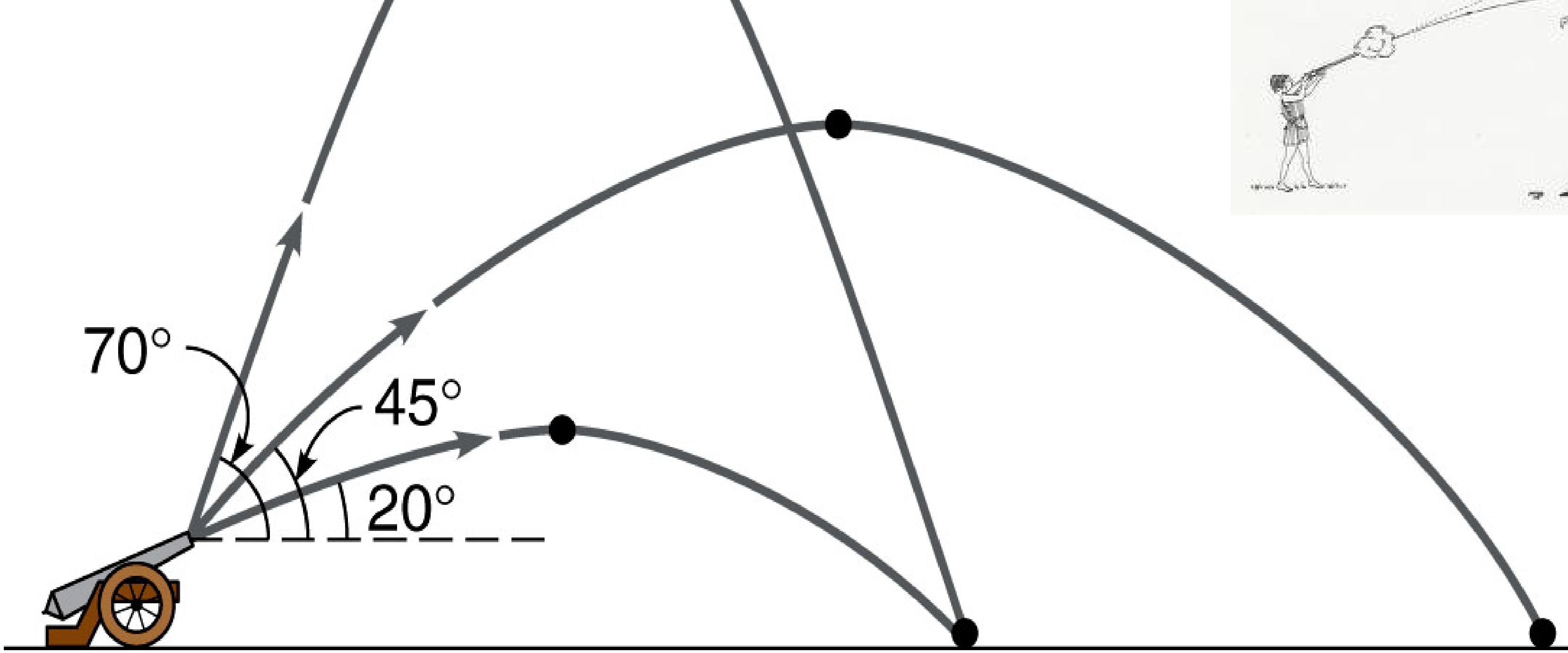
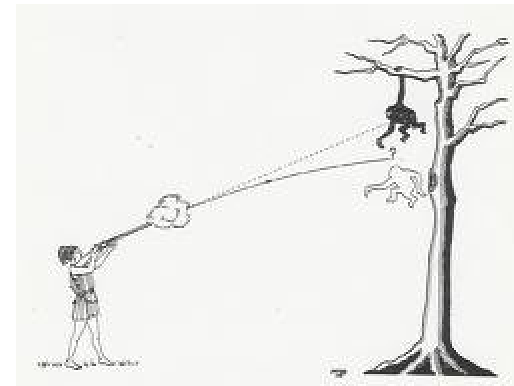
Projectile motion
slide 2

uniform circular motion
slide 32

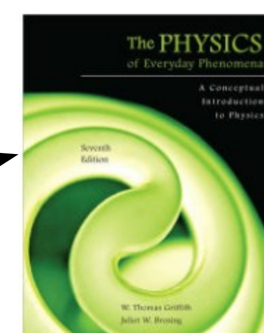
Projectile motion

Movie projectile sacre coeur



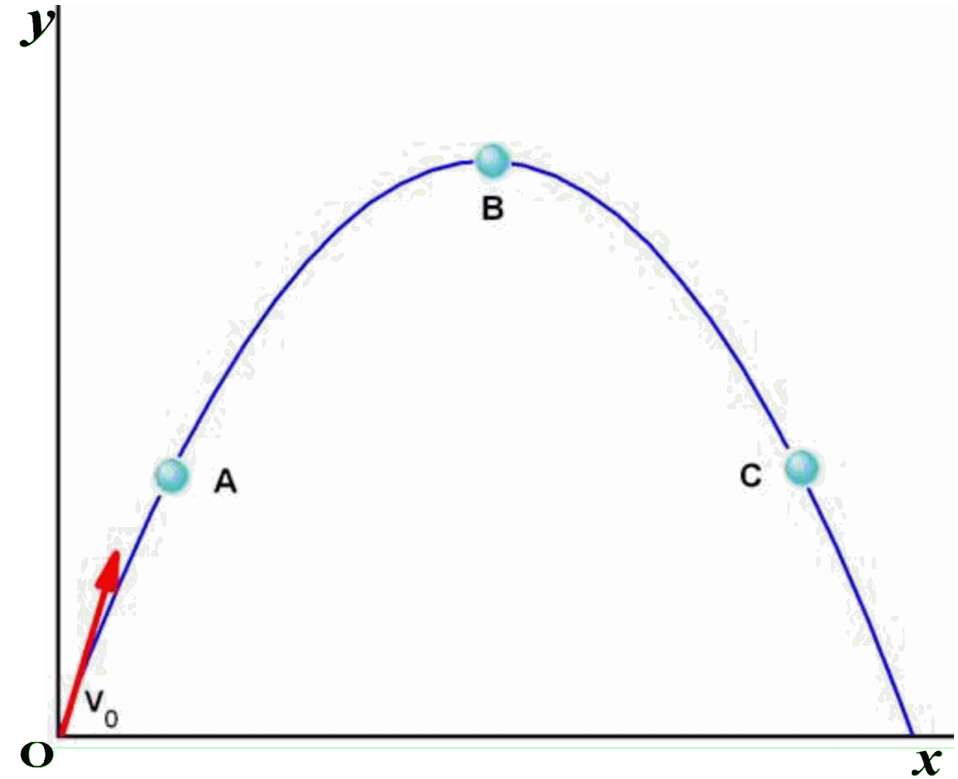


References for this unit:
-The physics of everyday phenomena: a conceptual introduction to physics by W. Thomas Griffith, McGraw-Hill
- Physics / Edition 8 by John D. Cutnell, Kenneth W. Johnson, Cutnell



Remember that acceleration is force per unit mass. As long as there is a force acting -. acceleration
The acceleration due to gravity is the weight per unit mass.

A ball is launched with an initial velocity v_0 as shown. Which one of the following arrows best represents the direction of the acceleration at point A? B ? C?



a) →

b) ↓

c) ↗

d) ↑

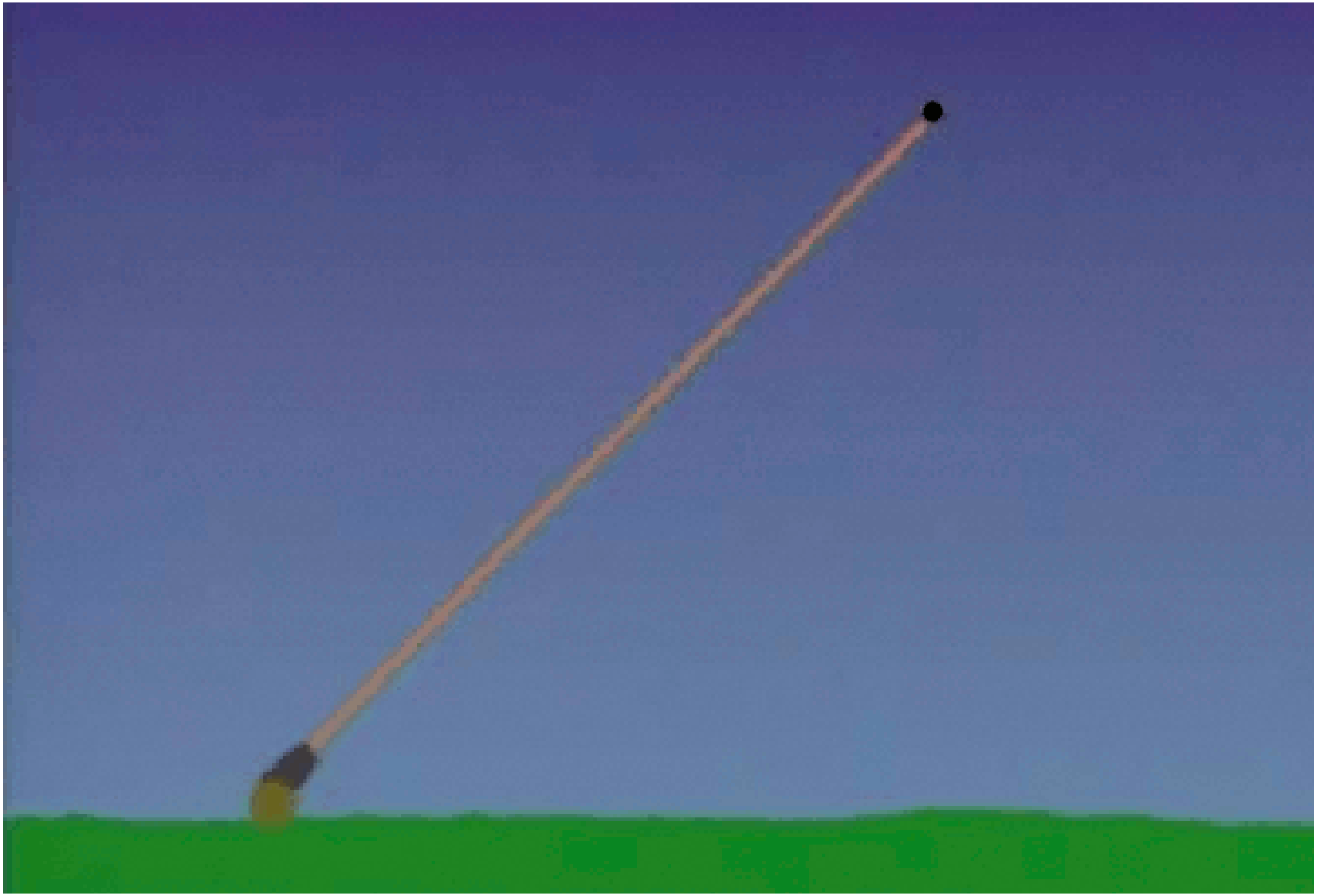
e) The acceleration at point A is zero m/s^2.

Projectile motion

BIG IDEA

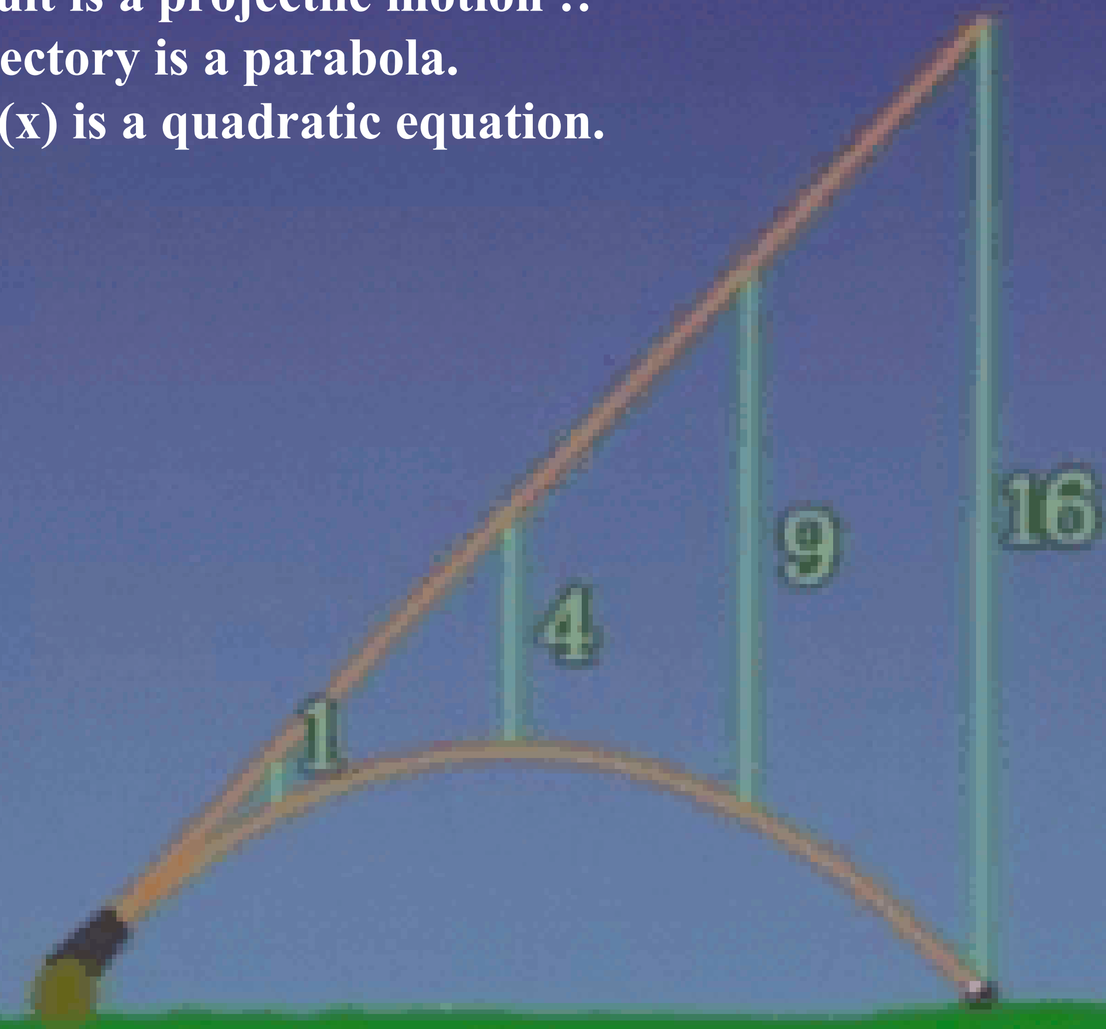
A two-dimensional problem
can be considered as 2
one-dimensional problems.
A body motion has 2 components
completely independent of each other.

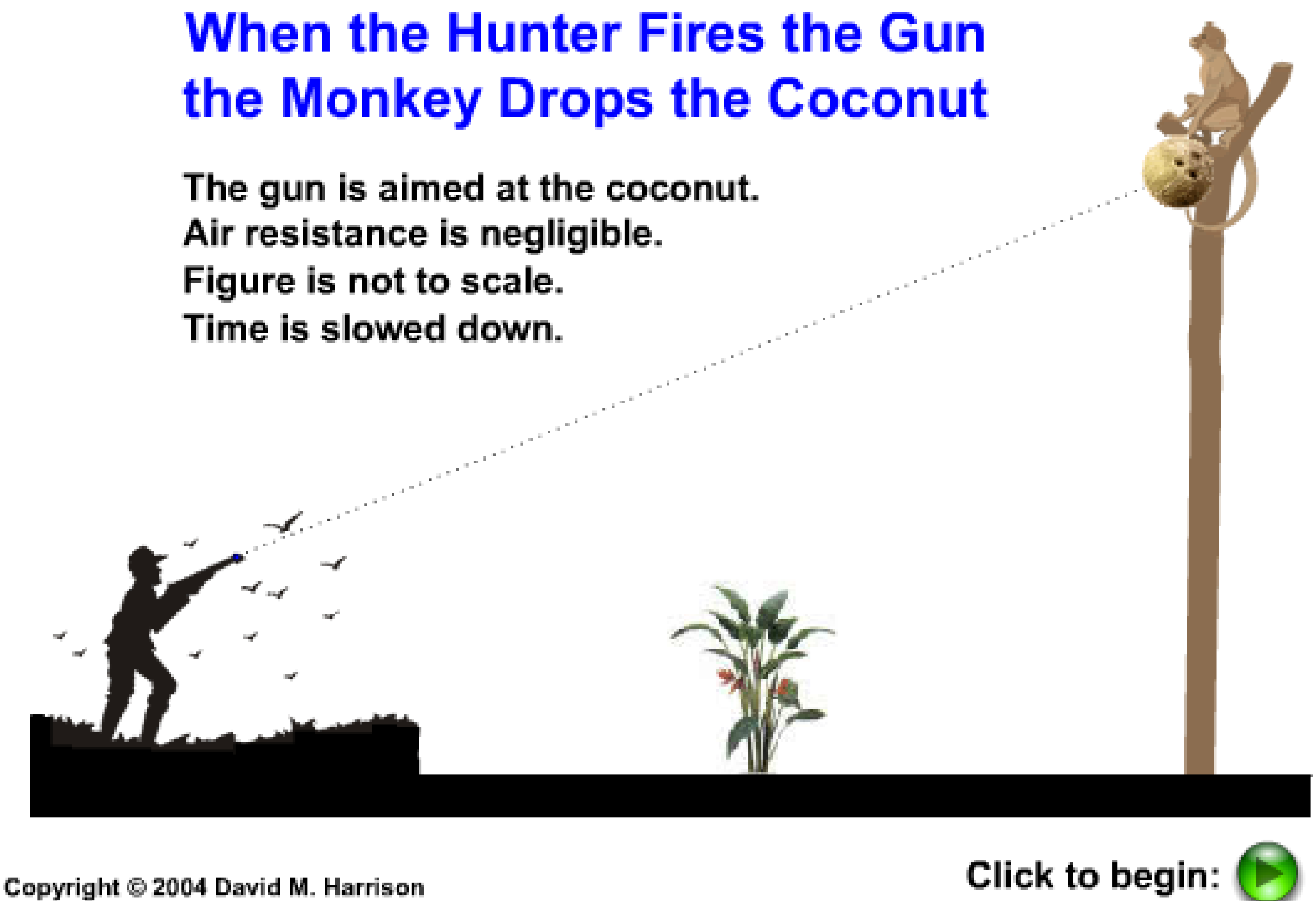
2 components means : we will use trig to find the components of the velocities along the path.
A horizontal component (cosine if you the standard notation) and vertical component
(sine if you use the standard notation).



Motion of a cannon ball without gravity. It goes in a straight line, at a constant velocity (same magnitude and same direction) if we neglect air resistance (drag). (Newton's 1st law = an object retains Its motion unless a force acts on it).

At the same time the ball is moving in a straight line it is also falling
The result is a projectile motion !!
The trajectory is a parabola.
So y = F(x) is a quadratic equation.

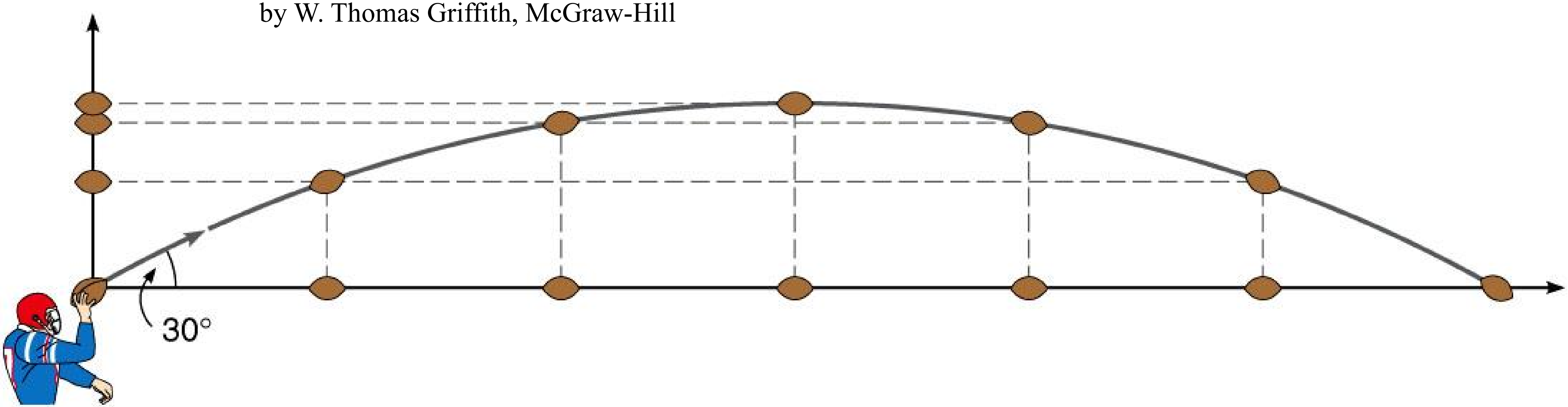




https://faraday.physics.utoronto.ca/GeneralInterest/Harrison/Flash/ClassMechanics/MonkeyHunter/MonkeyHunter.html

See exploration software # 7   and video monkey @ MIT

-The physics of everyday phenomena: a conceptual introduction to physics
by W. Thomas Griffith, McGraw-Hill



http://www.phy.hk/wiki/j/Eng/projectile/projectile_js.htm
Try also angle =0

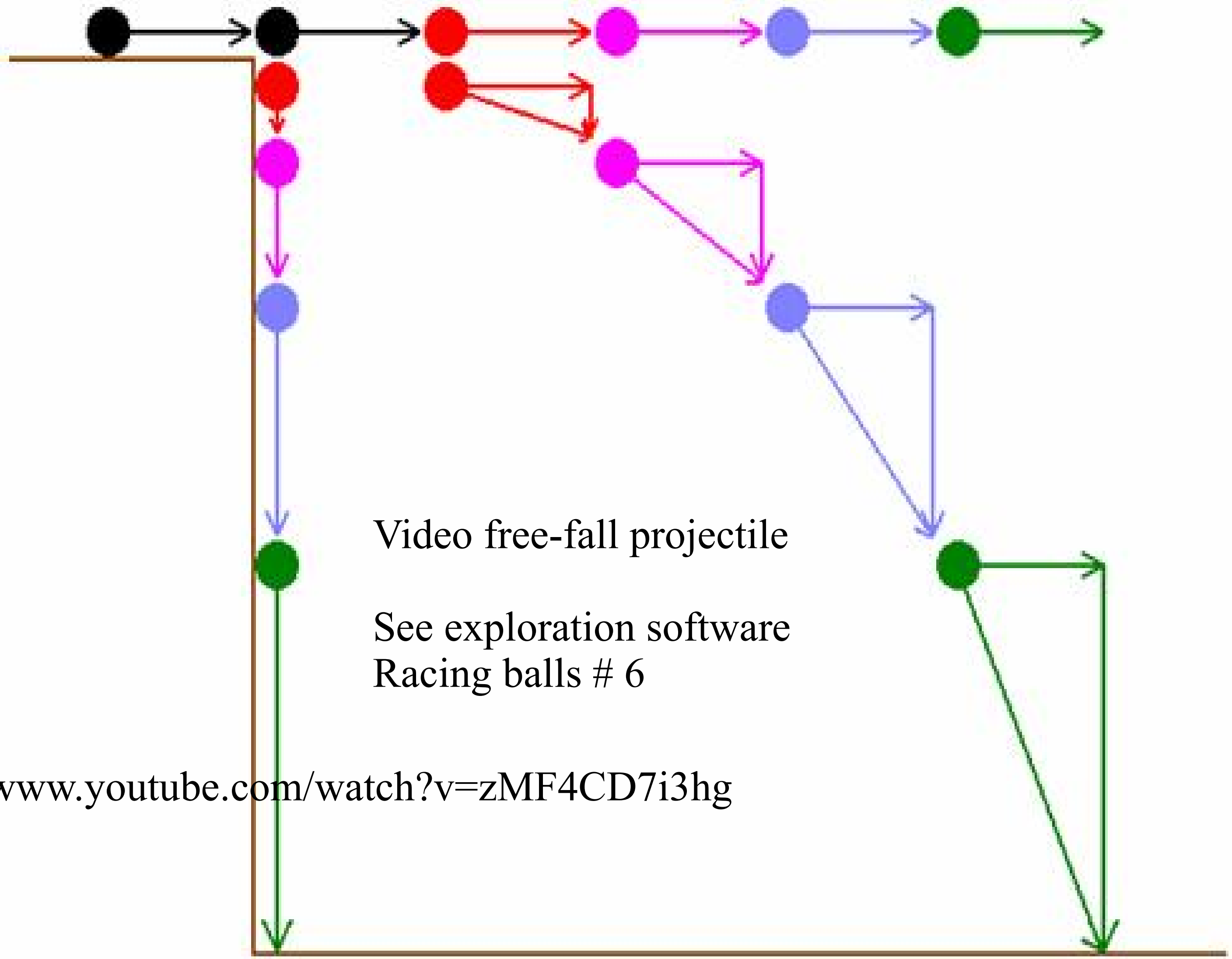
http://highered.mcgraw-hill.com/olcweb/cgi/pluginpop.cgi?it=swf::800::600::/sites/dl/free/0072482621/78778/Gravity_Nav.swf::Gravity%20Variations%20Interac

THE projectile motion has 2 components.
Along the horizontal the acceleration is 0 (no force acting on the ball)
Along the vertical the acceleration = -10m/s/s

The horizontal motion ignores the vertical motion

See applet projectile motion - Use angle 70 degrees, initial position 0
observe the components of the velocity, the vector acceleration, force, the position

http://www.walter-fendt.de/html5/phen/projectile_en.htm http://www.upscale.utoronto.ca/GeneralInterest/Harrison/Flash/ClassMechanics/TwoBallsGravity/TwoBallsGravity.html

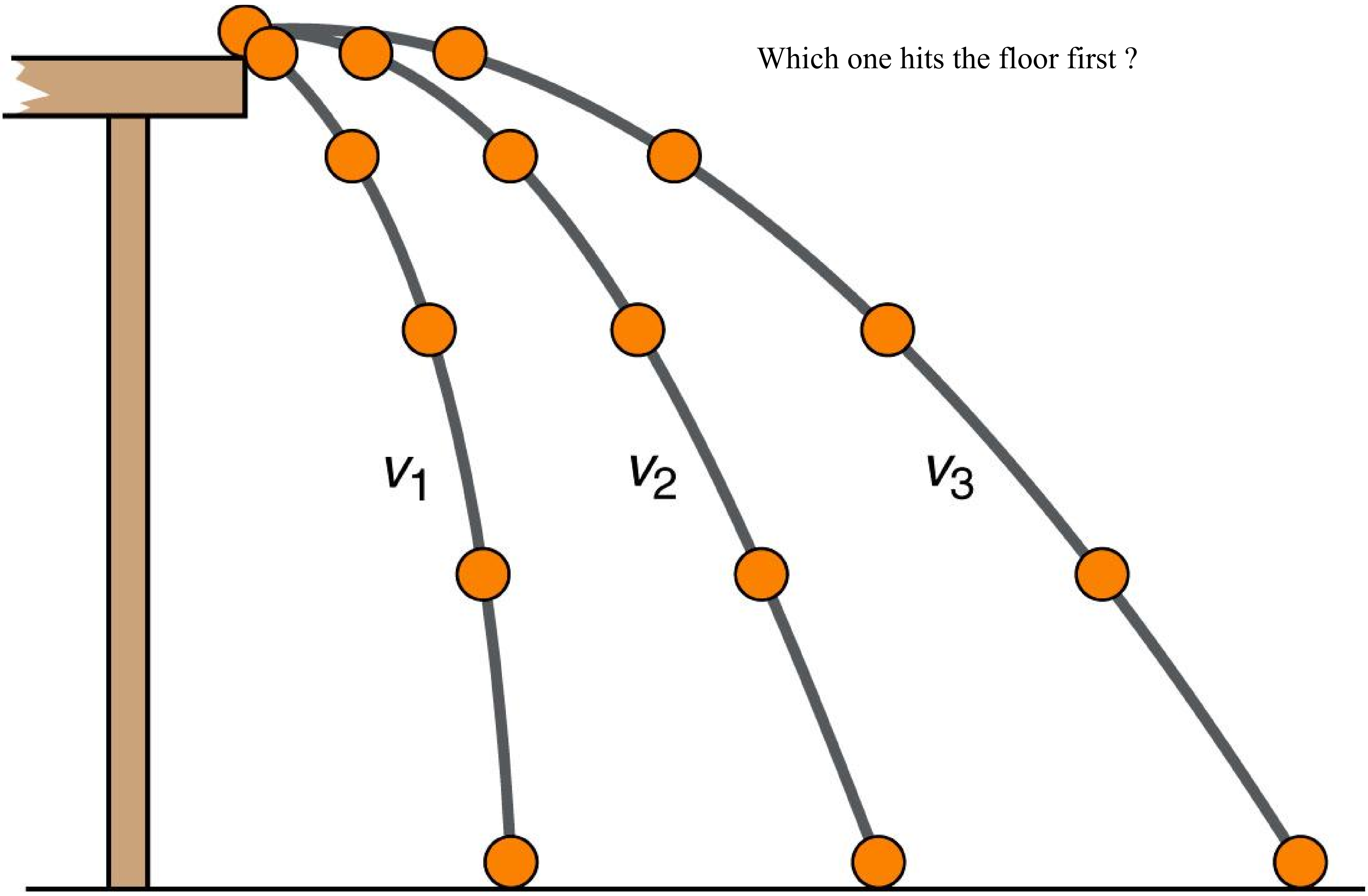


Video free-fall projectile

See exploration software
Racing balls # 6

https://www.youtube.com/watch?v=zMF4CD7i3hg

1 motion but 2 independent components.



Which one hits the floor first ?

A football is kicked at an angle 25° with respect to the horizontal. Which one of the following statements best describes the acceleration of the football during this event if air resistance is neglected?

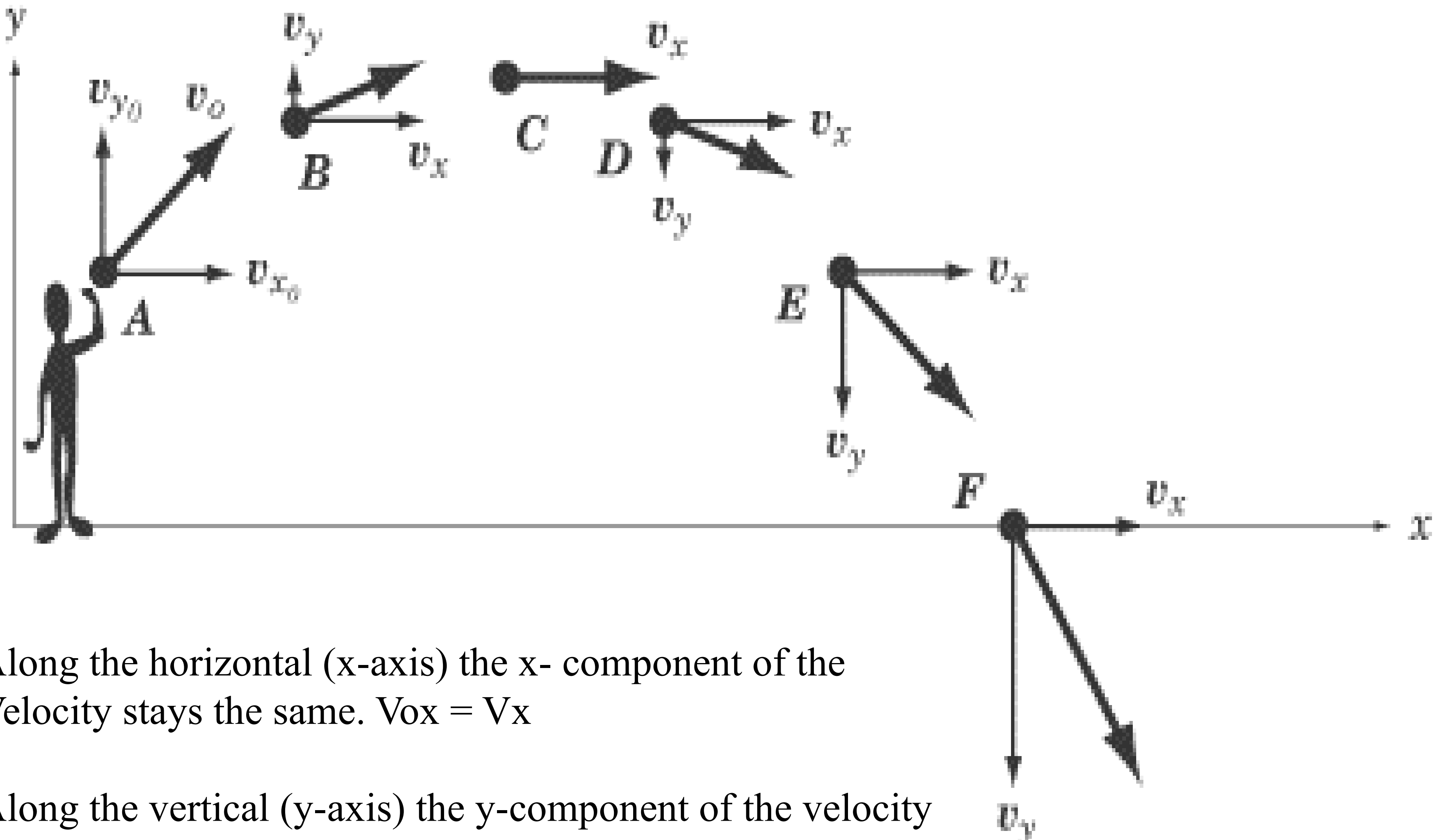
a) The acceleration is zero m/s^2 at all times.

b) The acceleration is zero m/s^2 when the football has reached the highest point in its trajectory.

c) The acceleration is positive as the football rises, and it is negative as the football falls.

d) The acceleration starts at 9.8 m/s^2 and drops to some constant lower value as the ball approaches the ground.

e) The acceleration is 9.8 m/s^2 at all times.



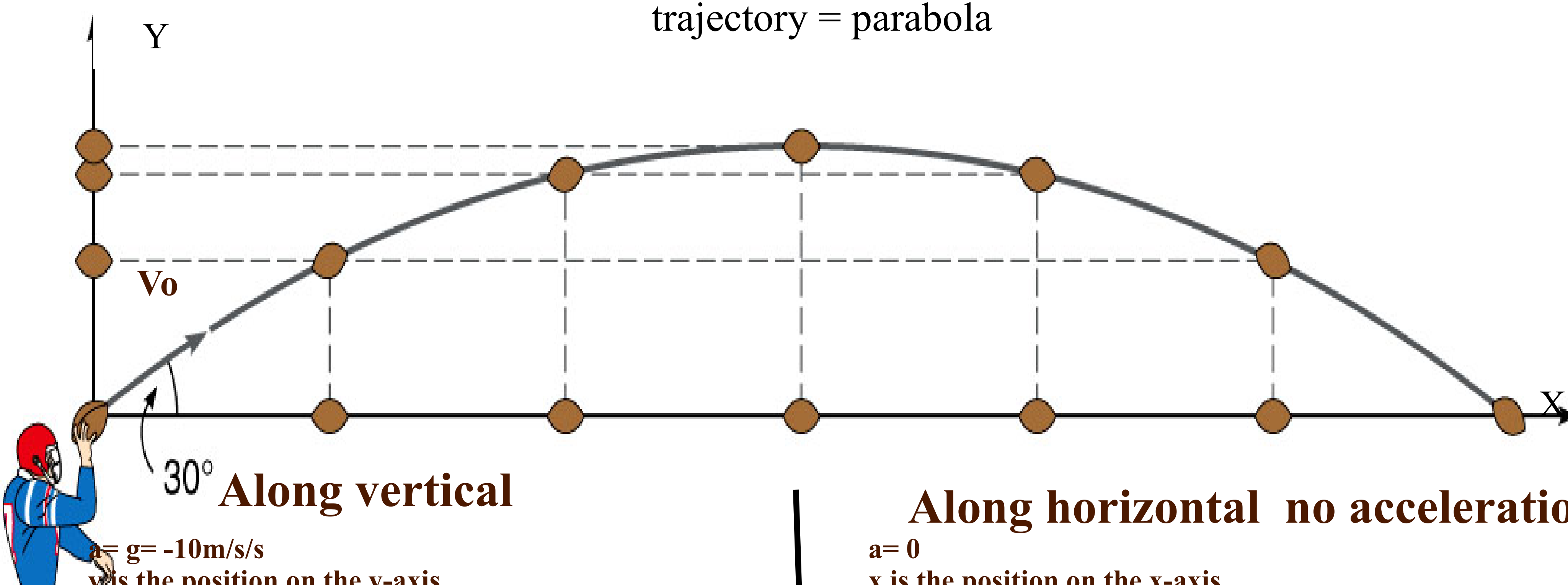
Along the horizontal (x-axis) the x- component of the Velocity stays the same. Vox = Vx

Along the vertical (y-axis) the y-component of the velocity Changes at a <u>constant rate</u> : Vy= Voy – 10t so decreases at a rate of 10m/s/s

Along the horizontal = no acceleration because no force

Along the vertical = acceleration = -10m/s/s

trajectory = parabola



Along vertical

a= g= -10m/s/s

y is the position on the y-axis.

We suppose initial position is 0.

Voy is the y-component of Vo the initial velocity.

Voy=Vo sin(30) y-component initial velocity

y= Voy t – 5 t^2 (like free-fall)

V = Voy – 10t (like free-fall)

V^2 = Voy^2 + 2(-10)(y)

a	y	Voy	Vy	t
-10				

Along horizontal no acceleration

a= 0

x is the position on the x-axis.

We suppose initial position is 0.

Vox is the x-component of Vo the initial velocity.

Vox=Vo cos(30)

(constant speed along horizontal)

x= Vox t

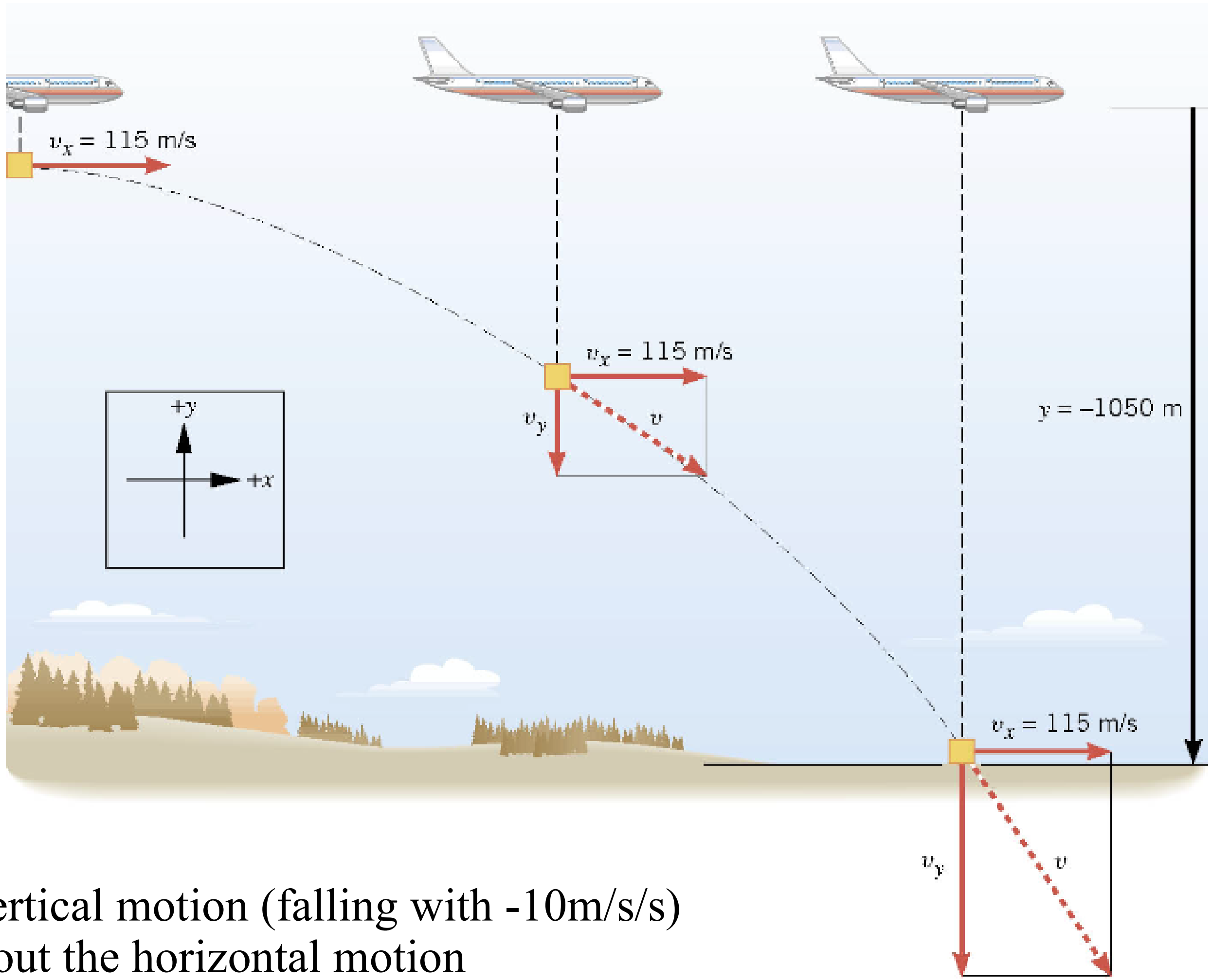
Vx = Vox

Velocity stays constant.

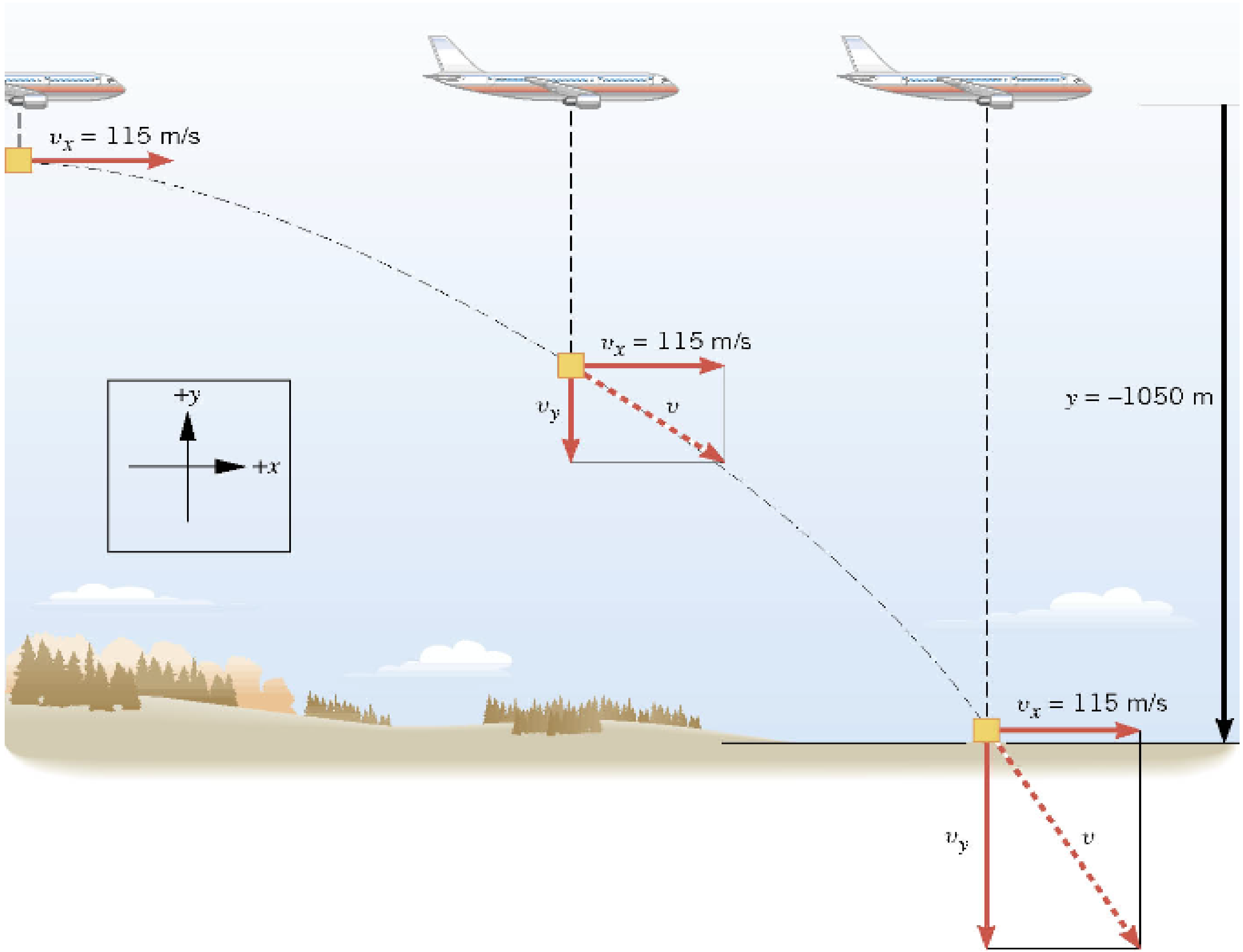
Time t is the same !

Example A Falling Care Package

The airplane is moving horizontally with a constant velocity of +115 m/s at an altitude of 1050m. Determine the time required for the care package to hit the ground.



Remember the vertical motion (falling with -10m/s/s)
Does not care about the horizontal motion



Note that the pilot always sees the package below the plane.
The plane and the package have the velocity along the horizontal.
Find the range of the package (horizontal displacement)
The vertical and horizontal parts of the motion occurs independently
FIND THE FINAL Vy and the FINAL VELOCITY V !! review vector.

An airplane is flying horizontally at a constant velocity when a package is dropped from its cargo bay. Assuming no air resistance, which one of the following statements is correct?

a) The package follows a curved path that lags behind the airplane.

b) The package follows a straight line path that lags behind the airplane.

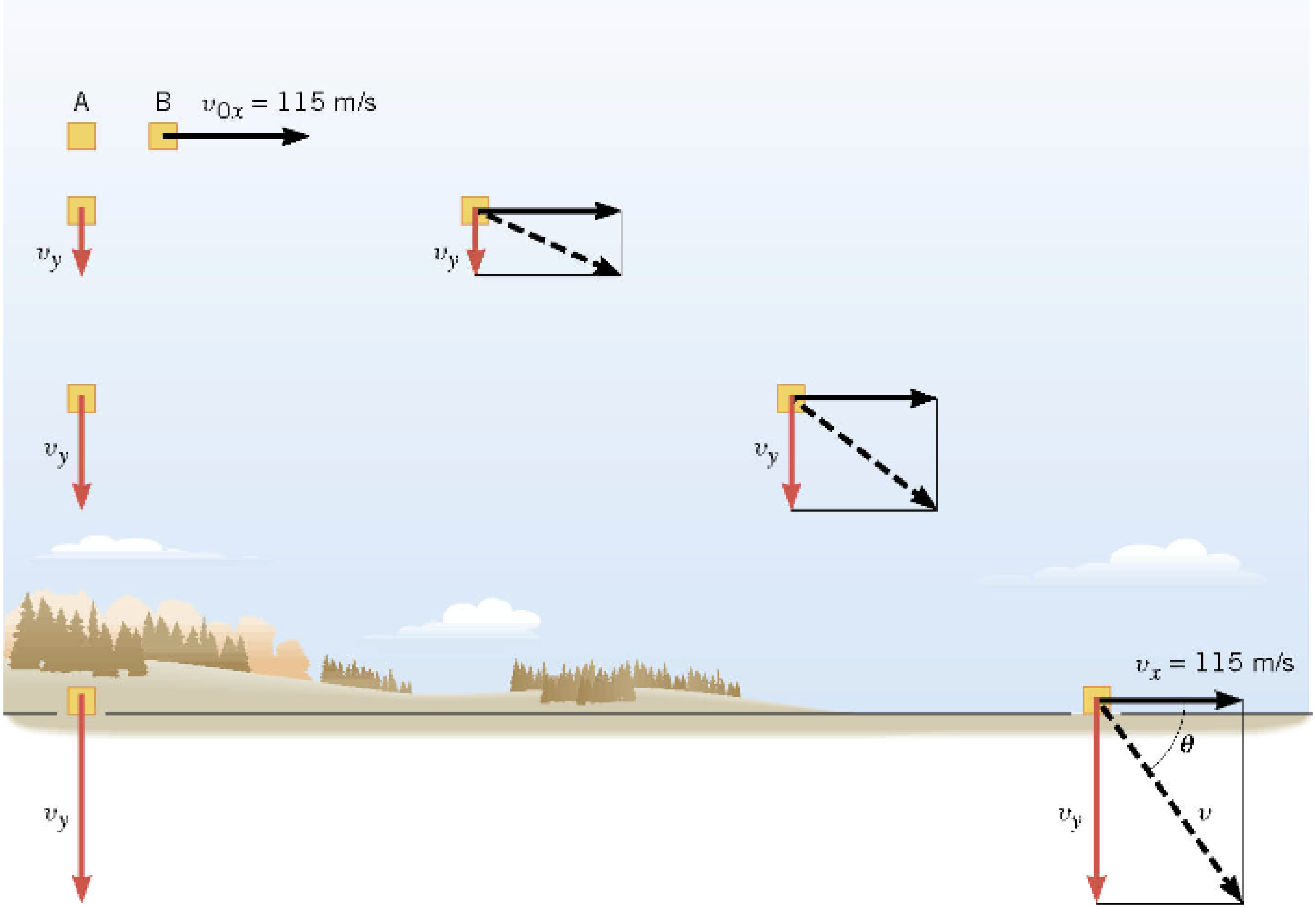
c) The package follows a straight line path, but it is always vertically below the airplane.

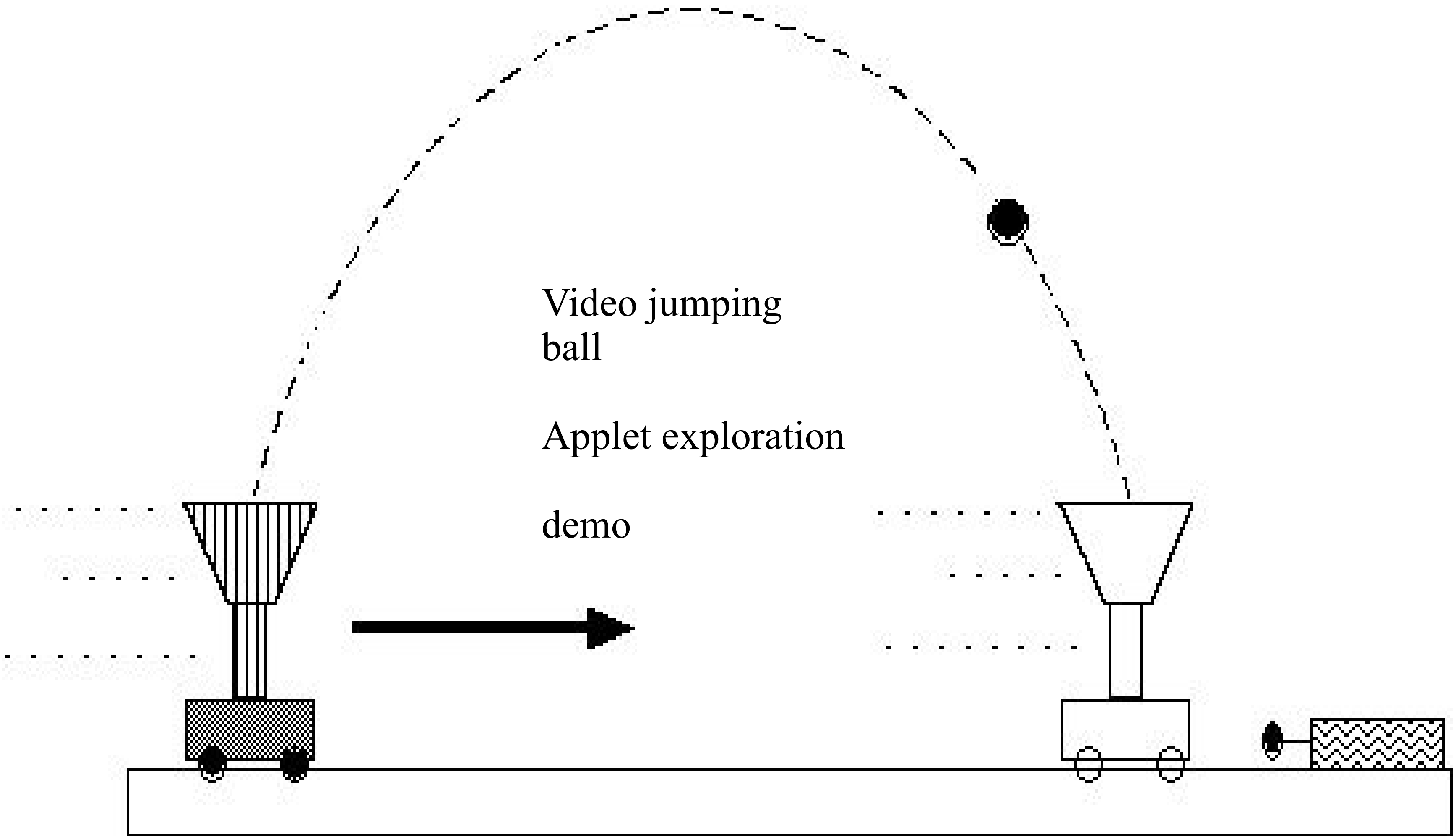
d) The package follows a curved path, but it is always vertically below the airplane.

e) The package follows a curved path, but its horizontal position varies depending on the velocity of the airplane.

See exploration software

Note that if a package A was dropped from a stationary balloon
At the same time package B was dropped from the plane
They will hit the ground at the same time and they will have
The same y-component of the velocity Vy. Will the final
Velocity V (when they hit the ground) be the same ?





MOTION DEPENDS ON THE FRAME OF THE REFERENCE.
If you stand on the car, the ball goes up and down along a straight line/
If you stand in the lab watching the car, the ball has a projectile motion.
It follows a parabola.

In two-dimensional motion in the x-y plane, what is the relationship between the x part of the motion to the y part of the motion?

a) The x part of the motion is independent of the y part of the motion.

b) The y part of the motion goes as the square of the x part of the motion.

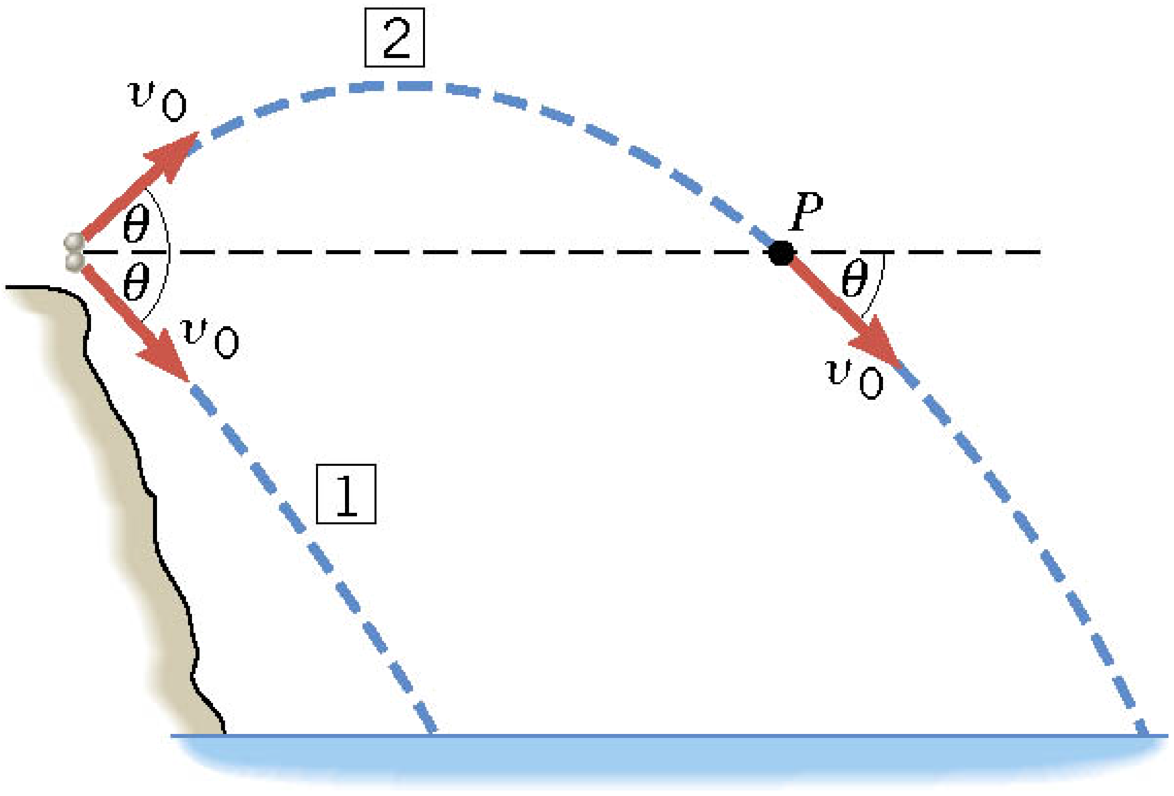
c) The x part of the motion is linearly dependent on the y part of the motion.

d) The x part of the motion goes as the square of the y part of the motion.

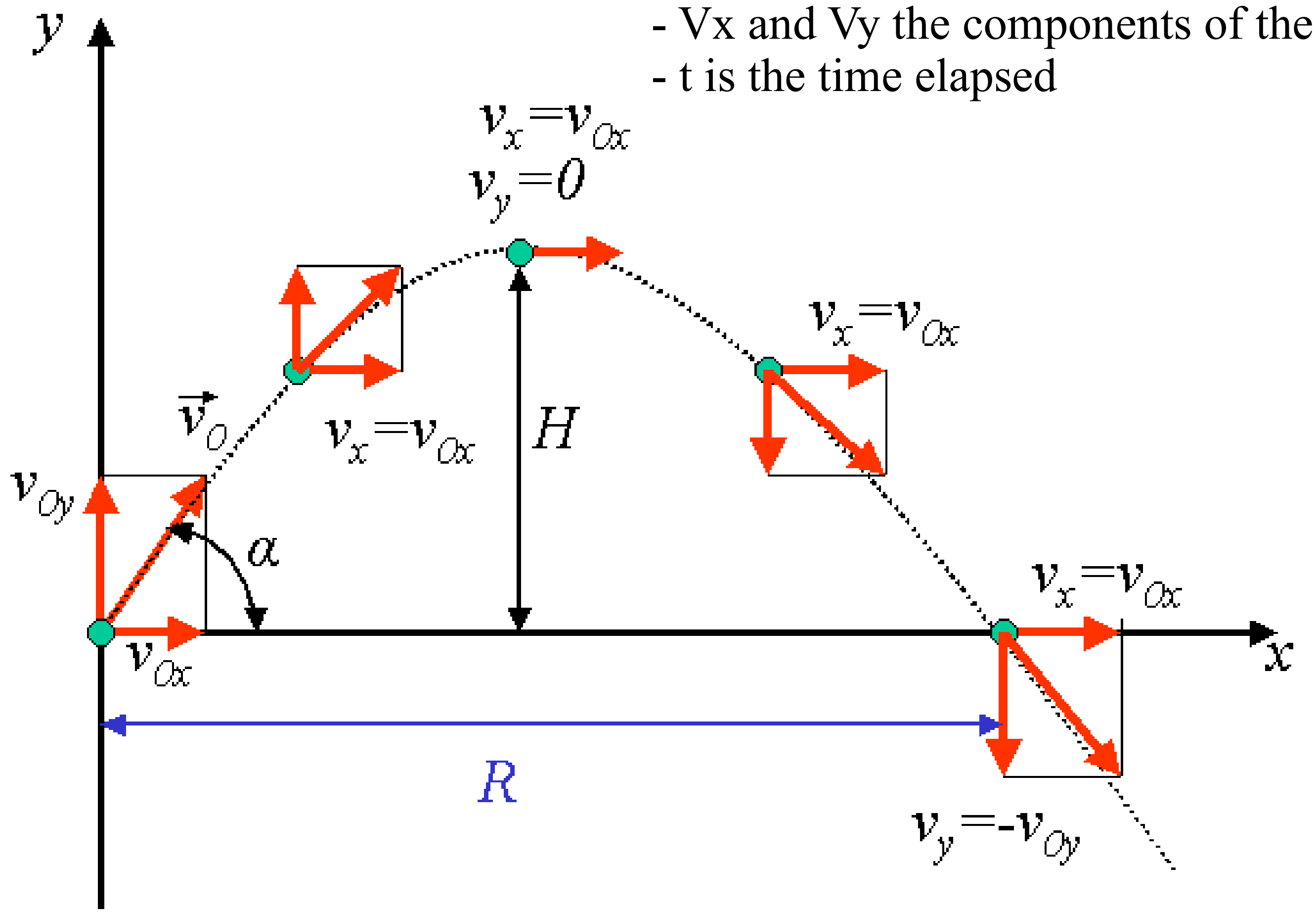
e) If the y part of the motion is in the vertical direction, then x part of the motion is dependent on the y part.

Conceptual Example 10 Two Ways to Throw a Stone

From the top of a cliff, a person throws two stones. The stones have identical initial speeds, but stone 1 is thrown downward at some angle above the horizontal and stone 2 is thrown at the same angle below the horizontal. Neglecting air resistance, which stone, if either, strikes the water with greater velocity?



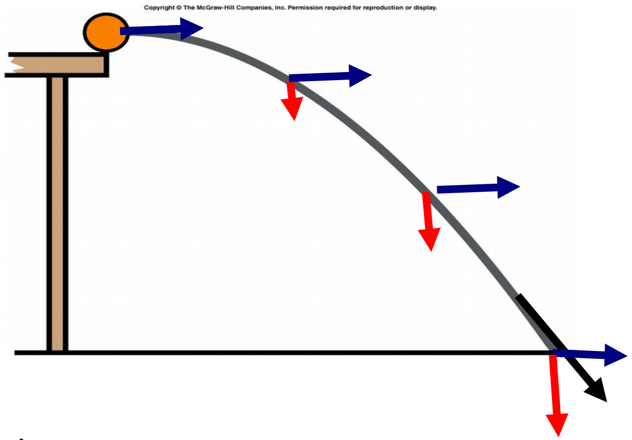
Here are the physical quantities that describe Projectile motion:
- x is the horizontal position, y the vertical position
- R is the range of the projectile
- H is the maximum height reached by projectile
- Vox and Voy the components of the initial velocity Vo
- Vx and Vy the components of the velocity along the path
- t is the time elapsed



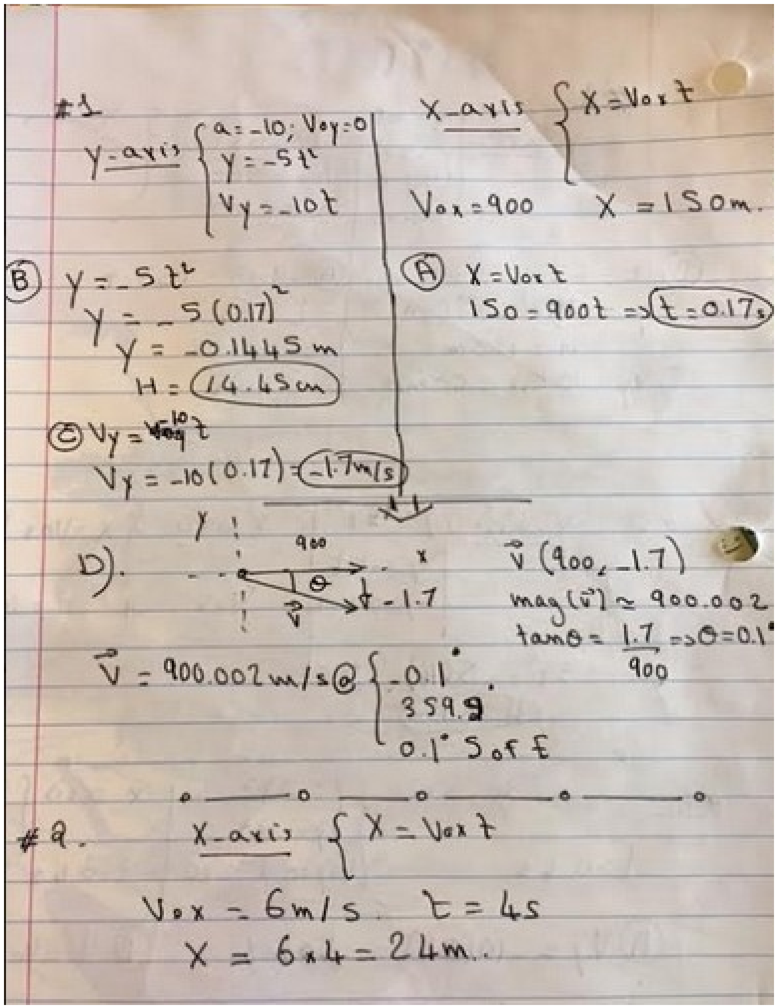
1) A bullet is fired horizontally with an initial velocity of Vox=900m/s at a target x=150m
From the rifle.
A) How much time is required for the bullet to reach the target?
Hint: Use the x-axis equations. along the horizontal, there is no acceleration, the velocity stays the same)
B) Using the approximation g= -10m/s/s , how far does the bullet fall in time.
Hint: Use the y-axis equations. along the vertical, acceleration is g and the initial velocity is 0
C) Find the final y-component Voy
D) Find the vector final velocity Vo and its direction
Hint: Use the components Vox and Voy to find the vector V.

2) A ball rolls off a shelf with a horizontal velocity of Vox=6m/s. At what horizontal distance x
From the shelf does the ball land if it takes 4s to reach the ground ?
Hint: Use the x-axis equations

3) A ball rolls off a table with a horizontal velocity of Vox=4m/s/ If it takes 0.4s for the ball to reach
The ground , how high above the floor is the tabletop ?
Hint: Voy=0 Use the y-axis equations. Vox is extra information.



4) A boll rolls off a table with a horizontal velocity of Vox=5m/s.
It it takes 0.4s for it to reach the floor:
A) What is the vertical component of the ball's velocity Vy just
before it hits the floor (g=-10). Use y-axis equations
B) What is the horizontal component of the ball's velocity
just before it hits the floor ? Vx
Use the x-axis equations C) what is the final velocity of the ball at t=0.4s



#1

Y-axis { $a = -10$; $V_{oy} = 0$; $y = -5t^2$; $V_y = -10t$

X-axis { $X = V_{ox} t$

$V_{ox} = 900$ $\quad X = 150\text{ m}$.

(A) $X = V_{ox} t$

$150 = 900t \Rightarrow \boxed{t = 0.17\text{ s}}$

(B) $y = -5t^2$

$y = -5(0.17)^2$

$y = -0.1445\text{ m}$

$H = 14.45\text{ cm}$

(C) $V_y = -10t$

$V_y = -10(0.17) = -1.7\text{ m/s}$

D). $\vec{v}\,(900, -1.7)$

$\text{mag}(\vec{v}) \simeq 900.002$

$\tan\theta = \frac{1.7}{900} \Rightarrow \theta = 0.1°$

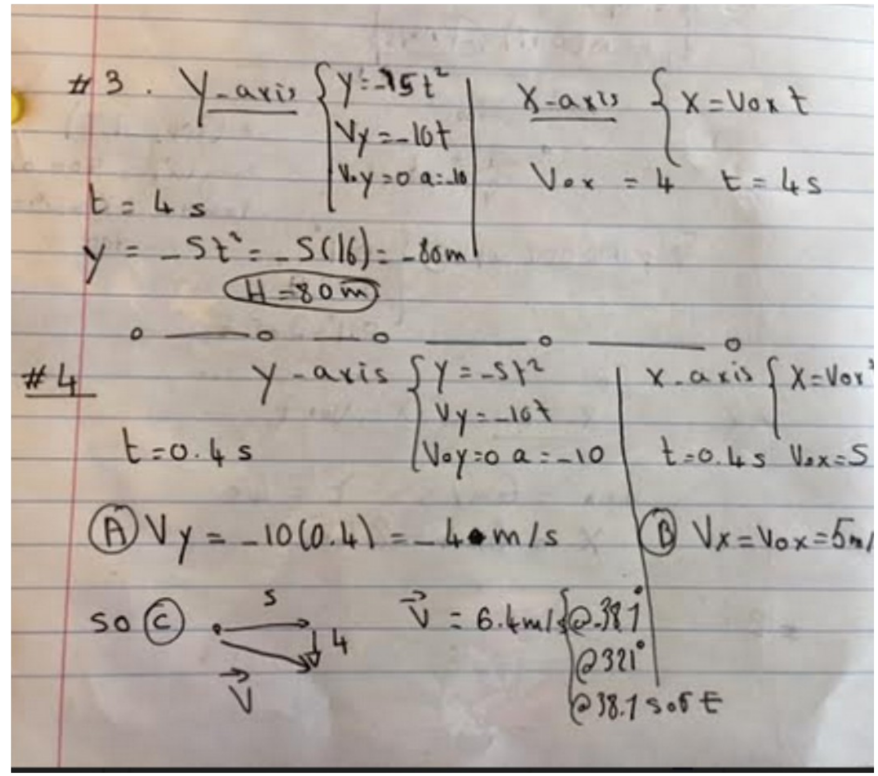
$\vec{v} = 900.002\text{ m/s} @ \{ -0.1°;\ 359.9°;\ 0.1°\text{ S of E}$

#2.

X-axis { $X = V_{ox} t$

$V_{ox} = 6\text{ m/s}$ $\quad t = 4\text{ s}$

$X = 6 \times 4 = 24\text{ m}$.



#3. Y-axis { $y = -5t^2$; $V_y = -10t$; $V_{oy} = 0$ $a = -10$

X-axis { $X = V_{ox} t$

$V_{ox} = 4$ $\quad t = 4\text{ s}$

$t = 4\text{ s}$

$y = -5t^2 = -5(16) = -80\text{ m}$

$H = 80\text{ m}$

#4 Y-axis { $y = -5t^2$; $V_y = -10t$; $V_{oy} = 0$ $a = -10$

X-axis { $X = V_{ox} t$

$t = 0.4\text{ s}$ $\quad t = 0.4\text{ s}$ $\quad V_{ox} = 5$

(A) $V_y = -10(0.4) = -4\text{ m/s}$

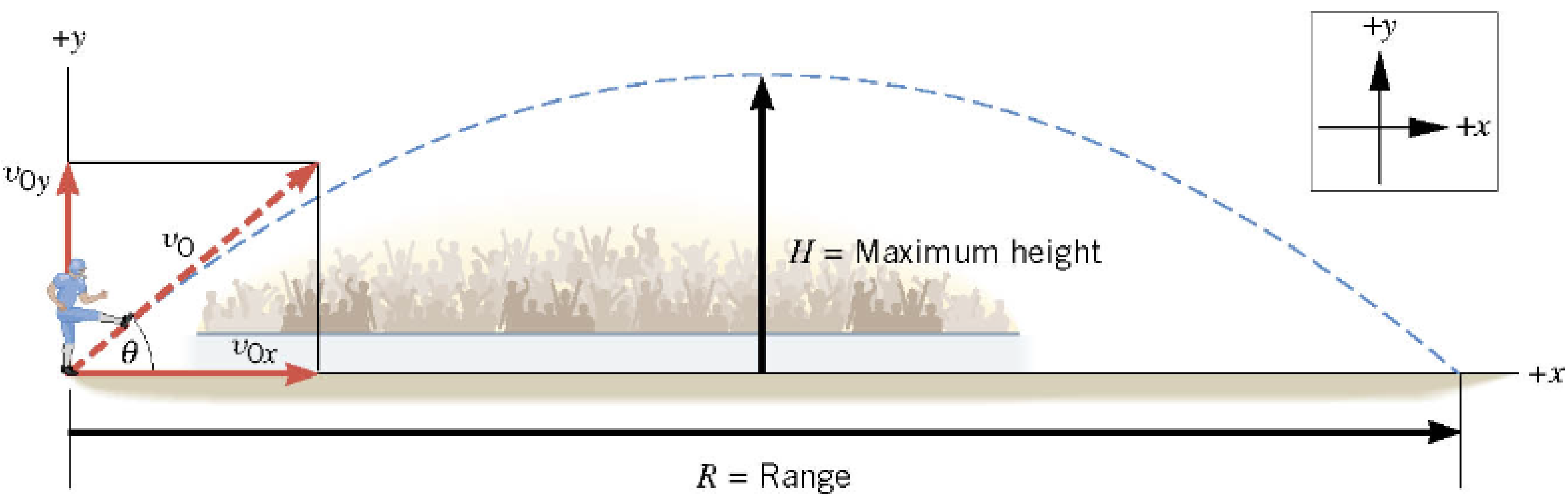
(B) $V_x = V_{ox} = 5\text{ m/s}$

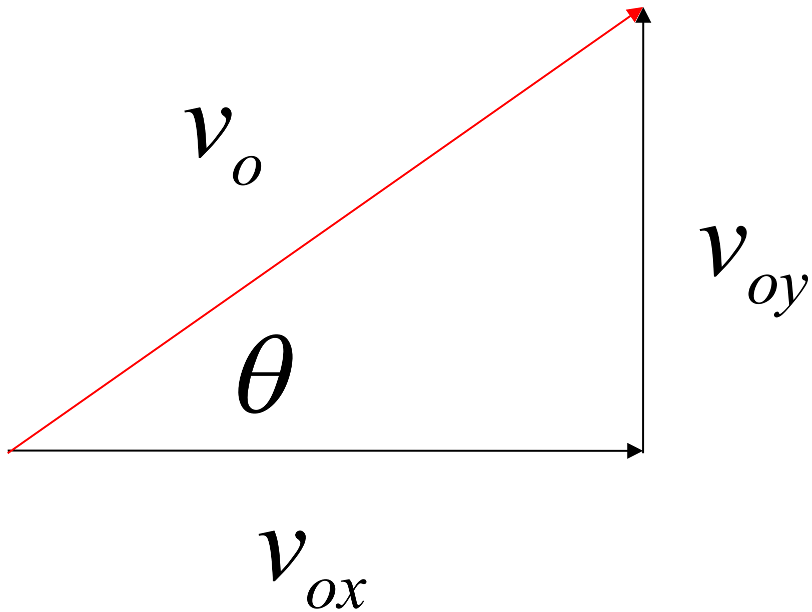
so (C) $\vec{v} = 6.4\text{ m/s} \{ @ -38.1°;\ @321°;\ @38.1\text{ S of E}$

Assignment: virtual lab projectile motion

Example 6 The Height of a Kickoff

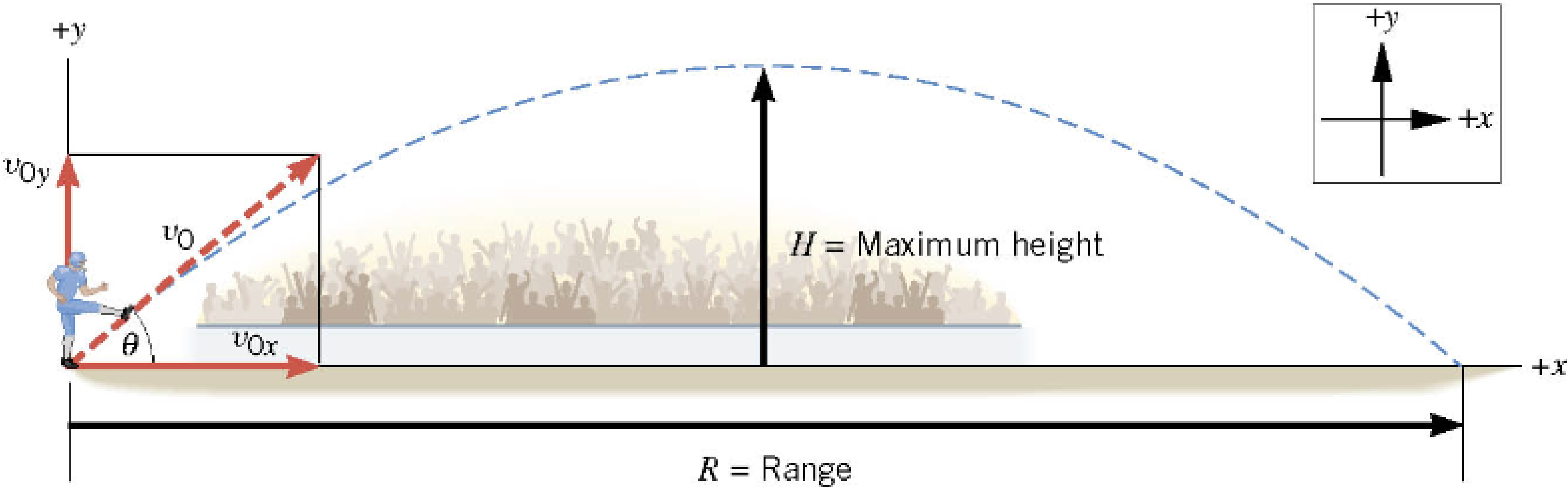
A placekicker kicks a football at and angle of 40.0 degrees and the initial speed of the ball is 22 m/s. Ignoring air resistance
1) find the components of the initial velocity Vo





$$v_{oy} = v_o \sin\theta = (22\,\mathrm{m/s})\sin 40^\circ = 14\,\mathrm{m/s}$$

$$v_{ox} = v_o \sin\theta = (22\,\mathrm{m/s})\cos 40^\circ = 17\,\mathrm{m/s}$$



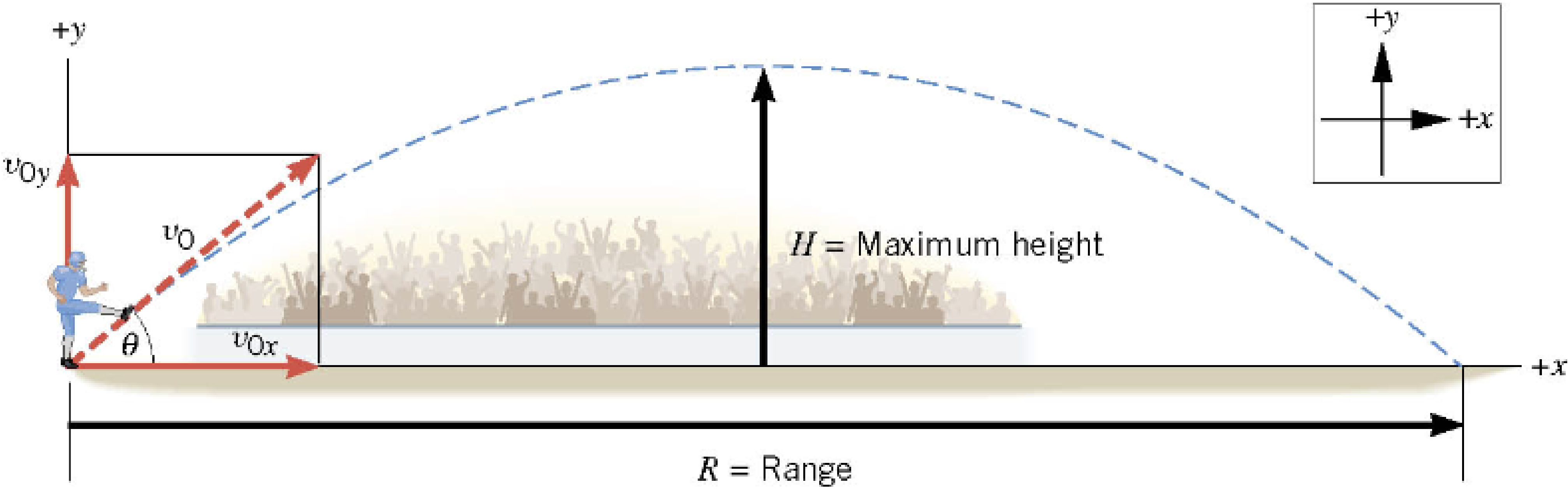
B) determine the maximum height that the ball attains.
Hint: focus on the vertical motion. The ball rises and falls.
It is like throwing a ball in the air.
Remember at the top the (vertical) velocity is 0

y	a_y	v_y	v_{oy}	t
?	--------	--------	----------	

y	a_y	v_y	v_{oy}	t
?	-9.80 m/s²	0	14 m/s	

$$v_y^2 = v_{oy}^2 + 2a_y y \quad \Longrightarrow \quad y = \frac{v_y^2 - v_{oy}^2}{2a_y}$$

$$y = \frac{0 - (14\,\mathrm{m/s})^2}{2(-9.8\,\mathrm{m/s^2})} = +10\,\mathrm{m}$$



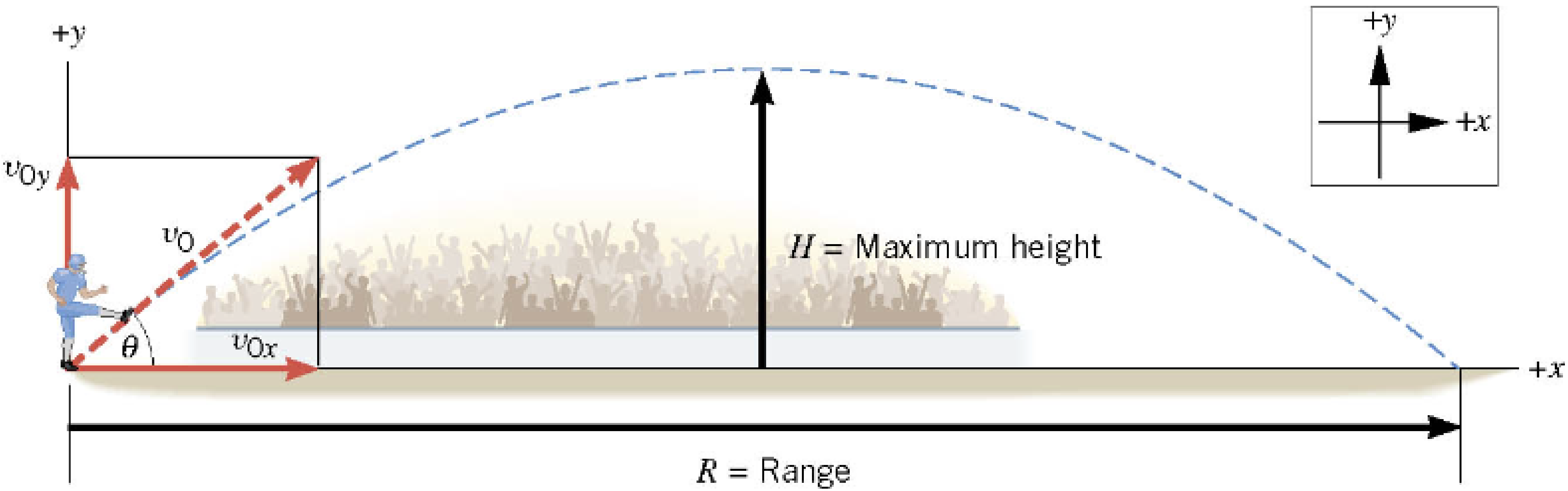
ay	Voy	Vy	Y	t

C) find the time of flight.
Hint: 2 ways to to it. You can find the time it takes to reach the highest point (y=H and Vy=0)
And you multiply this time by 2. This is the easiest way.
You can consider the whole path for which y = 0 (it comes back to a vertical position =0
and Vy = -Voy)

D) Calculate the range R of the projectile.

Hint: it is moving at a constant velocity along the x-axis

This is our old equation X = Vt. Time is given by the vertical component.



ax	Vox	Vx	X	t
0			?	

5) A ball rolls off a platform that is 5m above the ground.
The ball horizontal velocity as it leaves the platform is Vox =6m/s.
A) How much time does it take for the ball to hit the ground?
B) How far from the base of the platform does the ball hits the floor ? (range or x)

6) A projectile is fired at an angle of 45 degrees. The initial velocity is 30m/s
A) Using g=-10, how long does it take the ball to reach its high point ?
B) What horizontal distance does the ball travel in this time ?

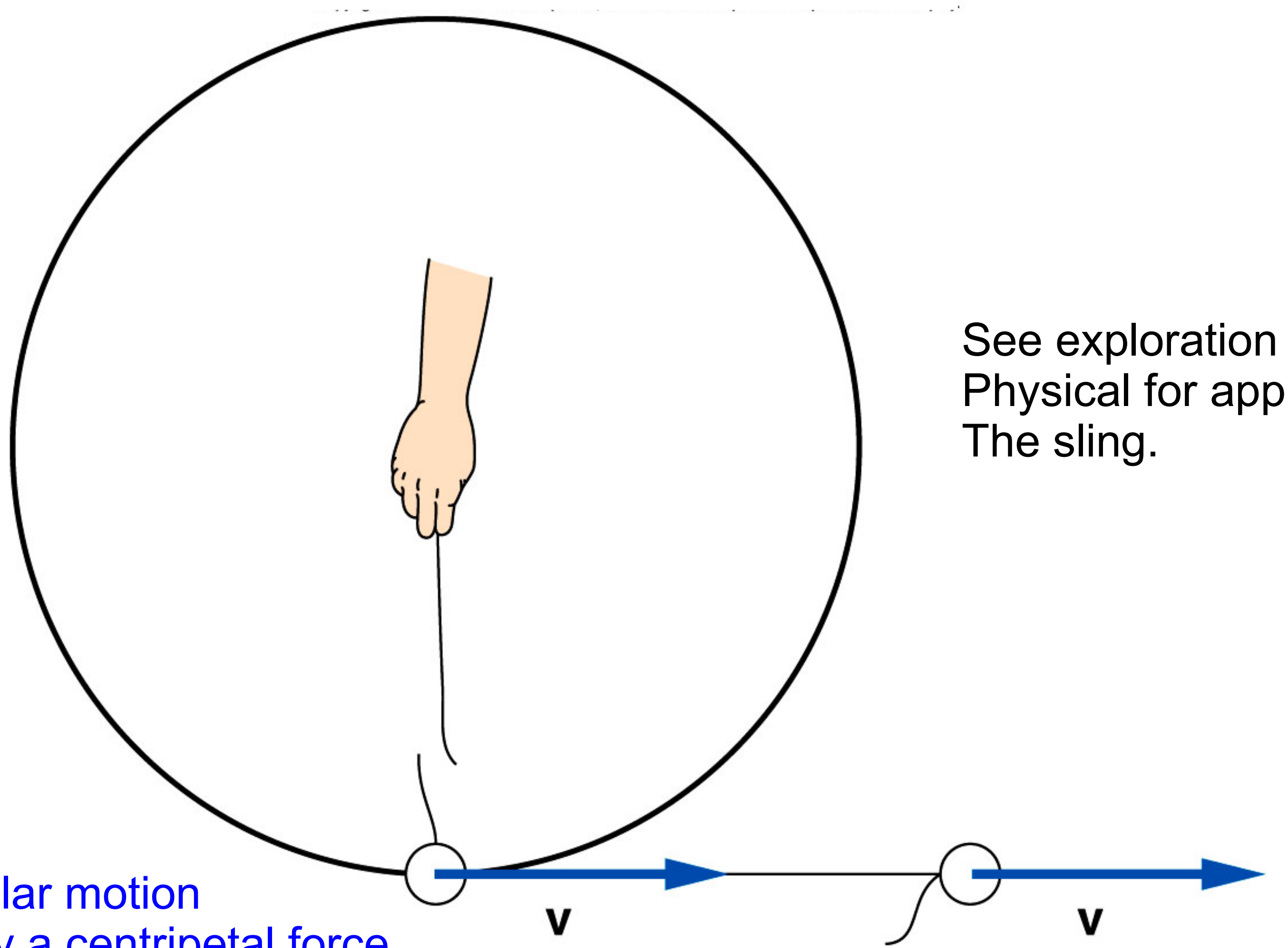
7)
A cannon is fired over level ground at an angle of 30 degrees to the horizontal.
The initial velocity of the cannon ball is 400m/s.

A) How long is the cannonball is the air ? (g=-10). Use the y-components.
Remember the total time of flight is twice the time required to reach the high point.
B) How far does the cannonball travel horizontally ?. Use the x-omponents.
C) Repeat these calculations assuming the cannon is fired at a 60 degrees angle to the horizontal.
In this case the horizontal component Is 400cos(60) and the vertical component is 400 sin(60).
D) How does the distance traveled compared to the earlier result ?

8)2 balls are rolled off a tabletop that is 0.8m above the floor.
Ball A has a horizontal velocity of Vox=3m/s and that of ball B is Vox=5m/s.
A) Assuming g=-10, how long does it take each ball to reach he floor ?
B) How far does each ball travel horizontally before hitting the floor ?

Assignments = take home quizzes to do

You spin a ball attached to a string. Which what will it go if you let go ? How this related to inertia ? And what if the Sun was to disappear , which way Will the Earth go ?



See exploration of
Physical for applet.
The sling.

See 2 video circular motion
You need to apply a centripetal force
To keep the object on the track
(demo: glass of water and the broom)

Source: the physics of every day phenomena / Mc Graw Hill

REMEMBER ACCELERATION = RATE of CHANGE of velocity

VELOCITY is a vector. VELOCITY CHANGES IF :

- magnitude changes (speed changes) or if

- direction changes !!!

Or BOTH

NOTE: **ACCELERATION IS PUSH/PULL per unit mass.**
ACCELERATION and FORCE have the same DIRECTION !!!

Acceleration is force per unit mass.

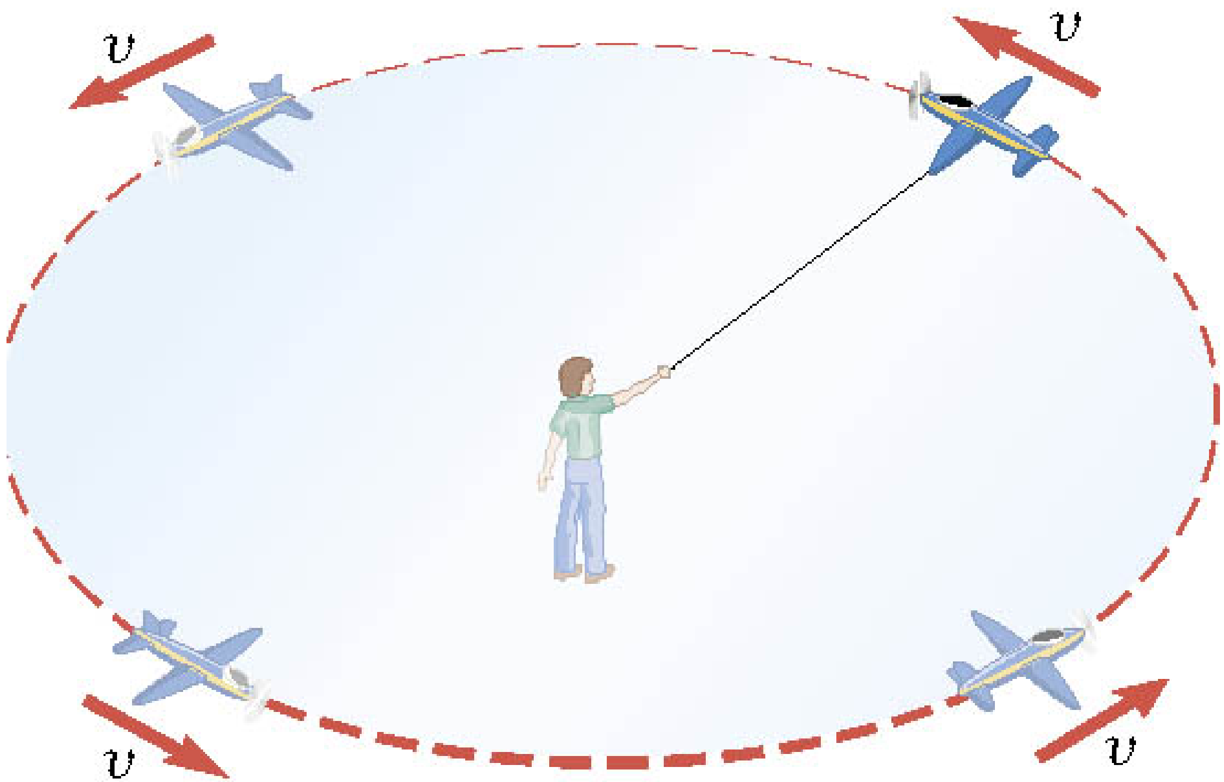
DEFINITION OF UNIFORM CIRCULAR MOTION

Uniform circular motion is the motion of an object traveling at a constant speed on a circular path.

The velocity is a vector.

The speed is constant
But not the velocity !!!
Can you see why ?

The velocity changes in direction.
You need a force to keep the
Plane on the track.
So there is an acceleration !
Force = acceleration !



See applets from exploration of physical

For an object in uniform circular motion, which one of the following statements is false ?

a) The velocity of the object is constant.

b) The magnitude of the acceleration of the object is constant.

c) The acceleration is directed radially inward.

d) The magnitude of the velocity is constant.

e) The velocity is directed in a direction that is tangent to the circular path.

Video circular motion (which path)

When using the term “uniform circular motion,” what do we mean by the term “uniform?”

a) The direction of the object’s velocity is constant.

b) The net force on the moving object is zero newtons.

c) The forces acting on the object are uniformly applied from all directions.

d) The motion occurs without the influence of the gravitational force.

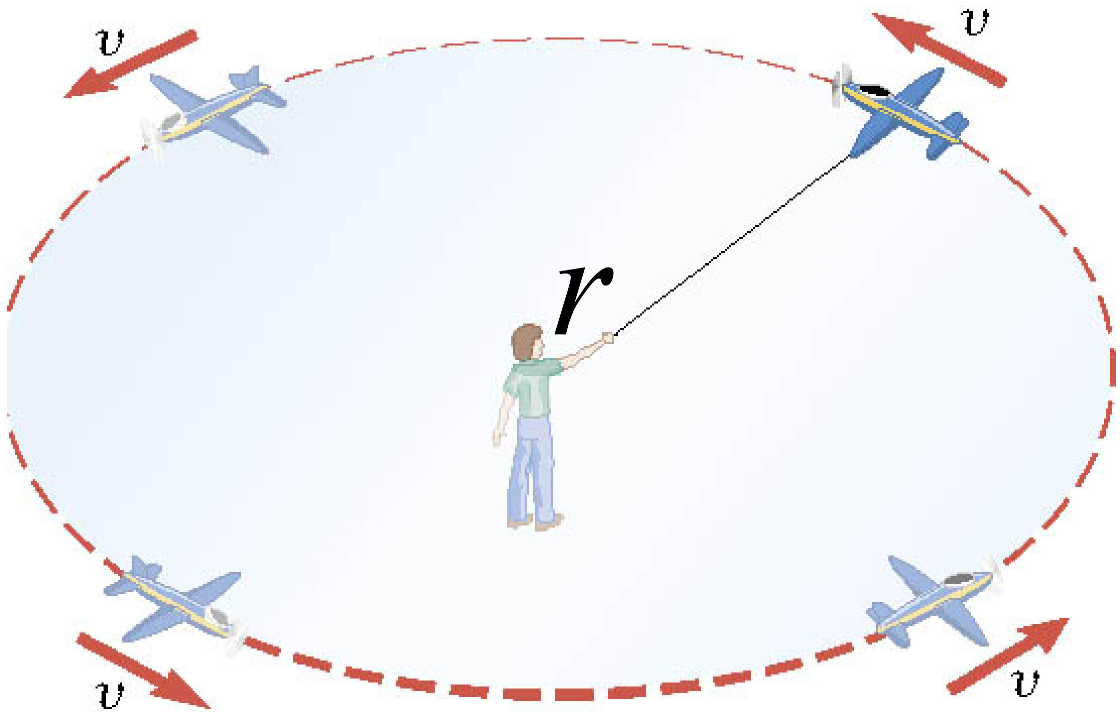
e) The motion of the object is at a constant speed.

Let *T* be the time it takes for the object to travel once around the circle. T is the period (s). The distance around a circle= 2 pi r
(or circumference – 360 degrees)
Note: The distance around half a circle = pi r (180 degrees)
The distance around ¼ a circle = 2 pi r /4 (90 degrees)

V is a velocity and its direction changes
Its magnitude is constant and is noted v
Here (lower case v in script)

V the angular speed.
It is not a velocity.
It is not a vector but a scalar.

Speed = distance / time
Speed = 2 pi R / time



An airplane flying at 115 m/s due east makes a gradual turn while maintaining its speed and follows a circular path to fly south. The turn takes 15 seconds to complete. What is the radius of the circular path?

a) 410 m

b) 830 m

c) 1100 m

d) 1600 m

e) 1098 m

Round to the nearest hundred.

From east to south that's ¼ of a circular path (circumference of a circle is 2 pi R)

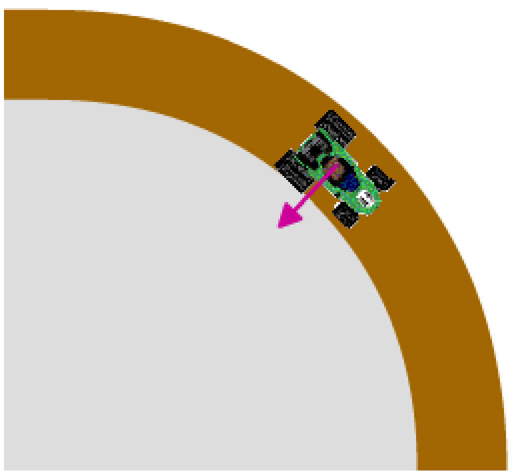
Example

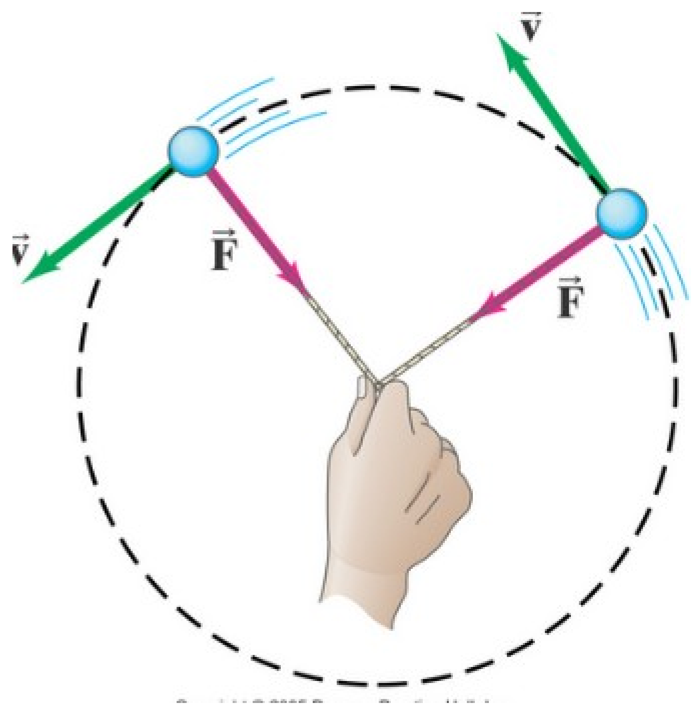
The wheel of a car has a radius of 0.29m and it being rotated at 830 revolutions per minute on a tire-balancing machine. Determine the speed at which the outer edge of the wheel is moving.

Centripetal acceleration

In uniform circular motion, the speed is constant, but the direction of the velocity vector is *not constant*.

$$a_c = \frac{v^2}{r}$$





The direction of the centripetal acceleration is towards the center of the circle; in the same direction as the change in velocity.

A bicycle racer is traveling at constant speed v around a circular track. The centripetal acceleration of the bicycle is a_c. What happens to the centripetal acceleration of the bicycle if the speed is doubled to $2v$?

a) The centripetal acceleration increases to $4a_c$.

b) The centripetal acceleration decreases to 0.25 a_c.

c) The centripetal acceleration increases to $2a_c$.

d) The centripetal acceleration decreases to $0.5a_c$.

e) The centripetal acceleration does not change.

A satellite orbits the Earth in uniform circular motion. What is the direction of centripetal acceleration of the satellite?

a) Because the centripetal acceleration is a scalar quantity, it doesn't have a direction.

b) The centripetal acceleration vector points radially outward from the Earth.

c) The centripetal acceleration vector points radially inward toward the Earth.

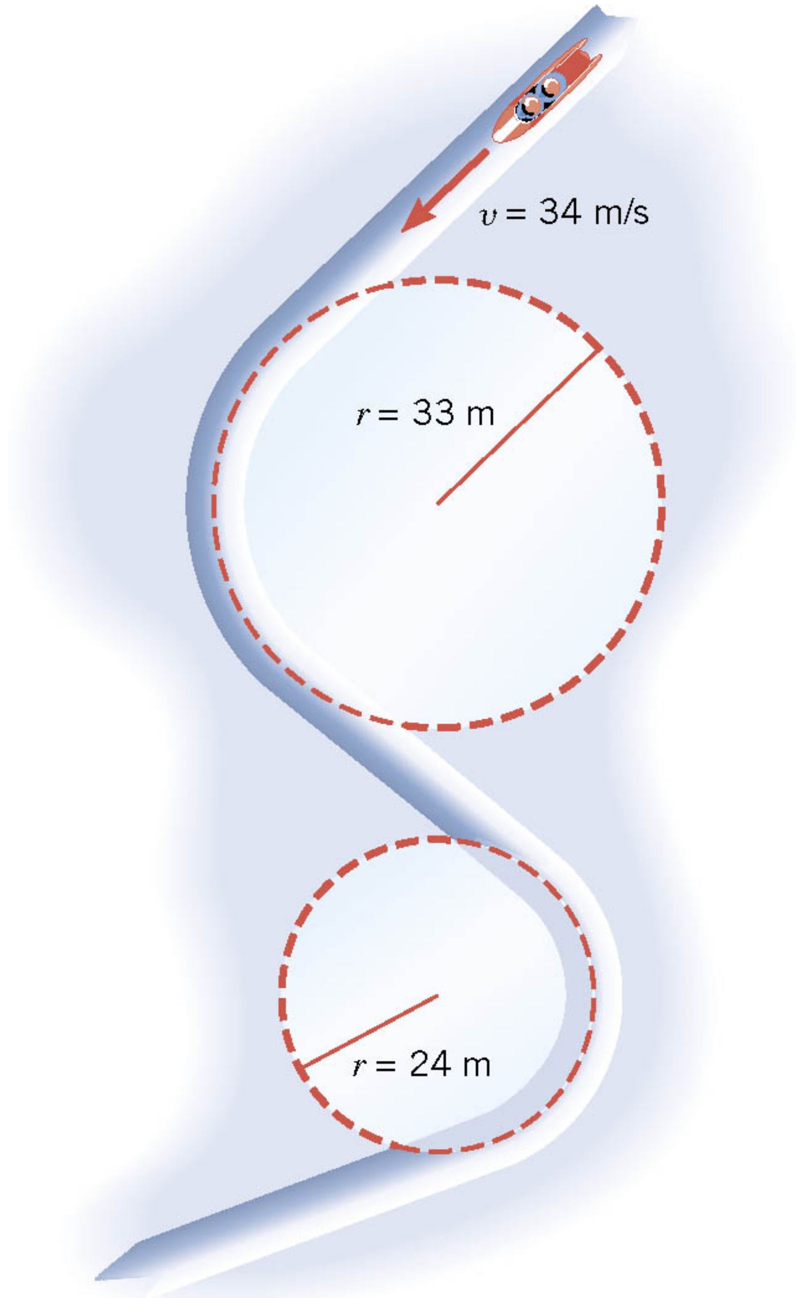
d) The centripetal acceleration vector points in the direction of the satellite's velocity.

e) The centripetal acceleration vector points in the direction opposite that of the satellite's velocity.

Example : The Effect of Radius on Centripetal Acceleration

The bobsled track contains turns with radii of 33 m and 24 m. Find the centripetal acceleration at each turn for a speed of 34 m/s. Express answers as multiples of

$$g = 9.8\,\mathrm{m/s^2}.$$



A truck is traveling with a constant speed of 15 m/s. When the truck follows a curve in the road, its centripetal acceleration is 4.0 m/s^2. What is the radius of the curve?

a) 3.8 m

b) 14 m

c) 56 m

d) 120 m

e) 210 m

$$a_c = \frac{v^2}{r} \qquad v = \frac{2\pi r}{T}$$

$$a_c = \frac{4\pi^2 r}{T^2}$$

While we are in this classroom, the Earth is orbiting the Sun in an orbit that is nearly circular with an average radius of 1.50×10^{11} m. Assuming that the Earth is in uniform circular motion, what is the centripetal acceleration of the Earth in its orbit around the Sun?

a) 5.9×10^{-3} m/s^2

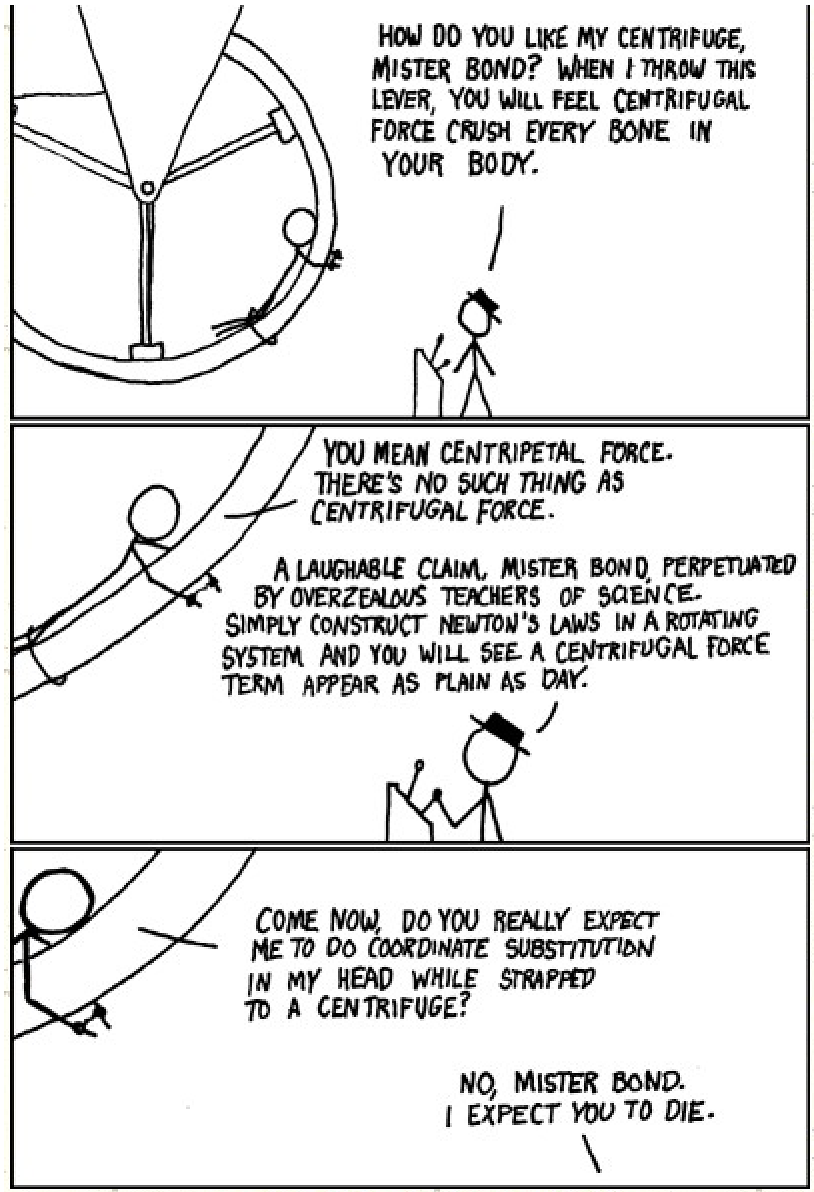
b) 1.9×10^{-5} m/s^2

c) 3.2×10^{-7} m/s^2

d) 7.0×10^{-2} m/s^2

e) 9.8 m/s^2

$$a_c = \frac{4\pi^2 r}{T^2}$$



source: http://xkcd.com/556/

1) What is the acceleration of a motorcycle going 28m/s on a circular track whose radius is 140m?

2) The clothes in a rotary drier whose radius is 0.25m are accelerated toward the center of the drier at 22m/s/s . How fast are the clothes moving ?

3) A ball is traveling at a constant speed of 5m/s in a circle with a radius of 0.8m. What is the centripetal acceleration of the ball?

4) A car rounds a curve with a radius of 25m at a speed of 20m/s. What is the centripetal acceleration of the car ?

5) A ball traveling in a circle with a constant speed of 3m/s has a centripetal acceleration of 9m/s/s. What is the radius of the circle.

6) How much larger is the required centripetal acceleration for a car rounding a curve at 60PMH then for one rounding the same curve at 30 MPH ?

7) A car with a mass of 1200kg is moving around a curve with a radius of 50m at a constant speed of 20m/s (45MPH)
What is the centripetal acceleration of the car?

optional

8) Object in orbit around the earth (artificial satellites, labs, the Moon..) are in free-fall. A satellite at low orbit has an acceleration of -10m/s/s = g. This acceleration is the centripetal acceleration of the satellite. So $|g|=v^2 / r$ with r = radius of Earth + altitude.

A) An artificial satellite is to be put in orbit a short distance above the surface of the Earth (r=6,400,000 m) . What speed must it be given
To make it go into a circular orbit ?

B) What is the acceleration due to gravity g ($|g|=v^2 / r$) at the surface of a planet with a radius of 1,600,000 m? If a satellite near the
Surface makes one circular orbit every 1700s ? (hint: v = speed = 2pi r / time)

C) How fast was the lunar orbiter traveling when it was in orbit around the moon, near its surface ? The radius of the Moon is 3,500,000 m,
And the acceleration due to gravity at its surface is 1.67m/s/s

slide 58

#1

$$a_c = \frac{(28)^2}{140} = \boxed{5.6\ m/s^2}$$

#2

$$\frac{22}{1} = \frac{v^2}{0.25} \Rightarrow v^2 = 0.25 \times 22$$

$$\boxed{v = 2.35\ m/s}$$

#3 $a_c = \frac{5^2}{0.8} = \boxed{31.25\ m/s^2}$

#4 $a_c = \frac{20^2}{25} = \boxed{16\ m/s^2}$

#5 $\frac{9}{1} = \frac{3^2}{R} \Rightarrow R = \frac{9}{9} = \boxed{1m}$

#6 $30 \xrightarrow{\times 2} 60$ But $a_c = \frac{v^2}{R}$

so when speed x 2 acceleration is x 4

#7

$$a_c = \frac{(20)^2}{50} = \boxed{8\ m/s^2}$$

#8

$$10 = \frac{v^2}{R}.$$

A $\frac{10}{1} = \frac{v^2}{6400{,}000} \Rightarrow v = \sqrt{10 \times 6400{,}000}$

$= \boxed{8000\ m/s}$

B $T = 1700 s$

$$a_c = \frac{4\pi^2 R}{T} = \frac{4\pi^2 (1600{,}000)}{(1700)^2}$$

$a_c = \cancel{21.856\ m/s^2}$.

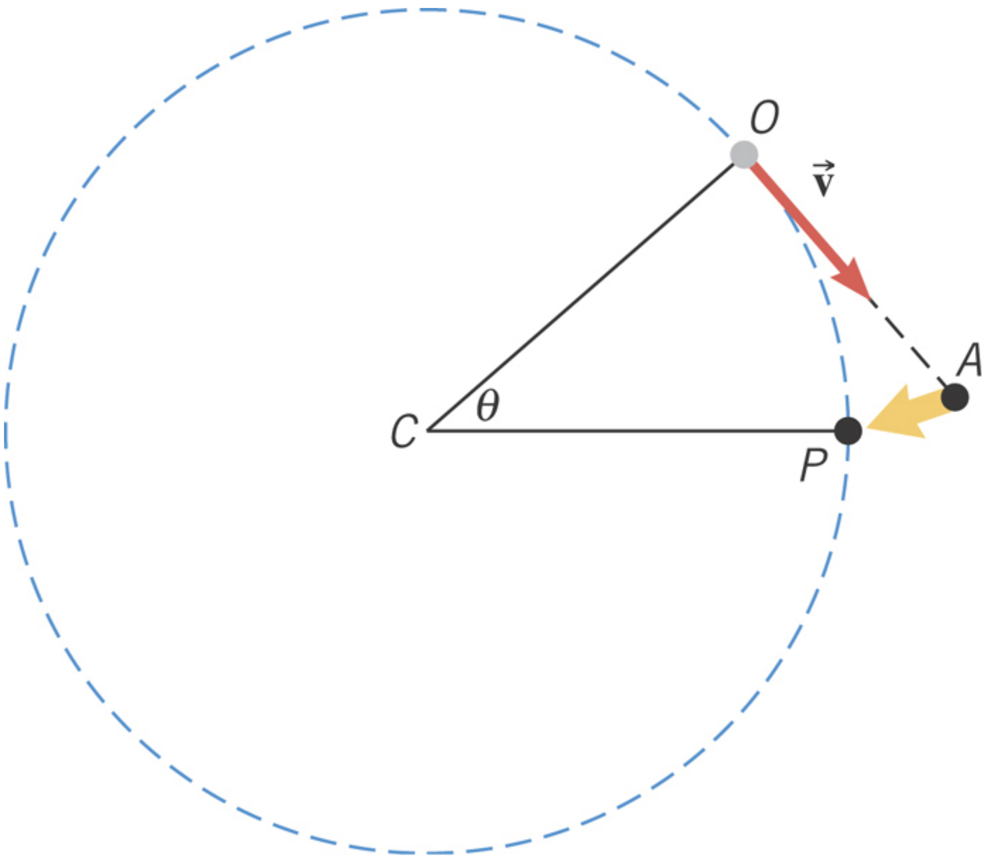
$\boxed{a_c = 22\ m/s^2}$

C $a_c = \frac{v^2}{R} \Rightarrow \frac{1.67}{1} = \frac{v^2}{3500{,}000}$

$v = \sqrt{3500{,}000 \times 1.67} = \boxed{2417.6\ m/s}$

Conceptual Example 2: Which Way Will the Object Go?

An object is in uniform circular motion. At point O it is released from its circular path. Does the object move along the straight path between O and A or along the circular arc between points O and P ?



Hint: to change a motion You need to apply a force.

Changing a motion means: changing direction, Speed or both.

Video: circular motion: which path

5.7.3. A steel ball is tied to the end of a string and swung in a vertical circle at constant speed. Complete the following statement: The direction of the instantaneous velocity of the ball is always

a) perpendicular to the circle.

b) toward the center of the circle.

c) tangent to the circle.

d) radially outward from the circle.

e) vertically downward.