University of Washington
Department of Computer Science and Engineering
CSE 417, Winter 2020

March 13, 2020

Homework 9, Problem 6

Problem 6 (10 Points):

(Kleinberg-Tardos, Based on exercise 9, Page 419) Network flow issues come up in dealing with natural disasters and other crises, since major unexpected events often require the movement and evacuation of large numbers of people in a short amount of time.

Consider the following scenario. Due to a virus outbreak in a region, paramedics have identified a set of n infected people distributed across the region who need to be rushed to hospitals. There are k hospitals in the region, and each of the n people needs to be brought to a hospital that is within a half-hour's driving time of their current location (so different people will have different options for hospitals, depending on where they are right now).

At the same time, one does not want to overload any one of the hospitals by sending too many patients its way. The paramedics are in touch by cell phone, and they want to collectively work out whether they can choose a hospital for each of the sick people in such a way that the load on the hospitals is balanced: Each hospital receives at most $\lceil n/k \rceil$ people.

Give a polynomial-time algorithm that takes the given information about the people's locations and determines whether this is possible.

Answer:

algorithm:

step 1: construct a directed graph with each patient and each hospital to be a node
step 2: if a patient is within s half-hour's driving time, add a edge from the patient to the hospital with capcity 1.
step 3: add one node s and n outgoing edge from s to each patient node, with capcity 1. add one node t and k incomming edge from each hospital node to t, with $\lceil n/k \rceil$ capcity.
step 4: conduct a Ford-Fulkerson algorithm to find whether there exists a s-t flow with value n. If it exists, it possble to sent patient to hospital in the context of the problem and visa versa.

proof:

The algorithm can solve the problem because the graph represent the capcity bound in the cotext of the problem. Each patient can only linked with the hospital if it within the reach of half-hour drive. And because from source to each patient node, the capcity is 1, this garantee any flow from source to patient is at most 1, which means each patient can only be dealt by one hospital. And dor the same reason, any flow out going from hospital node is at most $\lceil n/k \rceil$, this garantees the maximum dealing capcity of each hospital is at most $\lceil n/k \rceil$. Therefore, the problem can be reduced to a problem solving the maximum flow of directed graph. If the maxmium flow reachs value n, then that means all flow out from source node was flow to the terminal node, which means all patient can be dealt with given these boundary.

The time complexity is same as Ford-Fulkerson algorithm, accoring to according to Kleinberg textbook, p367 7.33, this algorithm can be solved in polynomial time.