

Homework 2 Problem 4

Problem 4 (10 points):

Let $G = (V, E)$ be an undirected graph with n vertices such that the degree of every vertex of G is at most k . Give an algorithm to color the edges of G with at most $2k - 1$ colors such that any pair of edges e and f which are incident to the same vertex have distinct colors.

Answer:

Algorithm:

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initiate a set with  $2k-1$  colors;
traverse each vertex use DFS;
iterate through all edge connected to this vertex:
    if edge not assigned color:
        assign a different color to that edge (differ to other edges connected to that vertex)
        label that edge been colored;
```

Proof:

Given a arbitrary edge $e = \{u, v\} \in E$, assume both vertices have maximum degree of k , that is $\deg(v) = k$ and $\deg(u) = k$. Therefore, the worst case total number of color used by the incident edges apart of $\{u, v\}$ of u (and the same as v) should be at most $k - 1$.

And since u and v are connected, therefore maximum number using by all incident edges of u and v should be $(k - 1) * 2 + 1$, where $\{u, v\}$ can only assigned one color. This holds true for all edge and vertices of edges, therefore the graph will require maximum colors number bound to $2k - 1$.