PLANE TRAVEL INVESTIGATION

MXB242 Group Project

STUDENT DECLARATION

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Abstract

The purpose of this report is to see how far a plane that is made out of various different materials and different size travel at a certain height above ground. The physics applied to a real aircraft can also be applied to planes that was experimented in this report. Using RStudio the data that was collected was analysed to perform an exploratory analysis about whether the data was reliable and accurate, model reduction on the full model to find the optimal model to showcase data, assumptions of the ANOVA that was calculated and the model assumptions. From the ANOVA analysis it shows that every height has an impact on the travel distance of the plane, size also has an impact and some materials have an impact while other do not. During the analysis, it was found that two interaction terms had a significant effect on distance travelled. In conclusion, baking paper had the furthest distance flown.

Contents

Introduction

Importance of Study

With technology advancing rapidly over the past century, one important aspect in society is travelling. One such travelling method, is via airplanes, which has allowed people to travel across the global for everyday needs such as visiting family, work, or leisure. This has resulted in the benefit of allowing people to travel quickly over a long distance. As a result, it has led to the study of this experiment, where the aim is to find how the travel distance of the plane is affected. This has resulted in three variables chosen to find the effect of travel distance of the plane. These three variables are: height at which the plane take-off from the ground, the material of the plane and size of the plane

Aims of Study

The aim of this study is to find how the travel distance of the plane is affected by the three variables. These variables comprise of three different height at which the plane take-off from the ground (height thrown). These are 150cm, 200cm and 250cm. There are five different materials used to create the planes, which are paper, baking paper, newspaper, aluminium, and cardboard. The size of the plane was completed with two different sized planes, they are 29.7cm (length) by 11cm (width) and 42cm (length) by 15.55cm (width). With all these variables in mind, the final goal is to optimize these variables and find out what combination of them result in the furthest distance travelled.

Questions to be investigated

The following questions are to be investigated:

Does the height at which the plane take-off from the ground effect the plane's travel distance?

Does the material used to make the plane affect the plane's travel distance?

Does the size of the plane affect the plane's travel distance?

What combination of these variables will yield the furthest distance travelled by the plane?

Variables

Variable Name	Name in R	Variable	Factor Type	Description
height thrown	Height	Factor	Categorical	150cm, 200cm, 250cm
Material	Material	Factor	Categorical	Paper, Baking paper, Cardboard, Aluminium, Newspaper
Size of Plane	Size	Factor	Categorical	A4 (29.7*11), A3(42*15)
Distance from throw	Distance	Numeric	Continuous	Centimetres

Height at which the plane take-off from the ground

There are three different heights, in which the plane is thrown from these comprise of 150cm, 200cm and 250cm. To measure these variables, we used a measuring tape and then found the heights. Next, we then kept the plane as straight as possible to the measuring tape height line and finally then threw the plane. This was done only for 150cm as we needed an aid to reach heights of 200cm and 250cm such as a stool and ladder, however, the process remained the same.

Limitations of this variable would be that the plane would not be thrown exactly at the heights mentioned as the planes were thrown by humans so there would be an element of human error in the height thrown. These limitations would cause the height thrown to be off by a couple of centimetres but the high number of tests (number of tests) would counteract this limitation.

Size of Plane

For the plane aerodynamics to remain constant, we kept the design of the plane the same such that ratio of the plane remained constant, so that the plane is not affected by having a different plane design.

There were two different size planes used, these being an A4 sized paper which would result in the planes length and width being 29.7cm by 11cm respectively and an A3 sized paper which would result in the planes length and width being 42cm and 15.55cm respectively

The error for this is that there can be error in folding the plane as some corners and folds will not be exactly the same due to hand eye coordination, so to reduce this error, same person folded the plane.

Material

There were five different materials being used to create the planes, these being paper, baking paper, newspaper, aluminium, and cardboard. These materials were chosen as they are common household items.

The error for this is that some of the sizes require use to cut some of material before the plane can be folded. Especially working with aluminium and baking paper was difficult so getting them to be the correct A4 or A3 size resulted in not perfectly straight lines, probably throwing off the dimensions by a few millimetres which might of effected the aero dynamics of the plane but the material was made sure to not have any glaring deformities.

Distance thrown

The distance thrown by each plane was measured with a tape measure by anchoring to where our feet were when we threw it and measured the distance from there to where the plane landed.

The error for this response variable would be that the tape measure result recorded by a human wouldn't be 100% accurate as we had to throw the plane, walk out to where the plane landed with the tape measure and record the distance flown.

Experiment Assumption and Limitation

Size ratio is not exactly the same for size of plane as for the A4 paper plane the dimensions was 29.7*11, while the dimensions for an A3 paper plane was 42*15. So, for the A4 plane one cm width resulted in 2.7cm length and for an A3 plane a cm of width resulted in 2.8cm of length. Although the dimensions are similar the effect of a small difference in dimensions ratio are unknown and must be considered in the limitation of this experiment. Another limitation is that because the experiment only accounted for the distance travelled from A to B in a straight line there were a lot of errors in this method. For example, the flips, turns, travelling in a curve etc. were not taken in account for when measuring the distance travelled. As such there would a lot of the error in our data as that information was not recorded.

Discussion

Literature Review

In a real aircraft there are many factors that affect how the plane travels. The theories that is applied to real life aircraft can also be applied to paper planes. There are little investigation reports about our experiment however the science behind it are similar. A variable that affect the travel of distance of a plane is aerodynamics which is a study of how objects move around the air *(May, 2017)*. In an airplane there are four factors that is affected upon the plane: weight, lift, thrust and drag *(Reddy, 2017)*. Weight is the force that is acting towards the centre of the earth, lift is the force acting perpendicular to the direction of the relative motion, thrust is the force acting along the direction of the motion, in this case the amount of power we use to throw the paper plane, and drag is the force acting opposite to the relative motion of the aircraft which is known as air resistance *(Shaw, 2014)*.

The main factors that are investigated and experimented is height at which the plane is thrown above ground height, size of the plane's length and width and the material of the plane. The first factor is height. This is important as gravity is what pulls the plane towards the ground and keeps the plane from being in the air. Higher altitudes, mean more time until the plane touches the ground, allowing for the plane to travel a longer distance. For a plane to constantly travel in the air the plane needs to be constantly lifted. According to D. Anderson from the Fermi National Accelerator Laboratory and S. Eberhardt from the Department of Aeronautics and Astronautics textbook (ref 4) it is explained that this process is known as the "Popular Explanation of Lift" which is an inaccurate application of Bernoulli's principle. When the paper plane is thrown the wings generate lift as it travels through the air because the air is sped up on top which creates a region of low pressure *(Anderson and Eberhardt, 2015)*. It is speculated that the shape of the air foil changes the pressure in the air, but it is the air that separates the leading edge, the wind that goes over the top must meet at the wing's edge with the part that goes under the bottom. This phenomenon is known as "principle of equal transit times" *(Anderson and Eberhardt, 2015)*.

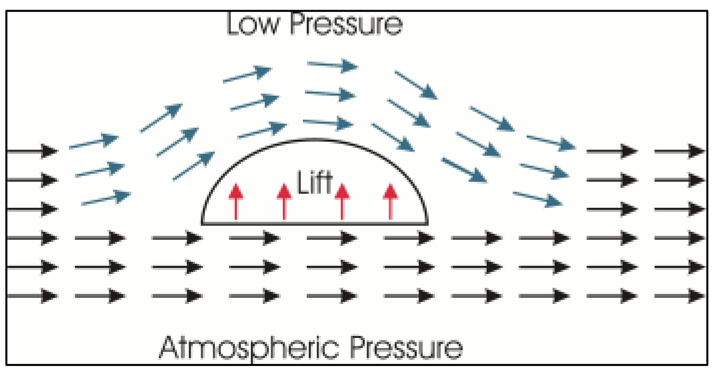


Figure 0.1 (Anderson and Eberhardt, 2015)

For the paper plane to maintain a longer duration of flight to travel further it also needs the force 'thrust' to help with its forward movement. Real aircraft's thrust comes from the engine that gives them the power to launch from the ground to maintain afloat. However, for paper planes the initial thrust is from throwing which then uses the remaining kinetic energy to glide for the rest of the flight. With unpredicted variables such as random twists, flips etc. its kinetic energy would be converted to potential energy while some energy is lost due to air resistance. This needs to be kept consistent for all trials in the experiment as no way to measure thrust.

The second factor is the size of plane. Where in this experiment scenario, increasing size is increasing the size of the wings and the length of the plane. Increasing the wing and the length of the plane does not necessarily mean the plane will travel longer distance. Different paper plane designs will also affect the distance travelled. As the plane travel through the air, the air is pushing it back against the plane which affects the distance travelled this is known as air resistance or drag. The bigger the plane is the more drag and air resistance it will experience and more thrust it will need to fly the heavier plane. Additionally, the weight of the plane makes a difference to the distance travelled as it is in flight the gravity will pull down the plane towards Earth. A larger plane will naturally have a bigger wing, having a bigger wing will increase the lift caused by the wing as there is more surface area. According to Nasa *(Hall, 2018)*, doubling the area of the plane will double the lift as lift is directly related to the surface area of the entire plane.

The last factor is material of the plane. Material that is heavier can result in a long distance travelled or a short distance travelled. For example, when throwing a rock, it is expected that the rock will travel further compared to throwing a tissue. However, if the rock was just free-falling the tissue can travel further due to its complicated shape. The tissue has many crevices that can affect the drag as it travels causing it to flip, glide, turn etc. In this case, the plane designs are kept the same to not change the aerodynamics of the plane too much. That is because every material has different density, weight and surface texture which affect the drag of the plane when it is the air. A source of drag on planes is skin friction where the molecules of the air and the solid surface of the aircraft meet. Since it is an interaction between a gas and a solid the magnitude of the friction depends on the properties of both gas and solid. The smoother the material is the less skin friction is produces. As for the gas, it depends on the viscosity of the air and the surrounding magnitude of the viscous forces *(Hall, 2015)*.

Methods of Data Collection

As mentioned in the introduction, this experiment has three treatments, this is due to the time frame of which the experiment can be conducted. The three treatments are paper plane material, height (cm) at which the plane was thrown and the ratio (cm:cm) of the plane's length and width, while the dependent variable is the distance (cm) the plane travelled.

The material of the paper planes was chosen first, which consists of five different types of: paper, baking paper, newspaper, aluminium, and cardboard. These materials were chosen as they are everyday objects that are readily available anywhere.

Next, we had to consider the size of plane, as a plane size that is too big, would result in there being not enough material. So, it was decided to use the size of 29.7cm by 11cm as it can be made using an A4 piece of paper without having to cut it. To not alter the aerodynamics of the plane by an unknown factor it was agreed to keep the ratio of the plane's length and the width in scalar multiple of each other. As such, the next size of the paper plane was decided to be an A3 paper with the size of 42cm by 15.55cm.

Following that, the height at which the paper plane was being thrown was decided at a different height of 150cm, 200cm and 250cm above ground height. As too low of a height would yield no results and too high of a height would result in dangerous situations. So, the heights of 150cm, 200cm and 250cm were selected, which can be accessed easily by using a stand or ladder without putting the thrower in a dangerous position.

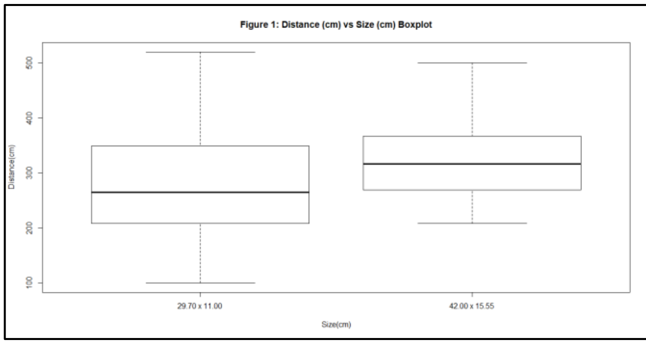
For this experiment, there were three trials of each combination conducted. The planes were then thrown at different heights. To keep external interference at a minimum the planes were thrown indoors with windows shut to minimize the effects of wind to a minimum and when throwing each plane, it was decided to only throw with a flick of the wrist to attempt to keep the force output on each throw equal. This resulted in ninety observations being made. This gave us enough data to analysis the results.

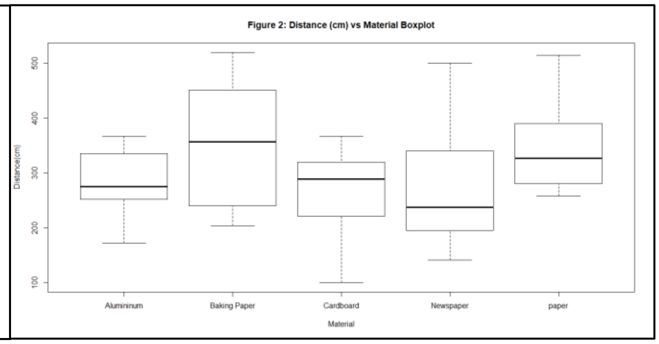
However, if one wished to improve on this, they can do the following: Firstly, they can increase the number of trials from three to a larger number, this will result in a better average, less affected by outliers. Secondly, they can increase the number of combinations by using more variable categories, this result in more data. Another improvement to this experiment would be to conduct it in an indoor facility where inconsistent wind speed would not affect the plane's flight duration. Lastly, they can find more independent variables, which would result in more data. However, a time constraint for the investigation has limited the amount of data that could be collected.

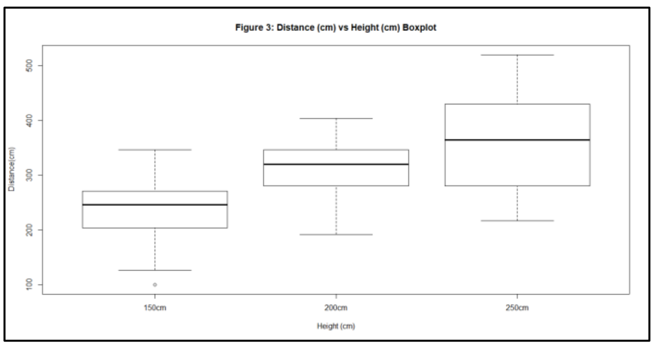
During the course of this experiment errors were bound to happen. As mentioned in the 'Introduction' when measuring the distance travelled flips, turns, travelling sideways etc. was not taking in account when measuring from point A to B in a straight line. This would create anomalies and errors within the data and be difficult to accurately process which material, size and height would be the best solution. Apart from these errors everything else went smoothly.

Exploratory Analysis

Figures 1, 2, 3: Distance vs Independent Variable boxplots







Figures 1 to 3 above each show box plots of one predictor variable, being size, material and height thrown against the response variable, distance flown. This is done to test for linearity, that a linear relationship between the response variable and predictor variables is reasonable, and from viewing the plots it is safe to assume so.

Each of the plots shows promising linear trends; figure 1 shows a general increase in distance flown with an increase in paper size, figure 2 shows that each material has a different effect on distance flown with baking paper on average flying the furthest and newspaper flying the shortest distance, figure 3 clearly shows a linear relationship with an increase in height thrown directly linking to an increase in distance shown. Although the plots and results look promising there is high variance in the materials newspaper and baking paper and the height thrown of 250cm which may indicate that a linear relationship might not be the best fit. Further investigation must be done to investigate the variables and see their relationships with other variables.

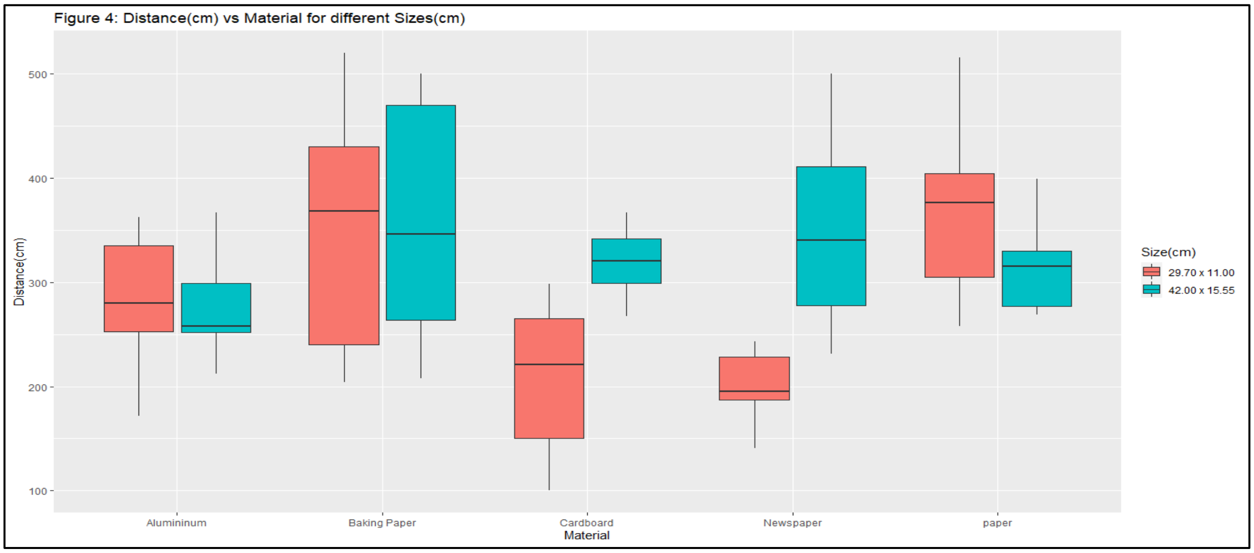


Figure 4 shows more information regarding the effect of material of different sizes on the response variable, it can be seen that there is seems to be no correlation between material and size of the plane in relationship to distance. This is evident by the materials Cardboard and Newspaper having increases in distance thrown with an increase in paper size but the opposite is true for the rest as Aluminium, baking paper and paper all show slight decreases in distance with an increase in size. This would suggest that the effects of paper size on material is not universal as the materials react differently to the size increase and indicates that this interaction term may be added to the model if statistically significant.

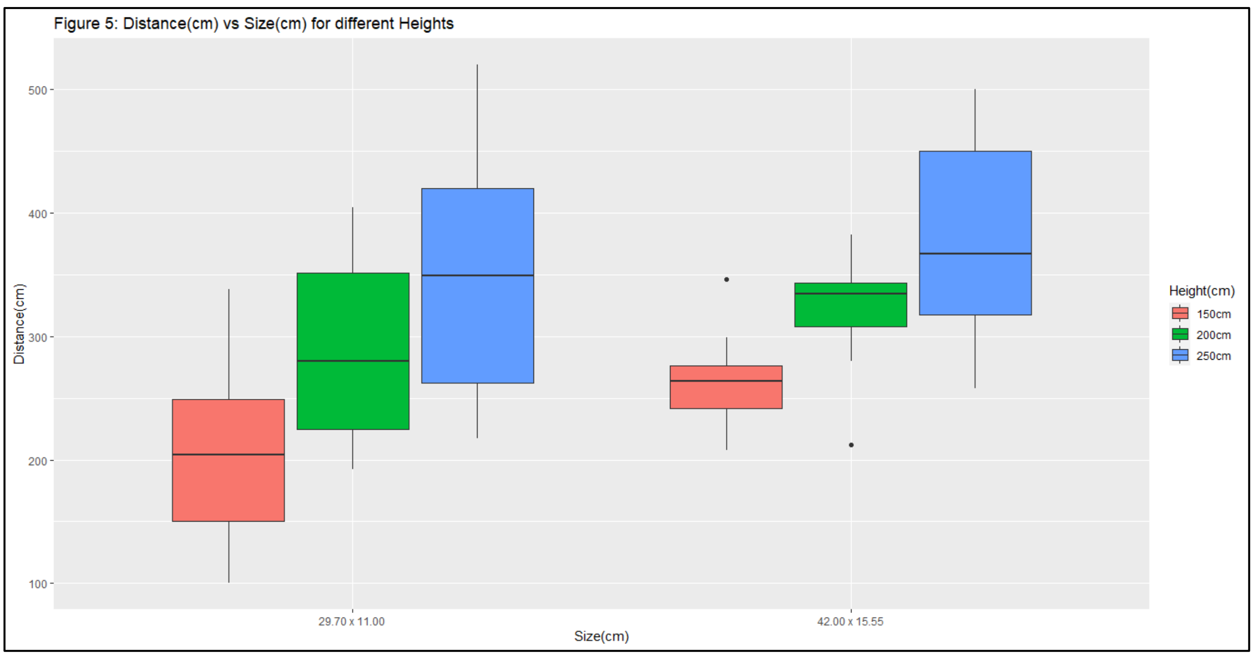


Figure 5 shows the effect of height and size on the response. From the plot it can be seen that height and size seem to be linearly correlated as an increase in height results in an increase of distance for both groups. Furthermore, the increase in size also resulted in an increase in distance for all heights so it is fair to assume that height and size are correlated and suggests that the final model may not need this interaction term if statistically insignificant.

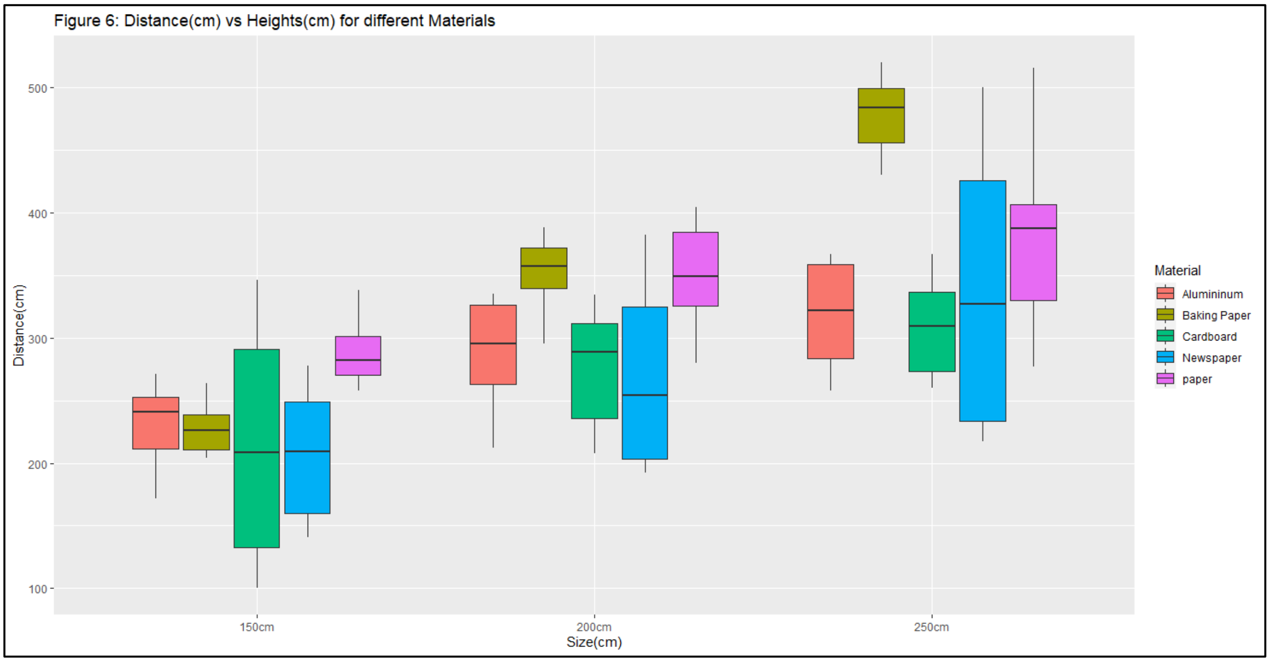


Figure 6 shows the response variable through the predictor variables material and height thrown. Although cluttered if you look at the key you can see that size and material are linearly correlated as the distance increases for every material with an increase of height thrown. Although there is a general trend in the plot not all the materials distances increase at the same rate with baking paper planes distance increasing the most with an increase in height. Therefore, it indicates that this interaction term may be added to the final model if significant.

Since, the independent variables are based on trials at certain categories, the independent variables are categorical. This means that an interaction plot can be used to get a better idea on how the variables interact with each other.

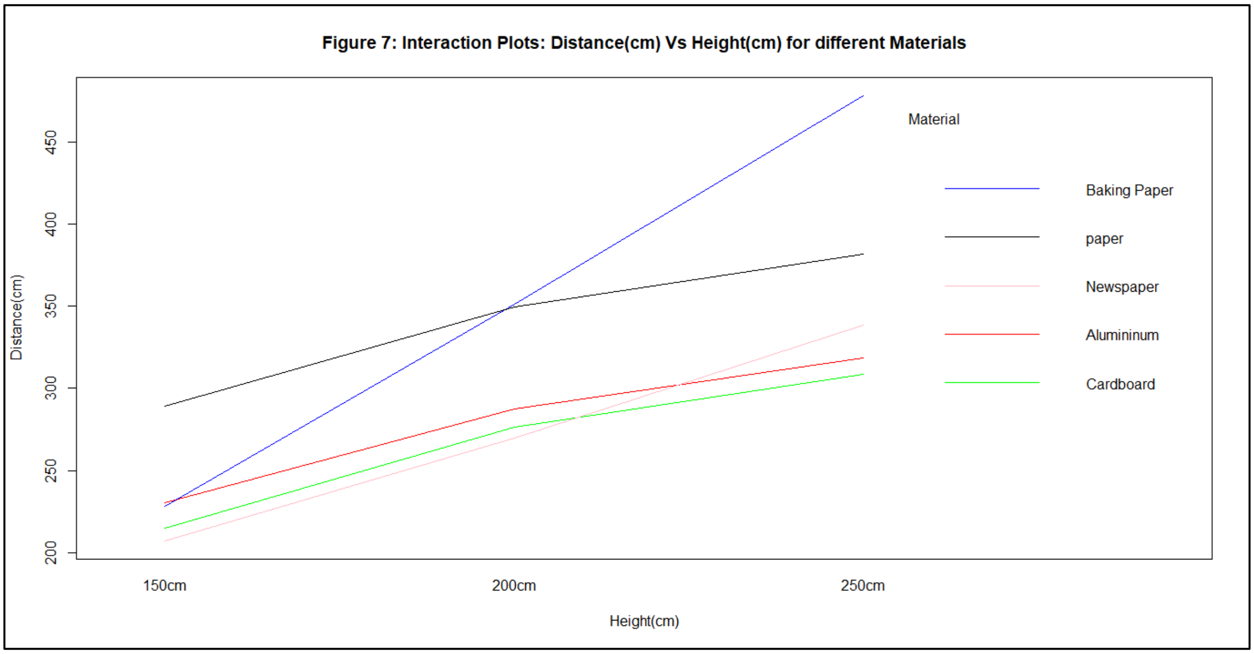


Figure 7 shows the interaction plot between Height and Materials Vs Distance and from the plot some trends are very evident. Height correlates to all materials so that an increase in height results in an increase of distance. Although the height correlates with all materials, the correlation is not the same for every material. Baking paper and newspaper appears to uncorrelated to other materials as their lines differ but looking at the plot paper, aluminium and cardboard appear to have parallel lines indicating that the interaction should be kept in the model if significant but the inclusion of cardboard, aluminium and paper in the model may be questioned as they seem to be highly correlated to each other and the inclusion of all three seems unnecessary.

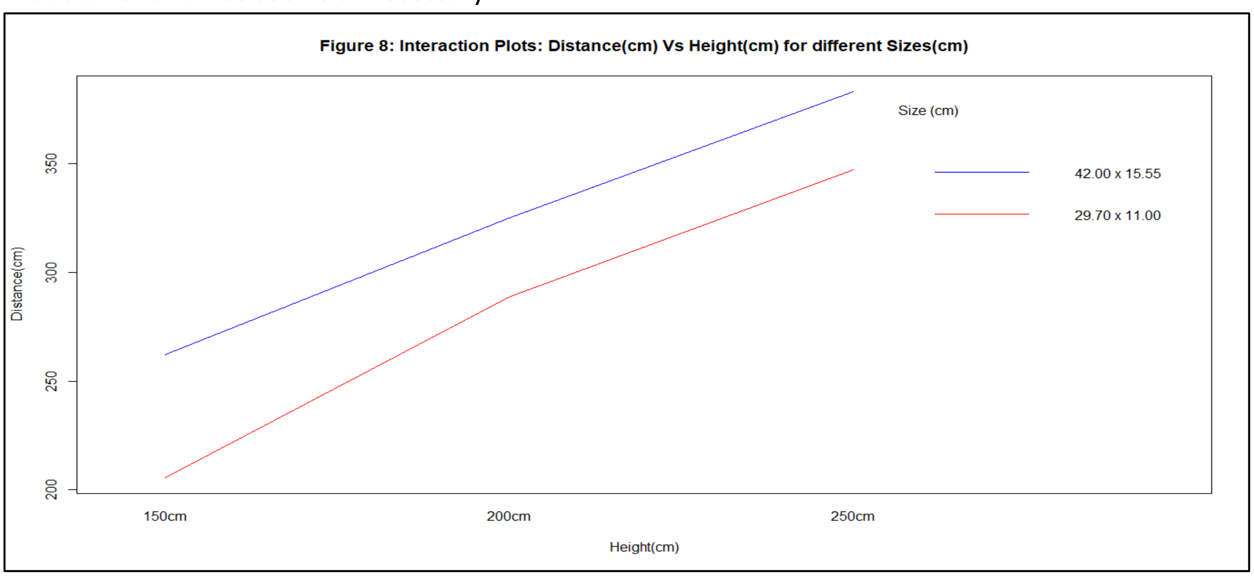


Figure 8 shows that Height and Size appear to be highly corelated with each other as an increase in height results in an increase in distance in both the “42.00x15.55” and “29.70x11.00” groups. Furthermore, lines are almost parallel with each other, suggesting that the final model should not include a Height and Size interaction term if statistically insignificant.

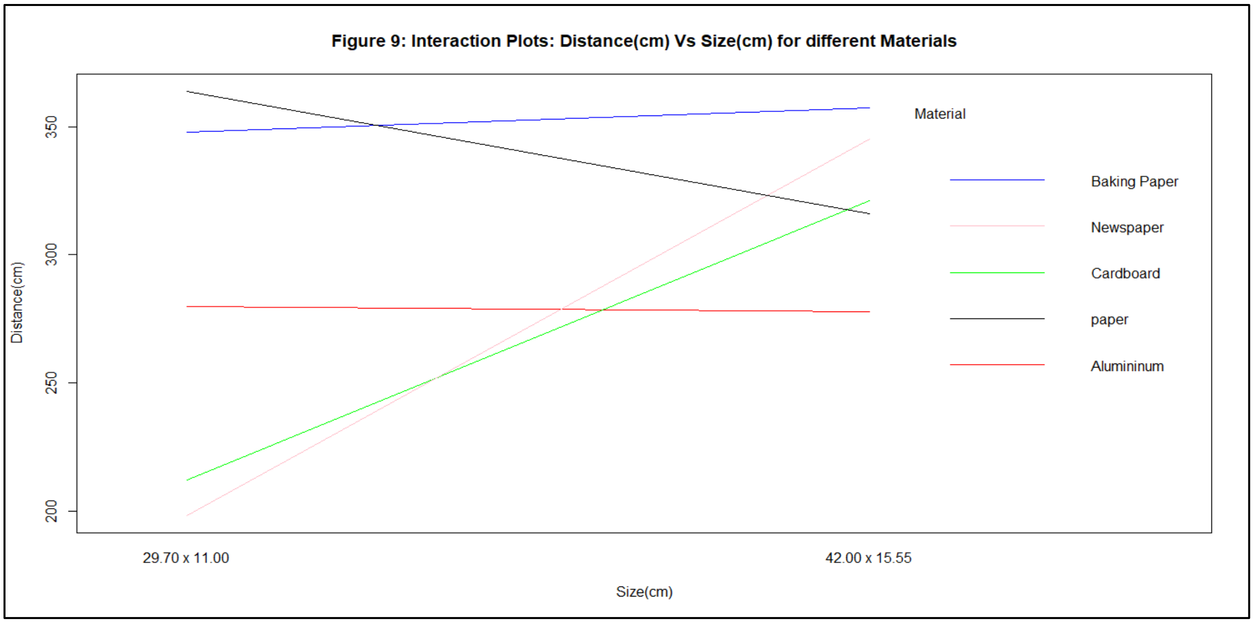
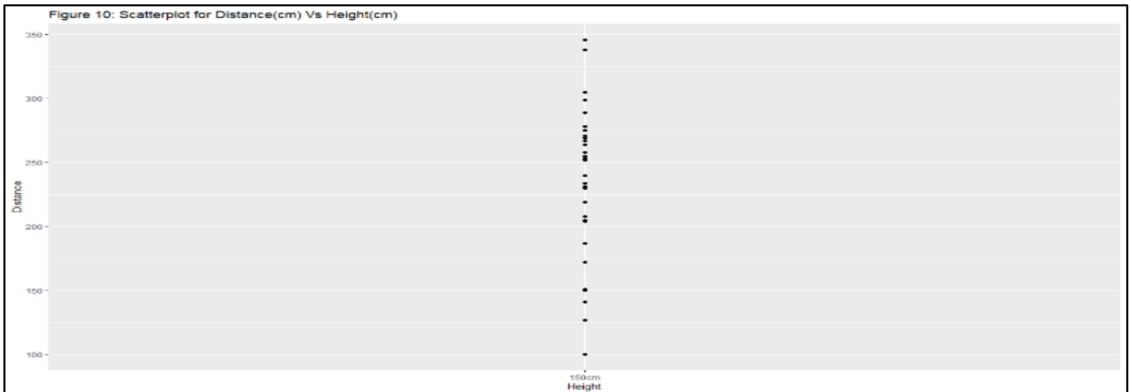
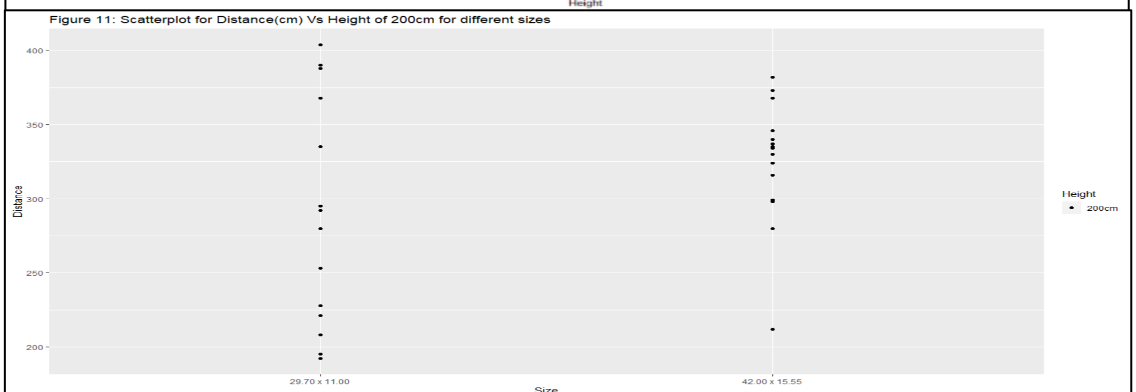


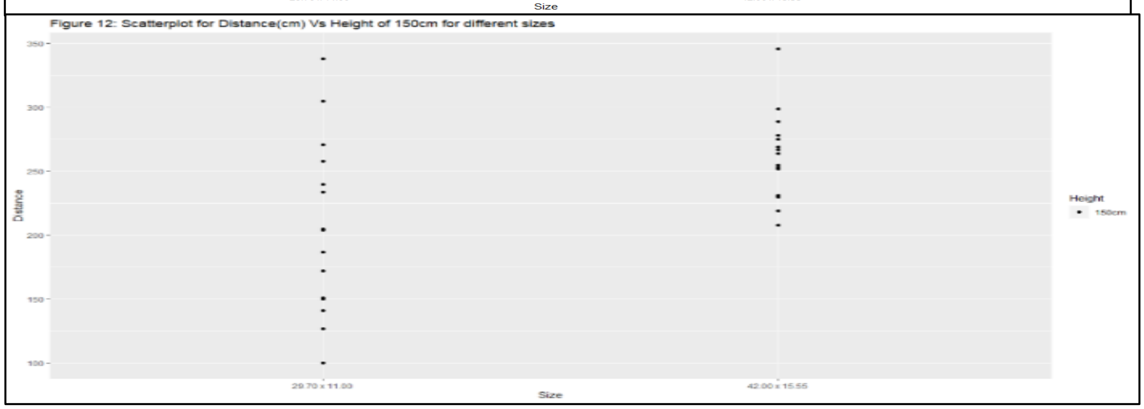
Figure 9 shows the interaction between distance vs size for different materials and unlike the figures before there does not seem to be much correlation between them. From the plot it shows that for an increase in size the materials newspaper, cardboard and baking paper all had an increase in distance while paper had a decrease in distance. aluminium has a very slight distance as it did not seem to be affected at all. These results suggest that this interaction term should be included in the final model if statistically significant.

Reliability and Accuracy

Figure 10, 11, 12: Outliers







The experiment was done with strict rules, such that the data collected is accurate and reliable. However, it can be seen in figure 3 and 5, that there are outliers. This is mostly due to human error that occurred during the experiment. So, to get a better view of these outliers, used a scatterplot to show this.

Although there were outliers their effect was not deemed significant enough to remove them from the data set used to find the final model.

Model reduction

Now use a model reduction on the full model to find the optimal model to showcase data. The full model consists of all the independent variables, their two and three-way interactions. Although figure 8 suggests that the interaction between Height:Size may be insignificant, we will still include for now, including all the other terms, and see if the step-wise regression decide its significance in the final model.

Now using the forward and backwards regression to determine the final model the two regressions produce two different results.

Firstly, the forward regression model produced

Distance = HeightX1 + SizeX2 + MaterialX3 + Height:MaterialX4 + Size:MaterialX5

Which doesn't include Height:Size, which follows the suggestion in figure 8 that this term might be insignificant. Also, Height:Size:Material isn't included which can be assumed because Height:Size was insignificant so adding a term to that interaction would do little to affect its significance. Finally, this model had an AIC value of 696.24 and an adjusted R squared value of 0.7733 meaning most of the variation is explained by the model.

Secondly, the backwards regression model produced

Distance = HeightX1 + SizeX2 + MaterialX3 + Height:MaterialX4 + Size:MaterialX5 + Height:SizeX6 + Height:Size:MaterialX7

Which includes all terms which suggests that they could all be significant, even those that were thought not to be. This model had an AIC value of 694.68 and an adjusted R squared value of 0.7919.

Now with two different models to choose from we'll first inspect the model produced by the backwards regression as it has a lower AIC value and a higher R squared value, meaning that more variation is explained by the model.

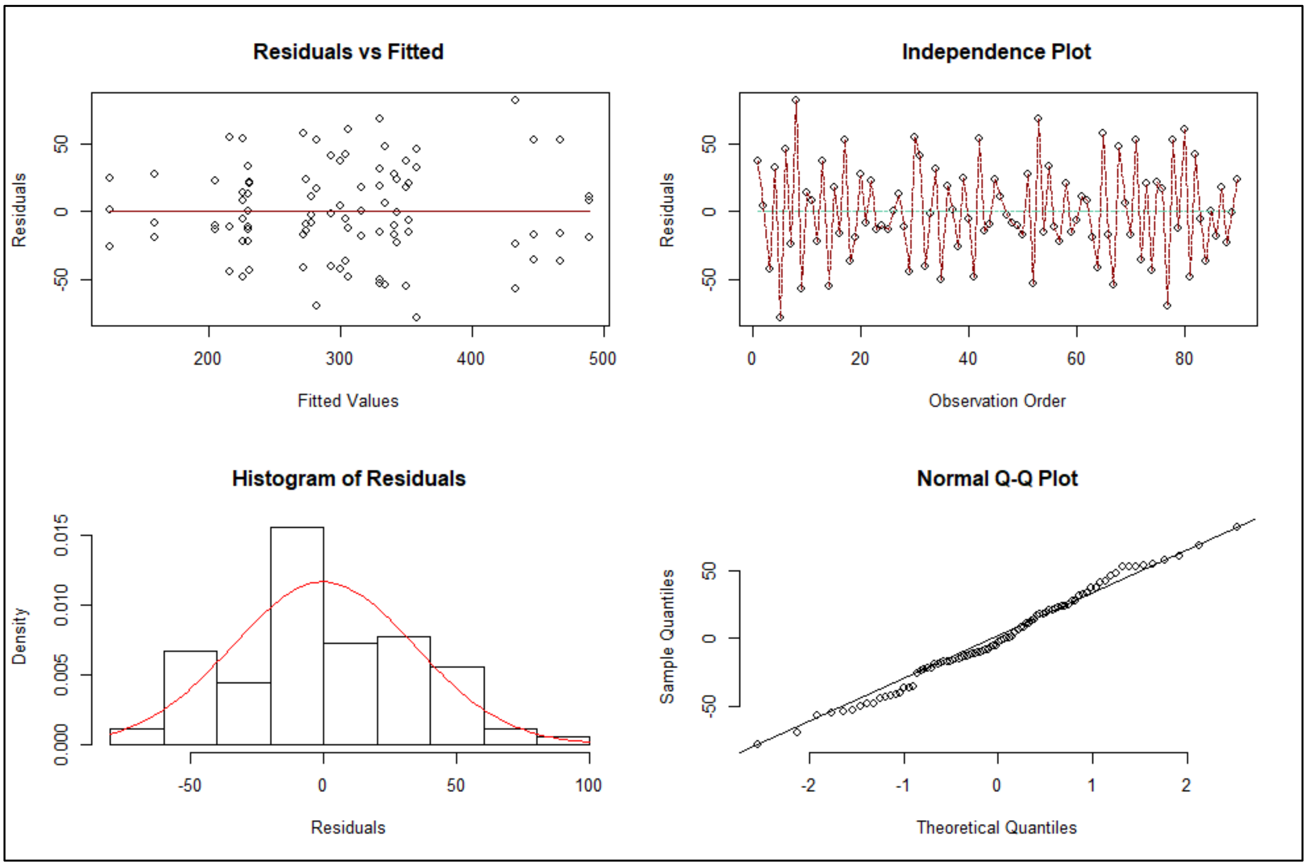
Assumptions of ANOVA

Before ANOVA can be used, the assumptions for ANOVA must be meet.

- Linearity: A Linear relationship between x, y is reasonable
- Homoscedasticity: Residuals have a mean of 0 and constant variance
- Independence: Residuals are independent of each other
- Normality: Residuals have a normal distribution

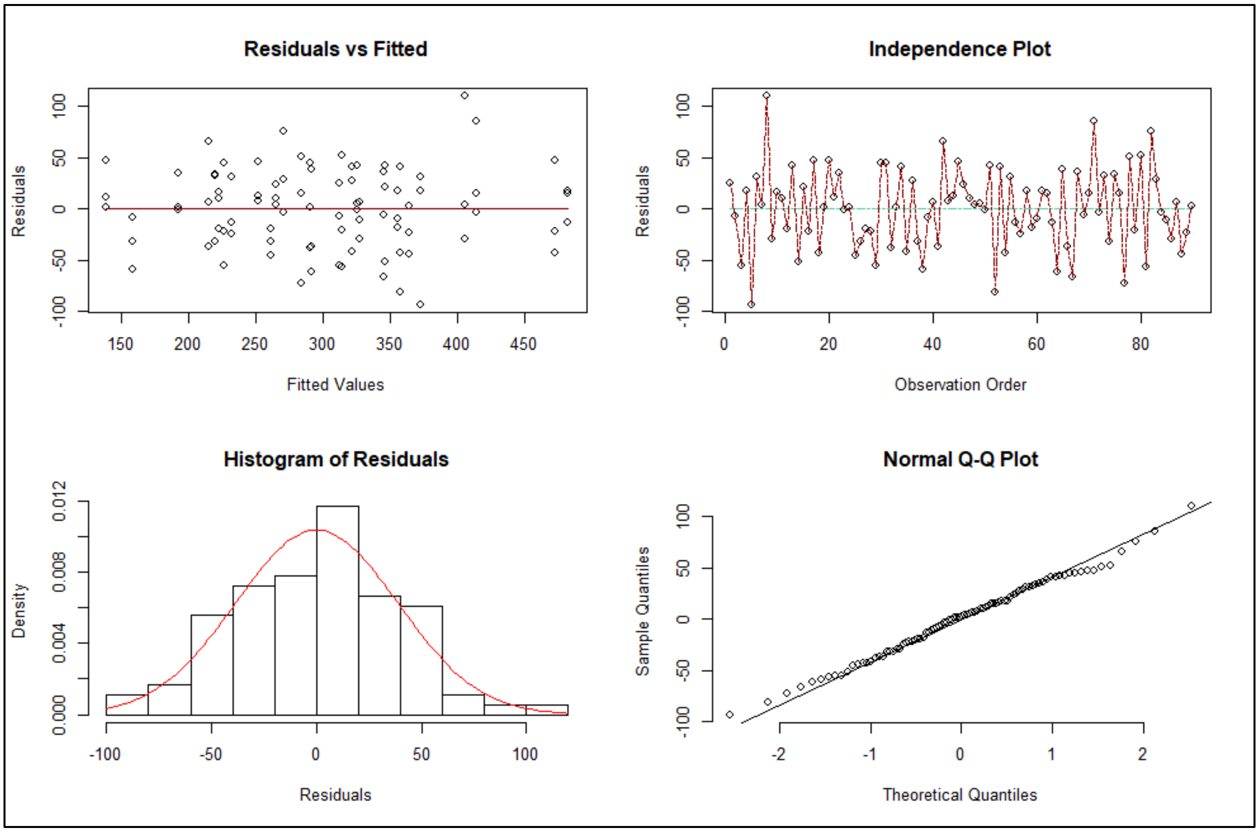
Firstly, from the figure 1, 2 and 3, linearity can be assumed as they all indicate a linear trend between x and y. In all three of the Response vs single Predictor variable it showed that an increase in predictor variable resulted in an increase in distance, thus confirming the assumption of linearity.

Figure 13: Plots using backwards regression model



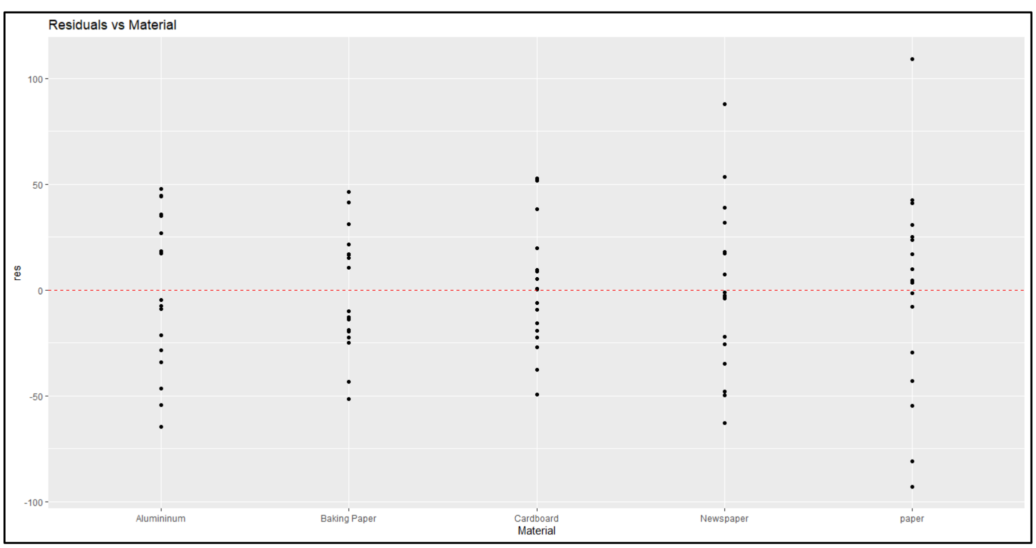
Looking at the plots from figure 13 the final model it can be seen that the even though the residuals have a mean of zero, they do not have constant variance towards larger fitted values, therefore the assumption of homoscedasticity is violated and suggests that this model should not be the final model. In light of this new evidence the other model produced will be inspected to see if it can be the final model.

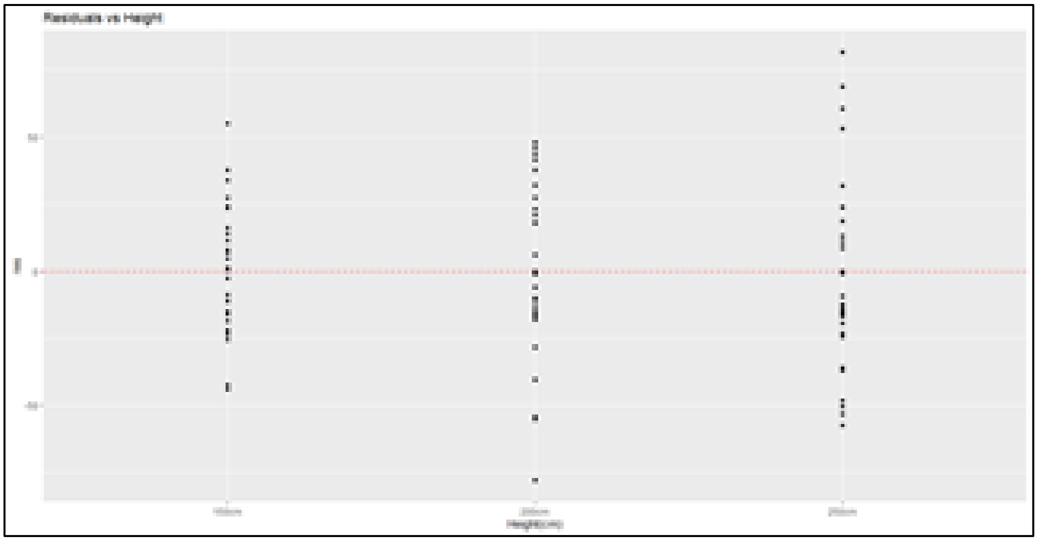
Figure 14: Plots using the forwards regression model

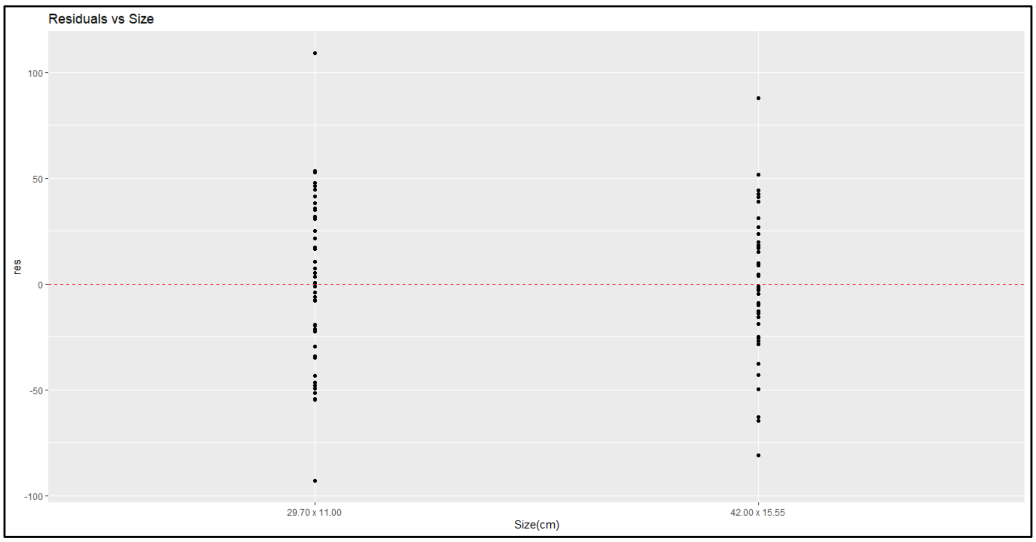


Looking at the new plots from figure 14 using the forwards regression model it is clear to see that the new model shows homoscedasticity as not only is the mean of the residuals zero but the variance is also constant, even towards larger fitted values. Now to check the other assumptions.

Figure 15, 16, 17: Residuals vs independent variables







Also, while checking homoscedasticity residuals vs independent variable plots were produced as seen in figures 15, 16, and 17. Viewing these plots it shows that each variable shows homoscedasticity as the residuals for each variable is centered

Figure 14's independence plot shows residuals vs the observation number. The plot shows no pattern between the points in the independence plot. If there is a pattern in the independence plot, it means that the model is not independent. However, the model has no pattern and therefore is independent. So, the model meets the assumption that residuals are independent of each other.

The Histogram and QQ-plot from figure 14 both confirm the assumption of normality, that the residuals have normal distribution. Firstly, the histogram shows that the values follows the normal distribution, so the histograms represents a normal distribution. Secondly, the points in the QQ-plot normally distributed around the QQ-line, indicating normality.

Results

Now with assumptions meet, ANOVA can be used.

Figure 18: ANOVA Table

```
Analysis of Variance Table

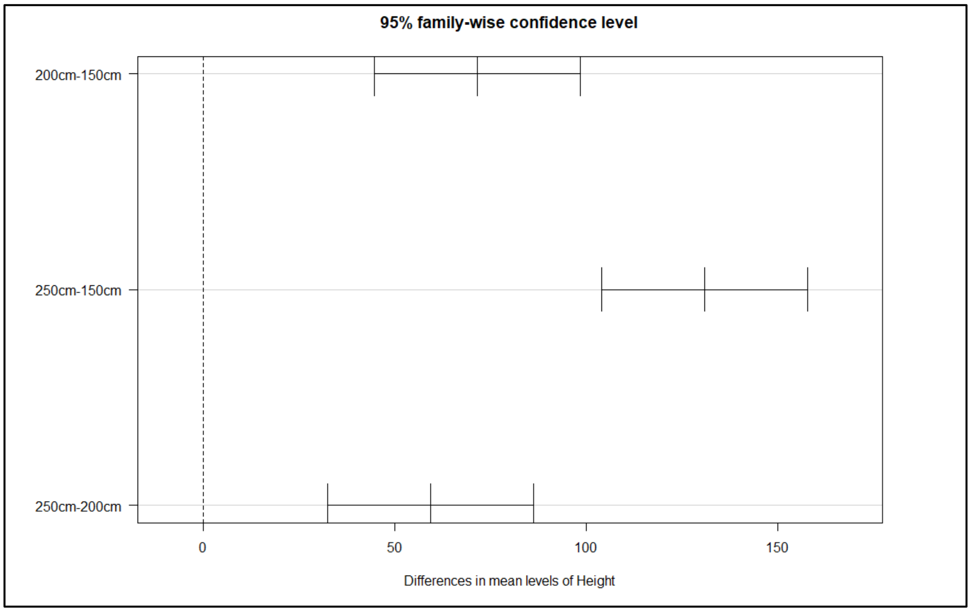
Response: (Distance)
                Df Sum Sq Mean Sq F value    Pr(>F)    
Height           2 257501  128750 68.2249 < 2.2e-16 ***
Size             1  43560   43560 23.0825 8.550e-06 ***
Material         4 121962   30491 16.1570 2.038e-09 ***
Height:Material  8  56892    7111  3.7684   0.00101 ** 
Size:Material    4 128980   32245 17.0866 8.068e-10 ***
Residuals       70 132100    1887                      
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

The ANOVA table from figure 18 shows the significance of all variables in the reduced model. The p value of height is 2.2e-16, material is 2.038e-09, size is 8.550e-06, Size:Material is 8.068e-10 and interaction of Height: Material is 0.00101 which are all lower than 0.05 therefore they are all significant and should be included in the model.

A Tukey’s honest significance difference test was run on the final model to check if the predictor variables have a significant effect on the response.

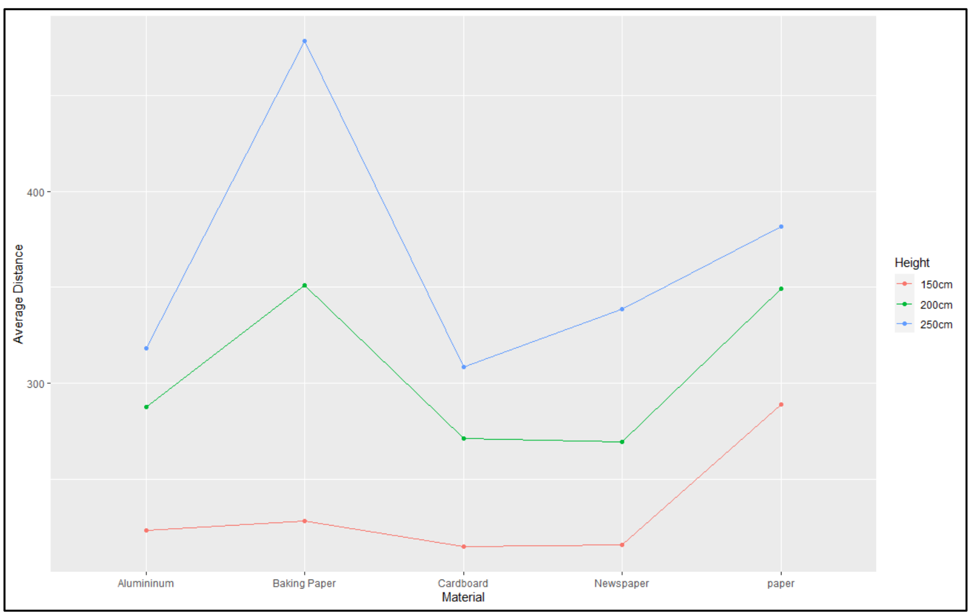
Does Height have an effect?

Figure 19: Tukey's HSD plot on Height



From the Figure 19 it shows that all the heights have an effect as their mean differences are not zero, with the difference between 250cm and 150cm (diff = 130.8) being the highest and 250cm and 200cm (diff = 32.47, p = 4e-06) being the lowest indicating that a larger difference in height results in a larger difference in means.

Figure 20: Distance against Material with Height groups



From the ANOVA from figure 18 it is clear to see that the effect height has (p = 0.00101) is significant effect in the interaction term with material and figure 20 above help visualize the effect height has in a Height:Material interaction term. From figure 20 it shows that height has a general effect as the distance flown increases with an increase in height for every material but the individual effects on the materials vary. For example, the difference for Aliminium:200cm and Aliminium:250cm (diff = 30.66, p =0.99) is much smaller than the same difference for baking paper, Baking Paper:200cm and Baking Paper:250cm (diff =127.00, p=0.0002). This shows the different interaction between Height and Material as material changes and hence the interaction.

Does Size have an effect?

Figure 21: Tukey's HSD on Size

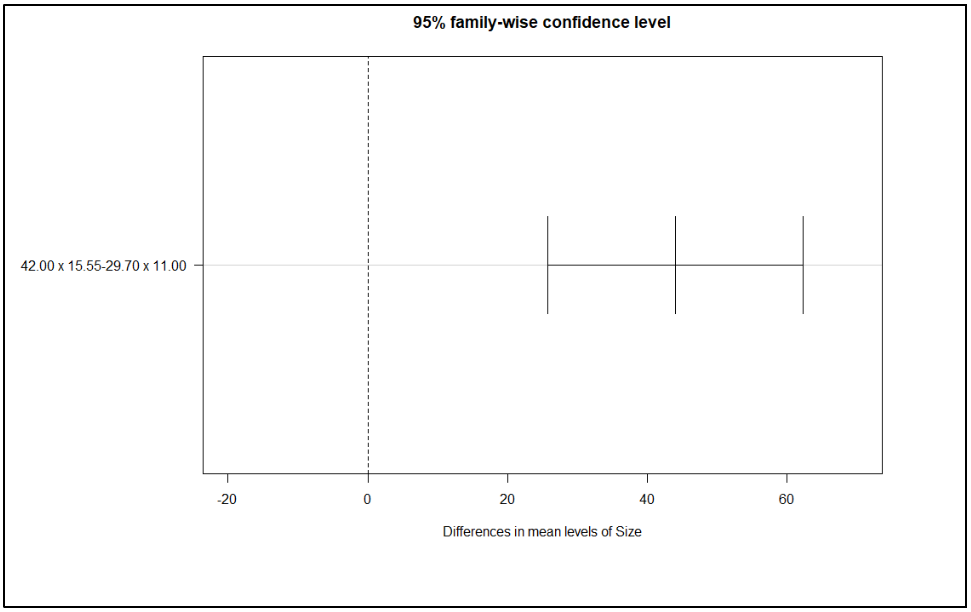
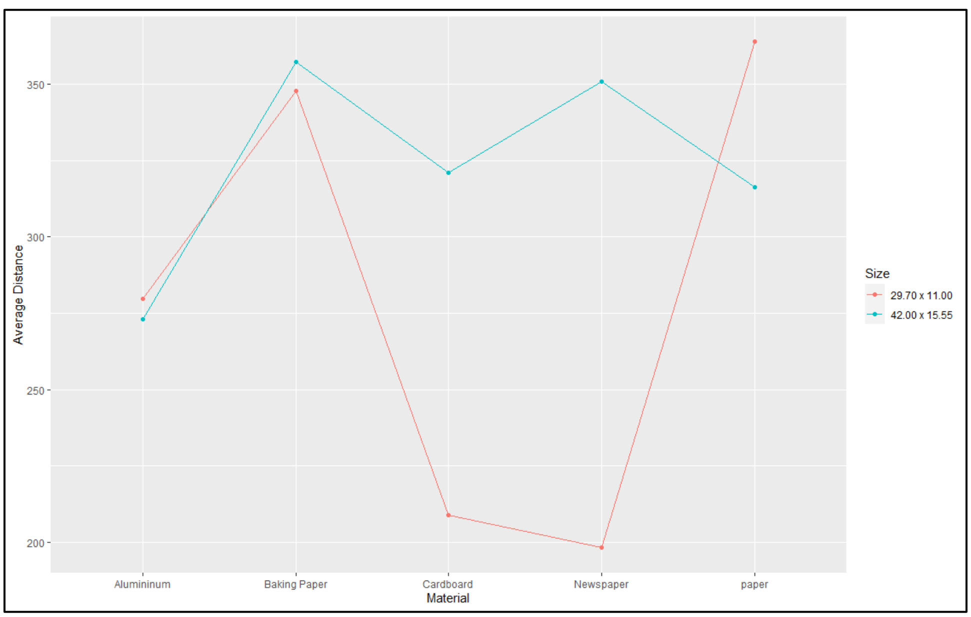


Figure 21 shows that the size does have an effect as the difference between the two size is not zero, with difference being 44 and p value being 8.5e-06, meaning that there is a difference between sizes.

Figure 22: Distance VS Material with size groups



From Figure 22 it is clear that the interaction term between Size and material is stronger than the effect of size on its own. This is evident as there are no general trends of size on distance with regards to material as they all interact differently. For example, both baking paper and newspaper's distance increases with an increase in size but the increase is drastically different with the difference between baking paper 42.00 x 15.55 and baking paper 29.70 x 11.00 (diff=9.44, p=0.99) is a lot smaller than the difference between Newspaper 42.00 x 15.55 and newspaper 29.70 x 11.00 (diff = 152.77, p = 0.00). Furthermore, the decrease in distance with an increase in size is not consistent as the difference between aluminium 42.00 x 15.55 and aluminium 29.70 x 11.00 (diff = -6.77, p = 0.99) and paper 42.00 x 15.55 and paper 29.70 x 11.00 (diff = -47.66, p = 0.38) is a lot different, hence the interaction effect between material and size.

Does Material have an effect?

Figure 23: Tukey's HSD on Material

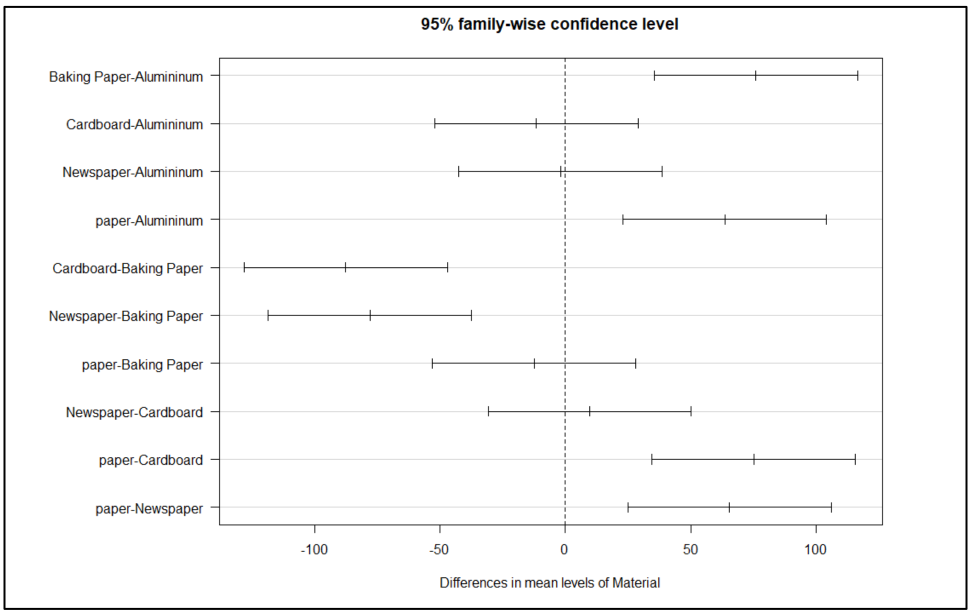
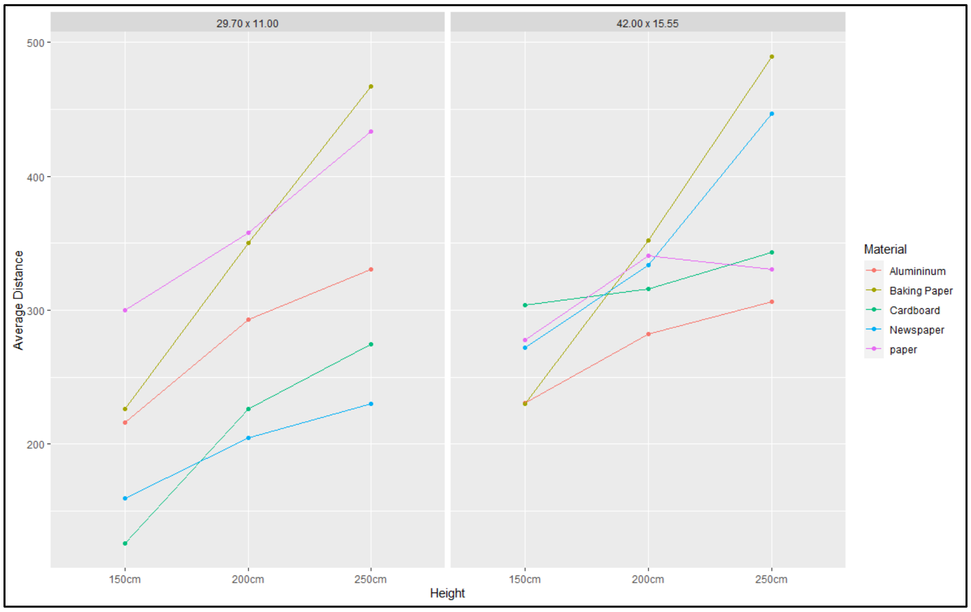


Figure 23 shows that materials effect on distance is mixed as some materials have an effect while others do not. The plot shows that Cardboard-Aluminium (diff = 11.61, p = 0.92), Newspaper-Aluminium (diff = -1.88, p = 0.99) and Newspaper-Cardboard (diff = 9.72, p = 0.96) have little to no effect on the response as even though their means are not zero, the 95% confidence interval of their means crosses zero indicating that the true mean might lie on zero or be close around it and their p-values are quite high indicating that they have little significance, further agreeing that their effect is 0. This information suggests that the materials are the same as each other and changing from one of those materials to another will not significantly affect the response. Furthermore, paper-Baking paper's confidence interval crosses the zero-line suggesting that those two are highly correlated and a change in them does not affect the response. Although there were unaffecting variables the rest of them did have a significant and varied effect on the response.

Figure 24: Distance VS Height & Size & Material



From the plots prior it was evident that Size and Height both had an effect on distance when interacting with material in both main and interacting effect, but figure 24 shows the effect that material has on both size and height.

Conclusion

In conclusion, this essay endeavoured to answer four questions:

Firstly, it was shown in the report that Height has an effect on distance as when height increased, the distance increased with 250cm resulting in the furthest distance flown for all materials and sizes. Although all planes increased distance with an increase in height, the distance increases were different for all plane types.

Secondly, it was shown that size has an effect except not always a positive one as an increase in size did not always lead to an increase in distance. Each material interacted differently with some planes distance increasing and others decreasing.

Thirdly, the report showed a variation of significance and insignificance of materials on distance indicating that some materials had an effect on distance, and some didn't. For example, cardboard, aluminium, and newspaper were all very similar and a change from one of these materials to another had very little effect on the distance thrown. Similarly baking paper and paper were also highly correlated and a change between these two materials had little effect on the distance.

Furthermore, in search for a model to best describe the relationship between the response, distance, and the variables it was found that not only were all the variables significant and had an effect on the distance but there were also two interaction terms that had a significant effect on distance flown, these being Height:Materials and Size:Material.

Lastly, it was found that the combination of baking paper, size 42.00x15.55 and height thrown 250cm resulted in the furthest distance flown.

More investigation and replicates must be done to find a deeper understanding of this topic.

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Appendix

Forward Regression

```
> fr <- step(lm(data=data.df,Distance~1),
+            scope=list(lower=~1, upper=~Height+Material+Size+Material:Size+Material:Height+Height:Size+Height:Size:Material),
+            direction="forward")
Start:  AIC=813.43
Distance ~ 1

           Df Sum of Sq    RSS    AIC
+ Height    2    257501 483494 779.01
+ Material  4    121962 619032 805.25
+ Size      1     43560 697434 809.98
<none>                  740994 813.43

Step:  AIC=779.01
Distance ~ Height

           Df Sum of Sq    RSS    AIC
+ Material  4    121962 361531 760.85
+ Size      1     43560 439934 772.51
<none>                  483494 779.01

Step:  AIC=760.85
Distance ~ Height + Material

                  Df Sum of Sq    RSS    AIC
+ Size             1     43560 317971 751.29
<none>                         361531 760.85
+ Height:Material  8     56892 304640 761.44

Step:  AIC=751.29
Distance ~ Height + Material + Size

                  Df Sum of Sq    RSS    AIC
+ Material:Size    4    128980 188992 712.47
+ Height:Material  8     56892 261080 749.55
<none>                         317971 751.29
+ Height:Size      2      2057 315914 754.71

Step:  AIC=712.47
Distance ~ Height + Material + Size + Material:Size

                  Df Sum of Sq    RSS    AIC
+ Height:Material  8     56892 132100 696.24
<none>                         188992 712.47
+ Height:Size      2      2057 186935 715.48

Step:  AIC=696.24
Distance ~ Height + Material + Size + Material:Size + Height:Material

              Df Sum of Sq    RSS    AIC
<none>                     132100 696.24
+ Height:Size  2    2057.3 130043 698.82
> 
```

Forward regression summary

```
> summary(fr)

Call:
lm(formula = Distance ~ Height + Material + Size + Material:Size + 
    Height:Material, data = data.df)

Residuals:
    Min      1Q  Median      3Q     Max 
-93.167 -28.181   2.056  27.861 109.333 

Coefficients:
                                     Estimate Std. Error t value Pr(>|t|)    
(Intercept)                           226.889     20.478  11.079  < 2e-16 ***
Height200cm                            64.167     25.081   2.558 0.012682 *  
Height250cm                            94.833     25.081   3.781 0.000325 ***
MaterialBaking Paper                   -3.444     28.961  -0.119 0.905668    
MaterialCardboard                     -68.167     28.961  -2.354 0.021397 *  
MaterialNewspaper                     -87.444     28.961  -3.019 0.003533 ** 
Materialpaper                          85.944     28.961   2.968 0.004105 ** 
Size42.00 x 15.55                      -6.778     20.478  -0.331 0.741654    
MaterialBaking Paper:Size42.00 x 15.55  16.222     28.961   0.560 0.577170    
MaterialCardboard:Size42.00 x 15.55   119.000     28.961   4.109 0.000106 ***
MaterialNewspaper:Size42.00 x 15.55   159.556     28.961   5.509 5.62e-07 ***
Materialpaper:Size42.00 x 15.55       -40.889     28.961  -1.412 0.162419    
Height200cm:MaterialBaking Paper       58.833     35.470   1.659 0.101652    
Height250cm:MaterialBaking Paper      155.167     35.470   4.375 4.15e-05 ***
Height200cm:MaterialCardboard          -7.833     35.470  -0.221 0.825855    
Height250cm:MaterialCardboard          -1.000     35.470  -0.028 0.977588    
Height200cm:MaterialNewspaper         -10.500     35.470  -0.296 0.768085    
Height250cm:MaterialNewspaper          27.833     35.470   0.785 0.435272    
Height200cm:Materialpaper              -3.833     35.470  -0.108 0.914246    
Height250cm:Materialpaper              -2.000     35.470  -0.056 0.955195    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 43.44 on 70 degrees of freedom
Multiple R-squared:  0.8217,	Adjusted R-squared:  0.7733 
F-statistic: 16.98 on 19 and 70 DF,  p-value: < 2.2e-16

> 
```

Backwards Regression

```
> br <- step(lm(data = data.df, Distance~Height+Material+Size+Material:Size+Material:Height+Height:Size+Height:Size:Material),
+           direction="backward")
Start:  AIC=694.68
Distance ~ Height + Material + Size + Material:Size + Material:Height + 
    Height:Size + Height:Size:Material

                       Df Sum of Sq    RSS    AIC
<none>                              103970 694.68
- Height:Material:Size  8     26073 130043 698.82
> 
```

Backwards Model Summary

```
> summary(br)

Call:
lm(formula = Distance ~ Height + Material + Size + Material:Size +
    Material:Height + Height:Size + Height:Size:Material, data = data.df)

Residuals:
    Min      1Q  Median      3Q     Max
-78.000 -19.167  -3.833  23.500  81.667

Coefficients:
                                          Estimate Std. Error t value Pr(>|t|)
(Intercept)                                 216.00      24.03   8.987 1.04e-12 ***
Height200cm                                  77.33      33.99   2.275  0.02648 *
Height250cm                                 114.33      33.99   3.364  0.00134 **
MaterialBaking Paper                         10.00      33.99   0.294  0.76961
MaterialCardboard                           -90.33      33.99  -2.658  0.01007 *
MaterialNewspaper                           -56.33      33.99  -1.657  0.10265
Materialpaper                                84.33      33.99   2.481  0.01591 *
Size42.00 x 15.55                            15.00      33.99   0.441  0.66056
MaterialBaking Paper:Size42.00 x 15.55      -10.67      48.07  -0.222  0.82514
MaterialCardboard:Size42.00 x 15.55         163.33      48.07   3.398  0.00121 **
MaterialNewspaper:Size42.00 x 15.55          97.33      48.07   2.025  0.04733 *
Materialpaper:Size42.00 x 15.55             -37.67      48.07  -0.784  0.43634
Height200cm:MaterialBaking Paper             47.00      48.07   0.978  0.33210
Height250cm:MaterialBaking Paper            126.67      48.07   2.635  0.01069 *
Height200cm:MaterialCardboard                23.33      48.07   0.485  0.62914
Height250cm:MaterialCardboard                34.33      48.07   0.714  0.47782
Height200cm:MaterialNewspaper               -32.00      48.07  -0.666  0.50813
Height250cm:MaterialNewspaper               -44.00      48.07  -0.915  0.36365
Height200cm:Materialpaper                   -19.67      48.07  -0.409  0.68389
Height250cm:Materialpaper                    18.67      48.07   0.388  0.69913
Height200cm:Size42.00 x 15.55               -26.33      48.07  -0.548  0.58583
Height250cm:Size42.00 x 15.55               -39.00      48.07  -0.811  0.42036
Height200cm:MaterialBaking Paper:Size42.00 x 15.55   23.67      67.98   0.348  0.72894
Height250cm:MaterialBaking Paper:Size42.00 x 15.55   57.00      67.98   0.839  0.40507
Height200cm:MaterialCardboard:Size42.00 x 15.55     -62.33      67.98  -0.917  0.36283
Height250cm:MaterialCardboard:Size42.00 x 15.55     -70.67      67.98  -1.040  0.30271
Height200cm:MaterialNewspaper:Size42.00 x 15.55      43.00      67.98   0.633  0.52942
Height250cm:MaterialNewspaper:Size42.00 x 15.55     143.67      67.98   2.113  0.03873 *
Height200cm:Materialpaper:Size42.00 x 15.55          31.67      67.98   0.466  0.64301
Height250cm:Materialpaper:Size42.00 x 15.55         -41.33      67.98  -0.608  0.54545
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 41.63 on 60 degrees of freedom
Multiple R-squared:  0.8597,	Adjusted R-squared:  0.7919
F-statistic: 12.68 on 29 and 60 DF,  p-value: 2.359e-16

>
```

Results appendix

obs#	Distance	Material	Height	Size
1	338	paper	150cm	29.70 x 11.00
2	305	paper	150cm	29.70 x 11.00
3	258	paper	150cm	29.70 x 11.00
4	390	paper	200cm	29.70 x 11.00
5	280	paper	200cm	29.70 x 11.00
6	404	paper	200cm	29.70 x 11.00
7	409	paper	250cm	29.70 x 11.00
8	515	paper	250cm	29.70 x 11.00
9	376	paper	250cm	29.70 x 11.00
10	240	Baking Paper	150cm	29.70 x 11.00
11	234	Baking Paper	150cm	29.70 x 11.00
12	204	Baking Paper	150cm	29.70 x 11.00
13	388	Baking Paper	200cm	29.70 x 11.00
14	295	Baking Paper	200cm	29.70 x 11.00
15	368	Baking Paper	200cm	29.70 x 11.00
16	451	Baking Paper	250cm	29.70 x 11.00
17	520	Baking Paper	250cm	29.70 x 11.00
18	430	Baking Paper	250cm	29.70 x 11.00
19	141	Newspaper	150cm	29.70 x 11.00
20	187	Newspaper	150cm	29.70 x 11.00
21	151	Newspaper	150cm	29.70 x 11.00
22	228	Newspaper	200cm	29.70 x 11.00
23	192	Newspaper	200cm	29.70 x 11.00
24	195	Newspaper	200cm	29.70 x 11.00
25	217	Newspaper	250cm	29.70 x 11.00
26	230	Newspaper	250cm	29.70 x 11.00
27	243	Newspaper	250cm	29.70 x 11.00
28	205	Aluminınum	150cm	29.70 x 11.00
29	172	Aluminınum	150cm	29.70 x 11.00
30	271	Aluminınum	150cm	29.70 x 11.00
31	335	Aluminınum	200cm	29.70 x 11.00
32	253	Aluminınum	200cm	29.70 x 11.00
33	292	Aluminınum	200cm	29.70 x 11.00
34	362	Aluminınum	250cm	29.70 x 11.00
35	280	Aluminınum	250cm	29.70 x 11.00
36	349	Aluminınum	250cm	29.70 x 11.00
37	127	Cardboard	150cm	29.70 x 11.00
38	100	Cardboard	150cm	29.70 x 11.00
39	150	Cardboard	150cm	29.70 x 11.00
40	221	Cardboard	200cm	29.70 x 11.00
41	178	Cardboard	200cm	29.70 x 11.00
42	280	Cardboard	200cm	29.70 x 11.00
43	260	Cardboard	250cm	29.70 x 11.00

44	265	Cardboard	250cm	29.70 x 11.00
45	298	Cardboard	250cm	29.70 x 11.00
46	289	paper	150cm	42.00 x 15.55
47	275	paper	150cm	42.00 x 15.55
48	269	paper	150cm	42.00 x 15.55
49	330	paper	200cm	42.00 x 15.55
50	324	paper	200cm	42.00 x 15.55
51	368	paper	200cm	42.00 x 15.55
52	277	paper	250cm	42.00 x 15.55
53	399	paper	250cm	42.00 x 15.55
54	315	paper	250cm	42.00 x 15.55
55	264	Baking Paper	150cm	42.00 x 15.55
56	219	Baking Paper	150cm	42.00 x 15.55
57	208	Baking Paper	150cm	42.00 x 15.55
58	373	Baking Paper	200cm	42.00 x 15.55
59	337	Baking Paper	200cm	42.00 x 15.55
60	346	Baking Paper	200cm	42.00 x 15.55
61	500	Baking Paper	250cm	42.00 x 15.55
62	498	Baking Paper	250cm	42.00 x 15.55
63	470	Baking Paper	250cm	42.00 x 15.55
64	231	Newspaper	150cm	42.00 x 15.55
65	330	Newspaper	150cm	42.00 x 15.55
66	255	Newspaper	150cm	42.00 x 15.55
67	280	Newspaper	200cm	42.00 x 15.55
68	382	Newspaper	200cm	42.00 x 15.55
69	340	Newspaper	200cm	42.00 x 15.55
70	430	Newspaper	250cm	42.00 x 15.55
71	500	Newspaper	250cm	42.00 x 15.55
72	411	Newspaper	250cm	42.00 x 15.55
73	252	Aluminimum	150cm	42.00 x 15.55
74	188	Aluminimum	150cm	42.00 x 15.55
75	253	Aluminimum	150cm	42.00 x 15.55
76	299	Aluminimum	200cm	42.00 x 15.55
77	212	Aluminimum	200cm	42.00 x 15.55
78	335	Aluminimum	200cm	42.00 x 15.55
79	294	Aluminimum	250cm	42.00 x 15.55
80	367	Aluminimum	250cm	42.00 x 15.55
81	258	Aluminimum	250cm	42.00 x 15.55
82	346	Cardboard	150cm	42.00 x 15.55
83	299	Cardboard	150cm	42.00 x 15.55
84	267	Cardboard	150cm	42.00 x 15.55
85	316	Cardboard	200cm	42.00 x 15.55
86	298	Cardboard	200cm	42.00 x 15.55
87	334	Cardboard	200cm	42.00 x 15.55
88	320	Cardboard	250cm	42.00 x 15.55
89	342	Cardboard	250cm	42.00 x 15.55

90	367	Cardboard	250cm	42.00 x 15.55

Tukeys HSD

```
tukey.test
  Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = tukeything)

$Height
                 diff       lwr       upr p adj
200cm-150cm  71.50000  44.64143  98.35857 1e-07
250cm-150cm 130.83333 103.97476 157.69190 0e+00
250cm-200cm  59.33333  32.47476  86.19190 4e-06

$Material
                              diff        lwr       upr     p adj
Baking Paper-Alumininum  76.000000   35.45259 116.54741 0.0000152
Cardboard-Alumininum    -11.611111  -52.15852  28.93629 0.9291909
Newspaper-Alumininum     -1.888889  -42.43629  38.65852 0.9999331
paper-Alumininum         63.555556   23.00815 104.10296 0.0003710
Cardboard-Baking Paper  -87.611111 -128.15852 -47.06371 0.0000006
Newspaper-Baking Paper  -77.888889 -118.43629 -37.34148 0.0000092
paper-Baking Paper      -12.444444  -52.99185  28.10296 0.9106085
Newspaper-Cardboard       9.722222  -30.82518  50.26963 0.9619129
paper-Cardboard          75.166667   34.61926 115.71407 0.0000190
paper-Newspaper          65.444444   24.89704 105.99185 0.0002328

$Size
                            diff     lwr     upr   p adj
42.00 x 15.55-29.70 x 11.00   44 25.7345 62.2655 8.5e-06

$`Height:Material`
                                              diff         lwr         upr     p adj
200cm:Alumininum-150cm:Alumininum        64.166667  -24.001579  152.334912 0.4148814
250cm:Alumininum-150cm:Alumininum        94.833333    6.665088  183.001579 0.0233306
150cm:Baking Paper-150cm:Alumininum       4.666667  -83.501579   92.834912 1.0000000
200cm:Baking Paper-150cm:Alumininum     127.666667   39.498421  215.834912 0.0002692
250cm:Baking Paper-150cm:Alumininum     254.666667  166.498421  342.834912 0.0000000
150cm:Cardboard-150cm:Alumininum         -8.666667  -96.834912   79.501579 1.0000000
200cm:Cardboard-150cm:Alumininum         47.666667  -40.501579  135.834912 0.8447594
250cm:Cardboard-150cm:Alumininum         85.166667   -3.001579  173.334912 0.0689975
150cm:Newspaper-150cm:Alumininum         -7.666667  -95.834912   80.501579 1.0000000
200cm:Newspaper-150cm:Alumininum         46.000000  -42.168245  134.168245 0.8757601
250cm:Newspaper-150cm:Alumininum        115.000000   26.831755  203.168245 0.0016867
150cm:paper-150cm:Alumininum             65.500000  -22.668245  153.668245 0.3805674
200cm:paper-150cm:Alumininum            125.833333   37.665088  214.001579 0.0003535
250cm:paper-150cm:Alumininum            158.333333   70.165088  246.501579 0.0000022
250cm:Alumininum-200cm:Alumininum        30.666667  -57.501579  118.834912 0.9957607
150cm:Baking Paper-200cm:Alumininum     -59.500000 -147.668245   28.668245 0.5427653
200cm:Baking Paper-200cm:Alumininum      63.500000  -24.668245  151.668245 0.4325018
250cm:Baking Paper-200cm:Alumininum     190.500000  102.331755  278.668245 0.0000000
150cm:Cardboard-200cm:Alumininum        -72.833333 -161.001579   15.334912 0.2203475
200cm:Cardboard-200cm:Alumininum        -16.500000 -104.668245   71.668245 0.9999967
250cm:Cardboard-200cm:Alumininum         21.000000  -67.168245  109.168245 0.9999355
150cm:Newspaper-200cm:Alumininum        -71.833333 -160.001579   16.334912 0.2390269
200cm:Newspaper-200cm:Alumininum        -18.166667 -106.334912   70.001579 0.9999890
250cm:Newspaper-200cm:Alumininum         50.833333  -37.334912  139.001579 0.7756085
150cm:paper-200cm:Alumininum              1.333333  -86.834912   89.501579 1.0000000
200cm:paper-200cm:Alumininum             61.666667  -26.501579  149.834912 0.4822632
250cm:paper-200cm:Alumininum             94.166667    5.998421  182.334912 0.0252481
150cm:Baking Paper-250cm:Alumininum     -90.166667 -178.334912   -1.998421 0.0400468
200cm:Baking Paper-250cm:Alumininum      32.833333  -55.334912  121.001579 0.9917630
250cm:Baking Paper-250cm:Alumininum     159.833333   71.665088  248.001579 0.0000017
150cm:Cardboard-250cm:Alumininum       -103.500000 -191.668245  -15.331755 0.0079519
200cm:Cardboard-250cm:Alumininum        -47.166667 -135.334912   41.001579 0.8544727
250cm:Cardboard-250cm:Alumininum         -9.666667  -97.834912   78.501579 1.0000000
150cm:Newspaper-250cm:Alumininum       -102.500000 -190.668245  -14.331755 0.0090433
200cm:Newspaper-250cm:Alumininum        -48.833333 -137.001579   39.334912 0.8207670
250cm:Newspaper-250cm:Alumininum         20.166667  -68.001579  108.334912 0.9999603
150cm:paper-250cm:Alumininum            -29.333333 -117.501579   58.834912 0.9972963
200cm:paper-250cm:Alumininum             31.000000  -57.168245  119.168245 0.9952801
250cm:paper-250cm:Alumininum             63.500000  -24.668245  151.668245 0.4325018
200cm:Baking Paper-150cm:Baking Paper   123.000000   34.831755  211.168245 0.0005364
250cm:Baking Paper-150cm:Baking Paper   250.000000  161.831755  338.168245 0.0000000
150cm:Cardboard-150cm:Baking Paper      -13.333333 -101.501579   74.834912 0.9999998
200cm:Cardboard-150cm:Baking Paper       43.000000  -45.168245  131.168245 0.9214069
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250cm:Cardboard-150cm:Baking Paper	80.500000	-7.668245	168.668245	0.1105809
150cm:Newspaper-150cm:Baking Paper	-12.333333	-100.501579	75.834912	0.9999999
200cm:Newspaper-150cm:Baking Paper	41.333333	-46.834912	129.501579	0.9411627
250cm:Newspaper-150cm:Baking Paper	110.333333	22.165088	198.501579	0.0032123
150cm:paper-150cm:Baking Paper	60.833333	-27.334912	149.001579	0.5053771
200cm:paper-150cm:Baking Paper	121.166667	32.998421	209.334912	0.0007004
250cm:paper-150cm:Baking Paper	153.666667	65.498421	241.834912	0.0000047
250cm:Baking Paper-200cm:Baking Paper	127.000000	38.831755	215.168245	0.0002973
150cm:Cardboard-200cm:Baking Paper	-136.333333	-224.501579	-48.165088	0.0000722
200cm:Cardboard-200cm:Baking Paper	-80.000000	-168.168245	8.168245	0.1160550
250cm:Cardboard-200cm:Baking Paper	-42.500000	-130.668245	45.668245	0.9277439
150cm:Newspaper-200cm:Baking Paper	-135.333333	-223.501579	-47.165088	0.0000842
200cm:Newspaper-200cm:Baking Paper	-81.666667	-169.834912	6.501579	0.0986221
250cm:Newspaper-200cm:Baking Paper	-12.666667	-100.834912	75.501579	0.9999999
150cm:paper-200cm:Baking Paper	-62.166667	-150.334912	26.001579	0.4685260
200cm:paper-200cm:Baking Paper	-1.833333	-90.001579	86.334912	1.0000000
250cm:paper-200cm:Baking Paper	30.666667	-57.501579	118.834912	0.9957607
150cm:Cardboard-250cm:Baking Paper	-263.333333	-351.501579	-175.165088	0.0000000
200cm:Cardboard-250cm:Baking Paper	-207.000000	-295.168245	-118.831755	0.0000000
250cm:Cardboard-250cm:Baking Paper	-169.500000	-257.668245	-81.331755	0.0000003
150cm:Newspaper-250cm:Baking Paper	-262.333333	-350.501579	-174.165088	0.0000000
200cm:Newspaper-250cm:Baking Paper	-208.666667	-296.834912	-120.498421	0.0000000
250cm:Newspaper-250cm:Baking Paper	-139.666667	-227.834912	-51.498421	0.0000431
150cm:paper-250cm:Baking Paper	-189.166667	-277.334912	-100.998421	0.0000000
200cm:paper-250cm:Baking Paper	-128.833333	-217.001579	-40.665088	0.0002261
250cm:paper-250cm:Baking Paper	-96.333333	-184.501579	-8.165088	0.0194911
200cm:Cardboard-150cm:Cardboard	56.333333	-31.834912	144.501579	0.6318135
250cm:Cardboard-150cm:Cardboard	93.833333	5.665088	182.001579	0.0262594
150cm:Newspaper-150cm:Cardboard	1.000000	-87.168245	89.168245	1.0000000
200cm:Newspaper-150cm:Cardboard	54.666667	-33.501579	142.834912	0.6776045
250cm:Newspaper-150cm:Cardboard	123.666667	35.498421	211.834912	0.0004865
150cm:paper-150cm:Cardboard	74.166667	-14.001579	162.334912	0.1970611
200cm:paper-150cm:Cardboard	134.500000	46.331755	222.668245	0.0000957
250cm:paper-150cm:Cardboard	167.000000	78.831755	255.168245	0.0000005
250cm:Cardboard-200cm:Cardboard	37.500000	-50.668245	125.668245	0.9729079
150cm:Newspaper-200cm:Cardboard	-55.333333	-143.501579	32.834912	0.6594468
200cm:Newspaper-200cm:Cardboard	-1.666667	-89.834912	86.501579	1.0000000
250cm:Newspaper-200cm:Cardboard	67.333333	-20.834912	155.501579	0.3357055
150cm:paper-200cm:Cardboard	17.833333	-70.334912	106.001579	0.9999913
200cm:paper-200cm:Cardboard	78.166667	-10.001579	166.334912	0.1380160
250cm:paper-200cm:Cardboard	110.666667	22.498421	198.834912	0.0030698
150cm:Newspaper-250cm:Cardboard	-92.833333	-181.001579	-4.665088	0.0295166
200cm:Newspaper-250cm:Cardboard	-39.166667	-127.334912	49.001579	0.9612762
250cm:Newspaper-250cm:Cardboard	29.833333	-58.334912	118.001579	0.9967871
150cm:paper-250cm:Cardboard	-19.666667	-107.834912	68.501579	0.9999708
200cm:paper-250cm:Cardboard	40.666667	-47.501579	128.834912	0.9479964
250cm:paper-250cm:Cardboard	73.166667	-15.001579	161.334912	0.2143526
200cm:Newspaper-150cm:Newspaper	53.666667	-34.501579	141.834912	0.7043337
250cm:Newspaper-150cm:Newspaper	122.666667	34.498421	210.834912	0.0005632
150cm:paper-150cm:Newspaper	73.166667	-15.001579	161.334912	0.2143526
200cm:paper-150cm:Newspaper	133.500000	45.331755	221.668245	0.0001116
250cm:paper-150cm:Newspaper	166.000000	77.831755	254.168245	0.0000006
250cm:Newspaper-200cm:Newspaper	69.000000	-19.168245	157.168245	0.2975565
150cm:paper-200cm:Newspaper	19.500000	-68.668245	107.668245	0.9999736
200cm:paper-200cm:Newspaper	79.833333	-8.334912	168.001579	0.1179277
250cm:paper-200cm:Newspaper	112.333333	24.165088	200.501579	0.0024430
150cm:paper-250cm:Newspaper	-49.500000	-137.668245	38.668245	0.8062560
200cm:paper-250cm:Newspaper	10.833333	-77.334912	99.001579	1.0000000
250cm:paper-250cm:Newspaper	43.333333	-44.834912	131.501579	0.9169831
200cm:paper-150cm:paper	60.333333	-27.834912	148.501579	0.5193513
250cm:paper-150cm:paper	92.833333	4.665088	181.001579	0.0295166
250cm:paper-200cm:paper	32.500000	-55.668245	120.668245	0.9925256

$`Material:Size`

	diff	lwr	upr	p adj
Baking Paper:29.70 x 11.00-Alumininum:29.70 x 11.00	67.888889	0.9692501	134.808528	0.0440089
Cardboard:29.70 x 11.00-Alumininum:29.70 x 11.00	-71.111111	-138.0307499	-4.191472	0.0283846
Newspaper:29.70 x 11.00-Alumininum:29.70 x 11.00	-81.666667	-148.5863055	-14.747028	0.0058866
paper:29.70 x 11.00-Alumininum:29.70 x 11.00	84.000000	17.0803612	150.919639	0.0040558
Alumininum:42.00 x 15.55-Alumininum:29.70 x 11.00	-6.777778	-73.6974166	60.141861	0.9999990
Baking Paper:42.00 x 15.55-Alumininum:29.70 x 11.00	77.333333	10.4136945	144.252972	0.0114953
Cardboard:42.00 x 15.55-Alumininum:29.70 x 11.00	41.111111	-25.8085277	108.030750	0.5967885
Newspaper:42.00 x 15.55-Alumininum:29.70 x 11.00	71.111111	4.1914723	138.030750	0.0283846
paper:42.00 x 15.55-Alumininum:29.70 x 11.00	36.333333	-30.5863055	103.252972	0.7486199
Cardboard:29.70 x 11.00-Baking Paper:29.70 x 11.00	-139.000000	-205.9196388	-72.080361	0.0000001
Newspaper:29.70 x 11.00-Baking Paper:29.70 x 11.00	-149.555556	-216.4751944	-82.635917	0.0000000
paper:29.70 x 11.00-Baking Paper:29.70 x 11.00	16.111111	-50.8085277	83.030750	0.9985722
Alumininum:42.00 x 15.55-Baking Paper:29.70 x 11.00	-74.666667	-141.5863055	-7.747028	0.0170823
Baking Paper:42.00 x 15.55-Baking Paper:29.70 x 11.00	9.444444	-57.4751944	76.364083	0.9999827
Cardboard:42.00 x 15.55-Baking Paper:29.70 x 11.00	-26.777778	-93.6974166	40.141861	0.9486167
Newspaper:42.00 x 15.55-Baking Paper:29.70 x 11.00	3.222222	-63.6974166	70.141861	1.0000000
paper:42.00 x 15.55-Baking Paper:29.70 x 11.00	-31.555556	-98.4751944	35.364083	0.8706862
Newspaper:29.70 x 11.00-Cardboard:29.70 x 11.00	-10.555556	-77.4751944	56.364083	0.9999552
paper:29.70 x 11.00-Cardboard:29.70 x 11.00	155.111111	88.1914723	222.030750	0.0000000
Alumininum:42.00 x 15.55-Cardboard:29.70 x 11.00	64.333333	-2.5863055	131.252972	0.0695714

Baking Paper:42.00 x 15.55-Cardboard:29.70 x 11.00	148.444444	81.5248056	215.364083	0.0000000
Cardboard:42.00 x 15.55-Cardboard:29.70 x 11.00	112.222222	45.3025834	179.141861	0.0000268
Newspaper:42.00 x 15.55-Cardboard:29.70 x 11.00	142.222222	75.3025834	209.141861	0.0000001
paper:42.00 x 15.55-Cardboard:29.70 x 11.00	107.444444	40.5248056	174.364083	0.0000662
paper:29.70 x 11.00-Newspaper:29.70 x 11.00	165.666667	98.7470279	232.586305	0.0000000
Alumininum:42.00 x 15.55-Newspaper:29.70 x 11.00	74.888889	7.9692501	141.808528	0.0165356
Baking Paper:42.00 x 15.55-Newspaper:29.70 x 11.00	159.000000	92.0803612	225.919639	0.0000000
Cardboard:42.00 x 15.55-Newspaper:29.70 x 11.00	122.777778	55.8581390	189.697417	0.0000035
Newspaper:42.00 x 15.55-Newspaper:29.70 x 11.00	152.777778	85.8581390	219.697417	0.0000000
paper:42.00 x 15.55-Newspaper:29.70 x 11.00	118.000000	51.0803612	184.919639	0.0000088
Alumininum:42.00 x 15.55-paper:29.70 x 11.00	-90.777778	-157.6974166	-23.858139	0.0013162
Baking Paper:42.00 x 15.55-paper:29.70 x 11.00	-6.666667	-73.5863055	60.252972	0.9999992
Cardboard:42.00 x 15.55-paper:29.70 x 11.00	-42.888889	-109.8085277	24.030750	0.5376201
Newspaper:42.00 x 15.55-paper:29.70 x 11.00	-12.888889	-79.8085277	54.030750	0.9997623
paper:42.00 x 15.55-paper:29.70 x 11.00	-47.666667	-114.5863055	19.252972	0.3849790
Baking Paper:42.00 x 15.55-Alumininum:42.00 x 15.55	84.111111	17.1914723	151.030750	0.0039837
Cardboard:42.00 x 15.55-Alumininum:42.00 x 15.55	47.888889	-19.0307499	114.808528	0.3783435
Newspaper:42.00 x 15.55-Alumininum:42.00 x 15.55	77.888889	10.9692501	144.808528	0.0105683
paper:42.00 x 15.55-Alumininum:42.00 x 15.55	43.111111	-23.8085277	110.030750	0.5302431
Cardboard:42.00 x 15.55-Baking Paper:42.00 x 15.55	-36.222222	-103.1418610	30.697417	0.7518815
Newspaper:42.00 x 15.55-Baking Paper:42.00 x 15.55	-6.222222	-73.1418610	60.697417	0.9999995
paper:42.00 x 15.55-Baking Paper:42.00 x 15.55	-41.000000	-107.9196388	25.919639	0.6004781
Newspaper:42.00 x 15.55-Cardboard:42.00 x 15.55	30.000000	-36.9196388	96.919639	0.9011123
paper:42.00 x 15.55-Cardboard:42.00 x 15.55	-4.777778	-71.6974166	62.141861	1.0000000
paper:42.00 x 15.55-Newspaper:42.00 x 15.55	-34.777778	-101.6974166	32.141861	0.7926494