

- definition of force – slide 1**
- revisiting vectors: components of forces – slide 7**
- adding vectors in 2D:
Tail to head method – Algebraic method – slide 10**
- types of forces: weight, normal force, tension – slide 27**
- static equilibrium – slide 46**
- frictional force – slide 51**
- coefficient of friction – slide 64**
- rotational equilibrium – slide 66**

A force is a push or pull. A force causes an object to change its state of motion:

- get it moving
- stopping it
- slowing down or speeding it up
- changing its direction (making it turn)

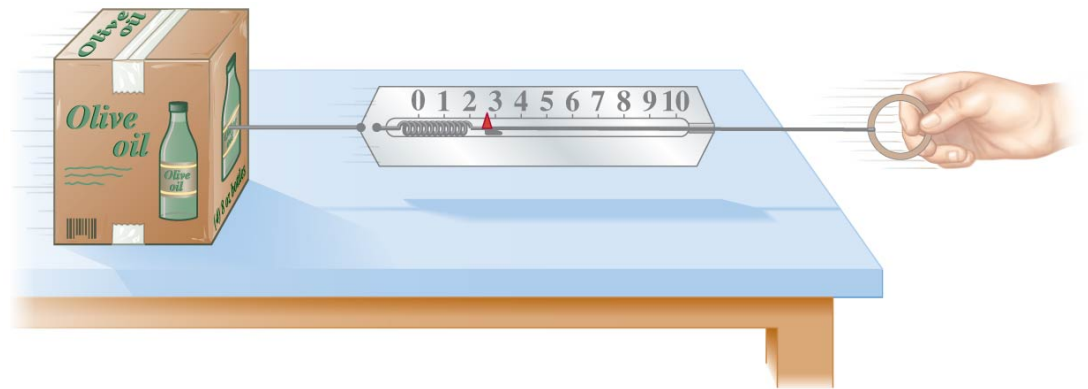


Force → change of velocity → acceleration

(Newton's 2nd law : $F = m a$ or acceleration is the force per unit mass)

We will see that once the object is moving, it will keep moving (with the same velocity) without push/pull . Unless there is friction which is a force.

The magnitude of a force can be measured using a spring scale.



Units of Force

- SI unit of force is a Newton (N)

$$1 \text{ N} \equiv 1 \frac{\text{kg m}}{\text{s}^2}$$

- US Customary unit of force is a pound (lb)
 - $1 \text{ N} = 0.225 \text{ lb}$

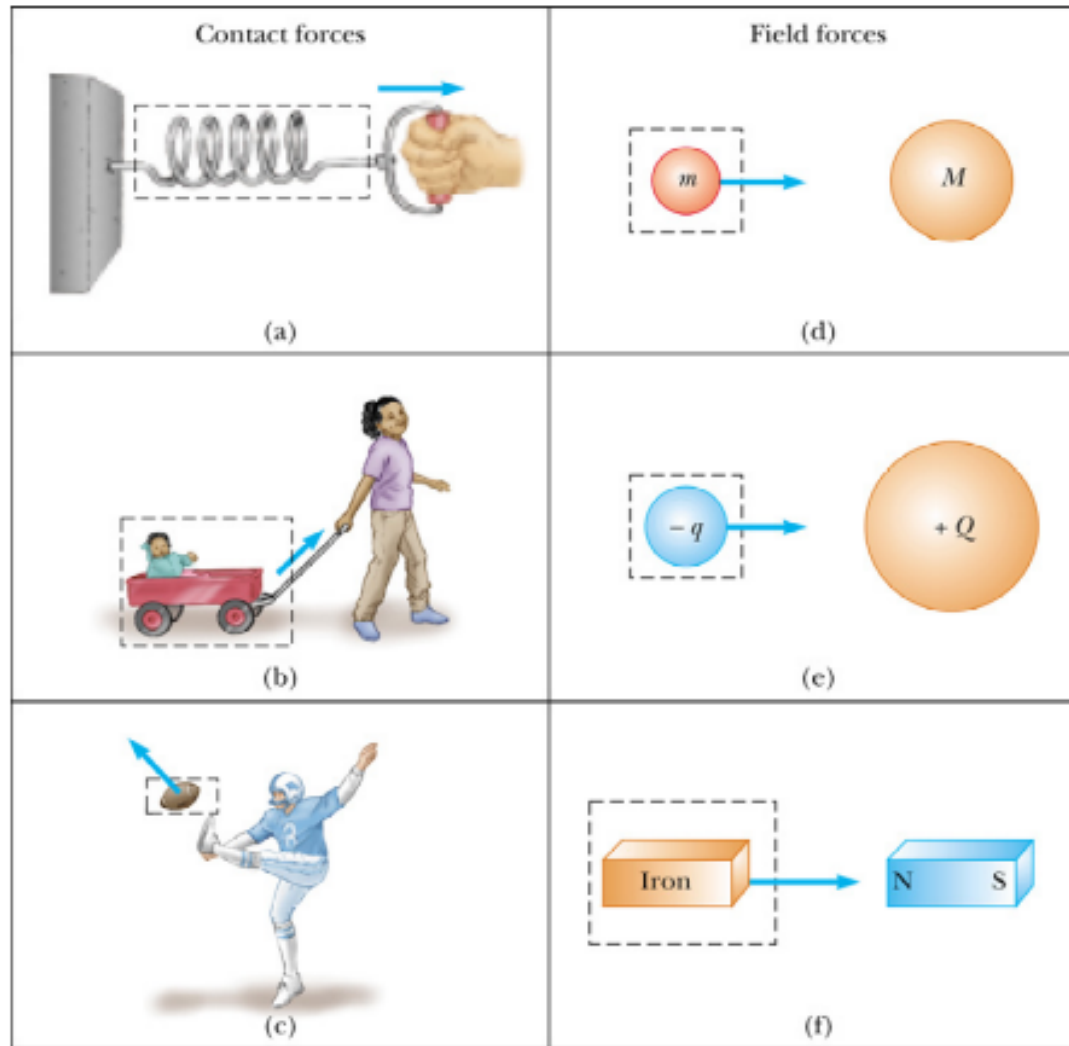
10 N is about 2 pounds which is about 1 kg
(to go from newtons to pounds divide by 5)

A ***force*** is a push or a pull.

Contact forces arise from physical contact .

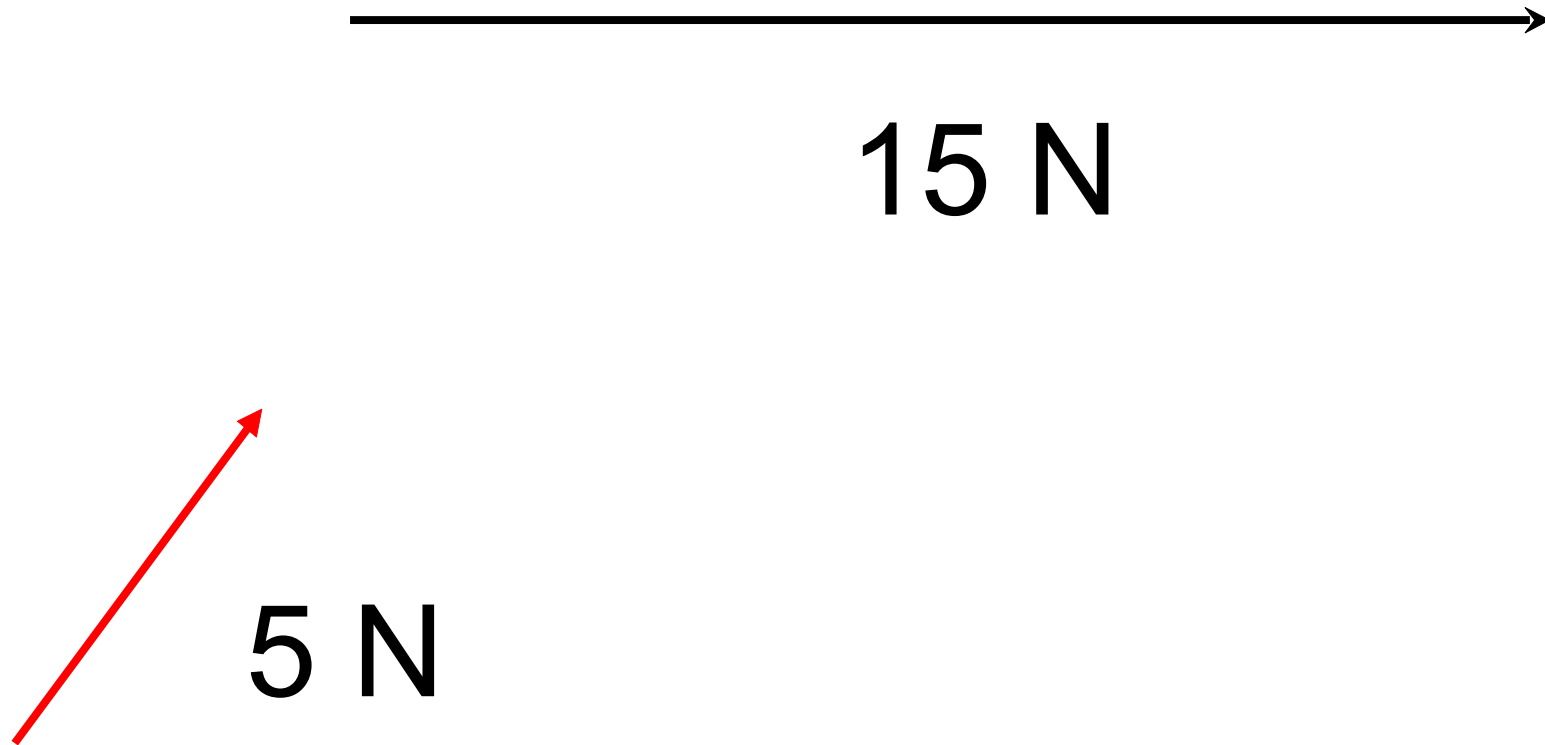
Action-at-a-distance forces do not require contact and include gravity and electrical forces.

Contact and Field Forces



Forces are vectors.

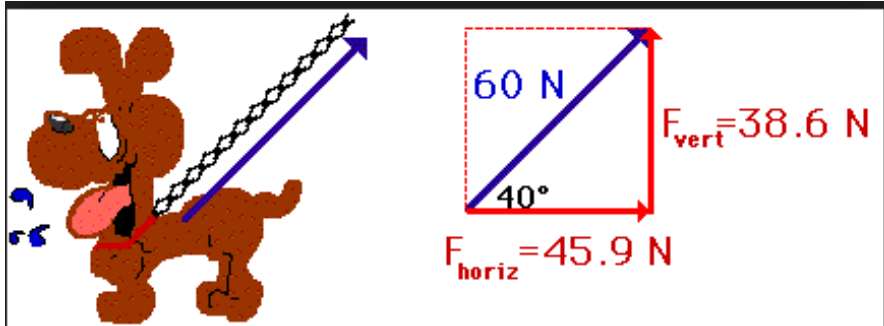
Forces have a magnitude (how much) and direction (which way)



REMEMBER VECTORS COME WITH COMPONENTS !!

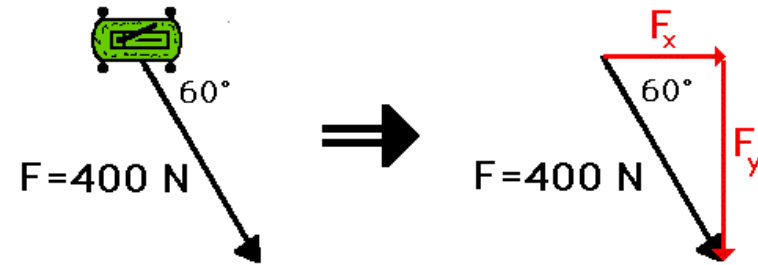
Components are independent from each other and have “different” job.

Source: <http://www.physicsclassroom.com/class/vectors/Lesson-3/Resolution-of-Forces>



Find the components of the force
Pulling the dog through the leash.

What is the “job “ of each component



Find the components of the force
ff the wind pushing the sail.

What is the “job “ of each component
($F_x = 200 \text{ N}$, $F_y = -173 \text{ N}$)

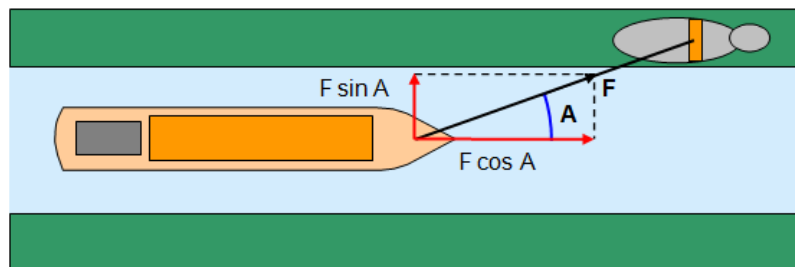
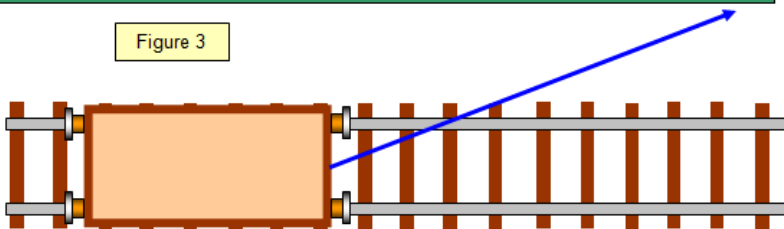
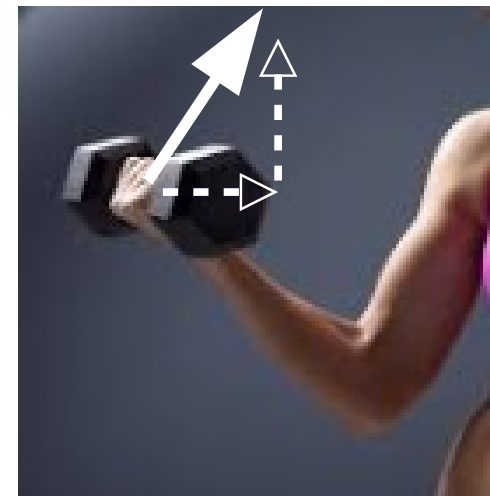


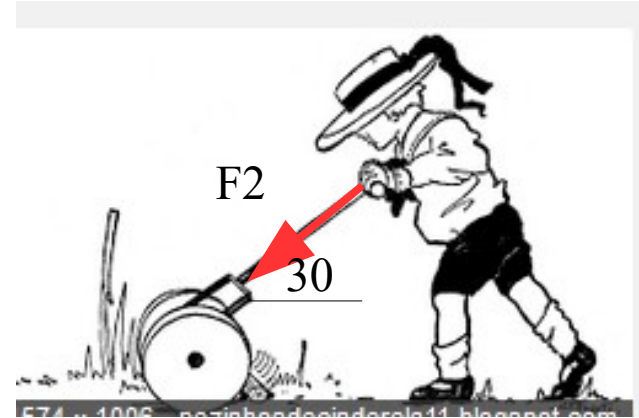
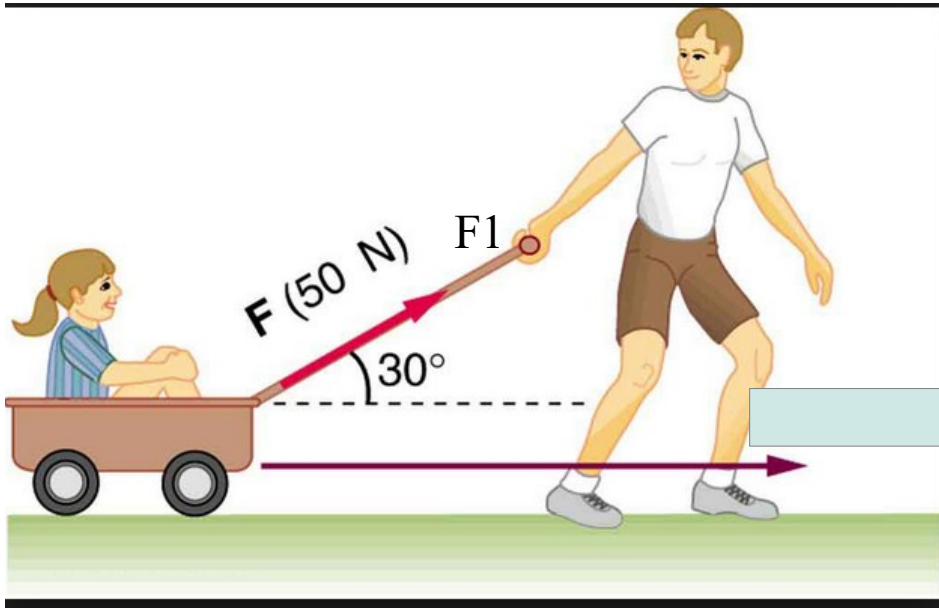
Figure 3



Describe in each
Case, what is the “job
Of each component.



If the vector (force) is in standard notation → cosine gives the x-component
→ sine gives the y-component



Draw each vector in a free-body diagram (x-y coordinate system). The tail is attached to the origin. Compute the components of the vectors. The magnitude of the force is the same but not its direction.

$F_1 = 50\text{N @ } \underline{\hspace{2cm}}$ but $F_2 = 50\text{N @ } \underline{\hspace{2cm}}$ (use standard notation)

The components are not the same : $F_1 (\quad , \quad)$ $F_2 (\quad , \quad)$

Describe what the x-component and the y-component are doing to each object.

The y-component of F_1 is the wagon with a force of .

The y-component of F_2 is

If there is friction. Which way is the best to pull with less effort ?

$F_1(43,25)$ $F_2(-43,-25)$

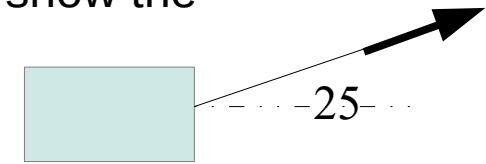
In a free-body diagram you find the components of the forces.
The x-axis is independent from the y-component

1) you are dragging a crate and the rope makes a 25 degrees angle with horizontal.

The tension in the rope is 250N. Make a free-body diagram and show the force.

A) How much force is being used to drag the crate ?
(find the horizontal component)

B) How much force (from you) is pulling the crate upward ?



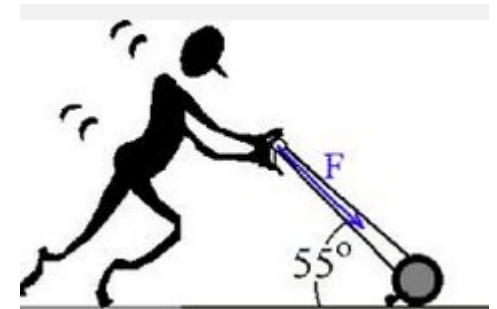
Solution: $F(226.6, 105.6)$

3) You push a lawn mower with a force of 160N, exerted directly along its shaft.

The shaft makes an angle of 55 degrees with the ground. Make a free body-diagram

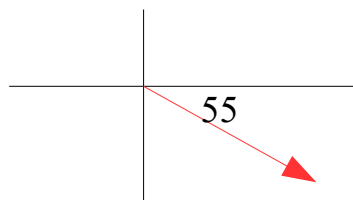
A) How much force is moving the lawn mower (along horizontal)

B) how much force is pushing the lawn mower toward the ground.



Solution:

$F(92, -131)$

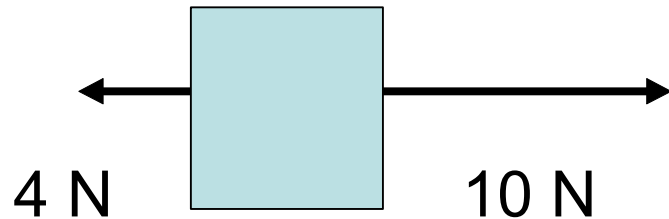


Adding vectors

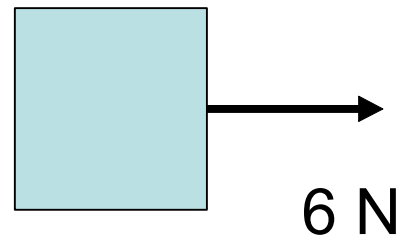
The net force on an object is the vector sum of all forces acting on that object.

The SI unit of force is the Newton (N).

Individual Forces

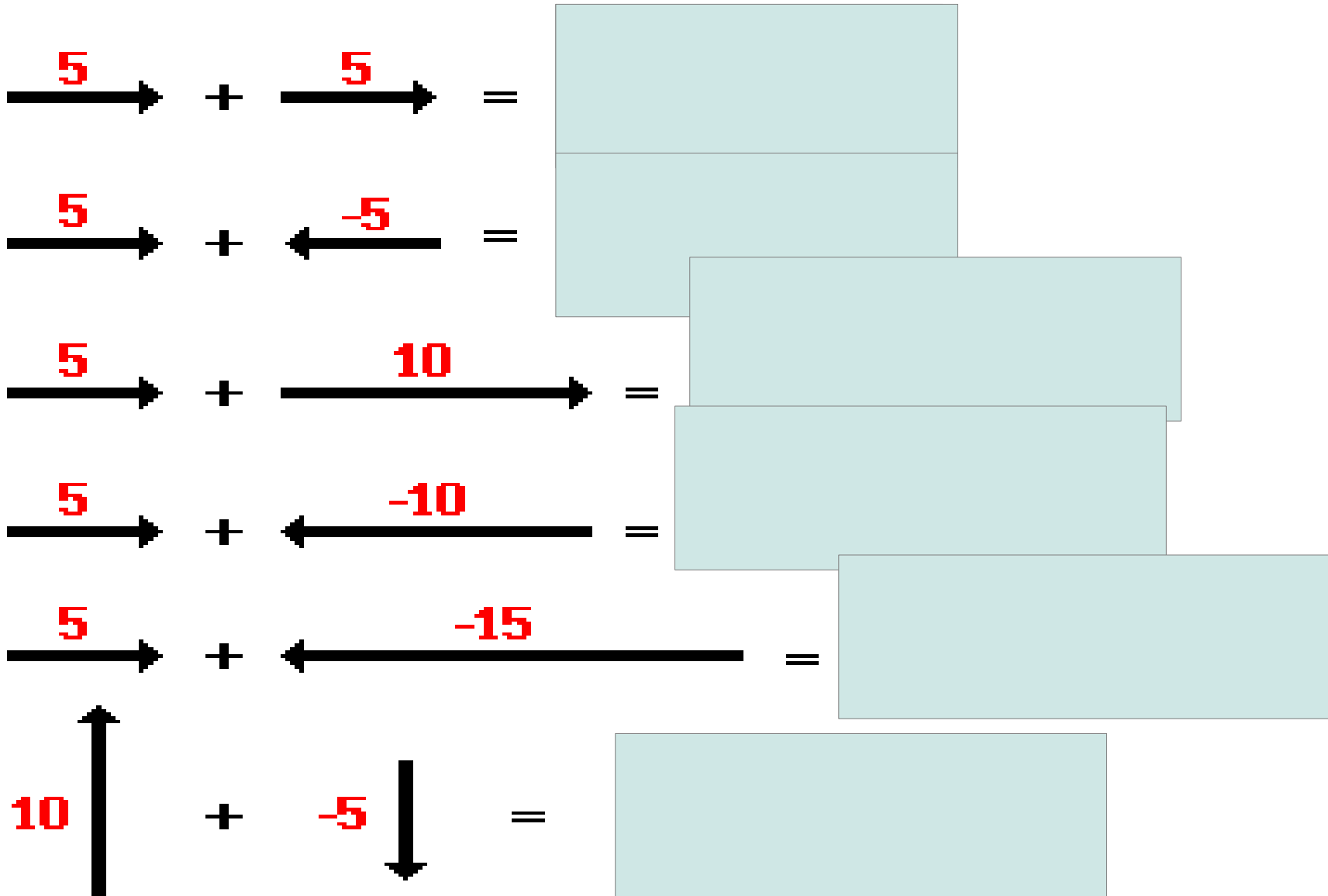


Net Force



Adding forces in 1D is easy:

The *net force (sum)* is the vector sum of all of the forces acting on an object.



Source:

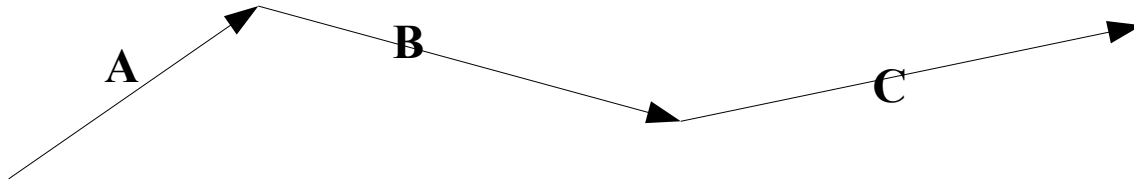
Mathematically, the net force is written as

$$\sum \mathbf{F}$$

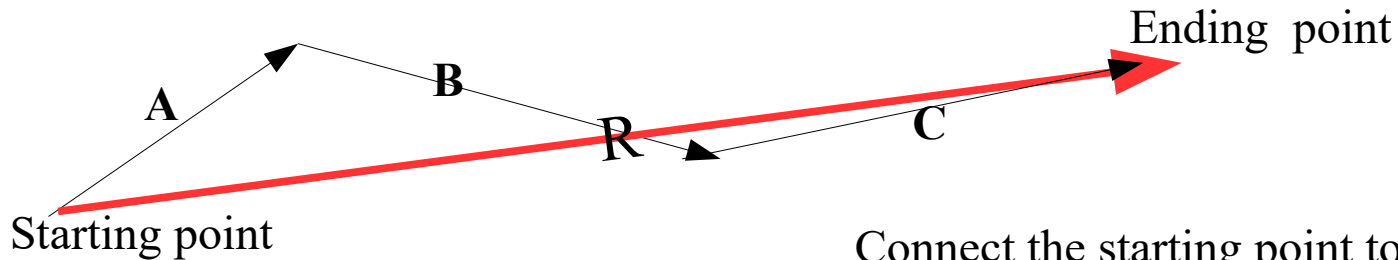
where the Greek letter sigma denotes the vector sum.

How to add vectors: method head to tail (tail to head)

Example: add the vectors A, B, C

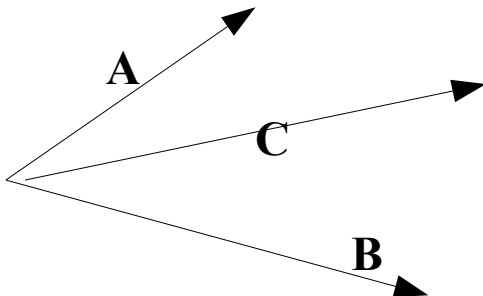


$A+B+C = R$ R is the vector sum (or resultant)

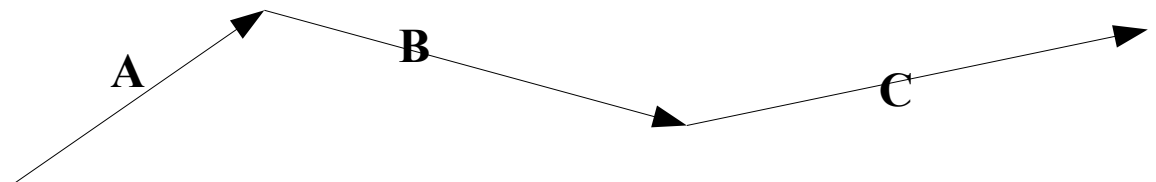


Connect the starting point to ending point
The vectors are tail to head (head to tail)

If the vectors are not tail to head then move them tail to head



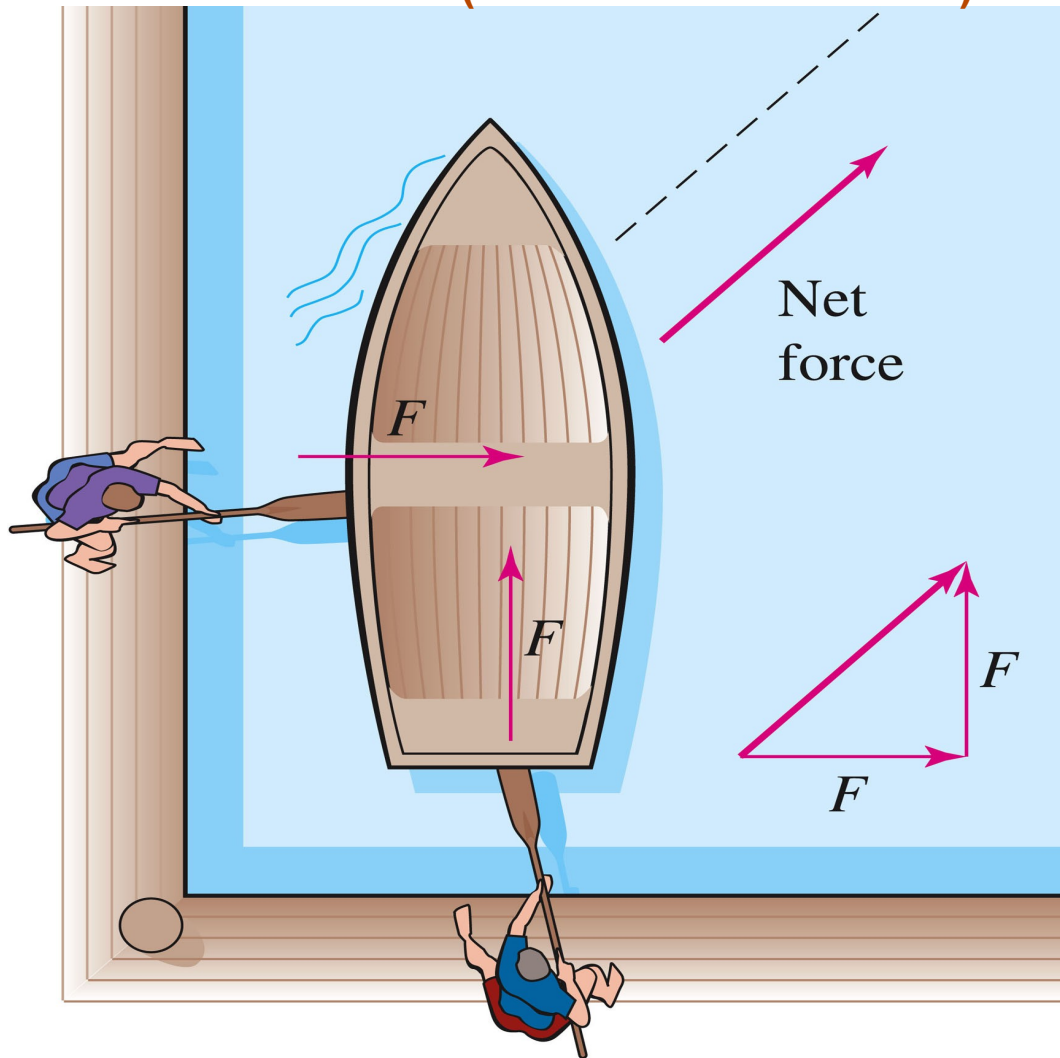
Move the tail of B on head of A and the tail of C on head of B



Try: <http://www.physicsclassroom.com/class/vectors/Lesson-1/Vector-Addition>

Special case: ITS EASY TO ADD VECTORS IF THEY ARE PERPENDICULAR

**In which direction do you think the boat will move ? =
Draw the resultant F_{net} (a vector). The boat will move in the same
Direction as F_{net} . (Newton's second law)**

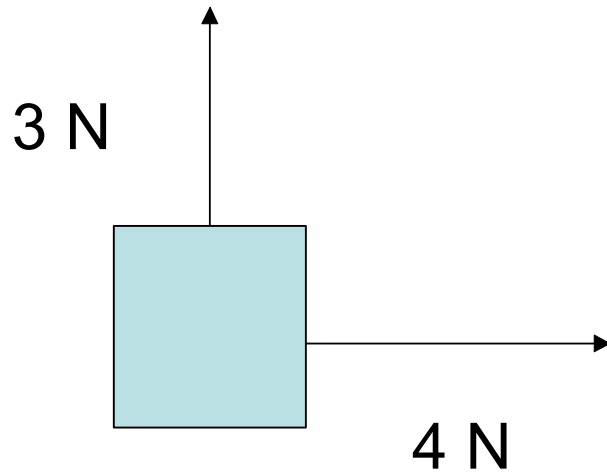


If $|F| = 100\text{N}$
Find R or F_{net}

First draw a free-body
Diagram :attach the tails
Of the forces at the origin.
The boat is reduced to
The origin.

Adding forces in 2D is easy if the individual forces are perpendicular.

Individual Forces



Net Force

FIND THE NET
FORCE.

DRAW it
Find direction and
Magnitude.

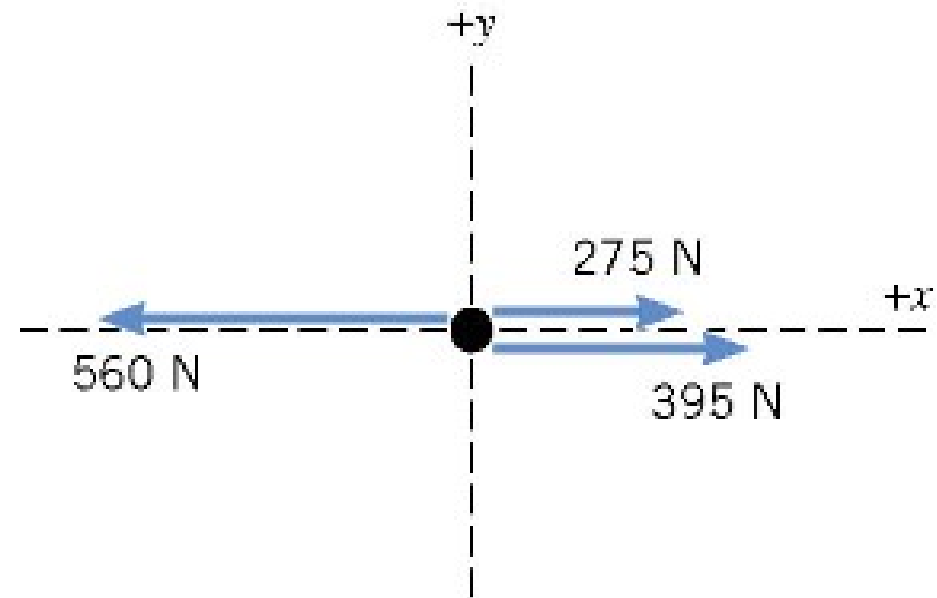
(add 3N@ right and 4N@ up)

A ***free-body-diagram*** is a diagram that represents the object and the forces that act on it.

Find the net force



(a)

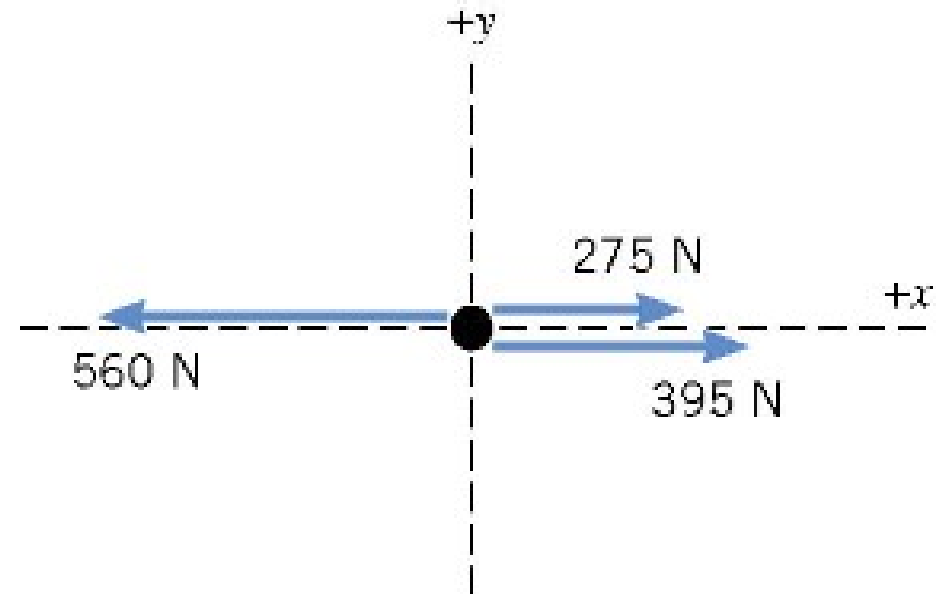


(b) Free-body diagram of the car

4.3 Newton's Second Law of Motion



(a)



(b) Free-body diagram of the car

The net force in this case is:

$$275 \text{ N} + 395 \text{ N} - 560 \text{ N} = +110 \text{ N}$$

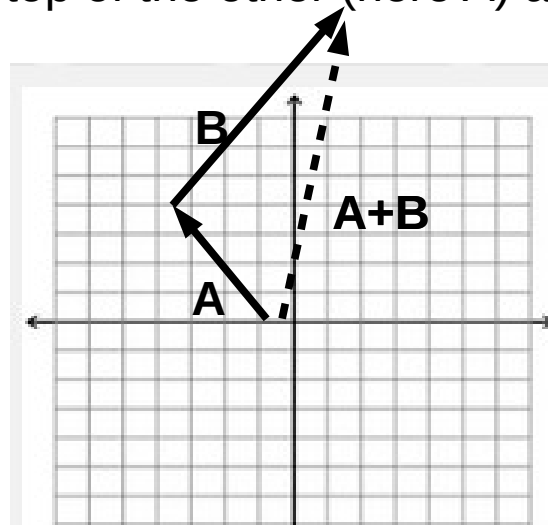
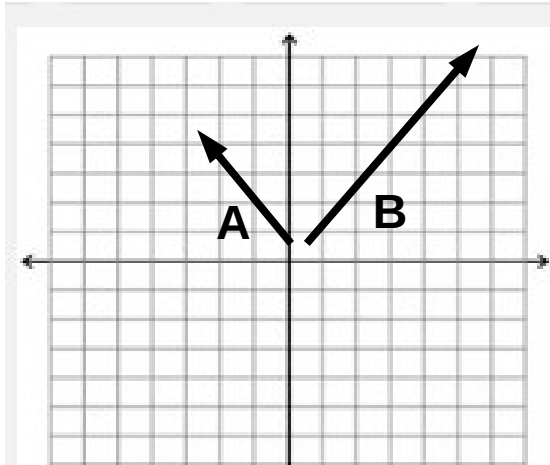
and is directed along the + x axis of the coordinate system.

Which one of the following tools is useful in representing the forces acting on an object and simplifies problem solving?

- a) free-body diagram
- b) scalar drawing
- c) vector analyzer
- d) Newton's ladder
- e) force monitor

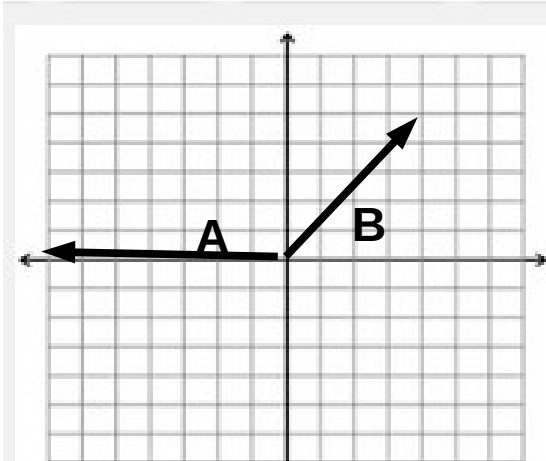
adding vectors in 2D geometrically

How to trace the vector sum of 2 vectors (or more) that are not perpendicular:
Place one vector (here B) on top of the other (here A) and connect the first tail to last head.

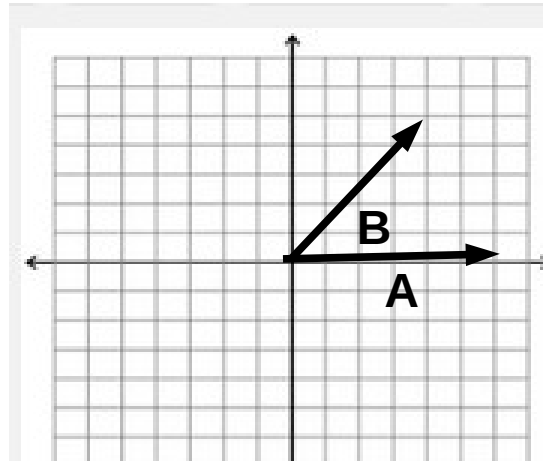


<https://phet.colorado.edu/en/simulation/vector-addition>

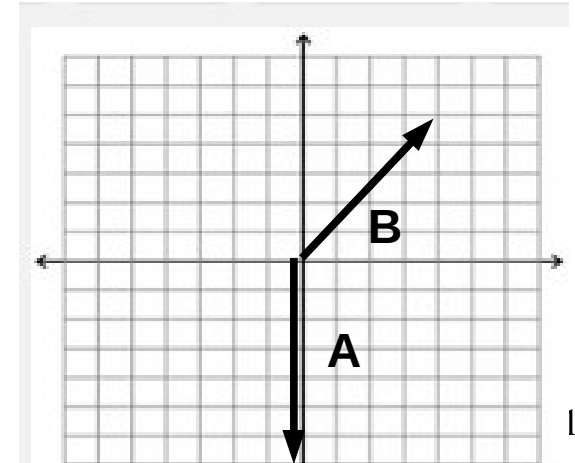
Below add A+B



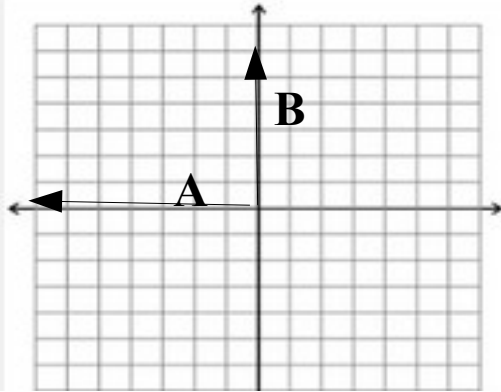
Below add A+B



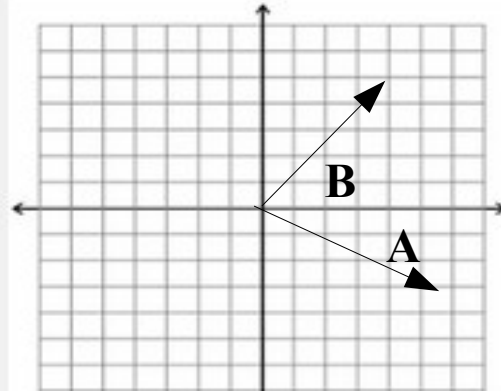
Below add A+B



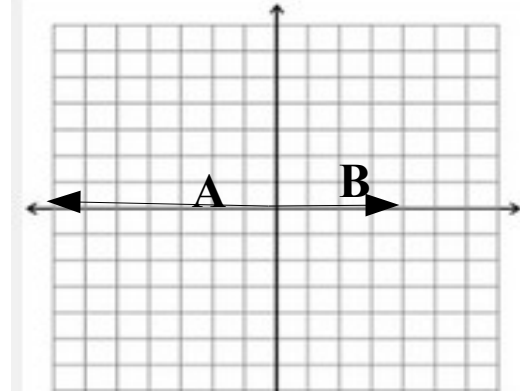
Below add $A+B$



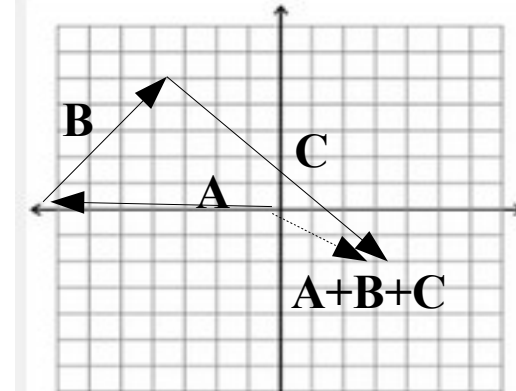
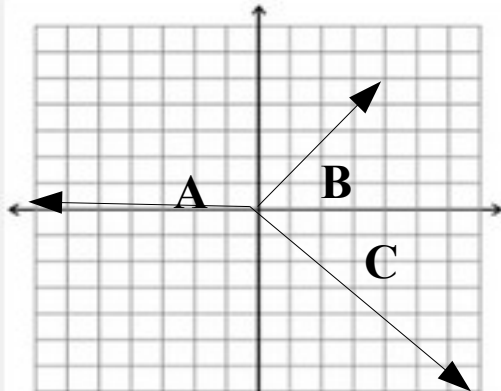
Below add $A+B$



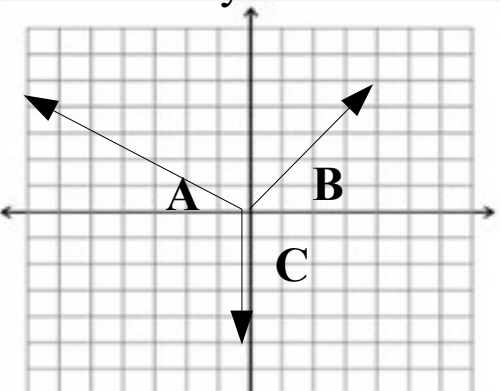
Below add $A+B$



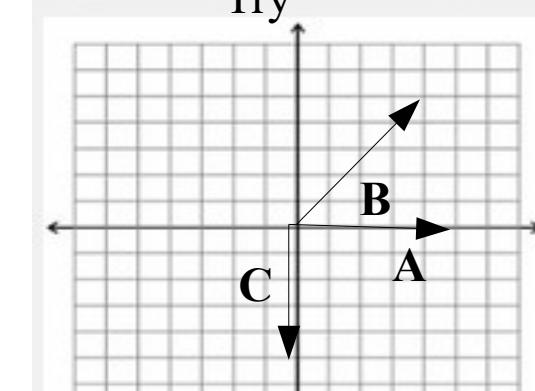
You can add more than 2 vectors :



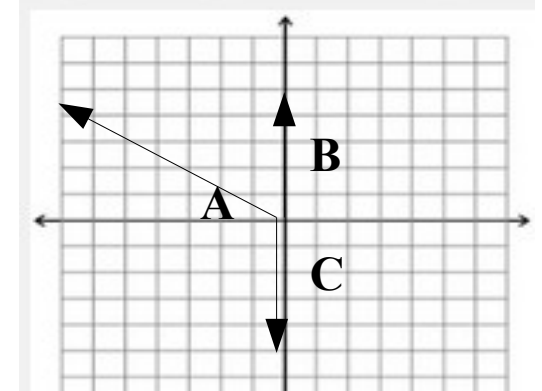
Try



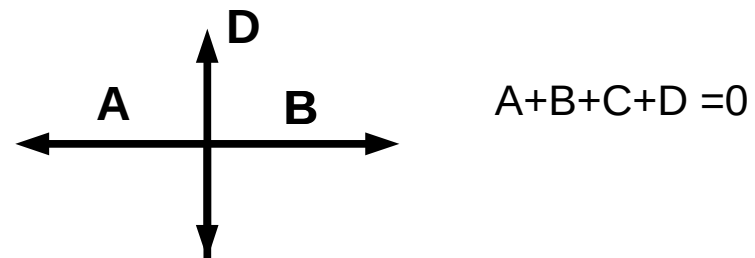
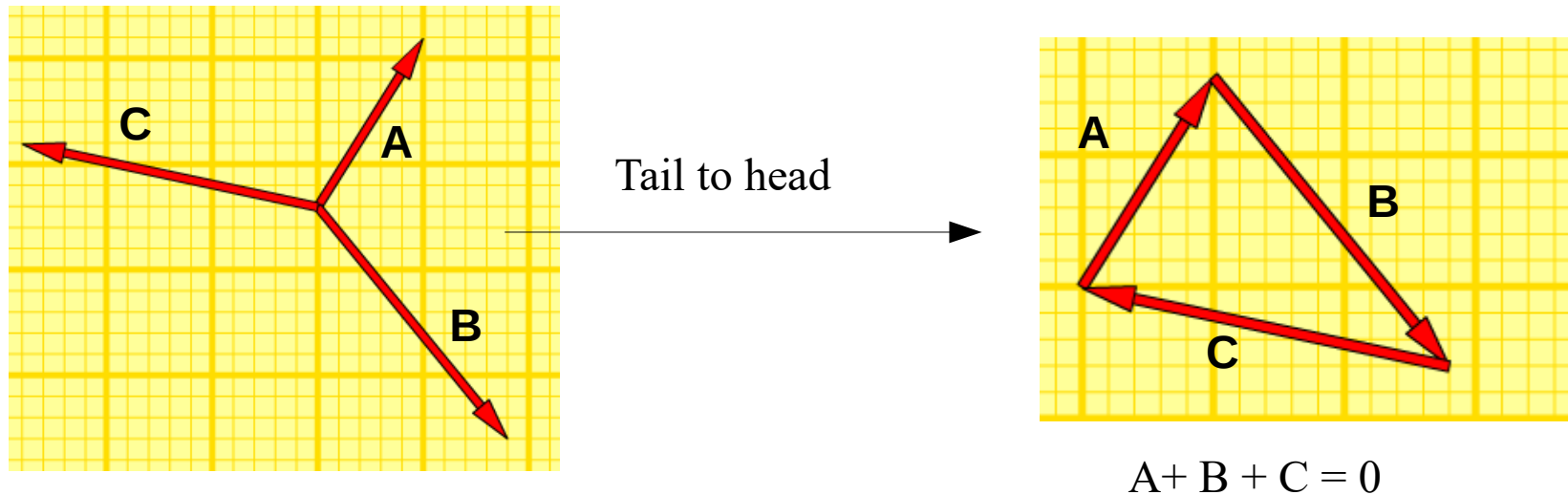
Try



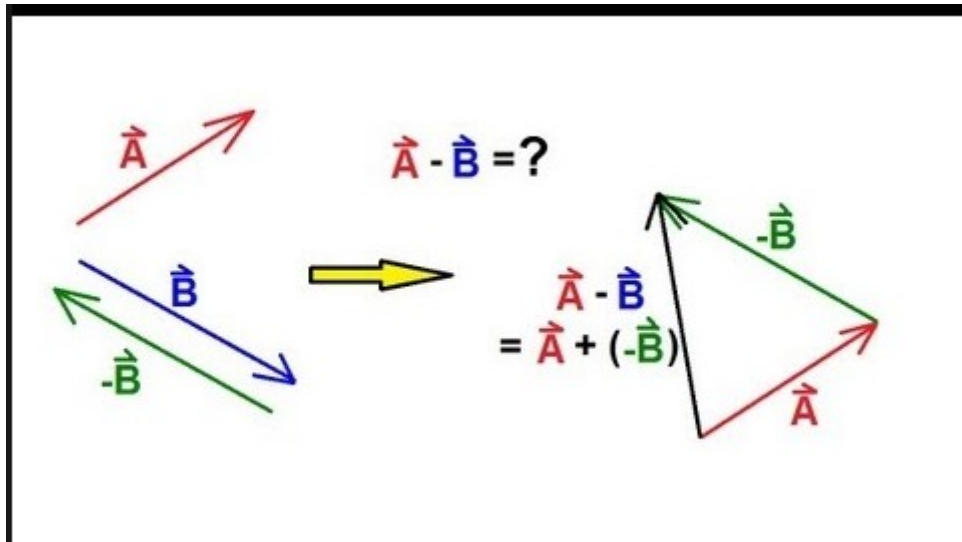
Try. Hint: B and C have the same magnitude.



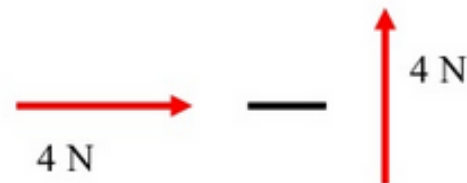
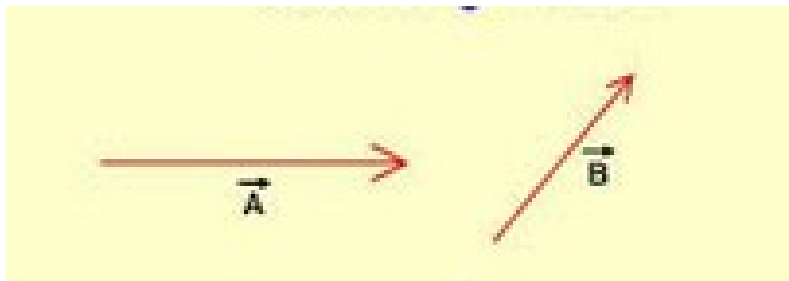
The vectors add up to zero
when the starting point = ending point
using the tail to head method/



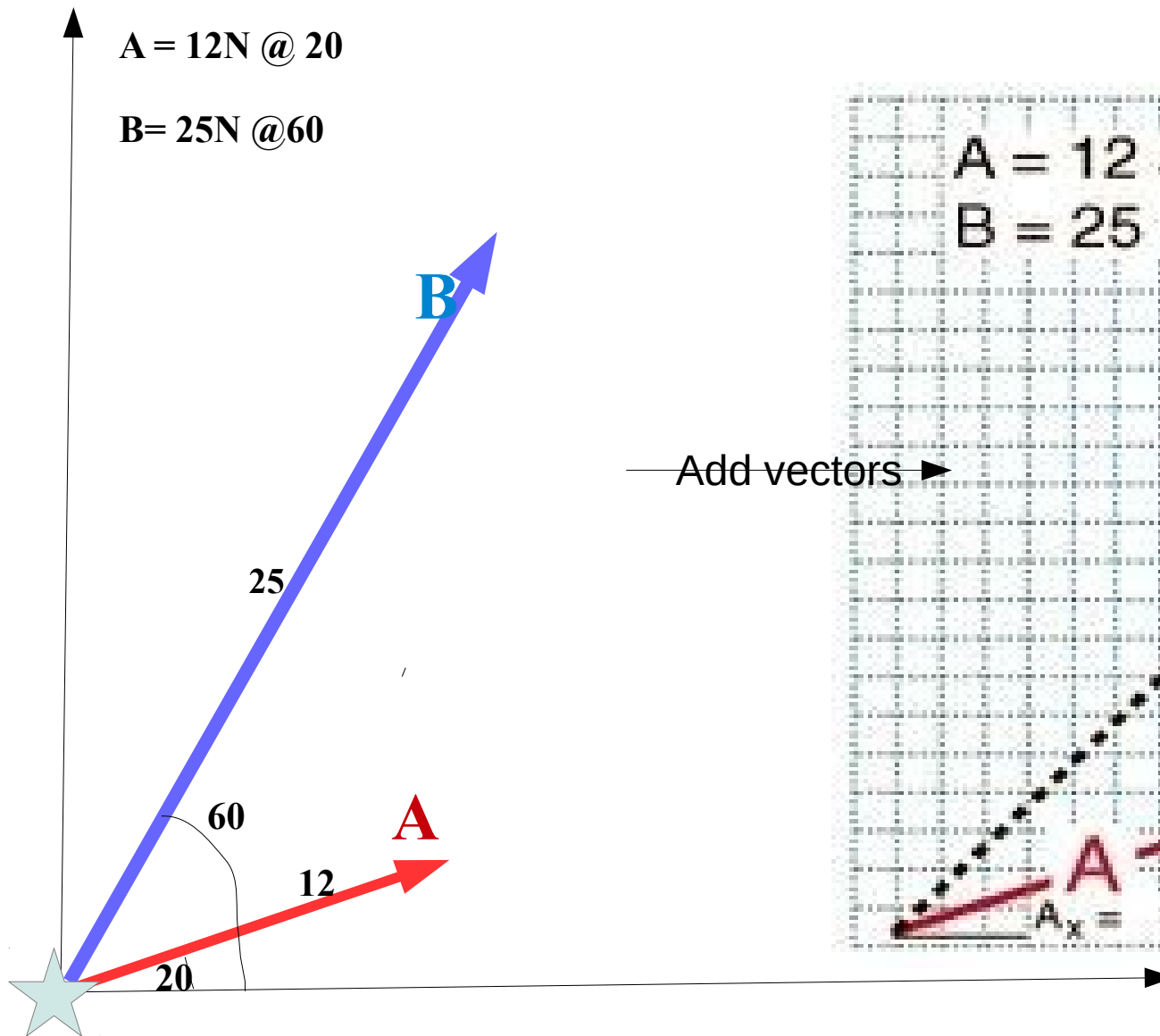
Optional: When you subtract vectors, you add an opposite. $A - B = A + (-B)$



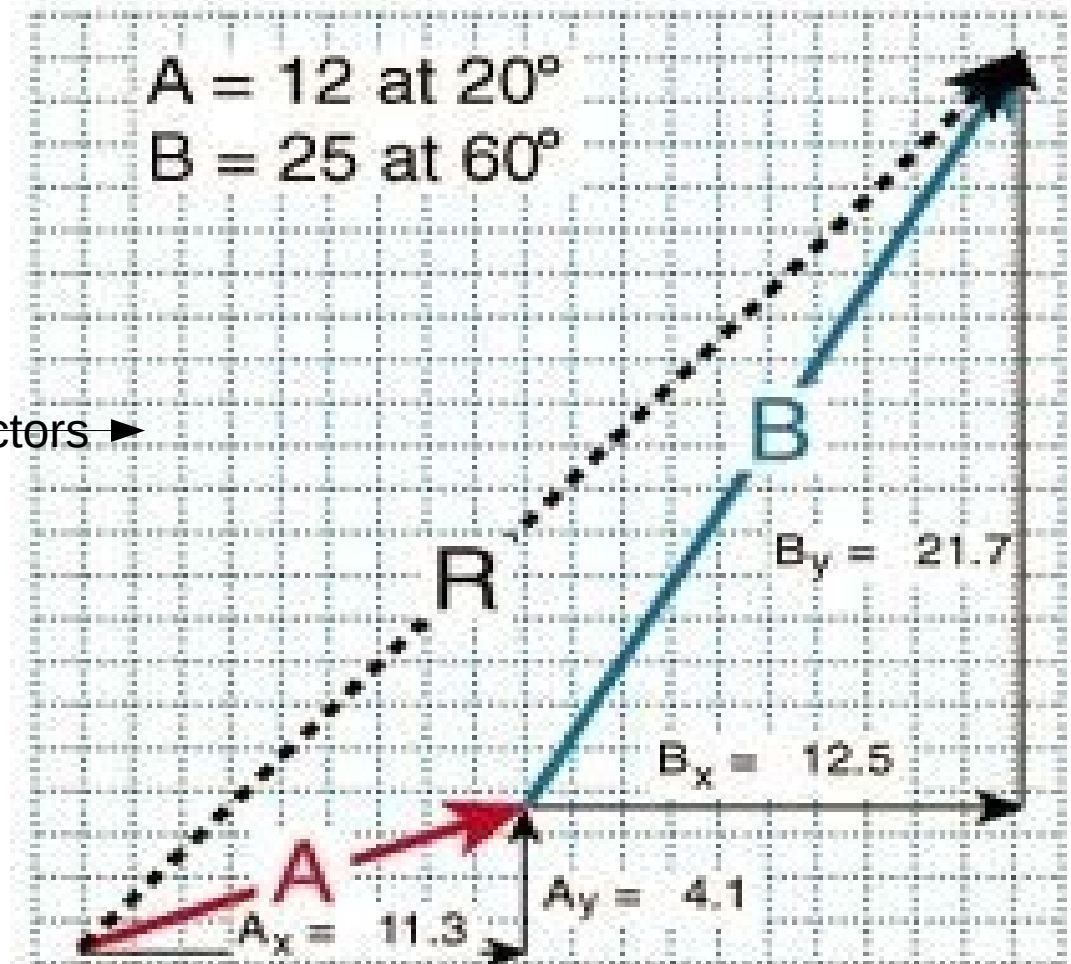
subtract vectors geometrically



Adding vector algebraically

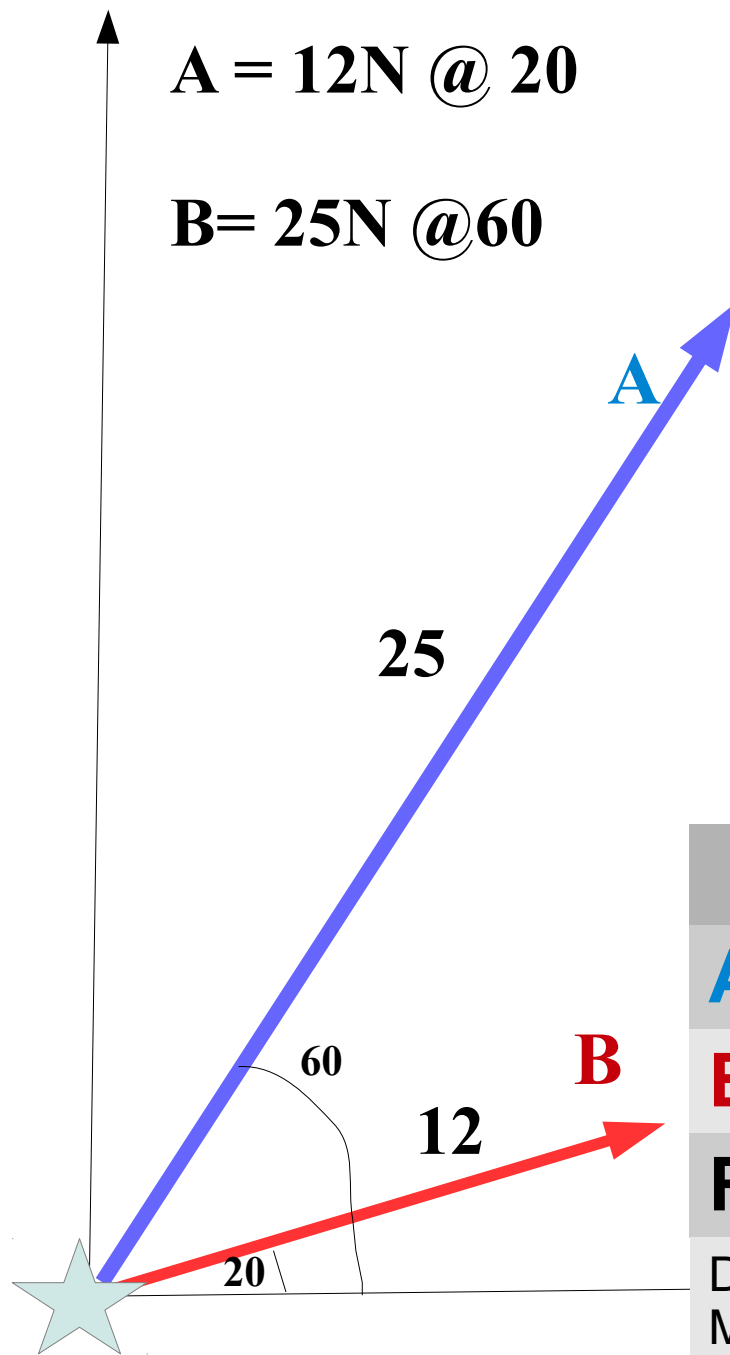


— Add vectors —>



$R_x = A_x + B_x = 12.5 + 11.3 = \underline{\hspace{2cm}}$ $R_y = A_y + B_y = \underline{\hspace{2cm}}$ so magnitude of $R = \underline{\hspace{2cm}}$

Direction = $\arctan(R_y/R_x)$ $\underline{\hspace{2cm}}$ Do the assignment in class.



When you add vectors you add the components
See exploration of physics.

How to add vector ?

Use the component method and trig.

$$A_x = 25 \cos(60) \quad A_y = 25 \sin(60)$$

$$B_x = 12 \cos(12) \quad B_y = 12 \sin(12)$$

$$R_x = A_x + B_x$$

$$R_y = A_y + B_y$$

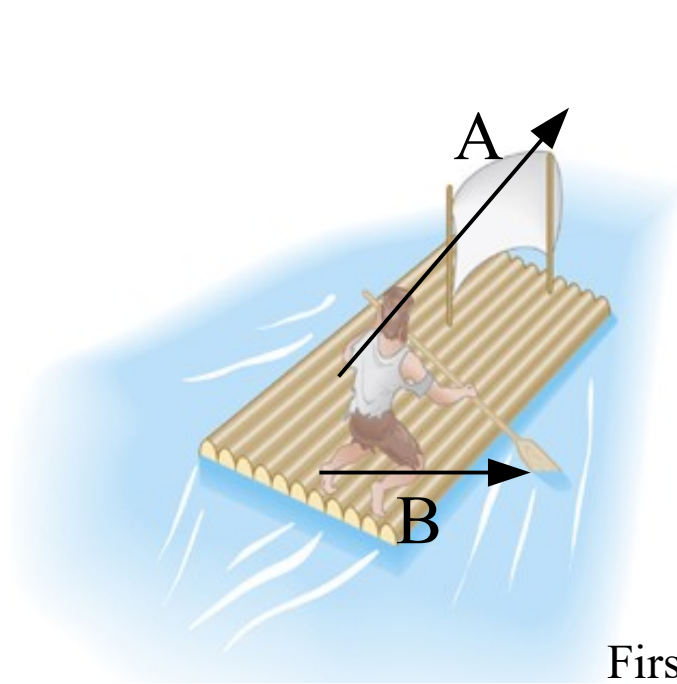
Fill the table.

To draw R move B on the head of A

And connect the tail of A to the head of B

	X	Y
A		
B		
R		
Direction = $\arctan(Y/X)$ =		
Magnitude =		

The raft is acted upon by 2 forces:
 from the current of the river @ right and from Robinson
Find the net force acting on the boat.



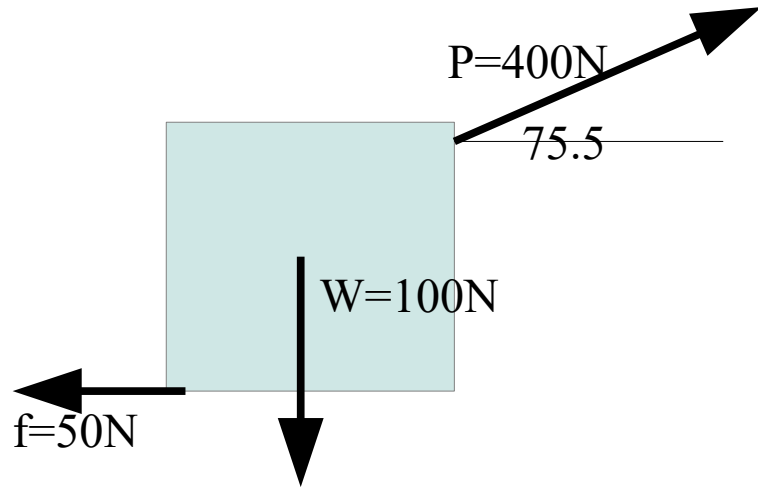
$$A = 15\text{N} @ 67^\circ$$

$$B = 17\text{N} @ 0^\circ$$

First build a free-body diagram
 Attach the tails to the origin.
 Find the net force $F_{\text{net}} = R = A + B$

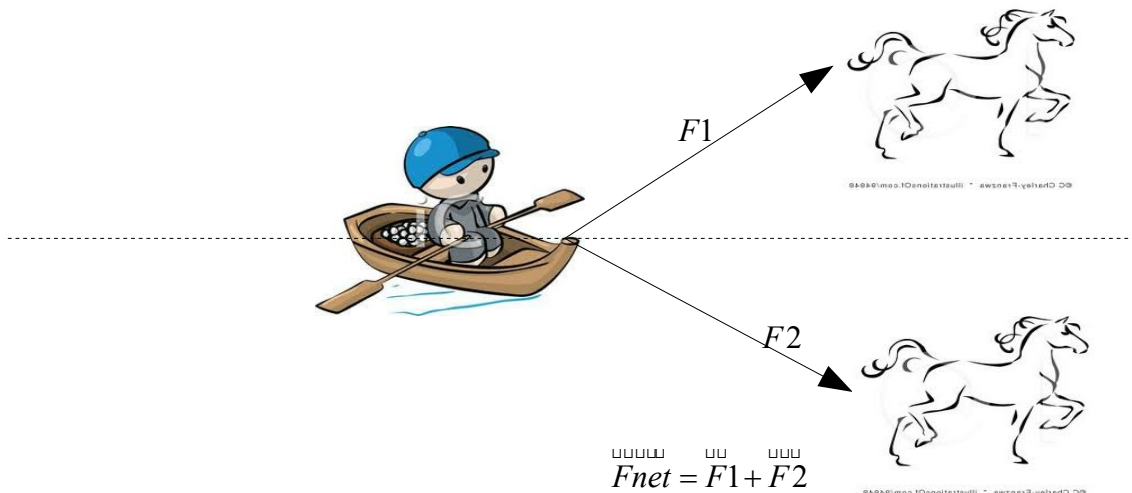
	X	Y
A		
B		
R = F_{net}		
Direction = $\arctan(Y/X) =$		
Magnitude =		

$$A(5.9, 13.8) \quad B(17, 0)$$



A box is pulled with a force of 400N @ 75.5

- 1) write each force in standard notation
- 2) in a table report the components of each force (see previous slide)
- 3) Find the components of the resultant R (sun)
- 4) draw R in a X-Y coordinate system
- 5) from 4) which way the box is moving ?

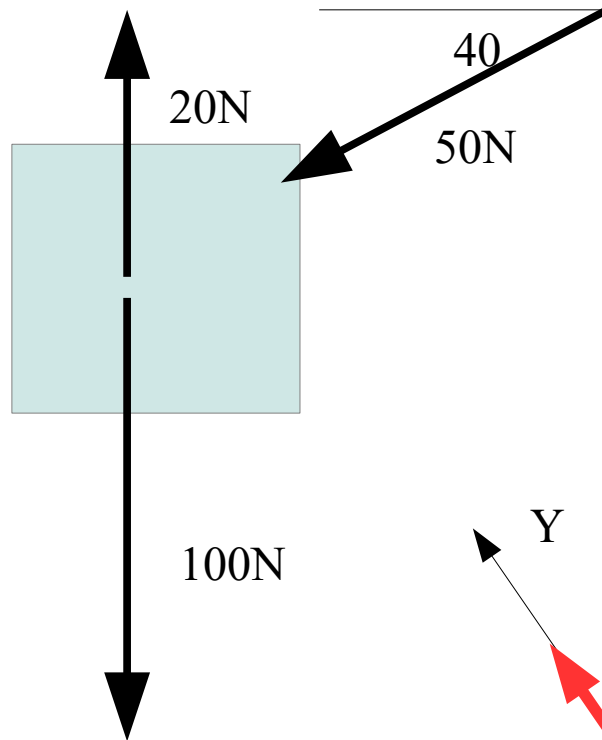


Same question.

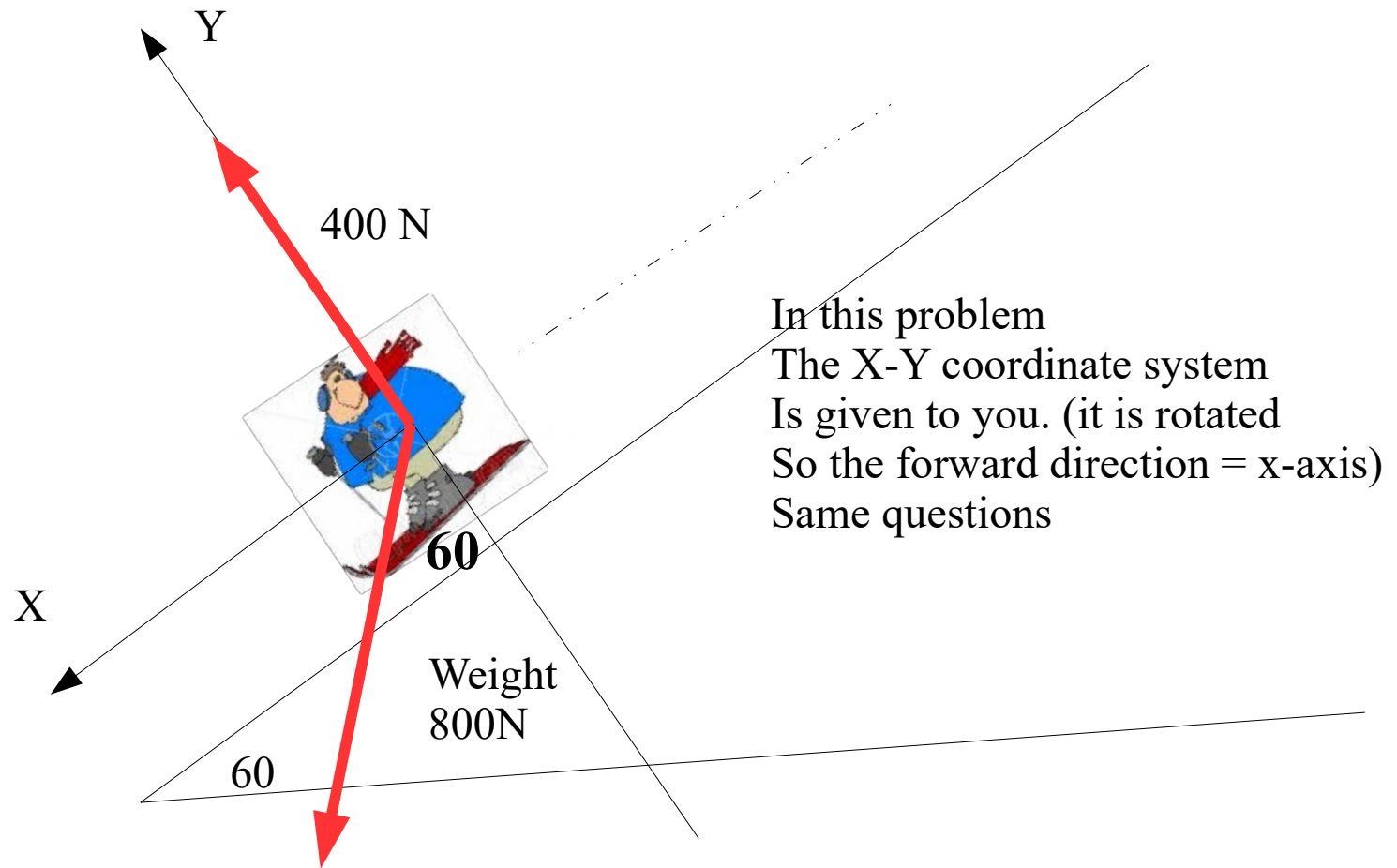
F1 and F2 have magnitude = 1000N

Direction is 45 degrees and - 45 degrees.

$F1(707,707)$ $F2(707,-707)$



- 1) write each force in standard notation
- 2) in a table report the components of each force (see previous slide)
- 3) Find the components of the resultant R (sum)
- 4) draw R in a X-Y coordinate system
- 5) from 4) which way the box is moving ?



KINDS of FORCES:

Tension in a rope

Normal force (also called support force)

Friction (static or kinetic)

Weight (fundamental force)

WEIGHT: Don't confuse weight and mass.

Mass is a measure of the amount of “stuff” contained in an object.

- Mass does not depend on the Planet the object is placed.

Unit is kilogram (kg)

WEIGHT IS THE PULL DUE TO GRAVITY.

WEIGHT is a FORCE but MASS is STUFF(INERTIA)
WEIGHT is measured in Newtons (or pounds).

**1 kg has a weight of 9.8N (about 10N)
or 2.2 pounds.**

Pound is the unit for force in the British system. The unit for Mass in the British system is the slug !

WEIGHT IS THE PULL DUE TO GRAVITY.
WEIGHT is a FORCE but MASS is STUFF(INERTIA)
WEIGHT is measured in Newtons (or pounds).

weight = mass x g **g is the acceleration due to gravity (m/s/s)**

Mass is in kg

Weight in Newton

On Earth g = 9.8 m/s/s about 10m/s/s

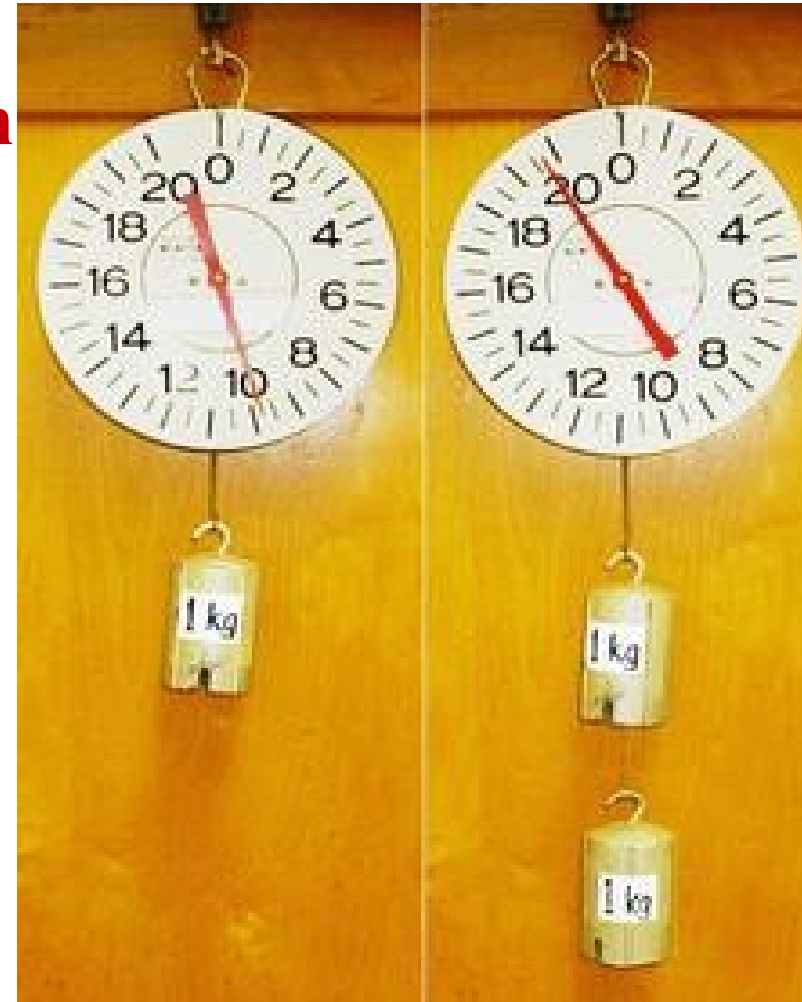
so weight = mass x 10

2.2 Pounds on Earth

1 kg has a weight of 10N

So 10N ↔ 2pounds

Divide by 5

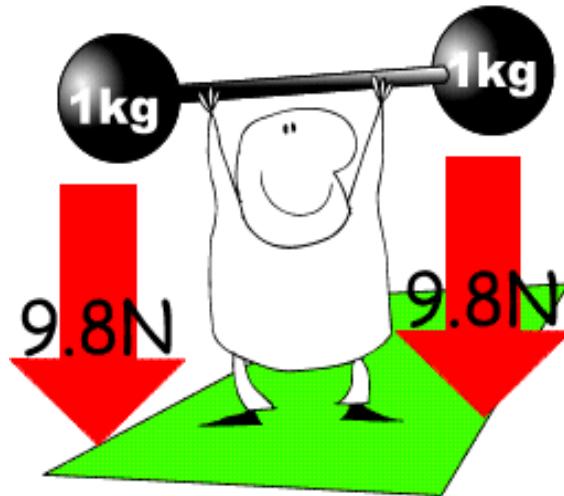


WEIGHT is a FORCE = pull due to gravity

It depends on the planet, star, object.. you are standing on.

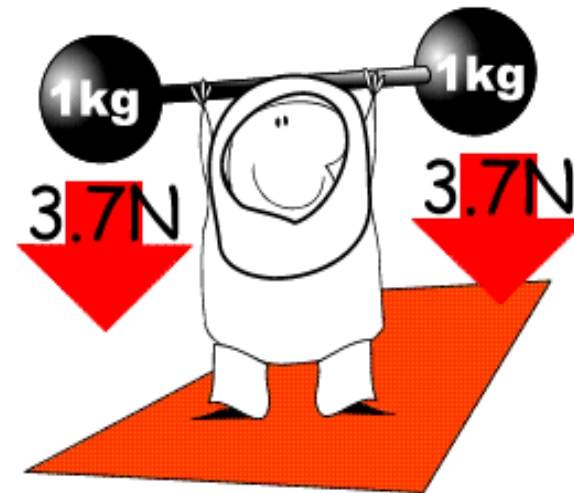
Earth

$$g = 9.8 \text{ N/kg}$$



Mars

$$g = 3.7 \text{ N/kg}$$

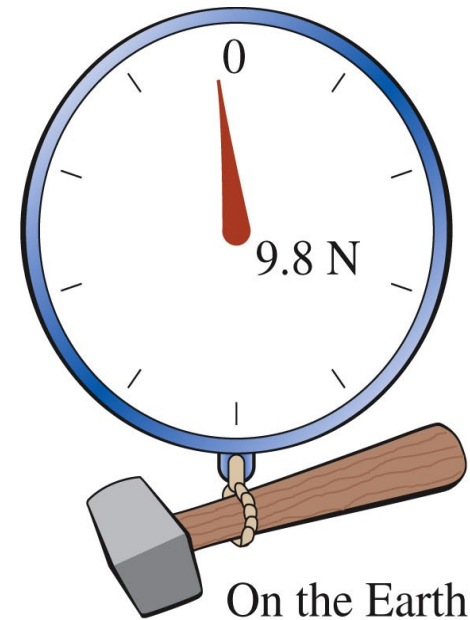
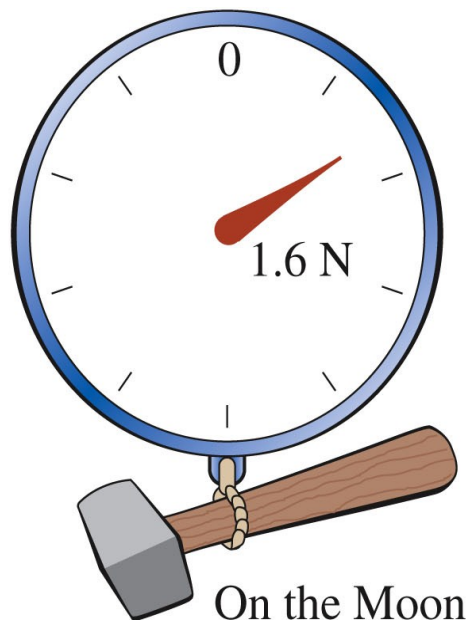
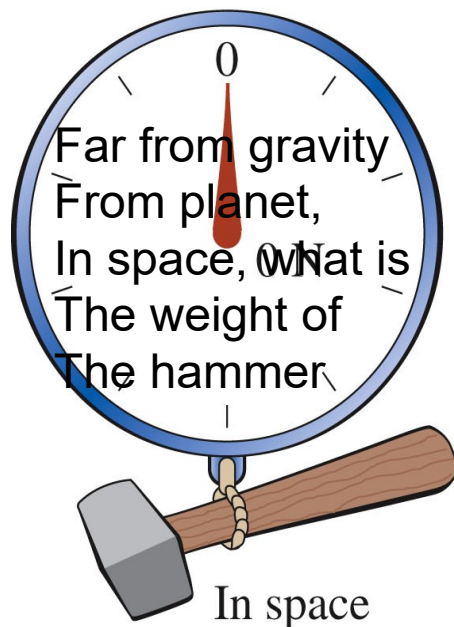


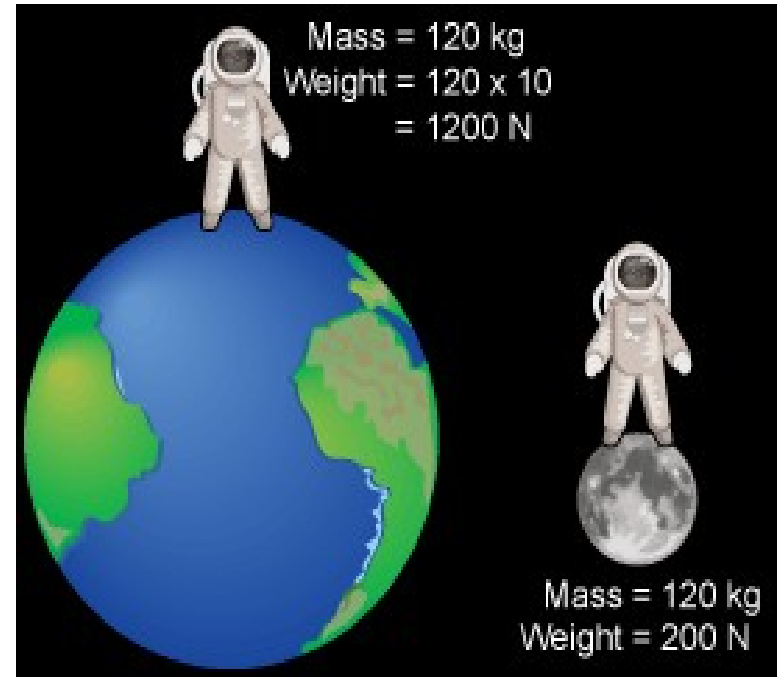
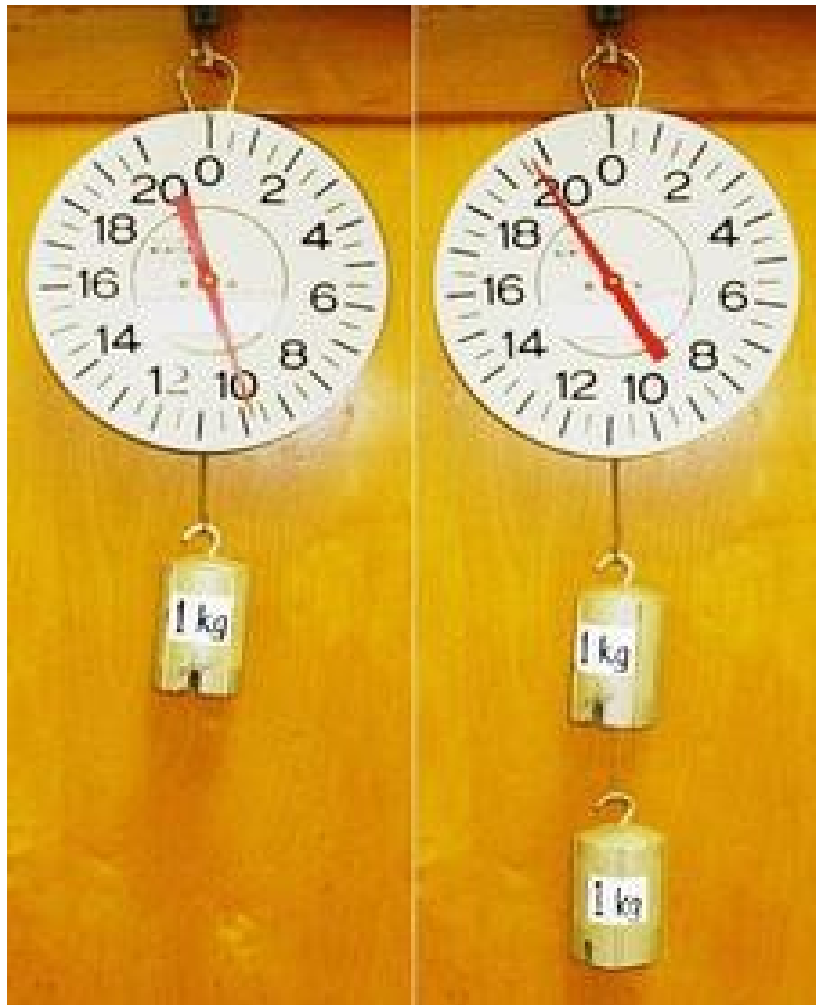
So if your mass is 80kg , your mass on the Moon
Will be : the same ? Greater ? Larger

What about your weight ?

Mass

- The weight of an object depends on which planet you measure the weight.
- The mass of the object is independent of the planet.





For a 70kg person :

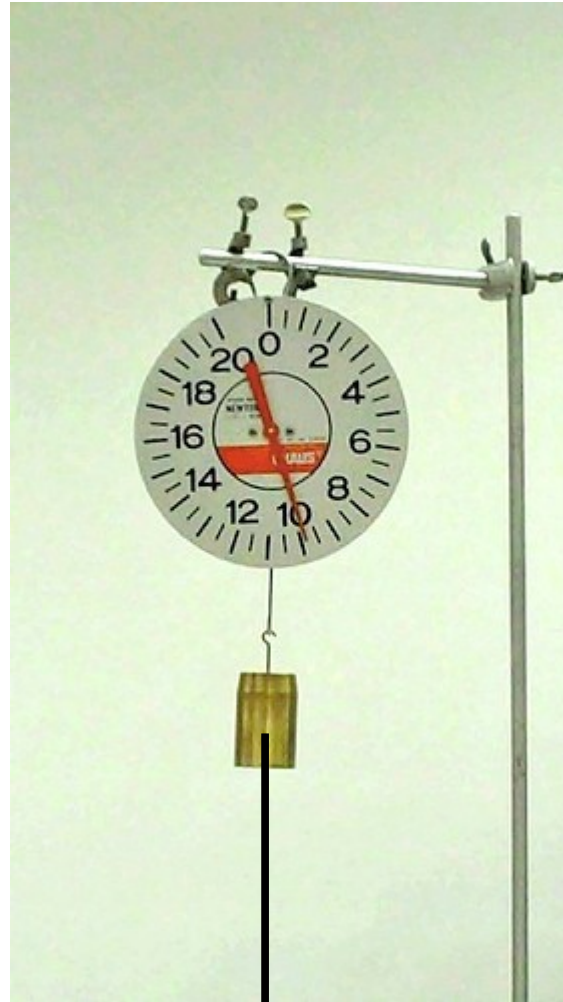
- 150 pounds on Earth
- 27 pounds on Moon
- 400 pounds on Jupiter
- 2 tons on the Sun
- ½ ounce on an asteroid (5 pennies)
- (6mph to escape, versus 7 miles per second for Earth = 25 200 mph)

Show applet elevator from website
And check weight on other planets.



- 1) If a hammer has a mass of 2.5kg, how much does it weigh on Earth ?
On the Moon/ (gravity is divided by 6 on the Moon)
- 2) What is the mass of a girl who weighs 340N on earth ?
- 3) what is the mass of a dog that weighs 75N ?
- 4) convert 10 pounds in newtons.
- 5) On Earth 10 pounds corresponds to what mass in kg ?
- 6) convert 10 newtons in pounds ? What is the mass ?

Weight W is straight down to center of Earth



Mass = 1kg

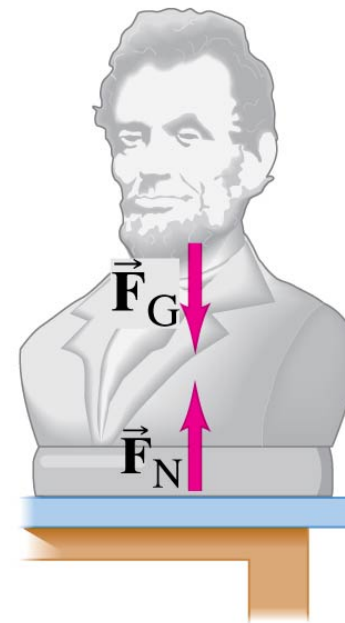
Weight is 10N@down

W

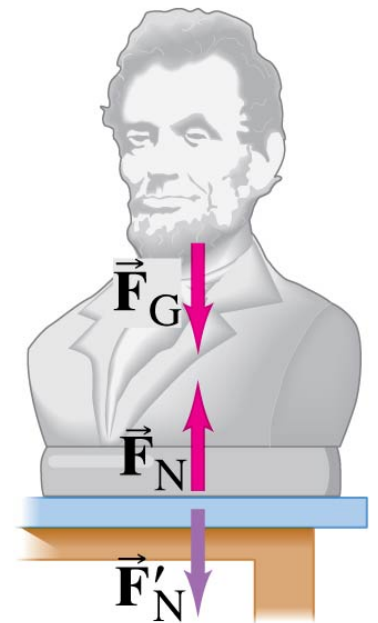
Definition of the Normal Force

The normal force is one component of the force that a surface exerts on an object with which it is in contact – namely, the component that is perpendicular to the surface.

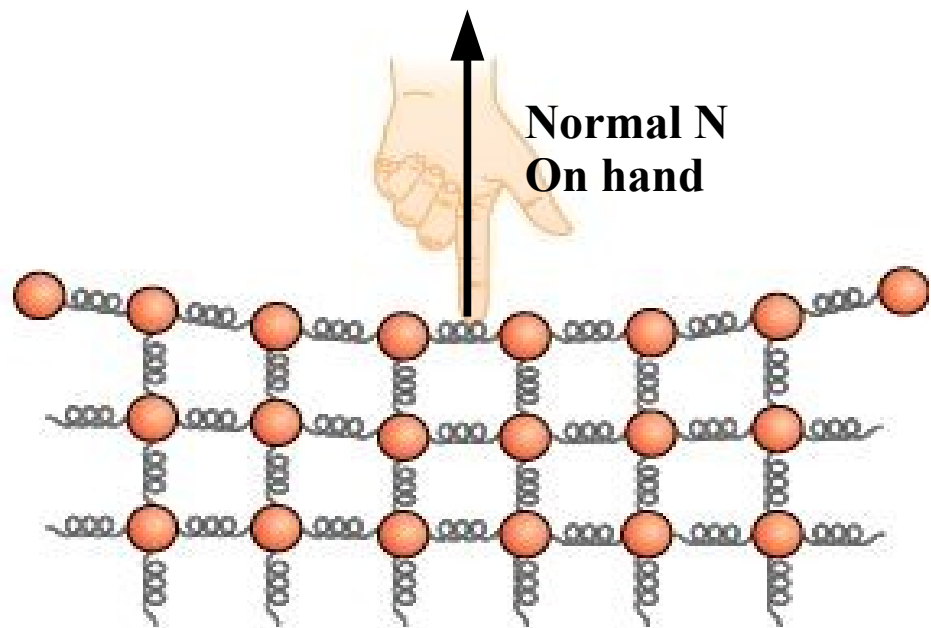
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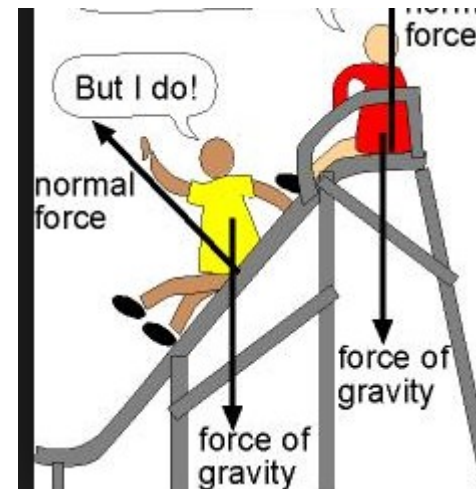
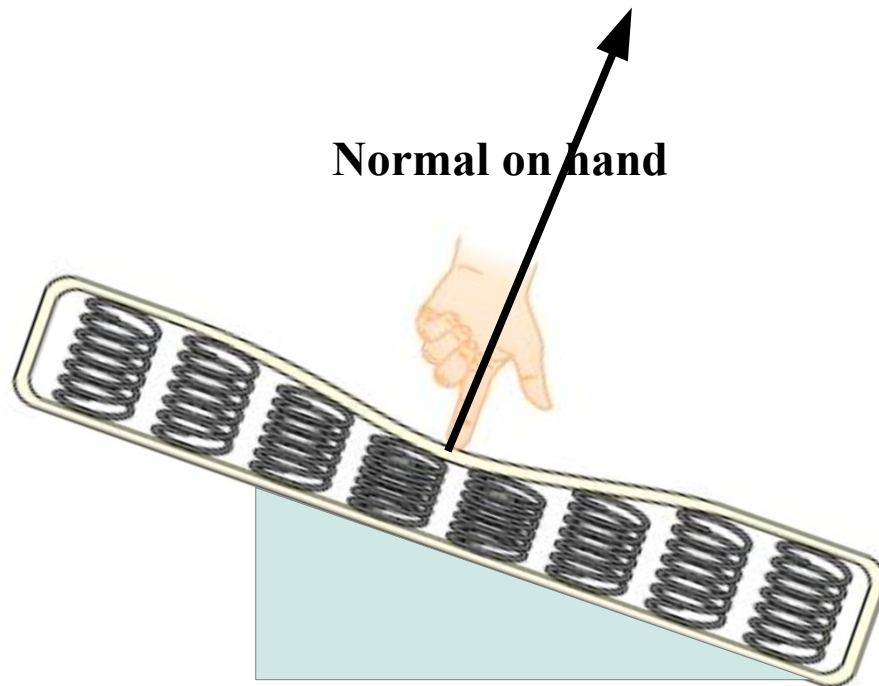
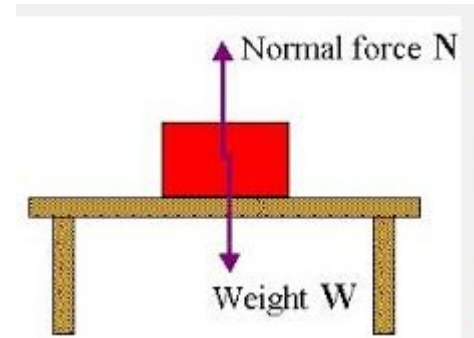
(a)

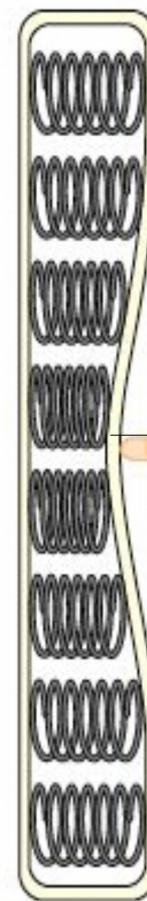
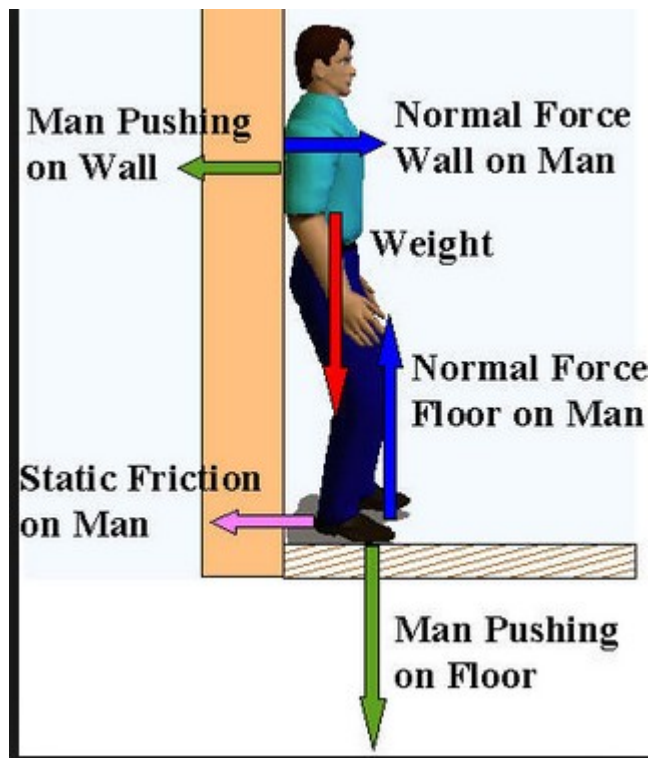


(b)



Normal force (also called support force)





Important the forces acting
On the man are:
Normal from wall, normal from floor,
friction

What is the meaning of the word “normal” in the term “normal force?”

- a) that it is in magnitude and opposite in direction to the weight of the object
- b) that it is one that is encountered in everyday life
- c) that it is directed perpendicular to a surface
- d) that it is measurable
- e) that it has a magnitude of 1 newton

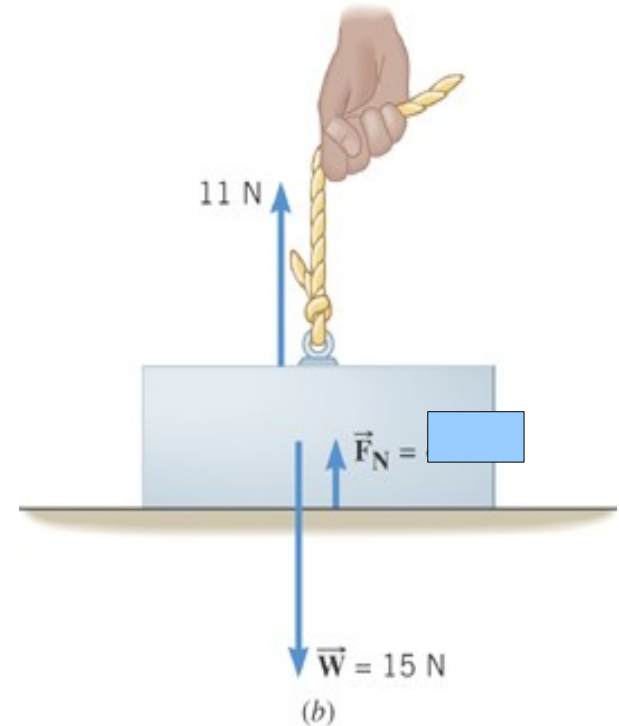
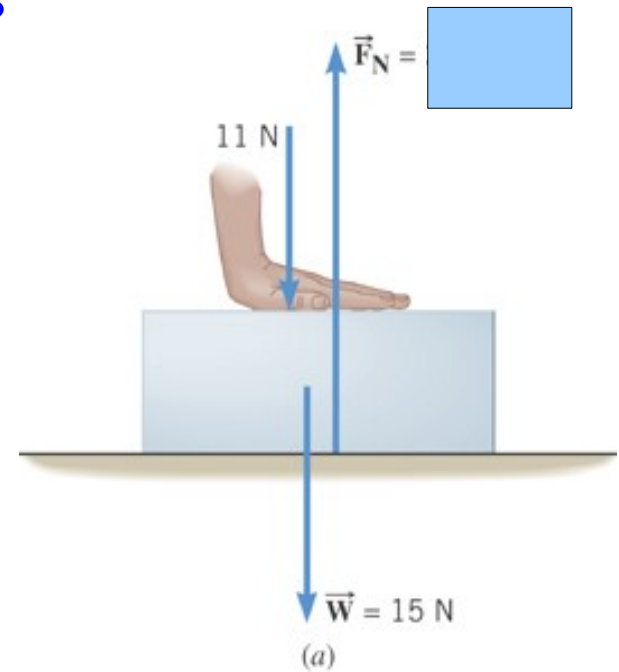
FIND The Normal Force ?

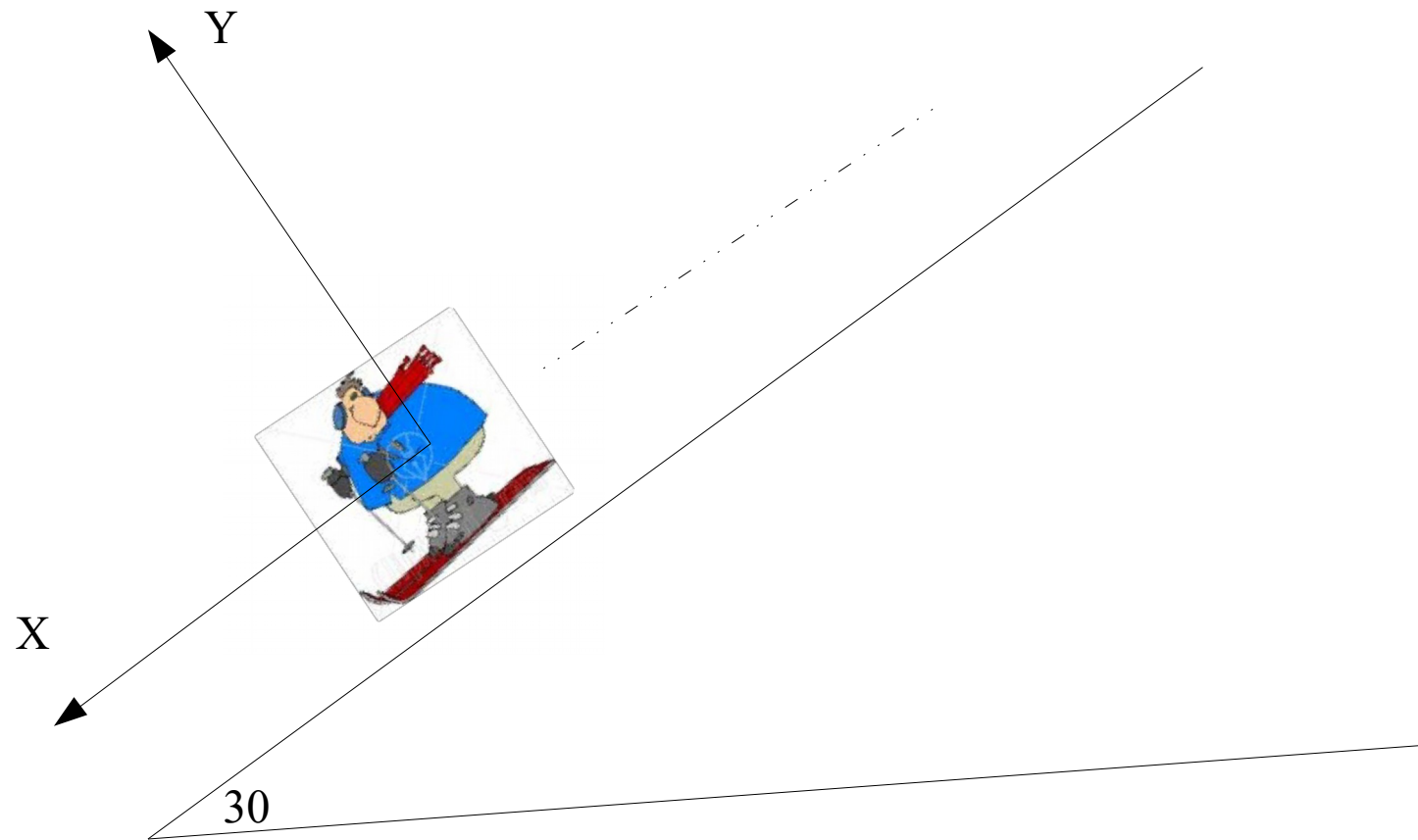
$$F_N - 11 \text{ N} - 15 \text{ N} = 0$$

$$F_N = \boxed{}$$

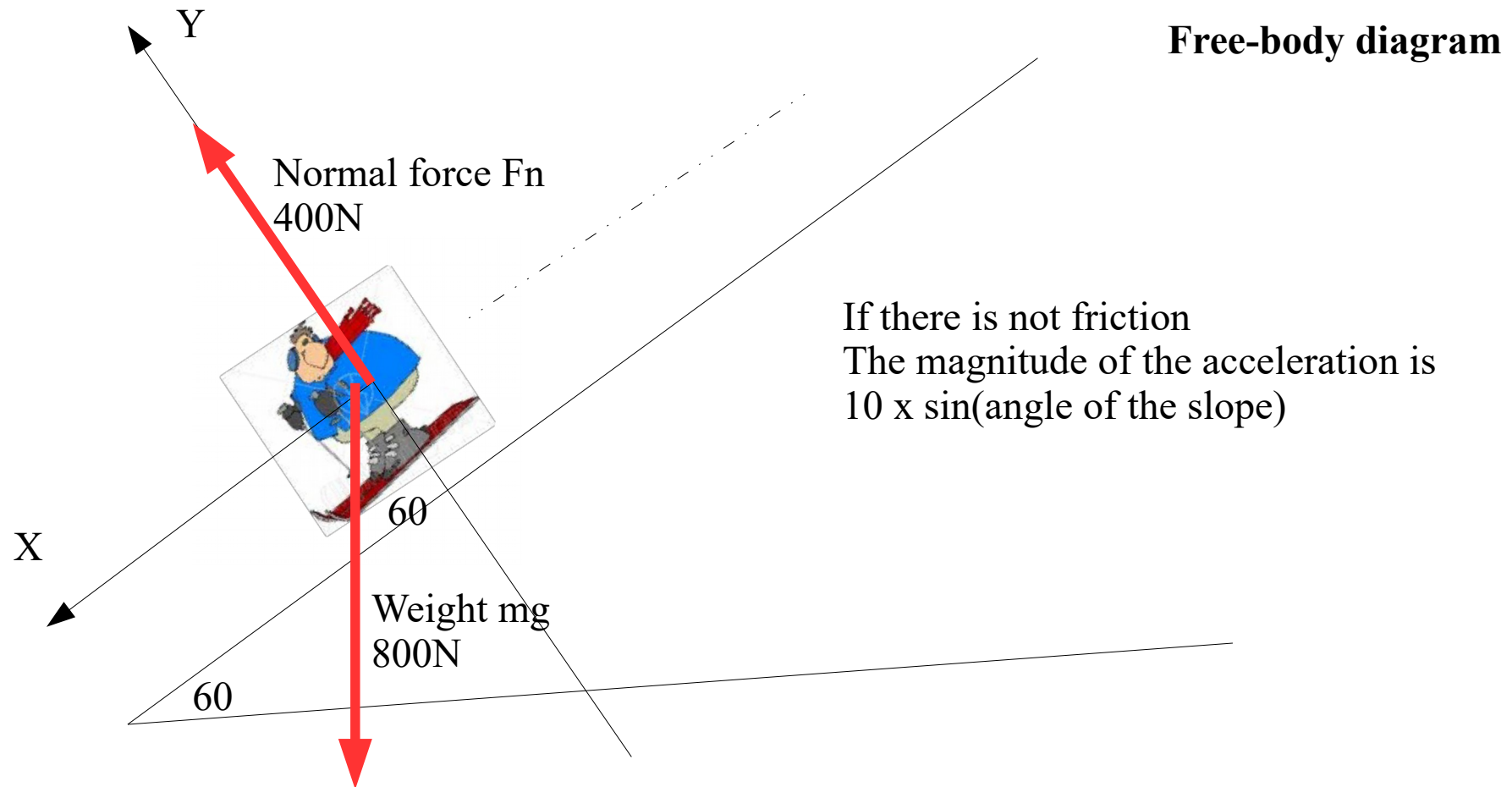
$$F_N + 11 \text{ N} - 15 \text{ N} = 0$$

$$F_N = \boxed{}$$



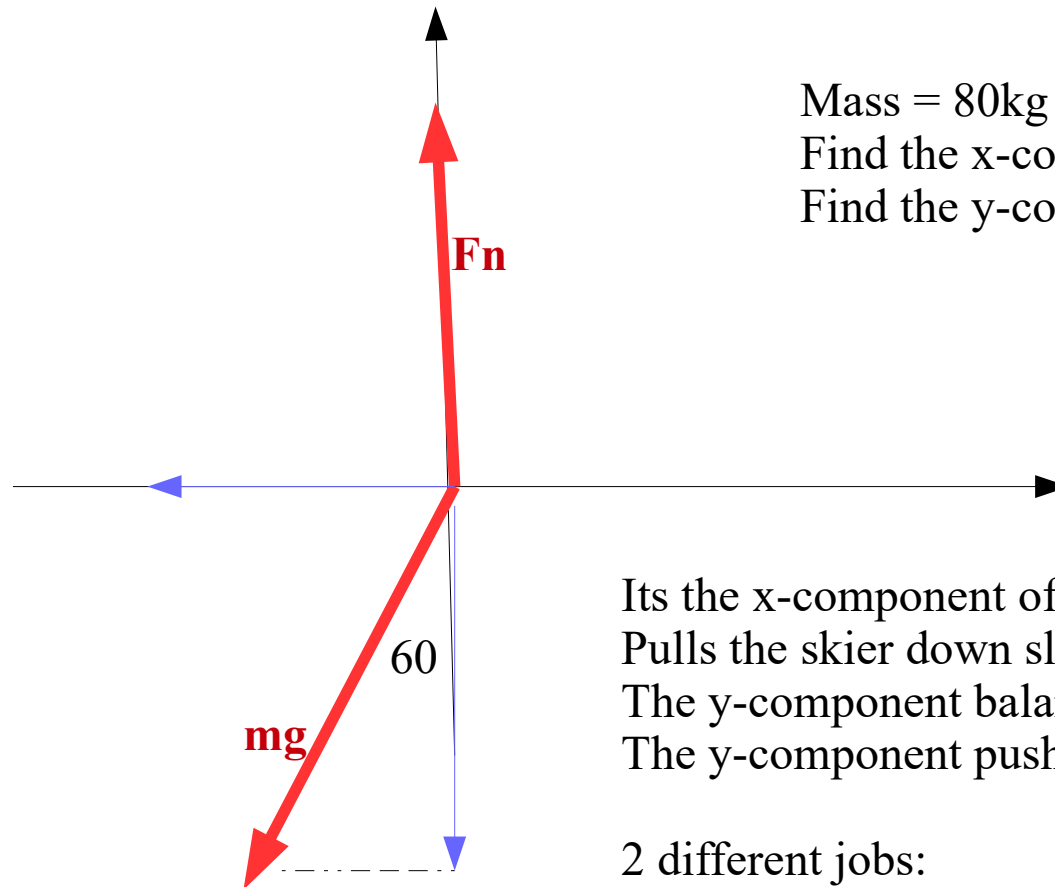


Draw the forces applied to the skier.
Then trace the forces in a free-body diagram



The skier has a mass of 80kg .

- 1) find the mass of the skier
- 2) draw a free body diagram with the forces
- 3) write the forces in standard notation
- 4) find the resultant R (standard notation) (R is the sum of F_n and W)
- 5) using newton's seconds law: $R = m a$ find the acceleration of the skier
- 6) compare the acceleration to $10 \sin(\text{angle})$



Mass = 80kg

Find the x-component of weight

Find the y-component of weight

It's the x-component of the weight that
pulls the skier down slope.

The y-component balances out the normal force.
The y-component pushes on the slope (snow).

2 different jobs:

X-component pulls him down

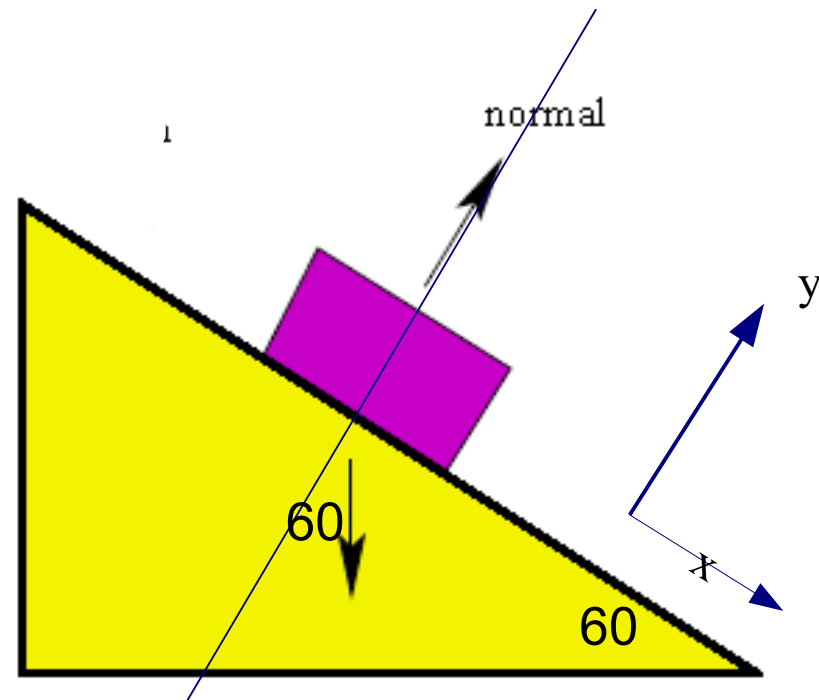
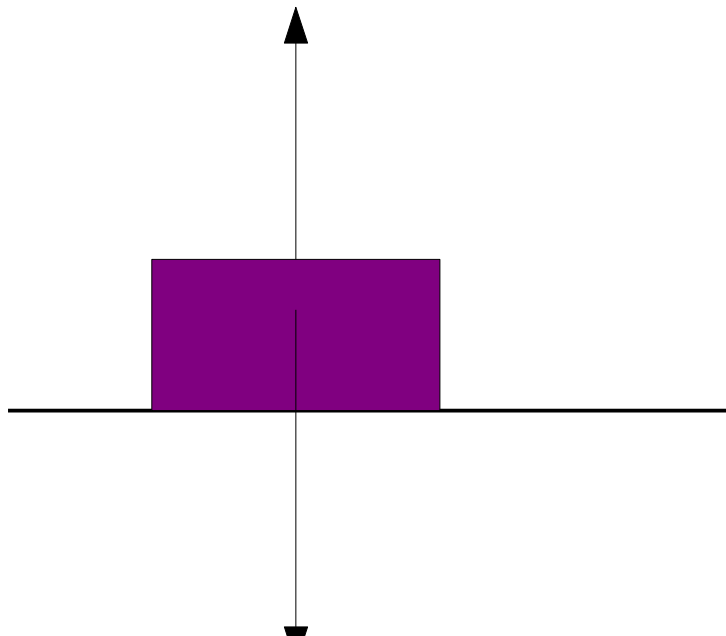
Y-component pushes on the ground

A brick (like skier) is resting on the surface of a flat board. As one end of the board is slowly raised, what change, if any, is there in the normal force exerted on the brick?

- a) The normal force increases.
- b) The normal force decreases.
- c) The normal force remains constant.
- d) Only the direction of the normal force changes.

What about the scale?

Same reading ?
Do demo

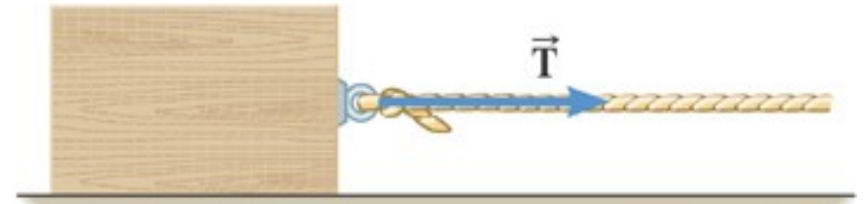


The Tension Force

Cables and ropes transmit forces through ***tension***.



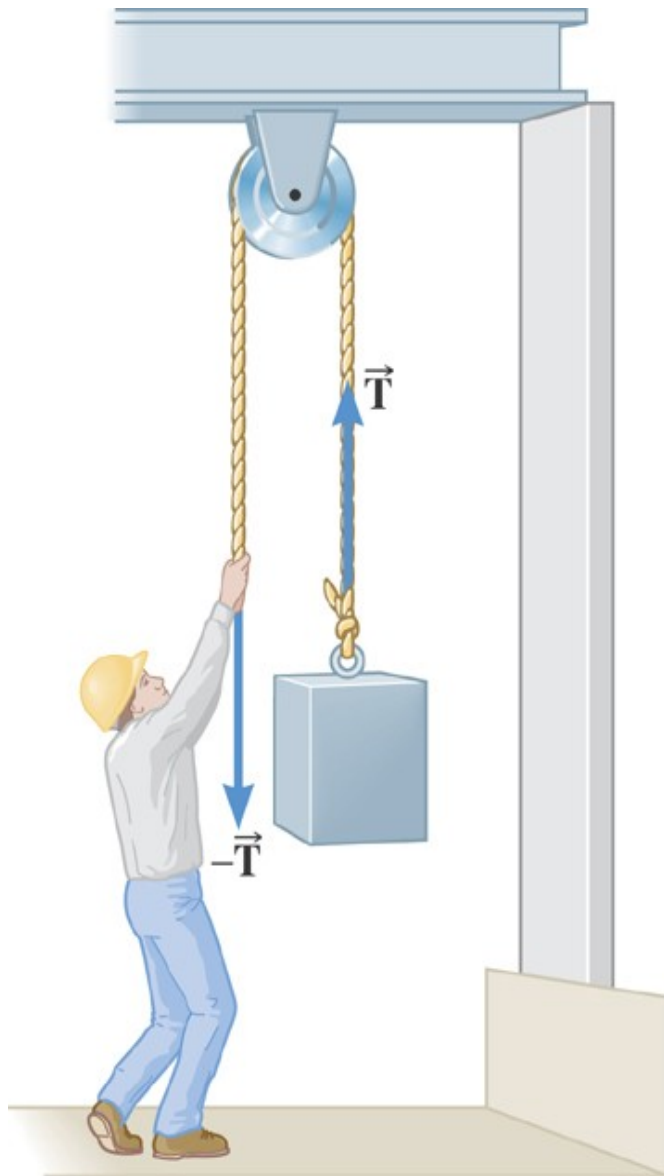
(a)



(b)



(c)



A massless rope will transmit tension undiminished from one end to the other.

If the rope passes around a massless, frictionless pulley, the tension will be transmitted to the other end of the rope undiminished.

demo

Static EQUILIBRIUM MEANS : no acceleration

MEANS : The net force is 0

$$\sum \vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_3 = 0$$

Algebraically (we use):

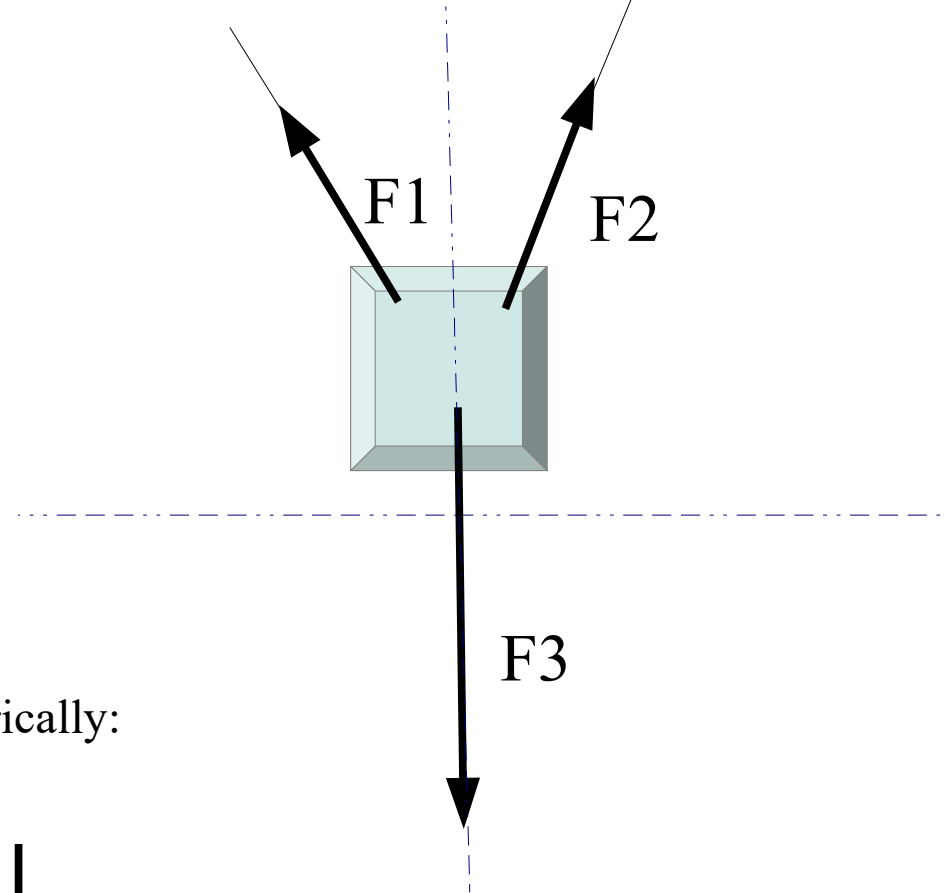
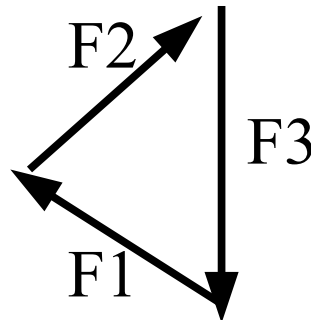
Sum of x-component = 0

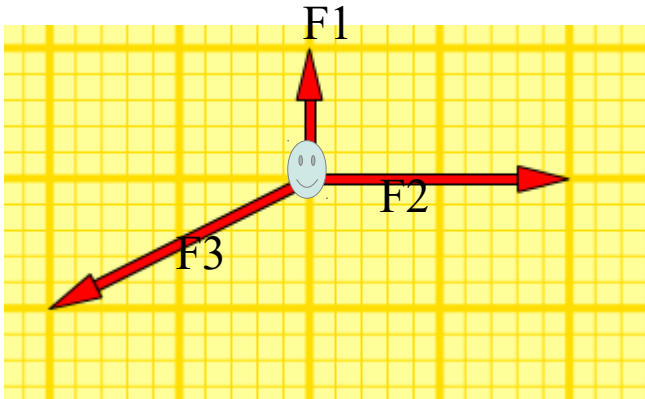
Sum of y-components = 0

$$F_{1x} + F_{2x} + F_{3x} = 0$$

$$F_{1y} + F_{2y} + F_{3y} = 0$$

Geometrically:





I) An object is in equilibrium. 2 forces are acting on it.

$F_1 = 5\text{N @ up}$ $F_2 = 10\text{N @ right}$ $F_3 = ?$

The sum of vectors is 0 = the net force is 0.

You are going to find the vector F_3 . (polar and Cartesian coordinates)

1) Fill the table

2) Use $F_{1x} + F_{2x} + F_{3x} = 0$ and $F_{1y} + F_{2y} + F_{3y} = 0$ to find F_{3x} and F_{3y}

3) Compute the magnitude of F_3 and its direction

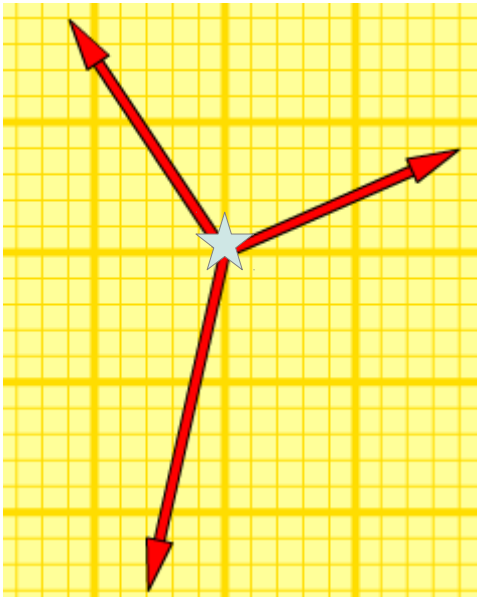
$F_3 = \underline{\hspace{1cm}} \text{ N @ } \underline{\hspace{1cm}}$

	x-axis	y-axis
F1		
F2		
F3	F3x	F3y
SUM=FNET	0	0

II) In this situation:

$F_1 = 11\text{N @ } 124^\circ$ $F_2 = 10\text{N @ } 24^\circ$ Find F_3

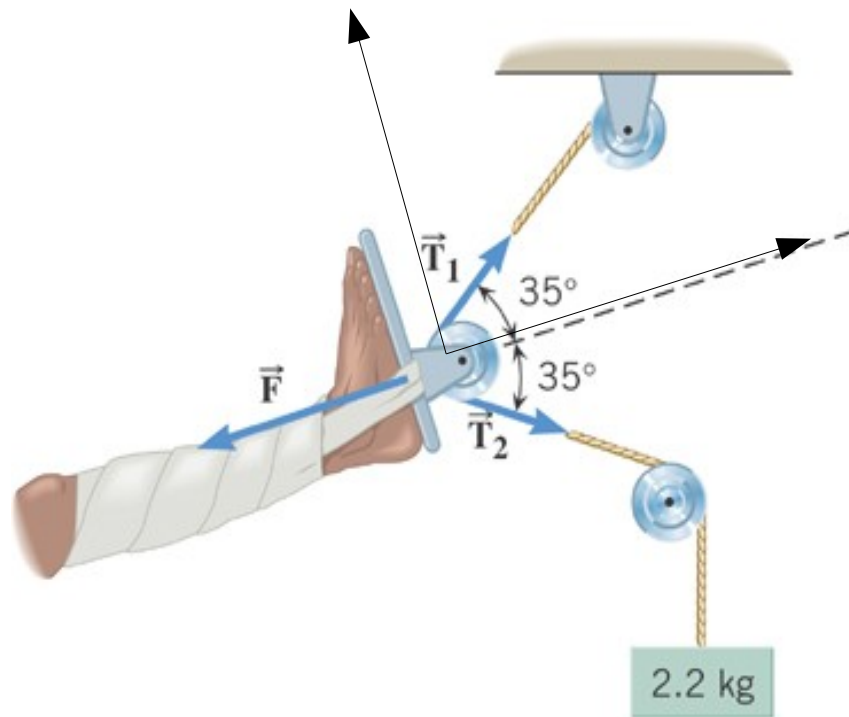
Follow the same steps but now you need to use trig to find the components of the vectors.



	x-axis	y-axis
F1		
F2		
F3	F3x	F3y
SUM=FNET	0	0

$F_1(-6.2, 4.5)$ $F_2(9.1, 4)$

4.11 Equilibrium Application of Newton's Laws of Motion



(a)

First draw a free-body diagram

Then $T_1 = T_2 = \text{weight} = 22\text{N}$

**Then find the magnitude of F
The force stretching the leg.**

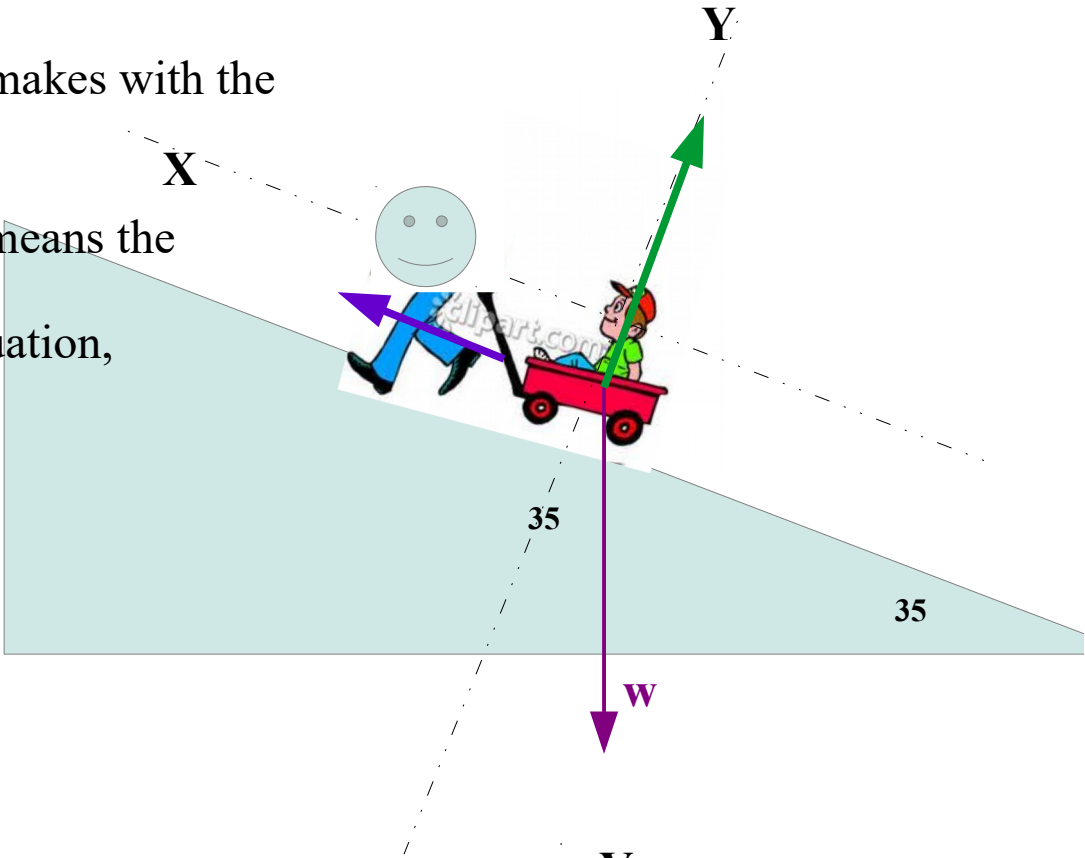
(b) Free-body diagram for the foot pulley

1) $|T_2|$ is a tension so $|T_2| = 2.2 \times 10 = \text{weight of the mass} = |T_1|$

	x-axis	y-axis
F	F_x	F_y
T1		
T2		
SUM=ENET	0	0

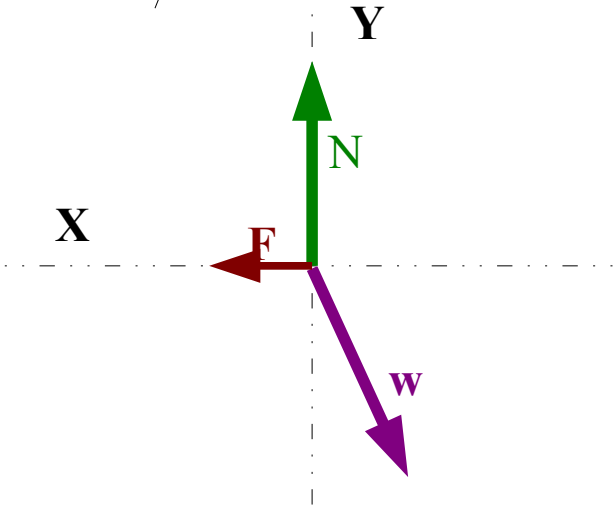
A wagon weighs 40 lb and the angle that the hill makes with the horizontal is 35 degrees. Keep pounds.

The wagon is moving at a constant speed . That means the acceleration is 0. That means the net force (or sum of the force is 0). It is an equilibrium situation, The goal is to find the normal force N.

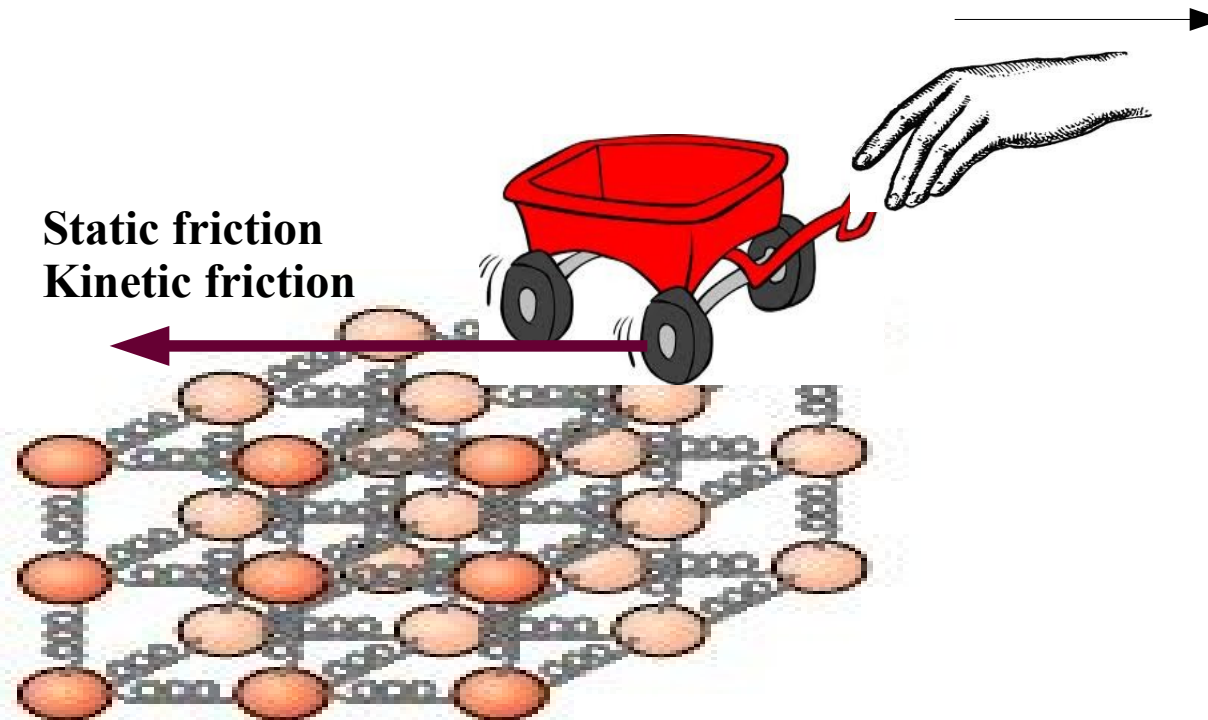


- 1) Draw a free-body diagram.
- 2) Write each force in standard notation
- 3) Fill the table below and find N

	x-axis	y-axis
N	?	?
F		
mg		
SUM=FNET	0	0

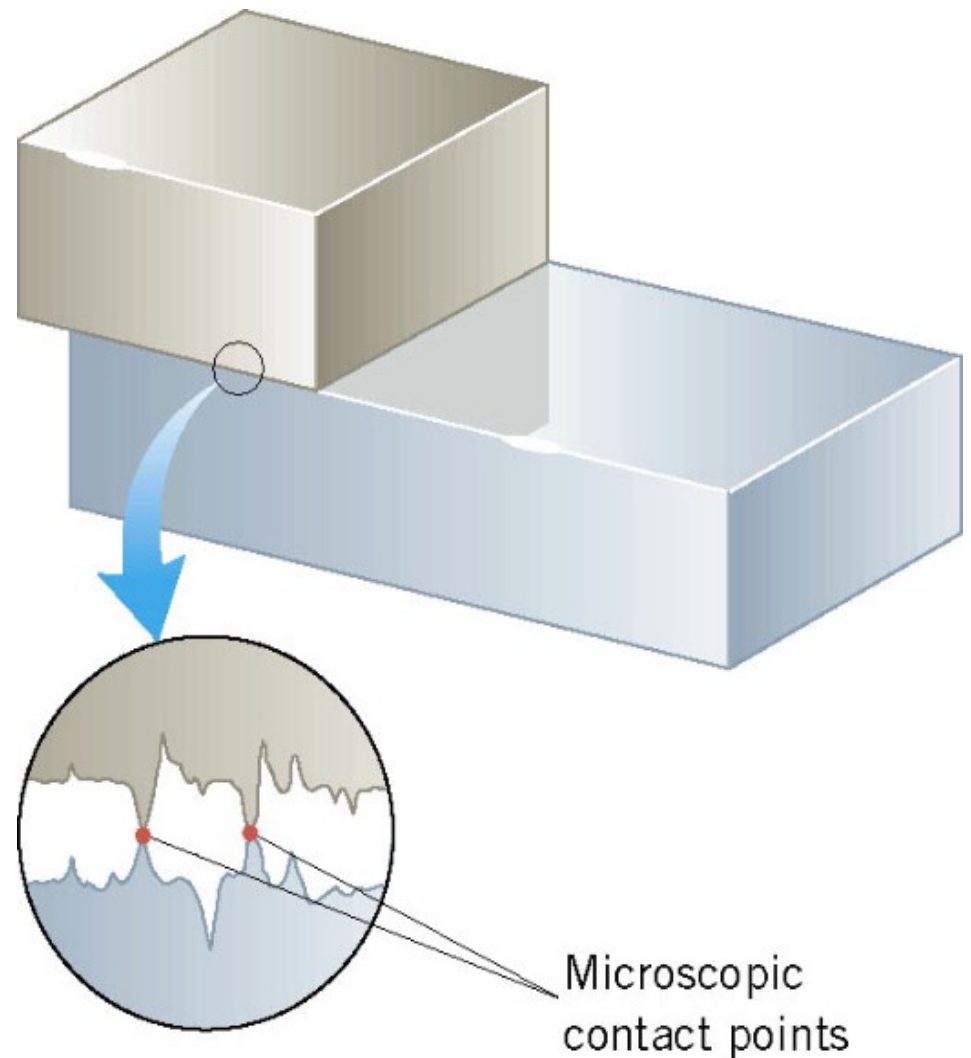


Frictional forces



<http://phet.colorado.edu/en/simulation/forces-and-motion-basics>

When an object is in contact with a surface there is a force acting on that object. The component of this force that is parallel to the surface is called the ***frictional force***.



When the two surfaces are not sliding across one another the friction is called ***static friction***.



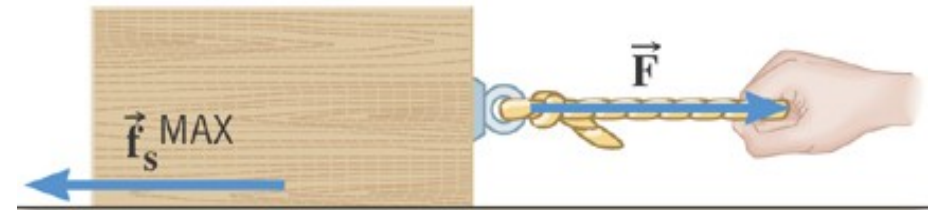
No movement

(a)



No movement

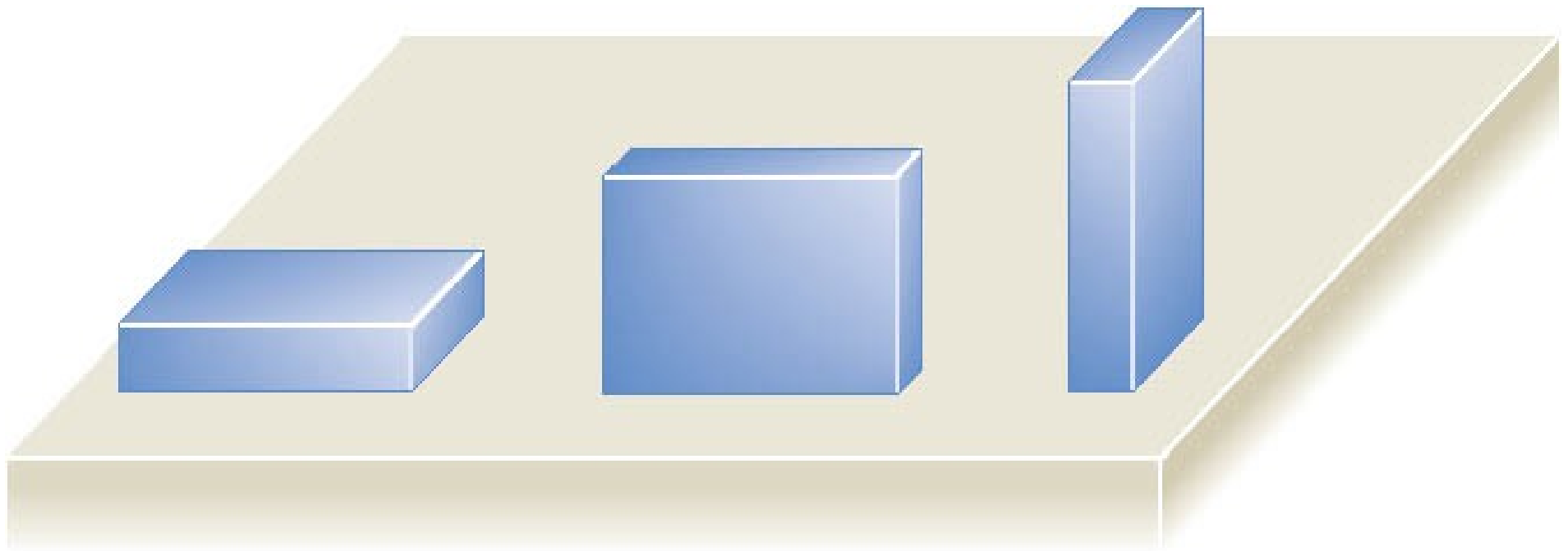
(b)

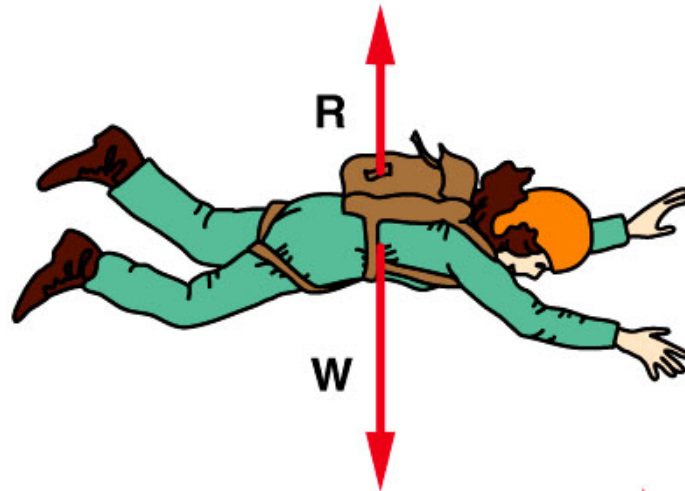
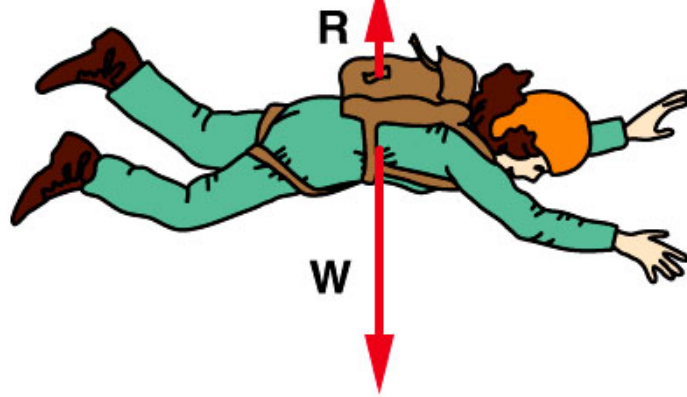


When movement just begins

(c)

Note that the magnitude of the frictional force does not depend on the contact area of the surfaces.

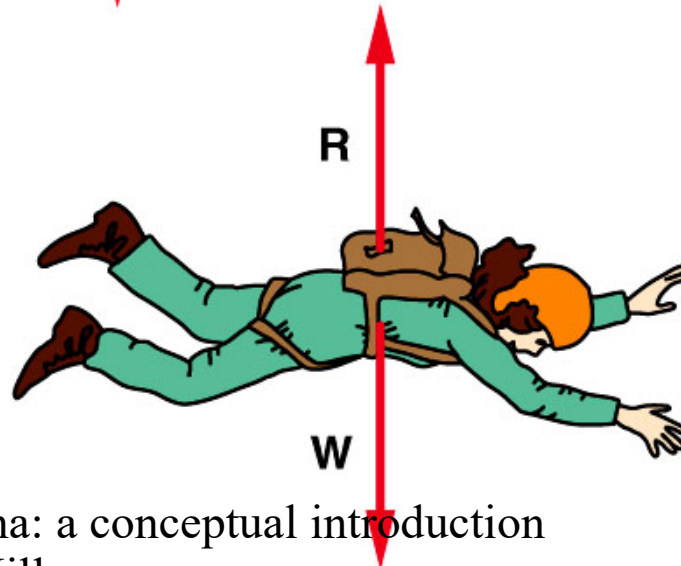




AIR RESISTANCE

And terminal speed

See exploration of physical



DRAW a free-body diagram for :

- 1) a parachutist speeding down to Earth (include air resistance)
- 2) a cockroach falling from the empire state building at a constant speed

- 3) A boy leaning with one hand on a tree. He stands on the ground (hint: all force come in pair)

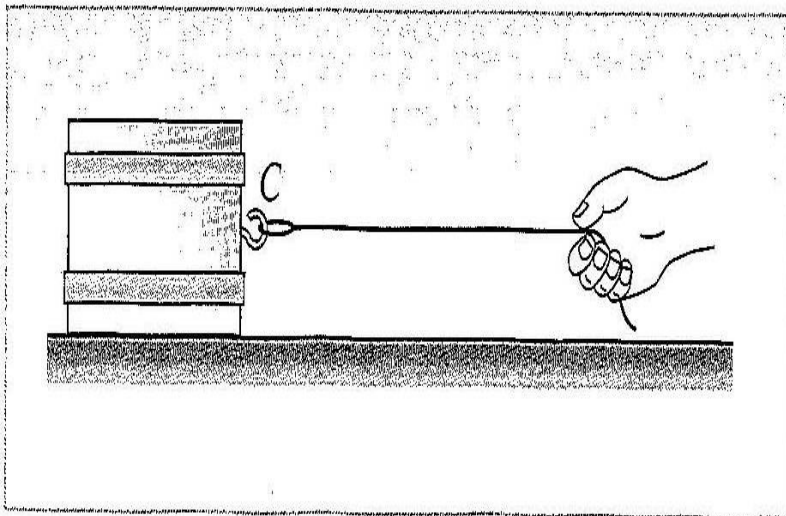
- 4) an airplane flying. Included lift force (from air pushing up) and thrust (motor).

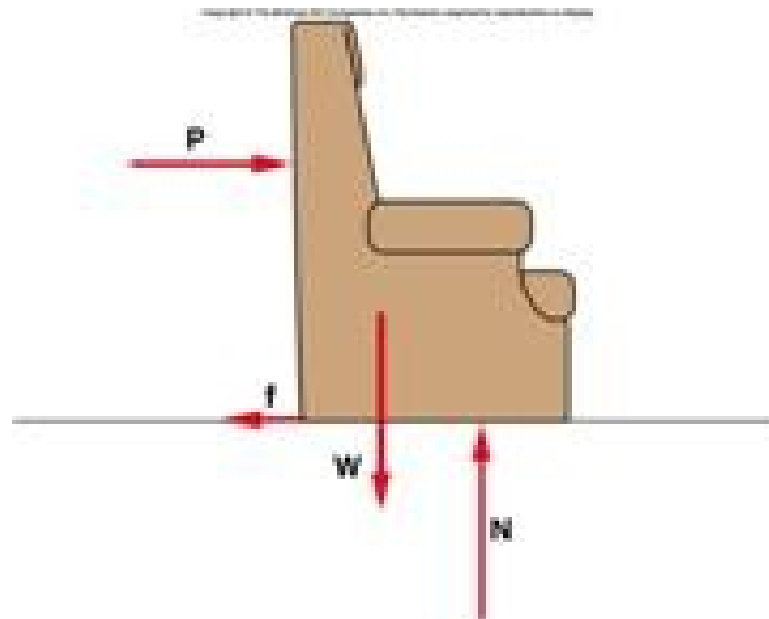
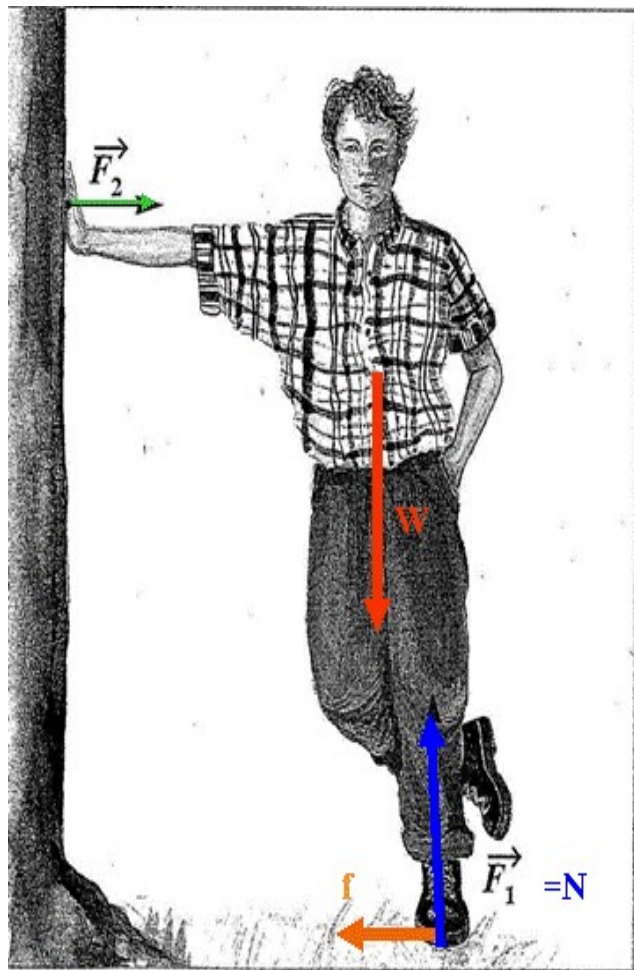
The plane moves at a constant speed

- 5) a block pulled by a string and sliding. (include friction)
Speeding up

- 6) a couch pushed over the floor

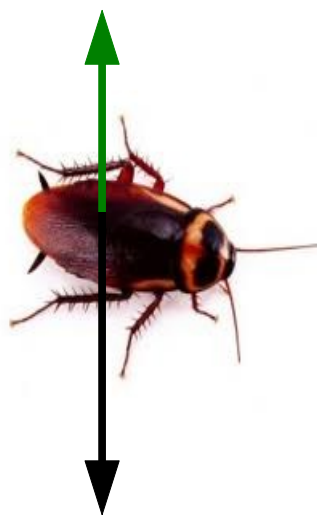
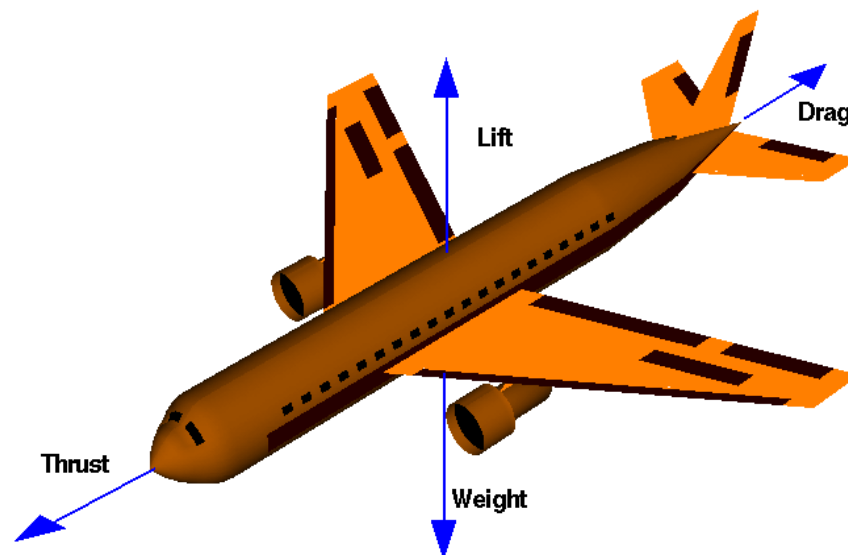
- 7) block on an inclined plane not sliding

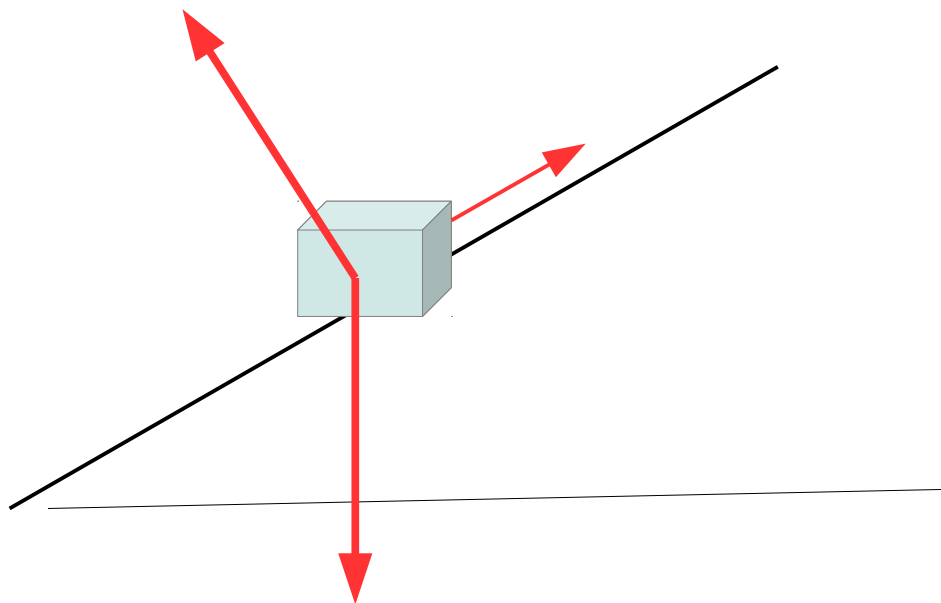
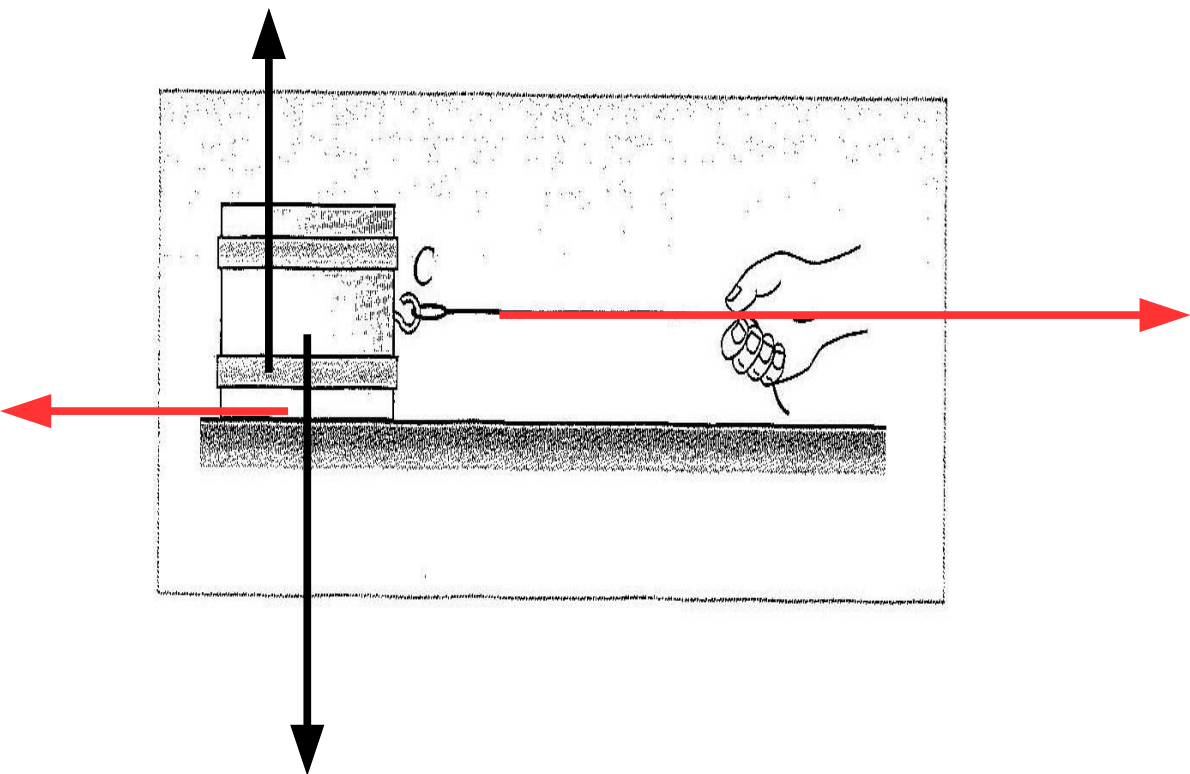


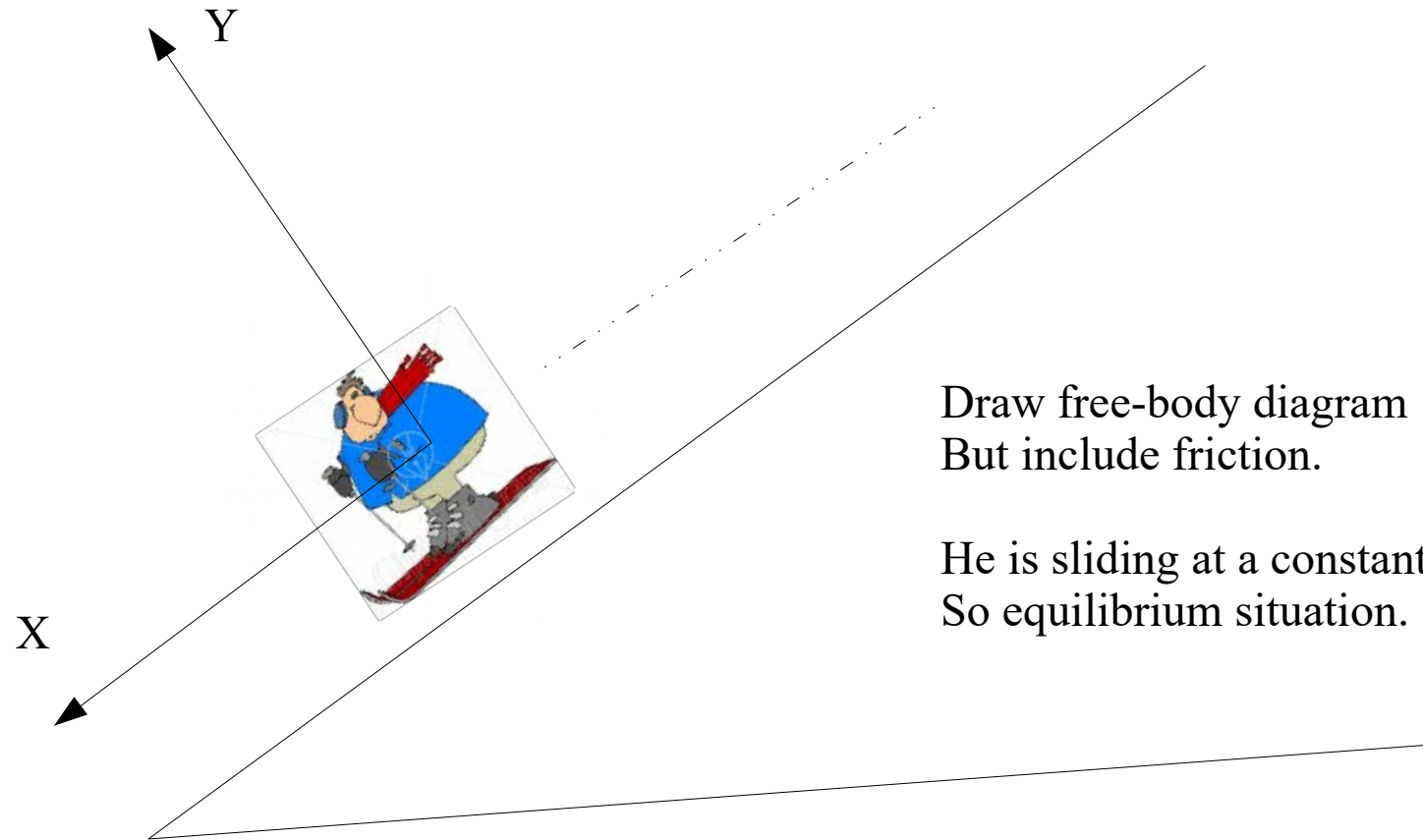


Four Forces on an Airplane

Glenn
Research
Center



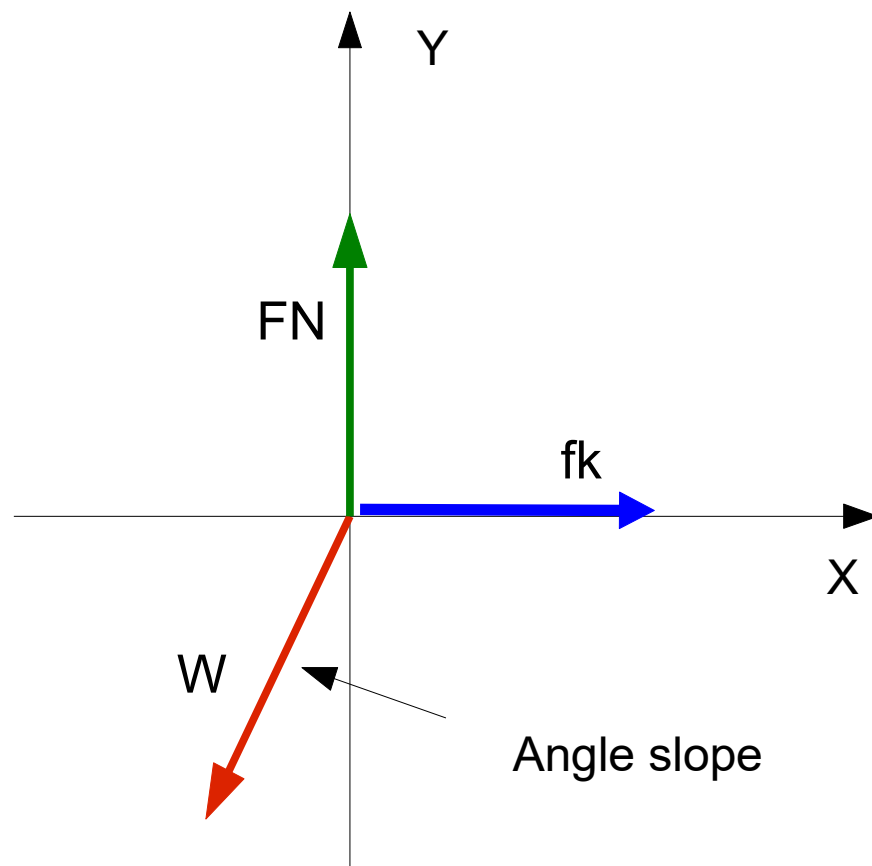
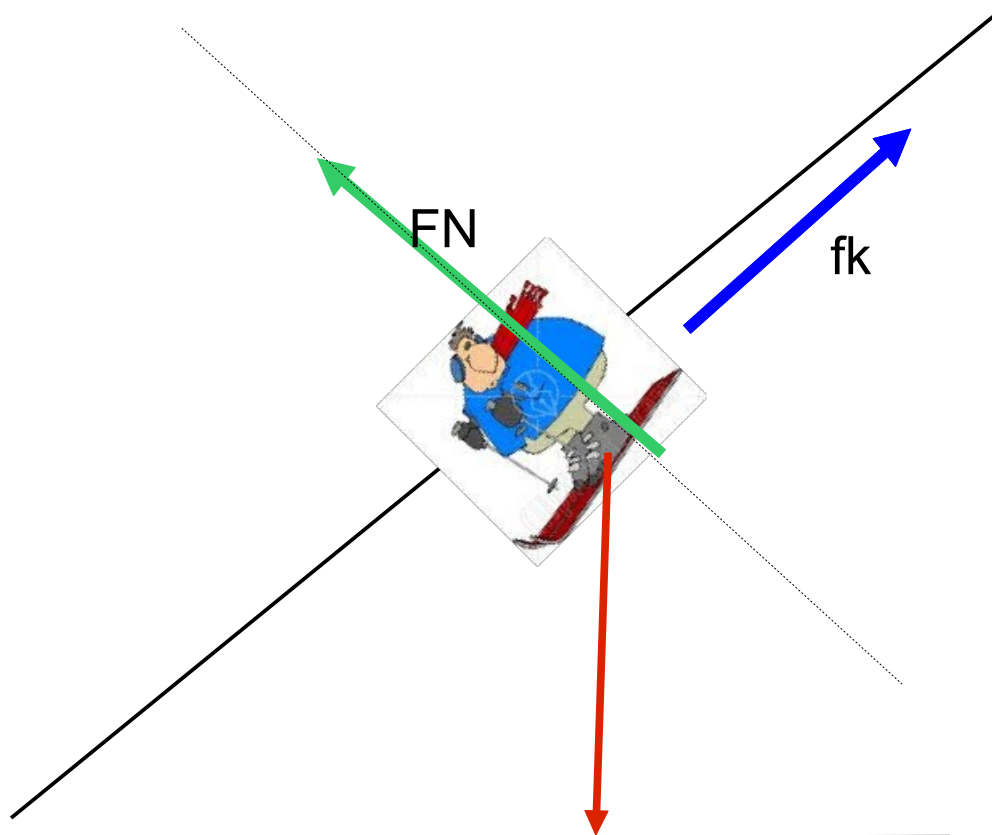




Draw free-body diagram
But include friction.

He is sliding at a constant speed.
So equilibrium situation.

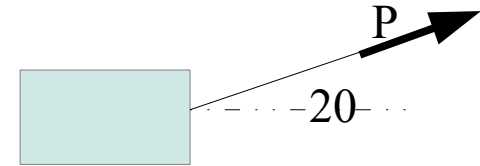
Draw the forces applied to the skier.
Then trace the forces in a free-body diagram



The effect that a force has in any direction can be found by calculating its component in that direction

1) A 2.5kg brick is being pulled by a cord (at a constant speed) that makes an angle of 20 degrees with the horizontal and has a 7N (P) of tension in it.

The forces are: tension, weight, normal, friction



A) find the weight of the brick

B) draw a free-body diagram with the 4 forces

C) write the forces in standard notation.

D) build a table with the components of the forces.

Use net force = 0 to find the normal force and the friction.

P

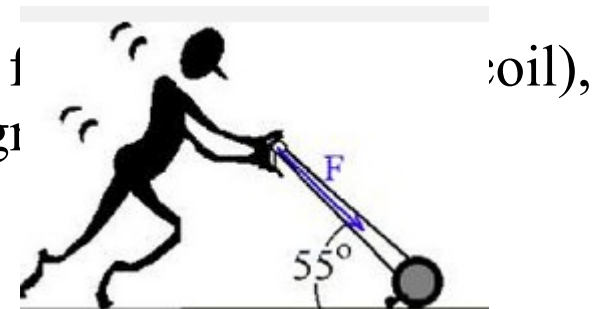
2) You are pushing a lawn mower at a constant speed. (so it is an equilibrium) The mass of the mower is 22 kg (weight is 220N) .

and is being pushed with a force of 150N (magnitude of P) against friction.

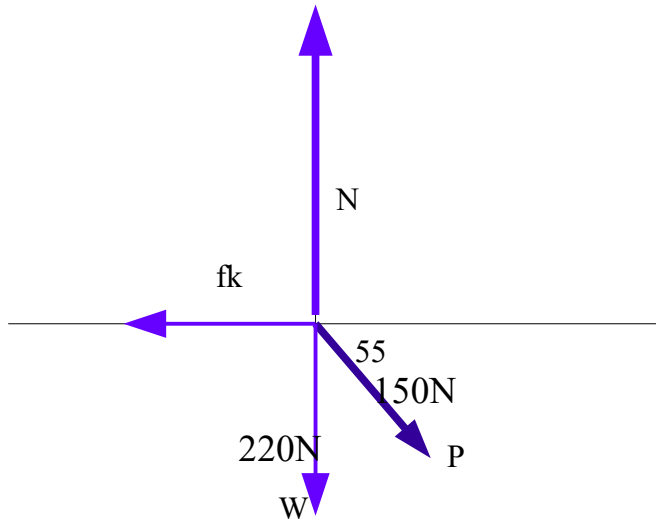
The shaft makes an angle of 55 degrees with ground. See figure.

So the forces are the push down along the shaft, the normal force, the weight and the friction opposing the motion along the ground.

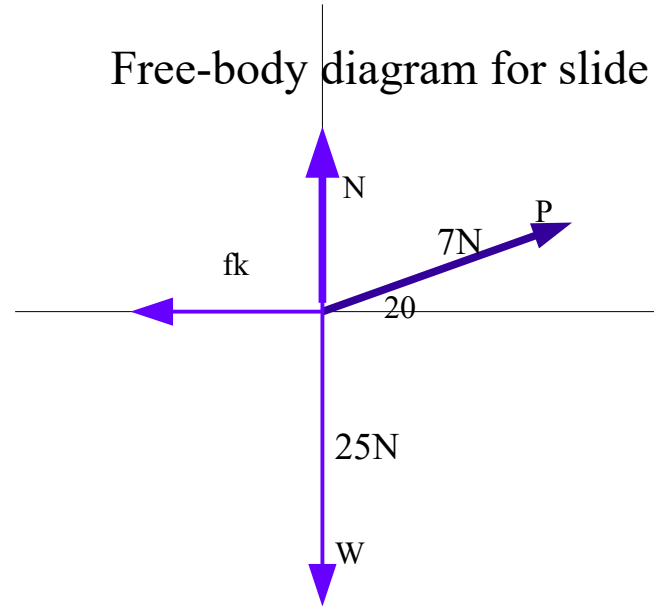
Same questions as above.



Free-body diagram for slide 61 Q2.



Free-body diagram for slide 61 Q1.



3) A helium balloon is in equilibrium as shown. The balloon weighs $W = 25\text{N}$ and is acted

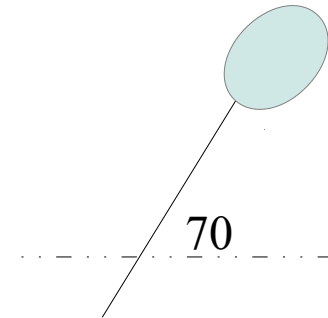
Upon by 4 forces. The weight (down), the buoyancy B (up) the tension T in the rope that is holding down the balloon. The force P of the wind pushing the Balloon to the right. The tension T is 16N @ 70 degrees.

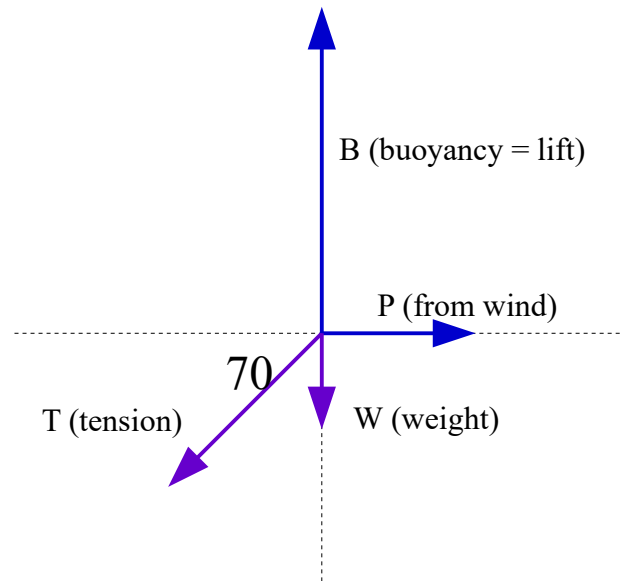
A) draw the forces on the balloon (solution next slide)

B) draw a free-body diagram

C) build the table with the components of the forces.

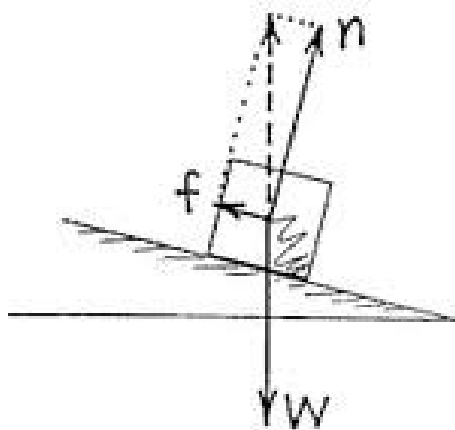
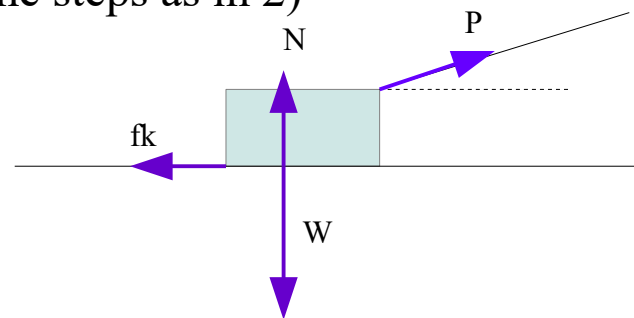
C) Find the buoyancy and the force exerted by the Wind.





4) A sled is being pulled along a horizontal road at a constant speed by means of a rope that makes an angle of 25 degrees with the horizontal. If the friction between the sled and the snow is 85N, how much is the tension P in the rope ? (free-body diagram please – follow same steps as in 2)

Use $\sum x\text{-components} = 0$ to find P .



5).

This block is not moving on the inclined plane. If the weight is 10N and the angle of the plane with horizontal 20 degrees:

A) find the frictional force f

B) find the normal N (recoil force from the ground)

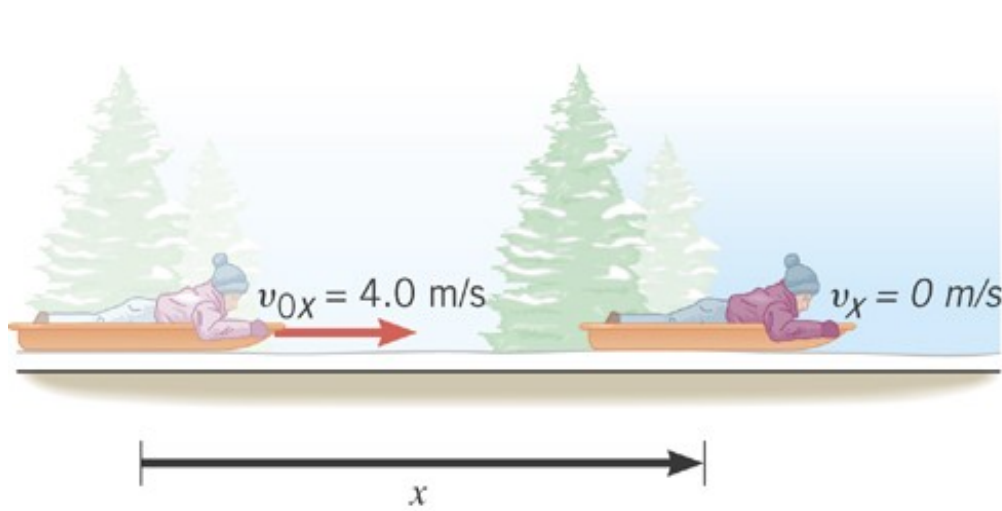
Static friction opposes the *impending* relative motion between two objects.

Kinetic friction opposes the relative sliding motion motions that actually does occur.

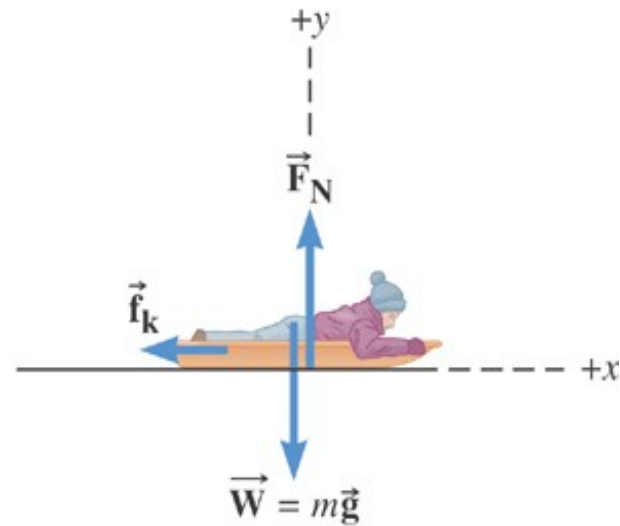
$$f_k = \mu_k F_N$$



$0 < \mu < 1$ is called the coefficient of kinetic friction.



(a)



(b) Free-body diagram
for the sled and rider

The sled comes to a halt because the kinetic frictional force opposes its motion and causes the sled to slow down.

Suppose the coefficient of kinetic friction is 0.05 and the total mass is 40kg. What is the kinetic frictional force?

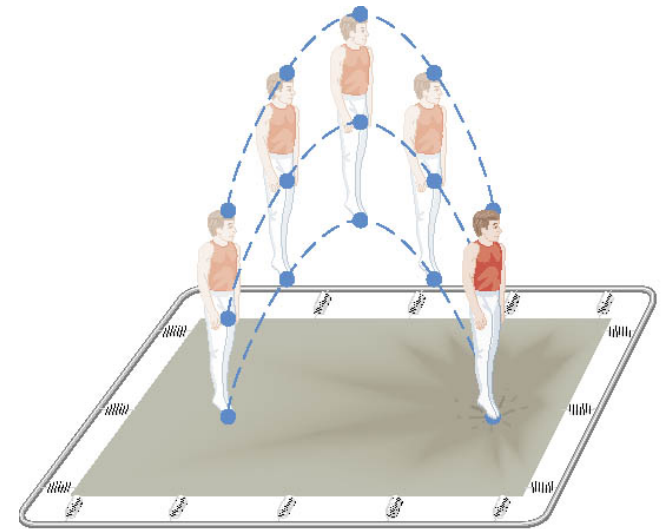
Rotational Dynamics



Source: Physics / Edition 8
by John D. Cutnell, Kenneth W. Johnson, Cutnell

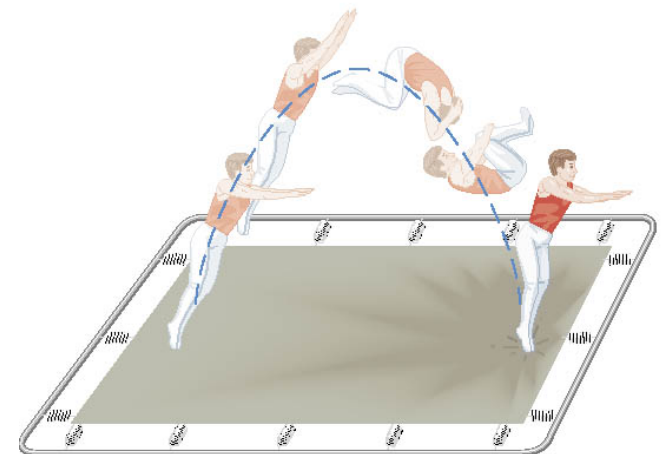
-The physics of everyday phenomena: a conceptual introduction to physics
by W. Thomas Griffith, McGraw-Hill

In pure translational motion, all points on an object travel on parallel paths.



(a) Translation

The most general motion is a combination of translation and rotation.



(b) Combined translation and rotation

According to Newton's second law, a net force causes an object to have a linear acceleration.

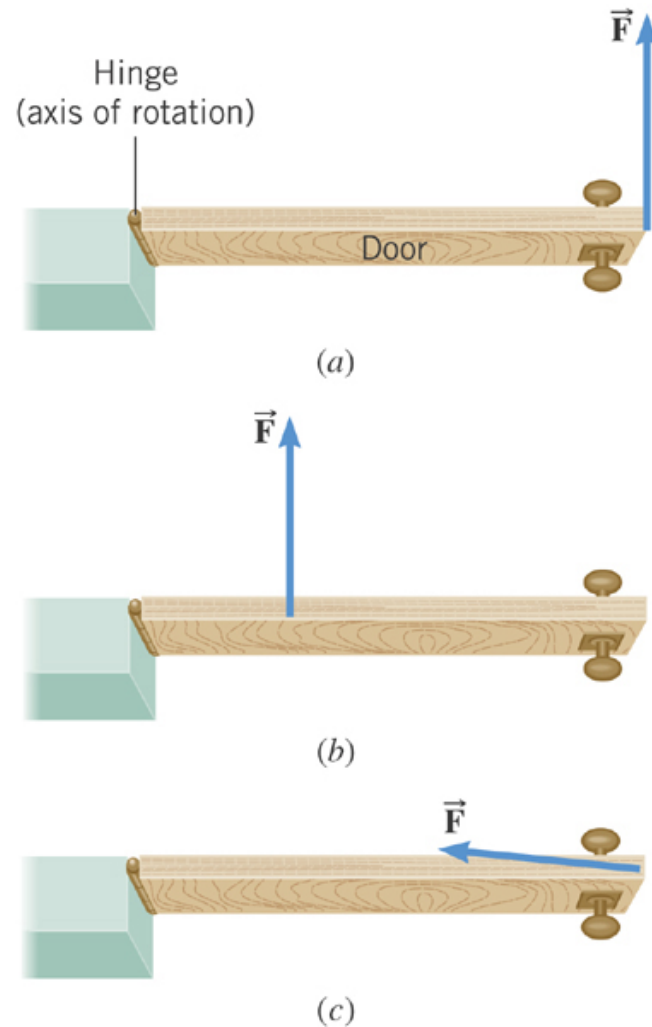
What causes an object to have an *angular acceleration*?

TORQUE τ

MEASURES HOW EFFICIENT A FORCE IS IN APPLYING A TWIST :

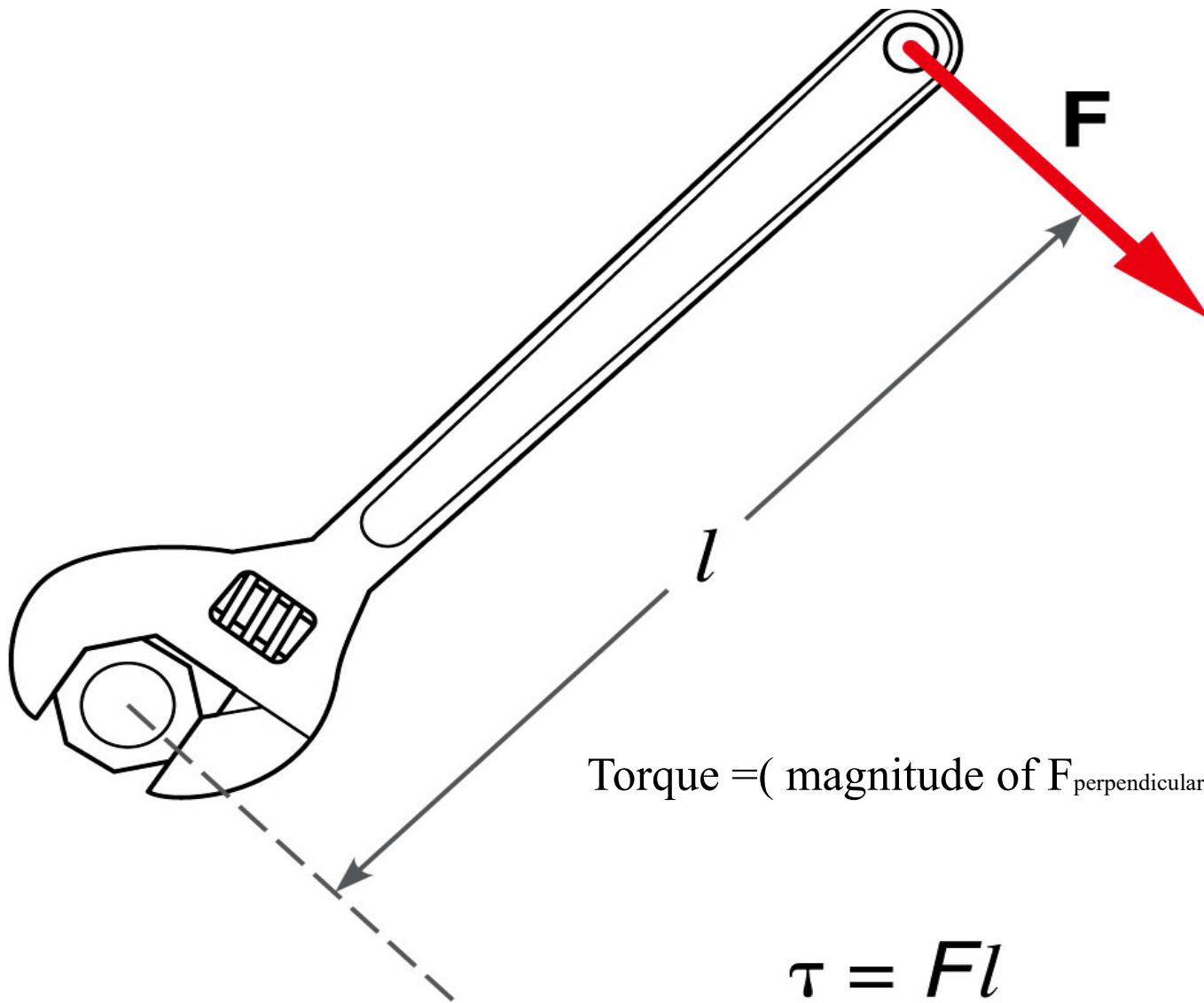
**Depend on the distant to the axis of rotation
and the direction of the force.**

Torque = how efficient is a force in rotating a rigid object (twist)
In which case the torque is the largest ? (more efficient in opening the door).



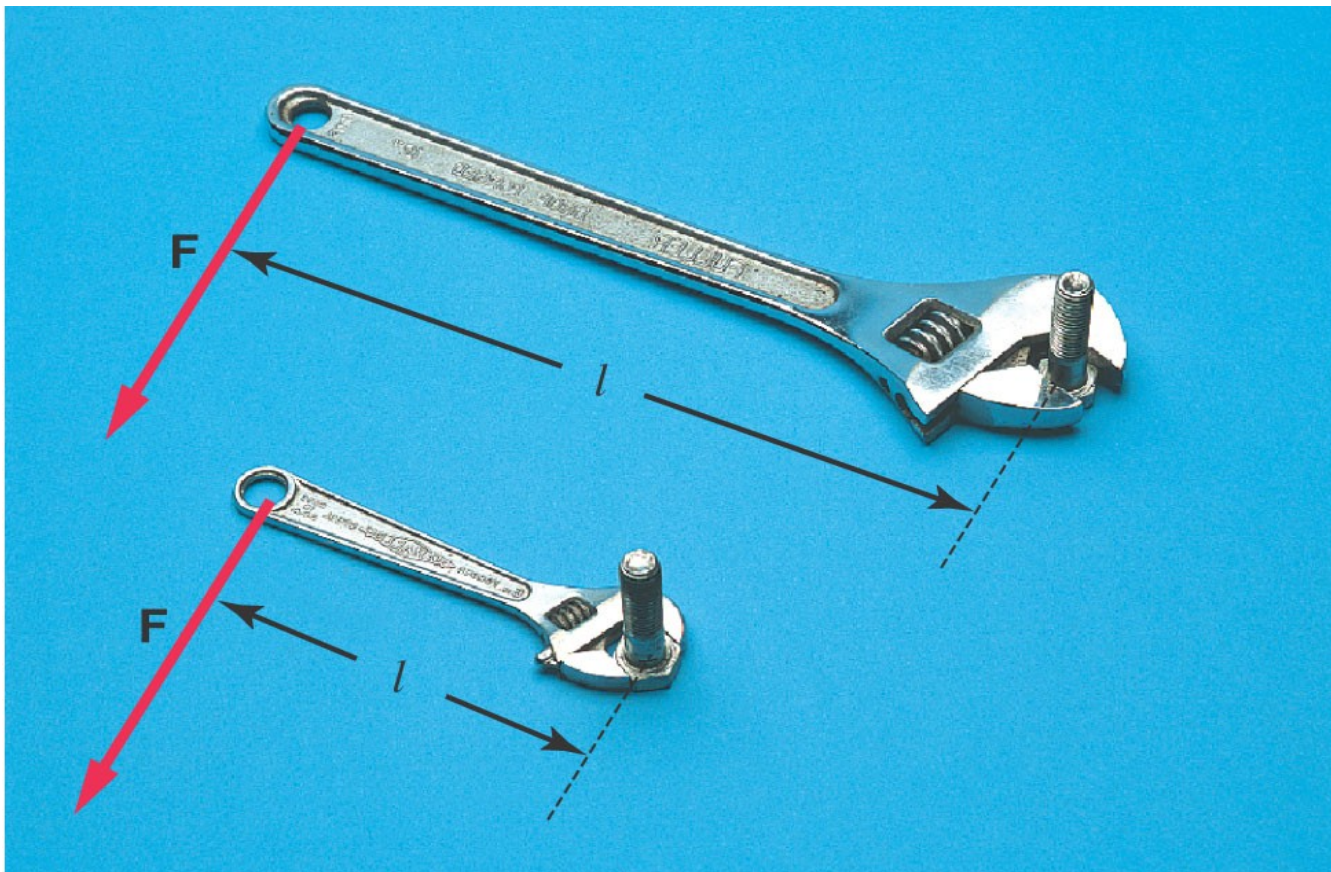
The amount of torque depends on where and in what direction the force is applied, as well as the location of the axis of rotation.

Any Suggestion for the expression of a torque when the force is normal
To the door ?



To be more efficient you can: increase F or increase F a better twist.

If $F=50\text{N}$ and the length of the wench is 24cm , find the torque. (cm to m)



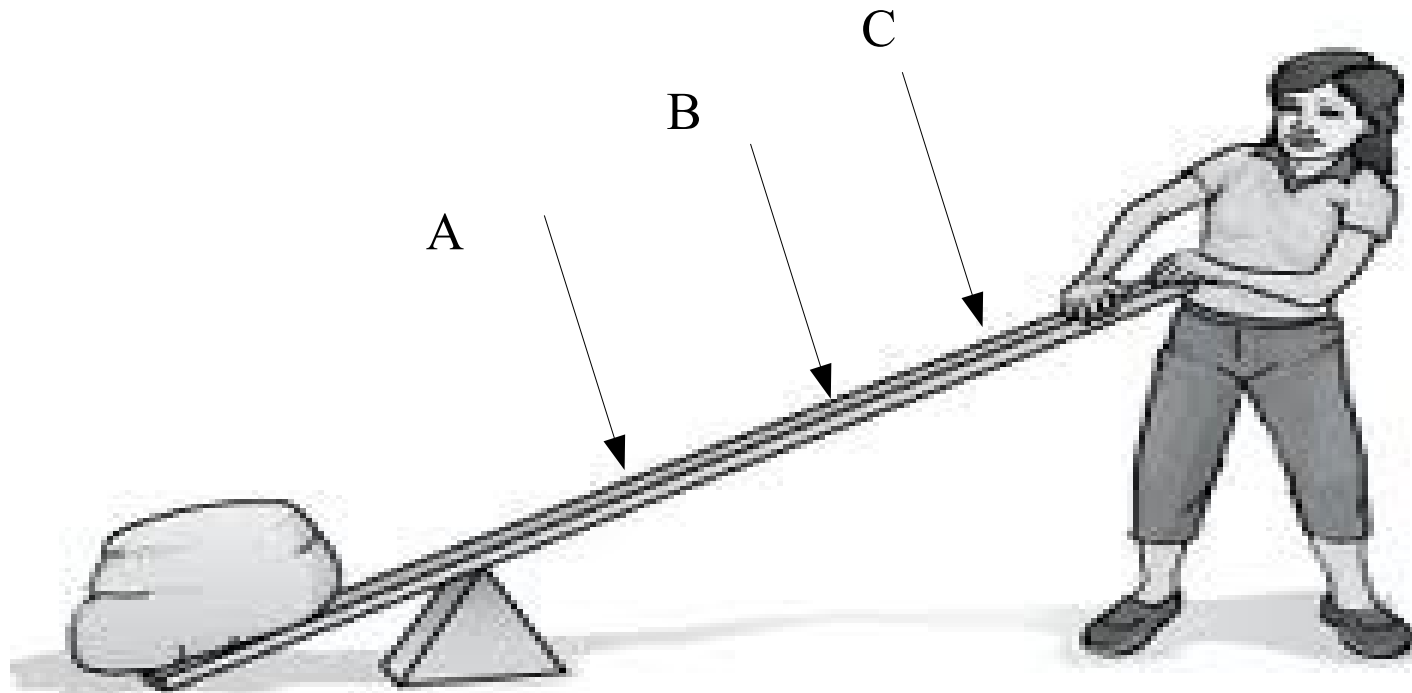
If l is divided by 2, the new torque :

- A) is the same
- B) is double
- C) is half

So why do we use long wrench ?

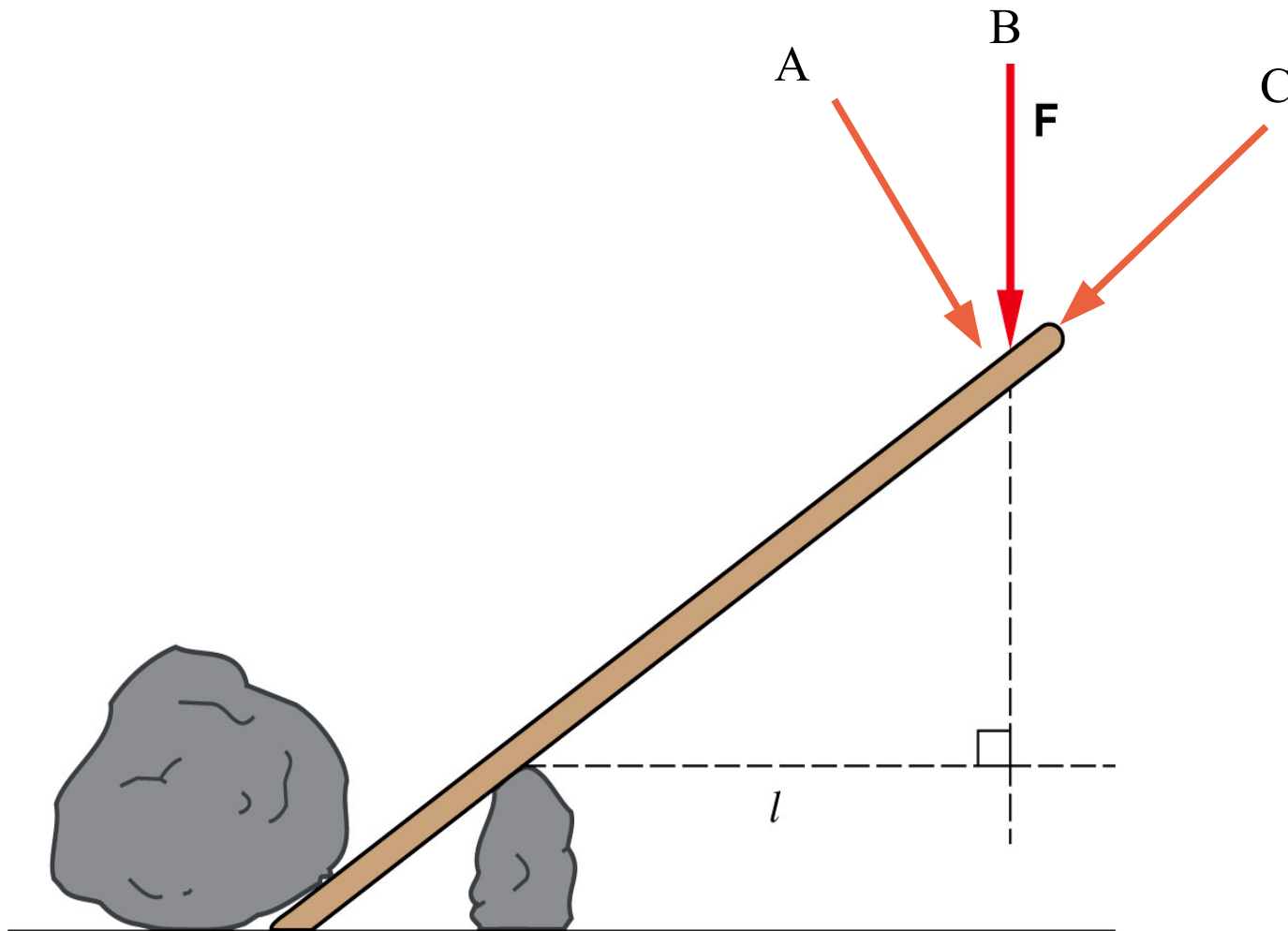
Where to push to make the torque the largest ?
(will make easier to push) A or B or C ?

The force applied is the same.



demo

Which direction will produce the larger twist ? A? B? C?
(more efficient for The same magnitude?)

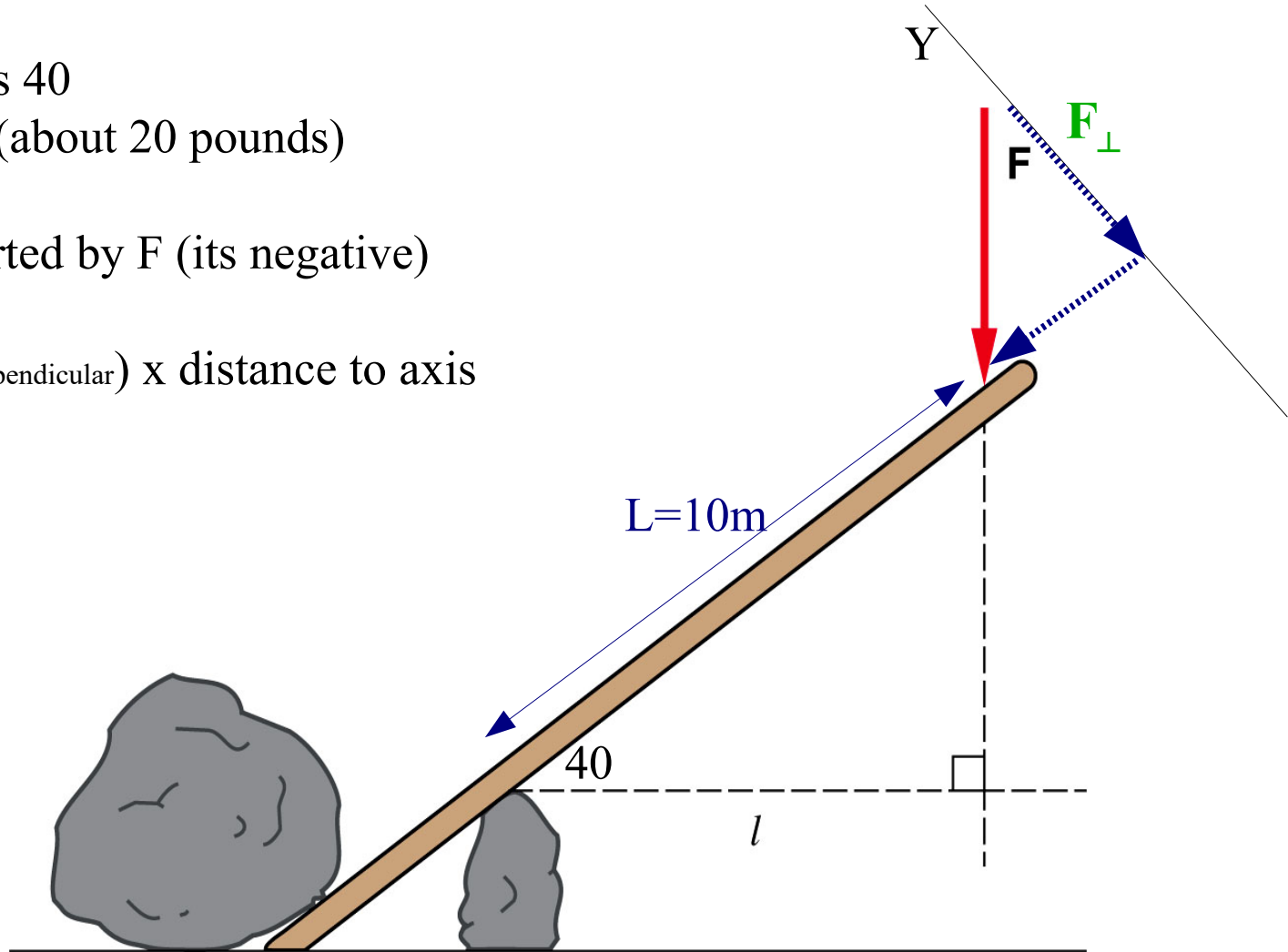


Suppose the angle is 40

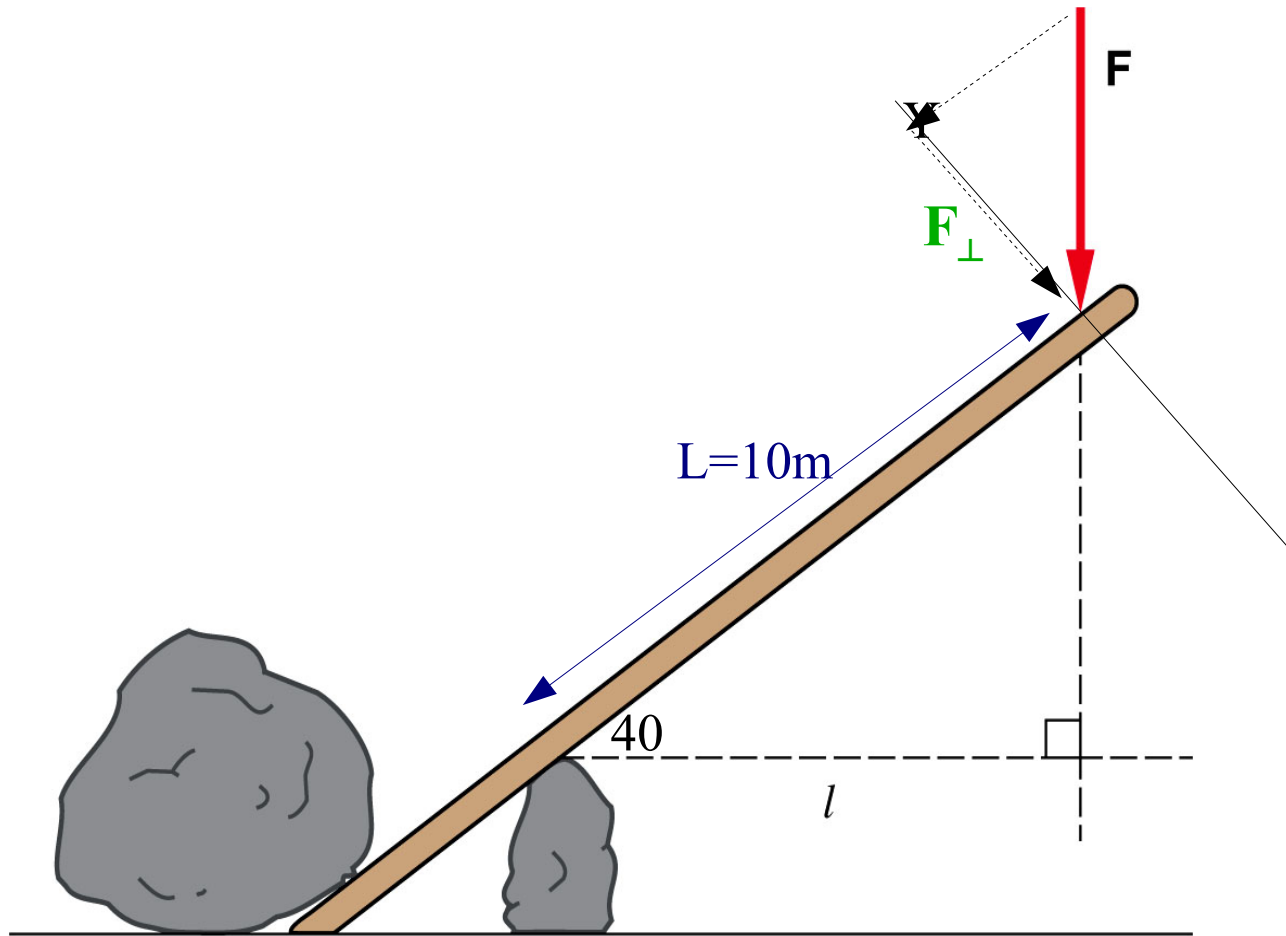
Suppose F is 100N (about 20 pounds)

Find the torque exerted by F (its negative)

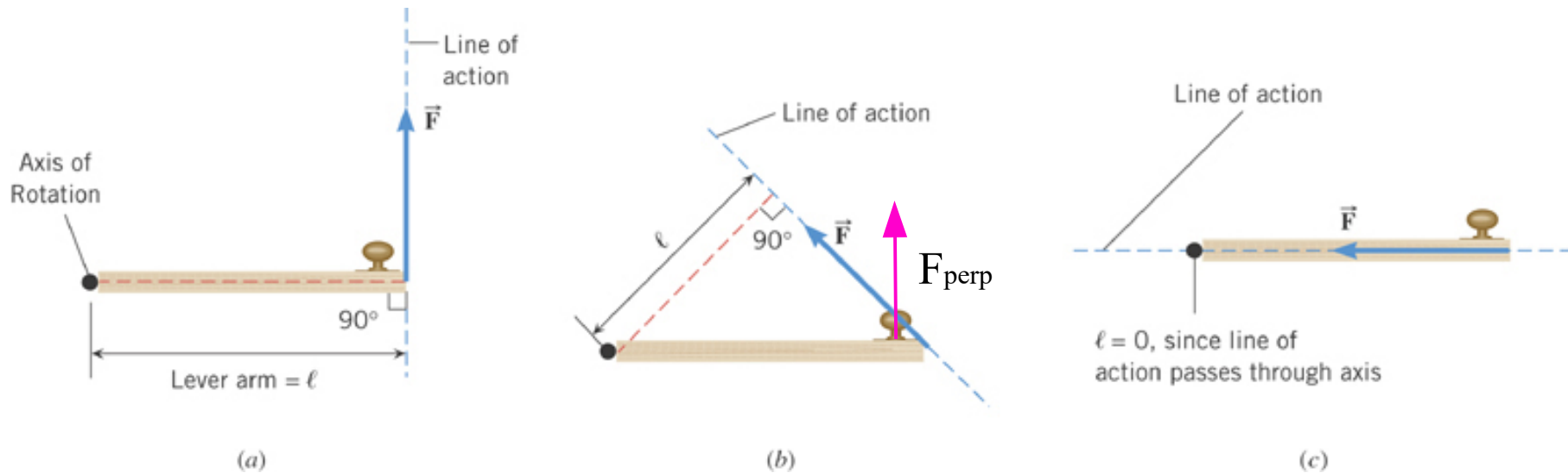
- (magnitude of $F_{\text{perpendicular}}$) x distance to axis



Only the component F_{\perp} (normal) is doing the job.



How to find ℓ the lever arm = perpendicular distance between the axis and the line of action of the force.

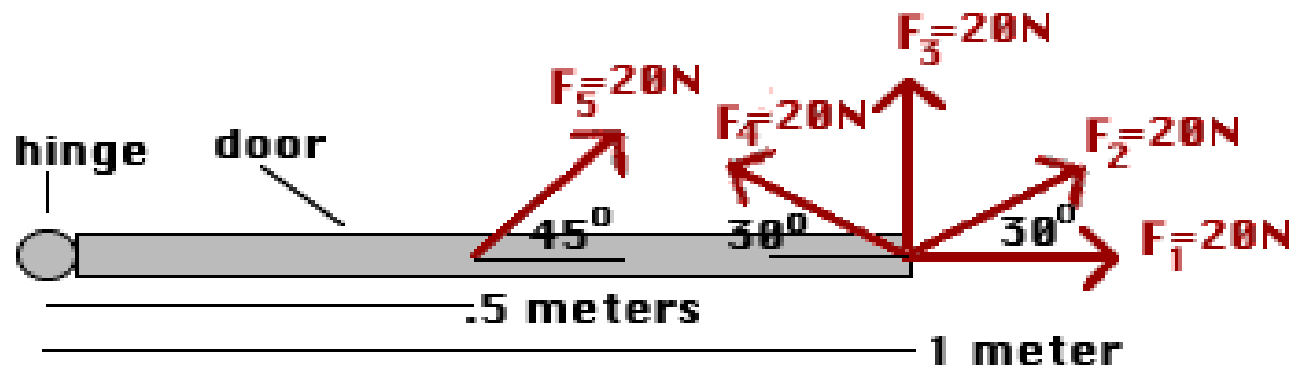


DEFINITION OF TORQUE

$$\tau = F_{\text{perpendicular component}} L$$

Direction: The torque is positive when the force tends to produce a counterclockwise rotation about the axis.

SI Unit of Torque: newton x meter (N·m)



Find the torque exerted by each force.

9.1.6. An object with a triangular cross-section is free to rotate about the axis represented by the black dot shown. Four forces with identical magnitudes are exerted on the object. Which one of the forces, if any, exerts the largest torque on the object?

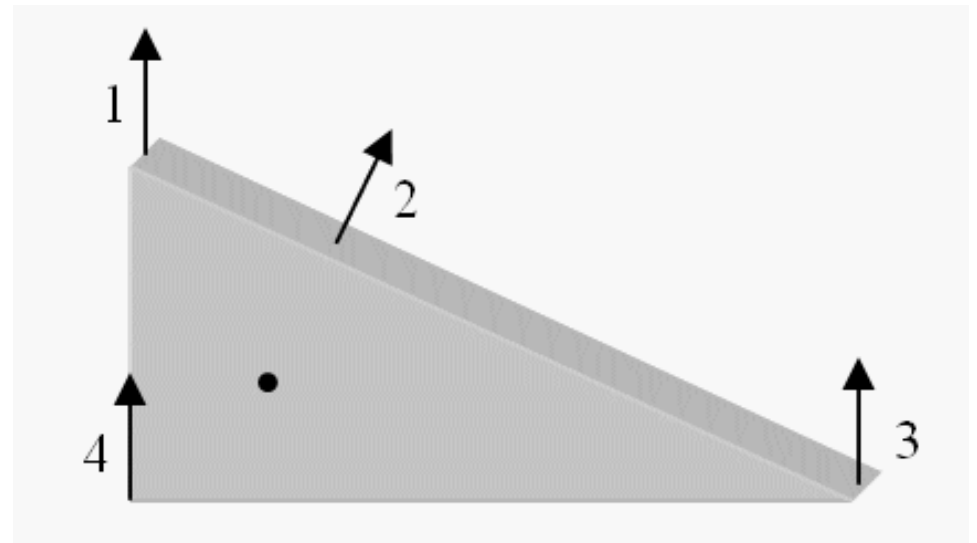
a) 1

b) 2

c) 3

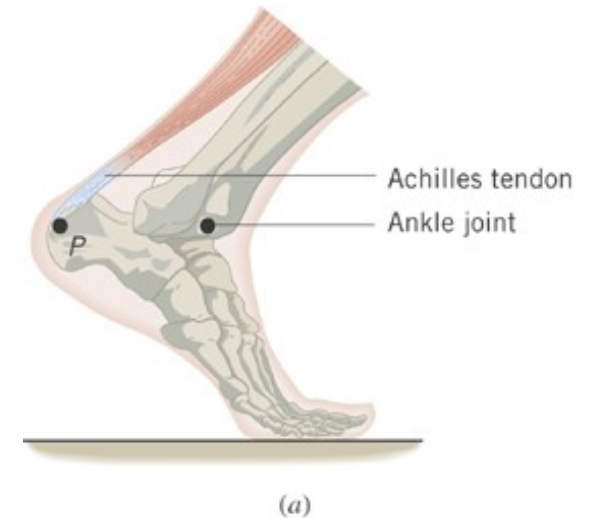
d) 4

e) The same torque is exerted by each force.

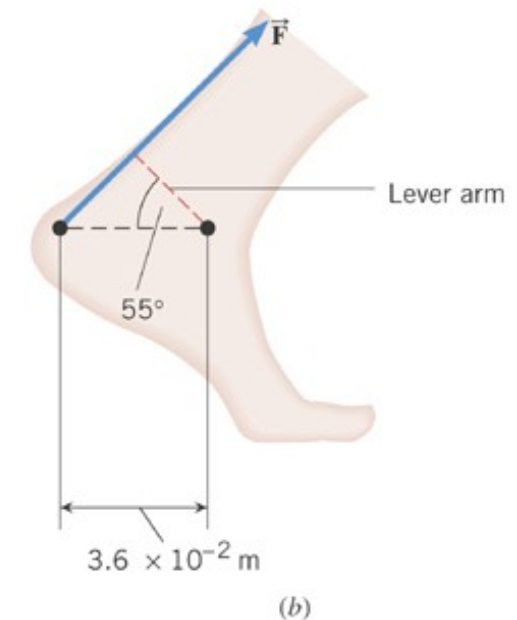


Example 2 The Achilles Tendon

The tendon exerts a force of magnitude 790 N. Determine the torque (magnitude and direction) of this force about the ankle joint.



$$\tau = F_{\text{perp}} \ell \text{ or}$$

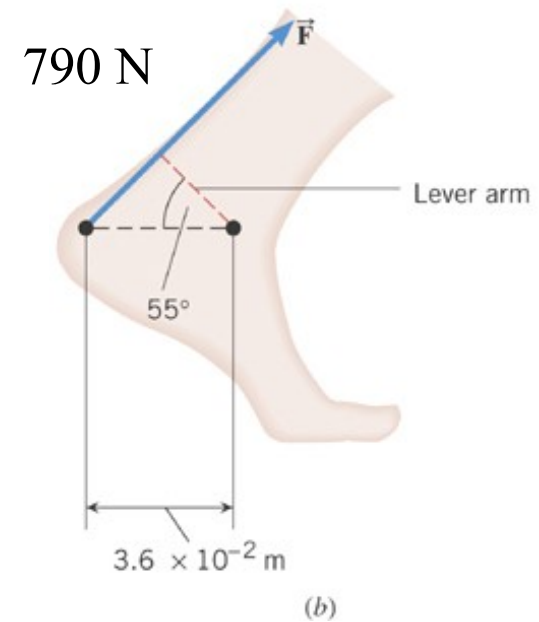
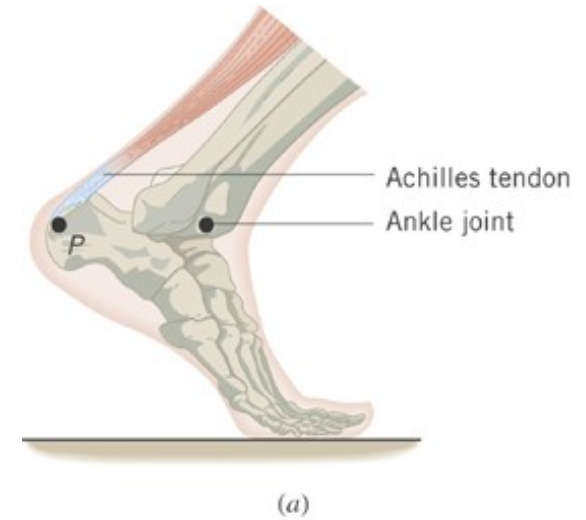


9.1 The Action of Forces and Torques on Rigid Objects

$$\tau = F\ell$$

$$\cos 55^\circ = \frac{\ell}{3.6 \times 10^{-2} \text{ m}}$$

$$\begin{aligned}\tau &= (720 \text{ N})(3.6 \times 10^{-2} \text{ m})\cos 55^\circ \\ &= 15 \text{ N} \cdot \text{m}\end{aligned}$$



1) The radius of the wheel of fortune is 1.2m, and the operator applies a force of 45N Tangentially to get it spinning. What torque has he supplied ?

2) a 32 kg child sits on a seesaw. If she is 2.2 m from the pivot, what is the torque That her weight exerts, making the seesaw rotate around the pivot ?

3) A torque of 30N.m is required to turn the steering wheel of a car. If the radius Of the wheel is 26cm/ How much force is needed.

4) You are using a crowbar 2.4 m long, pivoted at 0.20m from the end, to lift at the end A rock that weighs 750N. How much force do you have to exert ?

5) suppose that a merry-go-round is rotating at the rate if 10 rev/min

A) Express this rotational velocity in rev/s (rev means revolution = 1 tour)

B) Express this rotational velocity in rad/s (1 revolution = 2π rad)

6) The popular music record used to have a rate of 45RPM .

A) express this velocity in rev/s

B) Through how many revolutions does the record turn in a time of 5s?
(its a rate. Use proportion)

7) Suppose a disk rotates through 3 revolutions in 4 seconds

A) What is its displacement in radians in this time ? ((1 revolution = 2π rad)

B) What is its average rotational velocity in rad/s

8) A bicycle wheel is rotationally accelerated at the constant rate of 1.2 rev/s/s

A) If it starts from rest, what is its rotational velocity after 4s

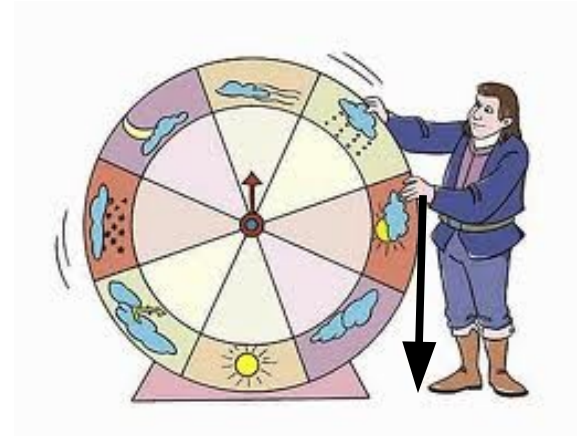
B) Through how many revolutions does it turn in this time ?

(like for kinematics : # of revolution = $\frac{1}{2}$ (acceleration in rev/s/s) x (time squared) + (initial rotational velocity) x (time)

9) The rotational velocity of a merry go round increases at a constant rate from 1.0 rad/s to 1.8 rad/s in a time of 4s.

What is the rotational acceleration of the merry go round ?

10) A force of 50N is applied at the end of a wrench handle that is 24cm long. The force is applied in a direction perpendicular To the handle . A) what is the torque applied to the nut by the wrench B) what would be the torque if the force were applied half Way up instead of at the end ?

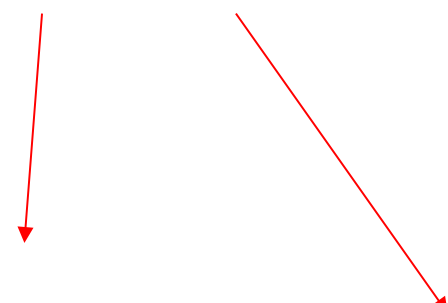


9.2 *Rigid Objects in Equilibrium*

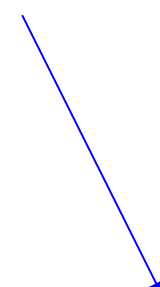
If a rigid body is in equilibrium, neither its linear motion nor its rotational motion changes.

$$a_x = a_y = 0$$

$$\alpha = 0$$


$$\sum F_x = 0$$

$$\sum F_y = 0$$


$$\sum \tau = 0$$

EQUILIBRIUM OF A RIGID BODY

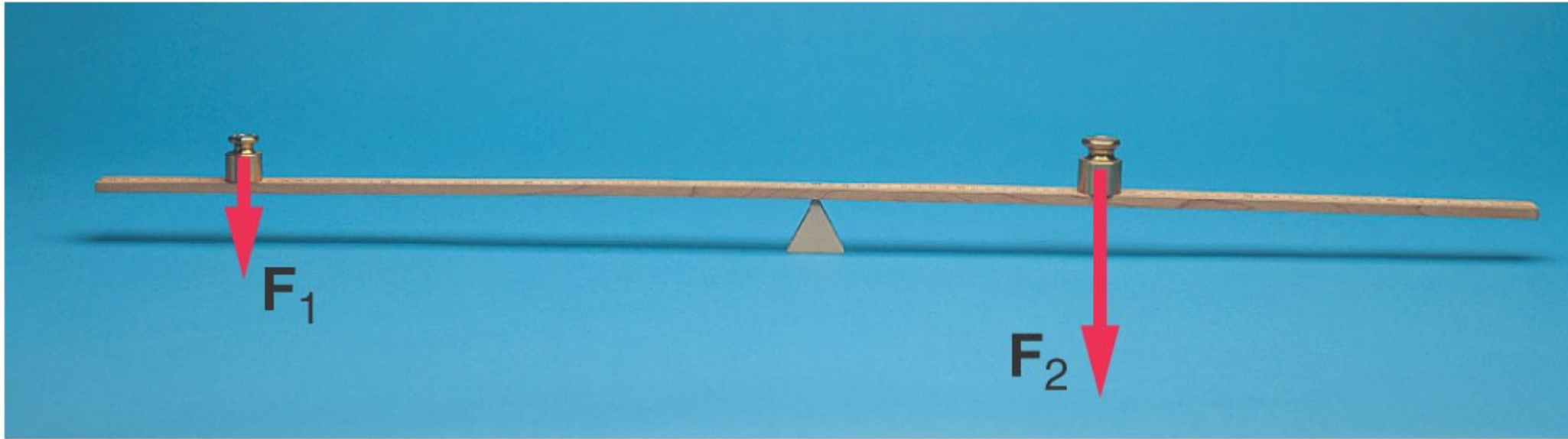
A rigid body is in equilibrium if it has zero translational acceleration and zero angular acceleration. In equilibrium, the sum of the externally applied forces is zero, and the sum of the externally applied torques is zero.

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum \tau = 0$$

Example of equilibrium:



https://phet.colorado.edu/sims/html/balancing-act/latest/balancing-act_en.html

If mass 2 = 1kg then $F_2 = \underline{\hspace{2cm}} \text{ N}$

If mass 1 = 500g, the $F_1 = \underline{\hspace{2cm}} \text{ N}$

If the masses are in equilibrium :

$$+ F_1 l_1 - F_2 l_2 = 0$$

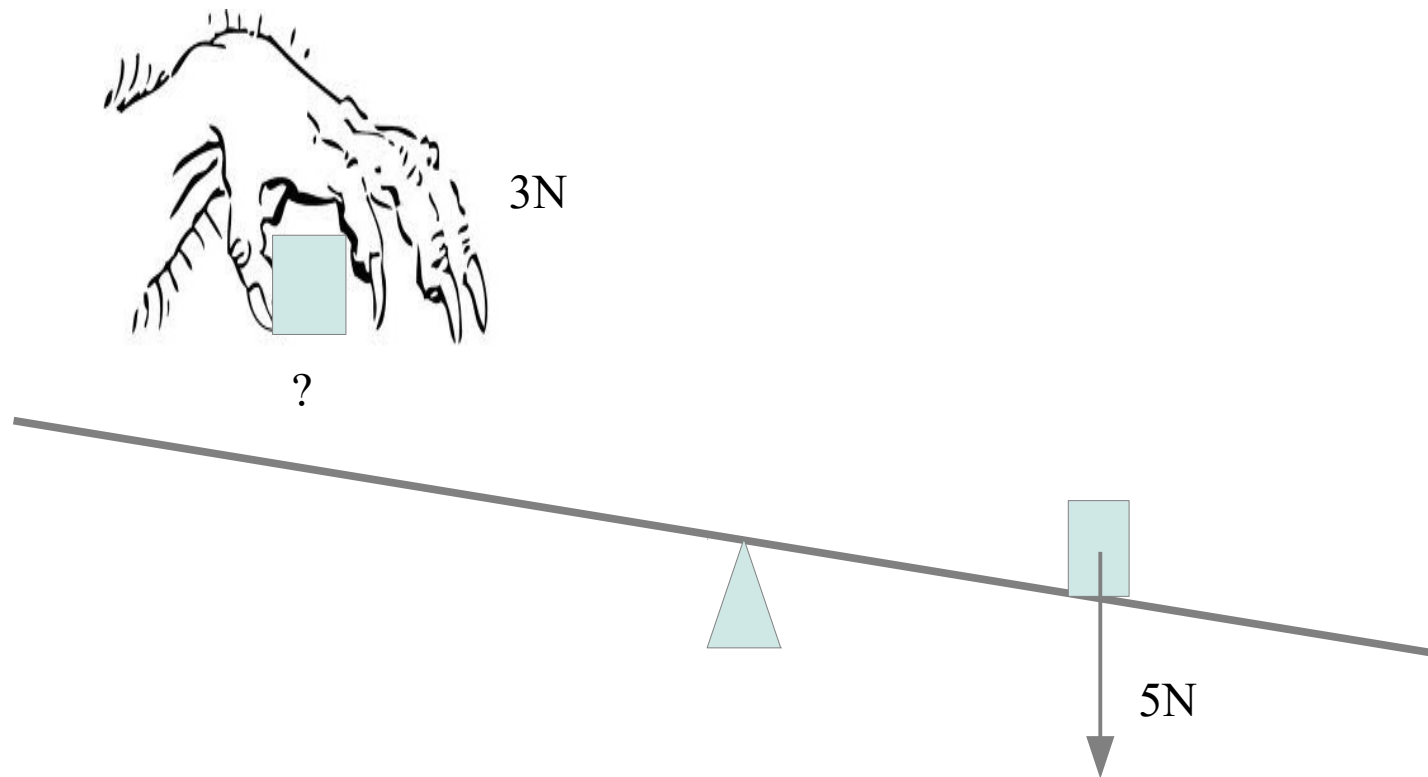
If $d_2 = 10 \text{ cm}$, find d_1 .

Draw a free body diagram and use up = right to find the force exerted by the Fulcrum on the ruler.

Suppose we have a 3N weight that we want to balance against a 5 N weight
On a stick, which is balanced when no mass are in place. The 5N weight is placed
20cm to the right of the fulcrum. (see below)

A) What is the torque produced by the 5N weight (cm to m and sign !!)

B) How far would we have to place the 3N weight from the fulcrum to balance the
System ?



REMEMBER: EQUILIBRUM MEANS :

I. $\sum(\text{torques}) = 0$

$$\pm F_1 l_1 \pm F_2 l_2 \pm F_3 l_3 + \dots = 0$$

With $F_1, F_2, F_3 \dots$ are the magnitude of the y-components of the forces
(perpendicular components)

l_1, l_2, l_3 are the distances from the fulcrum (axis).

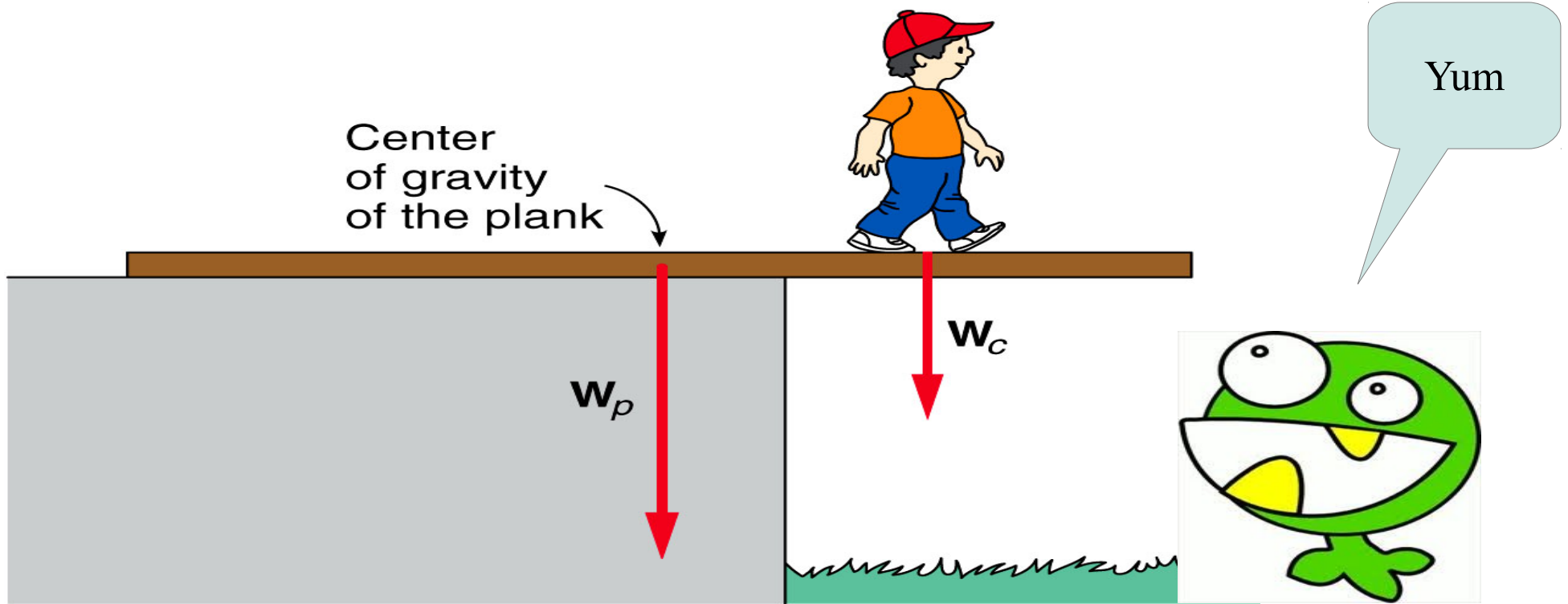
If you use this notation, you don't need to convert to meters or to newtons
As long as all the forces are expressed in the same unit and the
Distances are expressed in the same unit.

+ if counterclockwise motion and – if clockwise motion about the axis

II. $\sum (F_y) = 0$ up means $F_y > 0$ and down means $F_y < 0$

III. $\sum (F_x) = 0$ right means $F_x > 0$ and left means $F_x < 0$

OTHER example of equilibrium:

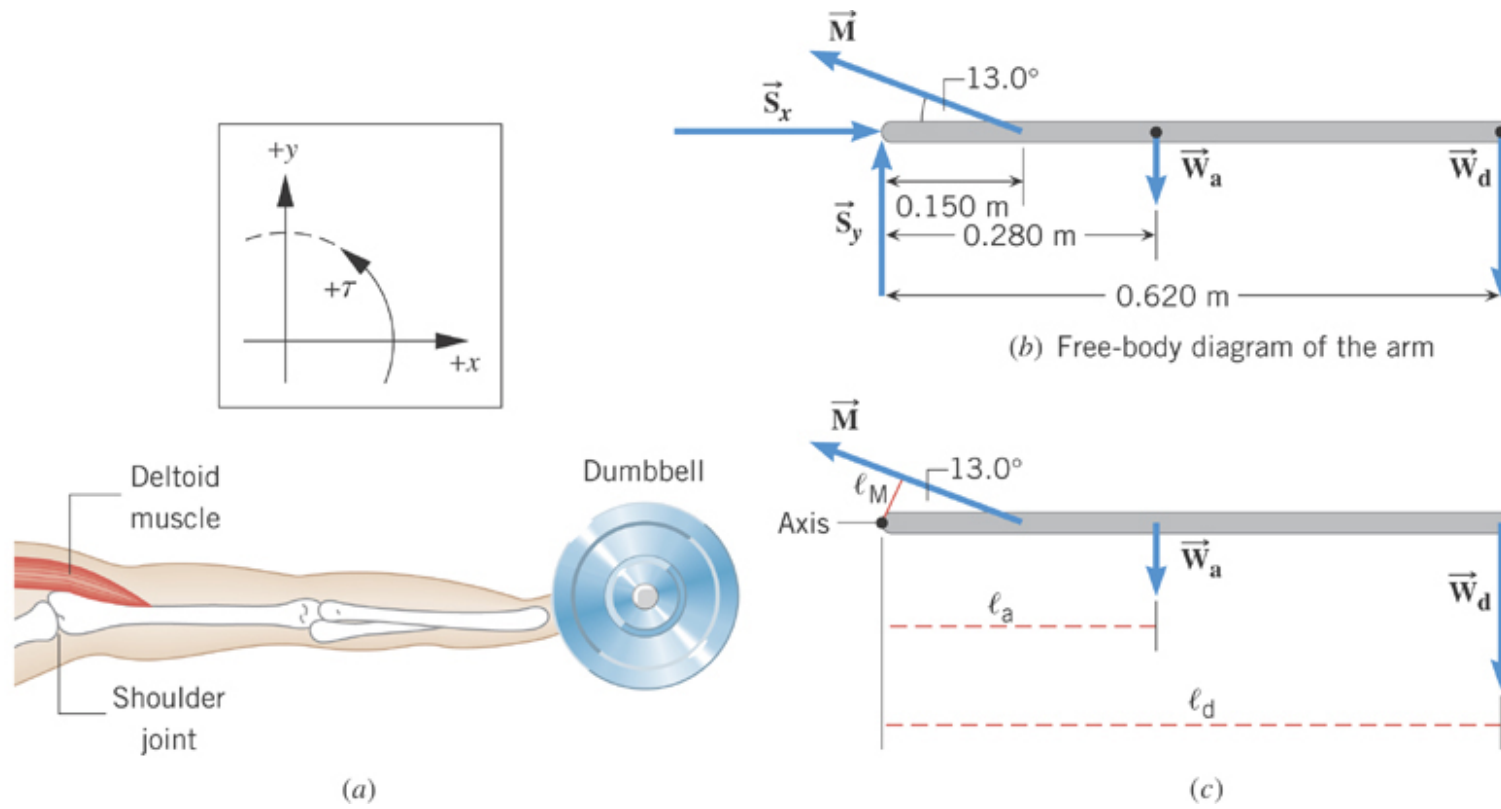


A 4m long plank with a weight of 80lb is placed on a dock with 1 m of its length extended Over a sea of deadly grass. A boy with a weight of 150lb is standing on the plank and moving out slowly from the Edge of the dock.

A) What is the torque exerted by the weight of the plank about the pivot point at the edge of the dock ? (treat all the weights as acting through the center of gravity of The plank).

B) How far from the edge of the dock can the boy move until the plank is just on the verge Of tipping ? (and the boy on the verge to be eaten)

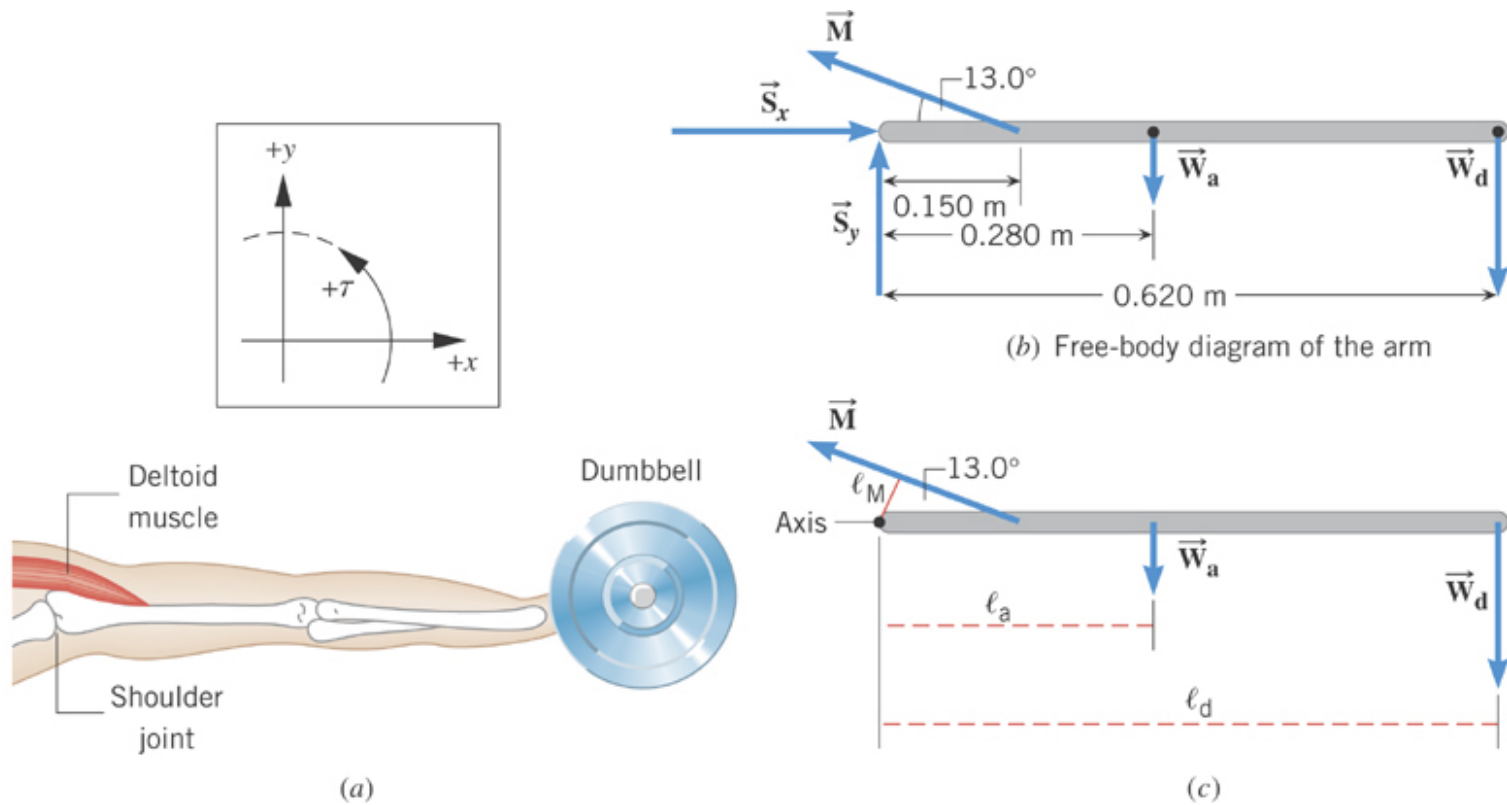
9.2 Rigid Objects in Equilibrium



Example 5 Bodybuilding

The arm is horizontal and weighs 31.0 N. The deltoid muscle can supply 1840 N of force. What is the weight of the heaviest dumbbell he can hold?

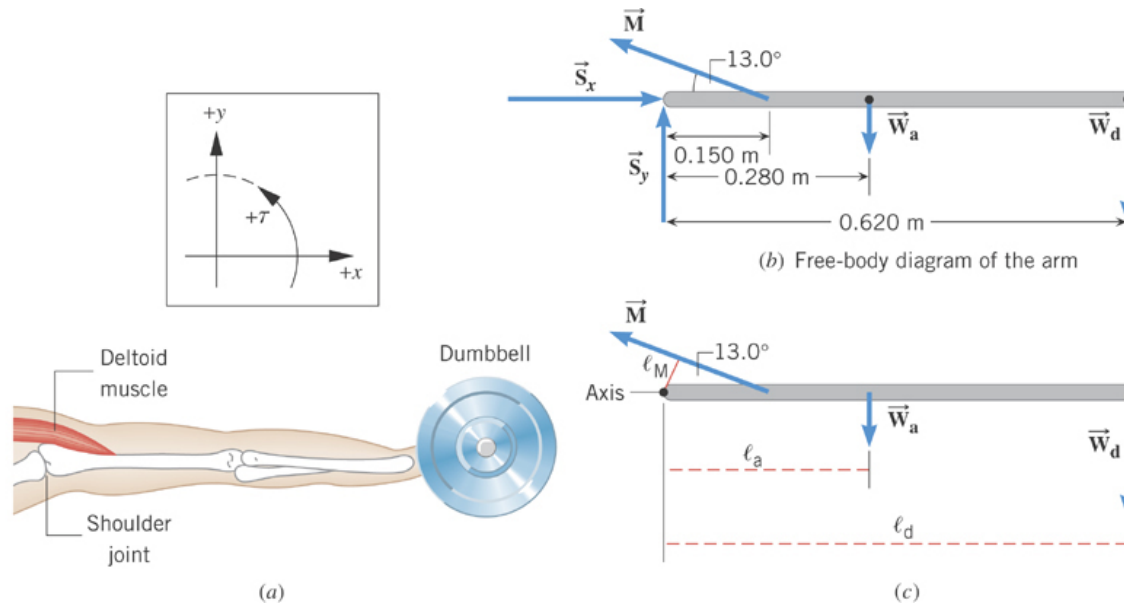
9.2 Rigid Objects in Equilibrium



$$\sum \tau = -W_a \ell_a - W_d \ell_d + M \ell_M = 0$$

$$\ell_M = (0.150 \text{ m}) \sin 13.0^\circ$$

9.2 Rigid Objects in Equilibrium



$$\begin{aligned}
 W_d &= \frac{-W_a \ell_a + M \ell_M}{\ell_d} \\
 &= \frac{-(31.0 \text{ N})(0.280 \text{ m}) + (1840 \text{ N})(0.150 \text{ m}) \sin 13.0^\circ}{0.620 \text{ m}} = 86.1 \text{ N}
 \end{aligned}$$

9.2.3. Consider the three situations shown in the figure. Three forces act on the triangular object in different ways. Two of the forces have magnitude F and one of the forces has a magnitude $2F$. In which case(s), if any, will the object be in equilibrium? In each case, the forces may act at the center of gravity or at the center of a corner.

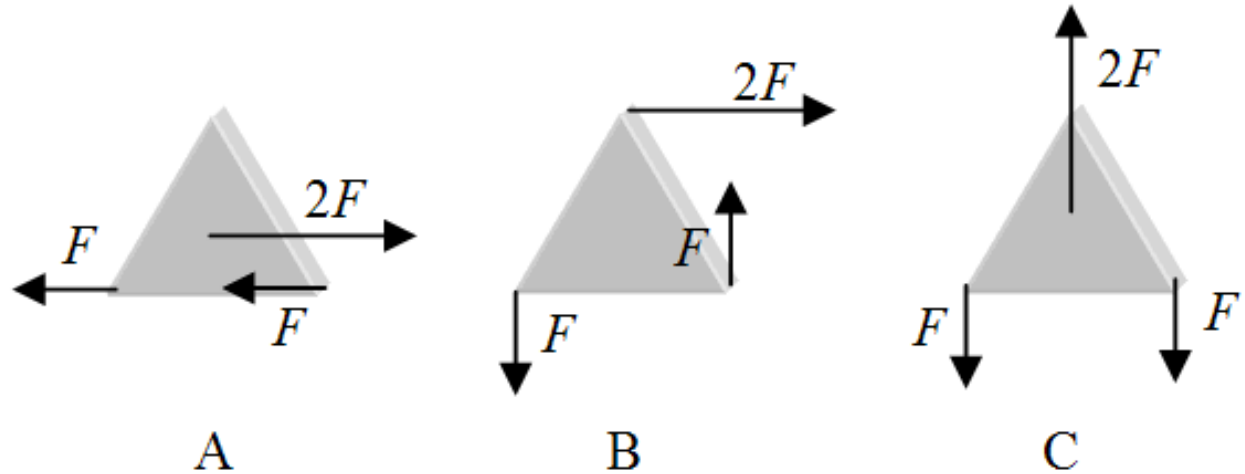
a) A only

b) B only

c) C only

d) A and C

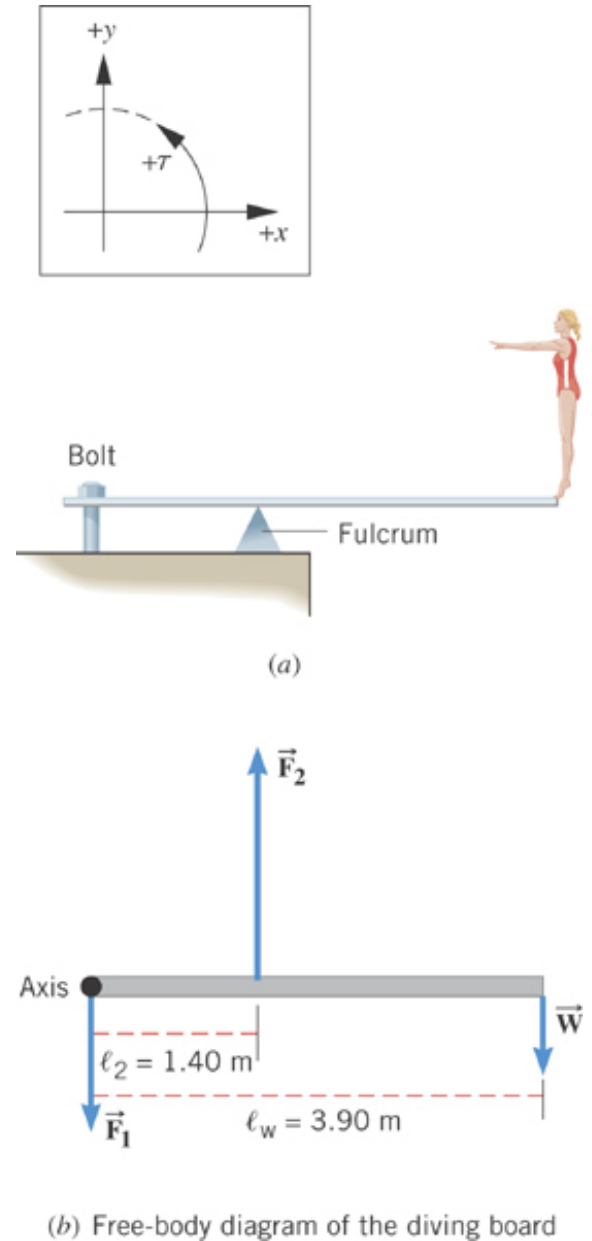
e) A and B



Example 3 A Diving Board

A woman whose weight is 530 N is poised at the right end of a diving board with length 3.90 m. The board has negligible weight and is supported by a fulcrum 1.40 m away from the left end.

Find the forces that the bolt and the fulcrum exert on the board.

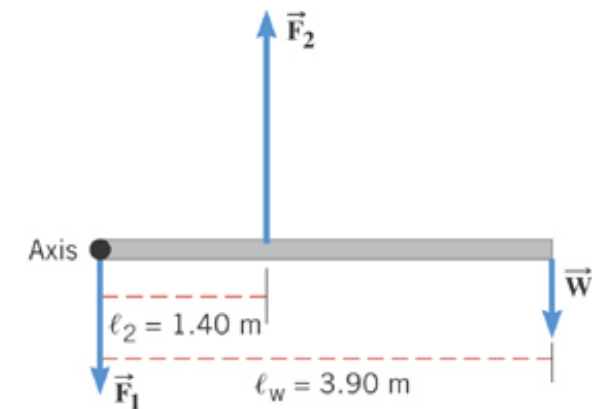
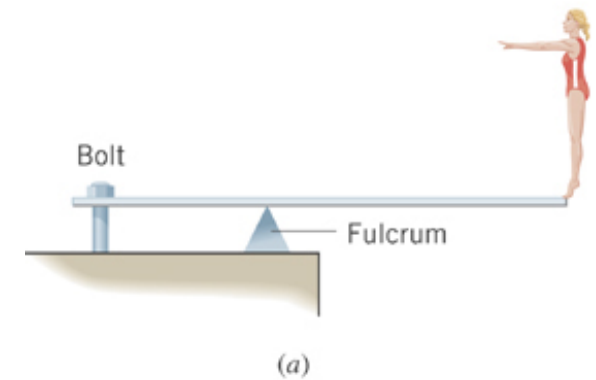
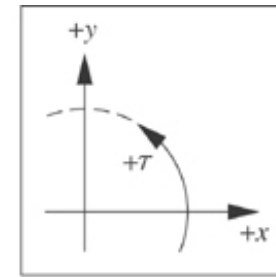


9.2 Rigid Objects in Equilibrium

$$\sum \tau = F_2 \ell_2 - W \ell_W = 0$$

$$F_2 = \frac{W \ell_W}{\ell_2}$$

$$F_2 = \frac{(530 \text{ N})(3.90 \text{ m})}{1.40 \text{ m}} = 1480 \text{ N}$$



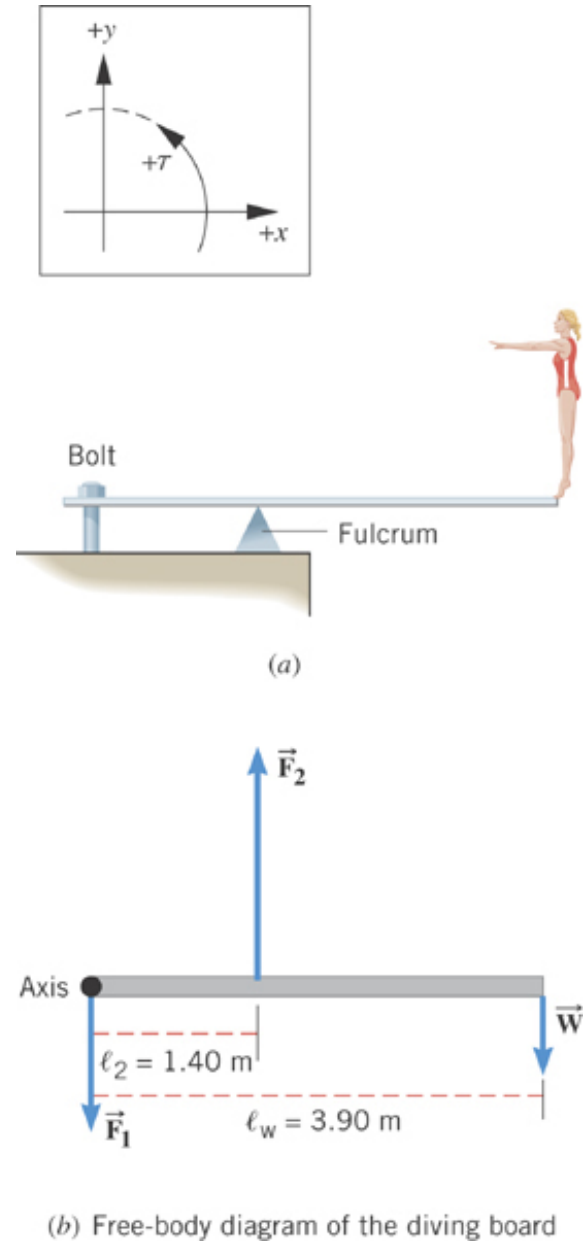
(b) Free-body diagram of the diving board

9.2 Rigid Objects in Equilibrium

$$\sum F_y = -F_1 + F_2 - W = 0$$

$$-F_1 + 1480 \text{ N} - 530 \text{ N} = 0$$

$$F_1 = 950 \text{ N}$$



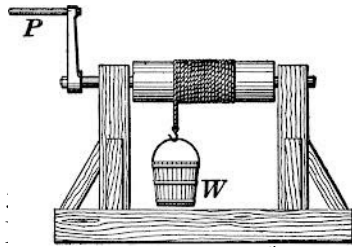
1) A weight of 30N is located at a distance of 10cm from the fulcrum of a simple balance beam. At what distance From the fulcrum should a weight of 20N be placed on the opposite side in order to balance the system ?

2) A weight of 5N is located at 10cm from the fulcrum of a simple balance beam. What weight should be placed at a point 4 cm from the fulcrum on the opposite side in order to balance the system.

3) A seesaw is 12 ft long and is pivoted at its center. George weighs 40 lb and is sitting at one end. His sister Alice, weighing 32 lb is 1 ft from him. Where does their 155 lb dad have to sit to balance the seesaw ?



4) The bucket (see figure) with the water in it has a mass of 6.0kg. The radius of the drum on which the rope is wound is 12cm. The length of the crank is 35 cm. How much force needed to lift the bucket ? (equilibrium between the 2 torques)



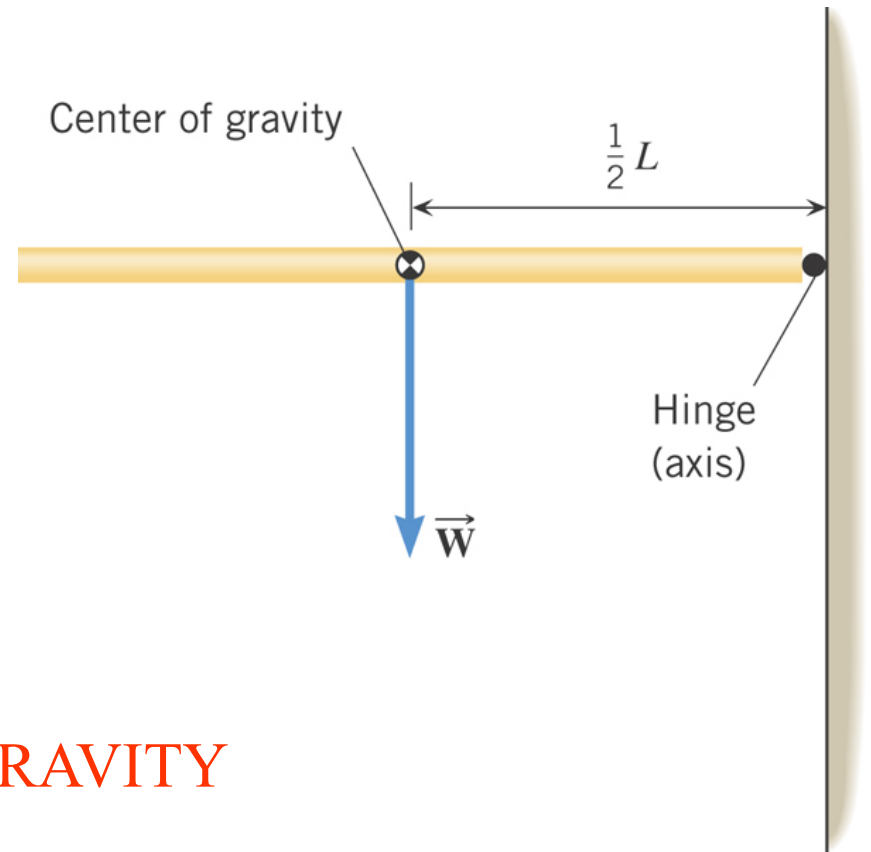
5) A plank 2.4 m long, pivoted at 0.2m from the end, to lift, at that end, a rock that weighs 750N. What force must be exerted at the other end to lift the rock?

6) You are free to choose a fulcrum (pivot) where it is most convenient to do computation. Take the example of a plank with a pile of books (see figure). The fulcrum can be either of the chairs. The mass of the books is 35kg and the plank is 3.7m long, and the books are centered 1.2 m from the left chair. What is the elastic recoil (that is the normal force). You find the solution in 2 steps. First take the fulcrum at the right chair and write the equilibrium equation. Then take the fulcrum at the left chair.



7) A crowbar 2.0m long is used to pry a rock out of the ground. By pivoting the crowbar 0.5m from the rock and pushing down on the other end. The end is pushed with a force of 250N. A) find the torque applied to the crow bar by the person doing the job. B) The force being applied to the rock.

9.3 Center of Gravity

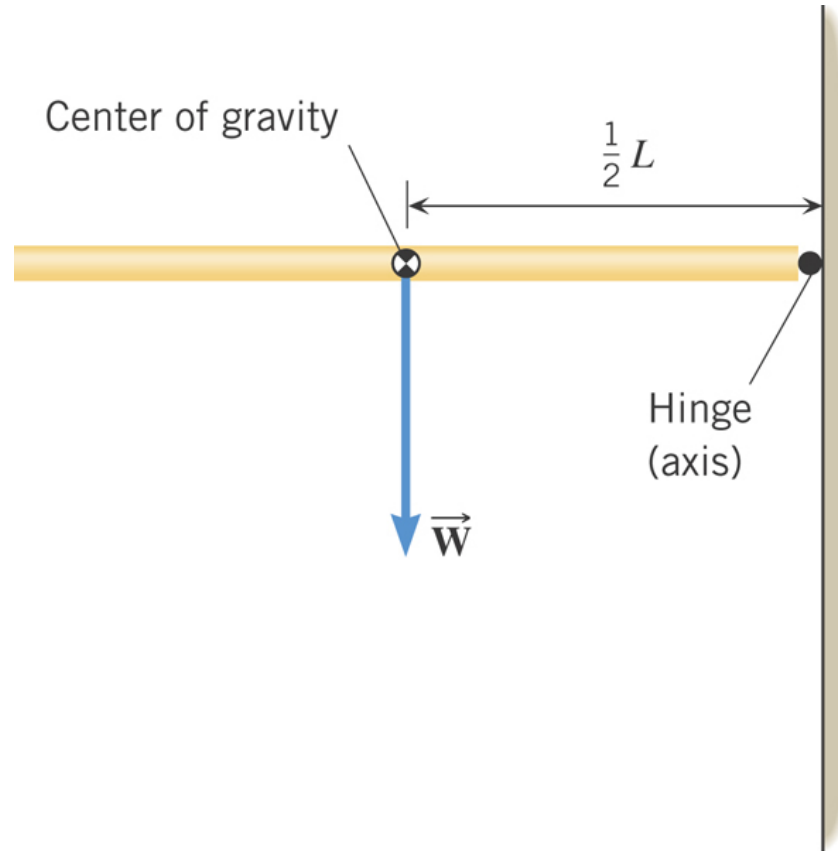


DEFINITION OF CENTER OF GRAVITY

The center of gravity of a rigid body is the point at which its weight can be considered to act when the torque due to the weight is being calculated.

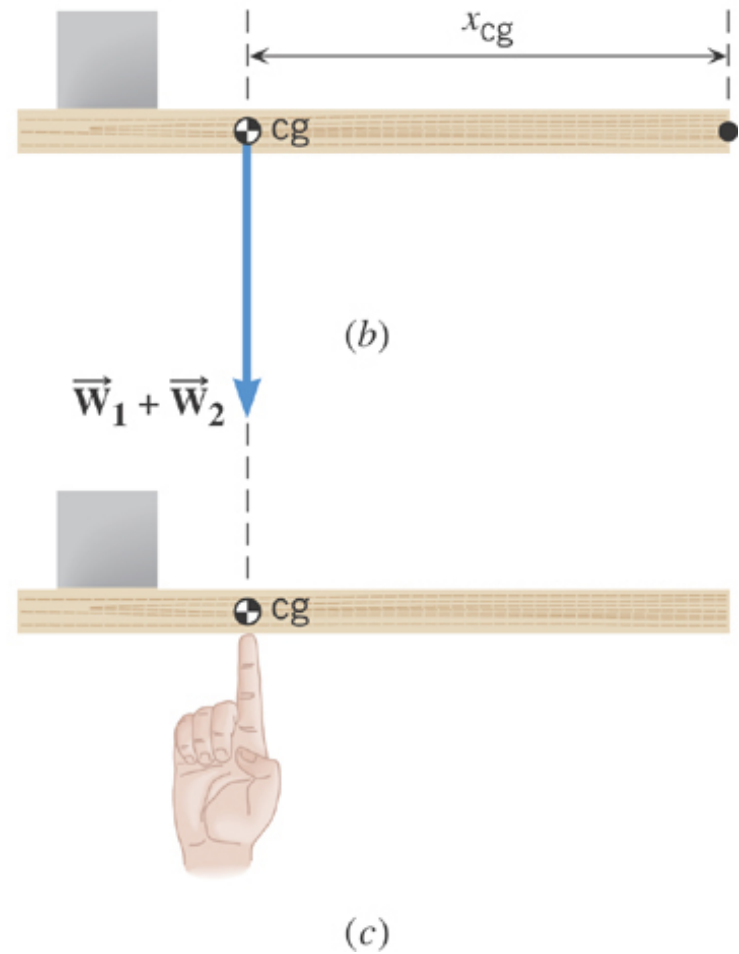
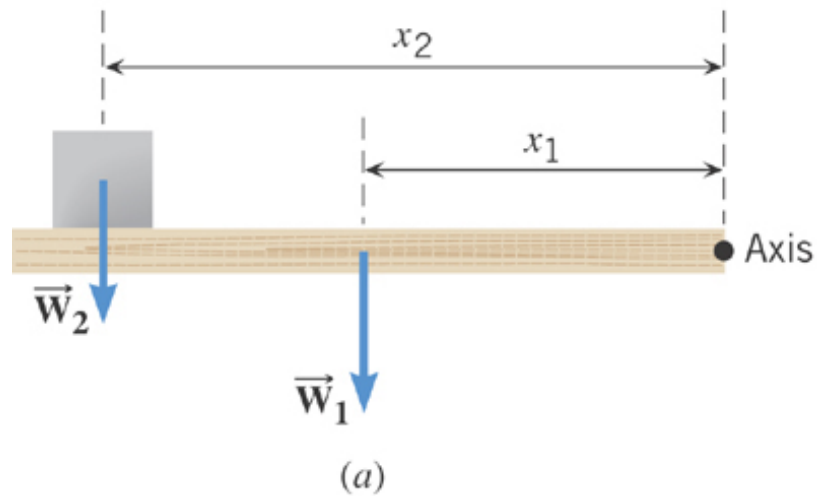
9.3 Center of Gravity

When an object has a symmetrical shape and its weight is distributed uniformly, the center of gravity lies at its geometrical center.



9.3 Center of Gravity

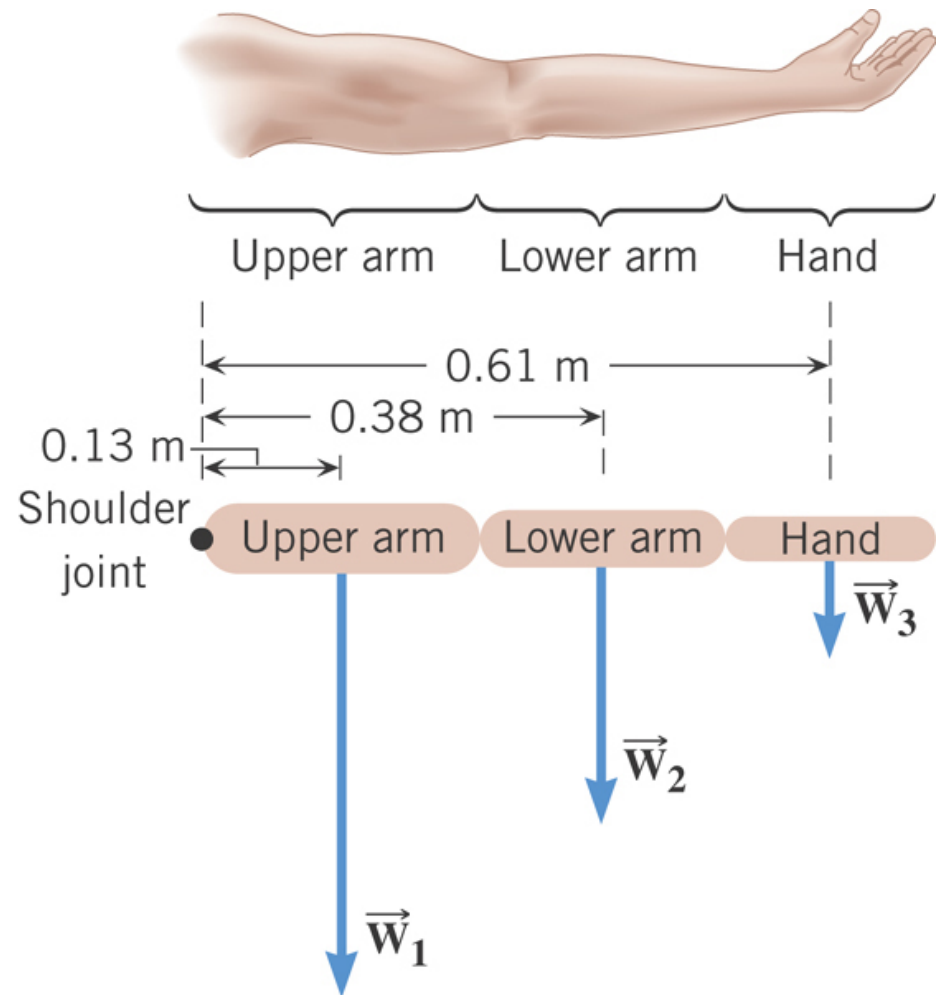
$$x_{cg} = \frac{W_1 x_1 + W_2 x_2 + \cdots}{W_1 + W_2 + \cdots}$$



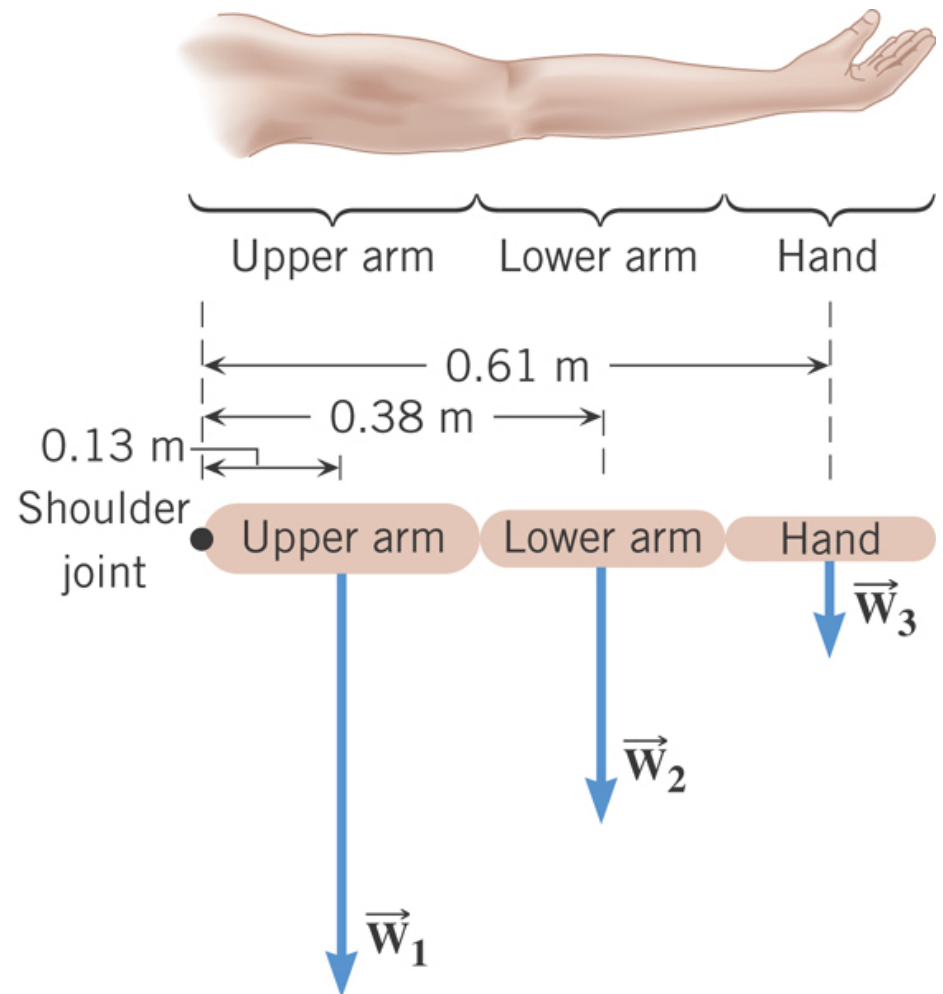
Example 6 The Center of Gravity of an Arm

The horizontal arm is composed of three parts: the upper arm (17 N), the lower arm (11 N), and the hand (4.2 N).

Find the center of gravity of the arm relative to the shoulder joint.



9.3 Center of Gravity

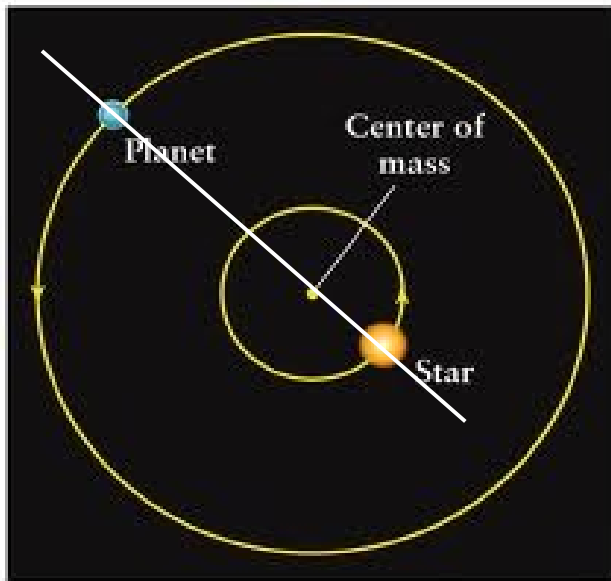


$$x_{cg} = \frac{W_1 x_1 + W_2 x_2 + \dots}{W_1 + W_2 + \dots}$$

$$x_{cg} = \frac{(17 \text{ N})(0.13 \text{ m}) + (11 \text{ N})(0.38 \text{ m}) + (4.2 \text{ N})(0.61 \text{ m})}{17 \text{ N} + 11 \text{ N} + 4.2 \text{ N}} = 0.28 \text{ m}$$

Remember : the star and the planet are always
Across from other (relative to the center of mass)

Also they move in opposite Direction



a

The wobble of the star is due to the gravitational tug from the planet. Increase the mass of the planet, increase the tug, increase the speed of the star.

You detect the wobble from the light emitted FROM THE STAR. Away the light is redshifted And the radial velocity is positive . Toward the Light is blueshifted and velocity is positive.