



2-1 分数 4

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Given a 3-SAT formula with  $k$  clauses, in which each clause has three variables, the MAX-3SAT problem is to find a truth assignment that satisfies as many clauses as possible. A simple randomized algorithm is to flip a coin, and to set each variable true with probability  $1/2$ , independently for each variable. Which of the following statements is FALSE?

- ☐ A. The expected number of clauses satisfied by this random assignment is  $7k/8$ .
- ☐ B. For every instance of 3-SAT, there is a truth assignment that satisfies at least a  $7/8$  fraction of all clauses.
- ☐ C. The probability that a random assignment satisfies at least  $7k/8$  clauses is at most  $1/(8k)$ .
- ☐ D. If we repeatedly generate random truth assignments until one of them satisfies  $\geq 7k/8$  clauses, then this algorithm is a  $8/7$ -approximation algorithm.

2-2 分数 4

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Which of the following statement is true ?

- ☐ A. A randomized algorithm for a decision problem with one-sided-error and correctness probability  $1/3$  (that is, if the answer is YES, it will always output YES, while if the answer is NO, it will output NO with probability  $1/3$ ) can always be amplified (放大) to a correctness probability of 99%.
- ☐ B. Let  $A$  and  $B$  be optimization problems where it is known that  $A$  reduces to  $B$  in polynomial time. Additionally, suppose that there exists a polynomial-time 2-approximation for  $B$ . Then there must exist a polynomial time 2-approximation for  $A$ .
- ☐ C. There exists a polynomial-time 2-approximation algorithm for the general Traveling Salesman Problem.
- ☐ D. Suppose that you have two deterministic online algorithms,  $A_1$  and  $A_2$ , with competitive ratios (the approximation ratio for an online algorithm is called competitive ratio)  $c_1$  and  $c_2$  respectively. Consider the randomized algorithm  $A^*$  that flips a fair coin once at the beginning; if the coin comes up heads, it runs  $A_1$  from then on; if the coin comes up tails, it runs  $A_2$  from then on. Then the expected competitive ratio of  $A^*$  is at least  $\min\{c_1, c_2\}$ .

2-3 分数 4

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weight of all edges crossing from one side of the partition to the other. Here we consider the case where all edges in the graph have the same weight 1.

Suppose that a partition of  $V$  into two disjoint sets  $A$  and  $B$  is given by a randomized algorithm, in which each vertex is randomly and independently assigned to one of the two sets.

Which of the following statements is FALSE?

- ☐ A. The expected approximation ratio of the algorithm is at most 2.
- ☐ B. There exists a partition  $A$  and  $B$  with at least  $m/2$  edges connecting the set  $A$  to the set  $B$ .
- ☐ C. The probability of finding a cut with value at least  $m/2$  is less than  $2/(m+2)$ .
- ☐ D. The expected number of edges in the partition generated by the algorithm is  $m/2$ .

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2-4 分数 3

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In the maximum satisfiability problem (MAX SAT), the input consists of  $n$  Boolean variables  $x_1, \dots, x_n$ ,  $m$  clauses  $C_1, \dots, C_m$  (each of which consists of a disjunction (that is an “or”) of some number of the variables and their negations, e.g.  $x_3 \vee \bar{x}_5 \vee x_{11}$ , where  $\bar{x}_i$  is the negation of  $x_i$ ), and a nonnegative weight  $w_j$  for each clause  $C_j$ . The objective of the problem is to find an assignment of the true/false to the  $x_i$  that maximizes the weight of the satisfied clauses.

A variable or a negated variable is a literal. The number of literals in a clause is called its length. Denote  $l_j$  to be the length of a clause  $C_j$ . Clauses of length 1 are called unit clauses.

**Randomized algorithm RA:** Setting each  $x_i$  to true with probability  $p$  independently.

Which of the following statement is false?

- ☐ A. Let  $p = 1/2$ , the randomized algorithm RA is a 2-approximation algorithm.
- ☐ B. If  $l_j \geq 3$  for each clause  $C_j$ . Let  $p = 1/2$ , the randomized algorithm RA is a  $9/8$ -approximation algorithm.
- ☐ C. If MAX SAT instances do not have unit clauses  $\bar{x}_i$ , we can obtain a randomized  $\frac{2}{\sqrt{5}-1} \approx 1.618$ -approximation algorithm for MAX SAT.
- ☐ D. One could obtain a better bound on optimal solution than  $\sum_{j=1}^m w_j$  for MAX SAT.

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2-5 分数 2

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Following is an EREW PRAM algorithm to solve the prefix-min problem.

```
1  for i , 1 ≤ i ≤ n pardo
2    B(0, i) = A(i)
3  for h = 1 to log n
4    for i , 1 ≤ i ≤ n/2h pardo
5      B(h, i) = min {B(h - 1, 2i - 1), B(h - 1, 2i)}
```

```

6   for h = log n to 0
7     for i even, 1 ≤ i ≤ n/2^h pardo
8       C(h, i) = C(h + 1, i/2)
9     for i = 1 pardo
10      C(h, 1) = B(h, 1)
11    for i odd, 3 ≤ i ≤ n/2^h pardo
12      C(h, i) = min {C(h + 1, (i - 1)/2), B(h, i)}
13    for i, 1 ≤ i ≤ n pardo
14      Output C(0, i)
15

```

This algorithm runs in \_\_ work?

- ☐ A.  $O(1)$
- ☐ B.  $O(N \log N)$
- ☐ C.  $O(\log N)$
- ☐ D.  $O(N)$

2-6 分数 3

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The following psuedo-code is for solving the Prefix Sums problem parallelly, where the input  $N$  numbers are stored in the array **A**. Which of the following gives the time and work load of the algorithm?

```

for Pi, 1 ≤ i ≤ n pardo
  B(0, i) := A(i)
for h = 1 to log(n)
  for i, 1 ≤ i ≤ n/(2^h) pardo
    B(h, i) := B(h-1, 2*i-1) + B(h-1, 2*i)
for h = log(n) to 0
  for i even, 1 ≤ i ≤ n/(2^h) pardo
    C(h, i) := C(h+1, i/2)
  for i = 1 pardo
    C(h, 1) := B(h, 1)
  for i odd, 3 ≤ i ≤ n/(2^h) pardo
    C(h, i) := C(h+1, (i-1)/2) + B(h, i)
for Pi, 1 ≤ i ≤ n pardo
  Output C(0, i)

```

- ☐ A.  $T(n) = O(\log n), W(n) = O(n \log n)$
- ☐ B.  $T(n) = O(n), W(n) = O(n)$
- ☐ C.  $T(n) = O(\log n), W(n) = O(n)$
- ☐ D.  $T(n) = O(n), W(n) = O(n \log n)$

2-7 分数 2

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When measure the performance of parallel algorithm, we often use work load ( $W(n)$ ) and worst-case running time ( $T(n)$ ). How many evaluation metrics are asymptotically equivalent to  $W(n)$  and  $T(n)$ ?

- $P(n) = W(n)/T(n)$  processors and  $T(n)$  time (on a PRAM)
- $W(n)/p$  time using any number of  $p \geq W(n)/T(n)$  processors (on a PRAM)
- $W(n)/p + T(n)$  time using any number of  $p$  processors (on a PRAM)

☐ A. 0

☐ B. 1

☐ C. 2

☐ D. 3

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2-8 分数 2

作者 bjj 单位 浙江大学

Which one of the following statements about the Ranking problem is TRUE? (Assume that both arrays contain  $N$  elements)

- ☐ A. Serial ranking algorithm has better time complexity comparing with the binary search algorithm.
- ☐ B. Using binary search algorithm to solve the problem will make the time complexity be  $O(N \log N)$
- ☐ C. It can be used in merging problem and make the merging problem solved in  $O(1)$  time.
- ☐ D. Parallel binary search algorithm will solve the problem in  $O(\log N)$  time with  $O(\log N)$  work load.

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2-9 分数 2

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Given 500 runs and 10 tapes. If simple k-way merges are used, what is the minimum number of passes we have to do?

☐ A. 5

☐ B. 6

☐ C. 7

☐ D. 8

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Given input { 25, 61, 5, 13, 32, 10, 30, 28, 69, 58, 70, 1, 21, 8, 16, 3, 42, 2 }. Suppose that the internal memory can handle  $M = 4$  records at a time. If the replacement selection is used, then  runs will be generated, and the longest run contains  records.

- ☐ A. 5, 4
  - ☐ B. 4, 6
  - ☐ C. 4, 8
  - ☐ D. 3, 9
- 

For a  $k$ -way merge in external sorting, the primary reason for  $k$  not assuming a large value is that:

- ☐ A.  $k$  has to be a finite integer
  - ☐ B.  $k$  is bounded above by the number of runs
  - ☐ C. during merging, the number of comparisons would increase
  - ☐ D. the I/O time would increase
- 

To sort  $N$  numbers by external sorting using a  $k$ -way merge and a  $k$ -size heap, which statement is TRUE about the total comparison times  $T(N, k)$  and  $k$ ?

- ☐ A.  $T(N, k)$  has nothing to do with  $k$ .
  - ☐ B.  $T(N, k)$  is  $O(k)$  for fixed  $N$ .
  - ☐ C.  $T(N, k)$  is  $O(k \log k)$  for fixed  $N$ .
  - ☐ D.  $T(N, k)$  is  $O(k^2)$  for fixed  $N$ .
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