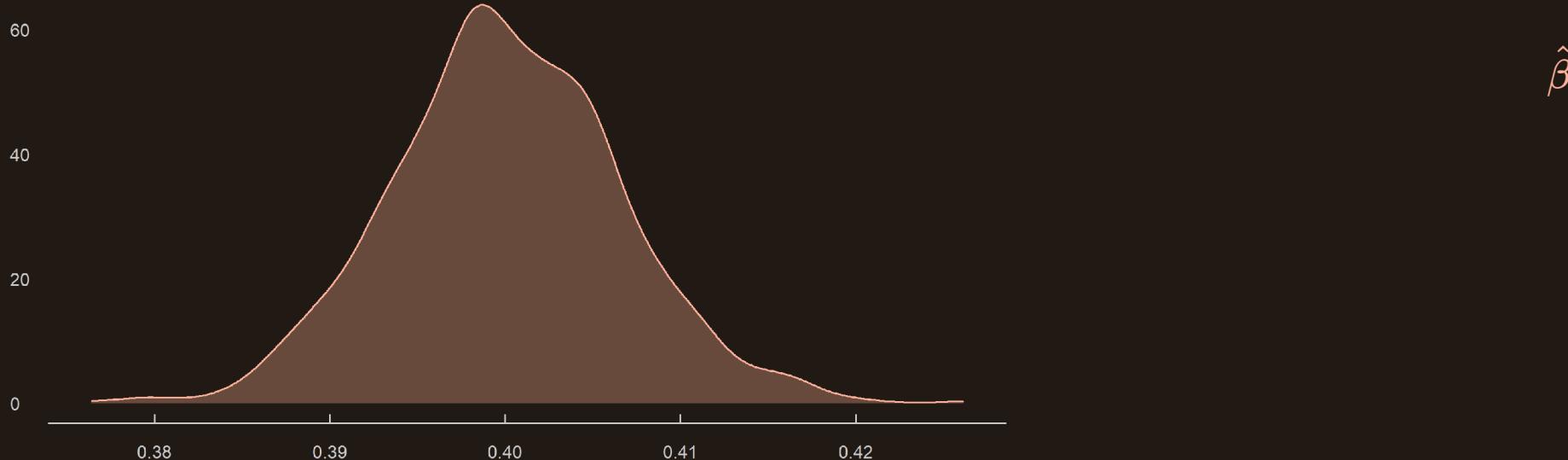


- Some random samples give higher estimates than others
- But **on expectation** we get the **right coefficient!**
- The $\hat{\beta}$ s actually follow a **normal distribution**
- And at the limit their mean would **converge towards β**

1. Asymptotic inference

1.2. Standardization

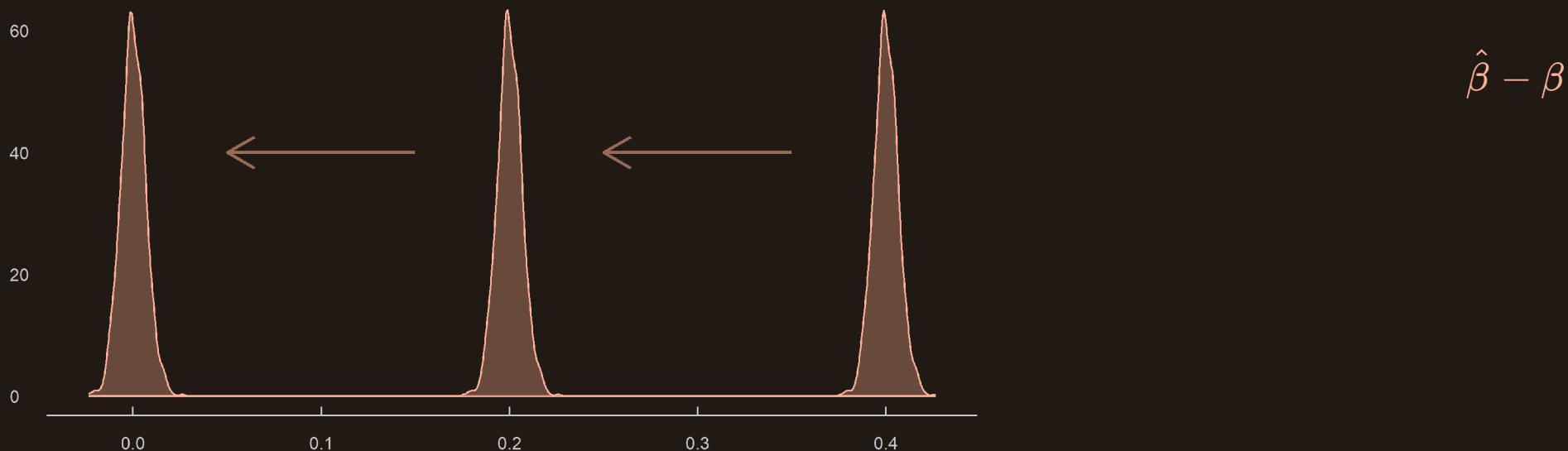
- That is crucial information because it allows to get back to something we know:
 -
 -
 -



1. Asymptotic inference

1.2. Standardization

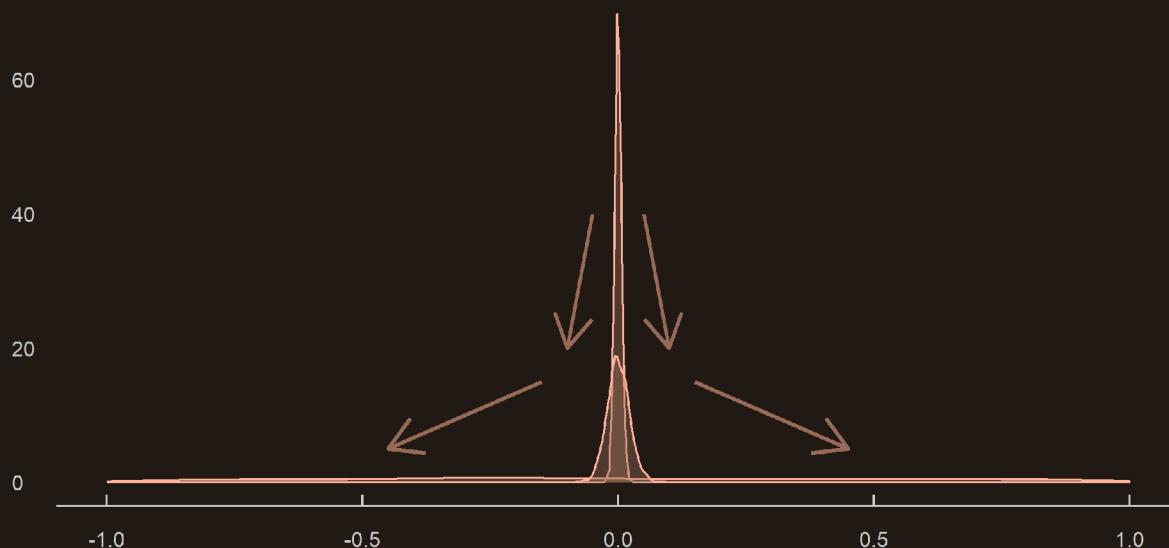
- That is crucial information because it allows to get back to something we know:
 - By subtracting β from the distribution of $\hat{\beta}$
 -
 -



1. Asymptotic inference

1.2. Standardization

- That is crucial information because it allows to get back to something we know:
 - By subtracting β from the distribution of $\hat{\beta}$
 - And dividing by the standard deviation of $\hat{\beta}$
 -

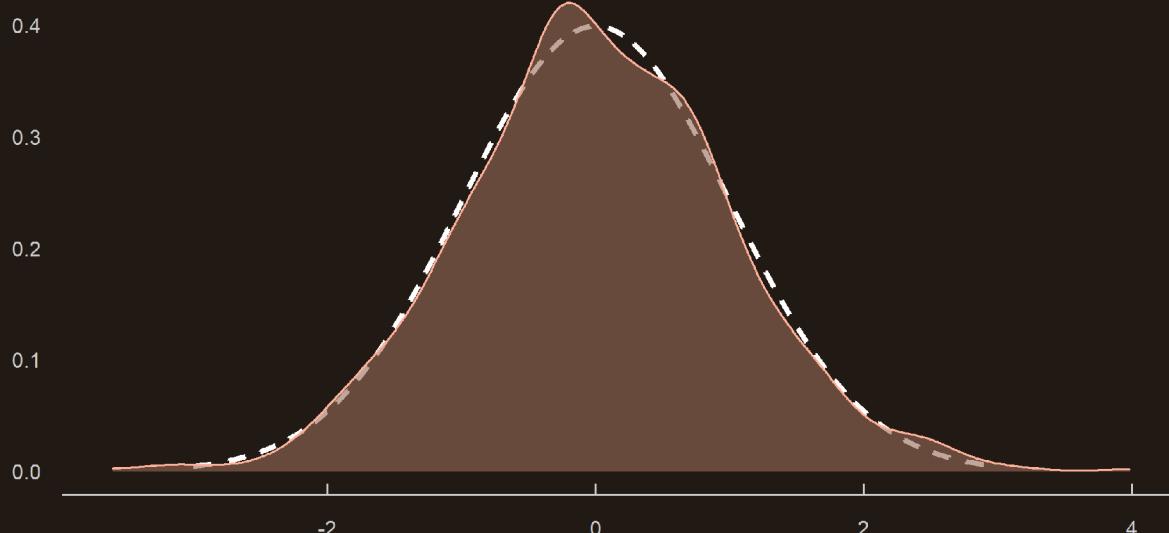


$$\frac{\hat{\beta} - \beta}{\text{SD}(\hat{\beta})}$$

1. Asymptotic inference

1.2. Standardization

- That is crucial information because it allows to get back to something we know:
 - By subtracting β from the distribution of $\hat{\beta}$
 - And dividing by the standard deviation of $\hat{\beta}$
 - With an infinite sample we would obtain the standard normal distribution

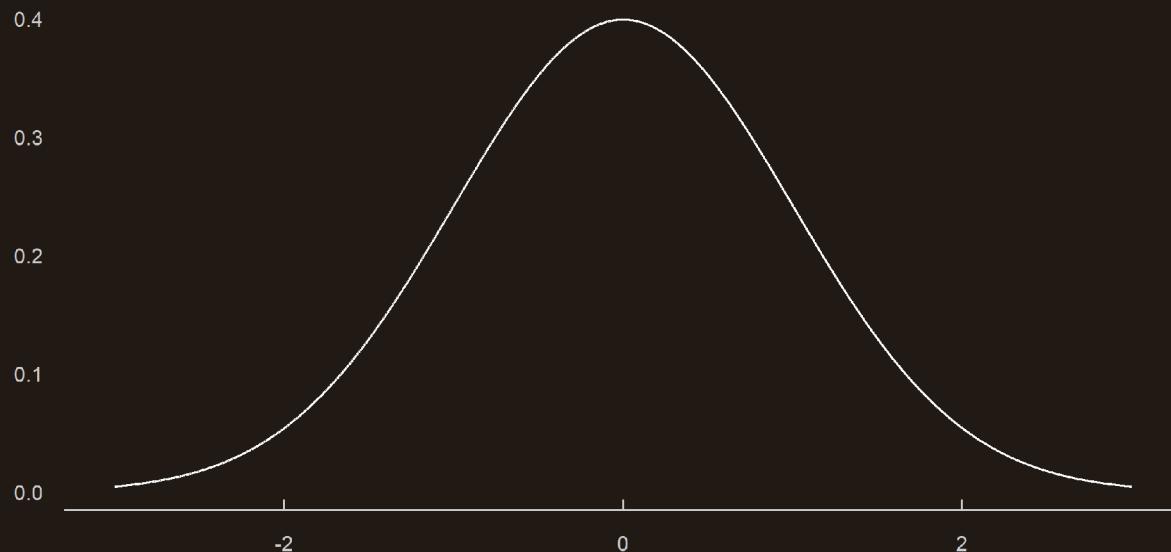


$$\frac{\hat{\beta} - \beta}{\text{SD}(\hat{\beta})} \sim \mathcal{N}(0, 1)$$

1. Asymptotic inference

1.3. Confidence interval

- We can use the fact that we know the standard normal distribution:
 -
 -
 -

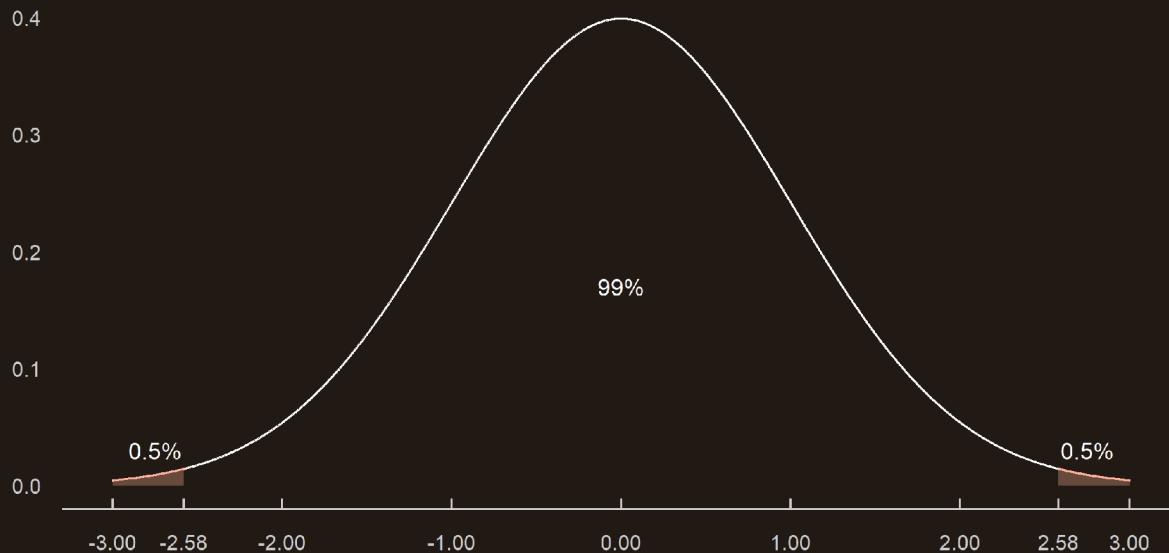


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1. Asymptotic inference

1.3. Confidence interval

- We can use the fact that we know the standard normal distribution:
 - That 99% of the distribution lie between ± 2.58
 -
 -



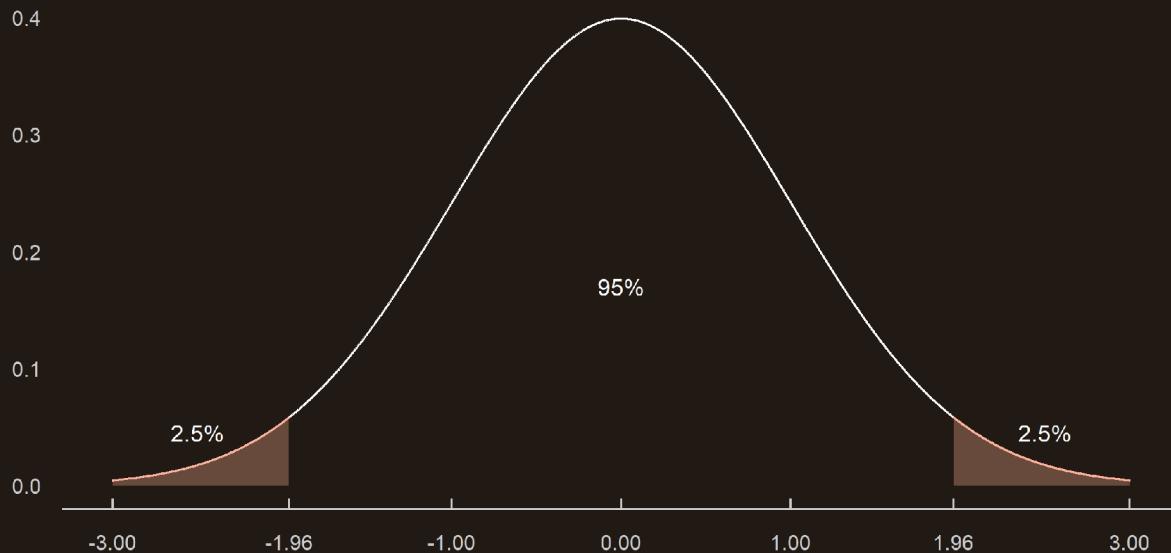
$$\frac{\hat{\beta} - \beta}{\text{SD}(\hat{\beta})} \sim \mathcal{N}(0, 1)$$

$$\Pr \left[-2.58 < \frac{\hat{\beta} - \beta}{\text{SD}(\hat{\beta})} < 2.58 \right] \approx 99\%$$

1. Asymptotic inference

1.3. Confidence interval

- We can use the fact that we know the standard normal distribution:
 - That 99% of the distribution lie between ± 2.58
 - That 95% of the distribution lie between ± 1.96
 -



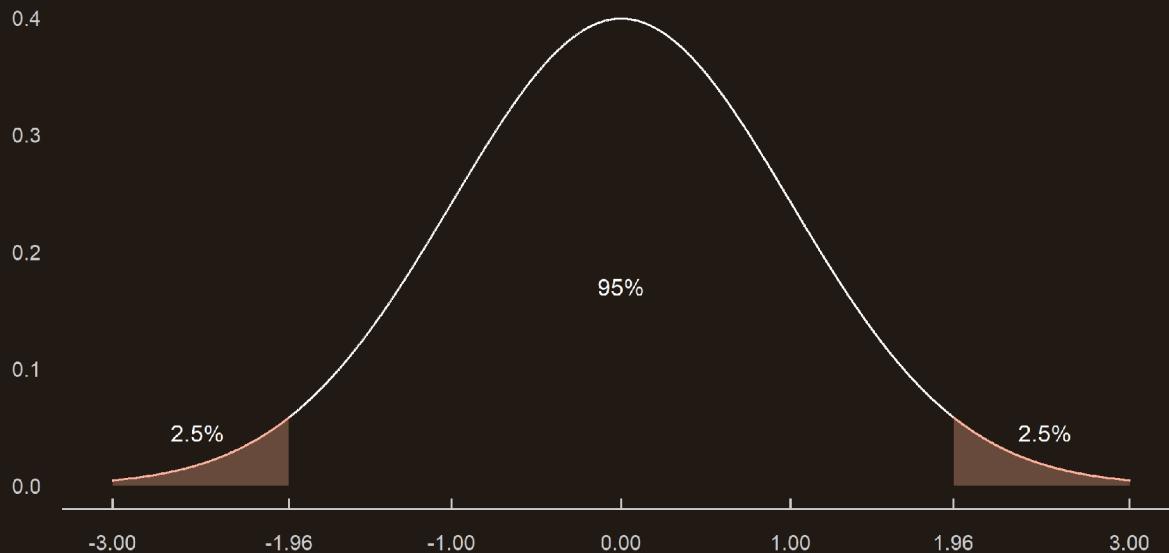
$$\frac{\hat{\beta} - \beta}{\text{SD}(\hat{\beta})} \sim \mathcal{N}(0, 1)$$

$$\Pr \left[-1.96 < \frac{\hat{\beta} - \beta}{\text{SD}(\hat{\beta})} < 1.96 \right] \approx 95\%$$

1. Asymptotic inference

1.3. Confidence interval

- We can use the fact that we know the standard normal distribution:
 - That 99% of the distribution lie between ± 2.58
 - That 95% of the distribution lie between ± 1.96
 - This is what allows to determine confidence intervals



$$\frac{\hat{\beta} - \beta}{\text{SD}(\hat{\beta})} \sim \mathcal{N}(0, 1)$$

$$\Pr \left[-1.96 < \frac{\hat{\beta} - \beta}{\text{SD}(\hat{\beta})} < 1.96 \right] \approx 95\%$$



Confidence interval



1. Asymptotic inference

1.3. Confidence interval

$$\Pr \left[-1.96 < \frac{\hat{\beta} - \beta}{\text{SD}(\hat{\beta})} < 1.96 \right] \approx 95\%$$

$$\Pr \left[-1.96 \times \text{SD}(\hat{\beta}) < \hat{\beta} - \beta < 1.96 \times \text{SD}(\hat{\beta}) \right] \approx 95\%$$

$$\Pr \left[-1.96 \times \text{SD}(\hat{\beta}) - \hat{\beta} < -\beta < 1.96 \times \text{SD}(\hat{\beta}) - \hat{\beta} \right] \approx 95\%$$

$$\Pr \left[+1.96 \times \text{SD}(\hat{\beta}) + \hat{\beta} > \beta > -1.96 \times \text{SD}(\hat{\beta}) + \hat{\beta} \right] \approx 95\%$$

$$\text{CI}_{95\%} : \hat{\beta} \pm 1.96 \times \text{SD}(\hat{\beta})$$



Overview

1. Asymptotic inference ✓

- 1.1. Data generating process
- 1.2. Standardization
- 1.3. Confidence interval

2. Exact inference

- 2.1. Standard error
- 2.2. Student-t distribution
- 2.3. Confidence interval

3. Hypothesis testing

- 3.1. P-value
- 3.2. linearHypothesis()

4. Wrap up!



Overview

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2. Exact inference

2.1. Standard error

- In the **previous section** I used phrases like "*at the limit*" or "**with an infinite sample**"
 - But **in practice** this is **not the case**, so things behave slightly differently
 - And this implies to make a few **statistical adjustments** to account for that

$$\hat{\beta} \pm 1.96 \times \text{SD}(\hat{\beta})$$

- First we **cannot measure** directly the **standard deviation** of $\hat{\beta}$
 - Indeed in practice we have **only one observation** of $\hat{\beta}$, not its whole distribution
 - But like for the mean, we can compute a **standard error** instead

Standard deviation

→ Measures the amount of variability, or dispersion, from the individual data values to the mean

Standard error

→ Measures how far an estimate from a given sample is likely to be from the true parameter of interest



2. Exact inference

2.1. Standard error

- We won't go through the theoretical computations together, but let's have a look at the formula:

$$\text{se}(\hat{\beta}) = \sqrt{\text{Var}(\hat{\beta})} = \sqrt{\frac{\sum_{i=1}^n \hat{\varepsilon}_i^2}{(n - \#\text{parameters}) \sum_{i=1}^n (x_i - \bar{x})^2}}$$

- Notice that the variance, and thus the standard error of our estimate, decreases as:



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- Notice that the variance, and thus the standard error of our estimate, decreases as:
 - The number of observations gets bigger



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- Notice that the variance, and thus the standard error of our estimate, decreases as:
 - The number of observations gets bigger
 - The number of parameters decreases
 - The sum of squared errors relative to the variance of x decreases



2. Exact inference

2.1. Standard error

- We won't go through the theoretical computations together, but let's have a look at the formula:

$$\text{se}(\hat{\beta}) = \sqrt{\widehat{\text{Var}}(\hat{\beta})} = \sqrt{\frac{\sum_{i=1}^n \hat{\varepsilon}_i^2}{(n - \#\text{parameters}) \sum_{i=1}^n (x_i - \bar{x})^2}}$$

- Notice that the variance, and thus the standard error of our estimate, decreases as:
 - The number of observations gets bigger
 - The number of parameters decreases
 - The sum of squared errors decreases relative to the variance of x
- And as the **standard error** gets **bigger, the confidence interval gets bigger**:

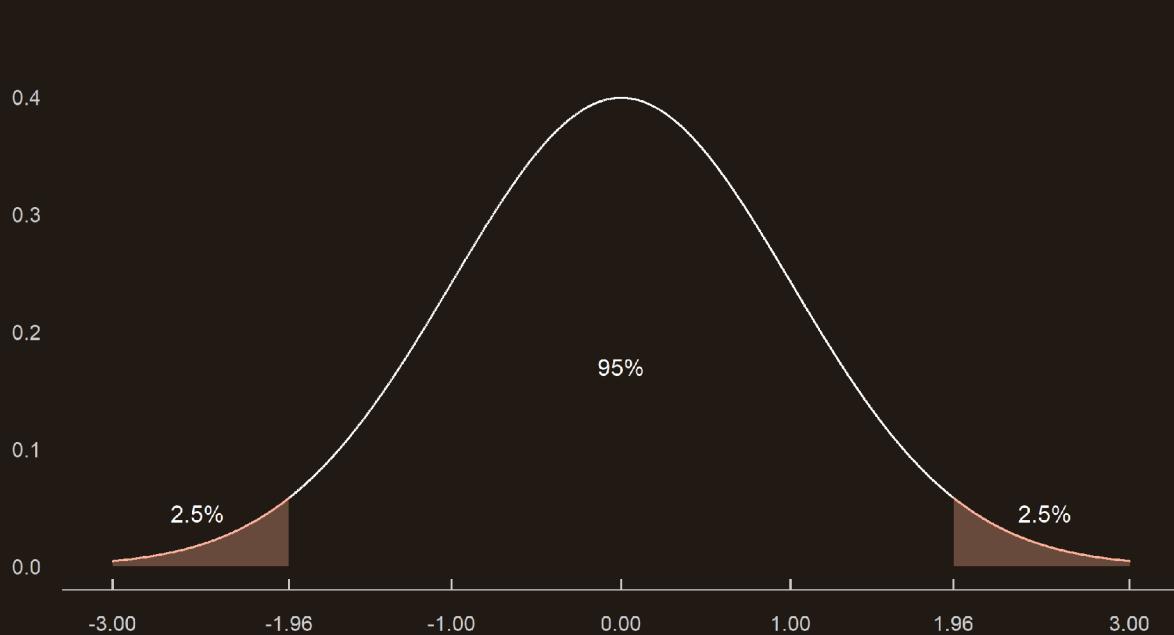
$$\hat{\beta} \pm 1.96 \times \text{se}(\hat{\beta})$$

2. Exact inference

2.2. Student-t distribution

- But that's not it, remember that we took the value **1.96** from the **normal distribution**

$$\hat{\beta} \pm 1.96 \times \text{se}(\hat{\beta})$$

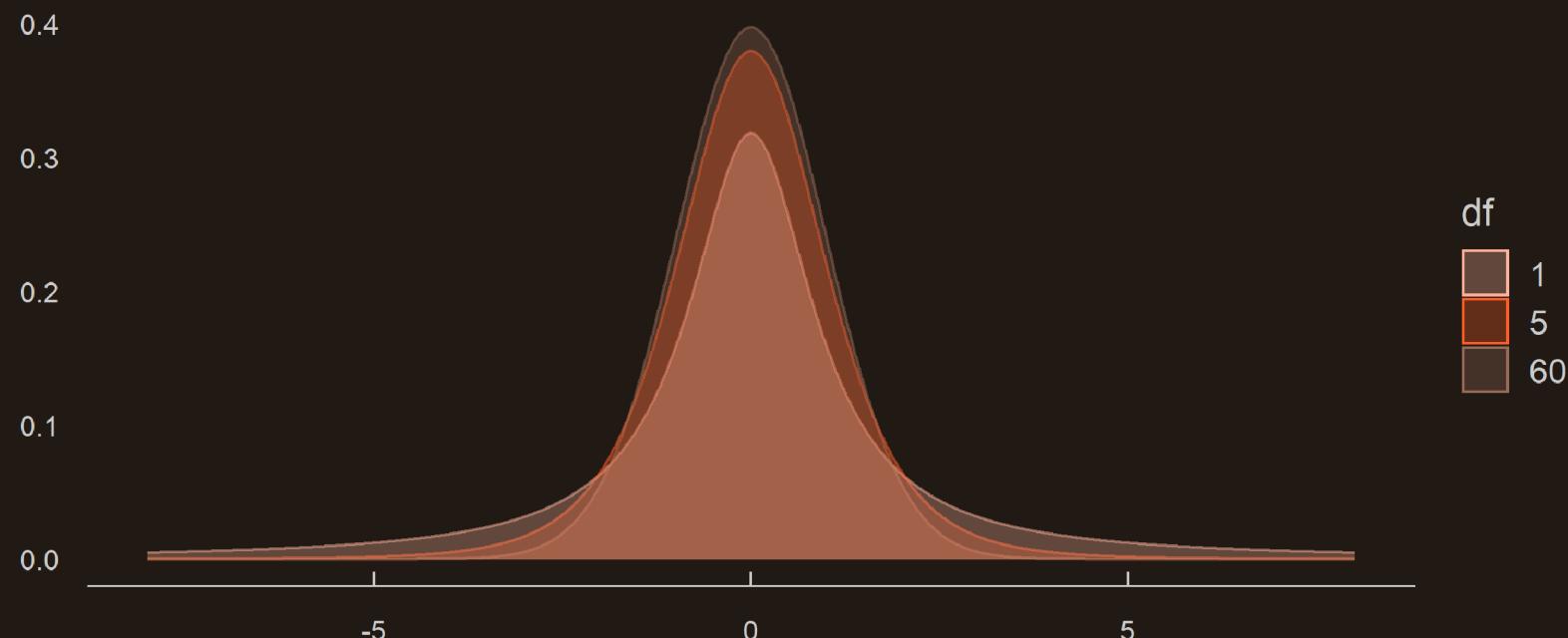


- But the **normal distribution** is what $\frac{\hat{\beta} - \beta}{\text{SD}(\hat{\beta})}$ converges to **at the limit**
- In the **finite** world, $\frac{\hat{\beta} - \beta}{\text{se}(\hat{\beta})}$ follows a slightly flatter distribution
- The **Student t distribution**, whose precise shape depends on the number of observations we have and parameters we estimate

2. Exact inference

2.2. Student-t distribution

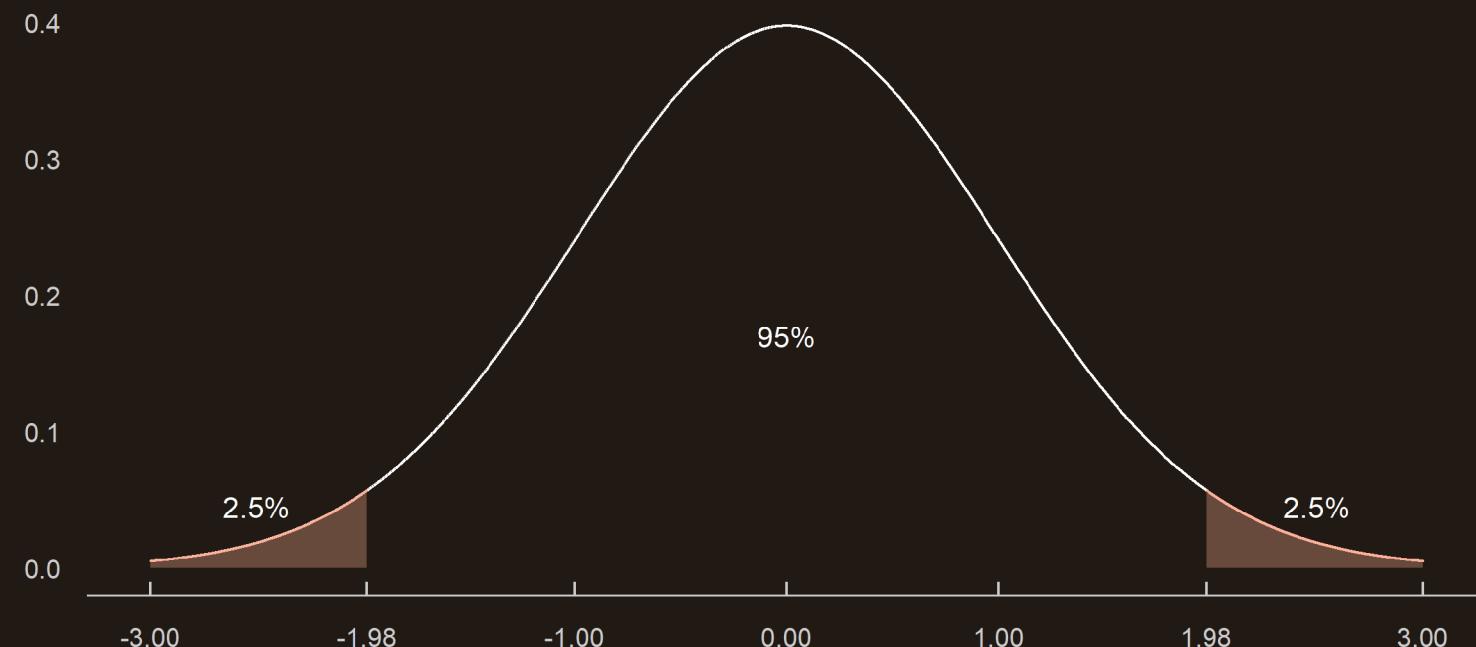
- The Student t distribution **accounts for** the fact that the **sample is finite**
 - The lower the number of **degrees of freedom** (#observations - #parameters) the flatter
 - And it **tends to a normal** distribution as the number of degrees of freedom $\rightarrow \infty$



2. Exact inference

2.2. Student-t distribution

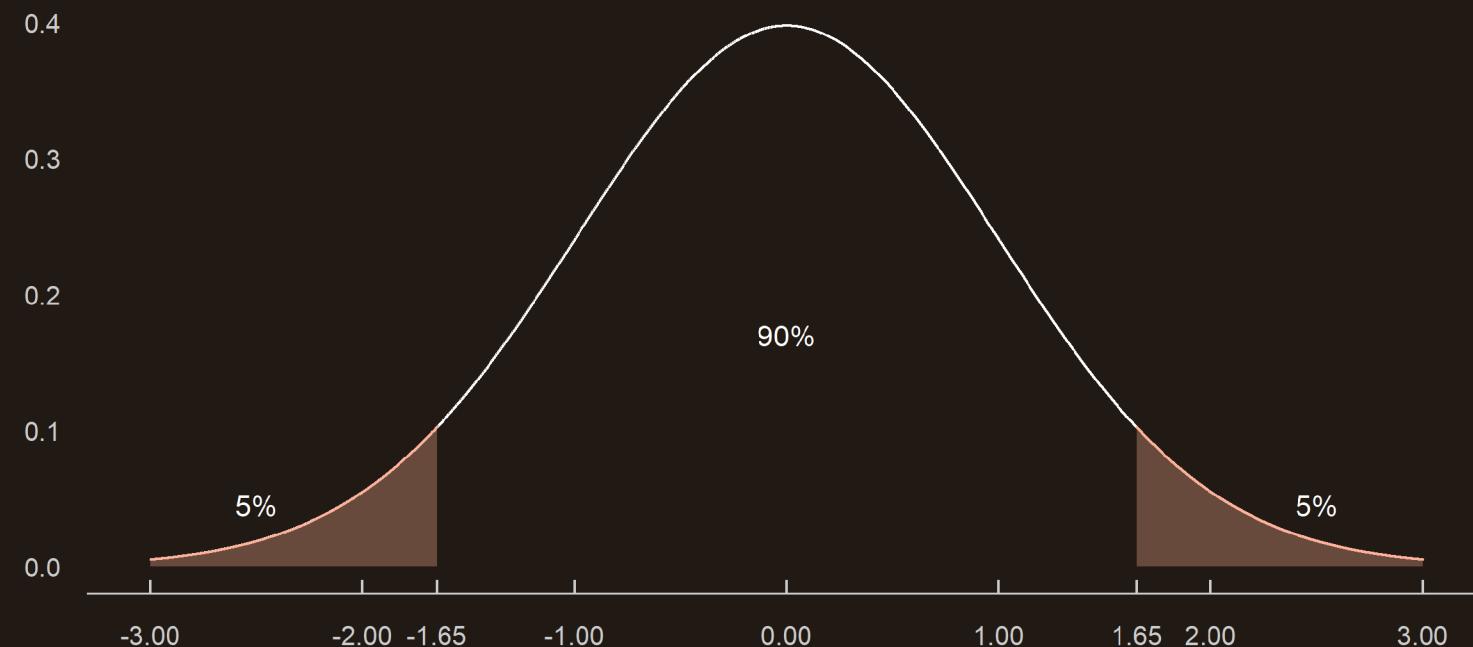
- But **we know the Student t distributions** just as well as the standard normal distribution
 - With 100 degrees of freedom, 95% of the distribution lie between ± 1.98
 -



2. Exact inference

2.2. Student-t distribution

- But **we know the Student t distributions** just as well as the standard normal distribution
 - With 100 degrees of freedom, 95% of the distribution lie between ± 1.98
 - With 3,000 degrees of freedom, 90% of the distribution lie between ± 1.65





2. Exact inference

2.2. Student-t distribution

- So **instead of 1.96**, we must use the **value such that**:
 - The **desired percentage** of the distribution is comprised within \pm that value...
 - For a Student t distribution with the relevant number of **degrees of freedom**
- We can get these values easily with the **qt()** function, indicating:
 -
 -

```
qt( , )
```



2. Exact inference

2.2. Student-t distribution

- So **instead of 1.96**, we must use the **value such that**:
 - The **desired percentage** of the distribution is comprised within \pm that value...
 - For a Student t distribution with the relevant number of **degrees of freedom**
- We can get these values easily with the **qt()** function, indicating:
 - The share of the distribution below the value we're looking for (e.g., 0.975 for a 95% CI)
 -

```
qt(.975, )
```



2. Exact inference

2.2. Student-t distribution

- So **instead of 1.96**, we must use the **value such that**:
 - The **desired percentage** of the distribution is comprised within \pm that value...
 - For a Student t distribution with the relevant number of **degrees of freedom**
- We can get these values easily with the **qt()** function, indicating:
 - The share of the distribution below the value we're looking for (e.g., 0.975 for a 95% CI)
 - The number of degrees of freedom of the Student t distribution (e.g., 88 observations - 2 parameters)

```
qt(.975, 86)
```

```
## [1] 1.987934
```

- Denote this value $t(df)_{1-\frac{\alpha}{2}}$
 - With α equal to 1 – the confidence level
 - And df the number of degrees of freedom

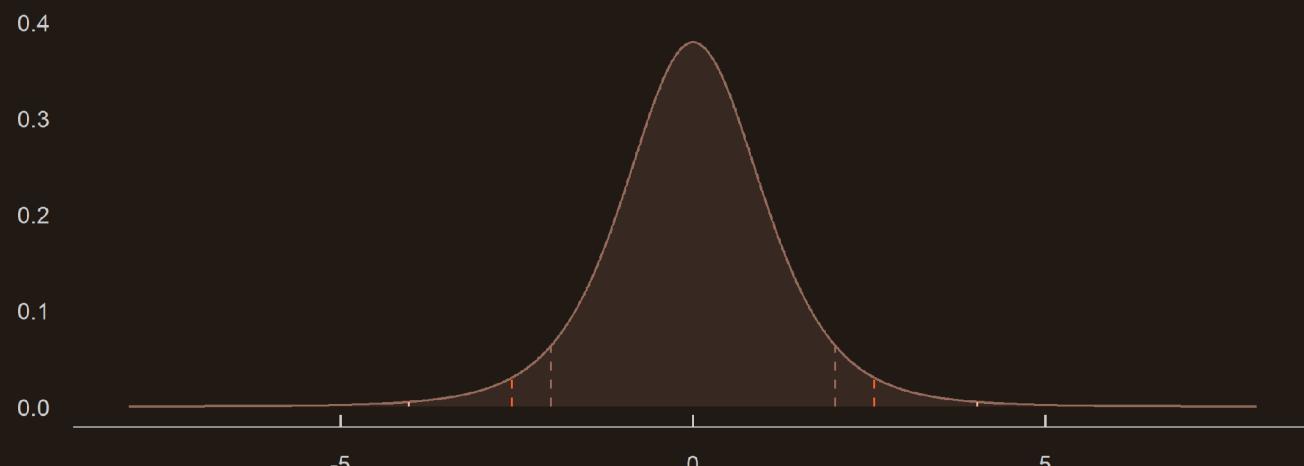
2. Exact inference

2.3. Confidence interval

- The formula for the confidence interval in finite sample hence writes:

$$\hat{\beta} \pm t(df)_{1-\frac{\alpha}{2}} \times \text{se}(\hat{\beta})$$

- The confidence interval increases as:
 - The confidence level increases
 -



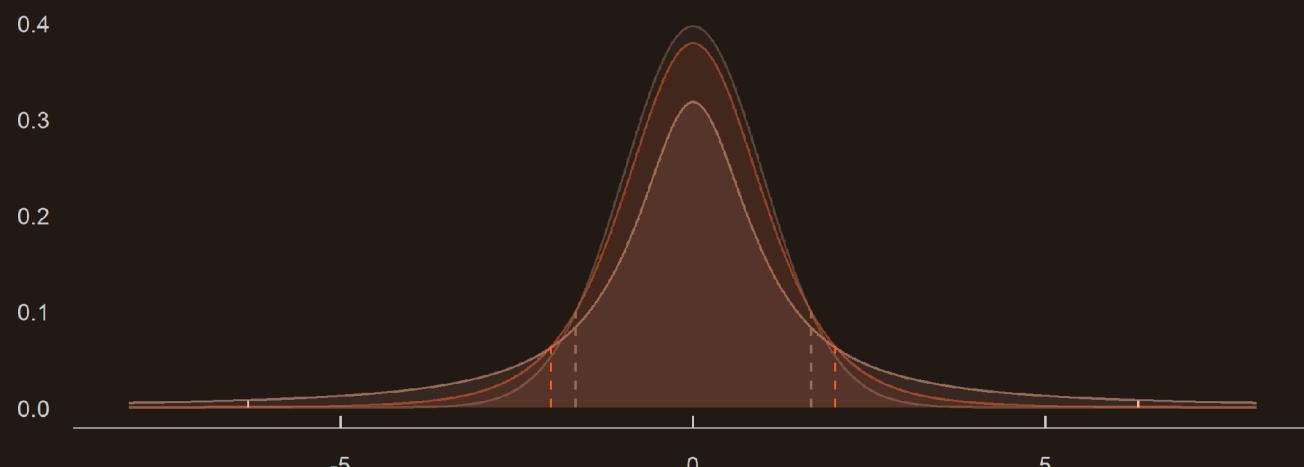
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- The formula for the confidence interval in finite sample hence writes:

$$\hat{\beta} \pm t(df)_{1-\frac{\alpha}{2}} \times \text{se}(\hat{\beta})$$

- The confidence interval increases as:
 - The confidence level increases
 - The number of degrees of freedom decreases



Practice

10 : 00

1) Import the `ggcurve.csv` dataset

2) Regress the IGE on the Gini coefficient and store the estimated regression parameters

3) Compute the 95% confidence interval of the regression slope

$$\hat{\beta} \pm t(\text{df})_{1-\frac{\alpha}{2}} \times \text{se}(\hat{\beta})$$

$$\text{se}(\hat{\beta}) = \sqrt{\frac{\sum_{i=1}^n \hat{\varepsilon}_i^2}{(n - \#\text{parameters}) \sum_{i=1}^n (x_i - \bar{x})^2}}$$

You've got 10 minutes!

Solution

1) Import the `ggcurve.csv` dataset

```
ggcurve <- read.csv("C:/User/Documents/ggcurve.csv")
```

2) Regress the IGE on the Gini coefficient and store the regression slope

```
model <- lm(ige ~ gini, ggcurve)
model
```

```
##
## Call:
## lm(formula = ige ~ gini, data = ggcurve)
##
## Coefficients:
## (Intercept)      gini
## -0.09129      1.01546
```

```
alpha <- model$coefficients[1]
beta <- model$coefficients[2]
```

Solution

3) Compute the 95% confidence interval of the regression slope

```
se_dat <- ggcurve %>%
  mutate(fit = alpha + gini * beta, e = ige - fit) %>%
  summarise(se = sqrt(sum(e^2)/((n()-2)*sum((gini-mean(gini))^2))))  
se_dat$se
```

```
## [1] 0.2642477
```

```
beta - se_dat$se * qt(.975, nrow(ggcurve) - 2)
```

```
##      gini  
## 0.4642511
```

```
beta + se_dat$se * qt(.975, nrow(ggcurve) - 2)
```

```
##      gini  
## 1.566673
```



Overview

1. Asymptotic inference ✓

- 1.1. Data generating process
- 1.2. Standardization
- 1.3. Confidence interval

2. Exact inference ✓

- 2.1. Standard error
- 2.2. Student-t distribution
- 2.3. Confidence interval

3. Hypothesis testing

- 3.1. P-value
- 3.2. linearHypothesis()

4. Wrap up!



Overview

1. Asymptotic inference ✓

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- 2.1. Standard error
- 2.2. Student-t distribution
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3. Hypothesis testing

- 3.1. P-value
- 3.2. linearHypothesis()

3. Hypothesis testing

3.1. P-value

- We now have the **95% confidence interval** for our estimate:
 - Our estimate of β is 1.02
 - And we are 95% sure that β lies between 0.46 and 1.57



- Note that in our **confidence interval** formula:
 - The **standard error** and the relevant Student **t distribution** are **given**
 - But the **confidence level** $1 - \alpha$ was **chosen arbitrarily**

→ Setting a **higher confidence** level would **widen the confidence interval**

→ Allowing for a **lower confidence** level would **narrow the confidence interval**



3. Hypothesis testing

3.1. P-value

- So far we framed the problem as:

"What are the values β is likely to take under a given confidence level?"

- But we could also think of it as:

"Under which confidence level is β is likely to take a given value?"

- And this is actually a very **practical way of framing the question:**
 - To (in)validate the predictions from a theoretical model
 - To know under which confidence level β is likely to be $\neq 0$ at all

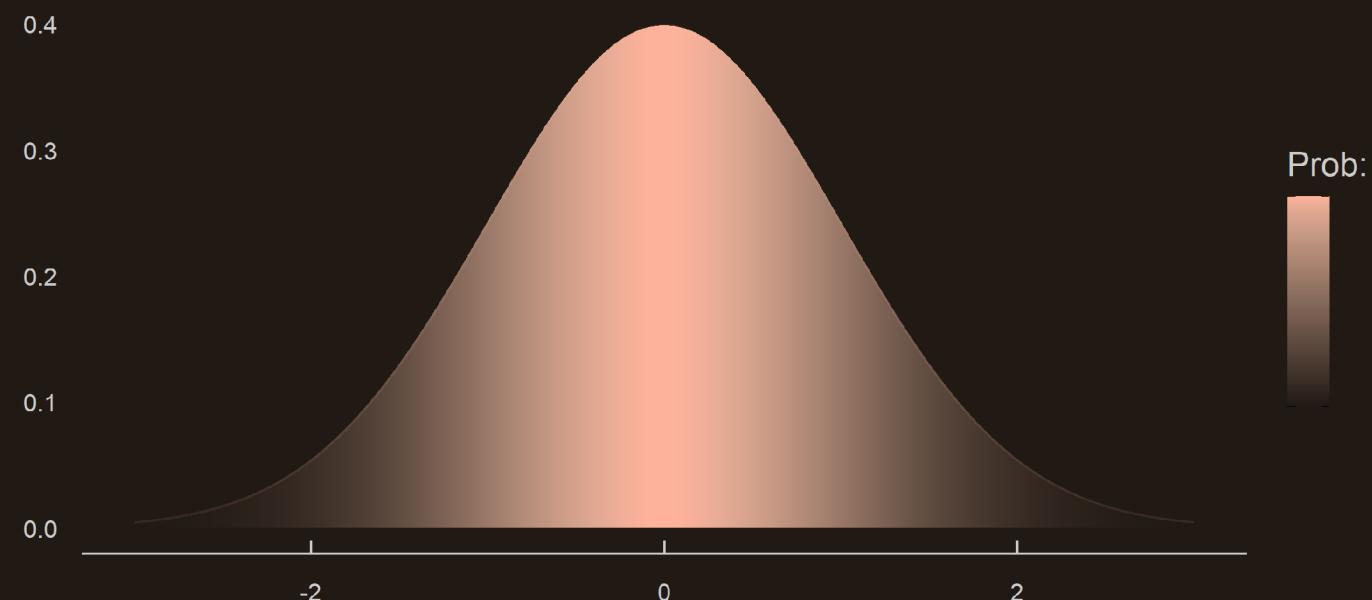
→ **But how to answer such questions in practice?**

3. Hypothesis testing

3.1. P-value

- We can start from the fact that even though we do not know β , we know that:

$$\frac{\hat{\beta} - \beta}{\text{se}(\hat{\beta})} \sim t(\text{df})$$



3. Hypothesis testing

3.1. P-value

- And that in this distribution some values are quite plausible:

$$\frac{\hat{\beta} - 1}{\text{se}(\hat{\beta})}$$

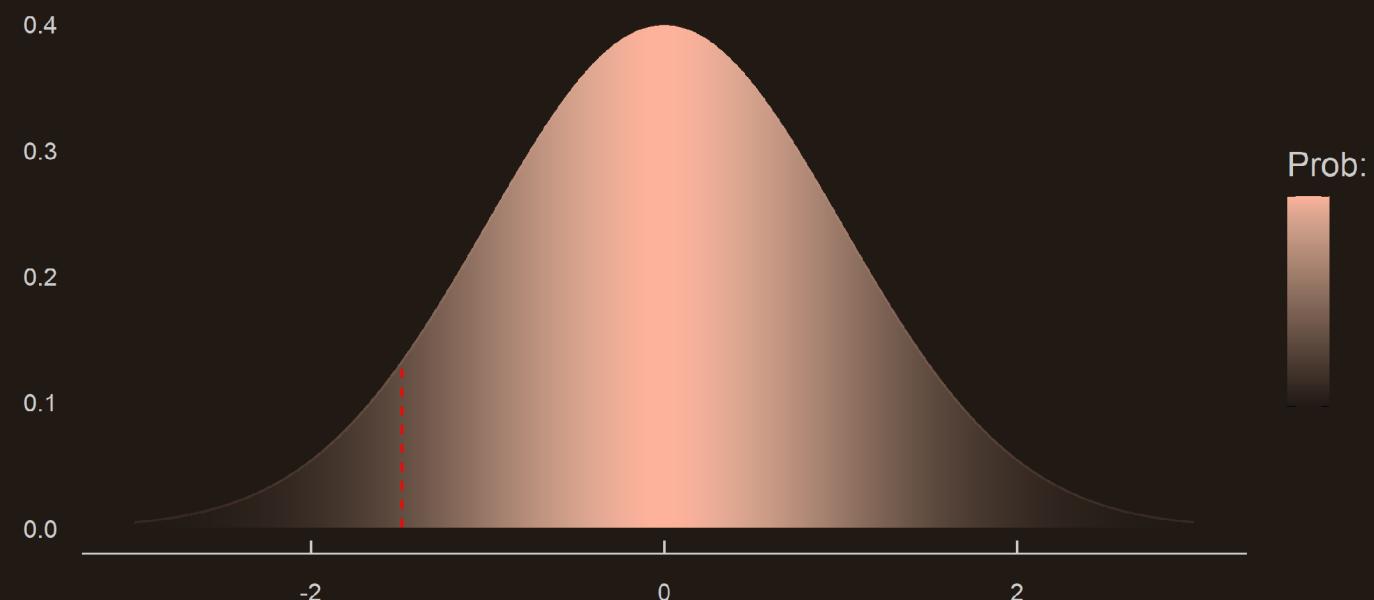


3. Hypothesis testing

3.1. P-value

- And some are way less plausible:

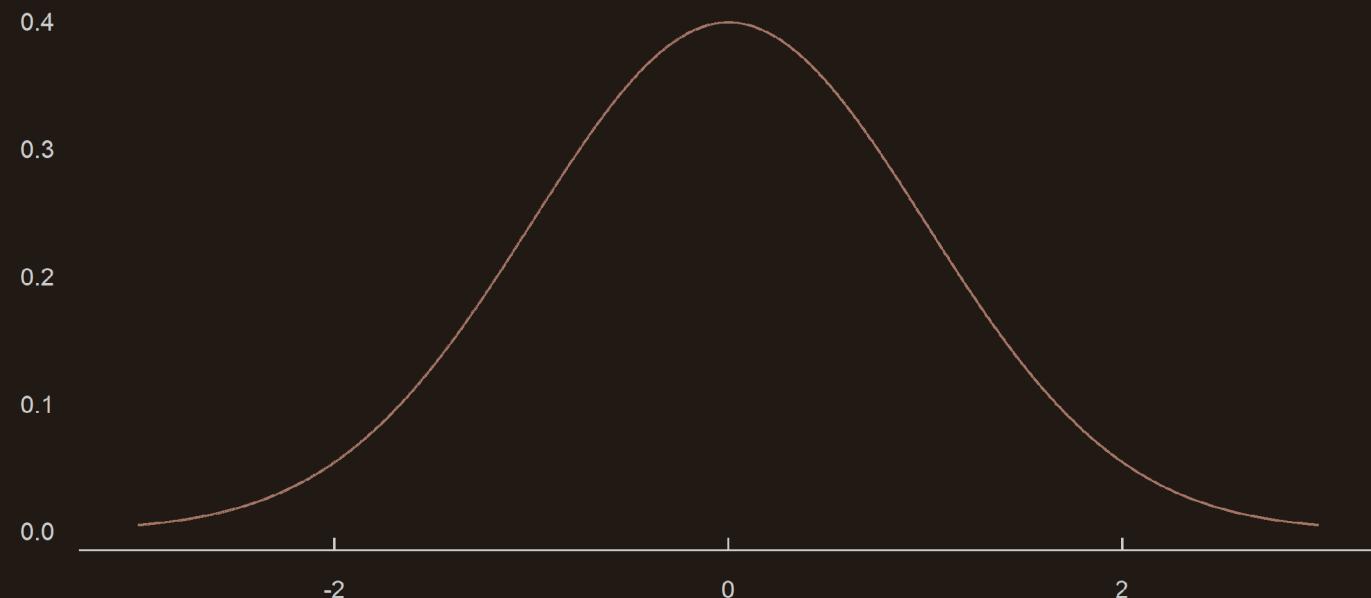
$$\frac{\hat{\beta} - 2.5}{\text{se}(\hat{\beta})}$$



3. Hypothesis testing

3.1. P-value

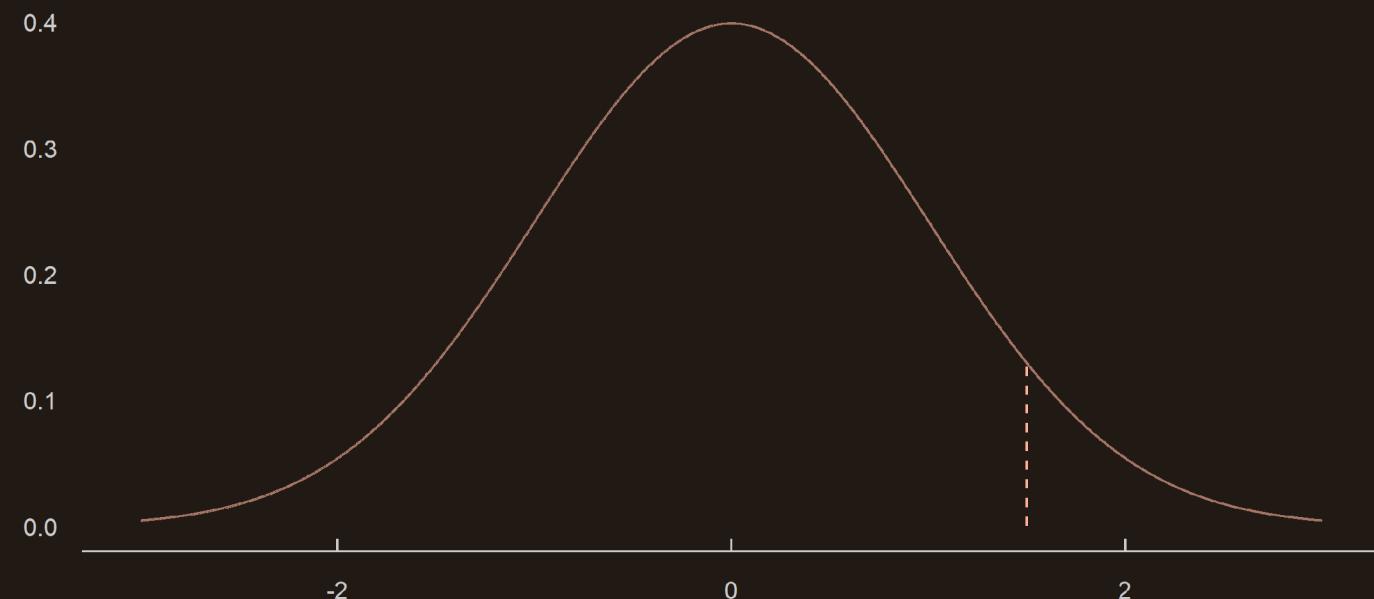
- But because the distribution is **continuous**:
 -
 -



3. Hypothesis testing

3.1. P-value

- But because the distribution is **continuous**:
 - The **probability** to draw any **exact value** would be **0**
 -



3. Hypothesis testing

3.1. P-value

- But because the distribution is **continuous**:
 - The **probability** to draw any **exact value** would be **0**
 - We can only compute the **probability to fall below** that value



3. Hypothesis testing

3.1. P-value

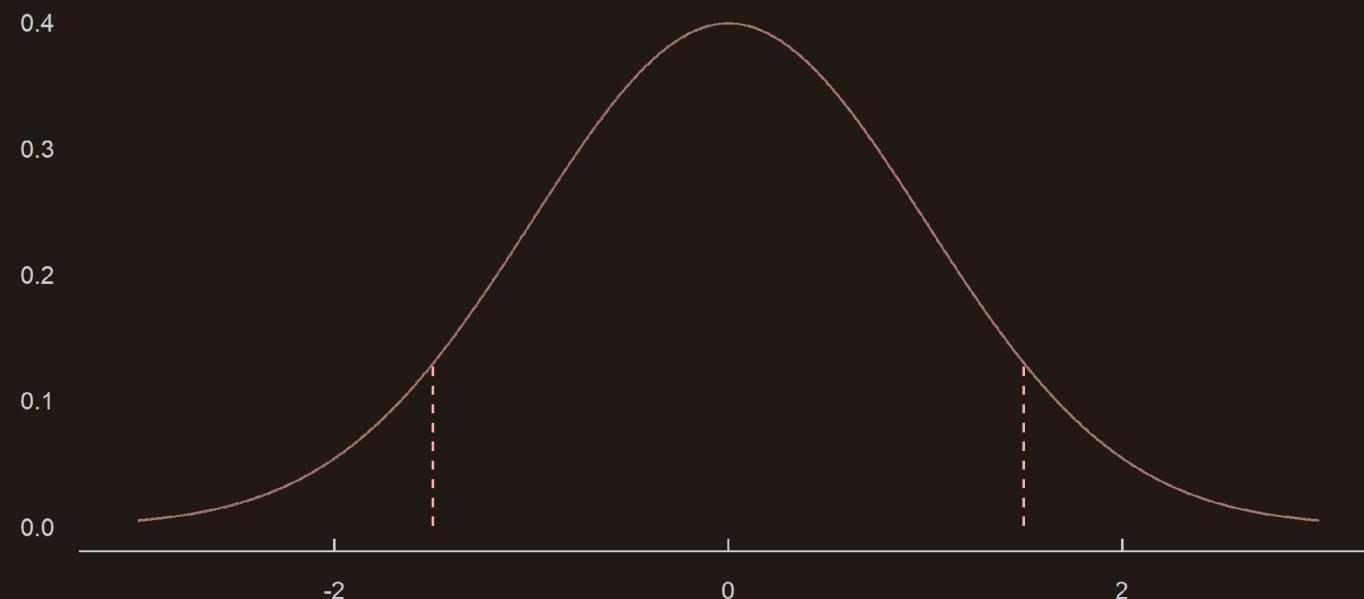
- But because the distribution is **continuous**:
 - The **probability** to draw any **exact value** would be **0**
 - **Or to fall above** that value **if it is negative**



3. Hypothesis testing

3.1. P-value

- But **generally** what makes sense is to know what are the **chances to fall *that far* from 0:**
 -
 -



3. Hypothesis testing

3.1. P-value

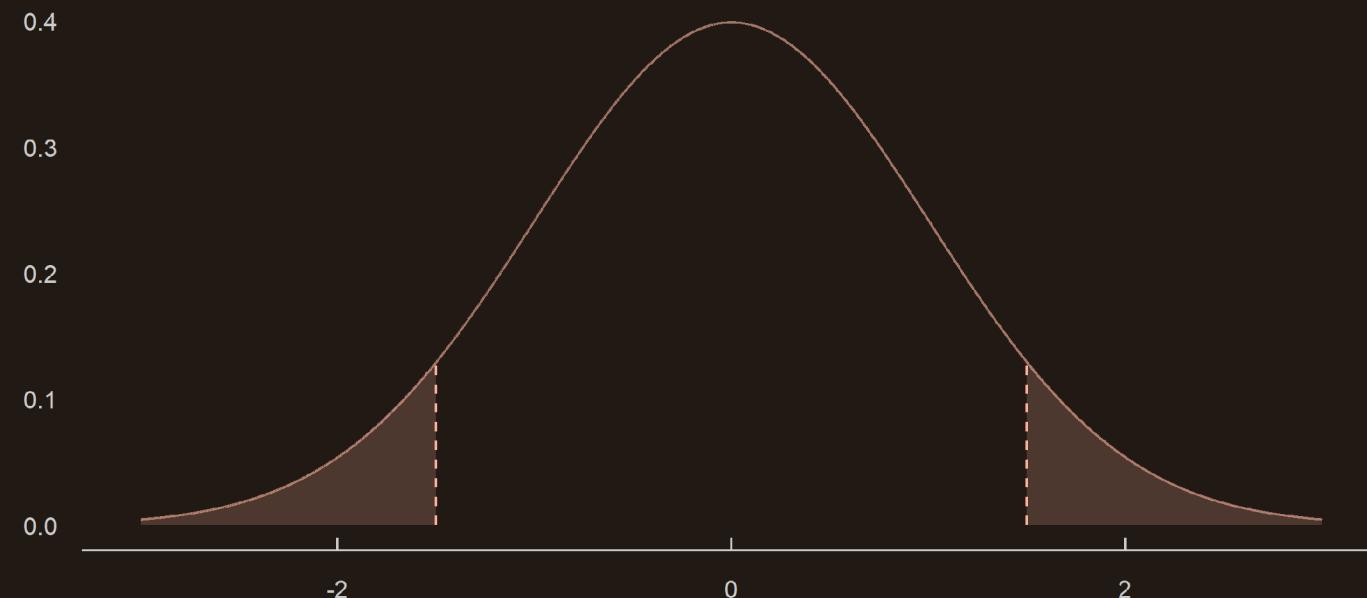
- But **generally** what makes sense is to know what are the **chances to fall *that far* from 0:**
 - So we take $1 -$ the probability to fall below the absolute value
 -



3. Hypothesis testing

3.1. P-value

- But **generally** what makes sense is to know what are the **chances to fall *that far* from 0:**
 - So we take $1 -$ the probability to fall below the absolute value
 - And we multiply it by 2





3. Hypothesis testing

3.1. P-value

- The **resulting area** is what we call a **p-value**
 - It is the probability that β falls at least as far from $\hat{\beta}$ as the hypothesized value
- Consider finding $\hat{\beta} = 4$ and a hypothesizing value of 3 for β
 - A p-value of 5% indicates that there is only a 5% chance to find a $\hat{\beta} = 4$ if $\beta = 3$
 - Below that threshold we would reject the hypothesis that $\beta = 3$ at the 95% confidence level
- Notice that in this example, the 95% confidence interval of $\hat{\beta}$ would not include the value 3
 - With a hypothesized value equal to the bound of a confidence interval the p-value would equal 1 - the corresponding confidence level
 - So a p-value lower than α means that the hypothesized value is outside the $(1 - \alpha)\%$ confidence interval

→ Let's go through a formal example with our data



3. Hypothesis testing

3.1. P-value

- Can we **reject** at the 95% confidence level that $\beta = 0$?

```
beta
```

```
##      gini  
## 1.015462
```

- We should start by hypothesizing that $\beta = 0$
 - This is what we call the "**null hypothesis**" H_0

$$H_0 : \beta = 0$$

Under H_0 :

$$\frac{\hat{\beta} - 0}{\text{se}(\hat{\beta})} \sim t(\text{df})$$



3. Hypothesis testing

3.1. P-value

- We should find the **area below** $(\hat{\beta} - 0)/\text{se}(\hat{\beta})$ in a Student t distribution we the right number of df
 - $(\hat{\beta} - 0)/\text{se}(\hat{\beta})$ is what we call **the t-stat**

```
(beta - 0) / se_dat$se
```

```
##      gini
## 3.842842
```

- While **qt()** gave us the **value** for a certain probability, **pt()** gives the the **probability** for a given value:
 -
 -

```
pt( , )
```



3. Hypothesis testing

3.1. P-value

- We should find the **area below** $(\hat{\beta} - 0)/\text{se}(\hat{\beta})$ in a Student t distribution we the right number of df
 - $(\hat{\beta} - 0)/\text{se}(\hat{\beta})$ is what we call **the t-stat**

```
(beta - 0) / se_dat$se
```

```
##      gini
## 3.842842
```

- While **qt()** gave us the **value** for a certain probability, **pt()** gives the the **probability** for a given value:
 - Put in the **t-stat**
 -

```
pt((beta - 0) / se_dat$se, )
```



3. Hypothesis testing

3.1. P-value

- We should find the **area below** $(\hat{\beta} - 0)/\text{se}(\hat{\beta})$ in a Student t distribution with the right number of df
 - $(\hat{\beta} - 0)/\text{se}(\hat{\beta})$ is what we call **the t-stat**

```
(beta - 0) / se_dat$se
```

```
##      gini  
## 3.842842
```

- While **qt()** gave us the **value** for a certain probability, **pt()** gives the the **probability** for a given value:
 - Put in the **t-stat**
 - And the **degrees of freedom**

```
pt((beta - 0) / se_dat$se, nrow(ggcurve) - 2)
```

```
##      gini  
## 0.9994921
```



3. Hypothesis testing

3.1. P-value

- We must then:
 - Take **1 - this probability** (area above the t-stat)
 -

```
1 - pt(abs((beta - 0) / se_dat$se), nrow(ggcurve) - 2)
```

```
##          gini
## 0.0005078528
```



3. Hypothesis testing

3.1. P-value

- We must then:
 - Take **1 - this probability** (area above the t-stat)
 - And **multiply it by 2** (consider the absolute distance and not the signed distance)

```
2 * (1 - pt(abs((beta - 0) / se_dat$se), nrow(ggcurve) - 2))
```

```
##      gini
## 0.001015706
```

- The **p-value** is **lower than 1%**:
 - We can **reject at the 99% confidence level** that $\beta = 0$
 - In that case we say that $\hat{\beta}$ is **significantly different from 0** at the 1% significance level
- But the **p-value** is **greater than 0.1%**:
 - We **cannot reject at the 99.9% confidence level** that $\beta = 0$
 - In that case we say that $\hat{\beta}$ is **not significantly different from 0** at the 0.1% significance level



3. Hypothesis testing

3.1. P-value

- By default, the **summary()** function **tests** whether or not each coefficient is significantly **different from 0**
 -

```
summary(lm(ige ~ gini, ggcurve))
```



3. Hypothesis testing

3.1. P-value

- By default, the **summary()** function **tests** whether or not each coefficient is significantly **different from 0**
 - You can **extract** the information from the **\$coefficient** attribute of the output

```
summary(lm(ige ~ gini, ggcurve))$coefficients
```

```
##             Estimate Std. Error    t value   Pr(>|t|)  
## (Intercept) -0.09129311  0.1287045 -0.7093234 0.486311455  
## gini        1.01546204  0.2642477  3.8428420 0.001015706
```

- For each coefficient it indicates:
 - The standard error
 - The t -stat ($H_0 : \beta = 0$)
 - The p-value ($H_0 : \beta = 0$)
- The output of the **summary()** function is great to have a **quick overview** of the model:

```
summary(lm(ige ~ gini, ggcurve))
```



3. Hypothesis testing

3.1. P-value

```
##  
## Call:  
## lm(formula = ige ~ gini, data = ggcurve)  
##  
## Residuals:  
##      Min       1Q   Median       3Q      Max  
## -0.188991 -0.088238 -0.000855  0.047284  0.252310  
##  
## Coefficients:  
##             Estimate Std. Error t value Pr(>|t|)  
## (Intercept) -0.09129    0.12870  -0.709  0.48631  
## gini        1.01546    0.26425   3.843  0.00102 **  
## ---  
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1  
##  
## Residual standard error: 0.1159 on 20 degrees of freedom  
## Multiple R-squared:  0.4247,    Adjusted R-squared:  0.396  
## F-statistic: 14.77 on 1 and 20 DF,  p-value: 0.001016
```



3. Hypothesis testing

3.1. P-value

```
##  
## Call:  
## lm(formula = ige ~ gini, data = ggcurve) ← Command  
##  
## Residuals:  
##      Min       1Q   Median       3Q      Max  
## -0.188991 -0.088238 -0.000855  0.047284  0.252310  
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## (Intercept) -0.09129    0.12870  -0.709  0.48631  
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```



3. Hypothesis testing

3.1. P-value

```
##  
## Call:  
## lm(formula = ige ~ gini, data = ggcurve) ← Command  
##  
## Residuals:  
##      Min       1Q   Median       3Q      Max  
## -0.188991 -0.088238 -0.000855  0.047284  0.252310 ← Residuals distribution  
##  
## Coefficients:  
##             Estimate Std. Error t value Pr(>|t|)  
## (Intercept) -0.09129    0.12870  -0.709  0.48631  
## gini        1.01546    0.26425   3.843  0.00102 **  
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```



3. Hypothesis testing

3.1. P-value

```
##  
## Call:  
## lm(formula = ige ~ gini, data = ggcurve) ← Command  
##  
## Residuals:  
##      Min       1Q   Median       3Q      Max  
## -0.188991 -0.088238 -0.000855  0.047284  0.252310 ← Residuals distribution  
##  
## Coefficients:  
##             Estimate Std. Error t value Pr(>|t|)  
## (Intercept) -0.09129    0.12870  -0.709  0.48631 ← Coefs, s.e., t-/p-values  
## gini        1.01546    0.26425   3.843  0.00102 **  
## ---  
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1  
##  
## Residual standard error: 0.1159 on 20 degrees of freedom  
## Multiple R-squared:  0.4247,    Adjusted R-squared:  0.396  
## F-statistic: 14.77 on 1 and 20 DF,  p-value: 0.001016
```



3. Hypothesis testing

3.1. P-value

```
##  
## Call:  
## lm(formula = ige ~ gini, data = ggcurve) ← Command  
##  
## Residuals:  
##      Min       1Q   Median       3Q      Max  
## -0.188991 -0.088238 -0.000855  0.047284  0.252310 ← Residuals distribution  
##  
## Coefficients:  
##             Estimate Std. Error t value Pr(>|t|)  
## (Intercept) -0.09129    0.12870  -0.709  0.48631 ← Coefs, s.e., t-/p-values  
## gini        1.01546    0.26425   3.843  0.00102 **  
## ---  
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 ← Significance  
##  
## Residual standard error: 0.1159 on 20 degrees of freedom  
## Multiple R-squared:  0.4247,   Adjusted R-squared:  0.396  
## F-statistic: 14.77 on 1 and 20 DF,  p-value: 0.001016
```



3. Hypothesis testing

3.1. P-value

```
##  
## Call:  
## lm(formula = ige ~ gini, data = ggcurve) ← Command  
##  
## Residuals:  
##      Min       1Q   Median       3Q      Max  
## -0.188991 -0.088238 -0.000855  0.047284  0.252310 ← Residuals distribution  
##  
## Coefficients:  
##             Estimate Std. Error t value Pr(>|t|)  
## (Intercept) -0.09129    0.12870  -0.709  0.48631 ← Coefs, s.e., t-/p-values  
## gini        1.01546    0.26425   3.843  0.00102 **  
## ---  
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 ← Significance  
##  
## Residual standard error: 0.1159 on 20 degrees of freedom  
## Multiple R-squared:  0.4247,    Adjusted R-squared:  0.396 ← df and advanced stats  
## F-statistic: 14.77 on 1 and 20 DF,  p-value: 0.001016
```



3. Hypothesis testing

3.2. linearHypothesis()

- But the **linearHypothesis()** function from the **car** package allows to **easily test other hypotheses**:
 -
 -

```
linearHypothesis( , )
```



3. Hypothesis testing

3.2. linearHypothesis()

- But the **linearHypothesis()** function from the **car** package allows to **easily test other hypotheses**:
 - You must provide the **model**
 -

```
linearHypothesis(lm(ige ~ gini, ggcurve), )
```



3. Hypothesis testing

3.2. linearHypothesis()

- But the **linearHypothesis()** function from the **car** package allows to **easily test other hypotheses**:
 - You must provide the **model**
 - And the **hypothesis** (referring to coefficients as in the summary)

```
linearHypothesis(lm(ige ~ gini, ggcurve), "gini = 0")
```

```
## Linear hypothesis test
##
## Hypothesis:
## gini = 0
##
## Model 1: restricted model
## Model 2: ige ~ gini
##
##   Res.Df      RSS Df Sum of Sq      F    Pr(>F)
## 1     21 0.46733
## 2     20 0.26883  1     0.1985 14.767 0.001016 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```



3. Hypothesis testing

3.2. linearHypothesis()

- You can also test **more complex hypotheses**
 - Like equality between coefficients

```
linearHypothesis(lm(ige ~ gini, ggcurve), "gini = (Intercept)")
```

```
## Linear hypothesis test
##
## Hypothesis:
## - (Intercept) + gini = 0
##
## Model 1: restricted model
## Model 2: ige ~ gini
##
##   Res.Df      RSS Df Sum of Sq      F    Pr(>F)
## 1     21 0.37634
## 2     20 0.26883  1   0.10751 7.9983 0.01039 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```



3. Hypothesis testing

3.2. linearHypothesis()

- You can also test **more complex hypotheses**
 - Like equality between coefficients, or joint hypotheses (relying a generalization of the t-test called *F-test*)

```
linearHypothesis(lm(ige ~ gini, ggcurve), c("gini = 0", "(Intercept) = 0"))
```

```
## Linear hypothesis test
##
## Hypothesis:
## gini = 0
## (Intercept) = 0
##
## Model 1: restricted model
## Model 2: ige ~ gini
##
##    Res.Df   RSS Df Sum of Sq      F    Pr(>F)
## 1     22 3.8841
## 2     20 0.2688  2     3.6153 134.48 2.523e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```



Overview

1. Asymptotic inference ✓

- 1.1. Data generating process
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- 1.3. Confidence interval

2. Exact inference ✓

- 2.1. Standard error
- 2.2. Student-t distribution
- 2.3. Confidence interval

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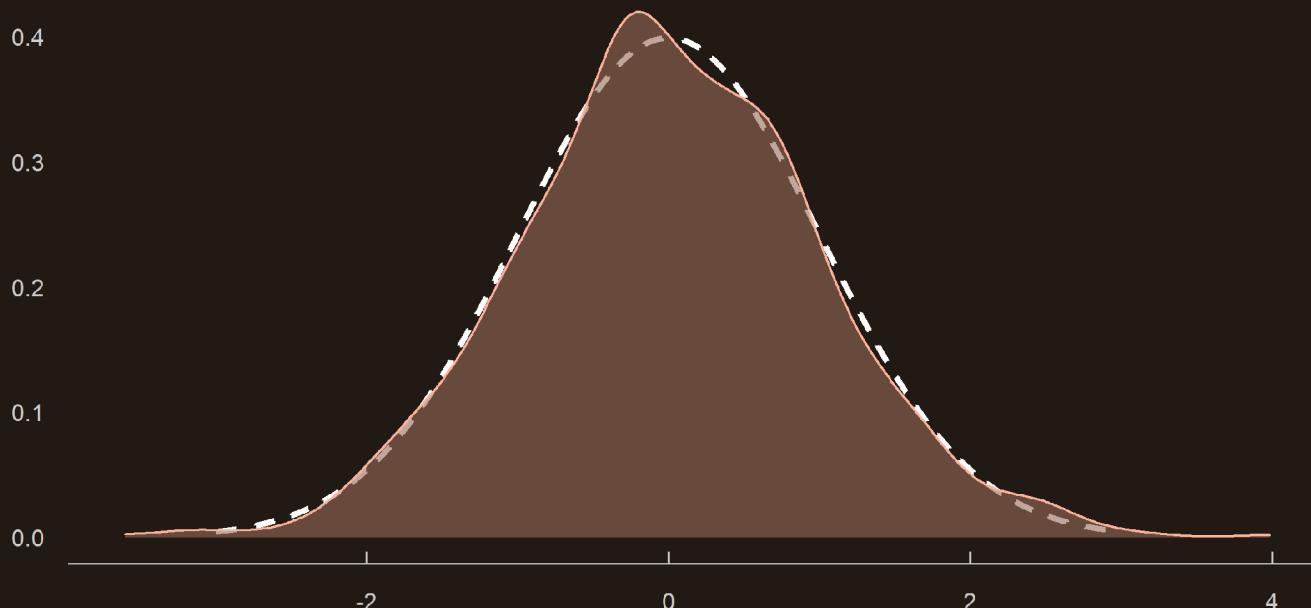
- 3.1. P-value
- 3.2. linearHypothesis()

4. Wrap up!

4. Wrap up!

Data generating process

- In practice we estimate coefficients on a **given realization of a data generating process**
 - So the **true coefficient** is **unobserved**
 - But our **estimation** is **informative** on the values the true coefficient is likely to take



$$\frac{\hat{\beta} - \beta}{\text{SD}(\hat{\beta})} \sim \mathcal{N}(0, 1)$$



4. Wrap up!

Confidence interval

- This allows to infer a **confidence interval**:

$$\hat{\beta} \pm t(\text{df})_{1-\frac{\alpha}{2}} \times \text{se}(\hat{\beta})$$

- Where $t(\text{df})_{1-\frac{\alpha}{2}}$ is the value from a **Student t distribution**
 - With the relevant number of **degrees of freedom** $\text{df}(n - \#\text{parameters})$
 - And the desired **confidence level** $1 - \alpha$
- And where $\text{se}(\hat{\beta})$ denotes the **standard error** of $\hat{\beta}$:

$$\text{se}(\hat{\beta}) = \sqrt{\text{Var}(\hat{\beta})} = \sqrt{\frac{\sum_{i=1}^n \hat{\varepsilon}_i^2}{(n - \#\text{parameters}) \sum_{i=1}^n (x_i - \bar{x})^2}}$$



4. Wrap up!

P-value

- It also allows to **test** how likely is β to be **different from a given value**:
 - If the **p-value** < 5%, we can **reject** that β equals the **hypothesized value** at the 95% confidence level
 - This threshold, very common in Economics, implies that we have 1 chance out of 20 to be wrong

```
linearHypothesis(lm(ige ~ gini, ggcurve), "gini = 0")
```

```
## Linear hypothesis test
##
## Hypothesis:
## gini = 0
##
## Model 1: restricted model
## Model 2: ige ~ gini
##
##   Res.Df      RSS Df Sum of Sq      F    Pr(>F)
## 1     21 0.46733
## 2     20 0.26883  1     0.1985 14.767 0.001016 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```