

- definition of force – slide 1
- revisiting vectors: components of forces – slide 7
- adding vectors in 2D:  
Tail to head method – Algebraic method – slide 10
- types of forces: weight, normal force, tension – slide 27
- static equilibrium – slide 46
- frictional force – slide 51
- coefficient of friction – slide 64
- rotational equilibrium – slide 66

A force is a push or pull. A force causes an object to change its state of motion:

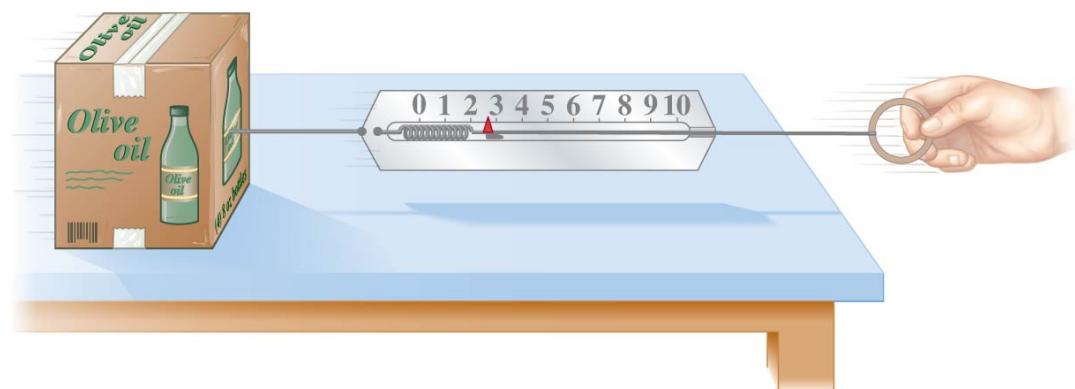
- get it moving
- stopping it
- slowing down or speeding it up
- changing its direction (making it turn)

**Force → change of velocity -> acceleration**

**(Newton's 2<sup>nd</sup> law : $F = m a$  or acceleration is the force per unit mass )**

We will see that once the object is moving, it will keep moving (with the same velocity) without push/pull . Unless there is friction which is a force.

The magnitude of a force can be measured using a spring scale.



# Units of Force

- SI unit of force is a Newton (N)

$$1 \text{ N} \equiv 1 \frac{\text{kg m}}{\text{s}^2}$$

- US Customary unit of force is a pound (lb)
  - $1 \text{ N} = 0.225 \text{ lb}$

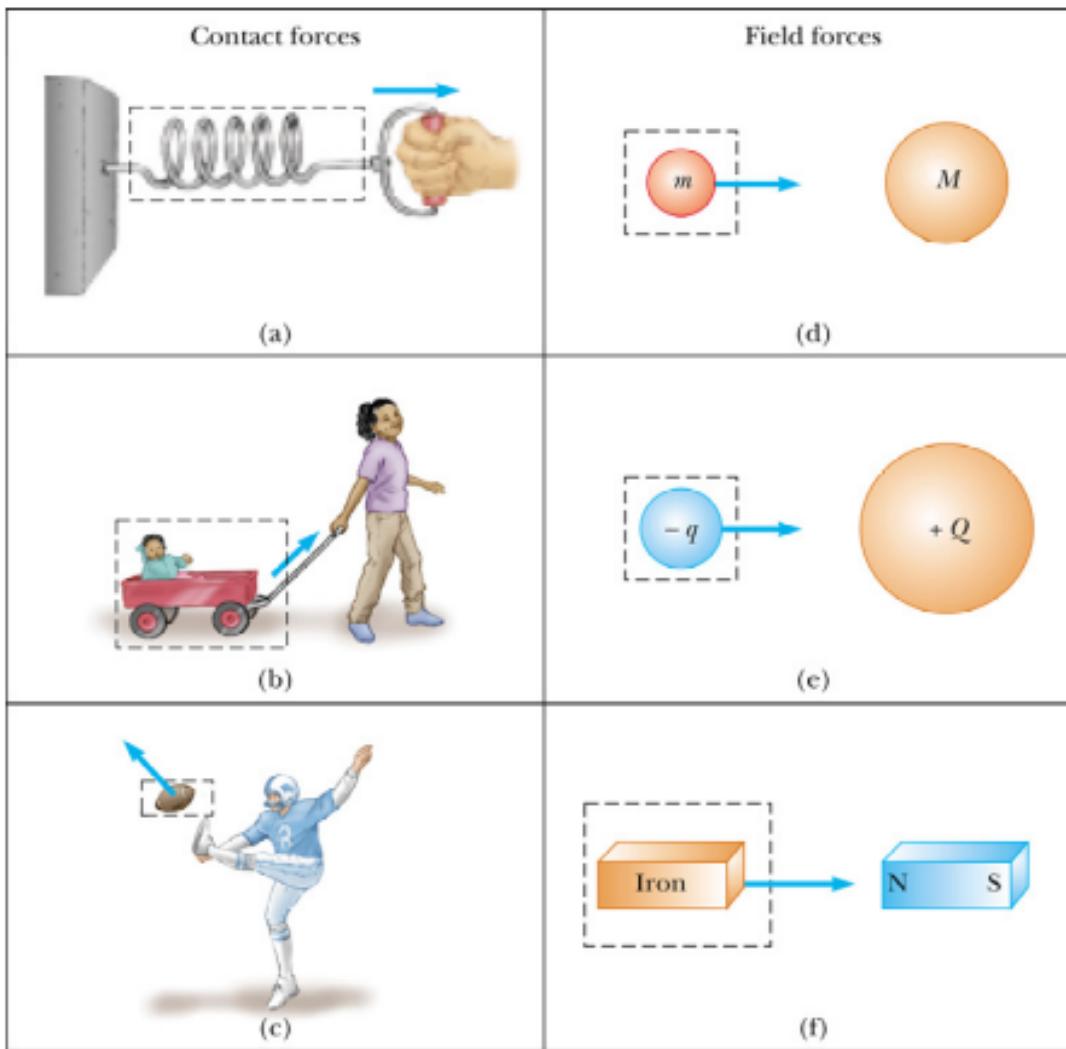
10 N is about 2 pounds which is about 1 kg  
(to go from newtons to pounds divide by 5)

A **force** is a push or a pull.

**Contact forces** arise from physical contact .

**Action-at-a-distance forces** do not require contact and include gravity and electrical forces.

# Contact and Field Forces

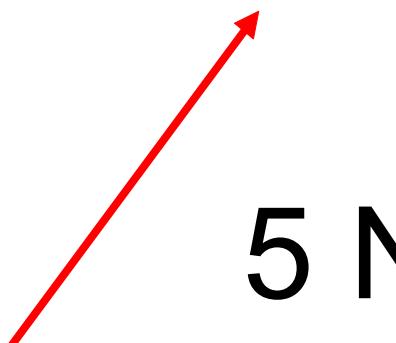


Forces are vectors.

Forces have a magnitude (how much) and direction (which way)



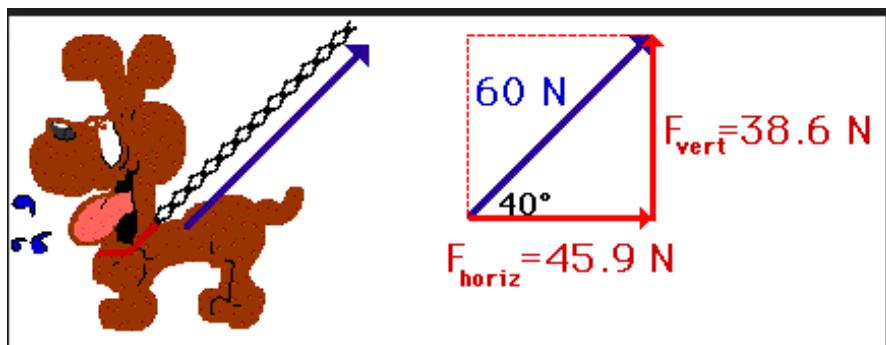
15 N



## REMEMBER VECTORS COME WITH COMPONENTS !!

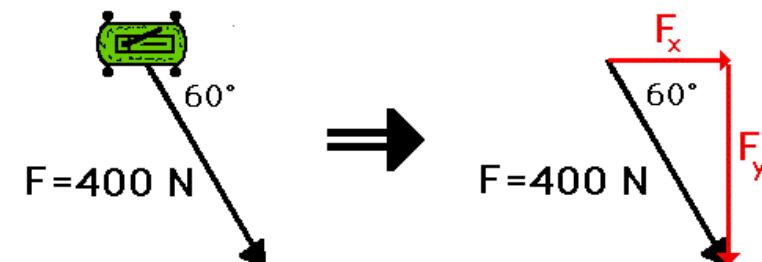
Components are independent from each other and have “different” job.

Source: <http://www.physicsclassroom.com/class/vectors/Lesson-3/Resolution-of-Forces>



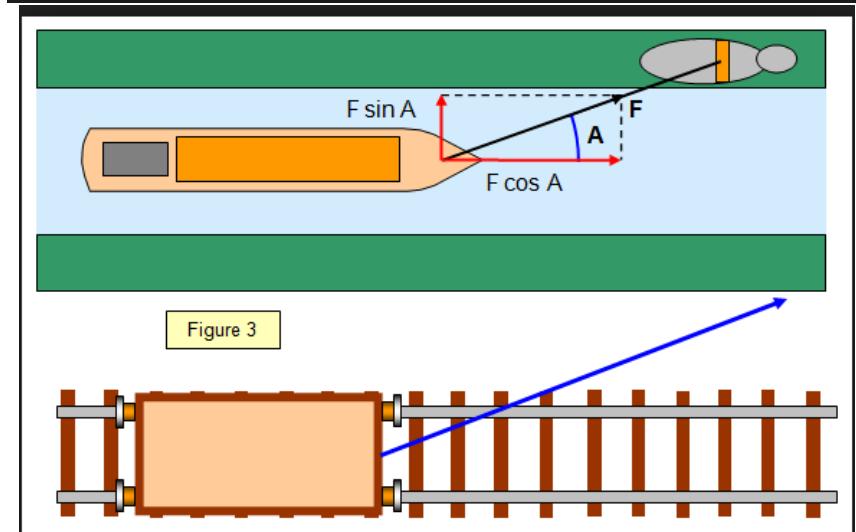
Find the components of the force  
Pulling the dog through the leash.

What is the “ job “ of each component



Find the components of the force  
if the wind pushing the sail.

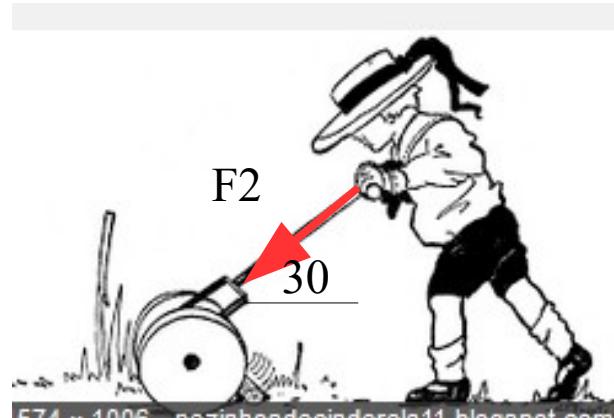
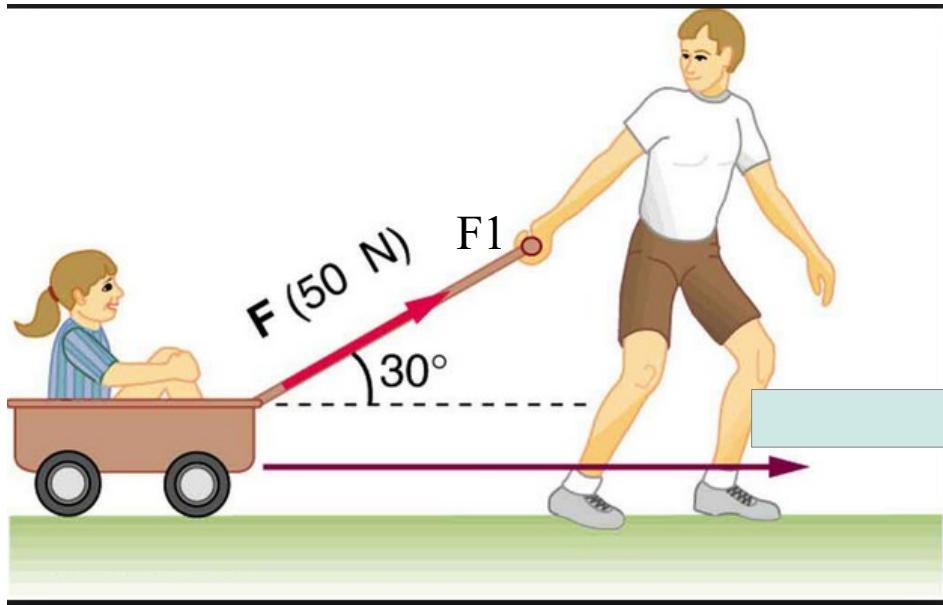
What is the “ job “ of each component  
( $F_x=200\text{N}$ ,  $F_y=-173\text{N}$ )



Describe in each  
Case, what is the “job  
Of each component.



If the vector (force) is in standard notation → cosine gives the x-component  
→ sine gives the y-component



Draw each vector in a free-body diagram (x-y coordinate system). The tail is attached to the origin. Compute the components of the vectors. The magnitude of the force is the same but not its direction.

$F_1 = 50\text{N}$  @ \_\_\_\_\_ but  $F_2 = 50\text{N}$  @ \_\_\_\_\_ (use standard notation)

The components are not the same :  $F_1( \ , \ )$     $F_2( \ , \ )$

Describe what the x-component and the y-component are doing to each object.

The y-component of  $F_1$  is \_\_\_\_\_ the wagon with a force of \_\_\_\_\_ .

The y-component of  $F_2$  is \_\_\_\_\_ .

If there is friction. Which way is the best to pull with less effort ?

$F_1(43,25)$     $F_2(-43,-25)$

In a free-body diagram you find the components of the forces.  
The x-axis is independent from the y-component

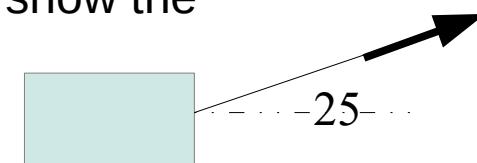
- 1) you are dragging a crate and the rope makes a 25 degrees angle with horizontal.

The tension in the rope is 250N. Make a free-body diagram and show the force.

A) How much force is being used to drag the crate ?

(find the horizontal component)

B) How much force (from you) is pulling the crate upward ?



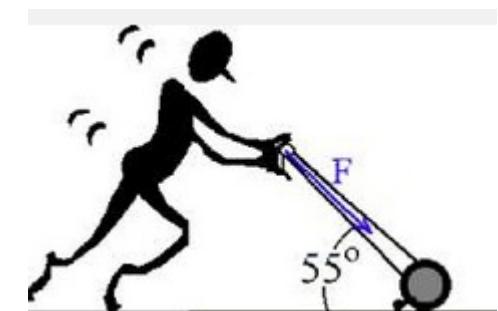
Solution:  $F(226.6, 105.6)$

- 3) You push a lawn mower with a force of 160N, exerted directly along its shaft.

The shaft makes an angle of 55 degrees with the ground. Make a free body-diagram

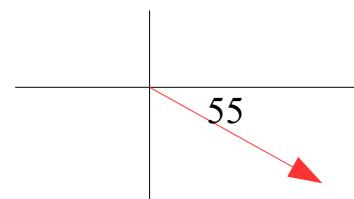
A) How much force is moving the lawn mower (along horizontal)

B) how much force is pushing the lawn mower toward the ground.



Solution:

$F(92, -131)$

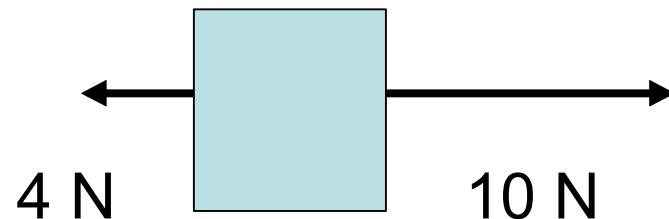


## Adding vectors

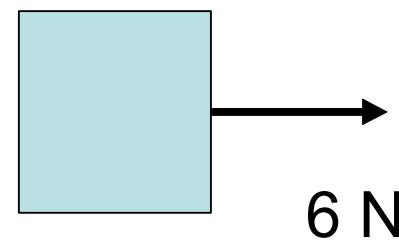
The net force on an object is the vector sum of all forces acting on that object.

The SI unit of force is the Newton (N).

Individual Forces



Net Force



## Adding forces in 1D is easy:

The *net force (sum)* is the vector sum of all of the forces acting on an object.

$$\begin{array}{c} \text{---} \\ \text{5} \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ \text{5} \\ \text{---} \end{array} =$$



$$\begin{array}{c} \text{---} \\ \text{5} \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ -5 \\ \text{---} \end{array} =$$



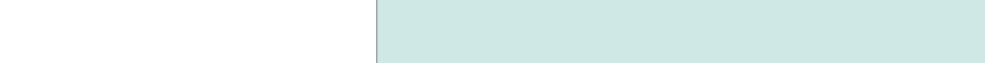
$$\begin{array}{c} \text{---} \\ \text{5} \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ \text{10} \\ \text{---} \end{array} =$$



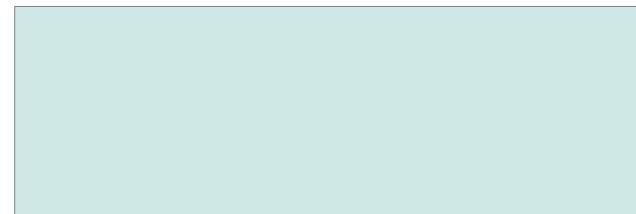
$$\begin{array}{c} \text{---} \\ \text{5} \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ -10 \\ \text{---} \end{array} =$$



$$\begin{array}{c} \text{---} \\ \text{5} \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ -15 \\ \text{---} \end{array} =$$



$$\begin{array}{c} \text{---} \\ \text{10} \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ -5 \\ \text{---} \end{array} =$$



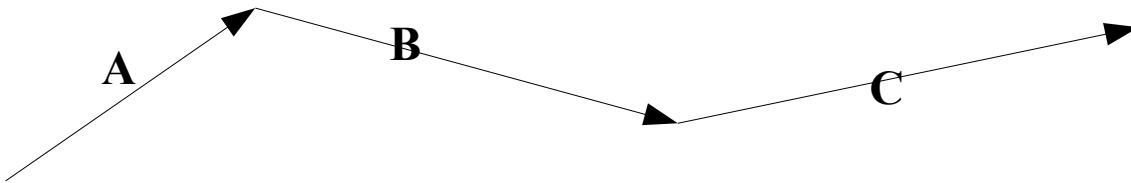
Source:

Mathematically, the net force is written as

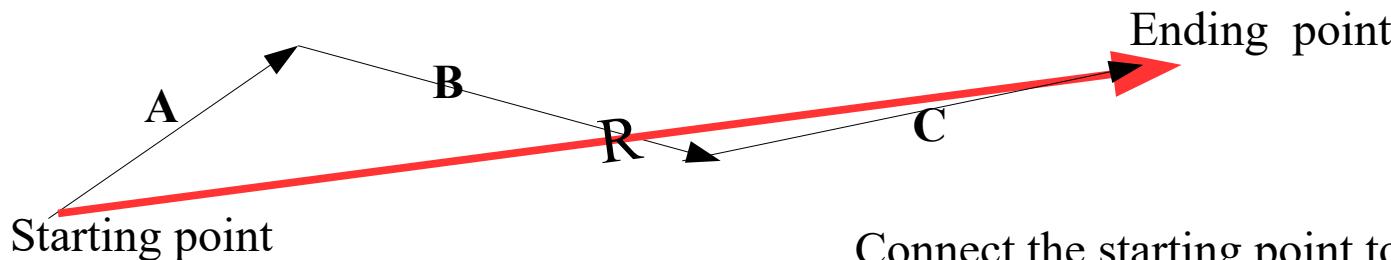
$$\sum \vec{F}$$

where the Greek letter sigma denotes the vector sum.

How to add vectors: method head to tail (tail to head)  
Example: add the vectors A, B, C

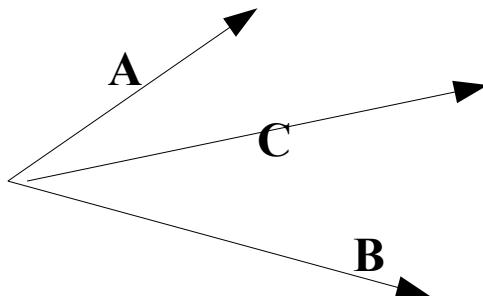


$$A + B + C = R \quad R \text{ is the vector sum (or resultant)}$$

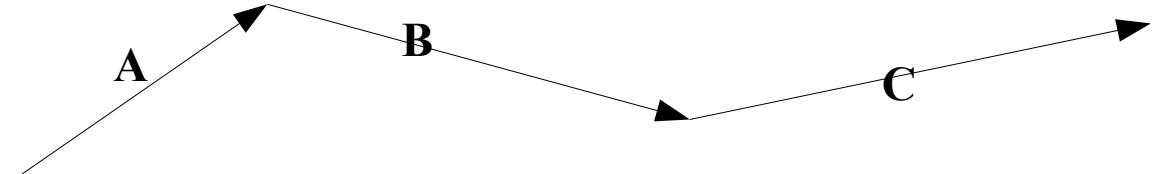


Connect the starting point to ending point  
The vectors are tail to head (head to tail)

If the vectors are not tail to head then move them tail to head



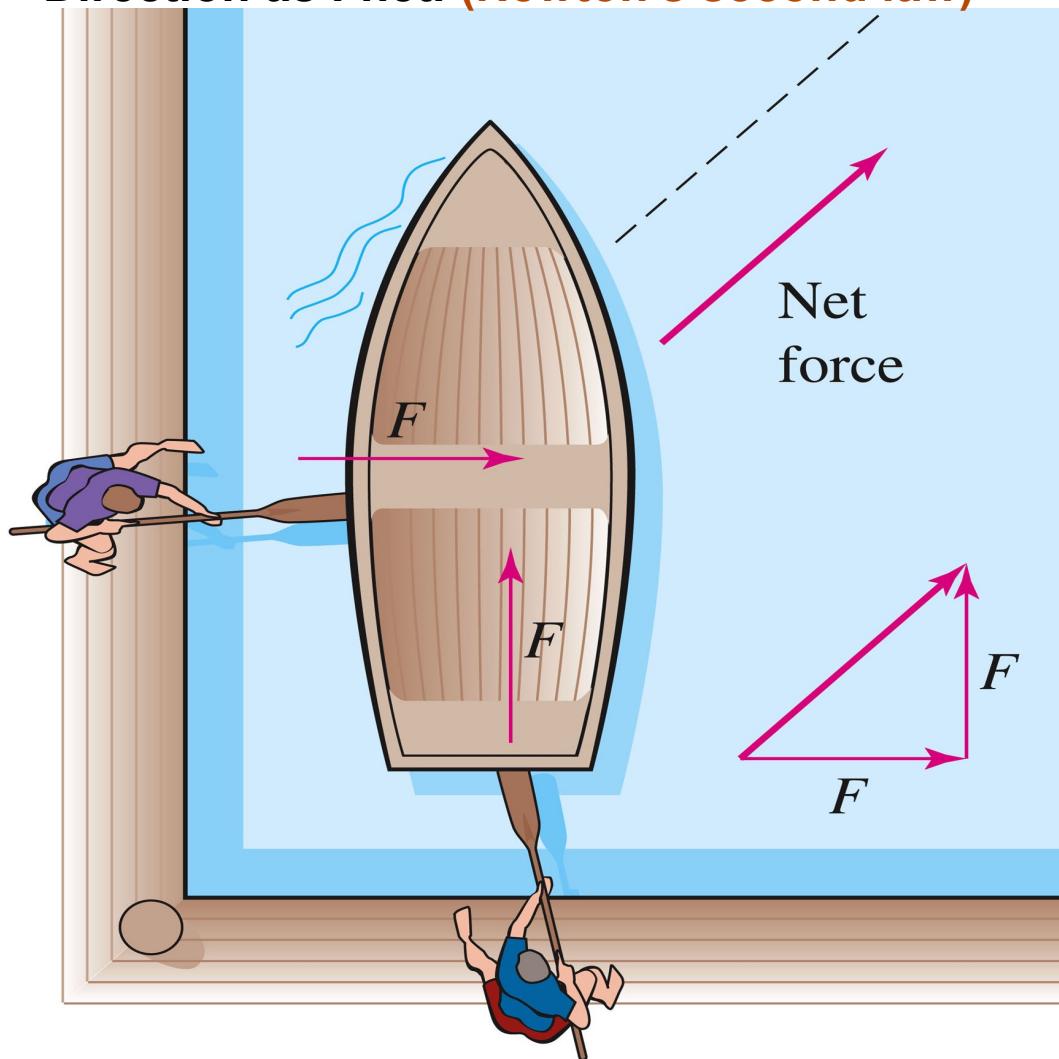
Move the tail of B on head of A and the tail of C on head of B



Try: <http://www.physicsclassroom.com/class/vectors/Lesson-1/Vector-Addition>

Special case: ITS EASY TO ADD VECTORS IF THEY ARE PERPENDICULAR

In which direction do you think the boat will move ? =  
Draw the resultant  $F_{net}$  ( a vector). The boat will move in the same  
Direction as  $F_{net}$ . (**Newton's second law**)



If  $|F| = 100\text{N}$   
Find R or  $F_{net}$

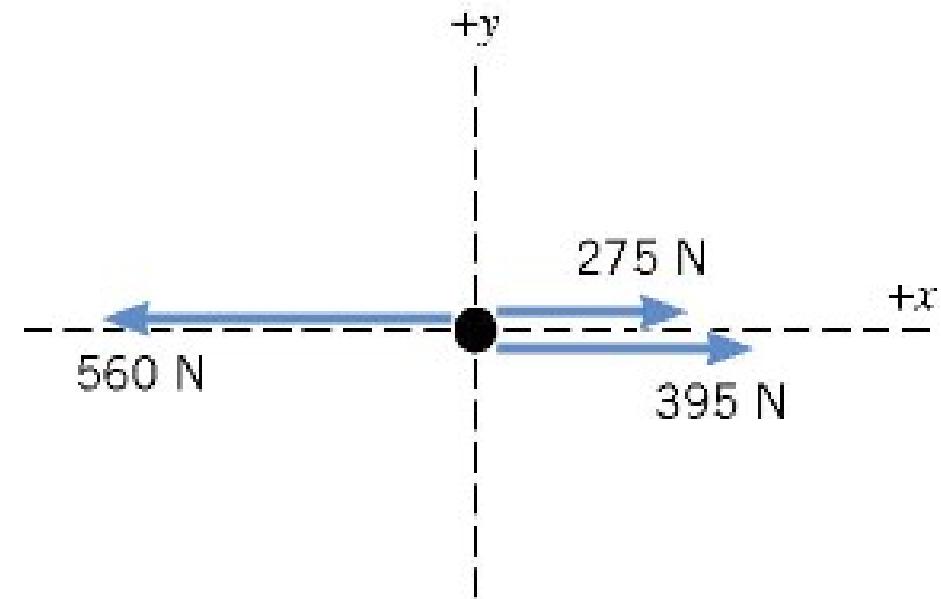
First draw a free-body  
Diagram :attach the tails  
Of the forces at the origin.  
The boat is reduced to  
The origin.

A **free-body-diagram** is a diagram that represents the object and the forces that act on it.

Find the net force



(a)

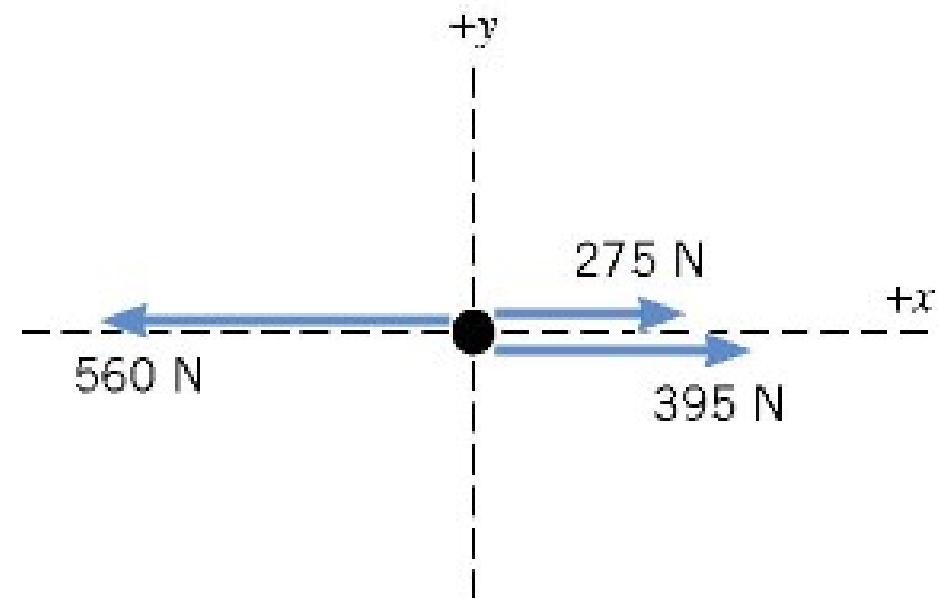


(b) Free-body diagram of the car

#### 4.3 Newton's Second Law of Motion



(a)



(b) Free-body diagram of the car

The net force in this case is:

$$275 \text{ N} + 395 \text{ N} - 560 \text{ N} = +110 \text{ N}$$

and is directed along the + x axis of the coordinate system.

Which one of the following tools is useful in representing the forces acting on an object and simplifies problem solving?

a) free-body diagram

b) scalar drawing

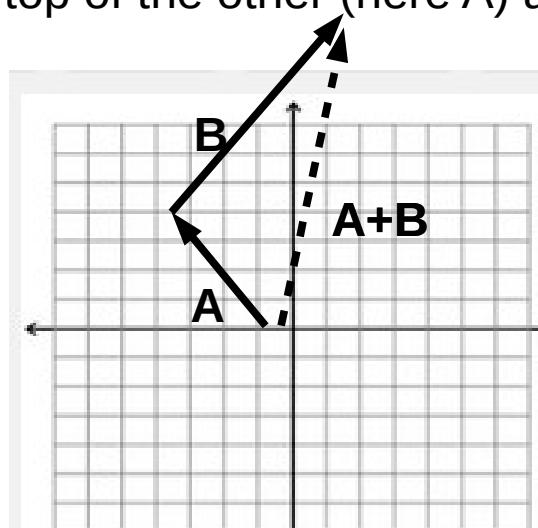
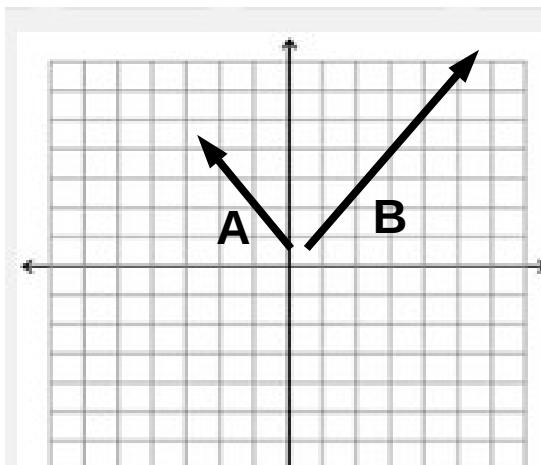
c) vector analyzer

d) Newton's ladder

e) force monitor

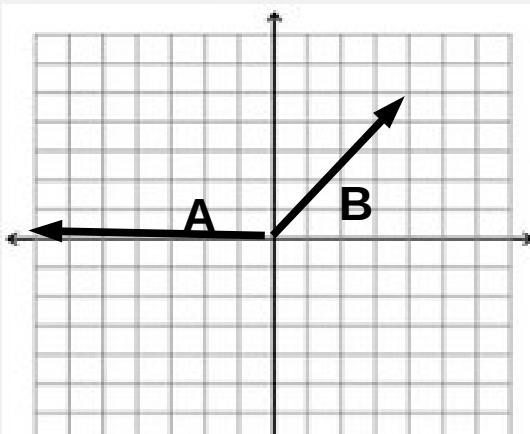
## adding vectors in 2D geometrically

How to trace the vector sum of 2 vectors (or more) that are not perpendicular:  
Place one vector (here B) on top of the other (here A) and connect the first tail to last head.

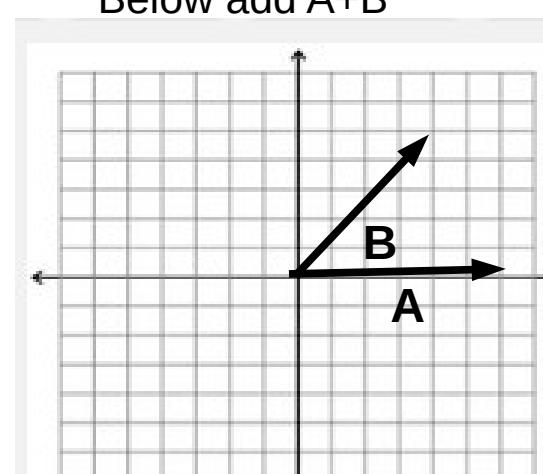


<https://phet.colorado.edu/en/simulation/vector-addition>

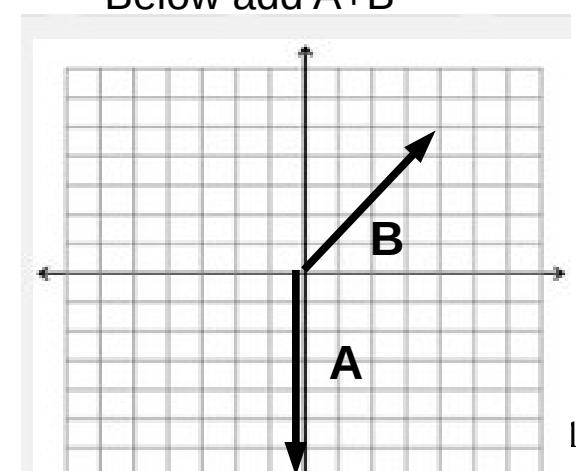
Below add  $A+B$



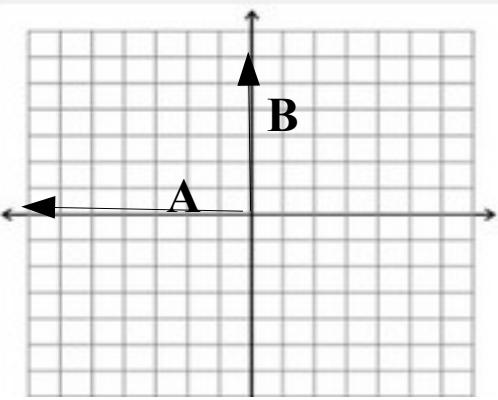
Below add  $A+B$



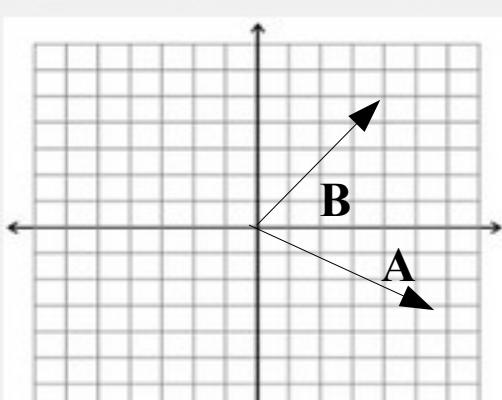
Below add  $A+B$



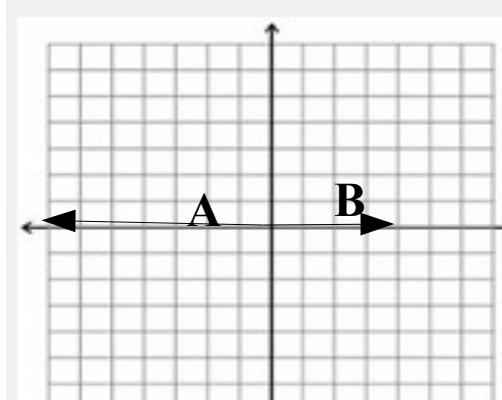
Below add A+B



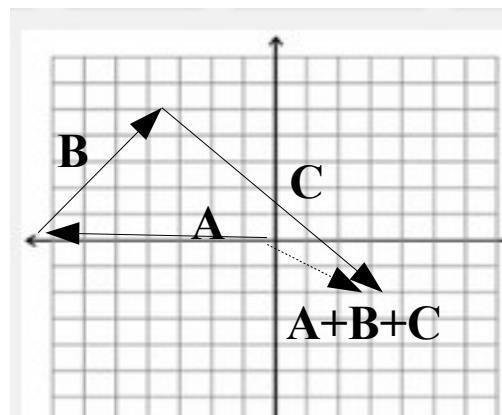
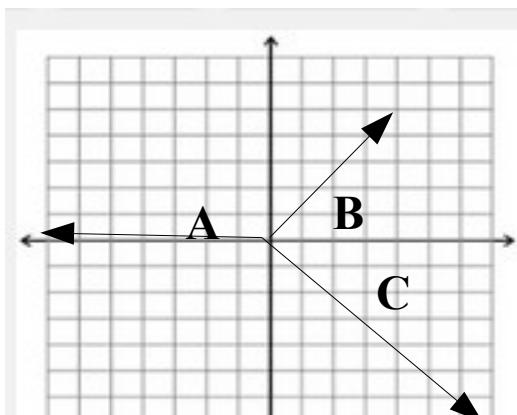
Below add A+B



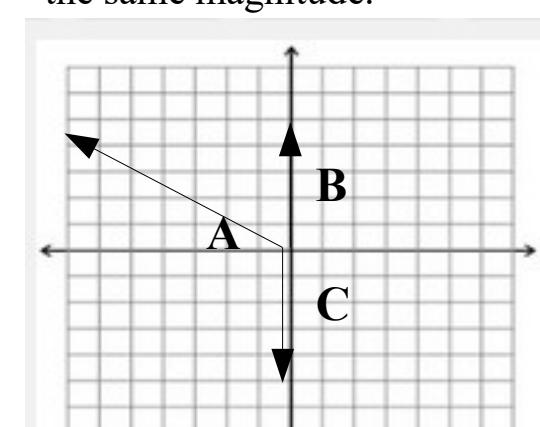
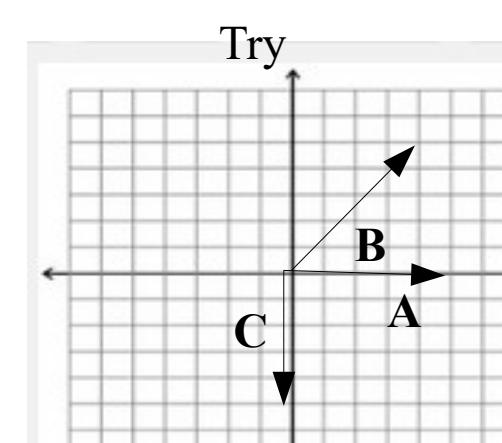
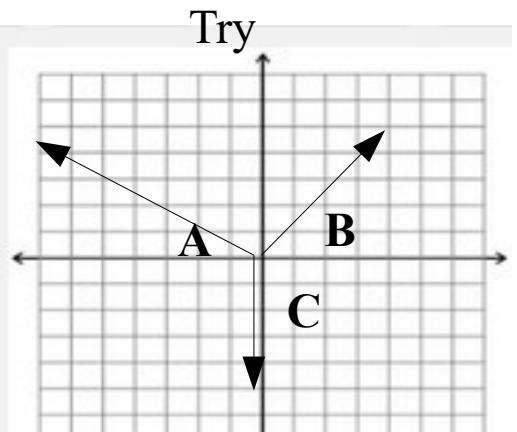
Below add A+B



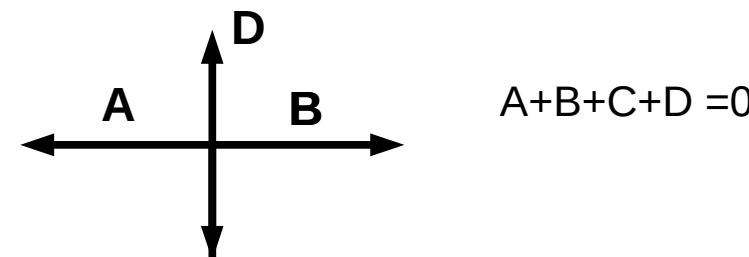
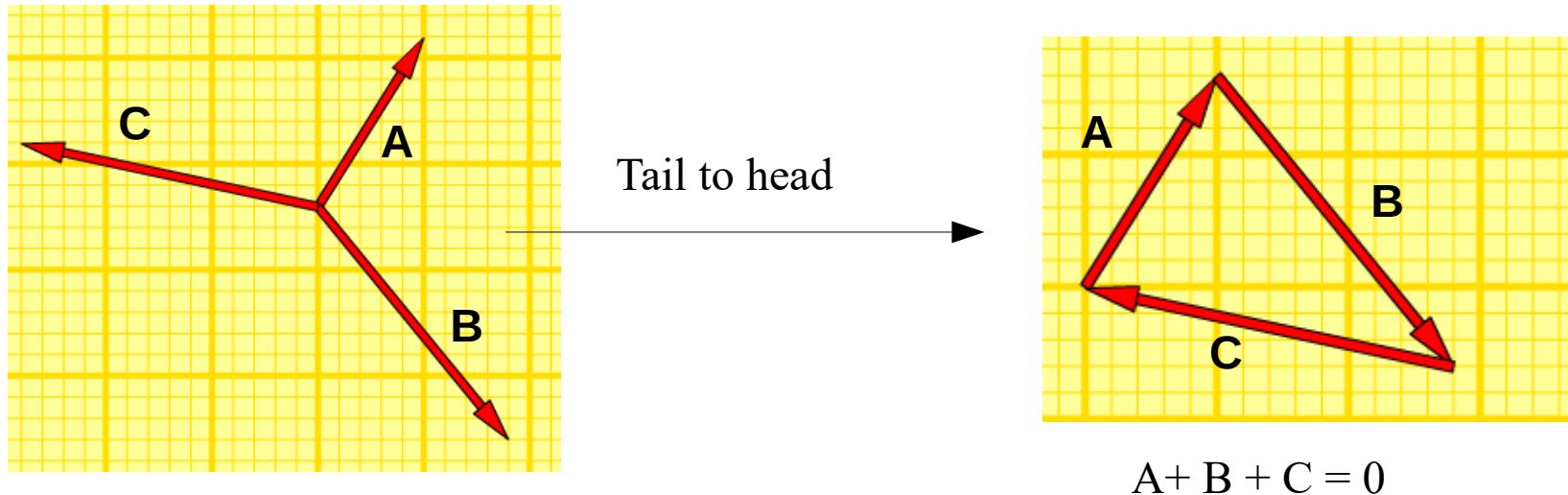
You can add more than 2 vectors :



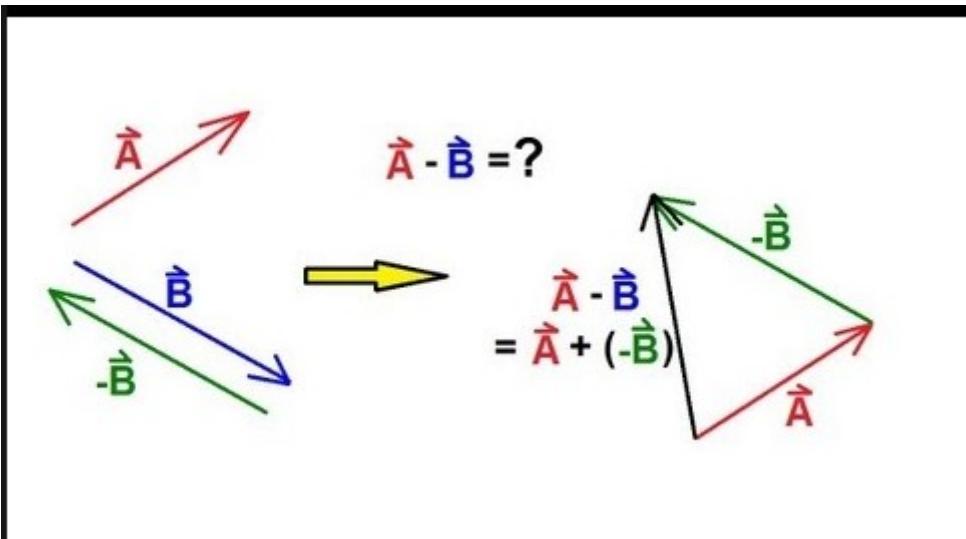
Try. Hint: B and C have the same magnitude.



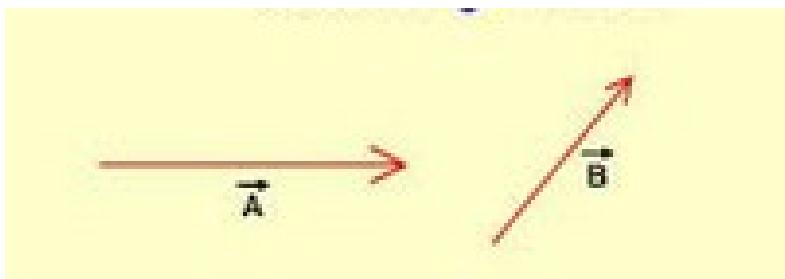
The vectors add up to zero  
when the starting point = ending point  
using the tail to head method/



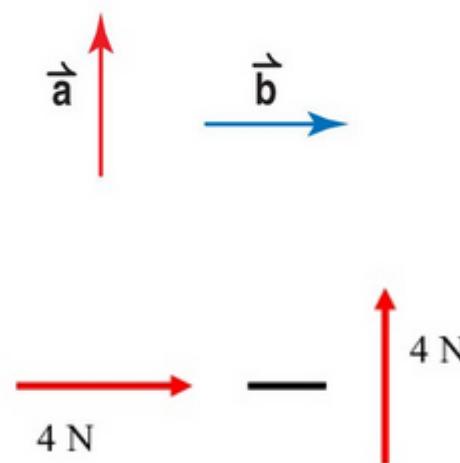
Optional: When you subtract vectors, you add an opposite.  $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$



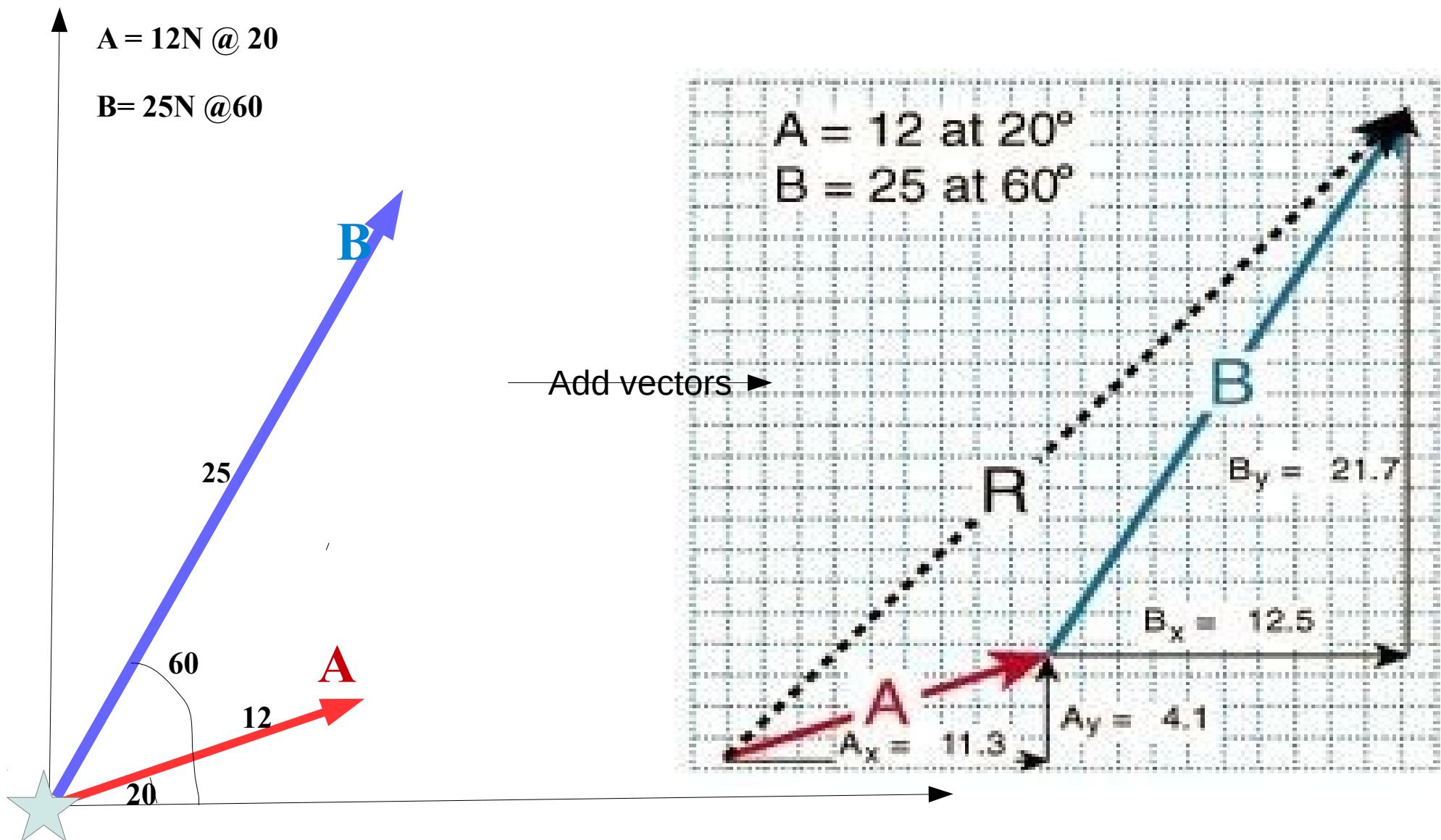
subtract vectors geometrically



$$\begin{array}{ccccccc} & & & & & & \\ \leftarrow & & \text{---} & & \rightarrow & & \text{---} \\ 6 \text{ m/s} & & & & 4 \text{ m/s} & & \end{array}$$



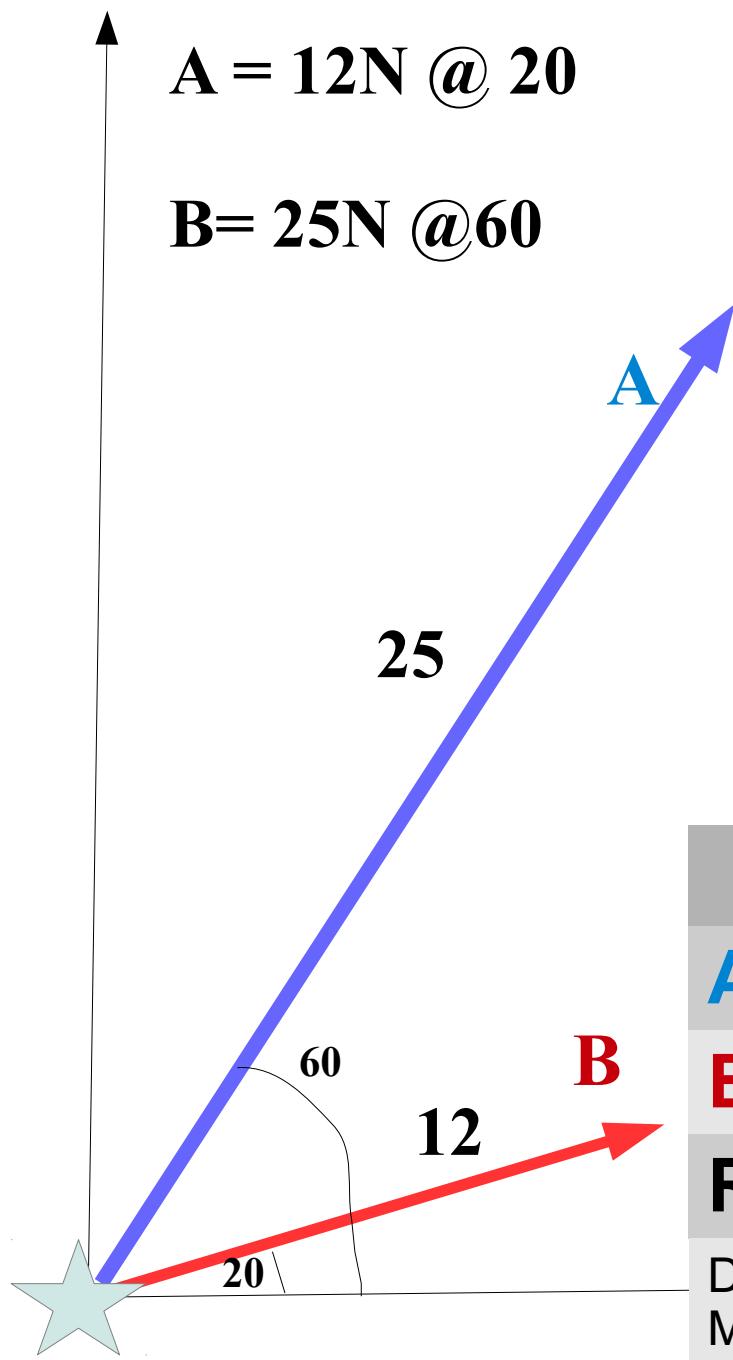
# Adding vector algebraically



$$Rx = Ax + Bx = 12.5 + 11.3 = \underline{\hspace{2cm}} \quad Ry = Ay + By = \underline{\hspace{2cm}} \quad \text{so magnitude of } R = \underline{\hspace{2cm}}$$

Direction =  $\arctan(Ry/Rx)$

Do the assignment in class.



When you add vectors you add the components  
See exploration of physics.

How to add vector ?

Use the component method and trig.

$$Ax = 25\cos(60) \quad Ay = 25\sin(60)$$

$$Bx = 12\cos(12) \quad By = 12\sin(12)$$

$$Rx = Ax + Bx$$

$$Ry = Ay + By$$

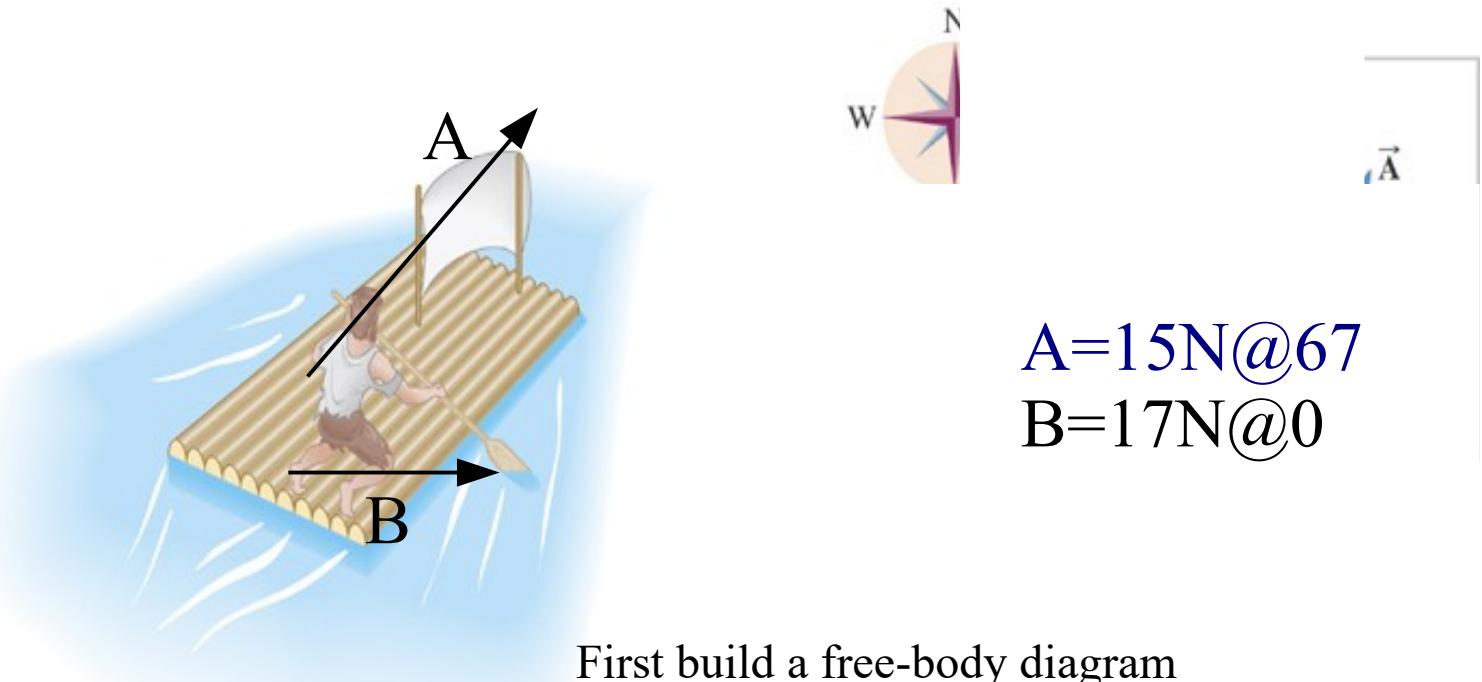
Fill the table.

To draw R move B on the head of A  
And connect the tail of A to the head of B

	X	Y
A		
B		
R		

Direction =  $\arctan(Y/X) =$   
Magnitude =

*The raft is acted upon by 2 forces:  
from the current of the river @ right and from Robinson  
Find the net force acting on the boat.*



$$A = 15\text{N}@67^\circ$$

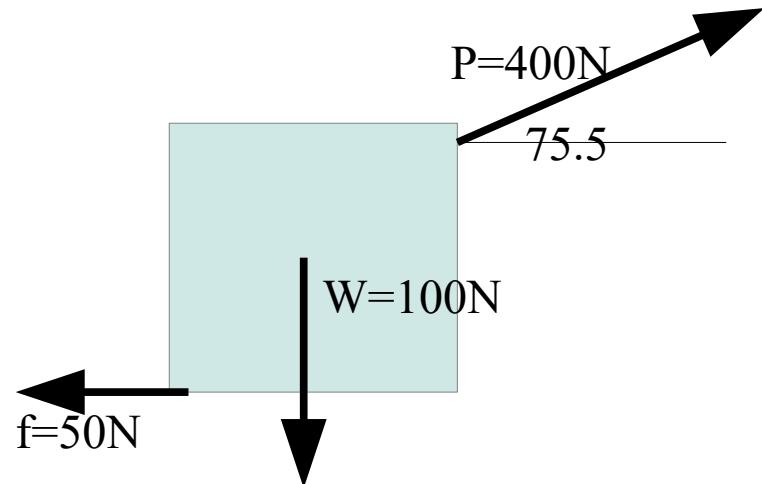
$$B = 17\text{N}@0^\circ$$

First build a free-body diagram  
Attach the tails to the origin.  
Find the net force  $F_{net} = R = A + B$

X	Y
A	
B	
<b>R=F<sub>net</sub></b>	
Direction = $\arctan(Y/X) =$	
Magnitude =	

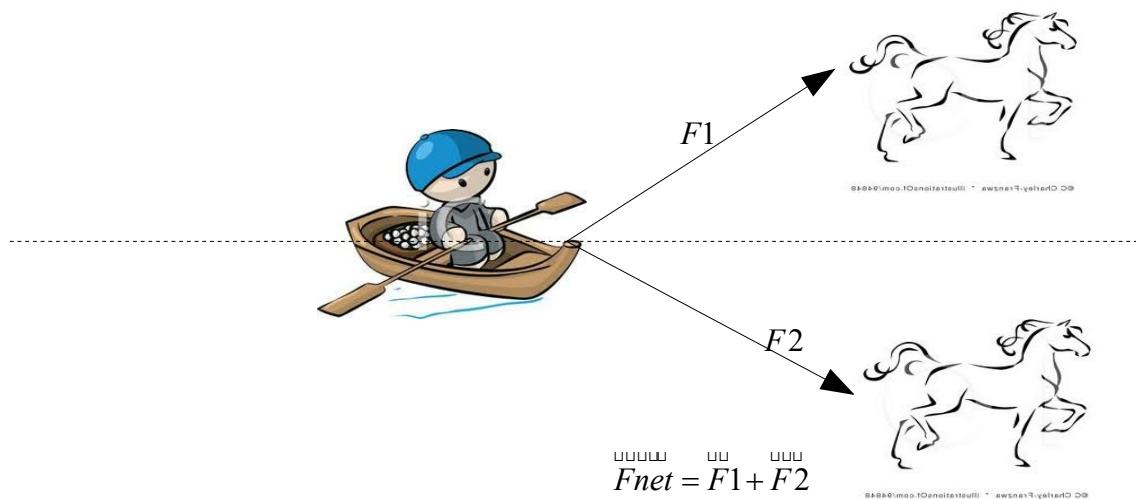
A(5.9,13.8)   B(17,0)

(c)



A box is pulled with a force of  $400\text{N} @ 75.5$

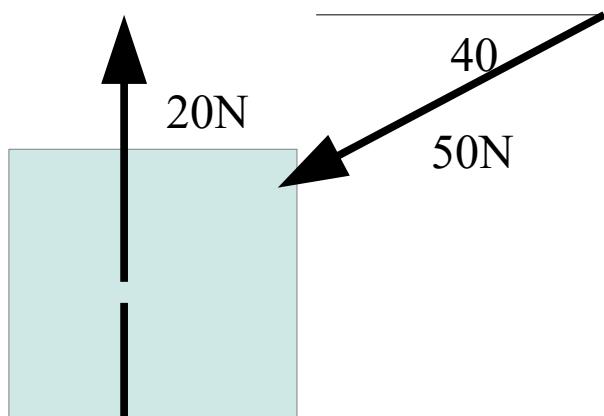
- 1) write each force in standard notation
- 2) in a table report the components of each force  
(see previous slide)
- 3) Find the components of the resultant R (sun)
- 4) draw R is a X-Y coordinate system
- 5) from 4) which way the box is moving ?



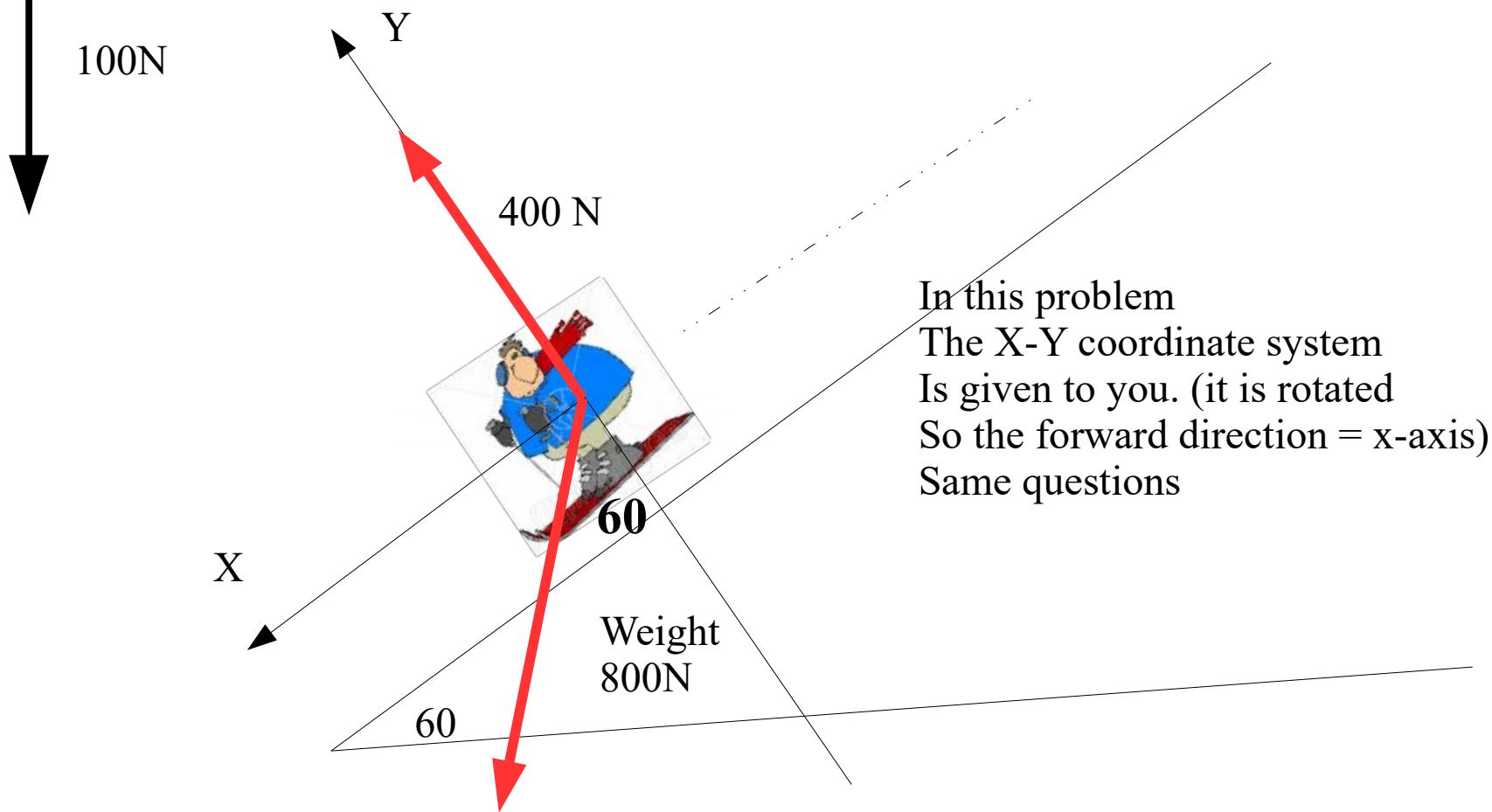
Same question.

$F_1$  and  $F_2$  have magnitude =  $1000\text{N}$   
Direction is  $45$  degrees and  $-45$  degrees.

$$F_1(707, 707) \quad F_2(707, -707)$$



- 1) write each force in standard notation
- 2) in a table report the components of each force  
(see previous slide)
- 3) Find the components of the resultant R (sum)
- 4) draw R in a X-Y coordinate system
- 5) from 4) which way the box is moving ?



## KINDS of FORCES:

Tension in a rope

Normal force (also called support force)

Friction (static or kinetic)

Weight (fundamental force)

**WEIGHT:** Don't confuse weight and mass.

**Mass** is a measure of the amount of “stuff” contained in an object.

- Mass does not depend on the Planet the object is placed.

Unit is kilogram (kg)

**WEIGHT IS THE PULL DUE TO GRAVITY.**

**WEIGHT is a FORCE but MASS is STUFF(INERTIA)**

**WEIGHT is measured in Newtons (or pounds).**

1 kg has a weight of 9.8N (about 10N)  
or 2.2 pounds.

Pound is the unit for force in the British system. The unit for Mass in the British system is the slug !

**WEIGHT IS THE PULL DUE TO GRAVITY.**  
**WEIGHT is a FORCE but MASS is STUFF(INERTIA)**  
**WEIGHT is measured in Newtons (or pounds).**

weight = mass x g      g is the acceleration due to gravity (m/s/s)  
                                Mass is in kg  
                                Weight in Newton

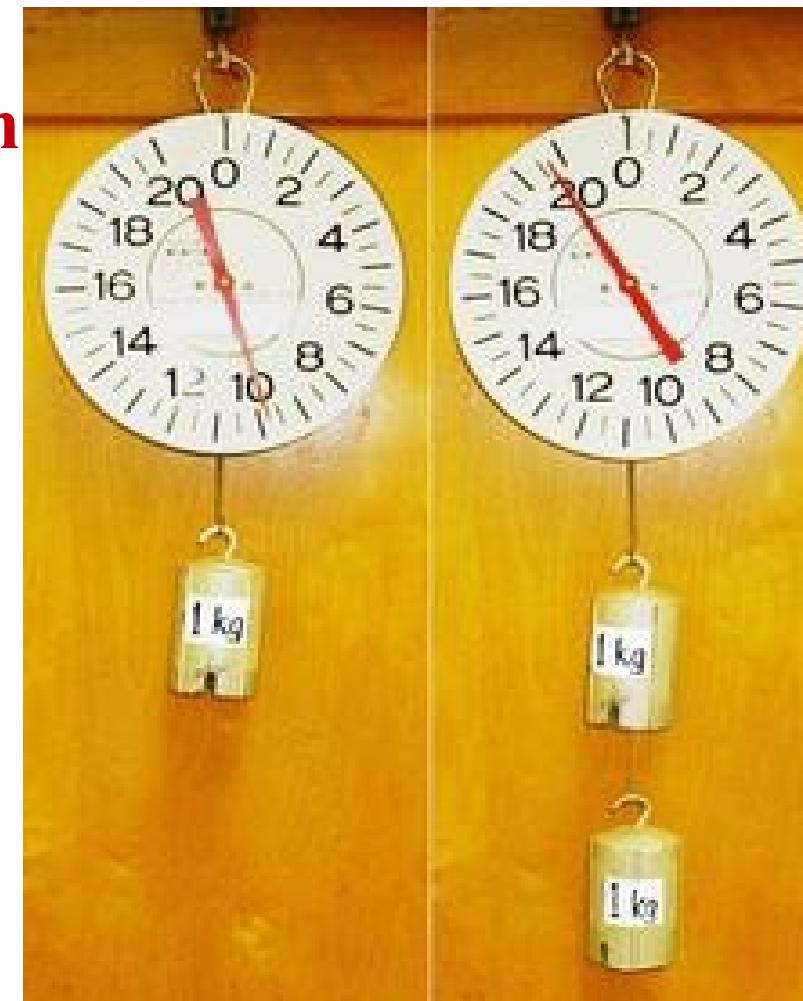
On Earth g = 9.8 m/s/s about 10m/s/s  
so weight = mass x 10

## 2.2 Pounds on Earth

1 kg has a weight of 10N

So 10N ↔ 2pounds

Divide by 5

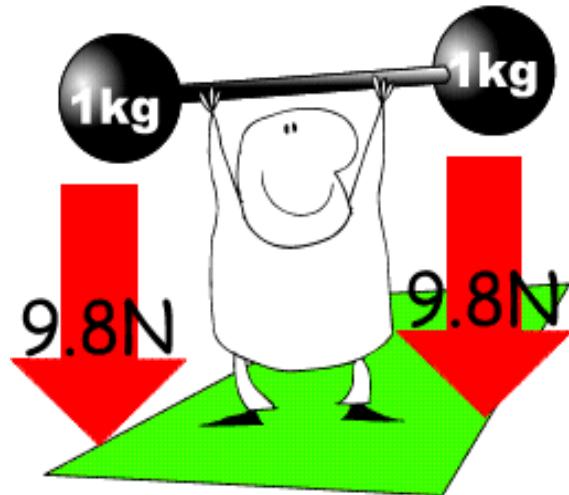


**WEIGHT is a FORCE = pull due to gravity**

**It depends on the planet, star, object.. you are standing on.**

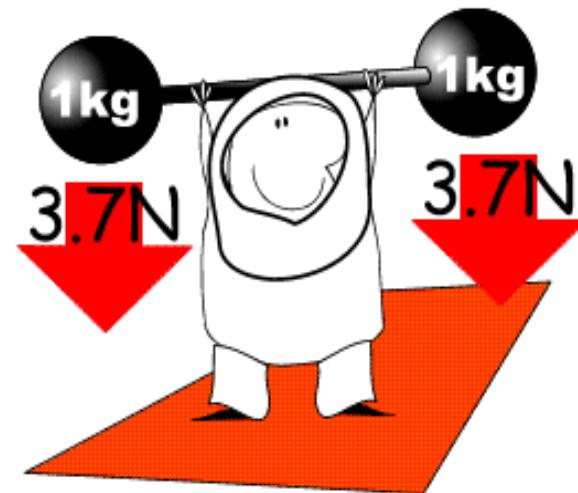
**Earth**

$$g = 9.8 \text{ N/kg}$$



**Mars**

$$g = 3.7 \text{ N/kg}$$

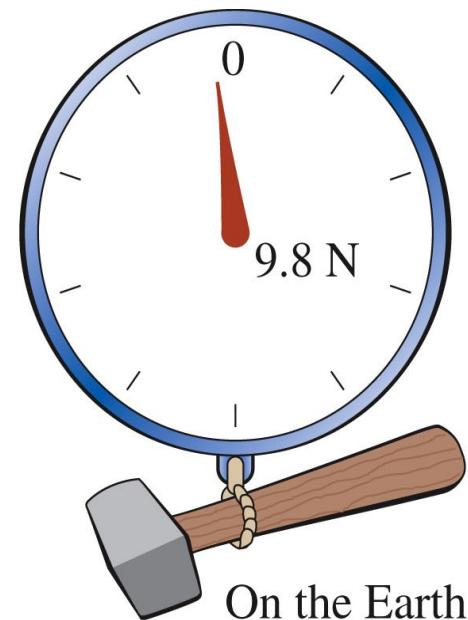
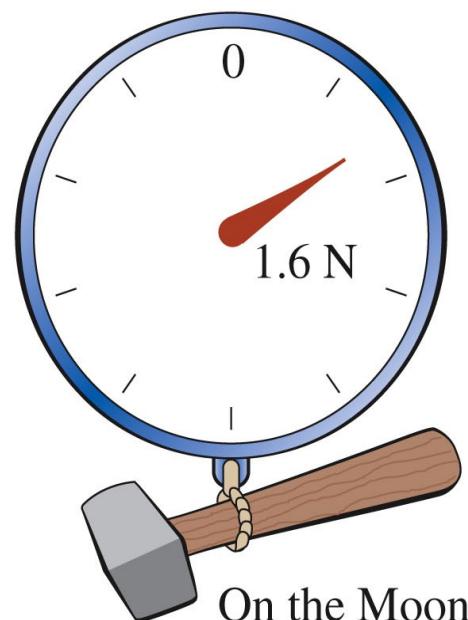
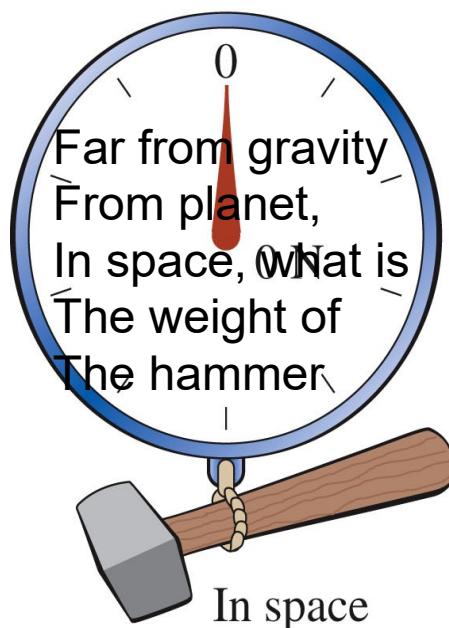


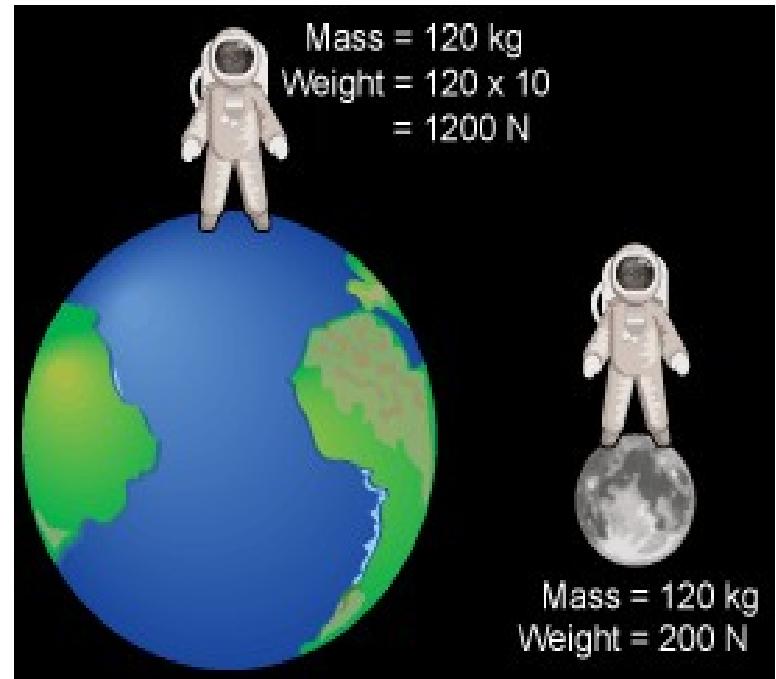
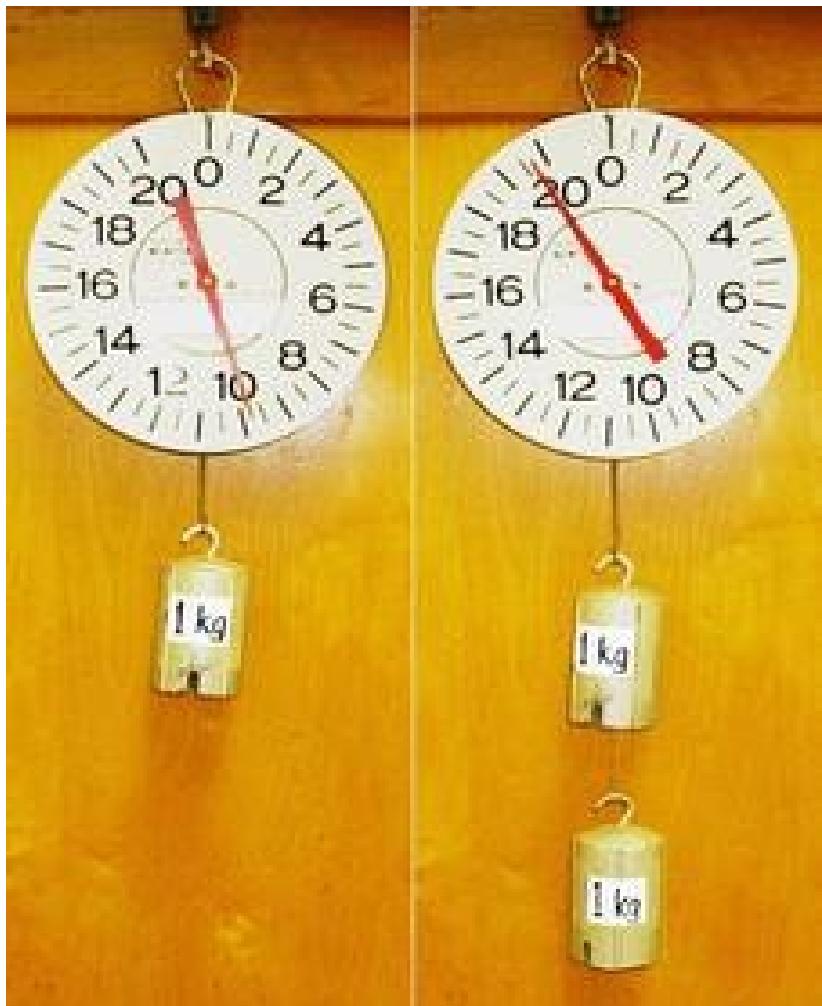
So if your mass is 80kg , your mass on the Moon  
Will be : the same ? Greater ? Larger

What about your weight ?

# Mass

- The weight of an object depends on which planet you measure the weight.
- The mass of the object is independent of the planet.





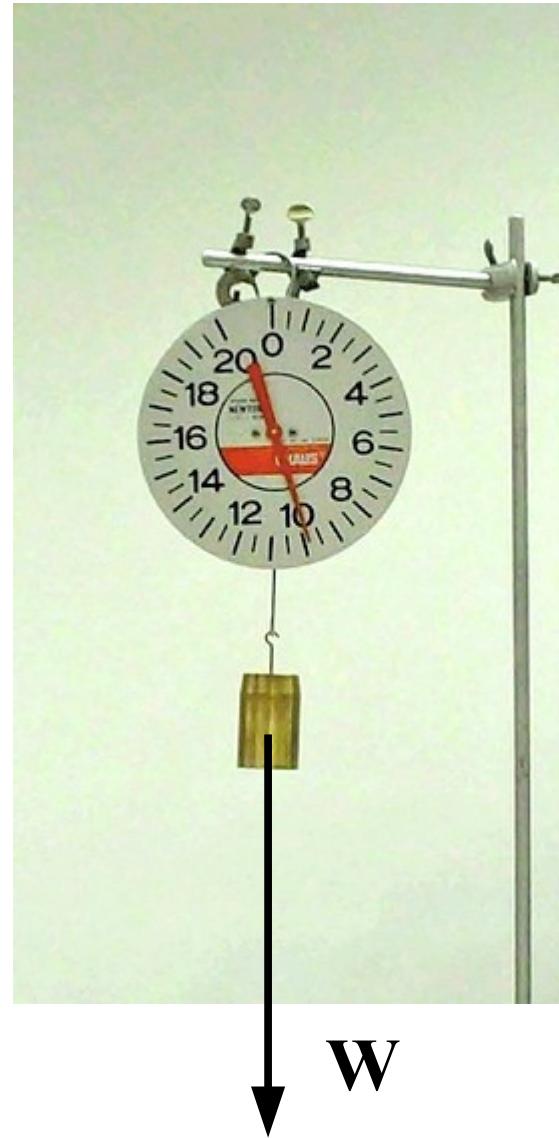
For a 70kg person :  
 150 pounds on Earth  
 27 pounds on Moon  
 400 pounds on Jupiter  
 2 tons on the Sun  
 ½ ounce on an asteroid (5 pennies)  
 (6mph to escape, versus 7 miles per second for Earth = 25 200 mph)

Show applet elevator from website  
 And check weight on other planets.



- 1) If a hammer has a mass of 2.5kg, how much does it weighs on Earth ?  
On the Moon/ (gravity is divided by 6 on the Moon)
- 2) What is the mass of a girl who weighs 340N on earth ?
- 3) what is the mass of a dog that weighs 75N ?
- 4) convert 10 pounds in newtons.
- 5) On Earth 10 pounds corresponds to what mass in kg ?
- 6) convert 10 newtons in pounds ? What is the mass ?

# **Weight W is straight down to center of Earth**

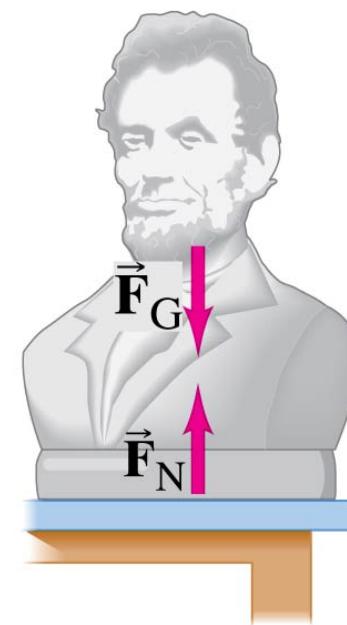
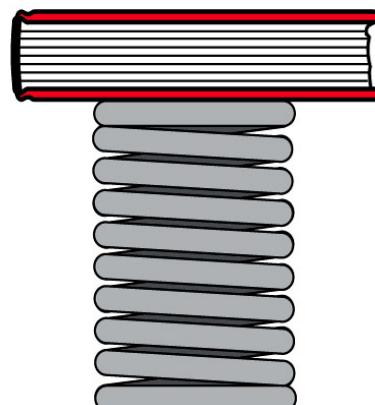
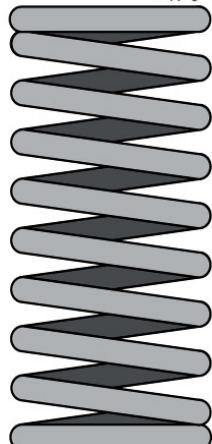


**Mass = 1kg  
Weight is 10N@down**

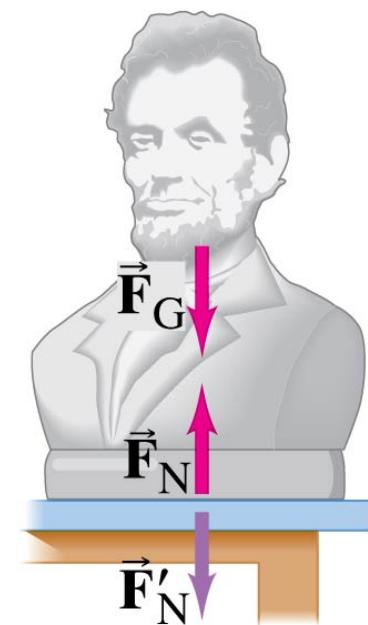
# Definition of the Normal Force

The normal force is one component of the force that a surface exerts on an object with which it is in contact – namely, the component that is perpendicular to the surface.

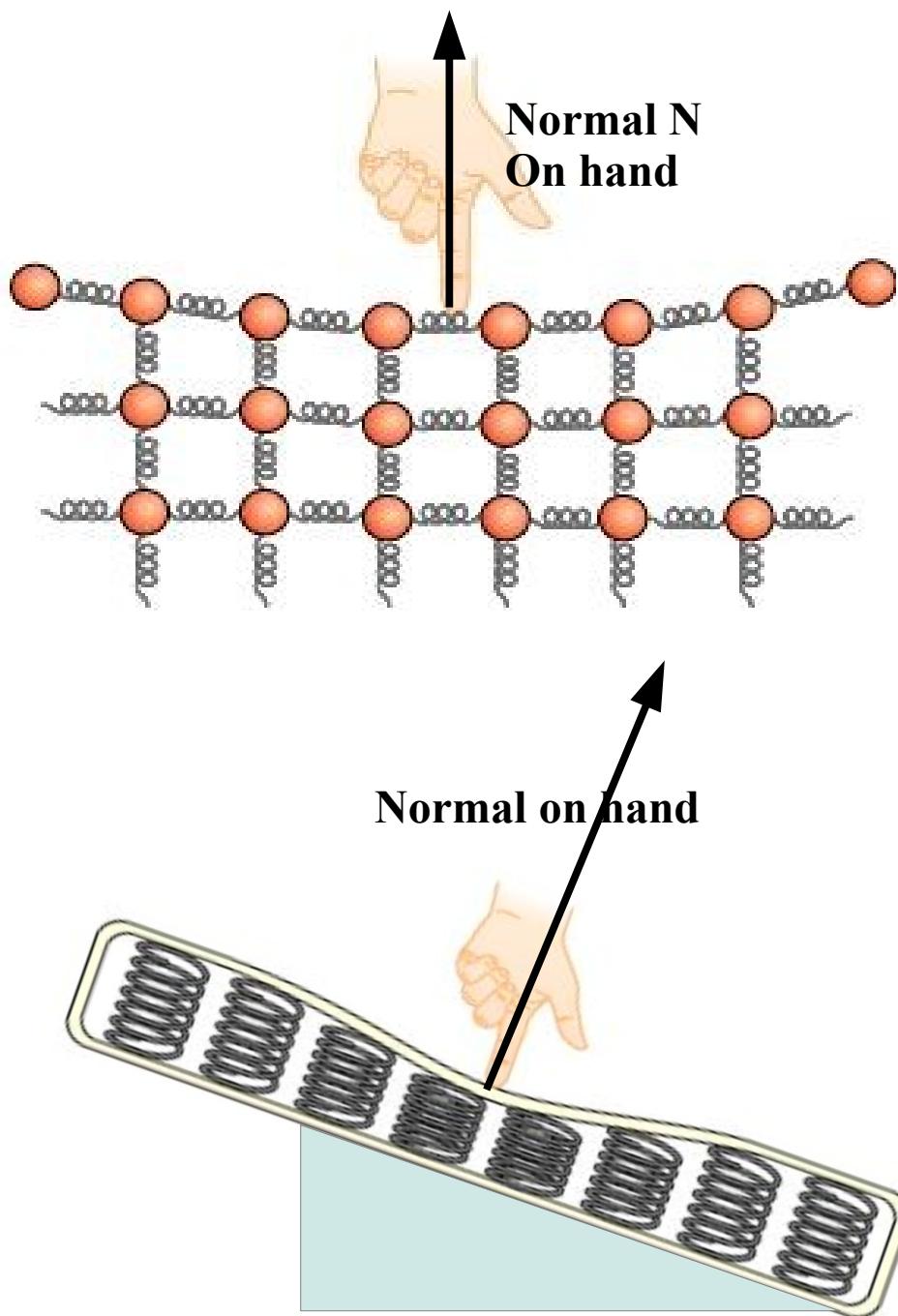
Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display.



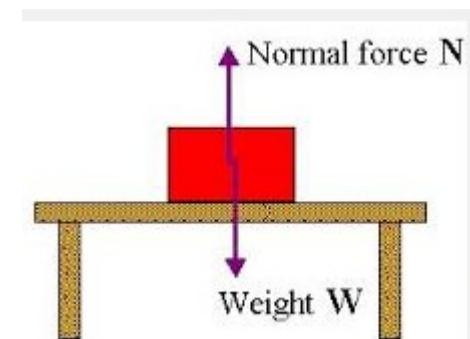
(a)

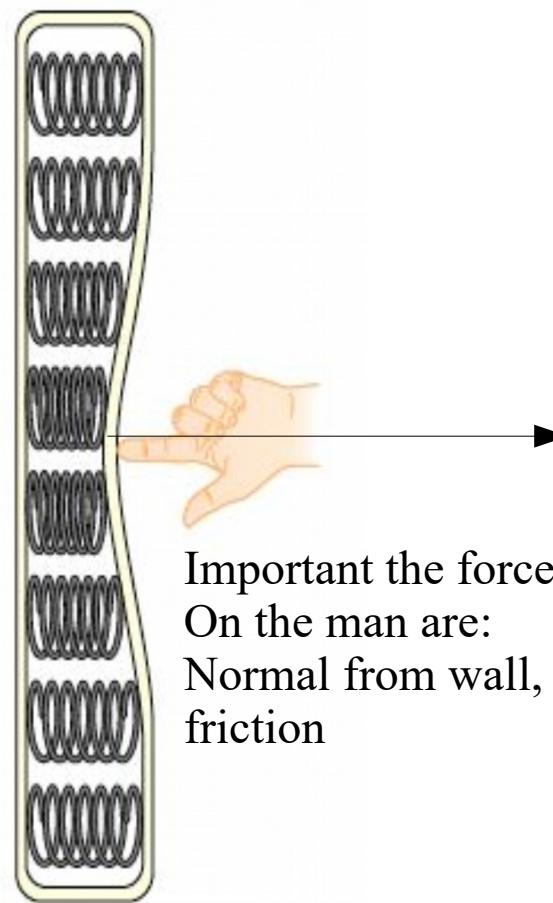
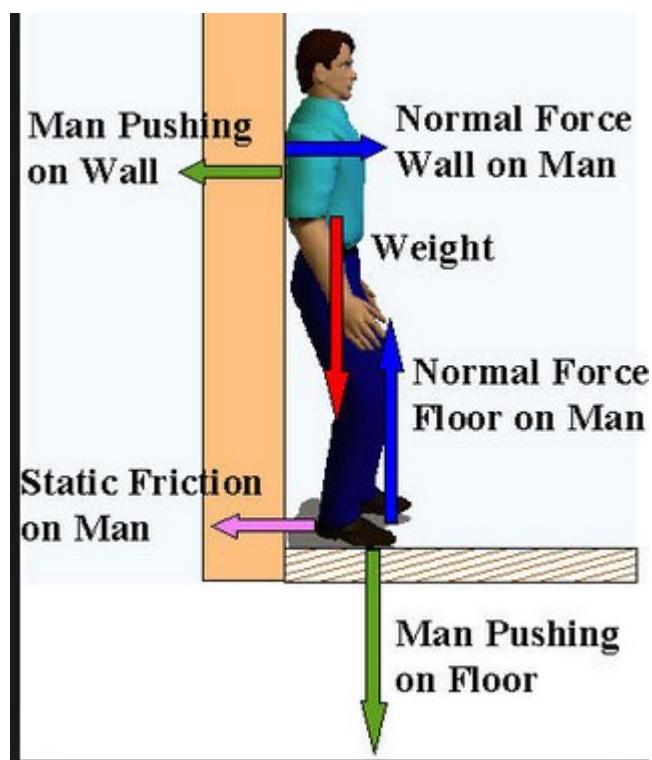


(b)



## Normal force (also called support force)





Important forces acting  
On the man are:  
Normal from wall, normal from floor,  
friction

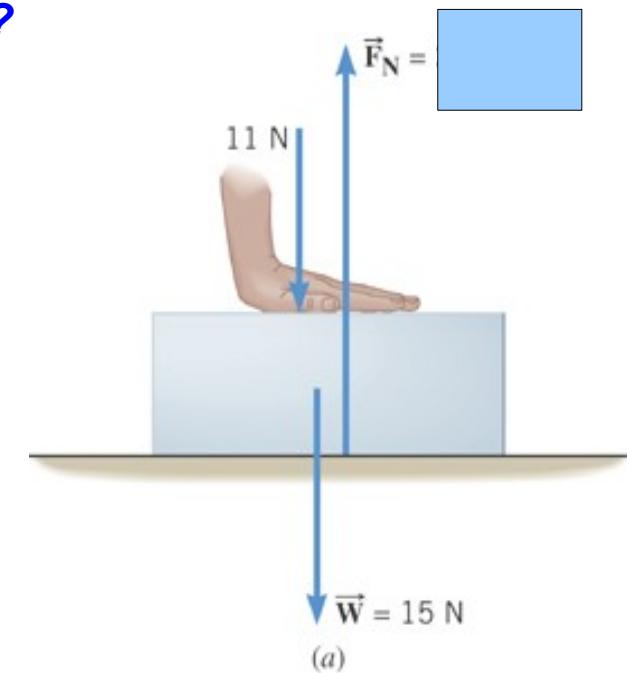
What is the meaning of the word “normal” in the term “normal force?”

- a) that it is in magnitude and opposite in direction to the weight of the object
- b) that it is one that is encountered in everyday life
- c) that it is directed perpendicular to a surface
- d) that it is measurable
- e) that it has a magnitude of 1 newton

## *FIND The Normal Force ?*

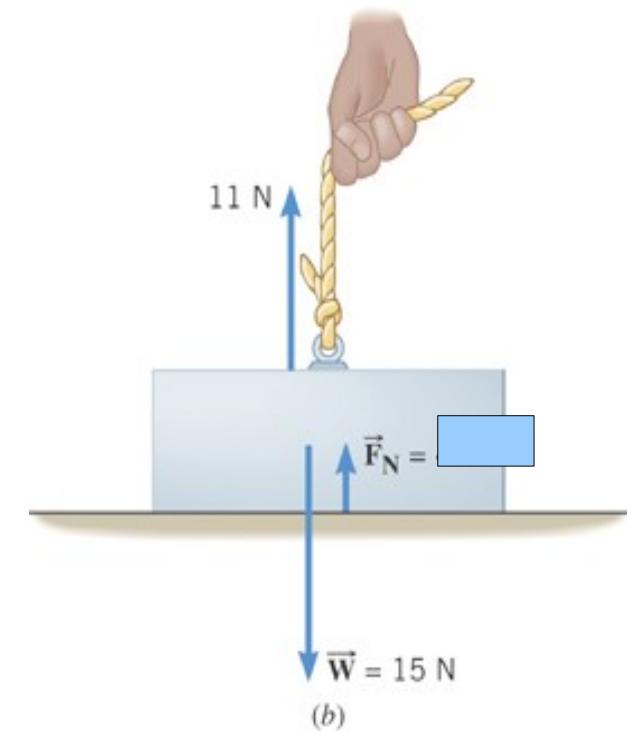
$$F_N - 11\text{ N} - 15\text{ N} = 0$$

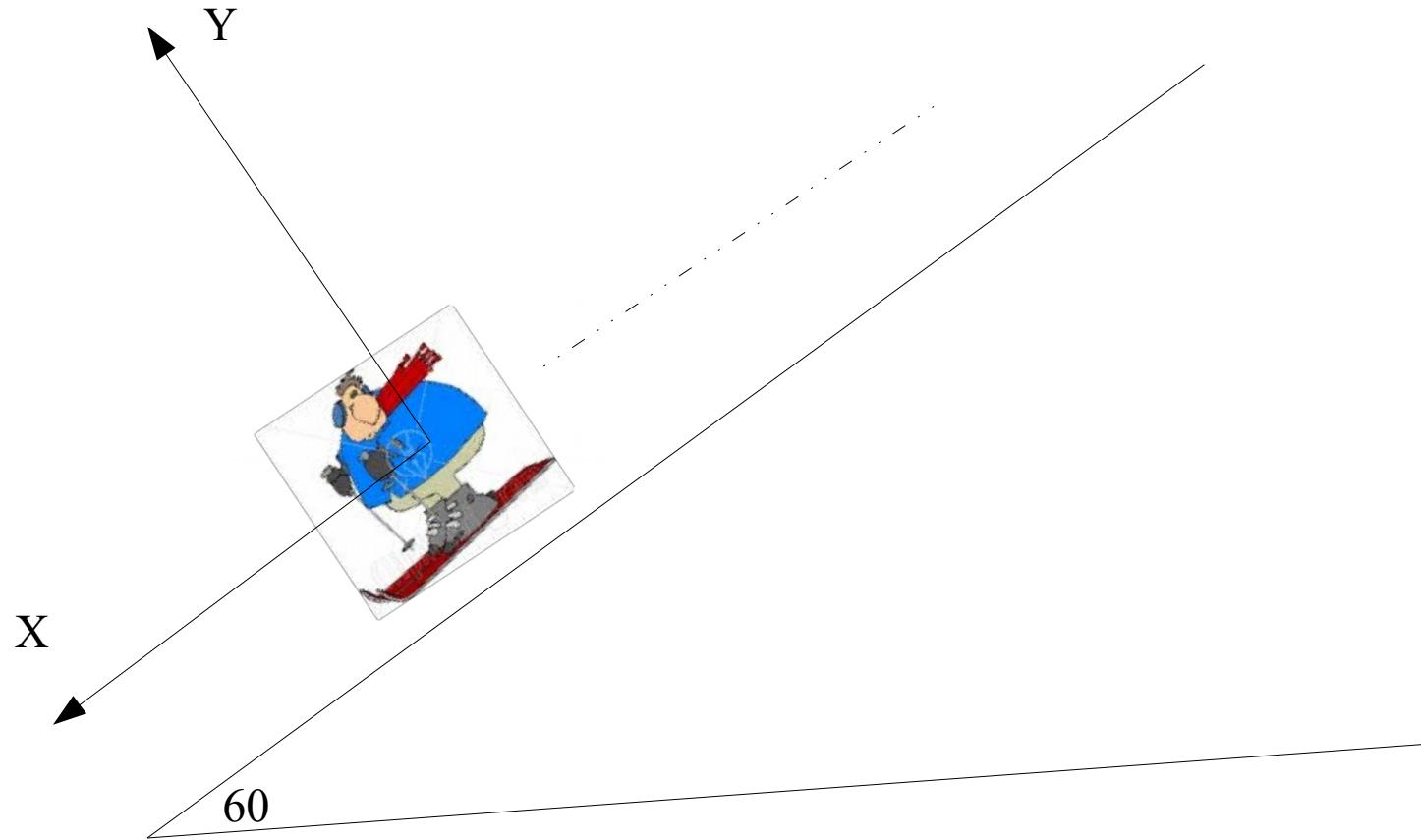
$$F_N = \boxed{\quad}$$



$$F_N + 11\text{ N} - 15\text{ N} = 0$$

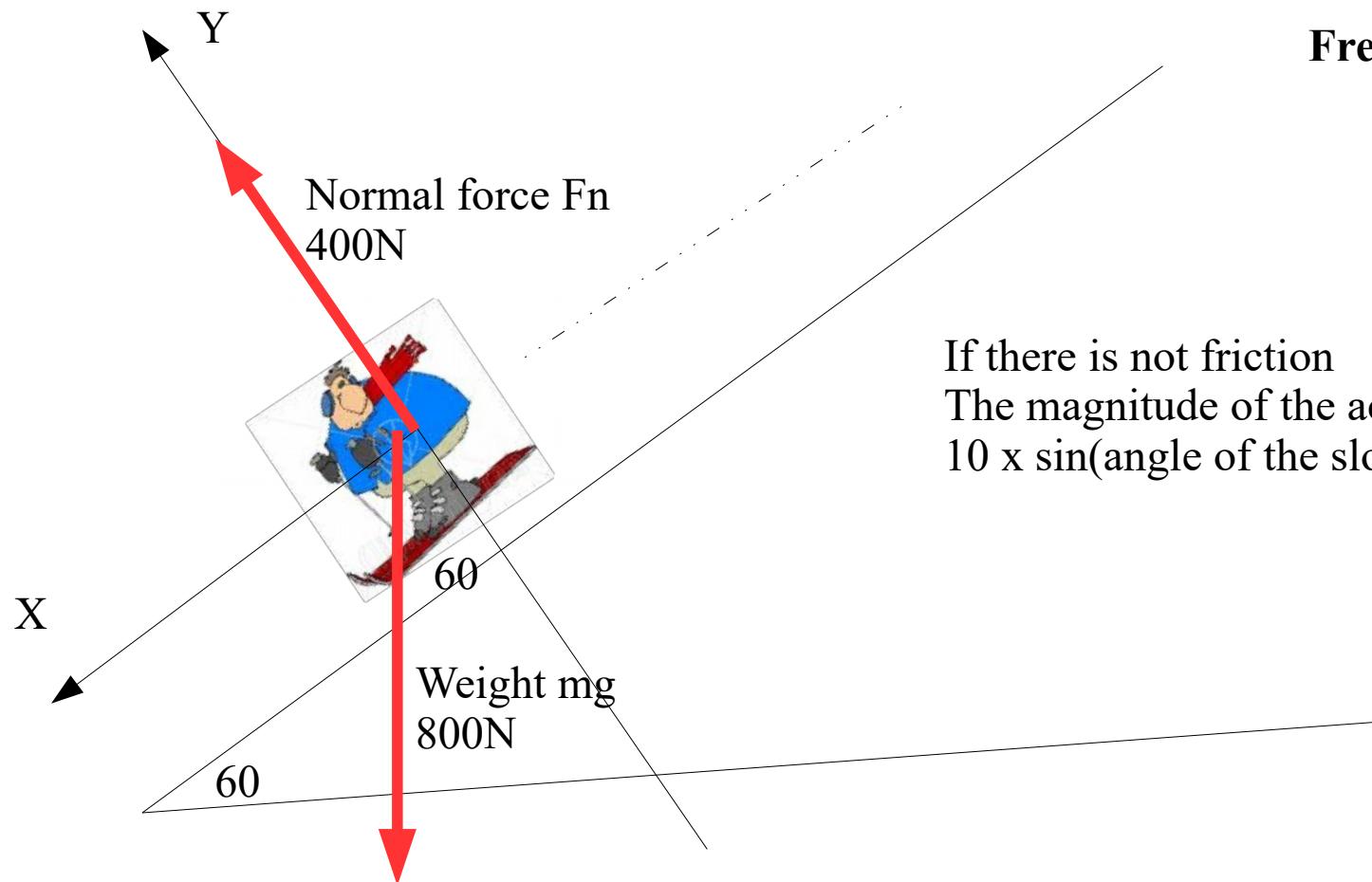
$$F_N = \boxed{\quad}$$





Draw the forces applied to the skier.  
Then trace the forces in a free-body diagram

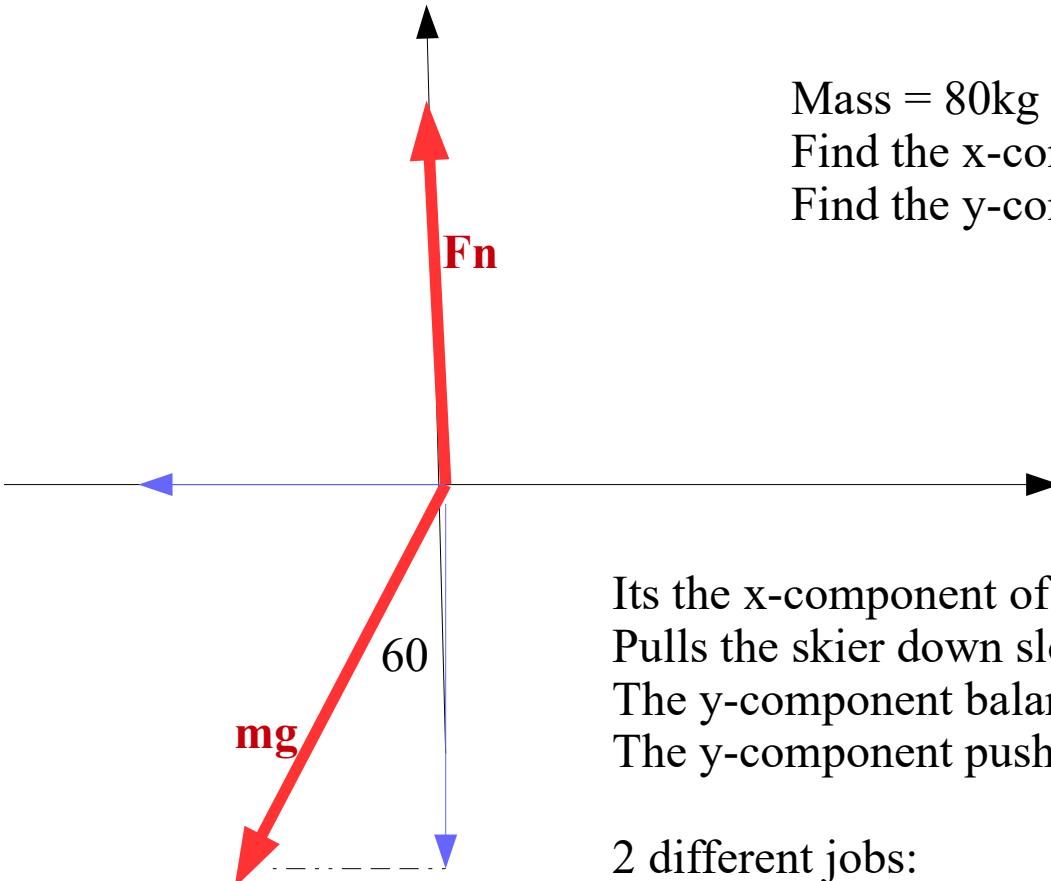
## Free-body diagram



If there is not friction  
The magnitude of the acceleration is  
 $10 \times \sin(\text{angle of the slope})$

The skier has a mass of 80kg.

- 1) find the mass of the skier
- 2) draw a free body diagram with the forces
- 3) write the forces in standard notation
- 4) find the resultant R (standard notation) (R is the sum of  $F_N$  and W)
- 5) using newton's seconds law:  $R = m a$  find the acceleration of the skier
- 6) compare the acceleration to  $10 \sin(\text{angle})$



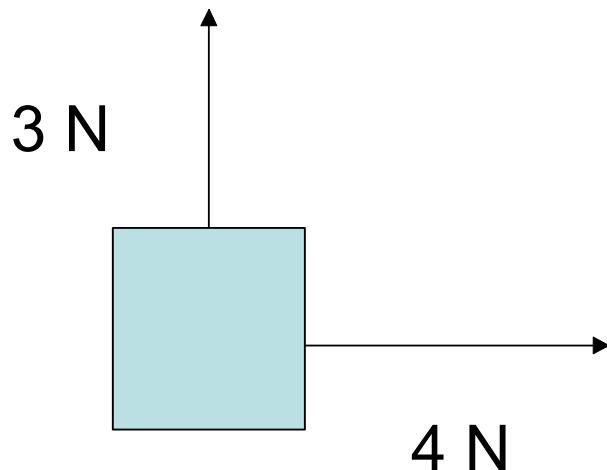
Mass = 80kg  
Find the x-component of weight  
Find the y-component of weight

Its the x-component of the weight that  
Pulls the skier down slope.  
The y-component balances out the normal force.  
The y-component pushes on the slope (snow).

2 different jobs:  
X-component pulls him down  
Y-component pushes on the ground

**Adding forces in 2D is easy if the individual forces are perpendicular.**

### Individual Forces



### Net Force

FIND THE NET  
FORCE.

DRAW it  
Find direction and  
Magnitude.

(add 3N@ right and 4N@ up)

A brick (like skier) is resting on the surface of a flat board. As one end of the board is slowly raised, what change, if any, is there in the normal force exerted on the brick?

a) The normal force increases.

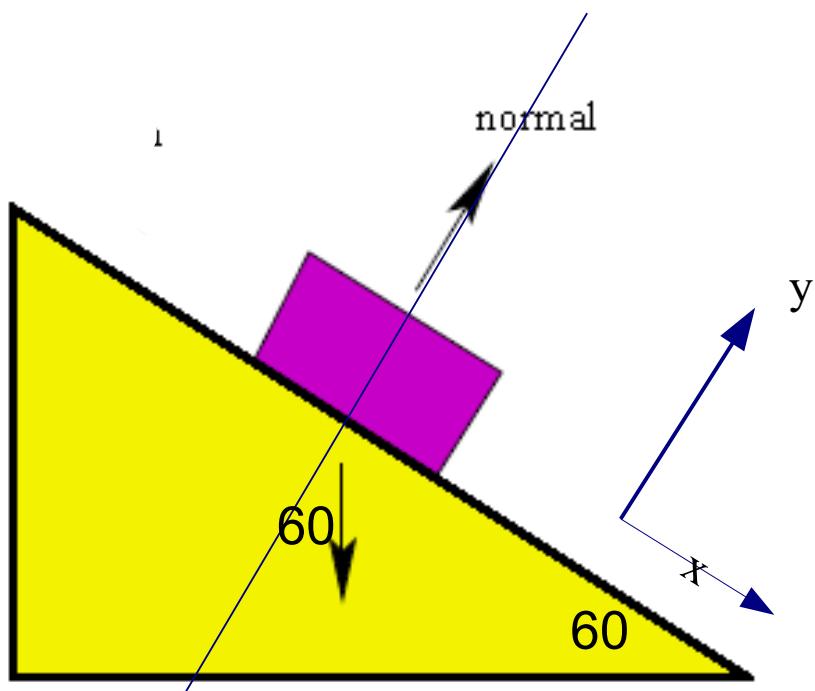
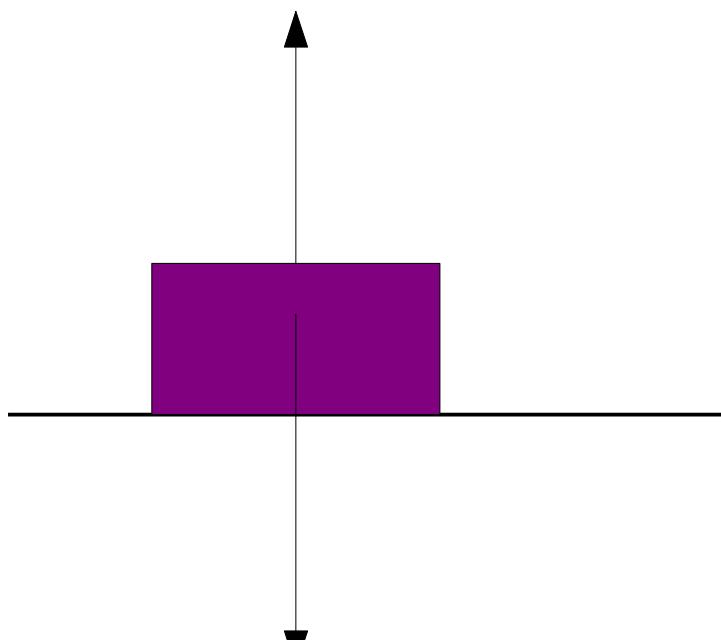
b) The normal force decreases.

c) The normal force remains constant.

d) Only the direction of the normal force changes.

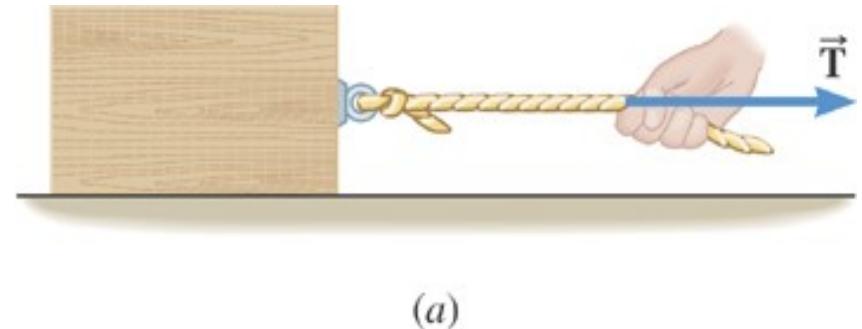
**What about the scale?**

Same reading ?  
Do demo

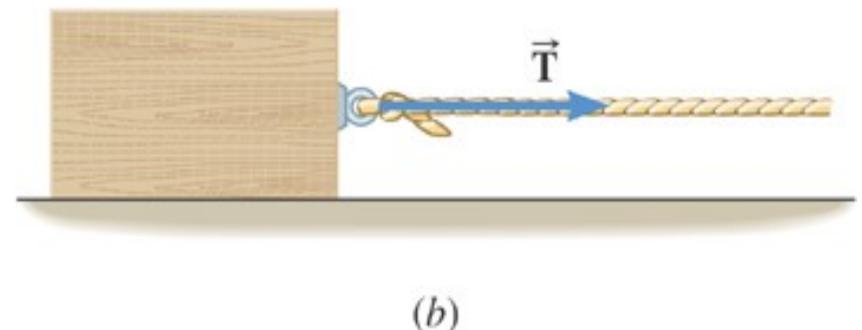


## *The Tension Force*

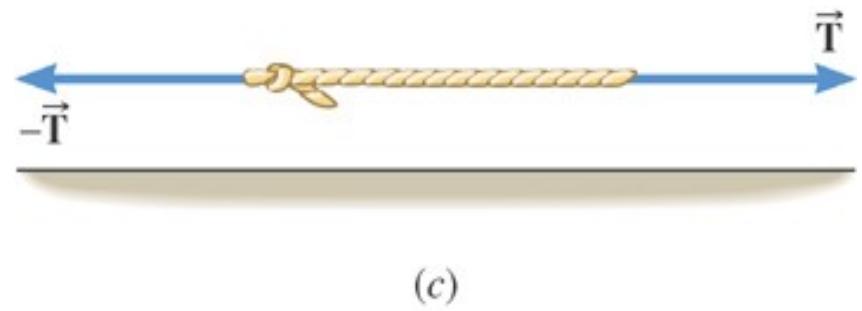
Cables and ropes transmit forces through ***tension***.



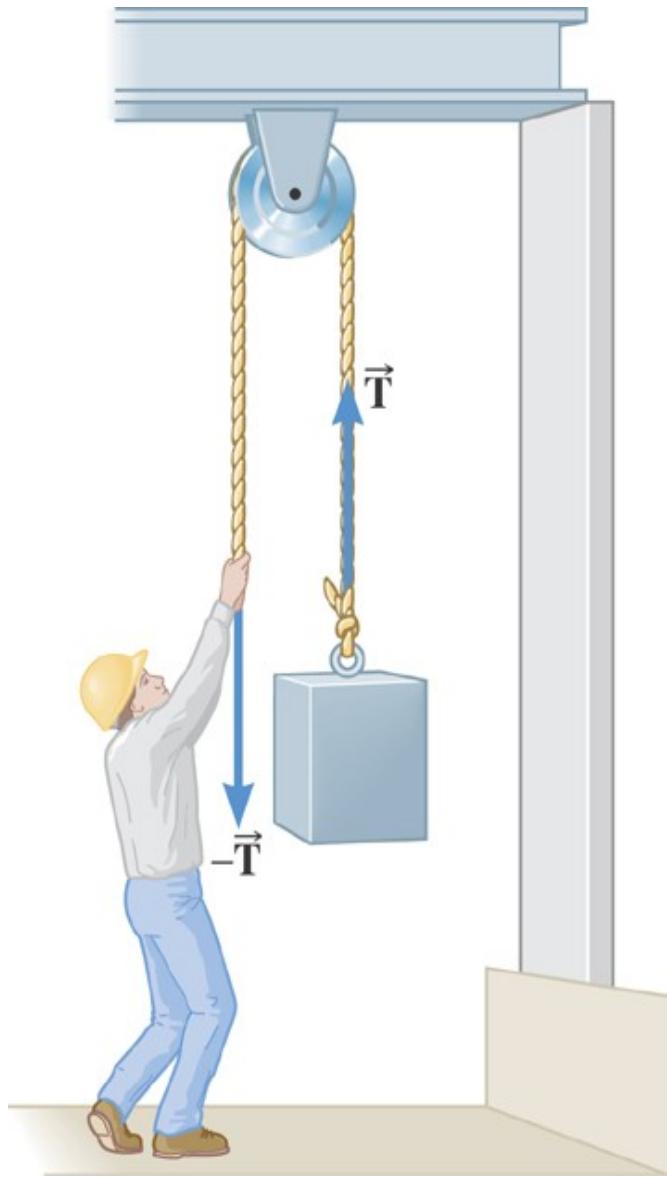
(a)



(b)



(c)



A massless rope will transmit tension undiminished from one end to the other.

If the rope passes around a massless, frictionless pulley, the tension will be transmitted to the other end of the rope undiminished.

demo

# Static EQUILIBRIUM MEANS : no acceleration

MEANS : The net force is 0

$$\sum \vec{F} = F_1 + F_2 + F_3 = 0$$

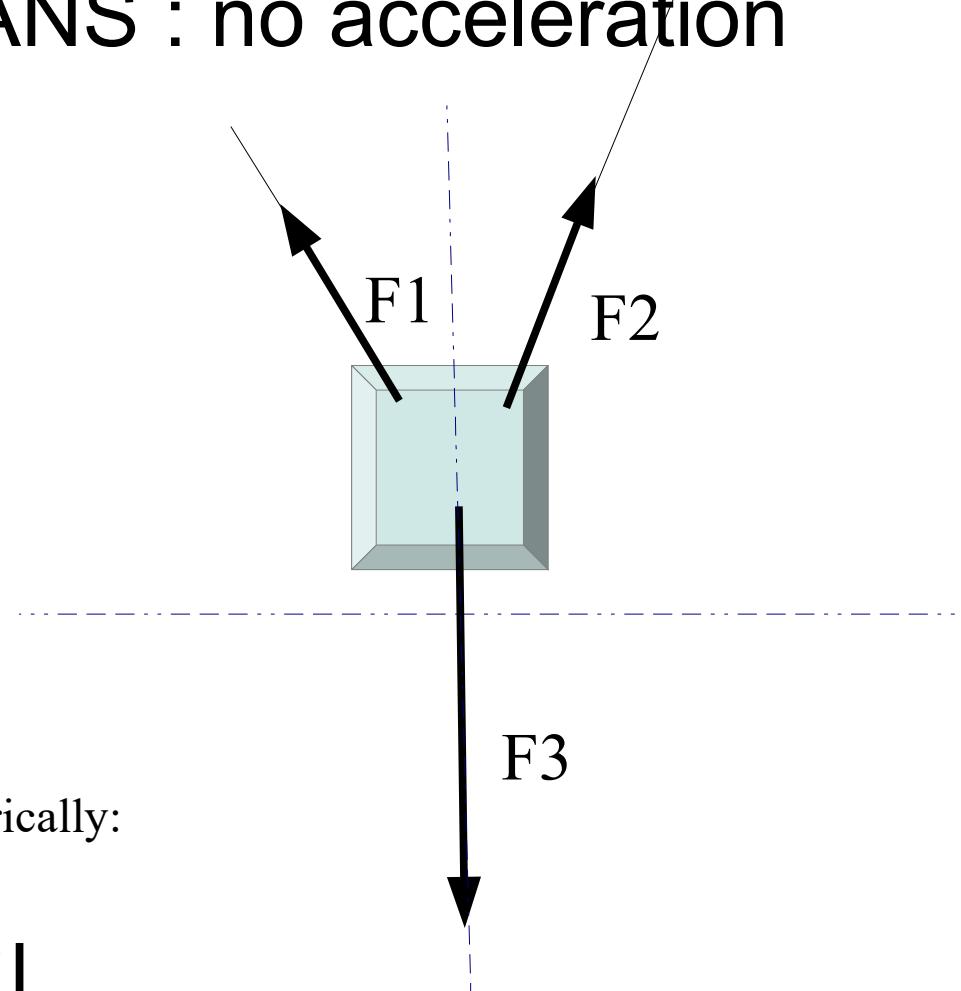
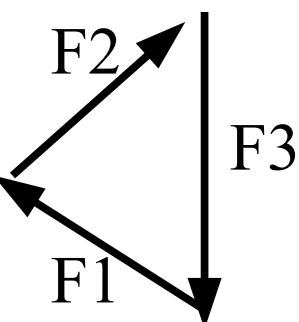
Algebraically (we use):

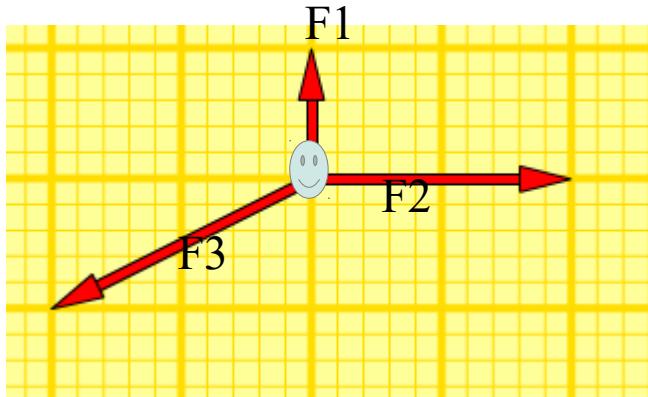
Sum of x-component = 0

Sum of y-components = 0

$$F_{1x} + F_{2x} + F_{3x} = 0$$

$$F_{1y} + F_{2y} + F_{3y} = 0$$





I) An object is in equilibrium. 2 forces are acting on it.

$$F_1 = 5\text{N} @ \text{up} \quad F_2 = 10\text{N} @ \text{right} \quad F_3 = ?$$

The sum of vectors is 0 = the net force is 0.

You are going to find the vector  $F_3$ . (polar and Cartesian coordinates)

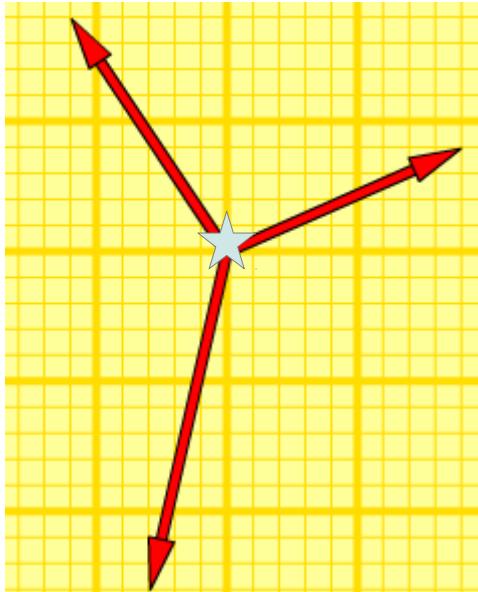
1) Fill the table

2) Use  $F_{1x} + F_{2x} + F_{3x} = 0$  and  $F_{1y} + F_{2y} + F_{3y} = 0$   
to find  $F_{3x}$  and  $F_{3y}$

3) Compute the magnitude of  $F_3$  and its direction

$$F_3 = \underline{\hspace{2cm}} \text{ N } @ \underline{\hspace{2cm}}$$

	x-axis	y-axis
<b>F1</b>		
<b>F2</b>		
<b>F3</b>	$F_{3x}$	$F_{3y}$
<b>SUM=FNET</b>	0	0



II) In this situation:

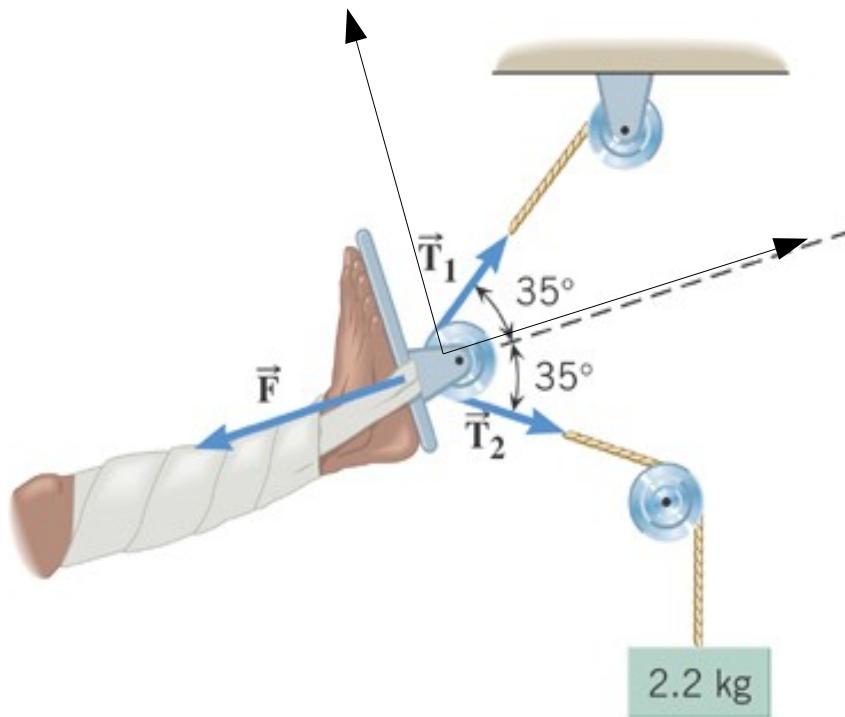
$$F_1 = 11\text{N} @ 124^\circ \quad F_2 = 10\text{N} @ 24^\circ \quad \text{Find } F_3$$

Follow the same steps but now you need to use trig to find the components  
Of the vectors.

	x-axis	y-axis
<b>F1</b>		
<b>F2</b>		
<b>F3</b>	$F_{3x}$	$F_{3y}$
<b>SUM=FNET</b>	0	0

$$F_1(-6.2, 4.5) \quad B(9.1, 4)$$

## 4.11 Equilibrium Application of Newton's Laws of Motion



(a)

First draw a free-body diagram

Then  $T_1 = T_2 = \text{weight} = 22\text{N}$

Then find the magnitude of  $F$   
The force stretching the leg.

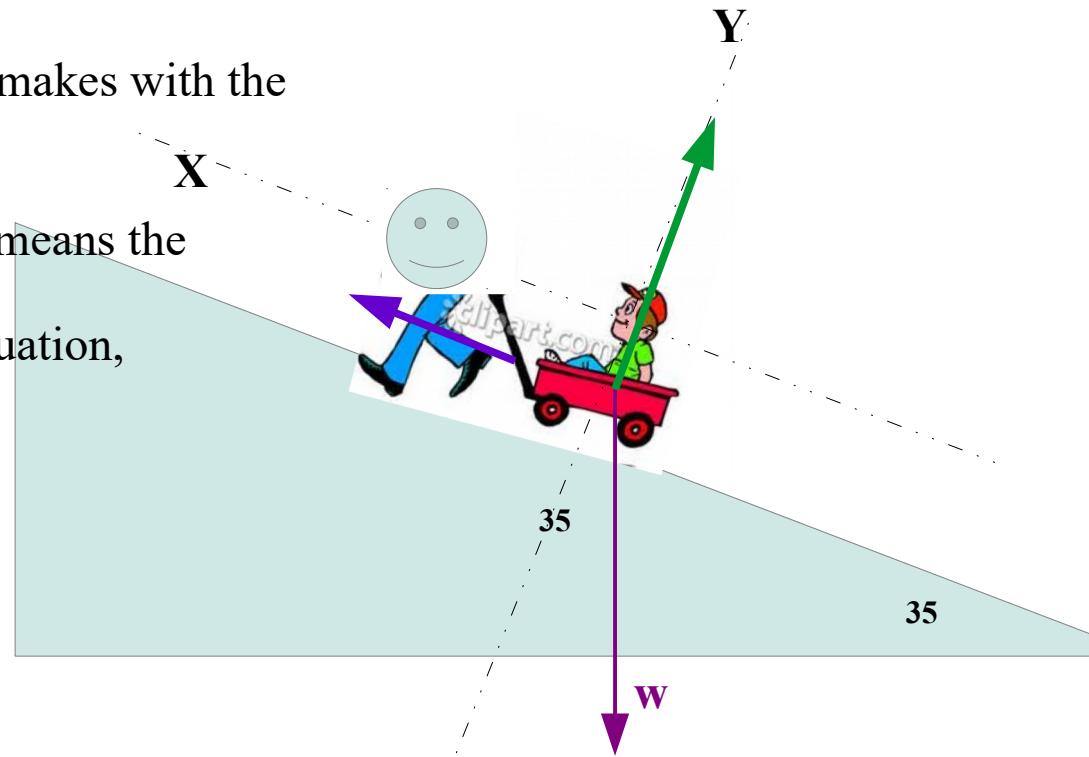
(b) Free-body diagram for the foot pulley

1)  $|T_2|$  is a tension so  $|T_2| = 2.2 \times 10 = \text{weight of the mass} = |T_1|$

	x-axis	y-axis
$F$	$F_x$	$F_y$
$T_1$		
$T_2$		
$\text{SUM}=\text{ENET}$	0	0

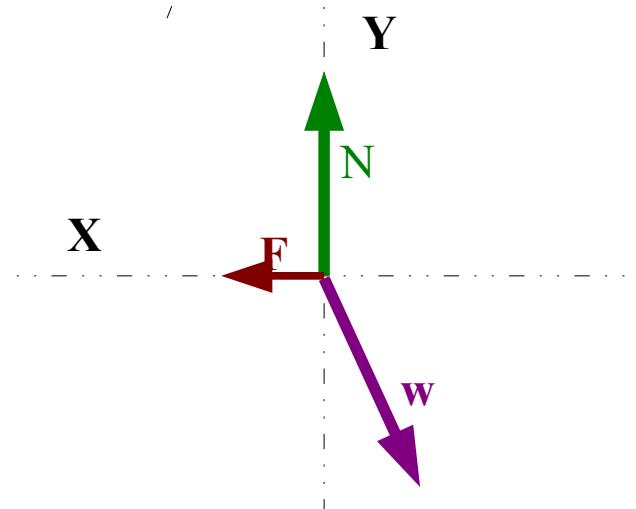
A wagon weighs 40 lb and the angle that the hill makes with the horizontal is 35 degrees. Keep pounds.

The wagon is moving at a constant speed . That means the acceleration is 0. That means the net force (or sum of the force is 0). It is an equilibrium situation, The goal is to find the normal force N.

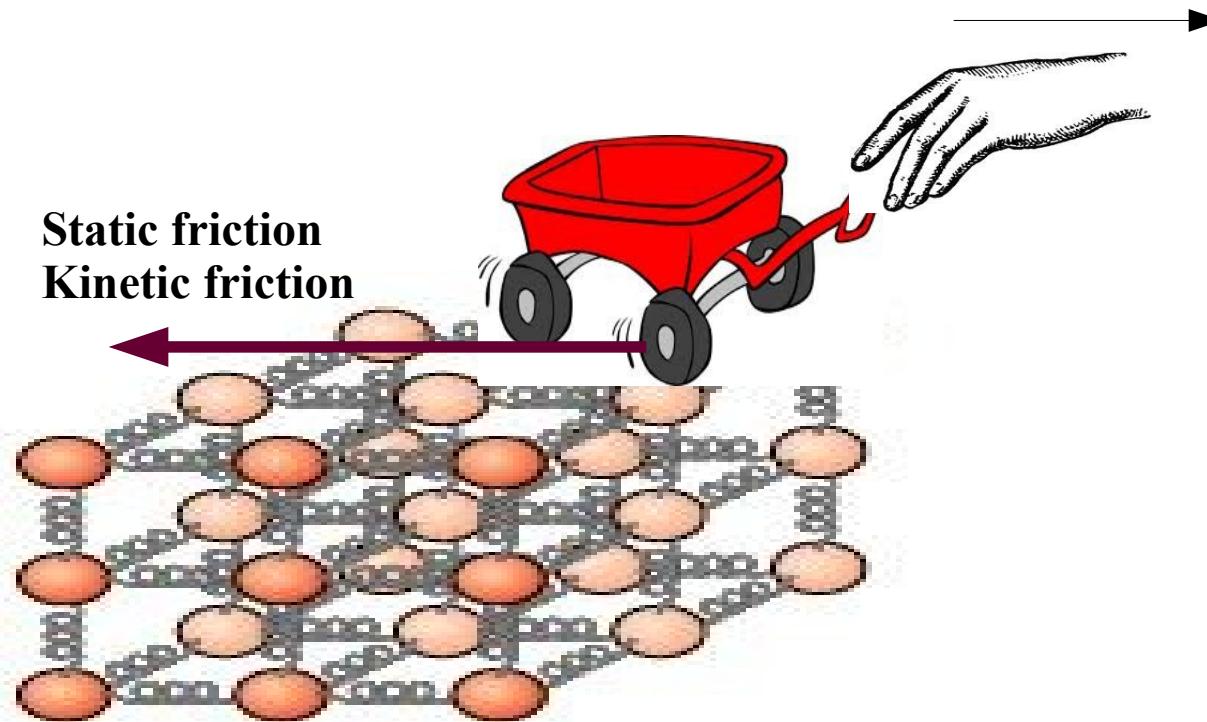


- 1) Draw a free-body diagram.
- 2) Write each force in standard notation
- 3) Fill the table below and find N

	x-axis	y-axis
N	?	?
F		
mg		
<b>SUM=FNET</b>	<b>0</b>	<b>0</b>

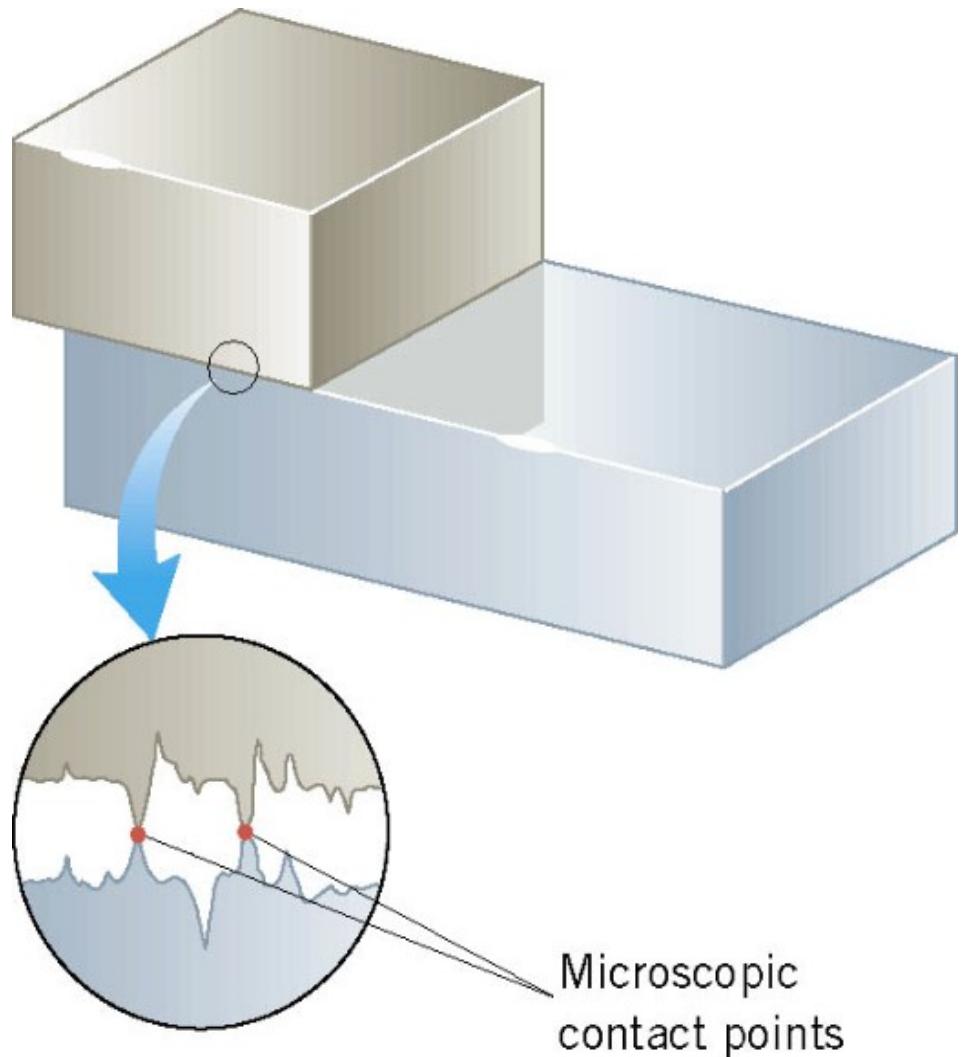


# Frictional forces

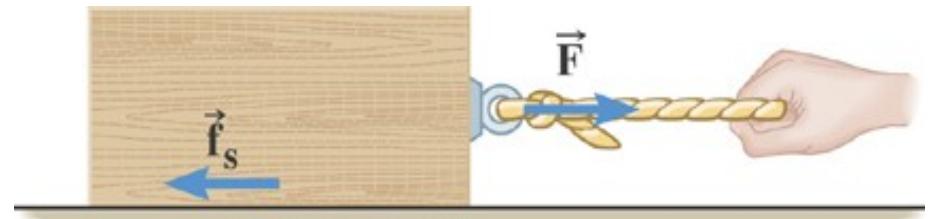
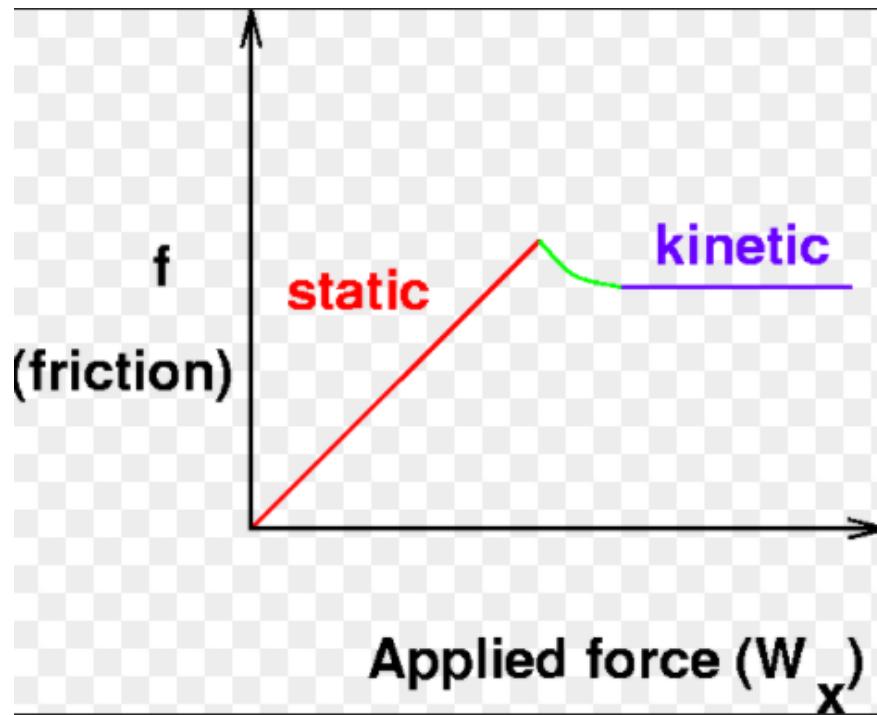


<http://phet.colorado.edu/en/simulation/forces-and-motion-basics>

When an object is in contact with a surface there is a force acting on that object. The component of this force that is parallel to the surface is called the ***frictional force***.

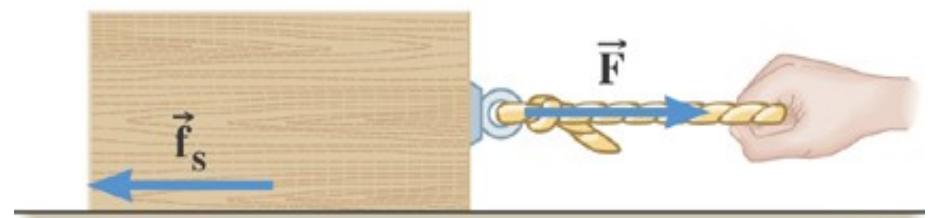


When the two surfaces are not sliding across one another the friction is called ***static friction***.



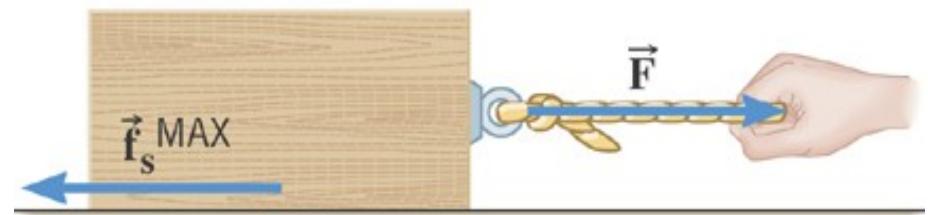
No movement

(a)



No movement

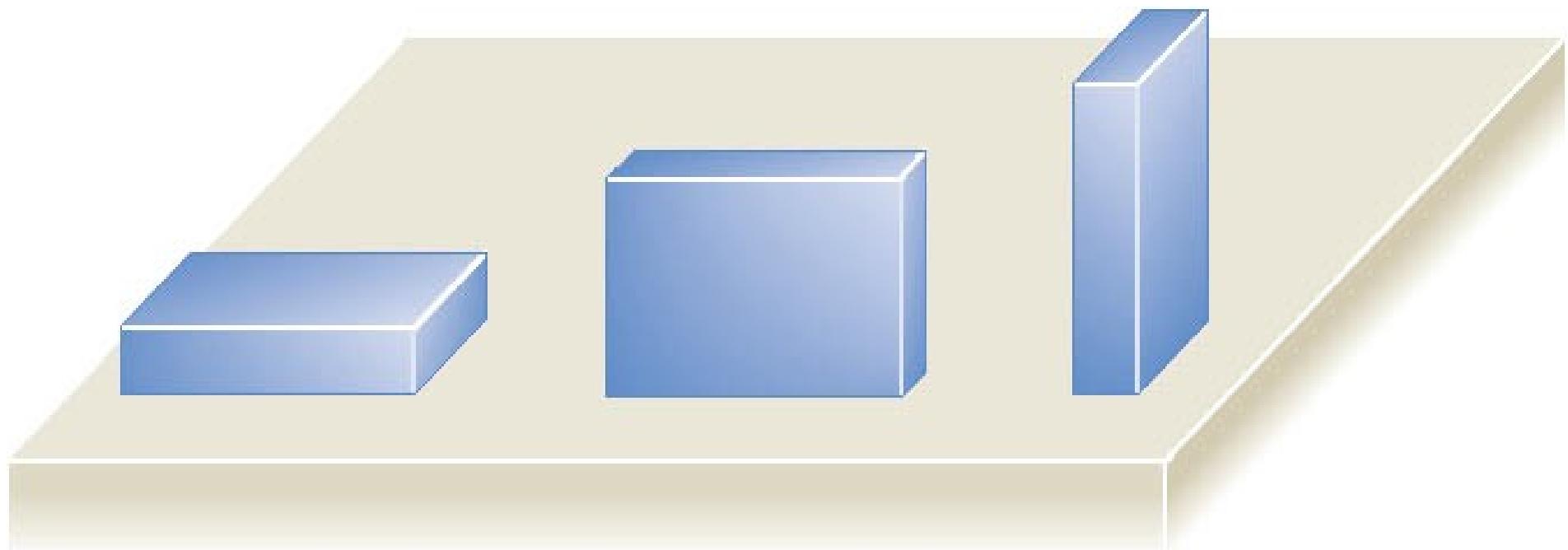
(b)

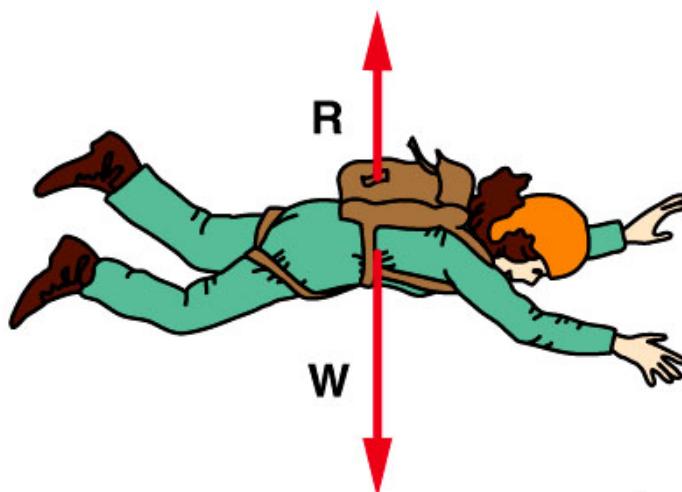
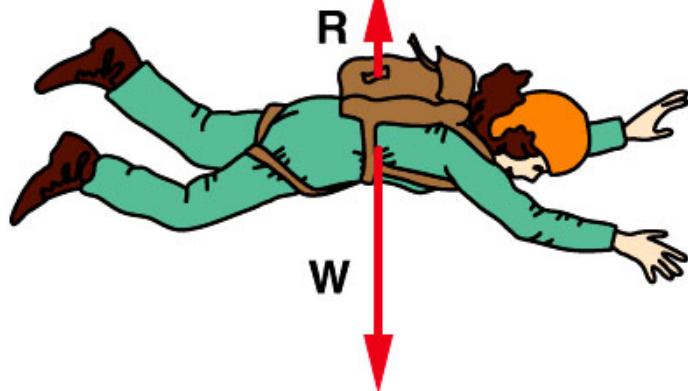


When movement just begins

(c)

Note that the magnitude of the frictional force does not depend on the contact area of the surfaces.

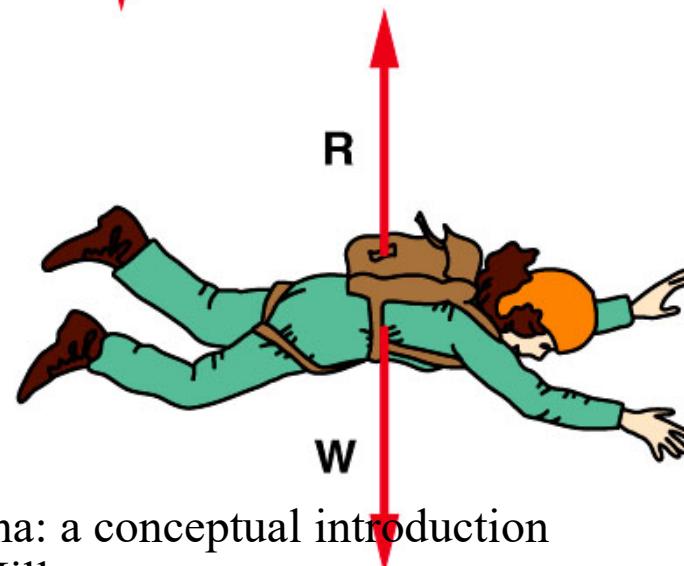




AIR RESISTANCE

And terminal speed

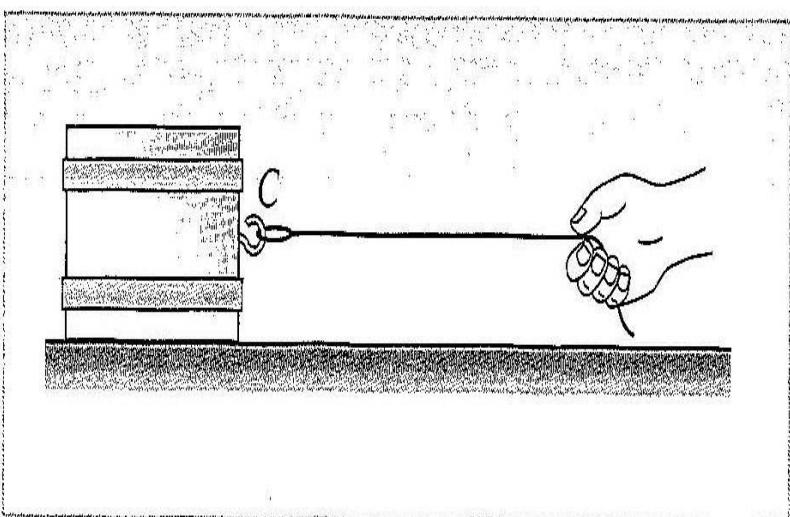
See exploration of physical

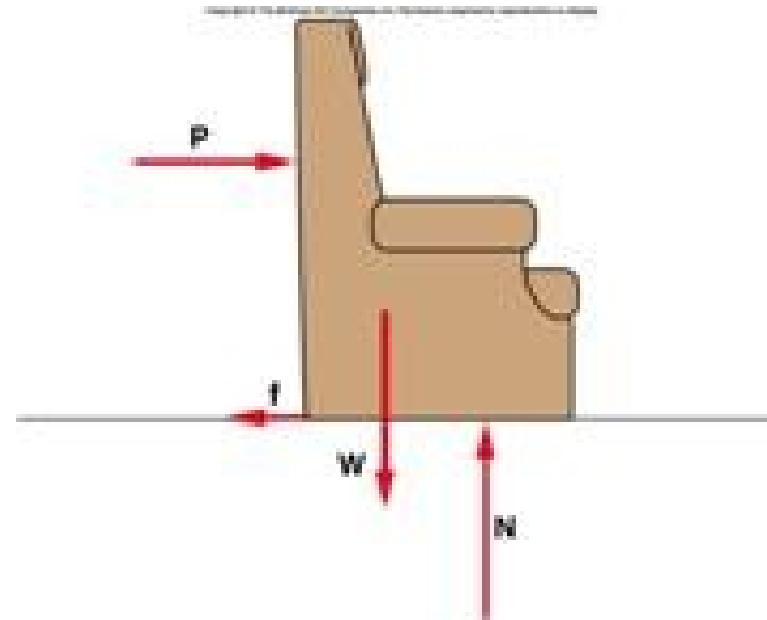
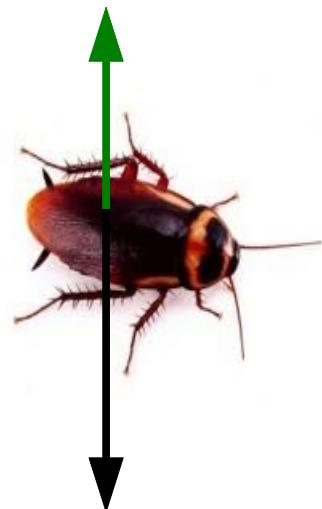
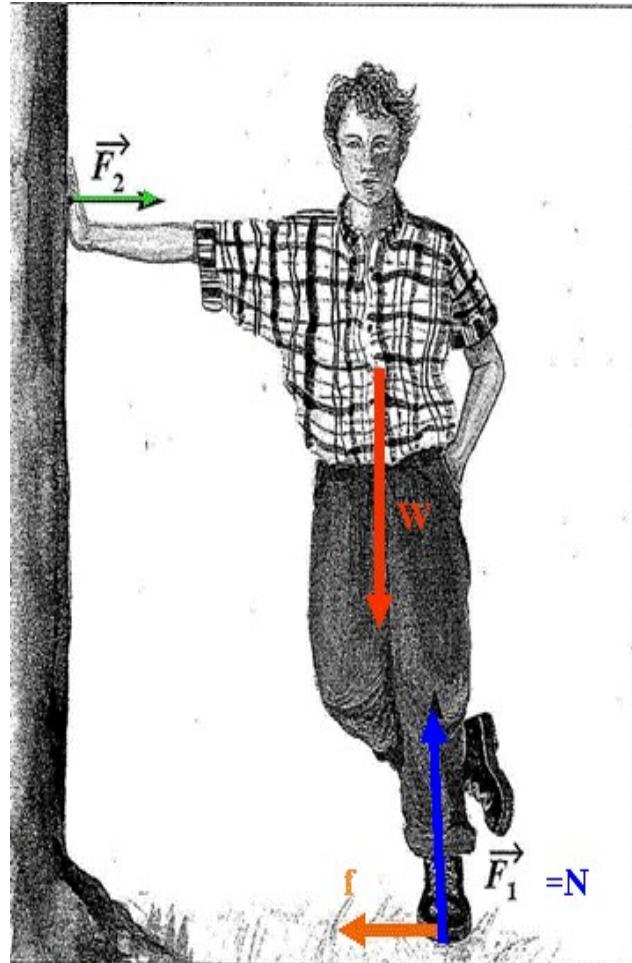


Reference: The physics of everyday phenomena: a conceptual introduction  
to physics by W. Thomas Griffith, McGraw-Hill

DRAW a free-body diagram for :

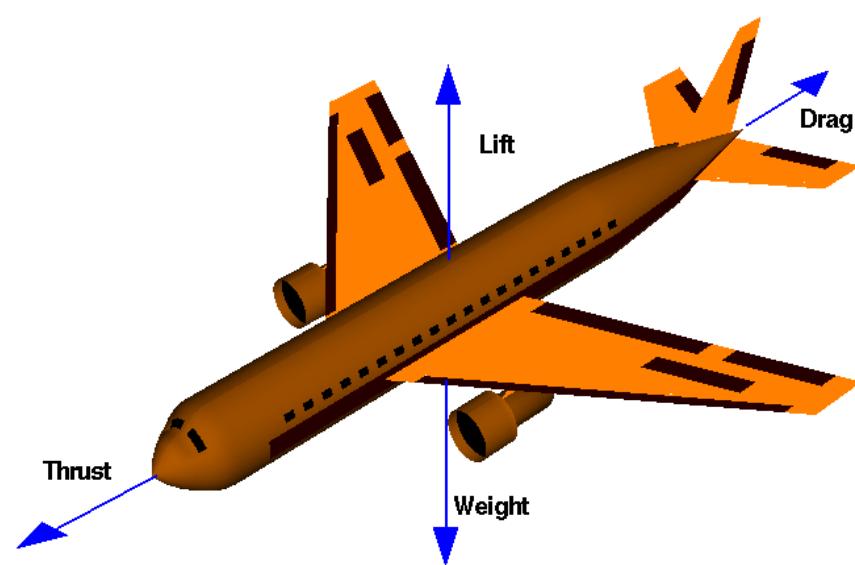
- 1) a parachutist speeding down to Earth (include air resistance)
- 2) a cockroach falling from the empire state building at a constant speed
- 3) A boy leaning with one hand on a tree. He stands on the ground  
(hint: all force come in pair)
- 4) an airplane flying. Included lift force  
(from air pushing up) and thrust (motor).  
The plane moves at a constant speed
- 5) a block pulled by a string and sliding. (include friction)  
Speeding up
- 6) a couch pushed over the floor
- 7) block on an inclined planed not sliding

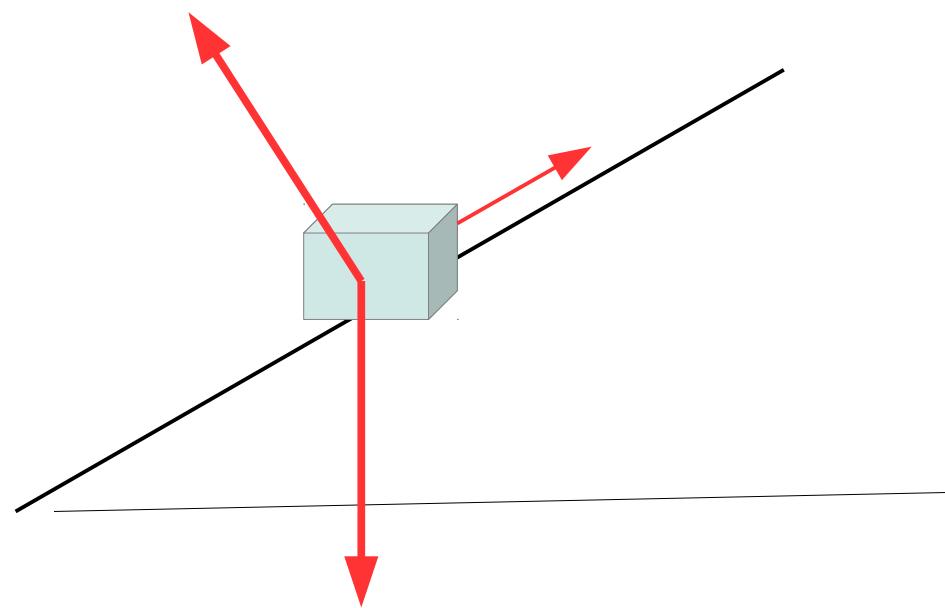
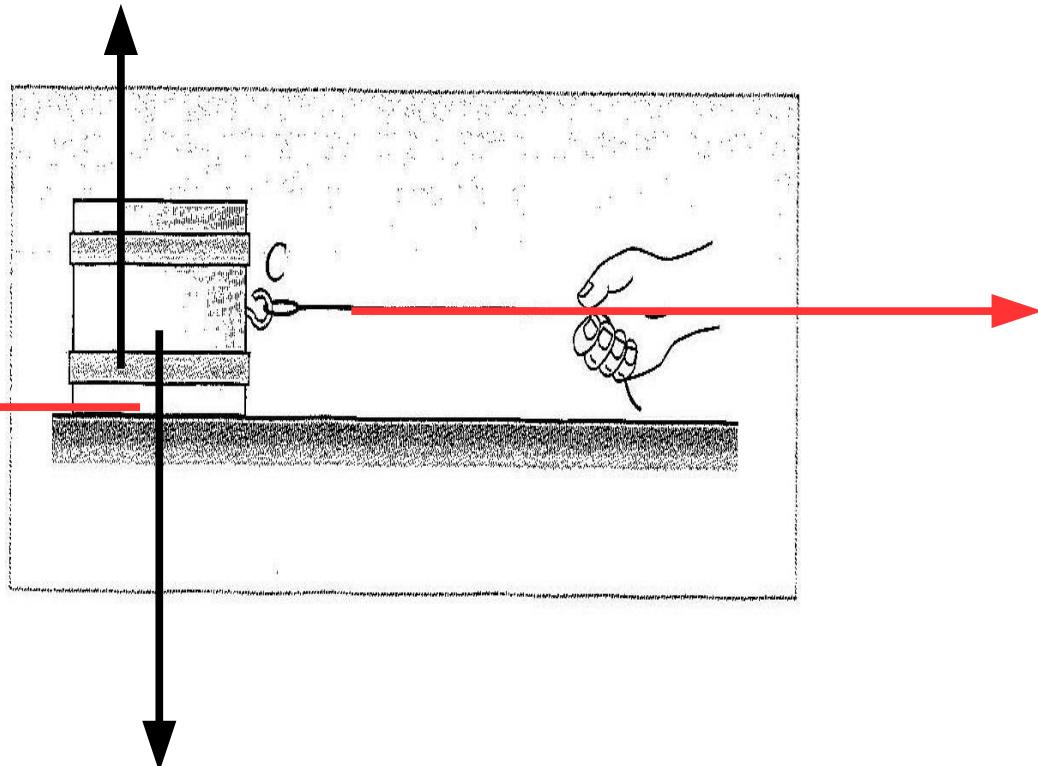


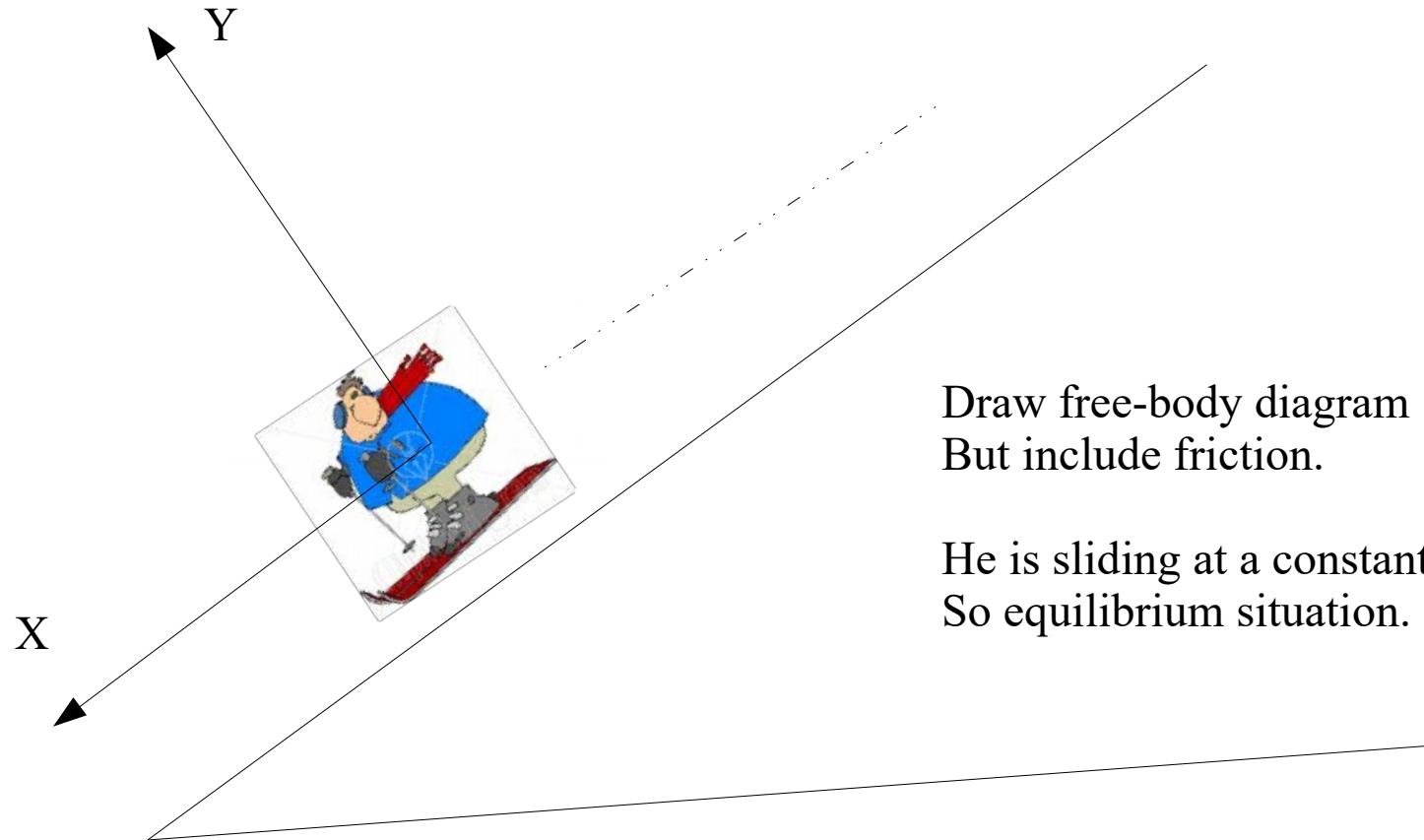


## Four Forces on an Airplane

Glenn  
Research  
Center



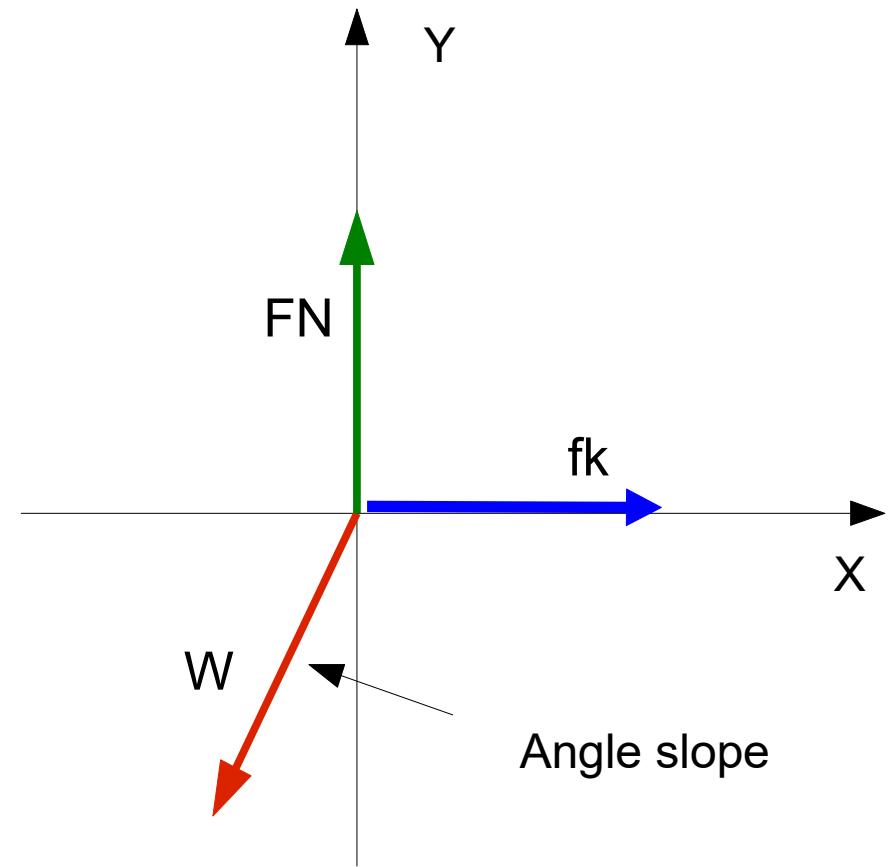
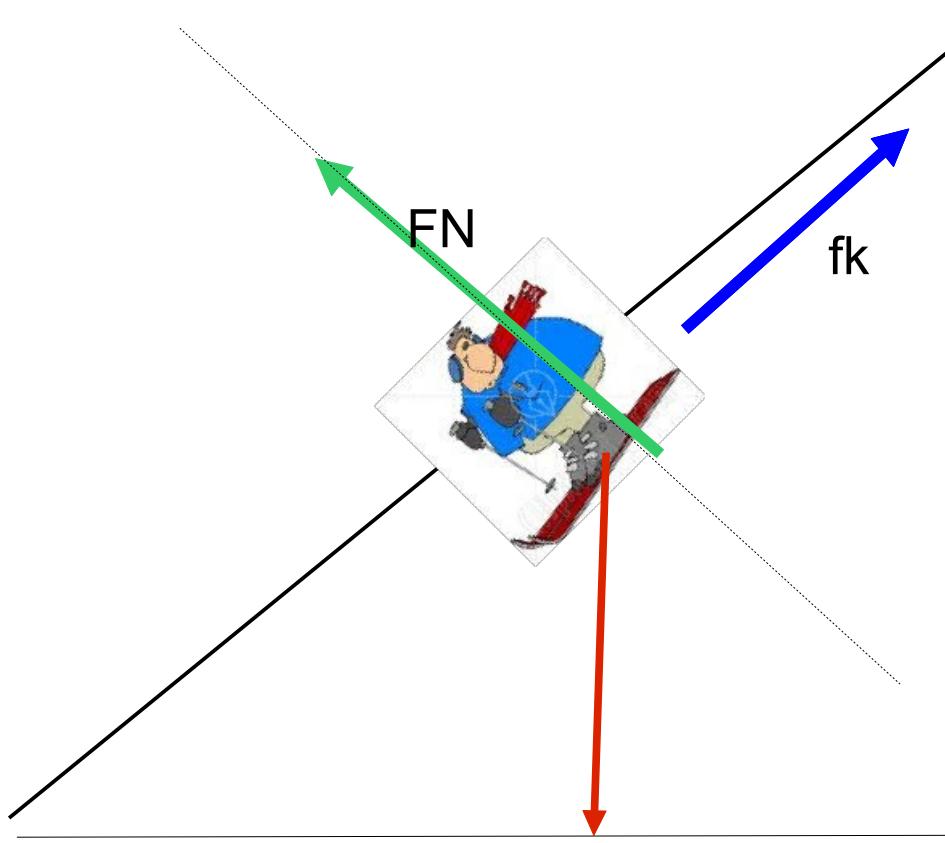




Draw the forces applied to the skier.  
Then trace the forces in a free-body diagram

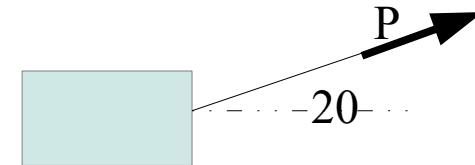
Draw free-body diagram  
But include friction.

He is sliding at a constant speed.  
So equilibrium situation.



The effect that a force has in any direction can be found by calculating its component in that direction

- 1) A 2.5kg brick is being pulled by a cord (at a constant speed) that makes an angle of 20 degrees with the horizontal and has a 7N (P) of tension in it.



The forces are: tension, weight, normal, friction

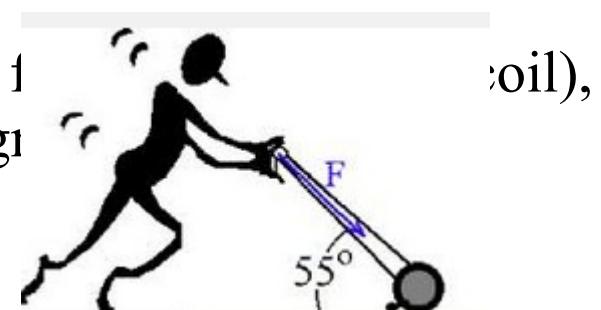
- A) find the weight of the brick
- B) draw a free-body diagram with the 4 forces
- C) write the forces in standard notation.
- D) build a table with the components of the forces.

Use net force = 0 to find the normal force and the friction.

P

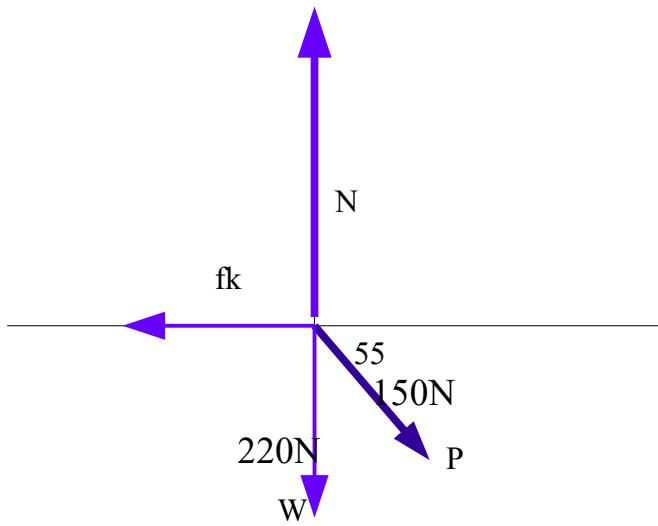
- 2) You are pushing a lawn mower at a constant speed. (so it is an equilibrium)  
The mass of the mower is 22 kg (weight is 220N).  
and is being pushed with a force of 150N (magnitude of P) against friction.  
The shaft makes an angle of 55 degrees with ground. See figure.

So the forces are the push down along the shaft, the normal force, the weight and the friction opposing the motion along the ground.

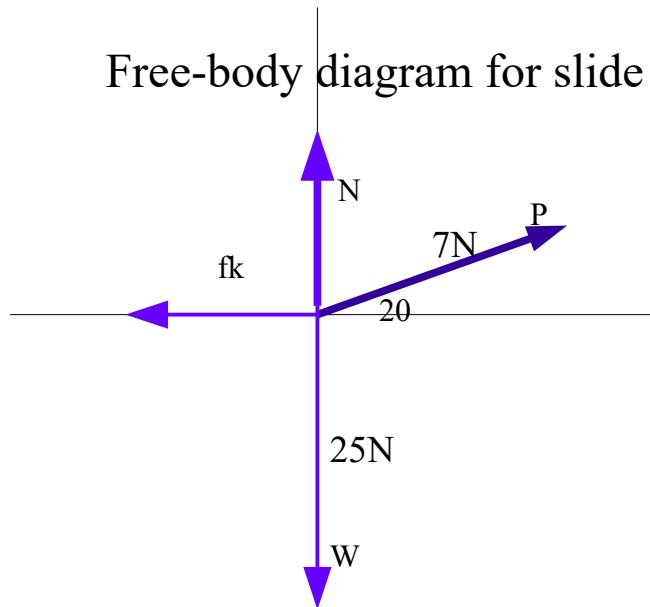


Same questions as above.

Free-body diagram for slide 61 Q2.

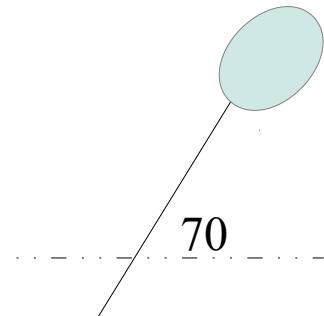


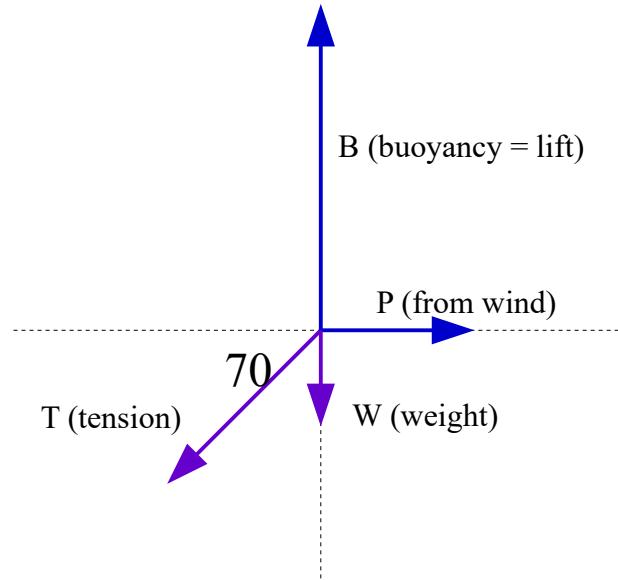
Free-body diagram for slide 61 Q1.



3) A helium balloon is in equilibrium as shown. The balloon weighs  $W= 25\text{N}$  and is acted upon by 4 forces. The weight (down), the buoyancy  $B$  (up) the tension  $T$  in the rope that is holding down the balloon. The force  $P$  of the wind pushing the balloon to the right. The tension  $T$  is  $16\text{N}$  @  $70$  degrees.

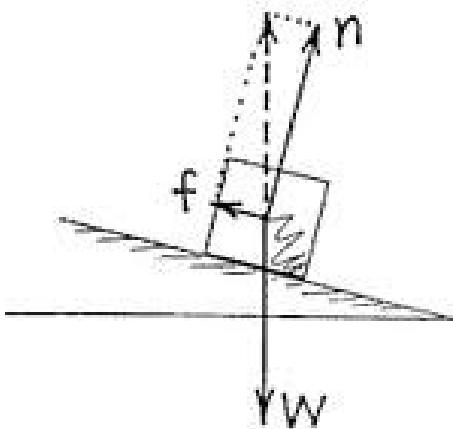
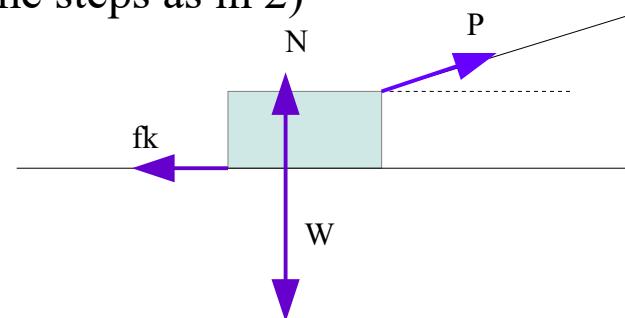
- A) draw the forces on the balloon (solution next slide)
  - B) draw a free-body diagram
  - C) build the table with the components of the forces.
- 
- C) Find the buoyancy and the force exerted by the Wind.





4) A sled is being pulled along a horizontal road at a constant speed by means of a rope that makes an angle of 25 degrees with the horizontal.  
 If the friction between the sled and the snow is 85N, how much is the tension P in the rope ? (free-body diagram please – follow same steps as in 2)

Use sum x-components =0 to find P.



5).  
 This block is not moving on the inclined plane. If the weigh is 10N and the angle of the plane With horizontal 20 degrees:  
 A) find the frictional force f  
 B) find the normal N (recoil force from the ground)

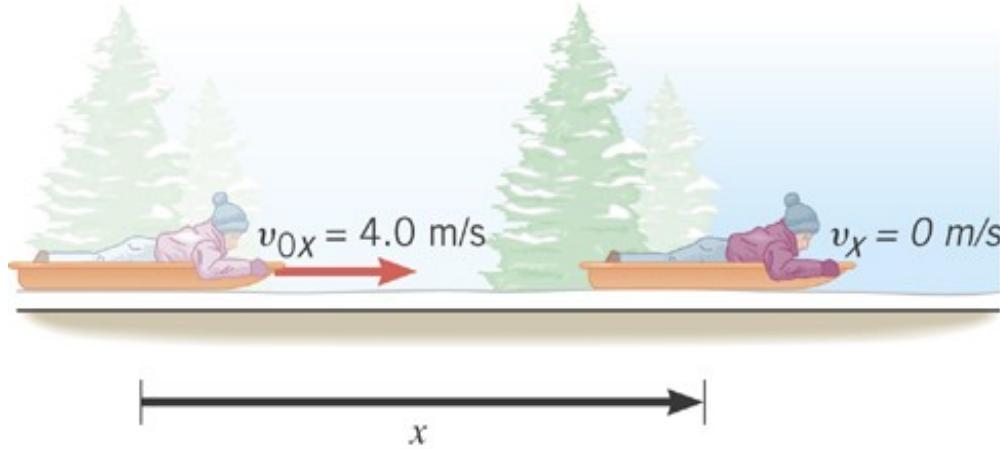
Static friction opposes the *impending* relative motion between two objects.

Kinetic friction opposes the relative sliding motion motions that actually does occur.

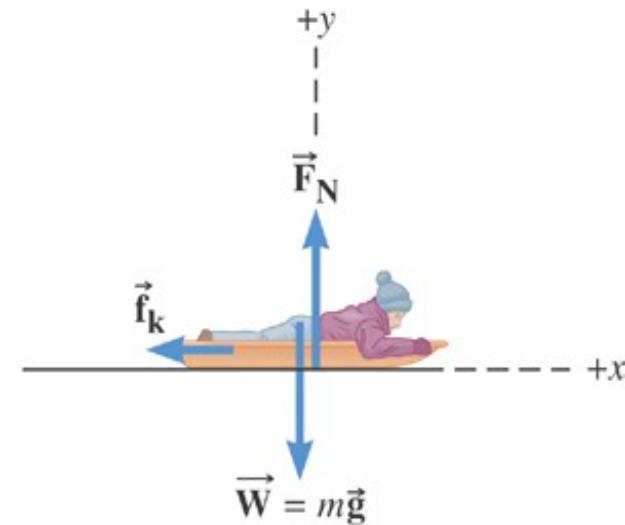
$$f_k = \mu_k F_N$$



$0 < \mu < 1$  is called the coefficient of kinetic friction.



(a)



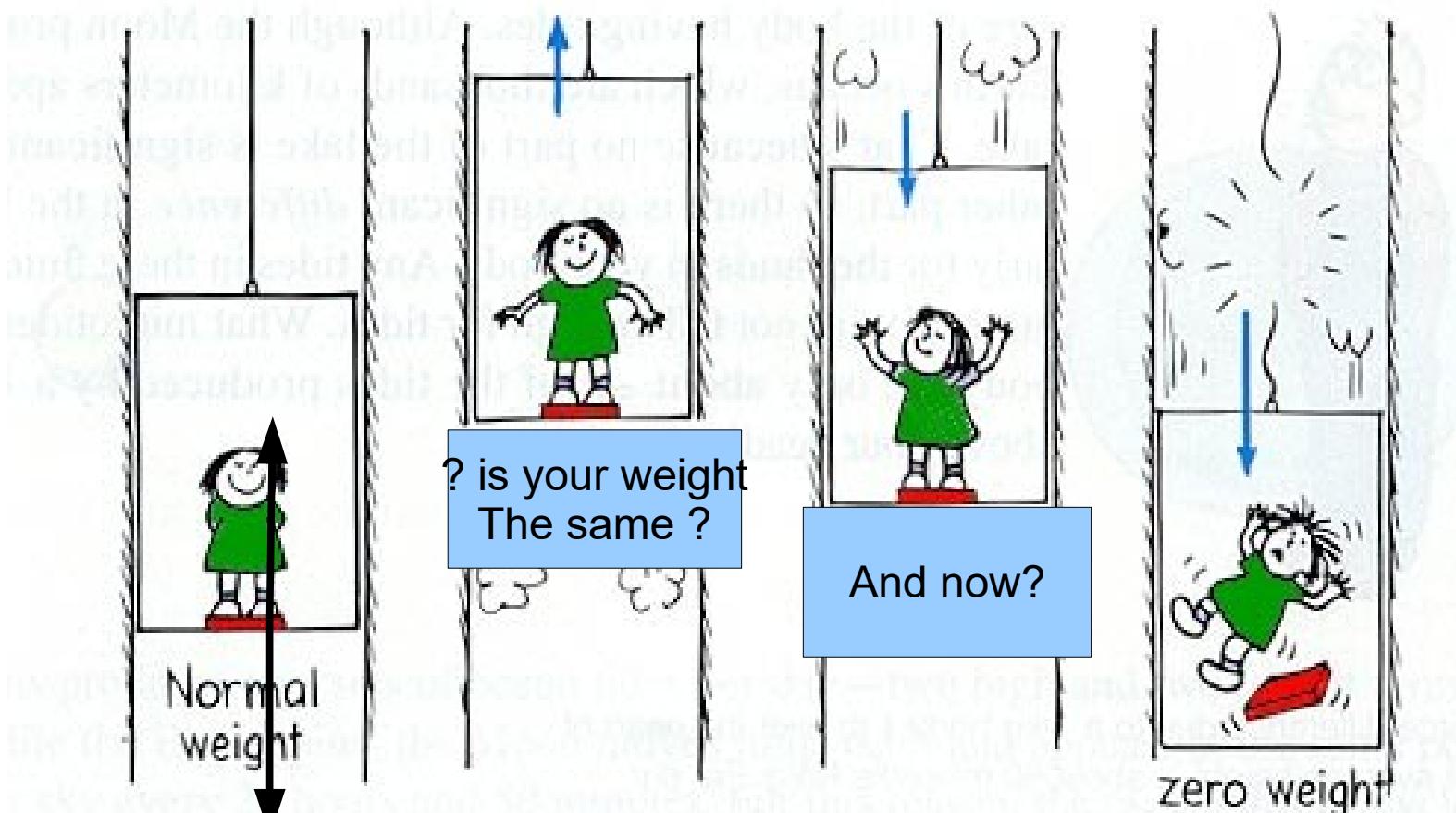
(b) Free-body diagram  
for the sled and rider

The sled comes to a halt because the kinetic frictional force opposes its motion and causes the sled to slow down.

Suppose the coefficient of kinetic friction is 0.05 and the total mass is 40kg. What is the kinetic frictional force?

## What is weightlessness ?

Gravity is still acting on you but you don't feel gravity anymore because There is no normal force. This is because your framework is falling With you. You are in free-fall.



The weight pulls the girl and  
The ground pushes the girl up  
This is the normal force. You feel  
The normal force not the weight..

The weight pulls the girl but  
There is no normal force.  
So the girl does not "feel" her weight

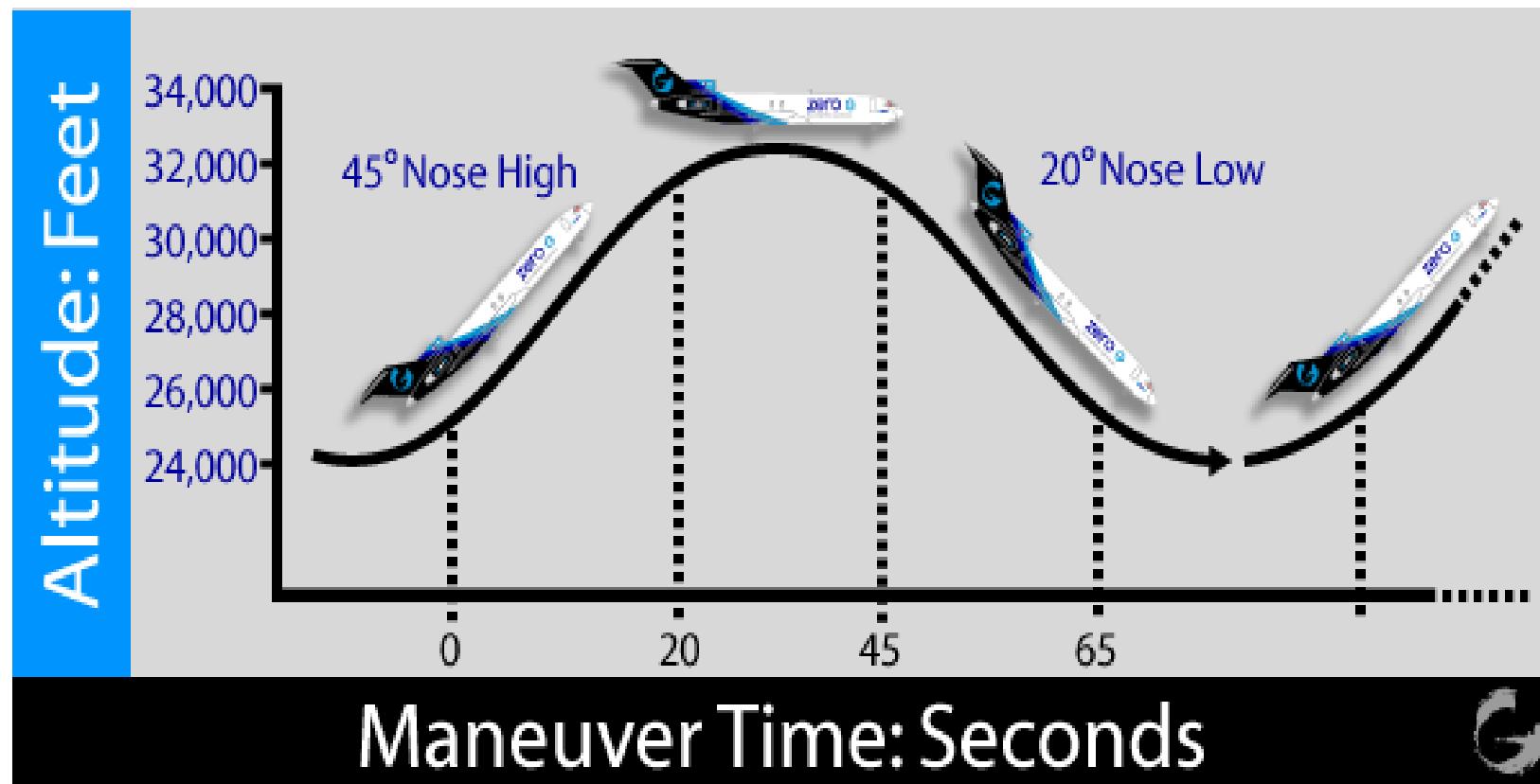
## Spinning in zero gravity

[http://www.youtube.com/watch?v=7tEkKhMVoS4&feature=player\\_embedded](http://www.youtube.com/watch?v=7tEkKhMVoS4&feature=player_embedded)

[http://www.youtube.com/watch?v=H0pGEq7bhLM&feature=player\\_embedded](http://www.youtube.com/watch?v=H0pGEq7bhLM&feature=player_embedded)

Zero gravity

<https://www.youtube.com/watch?v=LWGJA9i18Co>



While going down the plane and its inhabitants are in zero gravity for 10-15 seconds.

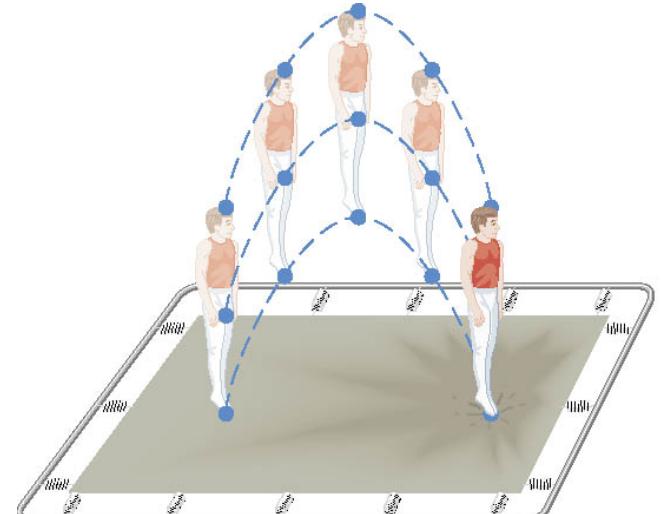
# *Rotational Dynamics*



Source: Physics / Edition 8  
by John D. Cutnell, Kenneth W. Johnson, Cutnell

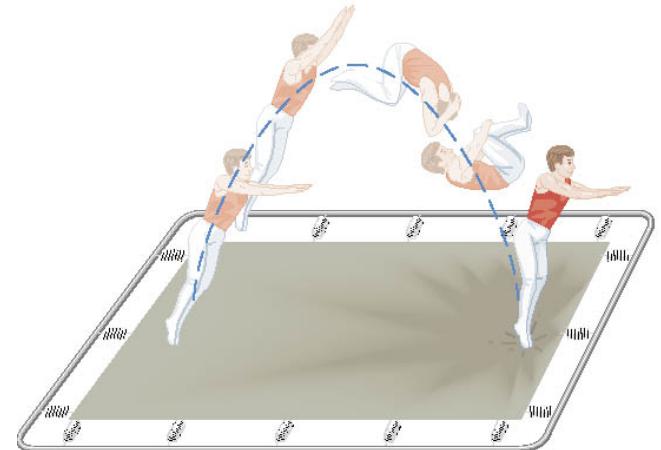
-The physics of everyday phenomena: a conceptual introduction to physics  
by W. Thomas Griffith, McGraw-Hill

In pure translational motion, all points on an object travel on parallel paths.



(a) Translation

The most general motion is a combination of translation and rotation.



(b) Combined translation and rotation

According to Newton's second law, a net force causes an object to have a linear acceleration.

What causes an object to have an *angular acceleration*?

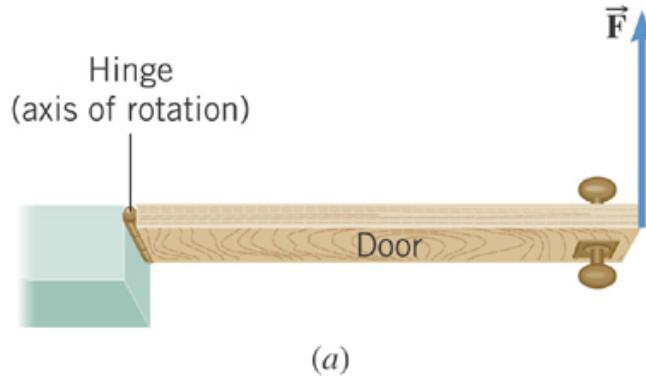
TORQUE       $\tau$

**MEASURES HOW EFFICIENT A FORCE IS IN APPLYING A TWIST :**

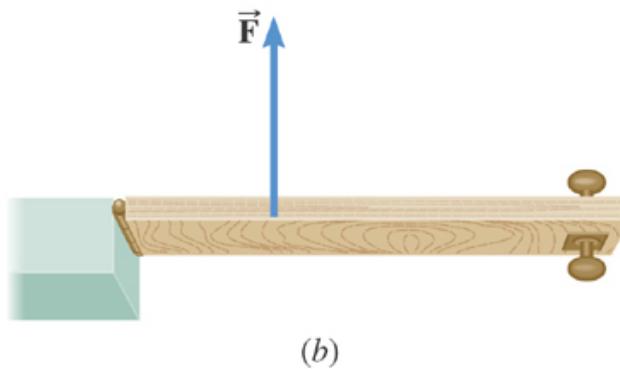
**Depend on the distance to the axis of rotation  
and the direction of the force.**

**Torque = how efficient is a force in rotating a rigid object (twist)**

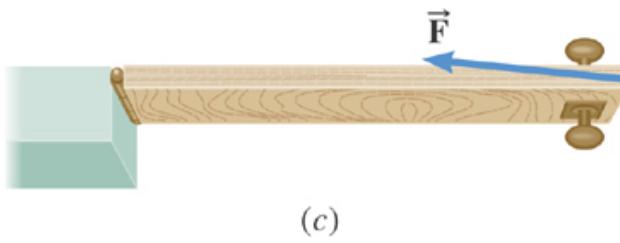
**In which case the torque is the largest ? (more efficient in opening the door).**



(a)



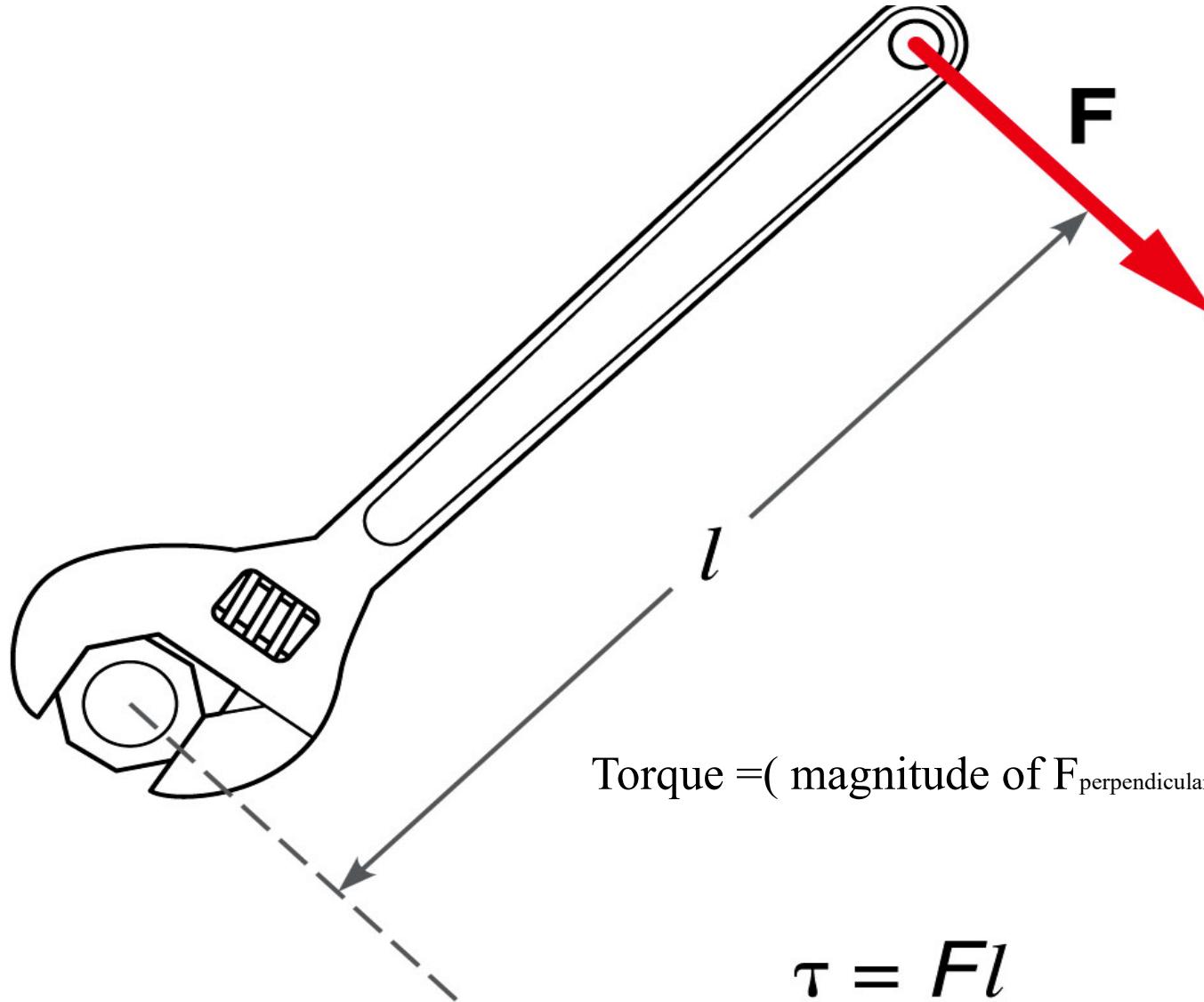
(b)



(c)

The amount of torque depends on where and in what direction the force is applied, as well as the location of the axis of rotation.

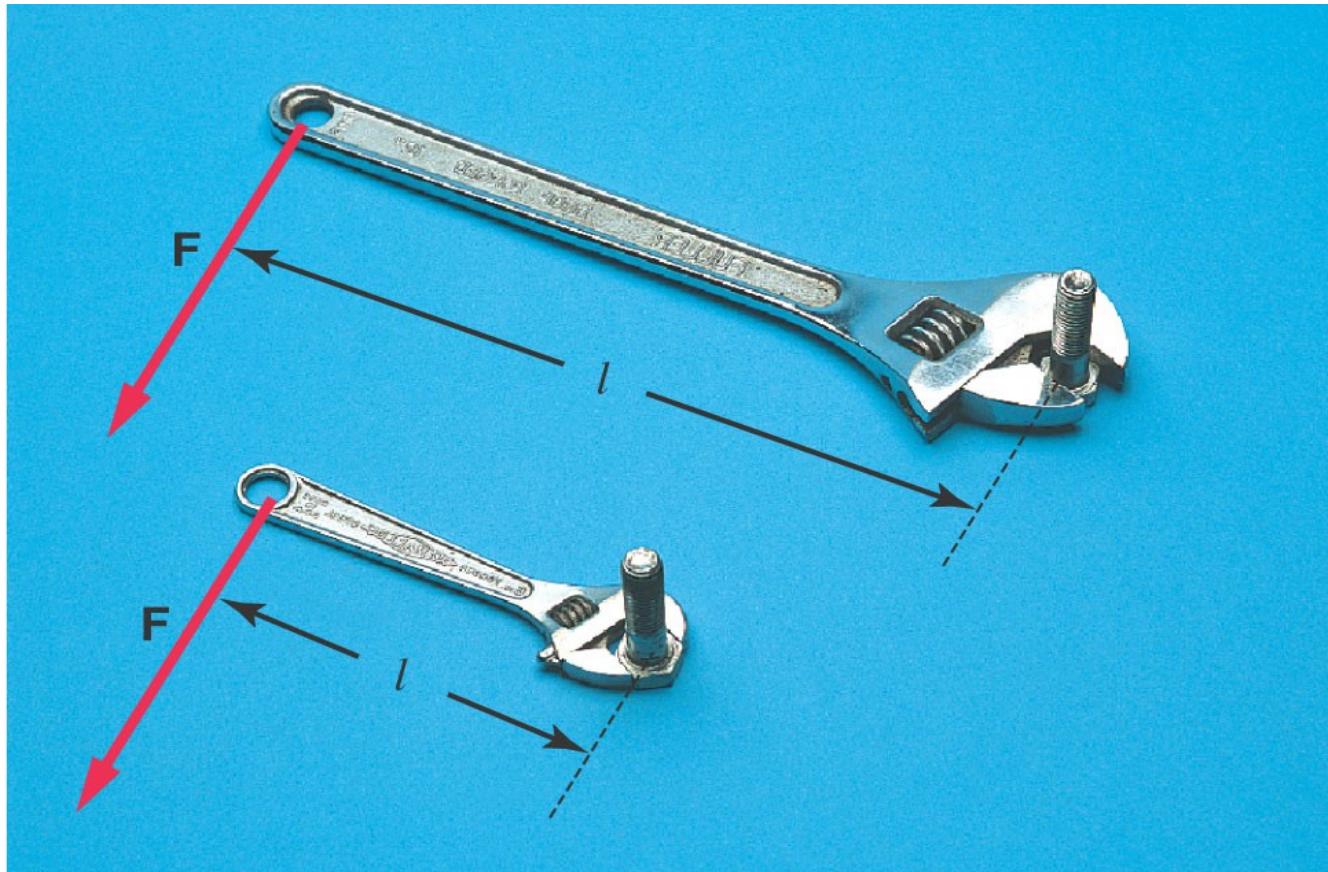
Any Suggestion for the expression of a torque when the force is normal  
To the door ?



Torque = ( magnitude of  $F_{\text{perpendicular}}$ ) x distance to axis

$$\tau = Fl$$

To be more efficient you can: increase  $F$  or increase  $F$  a better twist.  
If  $F=50\text{N}$  and the length of the wench is 24cm, find the torque. (cm to m)



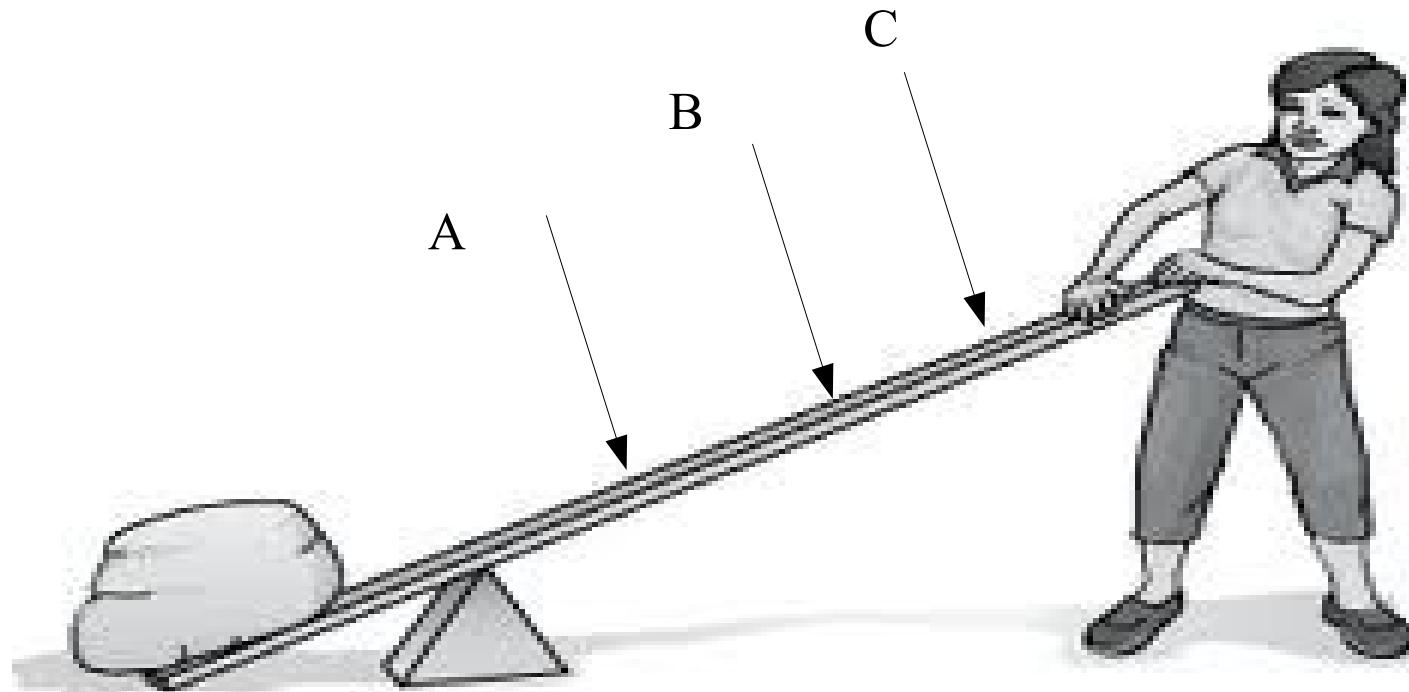
If  $l$  is divided by 2, the new torque :

- A) is the same
- B) is double
- C) is half

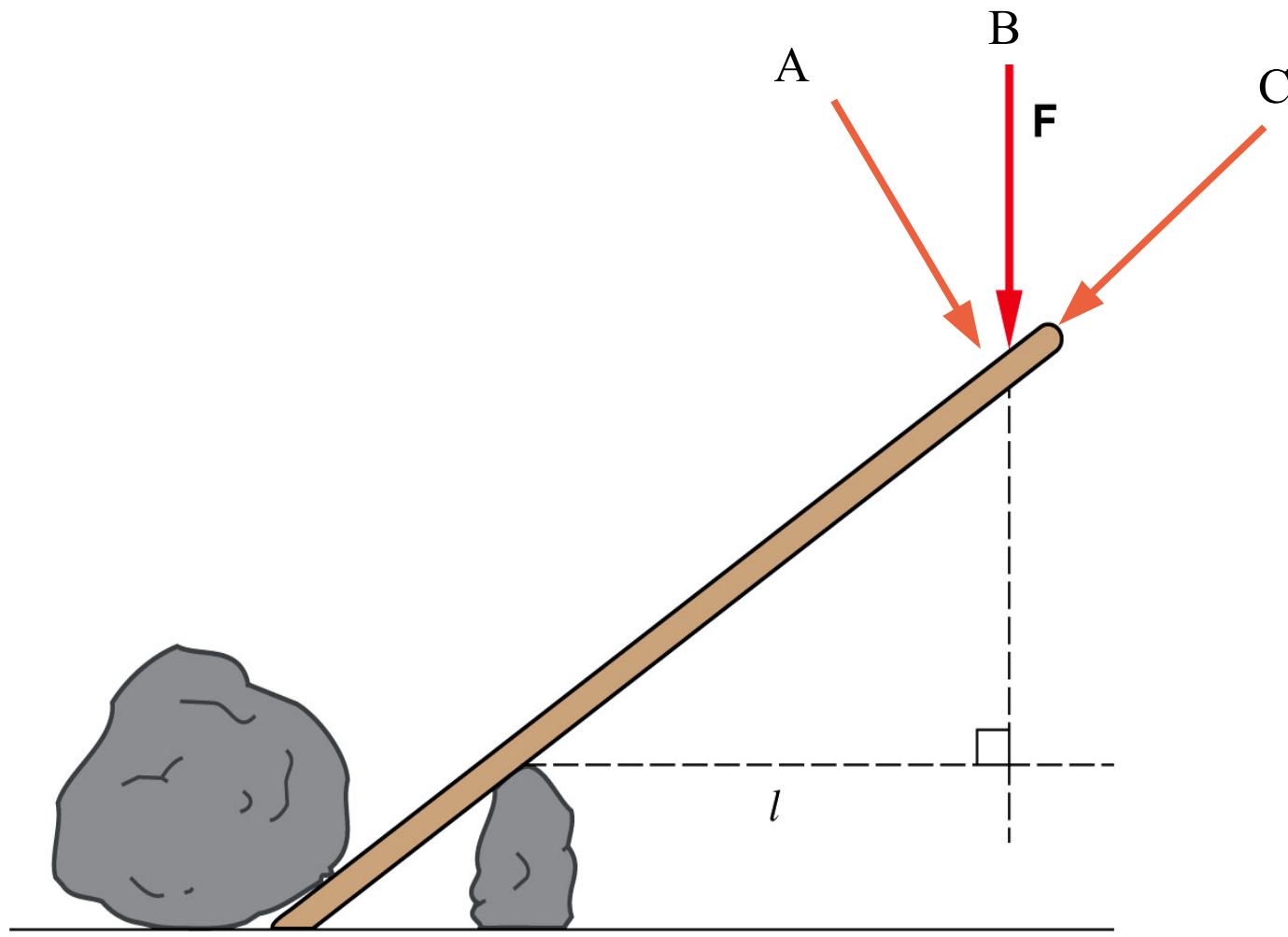
So why do we use long wrench ?

Where to push to make the torque the largest ?  
(will make easier to push) A or B or C ?

**The force applied is the same.**



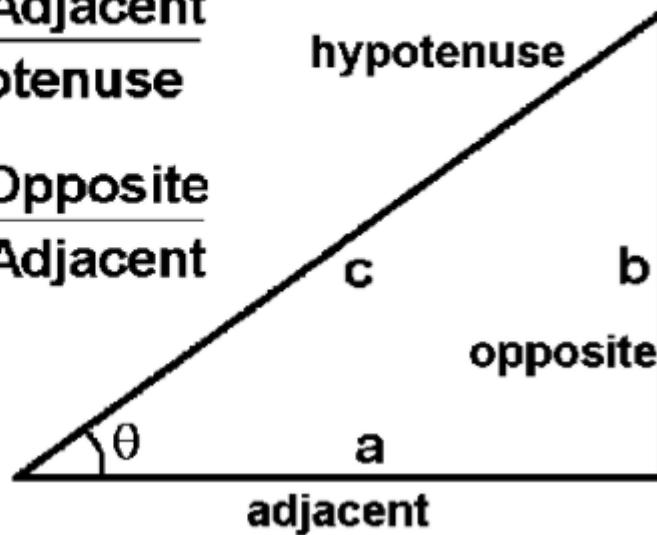
Which direction will produce the larger twist ? A? B? C?  
(more efficient for The same magnitude?)



$$\sin \theta = \frac{\text{Side Opposite}}{\text{Hypotenuse}}$$

$$\cos \theta = \frac{\text{Side Adjacent}}{\text{Hypotenuse}}$$

$$\tan \theta = \frac{\text{Side Opposite}}{\text{Side Adjacent}}$$



To find adjacent: use cosine(..) and multiply by hypotenuse

To find the opposite: use sine and multiply by the hypotenuse

[http://www.funmaths.com/worksheets/downloads/view.htm?ws0037\\_1.gif](http://www.funmaths.com/worksheets/downloads/view.htm?ws0037_1.gif)

[http://www.funmaths.com/worksheets/downloads/view.htm?ws0038\\_1.gif](http://www.funmaths.com/worksheets/downloads/view.htm?ws0038_1.gif) HW for WEDnesday 77

Suppose the angle is 40

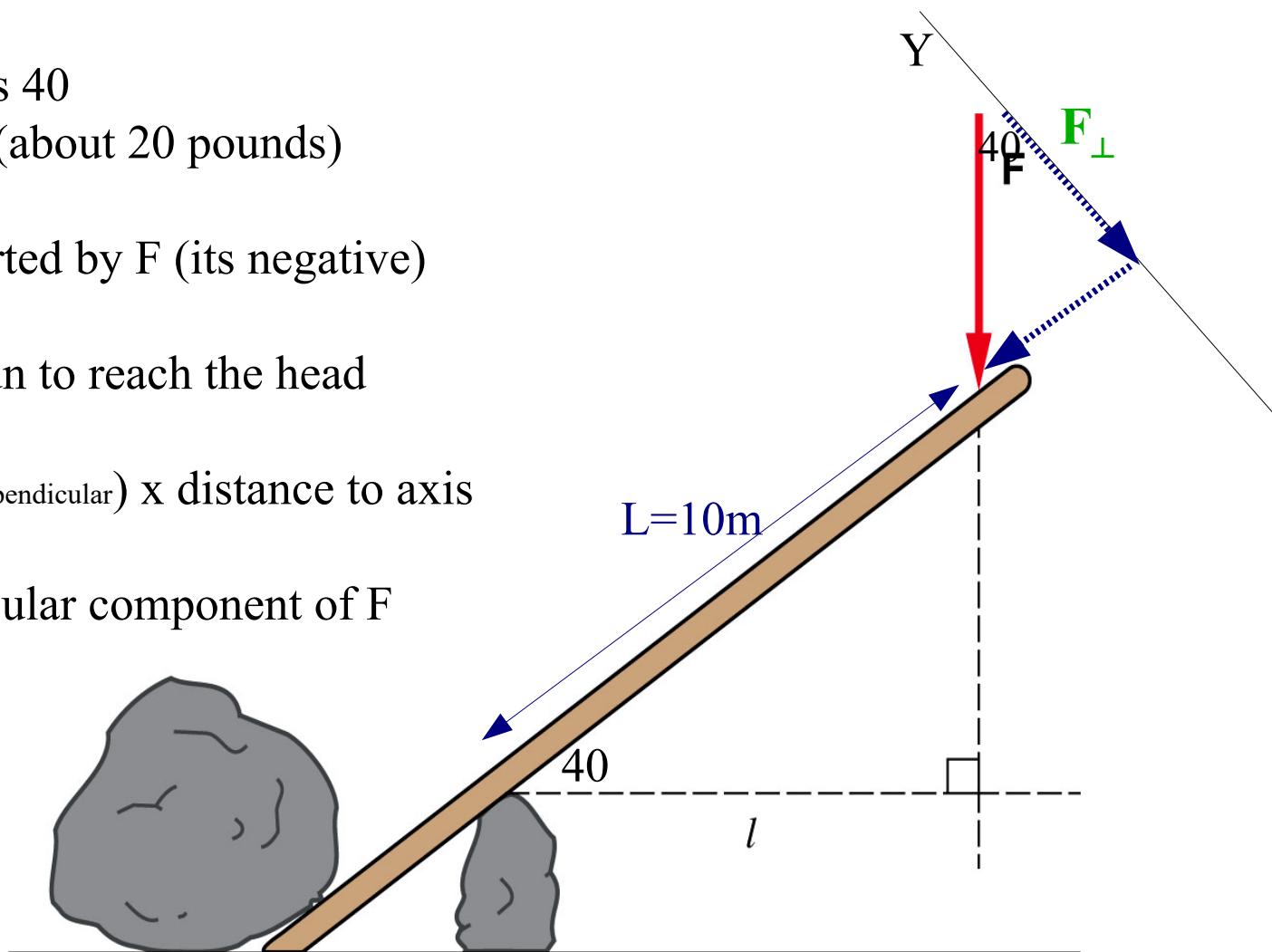
Suppose  $F$  is 100N (about 20 pounds)

Find the torque exerted by  $F$  (its negative)

From tail fall and run to reach the head

- (magnitude of  $F_{\text{perpendicular}}$ )  $\times$  distance to axis

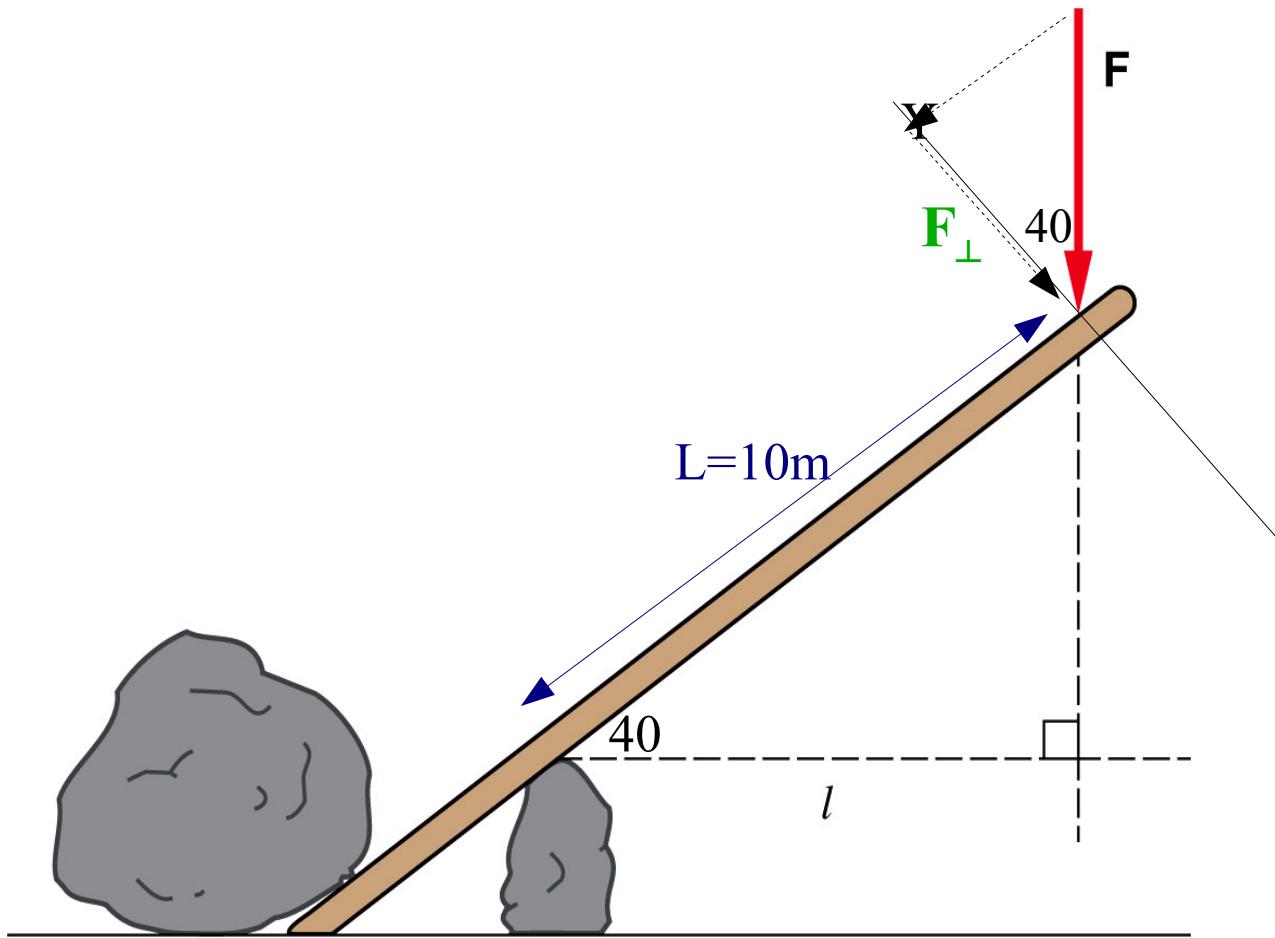
- only the perpendicular component of  $F$   
is doing the “job”



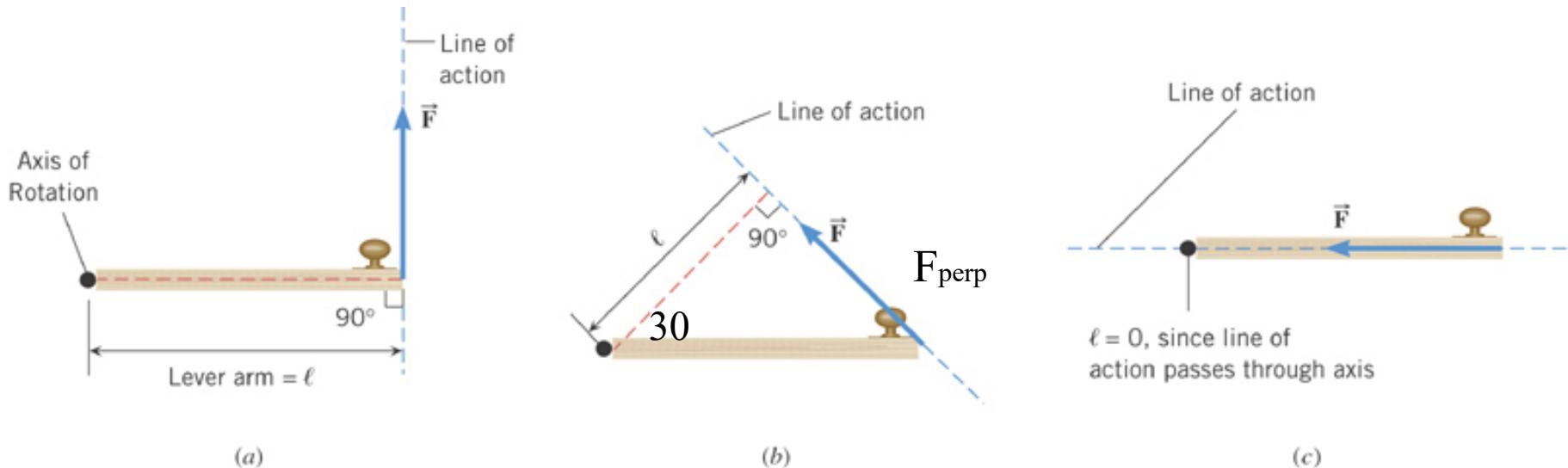
Only the component  $F_{\perp}$  (normal) is doing the job.

Or you can run and fall.

Same result.



*How to find  $l$  the lever arm = perpendicular distance between the axis and the line of action of the force.*



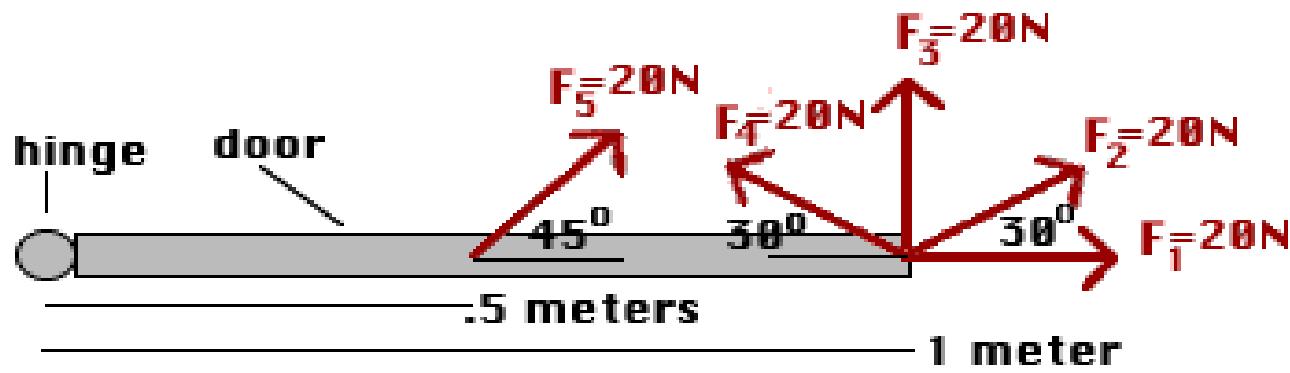
## DEFINITION OF TORQUE

$$\tau = F_{\text{perpendicular}} \cdot L$$

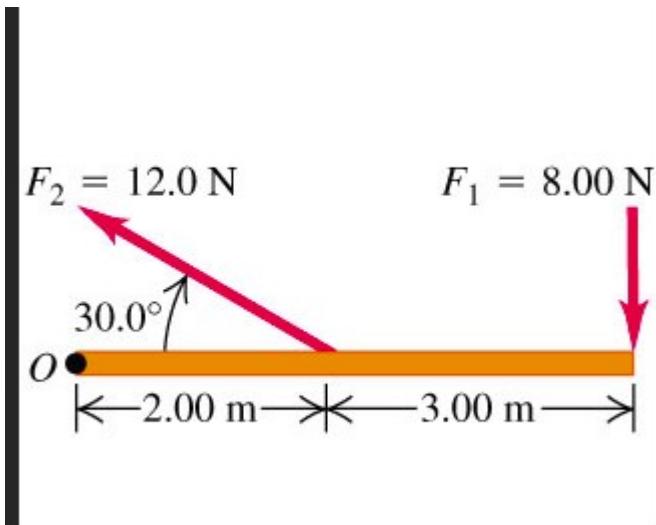
*perpendicular  
component*

**Direction:** The torque is positive when the force tends to produce a counterclockwise rotation about the axis.

**SI Unit of Torque:** newton x meter ( $\text{N} \cdot \text{m}$ )



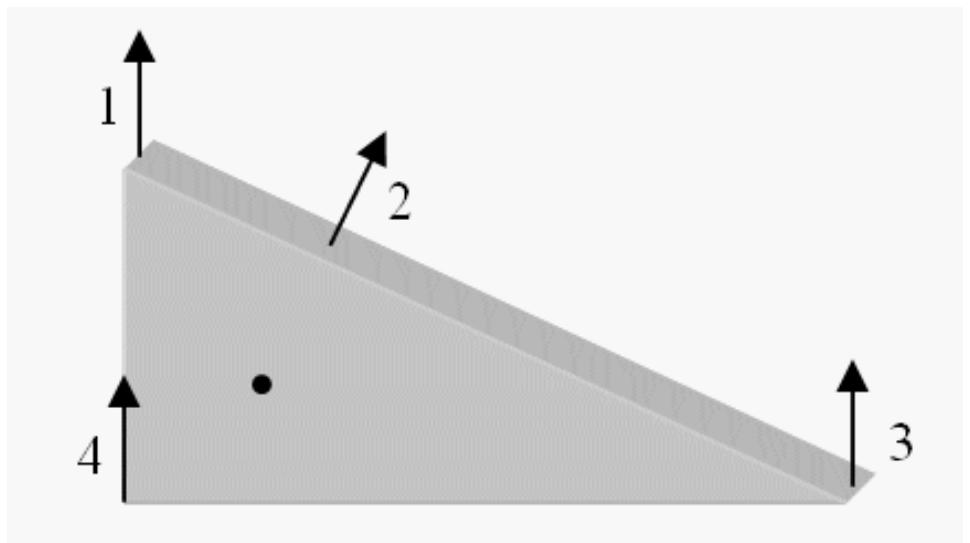
Find the torque exerted by each force.



9.1.6. An object with a triangular cross-section is free to rotate about the axis represented by the black dot shown. Four forces with identical magnitudes are exerted on the object. Which one of the forces, if any, exerts the largest torque on the object?

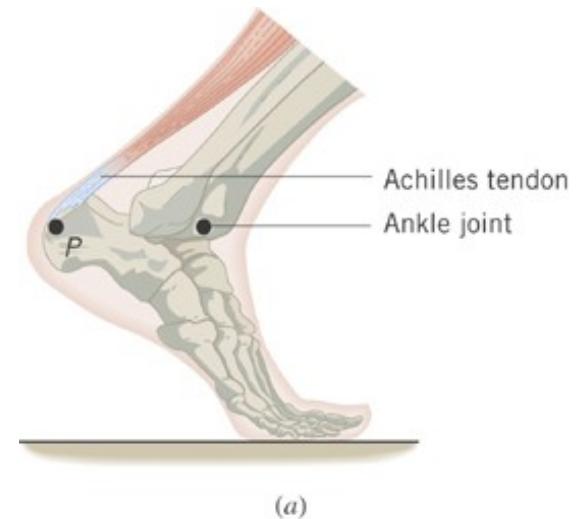
- a) 1
- b) 2
- c) 3
- d) 4

- e) The same torque is exerted by each force.



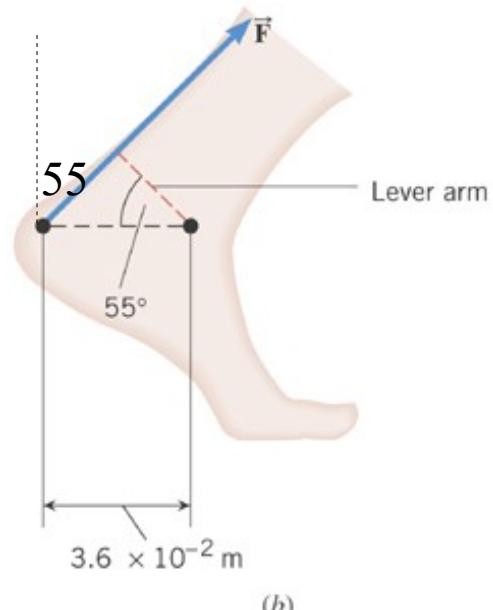
## *Example 2 The Achilles Tendon*

The tendon exerts a force of magnitude 790 N. Determine the torque (magnitude and direction) of this force about the ankle joint.



(a)

$$\tau = F_{perp} \ell \quad \text{or}$$



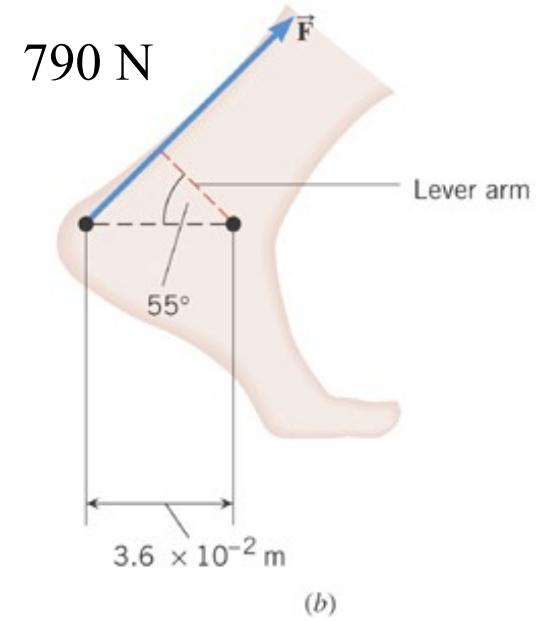
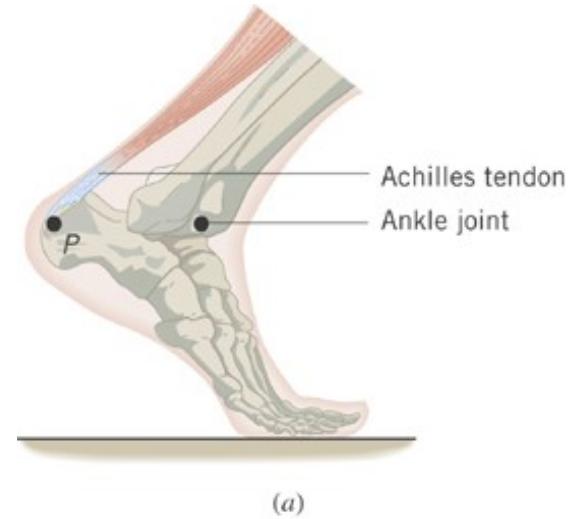
(b)

## 9.1 The Action of Forces and Torques on Rigid Objects

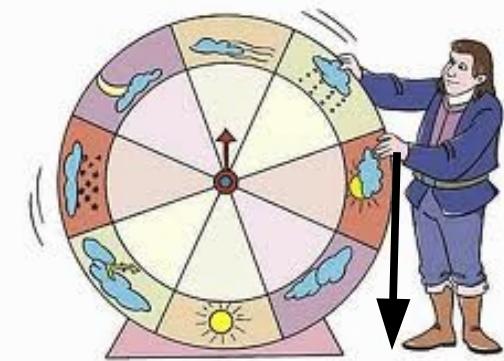
$$\tau = F\ell$$


$$\cos 55^\circ = \frac{\ell}{3.6 \times 10^{-2} \text{ m}}$$

$$\begin{aligned}\tau &= (720 \text{ N})(3.6 \times 10^{-2} \text{ m}) \cos 55^\circ \\ &= 15 \text{ N} \cdot \text{m}\end{aligned}$$



1) The radius of the wheel of fortune is 1.2m, and the operator applies a force of 45N tangentially to get it spinning. What torque has he supplied ?



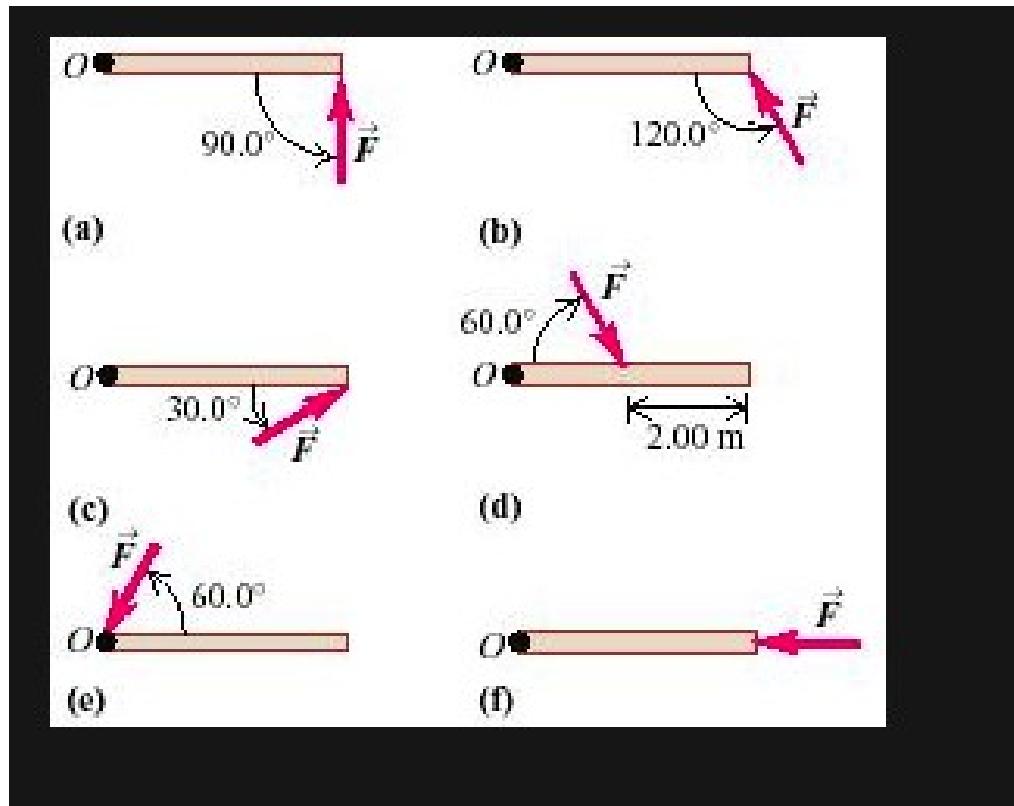
2) a 32 kg child sits on a seesaw. If she is 2.2 m from the pivot, what is the torque that her weight exerts, making the seesaw rotate around the pivot ?

3) A torque of 30N.m is required to turn the steering wheel of a car. If the radius of the wheel is 26cm/ How much force is needed.

4) You are using a crowbar 2.4 m long, pivoted at 0.20m from the end, to lift a rock that weighs 750N. How much force do you have to exert ?

10) A force of 50N is applied at the end of a wrench handle that is 24cm. The force is applied in a direction perpendicular to the handle . A) what is the torque applied to the nut by the wrench B) what would be the torque if the force were applied half way up instead of at the end ?





Find the torque of the forces  
The length of the door is 4m.

## OPTIONAL

5) suppose that a merry-go-round is rotating at the rate if 10 rev/min

- A) Express this rotational velocity in rev/s (rev means revolution = 1 tour)
- B) Express this rotational velocity in rad/s (1 revolution =  $2\pi$  rad)

6) The popular music record used to have a rate of 45RPM .

- A) express this velocity in rev/s
- B) Through how many revolutions does the record turn in a time of 5s?  
(its a rate. Use proportion)

7) Suppose a disk rotates through 3 revolutions in 4 seconds

- A) What is its displacement in radians in this time ? ((1 revolution =  $2\pi$  rad)
- B) What is its average rotational velocity in rad/s

8) A bicycle wheel is rotationally accelerated at the constant rate of 1.2 rev/s/s

- A) If it starts from rest, what is its rotational velocity after 4s
- B) Through how many revolutions does it turn in this time ?  
(like for kinematics : # of revolution =  $\frac{1}{2}$  (acceleration in rev/s/s) x (time squared) + (initial rotational velocity) x (time))

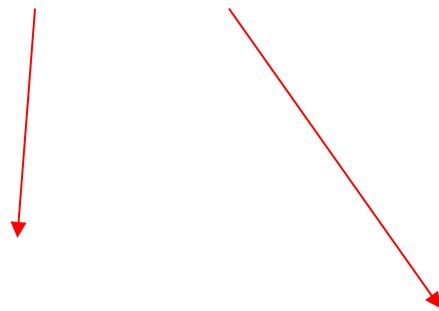
9) The rotational velocity of a merry go round increases at a constant rate from 1.0 rad/s to 1.8 rad/s in a time of 4s.

What is the rotational acceleration of the merry go round ?

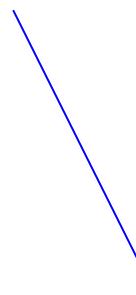
## 9.2 Rigid Objects in Equilibrium

If a rigid body is in equilibrium, neither its linear motion nor its rotational motion changes.

$$a_x = a_y = 0$$



$$\alpha = 0$$



$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum \tau = 0$$

### EQUILIBRIUM OF A RIGID BODY

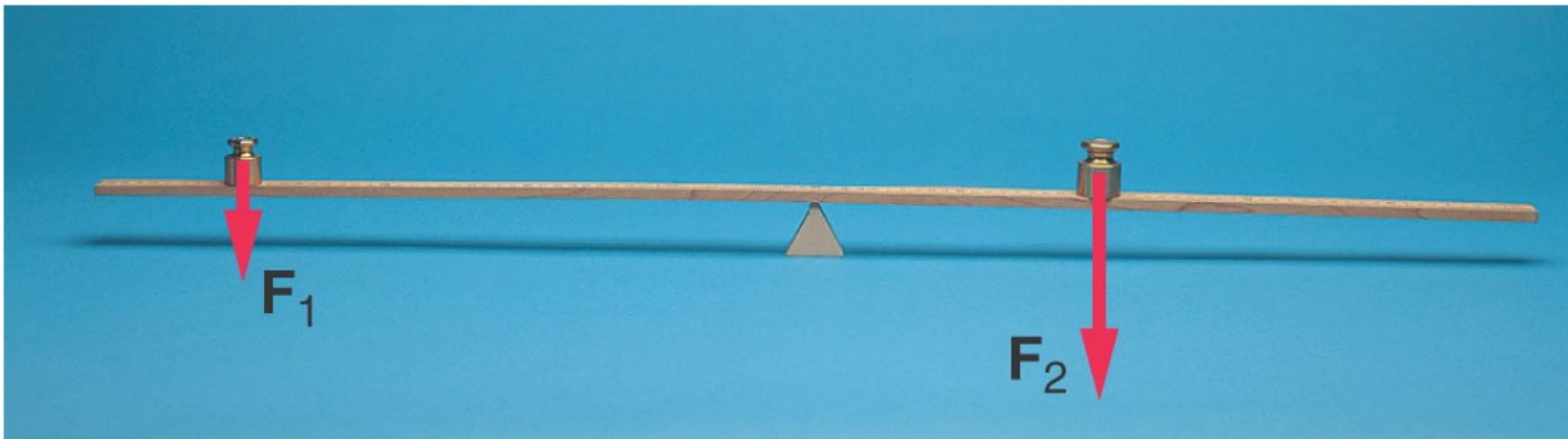
A rigid body is in equilibrium if it has zero translational acceleration and zero angular acceleration. In equilibrium, the sum of the externally applied forces is zero, and the sum of the externally applied torques is zero.

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum \tau = 0$$

## Example of equilibrium:



[https://phet.colorado.edu/sims/html/balancing-act/latest/balancing-act\\_en.html](https://phet.colorado.edu/sims/html/balancing-act/latest/balancing-act_en.html)

If mass  $2 = 1\text{kg}$  then  $F_2 = \underline{\hspace{2cm}}$  N

If mass  $1 = 500\text{g}$ , then  $F_1 = \underline{\hspace{2cm}}$  N

If the masses are in equilibrium :

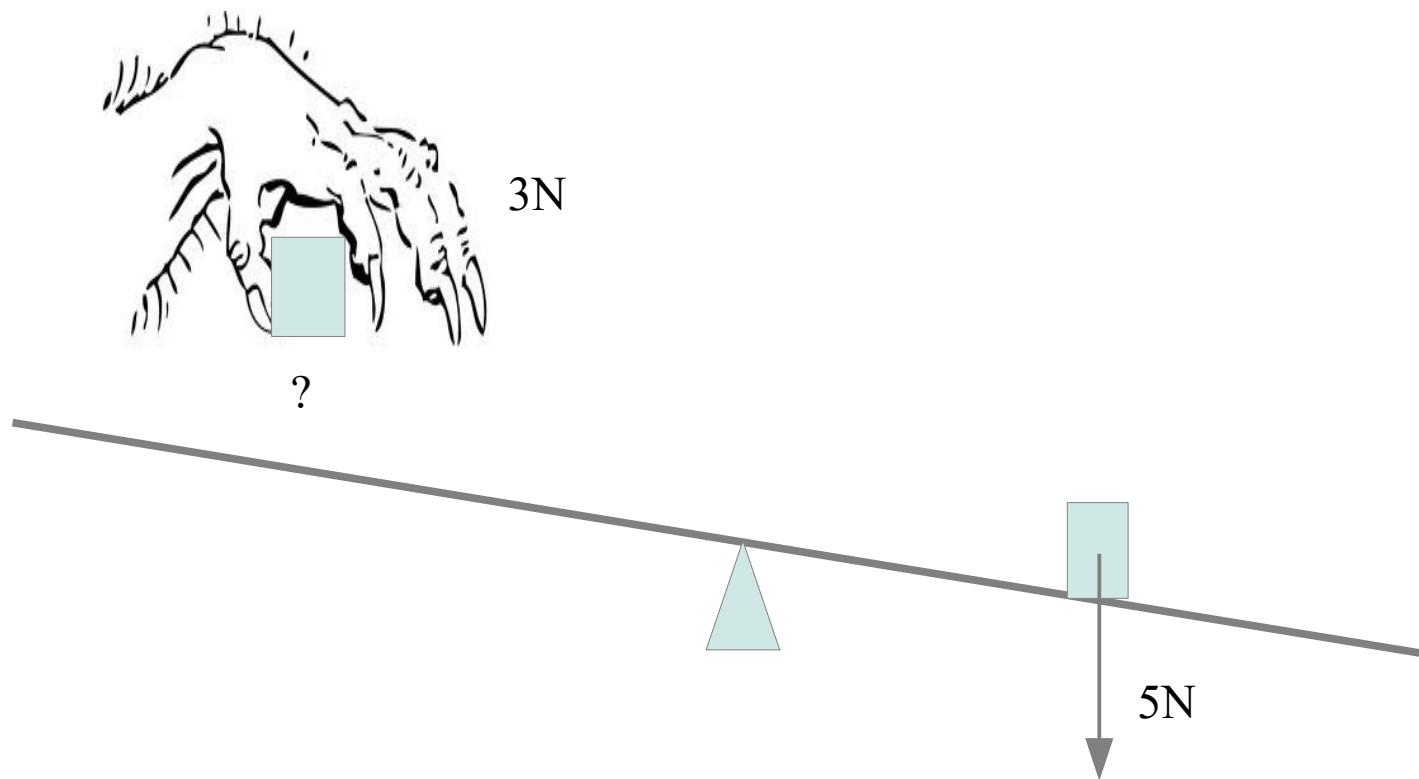
$$+ F_1 l_1 - F_2 l_2 = 0$$

If  $d_2 = 10\text{ cm}$ , find  $d_1$ .

Draw a free body diagram and use up = right to find the force exerted by the Fulcrum on the ruler.

Suppose we have a 3N weight that we want to balance against a 5 N weight  
On a stick, which is balanced when no mass are in place. The 5N weight is placed  
20cm to the right of the fulcrum. (see below)

- A) What is the torque produced by the 5N weight (cm to m and sign !!)  
B) How far would we have to place the 3N weight from the fulcrum to balance the System ?



## REMEMBER: EQUILIBRUM MEANS :

**I. sum(torques)= 0**

$$+/- \mathbf{F1} \mathbf{l1} +/- \mathbf{F2} \mathbf{l2} +/- \mathbf{F3} \mathbf{l3} + \dots = \mathbf{0}$$

With  $F_1, F_2, F_3 \dots$  are the magnitude of the y-components of the forces  
(perpendicular components)

$L_1, l_2, l_3$  are the distances from the fulcrum (axis).

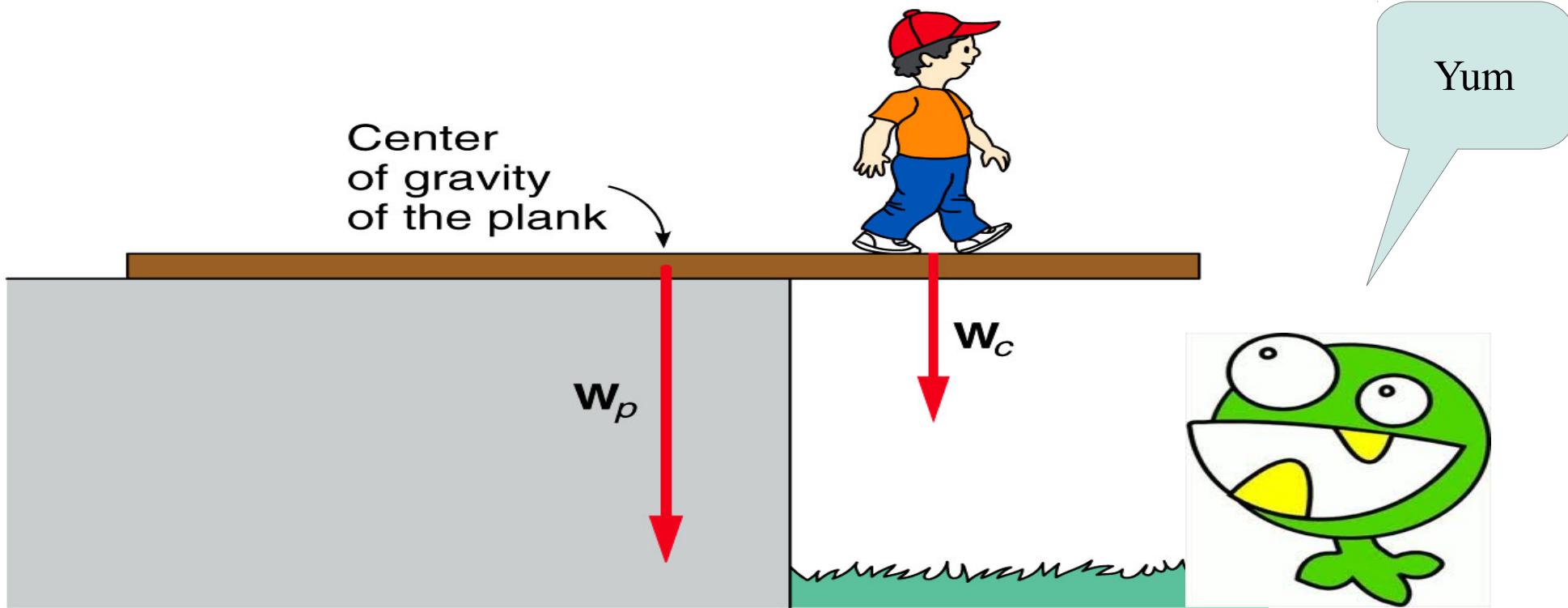
If you use this notation, you don't need to convert to meters or to newtons  
As long as all the forces are expressed in the same unit and the  
Distances are expressed in the same unit.

+ if counterclockwise motion and – if clockwise motion about the axis

**II. Sum (Fy) = 0** up means  $F_y > 0$  and down means  $F_y < 0$

**III. Sum (Fx) = 0** right means  $F_x > 0$  and left means  $F_x < 0$

OTHER example of equilibrium:

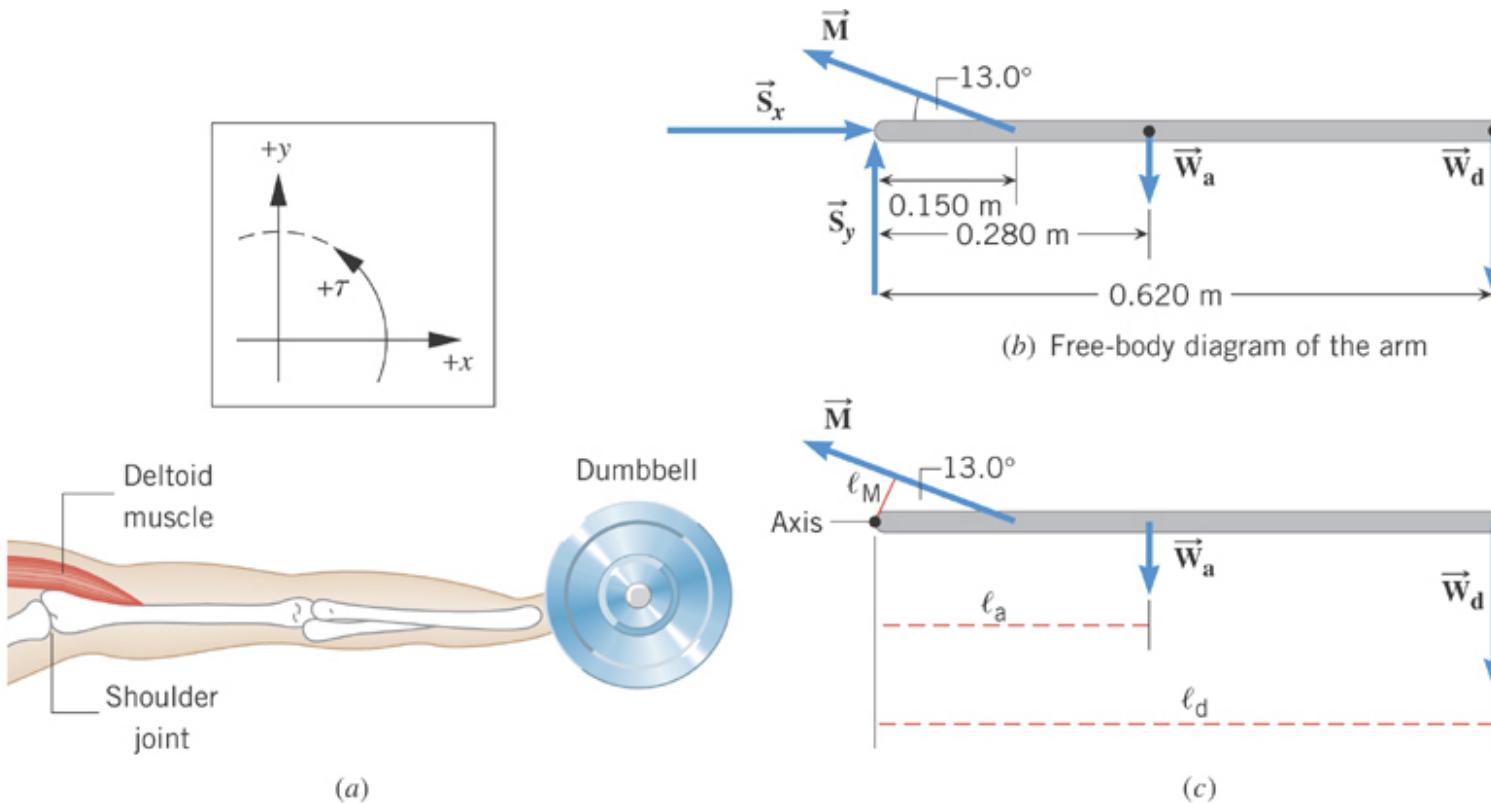


A 4m long plank with a weight of 80lb is placed on a dock with 1 m of its length extended Over a sea of deadly grass. A boy with a weight of 150lb is standing on the plank and moving out slowly from the Edge of the dock.

A) What is the torque exerted by the weight of the plank about the pivot point at the edge of the dock ? (treat all the weights as acting through the center of gravity of The plank).

B) How far from the edge of the dock can the boy move until the plank is just on the verge Of tipping ? (and the boy on the verge to be eaten)

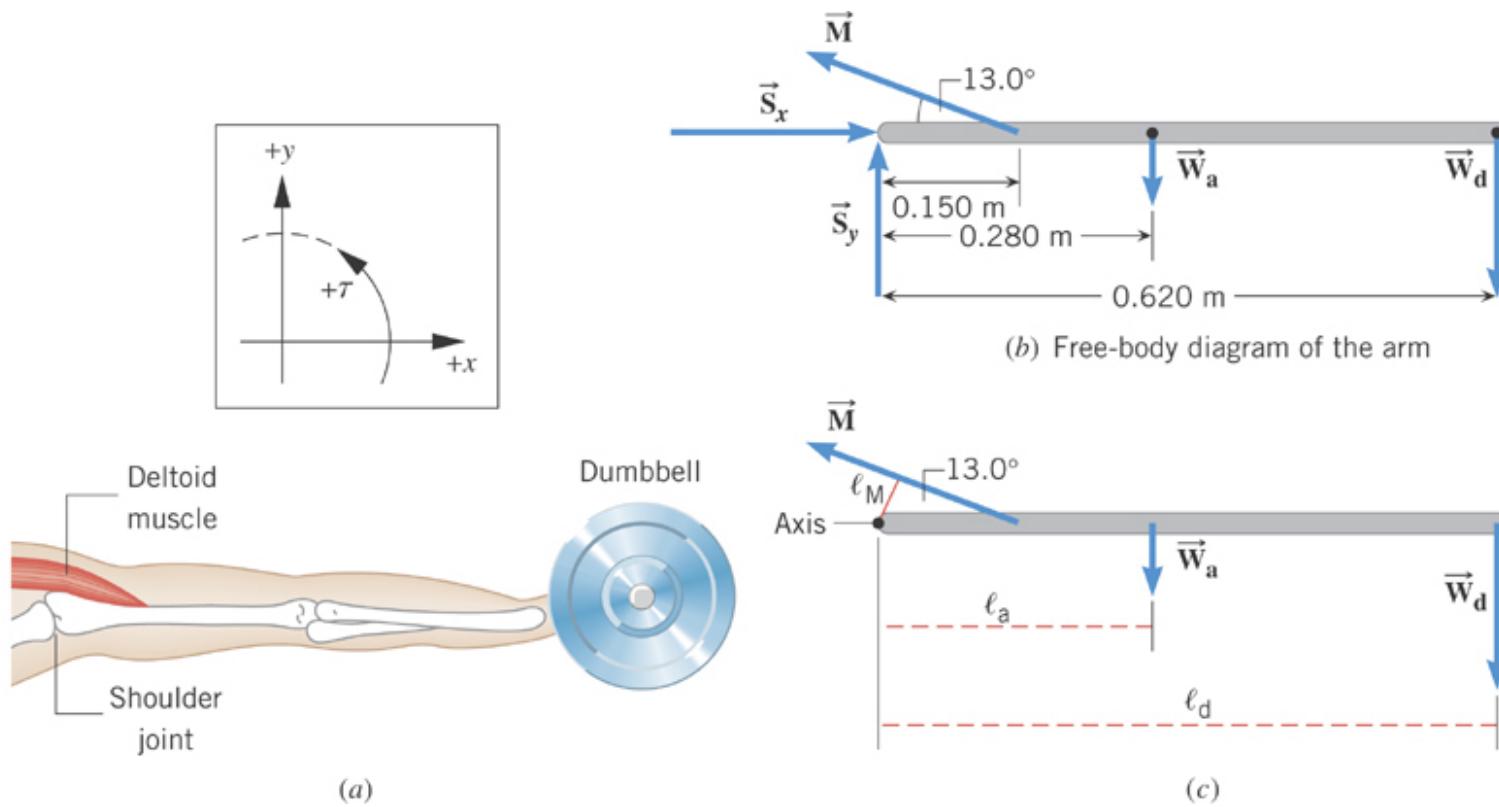
## 9.2 Rigid Objects in Equilibrium



### Example 5 Bodybuilding

The arm is horizontal and weighs 31.0 N. The deltoid muscle can supply 1840 N of force. What is the weight of the heaviest dumbbell he can hold?

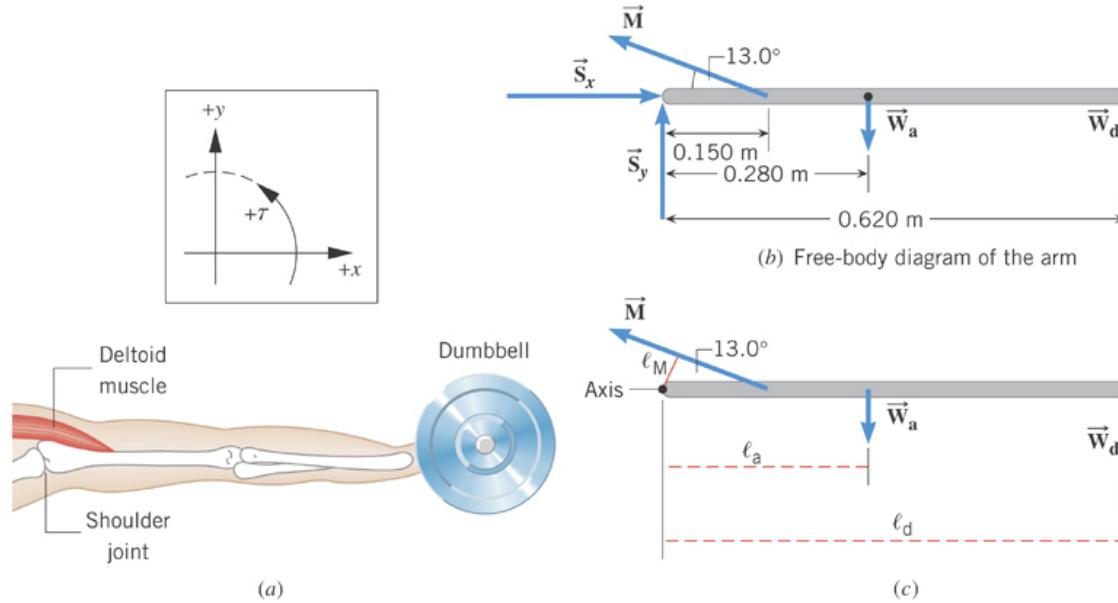
## 9.2 Rigid Objects in Equilibrium



$$\sum \tau = -W_a \ell_a - W_d \ell_d + M \ell_M = 0$$

$$\ell_M = (0.150 \text{ m}) \sin 13.0^\circ$$

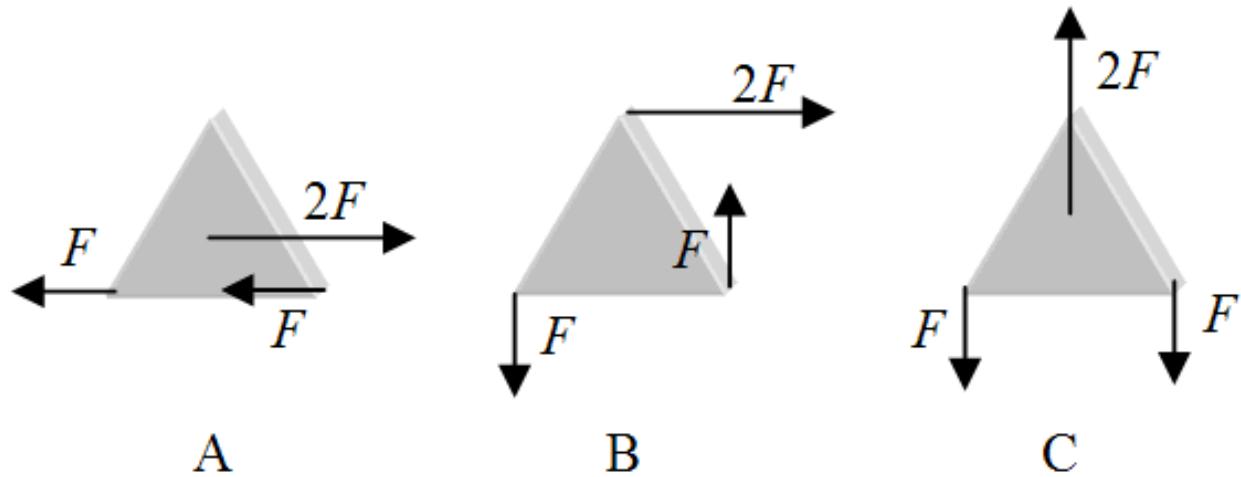
## 9.2 Rigid Objects in Equilibrium



$$\begin{aligned}
 W_d &= \frac{-W_a \ell_a + M \ell_M}{\ell_d} \\
 &= \frac{-(31.0 \text{ N})(0.280 \text{ m}) + (1840 \text{ N})(0.150 \text{ m}) \sin 13.0^\circ}{0.620 \text{ m}} = 86.1 \text{ N}
 \end{aligned}$$

9.2.3. Consider the three situations shown in the figure. Three forces act on the triangular object in different ways. Two of the forces have magnitude  $F$  and one of the forces has a magnitude  $2F$ . In which case(s), if any, will the object be in equilibrium? In each case, the forces may act at the center of gravity or at the center of a corner.

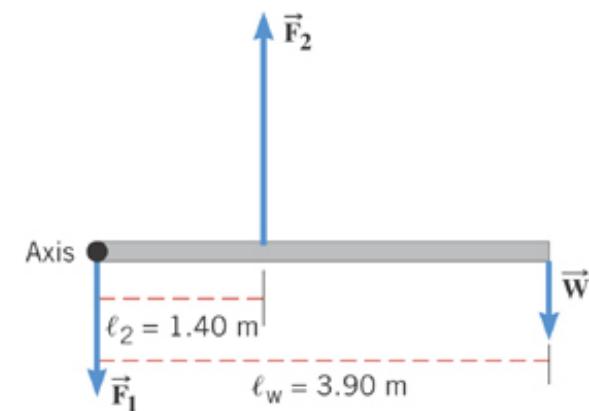
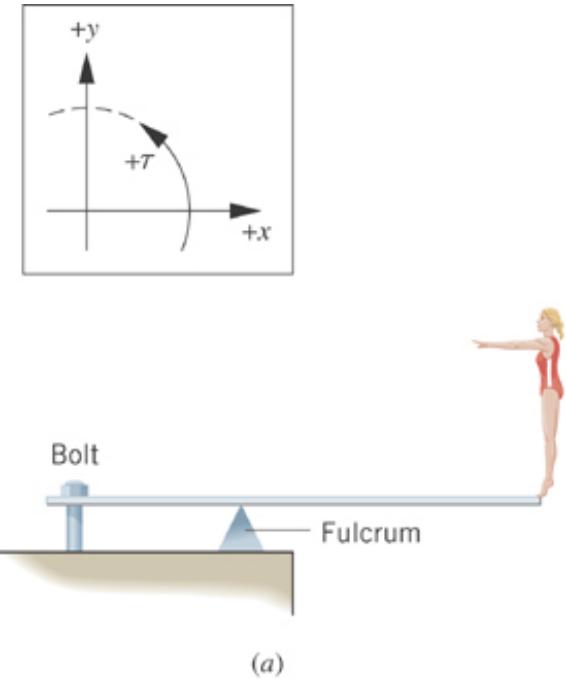
- a) A only
- b) B only
- c) C only
- d) A and C
- e) A and B



### *Example 3 A Diving Board*

A woman whose weight is 530 N is poised at the right end of a diving board with length 3.90 m. The board has negligible weight and is supported by a fulcrum 1.40 m away from the left end.

Find the forces that the bolt and the fulcrum exert on the board.



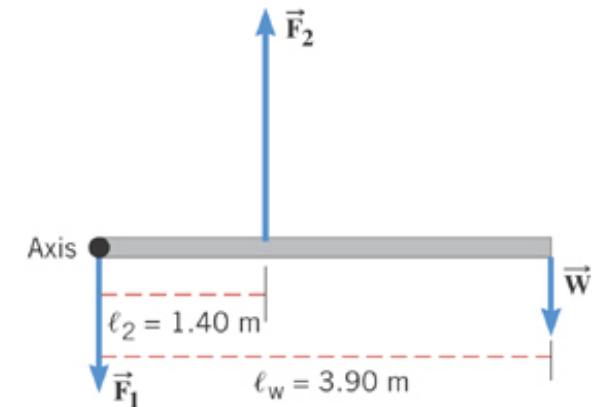
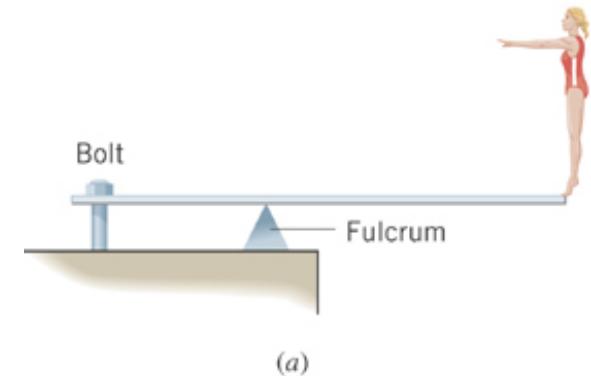
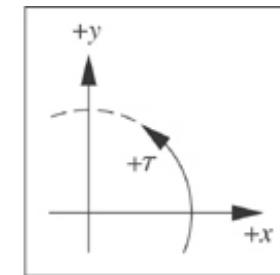
(b) Free-body diagram of the diving board

## 9.2 Rigid Objects in Equilibrium

$$\sum \tau = F_2 \ell_2 - W \ell_W = 0$$

$$F_2 = \frac{W \ell_W}{\ell_2}$$

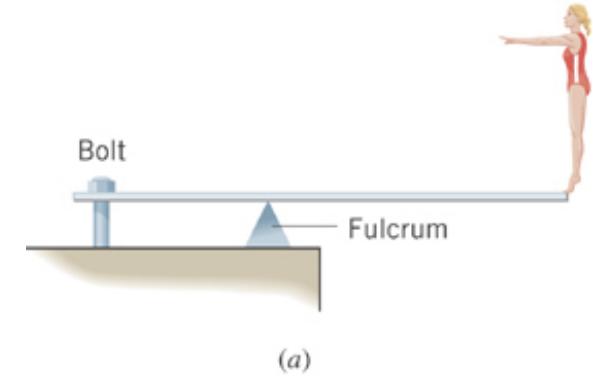
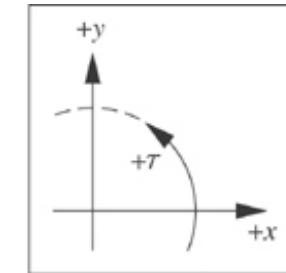
$$F_2 = \frac{(530 \text{ N})(3.90 \text{ m})}{1.40 \text{ m}} = 1480 \text{ N}$$



(b) Free-body diagram of the diving board

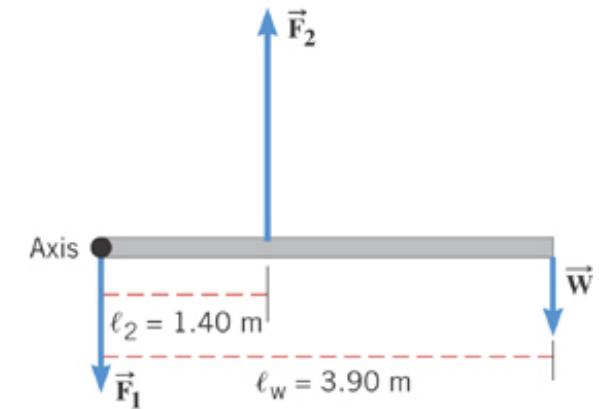
## 9.2 Rigid Objects in Equilibrium

$$\sum F_y = -F_1 + F_2 - W = 0$$



$$-F_1 + 1480 \text{ N} - 530 \text{ N} = 0$$

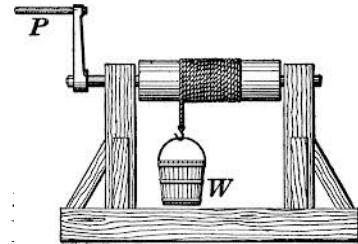
$$F_1 = 950 \text{ N}$$



(b) Free-body diagram of the diving board

- 1) A weight of 30N is located at a distance of 10cm from the fulcrum of a simple balance beam. At what distance From the fulcrum should a weight of 20N be placed on the opposite side in order to balance the system ?
- 2) A weight of 5N is located at 10cm from the fulcrum of a simple balance beam. What weight should be placed at a point 4 cm from the fulcrum on the opposite side in order to balance the system.

3) A seesaw is 12 ft long and is pivoted at its center. George weighs 40 lb And is sitting at one end. His sister Alice, weighing 32 lb is 1 ft from him. Where does their 155 lb dad have to sit to balance the seesaw ?



4) The bucket (see figure) with the water in it has a mass of 6.0kg. The radius of the drum on which the rope is wound is 12cm. The length of the crank is 35 cm. How much force needed to lift the bucket ? (equilibrium between the 2 torques)



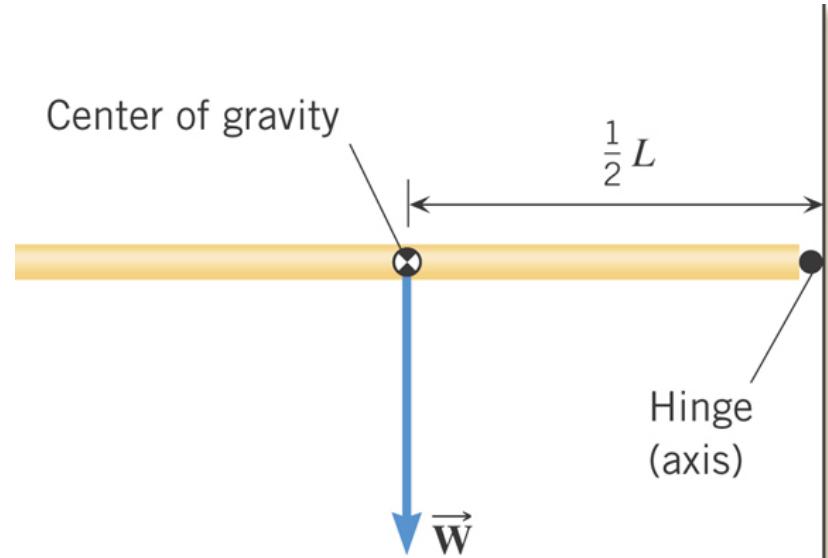
2.4 m long, pivoted at 0.2m from the end, to lift, at that end, a rock that weighs 750N. How much force do we have to exert ?

6) You are free to choose a fulcrum (pivot) where it is most convenient to Do computation. Take the example of a plank with a pile of books (see figure). The fulcrum can be either of the chairs. The mass of the books is 35kg and . The plank is 3.7m long, and the books are centered 1.2 m from the left chair. What is the elastic recoil (that is the normal force). You find the solution in 2 steps. First take the fulcrum at the right chair and then take the fulcrum at the left chair.



7) a crowbar 2.0m long is used to pry a rock out of the ground. By pivoting the crowbar 0.5m from the rock and pushing down On the other end. The end is pushed with a force of 250N. A) find the torque applied to the crow bar by the person doing the Job. B) The force being applied to the rock.

### 9.3 Center of Gravity

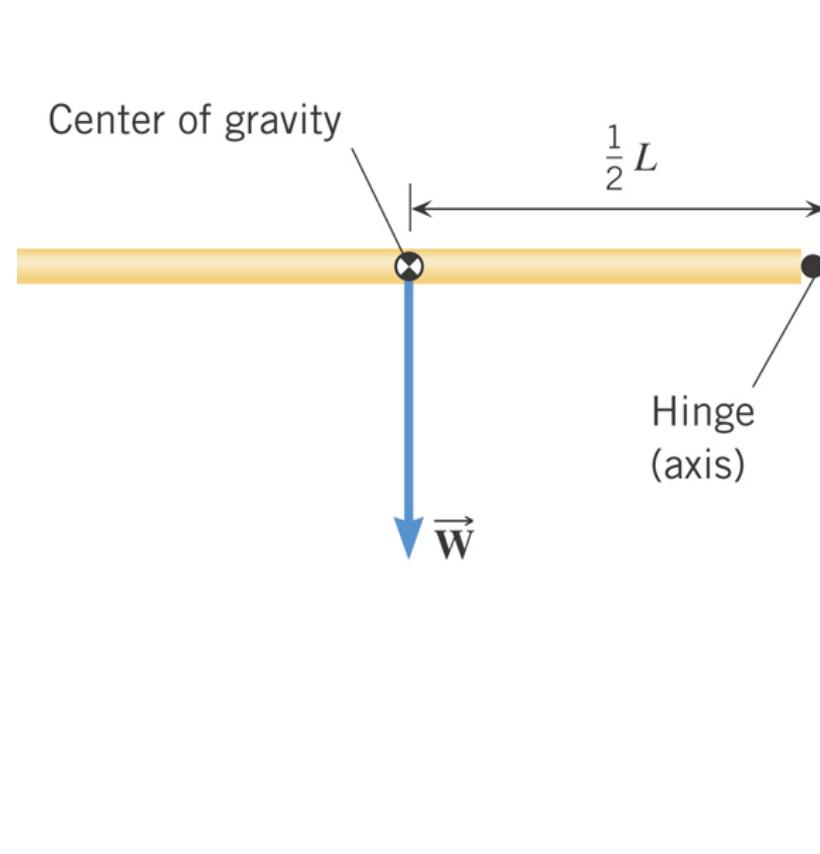


## DEFINITION OF CENTER OF GRAVITY

The center of gravity of a rigid body is the point at which its weight can be considered to act when the torque due to the weight is being calculated.

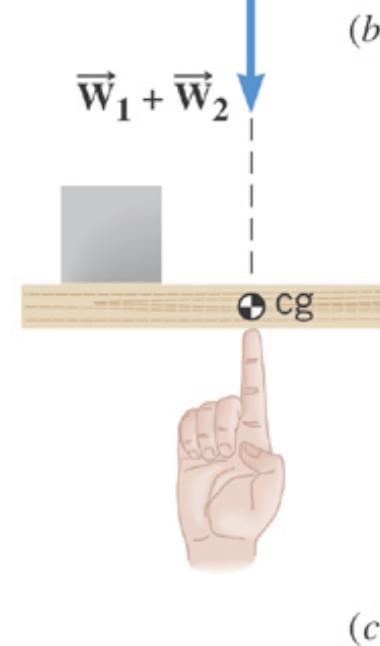
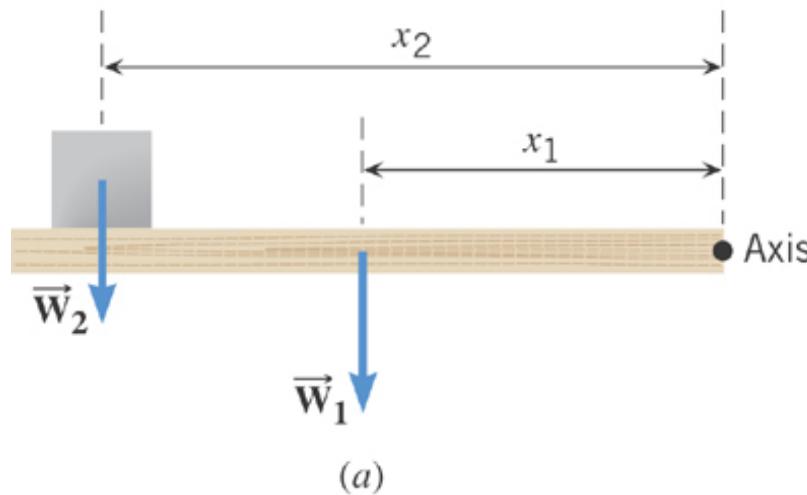
### 9.3 Center of Gravity

When an object has a symmetrical shape and its weight is distributed uniformly, the center of gravity lies at its geometrical center.



### 9.3 Center of Gravity

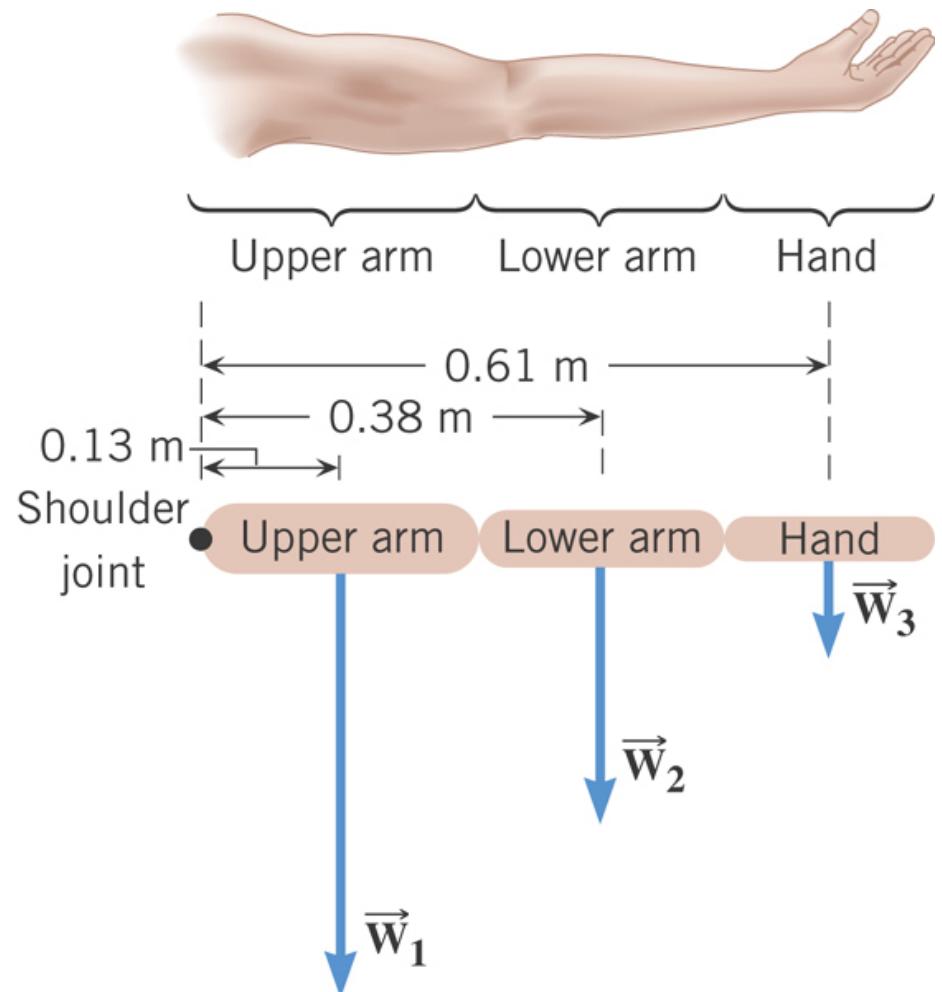
$$x_{cg} = \frac{W_1 x_1 + W_2 x_2 + \dots}{W_1 + W_2 + \dots}$$



## *Example 6 The Center of Gravity of an Arm*

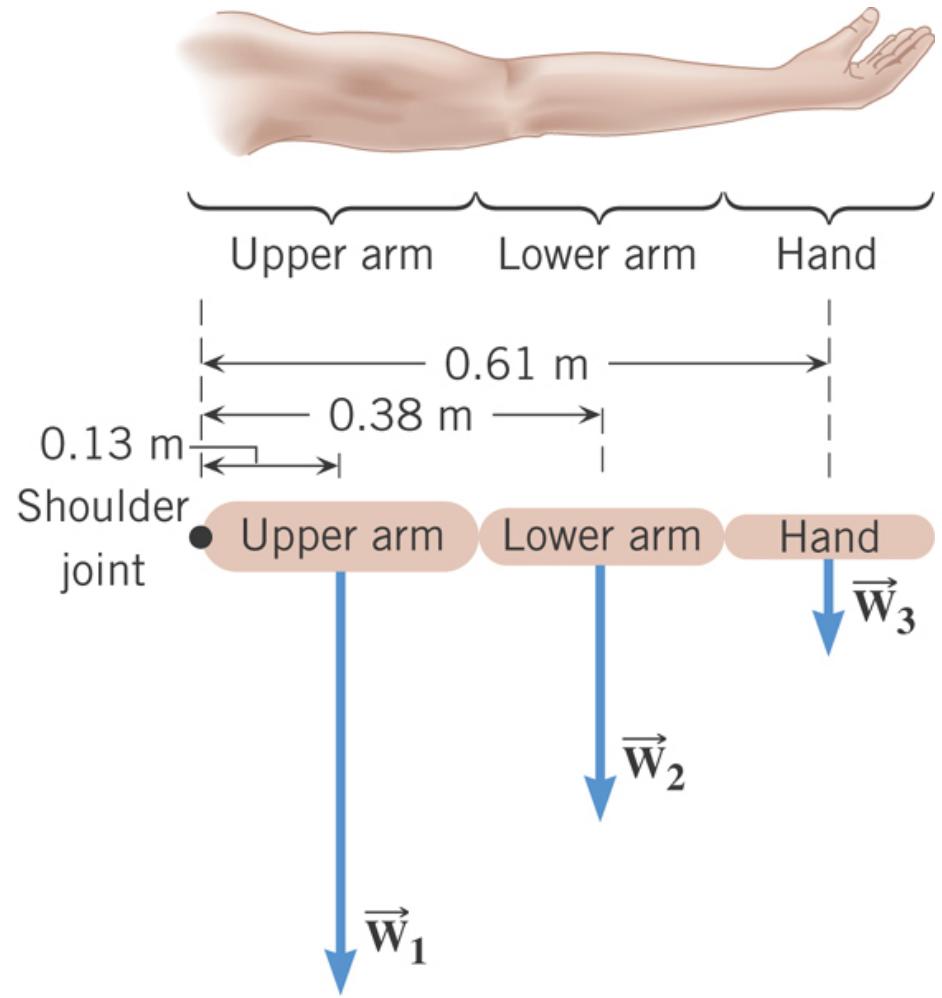
The horizontal arm is composed of three parts: the upper arm (17 N), the lower arm (11 N), and the hand (4.2 N).

Find the center of gravity of the arm relative to the shoulder joint.



### 9.3 Center of Gravity

$$x_{cg} = \frac{W_1 x_1 + W_2 x_2 + \dots}{W_1 + W_2 + \dots}$$



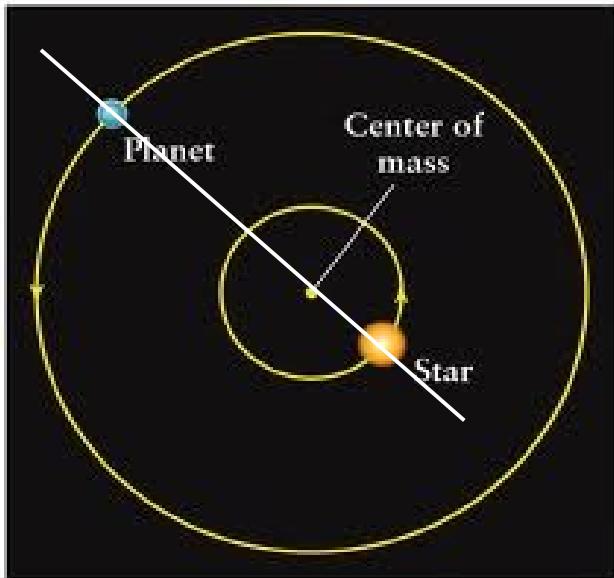
$$x_{cg} = \frac{(17 \text{ N})(0.13 \text{ m}) + (11 \text{ N})(0.38 \text{ m}) + (4.2 \text{ N})(0.61 \text{ m})}{17 \text{ N} + 11 \text{ N} + 4.2 \text{ N}} = 0.28 \text{ m}$$

Remember : the star and the planet are always  
Across from other (relative to the center of mass)

Also they move in opposite Direction

The wobble of the star is due to the gravitational tug  
from the planet. Increase the mass of the planet,  
increase the tug, increase the speed of the star.

You detect the wobble from the light emitted  
**FROM THE STAR**. Away the light is redshifted  
And the radial velocity is positive . Toward the  
Light is blueshifted and velocity is positive.



a