University of Washington March 18, 2020
Department of Computer Science and Engineering
CSE 417, Winter 2020

Homework 10, Problem 1

Problem 1 (10 points):

Answer the following questions with "yes", "no", or "unknown, as this would resolve the P vs. NP question." Give a brief explanation of your answer.

Define the decision version of the Interval Scheduling Problem as follows: Given a collection of intervals on a time-line, and a bound k, does the collection contain a subset of nonoverlapping intervals of size at least k?

a) Question: Is it the case that Interval Scheduling $\leq_P$ Vertex Cover?

b) Question: Is it the case that Independent Set $\leq_P$ Interval Scheduling?

Answer:

a) Yes:
Since it has been proven that Interval Scheduling can be solved in $O(n*log(n))$, it is in an polynomial time solvable problem. (Kleinberg page 121).
And it has been proven that Vertex Cover problem is an NP complete problem (Kleinberg page 472 8.16)
And since Interval Scheduling is polynomial time solvable and Vertex Cover is an NP complete problem, an NP complete problem is harder than a polynomial time solvable problem, therefore Interval Scheduling $\leq_P$ Vertex Cover (Vertex Cover is at least as hard as Interval Scheduling).

b) Unknown:
It has been proven that Independent Set is an NP complete problem (Kleinberg page 472 8.16).
And it has been proven that Interval Scheduling can be solved in $O(n*log(n))$, it is in an polynomial time solvable problem. (Kleinberg page 121).
Whether Independent Set $\leq_P$ Interval Scheduling is decided by whether P = NP. If P = NP, then any problem can be solved in polynomial time, therefore Independent Set is polynomial time reducible to Interval Scheduling problem, the statement is correct. If P not equal to NP, then Independent Set $\leq_P$ Interval Scheduling means the NP complete problem Independent Set can be reducible to a polynomial time solvable Interval Scheduling problem, which contradict P != NP. Therefore the statement is incorrect.