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Homework 2 Problem 3

Problem 3 (10 points):

The *diameter* of an undirected graph is the maximum distance between any pair of vertices. If a graph is not connected, its diameter is infinite. Let G be an n node undirected graph, where n is even. Suppose that every vertex has degree at least $n/2$. Show that G has diameter at most 2.

Answer:

For graph $G = (V, E)$, $\forall v_1$ and $v_2 \in V$:
There are only two conditions:
1. $\exists e \in E$ that $e = \{v_1, v_2\}$ v_1 and v_2 are connected, hence $dist(v_1, v_2) = 1$, therefore this condition guarantee diameter to less than 2.

2. $!\exists e \in E$ that $e = \{v_1, v_2\}$, assume neighbors set of N_1 and neighbors set of N_2.
since $deg(v_1) \geq n/2$ and $deg(v_2) \geq n/2$, and they are not connected, therefore v_1 (and v_2 itself) $\notin N_2$ and v_2 (and v_1 itself) $\notin N_1$, by pigeon hole principle, since $|n_1 \cup n_2|$ at most $n - 2$, and $n - 2 \leq |V|$, hence $n_1 \cap n_2 \neq \emptyset$, $\exists v$ that $v \in n_1$ and $v \in n_2$, therefore $dist(v_1, v_2) = 2$.

This two conditions hold true for any pair of vertices, so the diameter or distance between any two vertices as large as 2, therefore G has diameter at most 2.