University of Washington
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Homework 2 Problem 1

Problem 1 (10 points):

Order the following functions in increasing order by their growth rate:

1. n^3
2. $(\log n)^{\log n}$
3. $n^{\sqrt{\log n}}$
4. $2^{n/10}$

Explain how you determined the ordering.

Answer:

$1 < 2 < 3 < 4$

Firstly, for 1, 2 and 3:
$(\log n)^{\log n} = e^{(\log \log n)^{\log n}} = e^{\log n^{(\log \log n)}} = n^{\log \log n}$
Hence, sorting 1, 2, 3 equal to sorting:
1. n^3, 2. $n^{\log \log n}$, 3. $n^{\sqrt{\log n}}$
It is easy to prove that both 2 and 3 have larger growth rate than 1 since $\lim_{n\to\infty} \log(\log n) = +\infty$ and $\lim_{n\to\infty} \sqrt{\log n} = +\infty$, both $\exists n$ to make the exponent of n > 3.
Therefore, $1 < 2$ and $1 < 3$

Then for 2 and 3:
Since $\lim_{n\to\infty} n^{\log \log n} = \infty$ and $\lim_{n\to\infty} n^{\sqrt{\log n}} = \infty$, by applying L'Hospital rule:

$\lim_{n\to\infty} \frac{n^{\log \log n}}{n^{\sqrt{\log n}}} = \lim_{n\to\infty} \frac{\log \log n}{\sqrt{\log n}} = \lim_{n\to\infty} \frac{e^{\log n}}{e^{\sqrt{n}}}$
$= \lim_{n\to\infty} \frac{\log n}{\sqrt{n}} = \lim_{n\to\infty} \frac{2\sqrt{n}}{n} = 0 < \infty$
Therefore, $2 < 3$

Then for 3 and 4:
Since $\lim_{n\to\infty} n^{\sqrt{\log n}} = \infty$ and $\lim_{n\to\infty} 2^{n/10} = \infty$, by applying L'Hospital rule:
$\lim_{n\to\infty} \frac{2^{n/10}}{n^{\sqrt{\log n}}} = \lim_{n\to\infty} \frac{n \log 2}{10\sqrt{\log n} \log n} = \lim_{n\to\infty} \frac{2n^2}{3\sqrt{\log n}} = \lim_{n\to\infty} \frac{8n^2\sqrt{\log n}}{3} = +\infty$

Therefore, $3 < 4$

Therefore, $1 < 2 < 3 < 4$