

Homework 1 Problem 3

Problem 3 (10 points):

(Adapted from text, page 28, exercise 8.) For this problem, we explore the issue of *truthfulness* in the Gale-Shapley algorithm for Stable Matching. Show that a participant can improve its outcome by lying about its preferences. Consider $w \in W$. Suppose w prefers m to m' , but m and m' are low on w 's preference list. Show that it is possible that by switching the order of m and m' on w 's preference list, w achieves a better outcome, e.g., is matched with an m'' higher on the preference list than the one if the actual order was used.

Answer:

To simplify the situation, assume the $|M| = |W| = 3$ and the preference of w is $[m_1, m_2, m_3]$. If w wants to achieve a better outcome by switching its preference of m_2 and m_3 , there is a situation that given w does not switch the preference, w 's top rank potential partner m_1 will not be 'released' or unmatched from the standing proposal. Hence to construct this situation, w prefers m_3 to m_2 should make m_3 unmatched and then m_3 directly or indirectly cause m_1 unmatched from its current matching with, say w_{m_1} . Consider only the direct case, assume m_1 prefers w_2 over w_3 (the one making switch of preference) and w_2 prefers m_2 over m_1 , then that is the case where w_3 can be better off if the order was switched. The detail description is below.

the preferences for the M 's are given by the matrix:

$$\begin{bmatrix} w_2 & w_3 & w_1 \\ w_3 & w_2 & w_1 \\ w_3 & w_1 & w_2 \end{bmatrix}$$

the preferences for the W 's are given by the matrix:

$$\begin{bmatrix} m_3 & ? & ? \\ m_2 & m_1 & m_3 \\ m_1 & m_2 & m_3 \end{bmatrix}$$

$$(w_1 \ w_3 \ w_2)$$

Trace

m_1 proposes to w_2 Accepted

m_2 proposes to w_3 Accepted

m_3 proposes to w_3 where switch of order happens, unmatched (m_2, w_3)

m_2 proposes to w_2 where release of m_1 happens, unmatched (m_1, w_2)

m_1 proposes to w_3 unmatched (m_3, w_3)

m_3 proposes to w_1 Accepted