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Homework 3 Problem 3

Problem 3 (10 points):

Let $P = \{x_1, \dots x_n\}$ be points on the X-axis in increasing order, and R be a non-negative integer. Give an $O(n)$ time algorithm to determine the minimum number of intervals of length R to cover the points. Explain why your algorithm is correct. (This problem relates to Chapter 4 material on greedy algorithms, but should be doable before the material has been presented in class.)

Answer:

```
def find_min_intervals_number:
  counter = 0
  for x_i in P:
    # if current intervals cannot cover x_i
    if x_i > cur_intvl + R:
      # initialized a new interval
      cur_intvl = x_i + R
      counter += 1
  return counter
```

Proof of correctness

Suppose there is an minimum number of interval $|\{N'\}|$ and set of interval $\{N'\}$ which each individual interval cover optimal number of points, and that also mean each interval locates at its optimal position on X-axis. Also define that the result from algorithm above is $|\{N\}|$ and set of interval $\{N\}$.

Proof by showing $|\{N'\}| < |\{N\}|$ is wrong, then the algorithm disprove the optimality of optimal case, hence algorithm get optimal result:
x_1 is the leftmost point on X-axis, according to the algorithm, because no point to the left of x_1, $x_1 + R$ is the position that maximum the coverage of other points (if any) while cover x_1 at the same time. Therefore, the algorithm solve the subproblem that the optimal position interval cover x_1 with a result of $(x_1, x_1 + R)$, and $(x_1, x_1 + R)$ should appear in the optimal $\{N'\}$. Assume $(x_1, x_1 + R)$ cover k point where $k \geq 1$, then the next point should be considered is x_{k+1}. If the x_1 to x_k was removed from P, the new subproblem is $P = \{x_{k+1}, \dots x_n\}$ with same lenth R interval, and due to the same reason of x_1, the interval covers x_{k+1} should be $(x_{k+1}, x_{k+1} + R)$ according to algorithm. And for the same reason, this result is also interchange result between $\{N'\}$ and $\{N\}$.

And by applying to all subproblems, will get $|\{N'\}| < |\{N\}|$ is incorrect. Therefore the result of given algorithm is optimal.