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Homework 1 Problem 1

Problem 1 (10 points):

Let $I = (M, W)$ be an instance of the stable matching problem. Suppose that the preference lists of all $m \in M$ are identical, so without loss of generality, m_i has the preference list $[w_1, w_2, \ldots, w_n]$. Show that there is a unique solution to this instance.

Answer:

Given the identical preference list of m to w, the stable matching result is only decided by M. Every m will propose follows the same order, say P = $[w_1, w_2, \ldots, w_n]$, then w_1 will receive all proposal and be able to choose the 'top ranked' m_{w_1} in its preference list. And w_2 will receive all proposal except m_{w_1} since any m other than m_{w_1} will be rejected in their first round proposal with w_1 even if they are matched tentatively with w_1. And this ensures that for every $w \in W$, they faces a definitive proposals from the subset of m (if $w = w_n$, the proposal it will receive from $M - m_{w_1} - m_{w_2} \ldots - m_{w_{n-1}}$). And the since w and m have same size, there will always the last standing m to propose w_n. Therefore there is a unique solution to this instance.

In a formal expression:
preference list of all $m : P = [w_1, w_2, \ldots, w_n]$.

Base case:
w_1 receives $m \in M$ proposal, and final match with m_{w_1}, which m_{w_1} on top of P_{w_1}
Step 2:
w_2 receives $m \in M - m_{w_1}$ proposal, and final match with m_{w_2}, which m_{w_2} on top of $P_{w_2} \cap M - m_{w_1}$
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Step i:
w_i receives $m \in M - m_{w_1} - m_{w_2} \ldots - m_{w_{i-1}}$ proposal, and final match with w_i, which m_{w_i} on top of $P_{w_i} \cap M - m_{w_1} - m_{w_2} \ldots - m_{w_{i-1}}$
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Step n:
$\forall w_n \in M$ w_i receives $m \in M - m_{w_1} - m_{w_2} \ldots - m_{w_{n-1}}$, which is $|M - m_{w_1} - m_{w_2} \ldots - m_{w_{n-1}}| = w_i = 1$ proposal, and final match with w_n, which m_{w_n} on top of $P_{w_n} \cap M - m_{w_1} - m_{w_2} \ldots - m_{w_{n-1}}$
Since $|M| = |W| = n$, and $\forall w_n \in M$, w_n pair with only one m, $|M - m_{w_1} - m_{w_2} \ldots - m_{w_{n-1}}| = 1$, and since w_n must match one and m_{w_n} must on P_{w_n}, the only last two would finalized the matching.