University of Washington
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Homework 10, Problem 3

Problem 3 (10 points):

(Kleinberg-Tardos, Page 507, Problem 7). Since the 3-Dimensional Matching Problem is NP-complete, it is natural to expect that the corresponding 4-Dimensional Matching Problem is at least as hard. Let us define *4-Dimensional Matching* as follows. Given sets W, X, Y, and Z, each of size n, and a collection C of ordered 4-tuples of the form (w_i, x_j, y_k, z_l), do there exist n 4-tuples from C so that no two have an element in common?
Prove that 4-Dimensional Matching is NP-Complete.

Answer:

1) 4-Dimensional Matching is in NP:
Since given collection C of ordered 4-tuples, a certificate that there is a solution could be a collection of $C' \subseteq C$. And it is obvious that by travsing each matching triple in C' and using a n by 4 array to check whether it has been used in one visited matching, the verification can be done in polynomial time so the 4-Dimensional Matching problem is in class NP.

2) 3-Dimensional Matching $\leq_P$ 4-Dimensional Matching
Given an 3-Dimensional Matching with collection C of ordered triples of the form (w_i, x_j, y_k) and sets W, X, Y each of size n. One can replicate the sets Y and the last element of each triples in C to map the problem in to a 4-Dimensional Matching problem with collection C of ordered triples of the form (w_i, x_j, y_k, y_k) and sets W, X, Y, Y each of size n. Since the duplicated elements will not affect the result if the first three elements form a perfect matching, the last element must confrom the perfect matching since it was duplication of y_k. Hence by solving whether there is a pefect matching in the 4-Dimensional Matching problem, we can solve the 3-Dimensional Matching, with a polynomial change of problem size. Therefore 3-Dimensional Matching $\leq_P$ 4-Dimensional Matching.

It has been proven that 3-Dimensional Matching is an NP complete problem (Kleinberg page 482 8.20). And according to Kleinberg page 453 8.2: Suppose Y $\leq_P$ X. If Y cannot be solved in polynomial time, then X cannot be solved in polynomial time. And since 1) and 2), 4-Dimensional Matching is NP-Complete.