

Report

Question: Neural Network Approximation of $f(x) = \frac{1}{1+25x^2}$

1. Method:

In this project, we approximate the function:

$$f(x) = \frac{1}{1+25x^2} , \quad x \in [-1, 1]$$

using a simple fully connected feedforward neural network

implemented in PyTorch. The main steps are as follows:

(1) Data Preparation:

- Training data: 1000 evenly spaced points in $[-1, 1]$.
- Test data: 200 evenly spaced points in $[-1, 1]$.
- Dataset class converts numpy arrays to PyTorch tensors on the selected device (CPU or GPU).

(2) Network Architecture:

- Input layer: 1 neuron
- Hidden layer: 7 neurons, activation tanh
- Output layer: 1 neuron

(3) Training:

- Loss function: Mean Squared Error (MSE)
- Optimizer: Adam, learning rate 0.01
- Number of epochs: 2000

2. Results

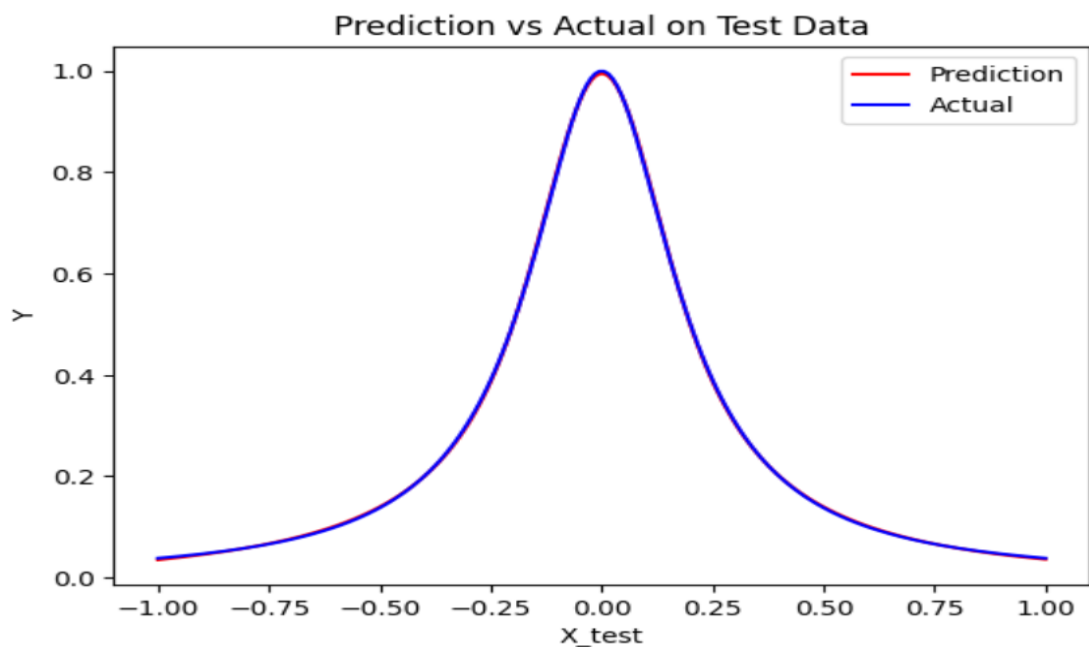
2.1 Training Loss Curve

The training loss decreases over epochs, showing convergence:



2.2 Function Approximation

The trained model predicts the function closely on the test set:



2.3 Test Error

test loss $4.6671989366586786 \times 10^{-6}$