Report

Question: Neural Network Approximation of f(x) = $\frac{1}{1+25x^2}$

1. **Method:**

   In this project, we approximate the function:

   $$f(x) = \frac{1}{1+25x^2} \ , \ x \in [-1, 1]$$

   together with its derivative:

   $$f'(x) = -\frac{50x}{(1+25x^2)^2}$$

   using a feedforward neural network trained with PyTorch.

   The model architecture consists of:

   - Input layer: 1 neuron
   - Hidden layer: 7 neurons, activation tanh
   - Output layer: 1 neuron

   The network is trained not only to approximate f(x), but also to match its derivative f′(x).

   This is achieved by combining two losses:

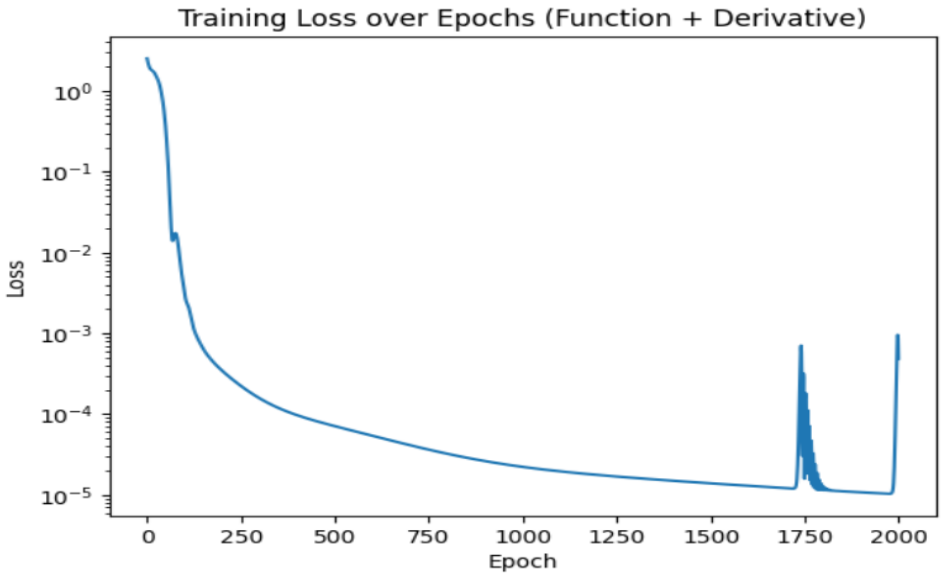
   $$\mathcal{L} = \mathrm{MSE}(\mathrm{f}_{\mathrm{pred}}, \mathrm{f}) + \mathrm{MSE}(\mathrm{f'}_{\mathrm{pred}}, \mathrm{f'})$$

   Training uses 1000 points, testing uses 200 points. Optimization is performed using Adam (η=0.01) for 2000 epochs.

2. **Results**

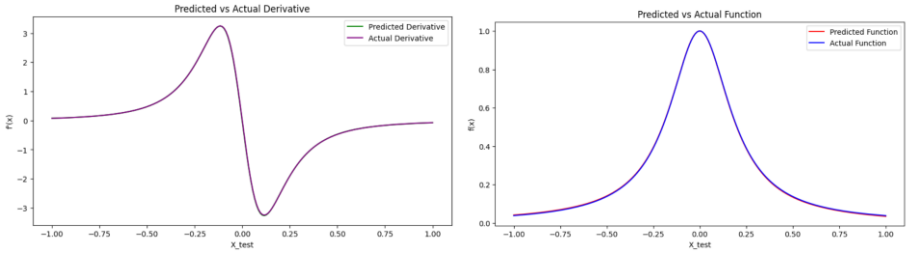
2.1 Training Loss Curve

The training loss decreases over epochs, showing convergence:



2.2 Function Approximation

The trained model predicts the function closely on the test set:



2.3 Test Error

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Test Function Loss: 9.80526783678215e-06
Test Derivative Loss: 9.93422872852534e-05
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